Examining Middle School Students' Methods of Justification

Leslie Reyes- Hernandez and Edward Mooney, Ph. D.

Department of Mathematics, Illinois State University, Normal, IL USA

Illinois State University — Illinois' first public university

Introduction

The *Common Core State Standards for Mathematics* (NGA Center, 2020) states that students should be able to construct mathematical arguments. In particular, the standards indicate that students should be able to justify any conclusions made and clearly communicate their justification. The purpose of this study was to examine the justifications provided by middle school students (Grades 5 to 8) on three mathematical tasks that required students to justify their reasoning. Particularly, we wanted to know,

1. What kind of justifications do middle school students produce?
2. What level of communication do middle school students use in their justifications?

Methods and Materials

Students ranging from fifth to eighth grade were given three different mathematical tasks. The tasks stated,

1. Amy and Stephen are trying out a number trick. Amy picks a starting number between 1 and 10. She adds it to 10 to the number and writes down the answer. She subtracts the starting number from 10 and writes down the answer. Then she adds the two answers from the first two steps.
   Stephan picks a starting number between 1 and 10. He adds it to 10 to the number and writes down the answer. He subtracts the starting number from 10 and writes down the answer. Then he adds the two answers from the first two steps.
   What do you notice about the two final answers?
   Will you always get the same final answer no matter what your starting number is?
   How would you convince a classmate that you would always get the same answer?
2. If you add any three odd numbers together, is your answer always odd? Provide an explanation that would convince your teacher that the answer is always odd.
3. Take a rectangle. Cut a rectangular piece from the upper right corner. What is the relationship between the perimeter of the rectangle and the perimeter of the new figure? Will this relationship be true regardless of the rectangle used? Why or why not?

Within our research, we analyzed 198 responses utilizing Balacheff's Taxonomy of Mathematical Proof and outlined the levels through which students' reasoning may progress as they encounter more complex mathematical problems.

Within students' mathematical reasoning, there are four main proofs and they are categorized in the justification levels: naive empiricism, crucial experiment, generic example, and thought experiment. Additionally, we added level 0 to further classify student work.

Methods and Materials (cont.)

We used Lo, Grant, and Flowers (2008) Classification of Student Justification to examine the level of explanation.

Justification Level	Description
4	Justification is based on mathematical reasoning solely using the mathematical properties and relationships in the situation (thought experiment)
3	Justification based on mathematical reasoning, but the inclusion of a case is provided as a general representation (generic example)
2	Justification is based on a **stated** generalized case (crucial experiment)
1	Justification is based on a few cases (Naïve empiricism)
0	Justification does not address mathematics or appears to indicate no understanding of task.

Explanation Level	Description
4	Justification is clear and mathematically correct
3	Justification is mostly clear and mathematically correct. Students may have glossed over, or omitted some aspects of the justification
2	Parts of the justification are mathematically incorrect or contain insufficient details.
1	Justifications are mainly descriptive or illustrative of the steps
0	Written work is missing or does not contain a valid reasoning strategy

The researchers scored each response individually and then met to compare scores. Any differences in scores were discussed and the final scores were then negotiated. We had 76.3% agreement on the levels of explanation and 82.3% agreement on the levels of justification.

Results

After studying the student responses, we classified them into the appropriate explanation and justification levels. The following table demonstrates how each grade level corresponded to each level of mathematical reasoning.

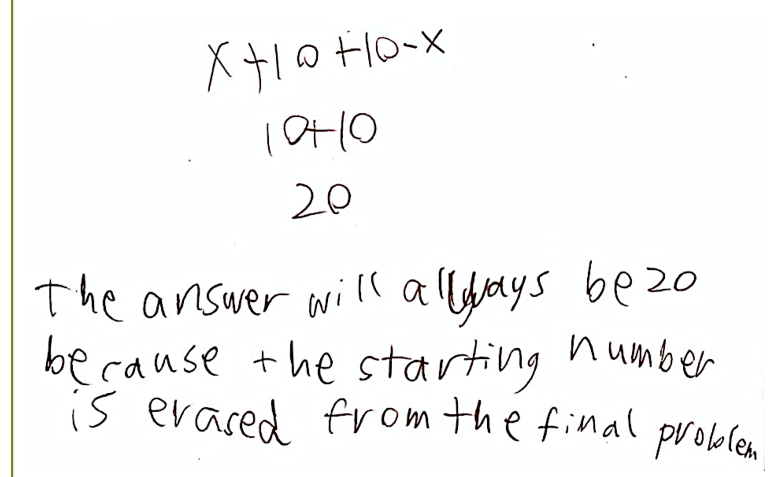
Justification	5th (N = 43)	6th (N = 43)	7th (N = 54)	8th (N = 58)	Overall (N=198)
0	19	25	30	17	46.0%
1	13	11	13	13	25.3%
2	2	0	4	2	4.0%
3	7	6	5	14	16.2%
4	2	1	2	12	8.6%

Explanation	5th (N = 43)	6th (N = 43)	7th (N = 54)	8th (N = 58)	Overall (N=198)
0	19	22	17	11	34.8%
1	16	12	25	19	36.4%
2	6	6	11	16	19.7%
3	0	2	1	10	6.6%
4	2	1	0	2	2.5%

Corresponding to question 1, the following is a sample of explanation level 3 and justification level 4.

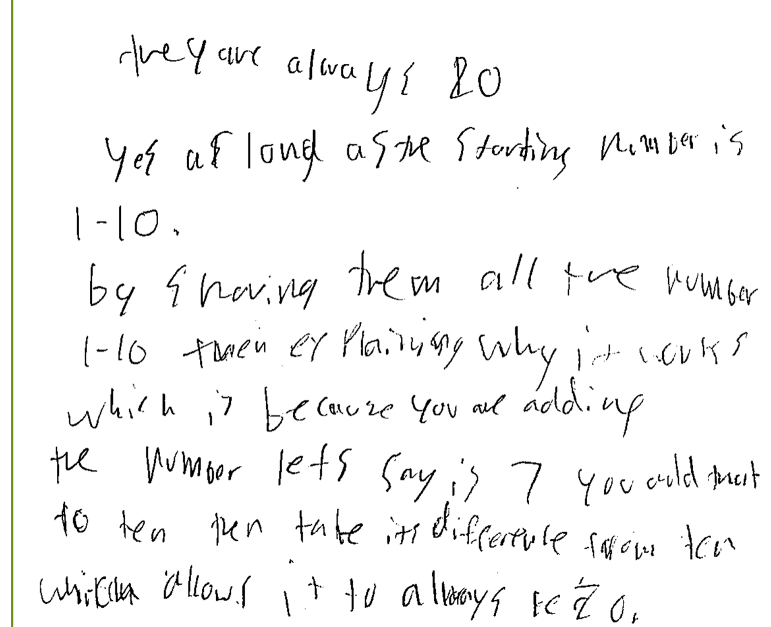
The student's work classifies as an explanation level of 3 because the justification is mathematically correct, but certain aspects are glossed over. This case is not considered an explanation level of 4 because the student does not state what value x stands for.

Results (cont.)



Furthermore, the justification level is considered a 4 because the student provides ideas based on mathematical reasoning. More specifically, this type of proof is a thought experiment because unlike the other levels, it is no longer a matter of demonstrating the outcome is valid because it works, but rather it establishes the necessary nature of its truth by providing thorough and cohesive mathematical reasoning.

Another sample of student work is the following. In this case, the explanation level is 2 and the justification level is also 2.



Within this response, the explanation level is deemed a 2 because it contains insufficient details. As for the justification level, it is considered a 2 because the reasoning utilizes a stated case as it mentions the example with the 7. This method of reasoning is considered a crucial experiment as it is based on the idea that if a situation applies here then it must always apply. Within mathematics, utilizing this method of reasoning may be flawed because "always" is a large statement and that may not always stand. Therefore, this reasoning classifies as a justification level of 2 since it makes a very large assumption.

Discussion

From our research, we found that for justification, almost half of the student responses had no reasoning or did not provide valid reasoning. Approximately one-fourth of the justifications were based on examples (Levels 1). Additionally, about one-fourth of justifications were grounded primarily on mathematical reasoning (Levels 3 and 4).

As for the explanation, students do not thoroughly communicate their reasoning. Over 90% of responses were missing a justification or insufficient details were provided (Levels 0 to 2).

Conclusions

Overall, our research demonstrated that students are unaccustomed to justifying their solutions. Without the explanations, the reasoning that drives the solution forward remains implicit.

This research is of high value to educators, parents, school administrators, and students throughout the world as it provides a sharper and more beneficial method of learning. Educational research is highly important as the future of the world lies within our classrooms today.

Contact

Leslie Reyes- Hernandez
Illinois State University
Email: lgreyes@ilstu.edu

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