

Article

Modeling of the Bitcoin Volatility through Key Financial Environment Variables: An Application of Conditional Correlation MGARCH Models

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Abstract: Since the launch of Bitcoin, there has been a lot of controversy surrounding what asset class it is. Several authors recognize the potential of cryptocurrencies but also certain deviations with respect to the functions of a conventional currency. Instead, Bitcoin's diversifying factor and its high return potential have generated the attention of portfolio managers. In this context, understanding how its volatility is explained is a critical element of investor decision-making. By modeling the volatility of classic assets, nonlinear models such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH) offer suitable results. Therefore, taking GARCH(1,1) as a reference point, the main aim of this study is to model and assess the relationship between the Bitcoin volatility and key financial environment variables through a Conditional Correlation (CC) Multivariate GARCH (MGARCH) approach. For this, several commodities, exchange rates, stock market indices, and company stocks linked to cryptocurrencies have been tested. The results obtained show certain heterogeneity in the fit of the different variables, highlighting the uncorrelation with respect to traditional safe haven assets such as gold and oil. Focusing on the CC-MGARCH model, a better behavior of the dynamic conditional correlation is found compared to the constant.

Keywords: Bitcoin; volatility; key financial environment variables; multivariate GARCH models; constant conditional correlation; dynamic conditional correlation; varying conditional correlation



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1. Introduction

In the midst of the global financial crisis of 2008, Satoshi Nakamoto published the Bitcoin project. The white paper by Nakamoto [1] described a digital currency based on a sophisticated peer-to-peer (p2p) protocol that allowed online payments to be sent directly to a recipient without going through a financial institution. At that time, a potential nonsovereign asset fully decentralized and isolated from the uncertainties of a specific country or market was presented as a great value proposition [2], which took even more value, if possible, during the European sovereign debt crisis of 2010–2013 [3]. Thus, in 2013, the popularity of cryptocurrencies clearly increased. The market capitalization closed the year exceeding USD 10 billion and the price of Bitcoin reached over a thousand dollars [4,5]. Thereafter, the cryptocurrency market size and the Bitcoin price have not stopped growing. In January 2018, capitalization peaked at over USD 800 billion [6] and, in December 2017, Bitcoin's price had reached close to USD 20,000, a revaluation to 2700% in 2017 alone [5]. Although, in September 2018, the cryptocurrencies market had lost 80% of its value and, at the end of the year, Bitcoin closed below USD 6 thousand, charting a market pattern of extreme price instability far removed from those of sovereign currencies [7,8]. In any case, since the launch of Bitcoin, cryptocurrencies have become a social and economic trending topic and the blockchain technology, that supports it, in one of the main disruptive innovations [9,10].

As of December 2020, there are more than 8265 cryptocurrencies with a market value of around USD 750 billion and a daily trading volume around USD 180 billion, with Bitcoin's market share being 69.2% [11].

At this point, the importance of Bitcoin as a new financial asset is obvious, as well as the controversy generated about its nature among market practitioners, academics, and in the economics press [12,13]. Particularly, in the financial literature, there is an intense debate about what type of asset Bitcoin is and what its real usefulness is. Thus, references such as [14–19] literally ask if it is really a currency, or others analyze its qualities as safe havens [20]. In this sense, although references such as [14,15] recognize some potential for Bitcoin to function as a currency, the common denominator of the literature on Bitcoin is to define it as a speculative investment with extreme volatility and a large average return [16–19]. In any case, Bitcoin is an investable asset [21] whose diversifying factor and high return potential motivate the attention of investment portfolio managers. Consequently, the interest of the Bitcoin analysis goes beyond a normalization of what type of asset it is, but rather a clear definition of its main attributes as an alternative investment, that is: Liquidity, risk, and return.

In the recent literature, some research has tried to understand how Bitcoin's liquidity is explained [22,23] and how efficient its market is [24–29]. However, if there is one aspect that has defined the Bitcoin market since its launch, it is the strong volatility it presents.

As Yu [30] indicates, volatility plays an important role in asset pricing, portfolio allocation, or risk management. Therefore, its forecasting and modelling have received special attention from the scientific community; see the reviews of Zhang and Li [31] and McAleer and Medeiros [32]. However, the study of the dynamic Bitcoin volatility is still an emerging line of research with some gaps still to fill. The subject has been treated from different perspectives; among others, Baek and Elbeck [18] analyze the Bitcoin relative volatility using detrend ratios with economic variables such as the Standard and Poor 500 (SP500) Index to underline its high speculation; Balcilar et al. [33] find that the volume can predict the Bitcoin returns (except in bear and bull market regimes); Bariviera [34] tests the presence of long memory in Bitcoin return time series, and on this basis [35], justifies the application of Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-type models [36,37]. Furthermore, the variability in Bitcoin volatility, alternating periods of extreme volatility with others of relative calm, indicates that the bitcoin price is suitable for GARCH modeling [36]. Thus, Dyhrberg [38] uses GARCH(1,1)—methodologically tested by Engle and Manganelli [39]—as a starting point to analyze the volatility of Bitcoin in relation to gold and the dollar. Recent research has used this univariate GARCH model as a benchmark to analyze and compare the volatility estimate of Bitcoin [40–44]. Although, as found in references such as [45–48], the Multivariate GARCH (MGARCH) approach [49] allows the model to test the sensitivity of Bitcoin with respect to variables of the economic and financial environment, improving the robustness in general terms.

More specifically, the application of nonlinear combinations of univariate GARCH models suggests suitable results in determining the conditional covariances of cryptocurrency volatility models. Thus, conditional correlation (CC) models have been applied by Chan et al. [45] in the Constant variant (CCC) and Kumar and Anandarao [46], Guesmi et al. [47], and Kyriazis et al. [48] in the Dynamic (DCC) [50] to study the Bitcoin volatility. Meanwhile, according to the literature review, the CC special case, that is, Varying (VCC) MGARCH [51], has not yet been tested in the cryptocurrencies context.

Therefore, starting from the GARCH(1,1) model as a benchmark, the main objective of this study is to model and assess the relationship between the Bitcoin volatility and key financial environment variables applying the three variants of the Conditional Correlation M-GARCH approach.

The nature of Bitcoin evolves as the cryptocurrency market matures and the critical mass of investors increases. Thus, this research contributes to the existing literature, first in the construction of the models. Thus, CC-MGARCH models have been developed combining traditional financial variables (such as the main commodities, exchange rates,

or indices), market benchmarks on blockchain technology, and means of payment such as Visa and Mastercard. On the one hand, this has made it possible to evaluate the Bitcoin connection with variables not yet tested. On the other, the correlation outcomes of the financial environment variables allow us to weigh the diversifying factor of Bitcoin and its usefulness as a safe haven asset. Second, the modeling of the three variants of the CC-MGARCH model makes it possible to measure how the volatility of Bitcoin behaves better, if with a constant or dynamic conditional correlation.

To do this, the daily prices of Bitcoin from the beginning of 2016 to the end of 2018 and the time series of the exogenous financial and economic variables on which the MGARCH models are based have been extracted from the Datastream® database.

The results of this research reveal that a better fit of the time-varying conditional correlation model is found compared to the constant. Like this, the DCC-MGARCH approach presents as best performing. The special case of the VCC model is also presented as a good candidate to explain the price movements and volatility of cryptocurrencies. In the same way, a certain heterogeneity in the fit of the different variables is shown, highlighting an uncorrelation with respect to traditional safe haven assets such as gold and oil. On the other hand, although a better fit is observed in the MGARCH models than in the standard GARCH model, the statistical analysis of precision confirms that GARCH(1,1) continues to be presented as a suitable alternative to model the Bitcoin volatility.

The rest of the paper is structured as follows. In Section 2, the literature review on the financial nature of Bitcoin and the forecast of its volatility is expanded. Section 3 presents the data sample used and the models developed. In Section 4, the main results of our research are described. Finally, Section 5 summarizes the key conclusions and contributions of this study.

2. Literature Review

The technological development of Bitcoin is supported on two key elements: Blockchain and mining. The blockchain is a smart combination of technologies such as p2p, timestamp, or cryptography, which build a data structure grouped in blocks, whose information can only be updated with the consensus of the system participants and can never be erased [1,9]. In practice, it results in an irrefutable and verifiable ledger of all transactions. However, as Crosby et al. [10] indicate, the blockchain goes beyond Bitcoin. Thus, Ali et al. [52] present a systematic review of academic research on the benefits and paradigm shift that the integration of blockchain technology in the financial sector is assuming. On the other hand, Bitcoins must be “mined” [53]. In essence, when a network of advanced computers solves an algorithm, it is rewarded with the Bitcoin units being mined; every 10 min, new coins are put into circulation [1]. Satoshi Nakamoto defined a limited stock of 21 million coins, so that around the year 2140, the issuance of Bitcoin would be completed. Through this cryptocurrency mining process, the Bitcoin developers establish a controlled currency issuance system inspired by the fixed supply of gold [17].

But is Bitcoin a commodity or a currency? As indicated in previous lines, it is still an open debate. As Bouri et al. [20] point out, cryptocurrencies were presented as a panacea that covered the weaknesses of the financial system at times such as the sovereign debt crisis of 2010–2013 or the Cypriot banking crisis of 2012–2013 [54]. Although, Bitcoin has not fulfilled its function as a currency, in a strict sense [17]. Yermack [17] performs an analysis of the Bitcoin deviations regarding the roles that a currency has to fulfill, that is, medium of exchange, unit of account, and store of value. The author points out a series of form problems in its operative, such as the unorthodox decimal price, delays in verifying transactions, or the acquisition process, among others, that limit Bitcoin to a payment method. In any case, he concludes that, in order for Bitcoin to function as a store of value and unit of account, its price must be more stable, since its volatility is more representative of the behavior of a speculative investment. In this same sense, Baur et al. [7] define Bitcoin as a speculative and uncorrelated asset and not as an alternative medium of exchange. There are other studies that classify it as a hybrid techno-financial instrument [19] or

suggest that Bitcoin behaves as an asset between currency-fiat and a crypto-commodity used for commercial and investment purposes [55].

In regulatory terms, under International Financial Reporting Standards (IFRS), the International Financial Reporting Interpretation Committee (IFRIC) transmitted to the International Accounting Standards Board (IASB) that cryptocurrencies are not financial assets or legal tender, but rather intangible assets and not monetary [56]. Under the Commodity Exchange Act (CEA), Bitcoin is a commodity for the US Commodity Futures Trading Commission (CFTC). Thus, future contracts with Bitcoin as an underlying asset have been traded in the Chicago Board Options Exchange (CBOE) or the Chicago Mercantile Exchange (CME) [57]. In this regard, Selgin [58] points out that Bitcoin has characteristics common to commodities and that it could be considered as synthetic commodity money. Like gold, Bitcoin offers an implicit return and presents some of the essence of traditional commodities; it is an exhaustible resource with a fixed supply [41,56]. However, Gronwald [41] finds that Bitcoin's price dynamics are particularly influenced by extreme price movements due to shocks to its demand. The study supports this differentiation in the immaturity of Bitcoin and, especially, in its certain and programmed supply against an uncertain short-term supply of gold and oil. In this sense, although the design of Bitcoin has been influenced by gold, it has less liquidity, its transactions are more complex and, above all, it is much more volatile [59].

Focusing on gold and its diversification, hedging, and safe haven properties [60], numerous academic studies have tried to contrast these same qualities in Bitcoin, and the conclusions have varied depending on the intensity of the stress scenario and the nature of the assets. Moreover, Urquhart and Zhang [61] define Bitcoin as a reasonable haven against sovereign currencies; Wang et al. [62] reach the same conclusions regarding the assets of the Chinese financial market; and Dyhrberg [63] indicates that it has the capacity to hedge positions on the Financial Times Stock Exchange (FTSE) Index and the US dollar. Selmi [64] places Bitcoin at the same level as gold as a safe haven and diversifier of the oil price. However, despite Bitcoin's weak correlation with traditional financial assets, some studies analyzed coincide in describing it as poor coverage as a safe-haven value in times of strong stress [65–67]. This weakness has been confirmed by Conlon et al. [68] in the COVID-19 crisis; Bitcoin's extreme volatility hinders its usefulness as a safe haven and even motivates that, paradoxically, it needs a stable coin as the Tether to cover itself [68]. In any case, studies such as that of Brieri et al. [65] show that a small weighting of Bitcoin can improve the risk–return trade-off of well-diversified portfolios; and others such as that of Ghabri et al. [69] observe that Bitcoin can lower the liquidity risk in the portfolio, under the Mean-Variance-Liquidity framework.

By consequence, inhibiting—for the time being—its usefulness as a currency or safe haven, its diversifying factor and high return potential motivate the interest of investment portfolio managers [7,16]. For that purpose, the volatility modeling has focused attention. Volatility is an important measure of market risk and, therefore, a crucial element on investors' decision-making in portfolio management [70]. In academic research, the work of Andersen and Bollerslev [71] signified a point of reference from which significant contributions have been made to model and forecast volatility using high-frequency data [30]. Thus, the study of volatility is a mature research topic [31,32], although the modelling of the cryptocurrency's volatility is a keyword with many gaps yet to be resolved.

In recent years, under the long memory assumption [34,35], a line of research has emerged focused on the development of GARCH-type models to explain and forecast the volatility of Bitcoin and of the cryptocurrencies, in general. Thus, the GARCH(q,m) model of Bollerslev [36] has been taken as a benchmark [44]. In one of the first works, Glaser et al. [16] successfully apply the standard GARCH(1,1) to determine through the modeling of Bitcoin's volume and price that the investors' intention when changing their national currency to digital is not to seek an alternative transaction system, but an alternative investment vehicle. In this line, Ardia et al. [40] show that the Markov-switching GARCH (MS-GARCH) models improve the results of the single-regime for Bitcoin Value-at-Risk

(VaR) forecasting. For his part, Gronwald [41] compares the GARCH(1,1) with various linear and nonlinear GARCH approaches to model the extreme price movements of Bitcoin. On the other hand, Hung et al. [42] incorporate jump-robust-realized measures into the simple GARCH model, obtaining a better forecast performance for Bitcoin volatility with the RGARCH(1,1) model built. Furthermore, Trucíos [43] starting from GARCH(1,1), carried out a profuse comparative of the Bitcoin volatility forecast applying different extensions of the several classical GARCH-type models. The author suggests the use of the generalized autoregressive score (GAS), particularly the t-GAS model [72], to forecast the volatility in the absence of high-frequency data. Other comparative studies, such as those of Katsiampa [35] and Conrad et al. [73], have also taken the simple GARCH models as the initial basis for forecasting Bitcoin movements.

Although the GARCH(1,1) model is presented as a suitable reference point, the development of MGARCH models (see [37,49]) can offer a superior capacity by allowing the integration of the relationships between the volatility processes of various time series with Bitcoin [46]. Chan et al. [45] analyze the hedging and diversification capacity of Bitcoin against stock indices such as the Euro STOXX, Nikkei, Shanghai A-Share, S&P 500, and the TSX index. To carry out their study, the authors use the GARCH(1,1), Constant Conditional Correlation (CCC) of Bollerslev [74], and the Dynamic Conditional Correlation (DCC) based in Engle [50], among others, models. The standard GARCH model and the CCC-MGARCH offer a significant fit and consistent results. Although, the study reveals a nonconvergence of the DCC model, especially for the monthly frequency data, and does not improve its logarithmic probability with respect to the CCC model. By contrast, Kumar and Anandarao [46] selected the DCC-MGARCH model to study the dynamics of the volatility spill of four cryptocurrencies, due to its ability to capture time-varying conditional-correlations and covariances. Similarly, Guesmi et al. [47] and Kyriazis et al. [48] successfully support the DCC process to explain the proprieties of Bitcoin in the financial markets and its relationship with other cryptocurrencies.

In this sense, the DCC-MGARCH models and the VCC-MGARCH model of Tse and Tsui [51] allow time-varying conditional correlations that can provide a better understanding of the dynamic structure of the conditional correlations [75]. Although the VCC-MGARCH offers suitable results in the forecast of the volatility of several commodities [76], no evidence has been found that the variable conditional correlation model has been tested in Bitcoin modeling.

Based on the review literature and on the Bitcoin time-series continuing to build and mature, this research contributes to the existing discussion in several ways. First, the outcomes of the developed models show an additional perspective on the degree of correlation of Bitcoin with the traditional benchmark of the financial market, calibrating its diversification factor. Likewise, in the structure of the models the relationship between the movements of Bitcoin returns and previously untested variables is integrated, but with a possible coherent connection. Second, taking GARCH(1,1) as a starting point, the adequacy and adjustment of the multivariate GARCH models CCC, DCC, and VCC are analyzed to model the volatility of Bitcoin.

3. Materials and Methods

3.1. Data and Variables

The data used to carry out the analysis have been the daily prices of Bitcoin (BTC), extracted from the Datastream® database. The sample focuses on the time window from December 2016 to the end of the year 2018, as the data prior to 2016 due to the still low critical mass of the market could distort the significance of the results. On the other hand, to build the GARCH models, a series of exogenous variables have been selected that, a priori, may have a certain causal relationship with the movements of Bitcoin. In this sense, this selection has been based on a review of the literature and on aspects related to the technology that supports Bitcoin. In particular, as a reference for the evolution of financial markets, the stock indexes of the Standard and Poor 500 (SP500) and the NIKKEI 225 (N225)

have been selected. Both indices are used as traditional variables in the financial literature. In the field of cryptocurrencies, for example, García-Jorcano and Benito [77] analyze the dependency structure between the SP500 and N225 with Bitcoin, underlining the hedging properties of Bitcoin against the stock markets considered. On the other hand, in line with recent studies [78], commodities like gold or crude oil futures (WTI) have been introduced in the model for their common qualities as a possible safe haven asset. On the basis that Bitcoin is presented as a currency and supported by previously highlighted literature (see Dyhrberg [38]), the reference exchange rate USD/EUR have been added. Likewise, due to the usefulness of Bitcoin as an alternative means of payment, the stock market evolution of the financial services multinationals Visa (VISA) and Mastercard (MAST) has been included in the model. Note that both companies have as a strategic axis the promotion and development of financial services based on bitcoins and cryptocurrencies.

Finally, variables related to technology companies that are in the blockchain value chain or are relying on it to develop disruptive innovations have been introduced. The companies considered have been Riot Blockchain Inc (RIOT), focused on bitcoin mining and blockchain construction; KBR American engineering and construction company; and Nvidia (NVDA), a multinational specialized in the development of graphic processing units and integrated circuit technologies for workstations, personal computers, and mobile devices. Table 1 shows descriptive statistics of the selected variables, and Figure 1 presents the volatility graphs of each of the variables used in the research.

Table 1. Descriptive analysis of the variables.

Variable	N	Mean	Std. Dev	Min	Max
BTC	757	55,660.54	3787.380	750.600	19,1870.00
SP500	757	11.623	3.615	7.800	22.120
RIOT	757	6.554	5.889	1.350	38.600
N225	757	21,179.261	1556.405	18,240.500	24,270.620
WTI	757	57.727	8.748	42.530	76.410
GOLD	757	1308.948	550.06	1134.550	1425.900
USD/EUR	757	1.151	0.55	10.39	1.251
KBR	757	17.479	20.52	13.630	21.910
VISA	757	113.281	20.901	75.430	150.790
MAST	757	156.609	36.897	10.180	223.770
NVDA	757	186.919	57.649	87.640	289.360

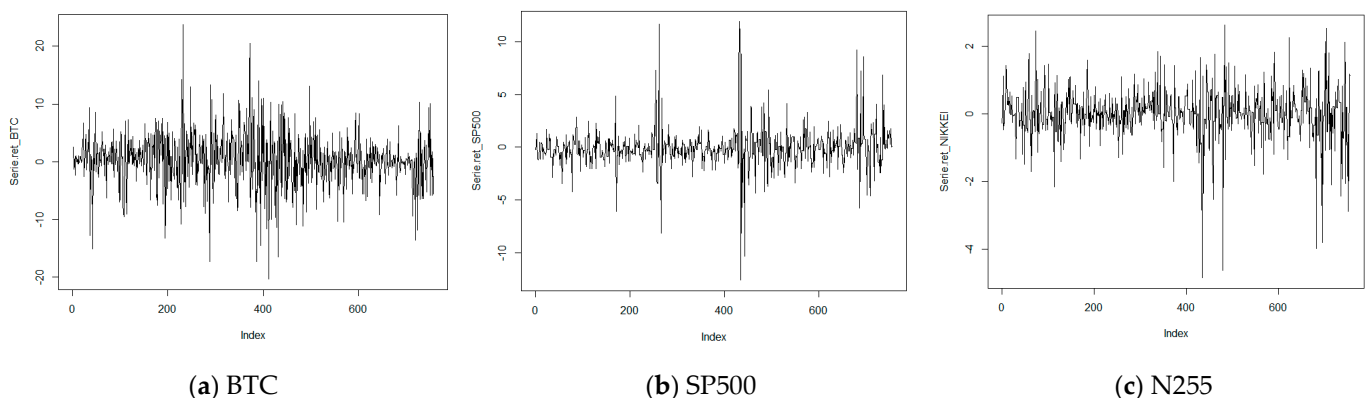


Figure 1. Cont.

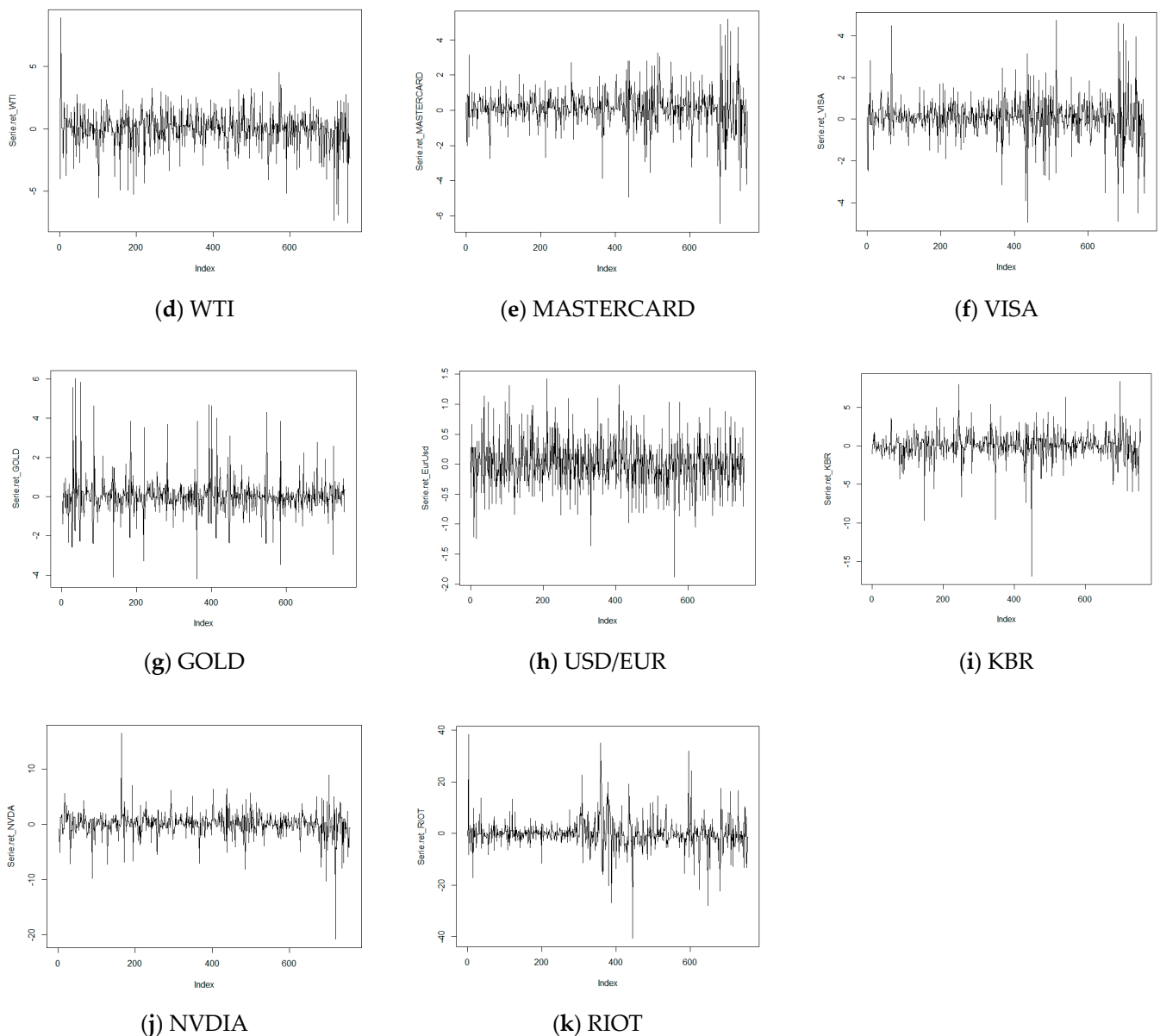


Figure 1. Time series plot of volatility data.

3.2. Methodology

First, the returns of the variables, l_t , have been calculated as a logarithmic rate:

$$l_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where p_t are the daily prices at market close of the variable in period t .

The Dickey–Fuller tests are carried out in order to determine the stationarity properties of the Bitcoin time series. Thus, starting from the model $X_t = \rho X_{t-1} + u_t$, where X_t is the Bitcoin return in a unit of time t , u_t is the error term, and the coefficient $-1 \leq \rho \leq 1$. The test is based on the following expression:

$$\Delta X_t = \delta X_{t-1} + u_t \tag{2}$$

where $\delta = (\rho - 1)$. In our study, the null hypothesis of the existence of a unit root is discarded in the Bitcoin series (p -value = 0); by consequence, there is stationarity.

In the same way, to detect whether there is independence in the residuals, the autocorrelation Ljung–Box test has been used. The Q statistic is defined by:

$$Q = T(T + 2) \sum_{h=1}^m \frac{\hat{\rho}_h^2}{T - r} \tag{3}$$

where T is the sample size, ρ_h the coefficient of autocorrelation of the residuals, r the number of estimated parameters, and m the lags being tested. Thus, under the null hypothesis where the statistic Q asymptotically follows a chi-squared distribution, the autocorrelation in the residuals is ruled out (p -value 2.2×10^{-16}).

The GARCH models are a generalization of the ARCH (autoregressive conditional heteroscedasticity) proposed by Engle [79] with the aim of collecting the episodes of volatility temporal grouping. The basic structure of the ARCH (q) model starts from:

$$y_t = \sigma_t \epsilon_t \tag{4}$$

where $\epsilon_t \sim N(0,1)$, $\sigma_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \dots + \alpha_q \epsilon_{t-q}^2 = \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2$, $\alpha_0 > 0$, and $\alpha_i > 0, i > 0$. If an autoregressive moving average model (ARMA) model is assumed for the error variance, the model is a GARCH [36].

In this sense, let $\{Y_t\}_{t \in I}$ be a stochastic process where T is a direct set of indices, where $\beta' = (\beta_0, \beta_1, \dots, \beta_k)$ and $\omega' = (\alpha_0, \alpha_1, \dots, \alpha_q, \gamma_1, \dots, \gamma_q)$ are parameter vectors to model the mean and variance, respectively; $z_t = (1, \epsilon_{t-1}^2, \dots, \epsilon_{t-q}^2, h_{t-1}, \dots, h_{t-q})$ is the vector of variables for the variance; and $x_t = (1, x_{t1}, \dots, x_{tk})$ is the vector of explanatory variables observed in time t. In this model, $e_t = y_t \beta$ and Ω_{t-1} are the information available up to time $t - 1$.

The GARCH(p,q) model in Bollerslev [36] regression is given by:

$$\begin{aligned} Y_t | \Omega_{t-1} &\sim N(\mu_t, h_t) \\ \mu_t &= x_t \beta \\ h_t = z_t \omega &= \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i h_{t-i}^2 \\ \epsilon_t &= y_t - x_t \beta \end{aligned} \tag{5}$$

where $p \geq 0, q \geq 0, \epsilon_t \sim IIDDS(0,1)_t$, and ω, α , and γ are parameters such that $\omega > 0, \alpha, \gamma \geq 0$ and $\alpha + \gamma < 1$.

MGARCH models are applied to capture the behavior of Bitcoin volatility, from the study of the dynamics of covariances or correlations. Generically, MGARCH models are defined as:

$$\begin{aligned} y_t &= Cx_t + \epsilon_t \\ \epsilon_t &= H_t^{1/2} v_t \end{aligned} \tag{6}$$

The conditional covariance matrix H_t is positive definite and supported by the product of the diagonal matrix of conditional variances, D_t , by a matrix of conditional correlations, R_t , and a matrix of time-invariant unconditional correlations of the standardized residuals $D_t^{-1/2} \epsilon_t$. The model works with the assumption that all conditional variance follows a univariate GARCH process and the parameterizations of R_t depend on each model. This two-step process results in an estimation of the parameters of high-dimensional systems [80].

If the conditional correlation matrix is assumed to be constant over time and only the conditional standard deviation varies over time, then $R_t = R$, where R is positive definite.

Following the mathematical notation defined above, the constant conditional correlation (CCC) model can be defined as:

$$\begin{aligned}
 y_t &= \mathbf{C}x_t + \varepsilon_t \\
 \varepsilon_t &= \mathbf{H}_t^{1/2}v_t \\
 \mathbf{H}_t &= \mathbf{D}_t^{1/2}\mathbf{R}\mathbf{D}_t^{1/2}
 \end{aligned}
 \tag{7}$$

\mathbf{D}_t is described by the following matrix:

$$\mathbf{D}_t = \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m,t}^2 \end{pmatrix}
 \tag{8}$$

where $\sigma_{i,t}^2$ is defined by

$$\sigma_{i,t}^2 = s_i + \sum_{j=1}^{p_i} \alpha_j \varepsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2
 \tag{9}$$

by default, or

$$\sigma_{i,t}^2 = \exp(\gamma_i z_{i,t}) + \sum_{j=1}^{p_i} \alpha_j \varepsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2
 \tag{10}$$

where α_j is an ARCH parameter and β_j is a GARCH parameter, and

$$\mathbf{R} = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{12} & 1 & \cdots & \rho_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m} & \rho_{2m} & \cdots & 1 \end{pmatrix}
 \tag{11}$$

The CCC-MGARCH could be an unrealistic model in certain scenarios. In this sense, if we assume that the correlation matrix, R_t , varies over time, H_t is positive definite if R_t is positive definite at a moment of time and the conditional variances, h_{ti} , $i = 1, \dots, n$, are well-defined [81]. On the basis of this dynamism, Engle [50] proposes the dynamic conditional correlation (DCC) MGARCH model. The DCC-MGARCH is based on the fact that the covariance matrix, H_t , can be decomposed into a conditional deviation, D_t , and a correlation matrix, R_t ; both D_t and R_t are considered time variables in the model. The DCC-MGARCH model is defined as:

$$\begin{aligned}
 y_t &= \mathbf{C}x_t + \varepsilon_t \\
 \varepsilon_t &= \mathbf{H}_t^{1/2}v_t \\
 \mathbf{H}_t &= \mathbf{D}_t^{1/2}\mathbf{R}\mathbf{D}_t^{1/2} \\
 \mathbf{R}_t &= \text{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t\text{diag}(\mathbf{Q}_t)^{-1/2} \\
 \mathbf{Q}_t &= (1 - \lambda_1 - \lambda_2)\mathbf{R} + \lambda_1\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}_{t-1}' + \lambda_2\mathbf{Q}_{t-1}
 \end{aligned}
 \tag{12}$$

where \mathbf{R}_t is a matrix of conditional quasi-correlations.

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m,t} & \rho_{2m,t} & \cdots & 1 \end{pmatrix}
 \tag{13}$$

In third place, a special case of the DCC-MGARCH model is the varying conditional correlation (VCC) MGARCH, introduced by Tse and Tsui [51]. The VCC-MGARCH skips normalization and obtains R_t in one step [82]:

$$R_t = (1 - \lambda_1 - \lambda_2)R + \lambda_1\psi_{t-1} + \lambda_2R_{t-1} \quad (14)$$

where λ_1 and λ_2 , as in DCC, are the parameters that govern the dynamics of conditional correlations; $\lambda_1 - \lambda_2$ are scalar parameters to weigh the effects of prior shocks and dynamic conditional correlations on the current [83]; thus:

$$\begin{aligned} y_t &= Cx_t + \varepsilon_t \\ \varepsilon_t &= H_t^{1/2}v_t \\ H_t &= D_t^{1/2}RD_t^{1/2} \end{aligned} \quad (15)$$

Finally, to compare the prediction model and, therefore, evaluate which prediction method is the most empirically effective, the following methods are calculated.

$$MAPE = \frac{\sum_{t=1}^n \frac{|Y_t - \hat{Y}_i|}{Y_t}}{n} * 100 \quad (16)$$

$$MAE = \frac{\sum_{t=1}^n \frac{|Y_t - \hat{Y}_i|}{Y_t}}{n} \quad (17)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n}} \quad (18)$$

where Y_t is the actual value and \hat{Y}_i are the forecast values. To support these regression metrics, the Diebold–Mariano (DM) test has been applied [84]. This tool is used to assess the predictive accuracy of the models. Thus, defining the forecast errors as $e_{t+h|t}^i = \hat{y}_{it} - y_t, i = a, b, \dots$, and where $L(\cdot)$ is the loss function, the differences between the forecast of both models can be determined as:

$$DM = L(e_{t+h|t}^a) - L(e_{t+h|t}^b) \quad (19)$$

The null hypothesis of the test assumes that the two forecasts have the same precision, that is:

$$H_0 : L(e_{t+h|t}^a) = L(e_{t+h|t}^b) \quad (20)$$

4. Results

Table A1 of the Appendix A shows the bivariate correlations between the price returns of each variable. If the correlation coefficients between Bitcoin and the different assets are observed, it is noted that technology companies such as RIOT (0.689 *), KBR (0.783 **), or NVDA (0.977 **) and electronic payment methods like VISA (0.831 **) are those that seem to be more related to cryptocurrency. This previous analysis helps to select the financial variables to consider for the construction of the multivariate models. Table 2 shows the conditional correlations between the returns of each of the variables with those of Bitcoin.

As expected in view of the bivariate correlation table, the commodities GOLD and OIL present a nonsignificant conditional correlation. The stock markets indices, SP500 and NIKKEI, with a negative correlation, are not significant in any of the MGARCH models. The USD/EUR exchange rate is also uncorrelated with the Bitcoin returns. On other hand, Table 3 shows the results obtained in the GARCH(1,1) and Table 4 describes the outcomes obtained in the three variants of the MGARCH model, using the RIOT Blockchain, VISA, MAST, NVDA, and KBR price returns as independent variables. The MAST correlation (0.578), although not significant, when introduced in the model gives us quality in the

results. All z coefficients are significant in each of the variables considered, both their ARCH and GARCH components, and in the constant term.

Table 2. Conditional correlations.

Correlation	CCC-MGARCH			DCC-MGARCH			VCC-MGARCH		
	Coef.	Std. Error	z	Coef.	Std. Error	z	Coef.	Std. Error	z
BTC-RIOT	0.715586	0.354285	20.2 **	0.61279	0.408964	1.50 *	0.772695	0.386975	20.0 **
BTC-VISA	0.789188	0.362937	2.17 **	0.702872	0.413236	1.70 **	0.809611	0.390046	20.8 **
BTC-MAST	0.63838	0.363874	1.75 *	0.602087	0.412589	1.46	0.65487	0.389324	1.68 *
BTC-NVDA	0.95818	0.364196	2.63 **	0.817696	0.413916	1.98 **	0.1004301	0.389384	2.58 **
BTC-KBR	0.812567	0.364059	2.23 **	0.707863	0.404163	1.75 *	0.814958	0.384864	2.12 **
BTC-SP500	−0.370483	0.365215	−10.1	−0.315056	0.382983	−0.82	−0.405142	0.406624	−1.00
BTC-N225	−0.136106	0.366807	−0.37	−0.105618	0.402157	−0.26	−0.080927	0.391535	−0.21
BTC-GOLD	−0.038485	0.365867	−0.11	−0.165848	0.441296	−0.38	−0.084389	0.410253	−0.21
BTC-WTI	0.298558	0.365755	0.82	0.414276	0.434215	0.95	0.330555	0.416259	0.79
BTC-USD/EUR	−0.1387	0.365587	−0.38	−0.17052	0.432998	−0.39	−0.113251	0.399557	−0.28

* significance level $\alpha = 0.1$; ** significance level $\alpha = 0.5$.

Table 3. Generalized autoregressive conditional heteroskedasticity (GARCH) (1,1) model.

GARCH(1,1)	Estimate	Std. Error	t Value	Pr (> t)
μ	0.22286	0.165905	1.3433	0.1792
ar1	−0.91074	0.38117	−23.8934	0.000
ma1	0.89966	0.3891	23.1218	0.000
ω	0.82175	0.396971	20.7	0.384
α_1	0.12374	0.36023	3.435	0.006
β_1	0.84611	0.40622	20.8287	0.000
Log-Likelihood		−2178.159		
AIC		5.7782		
BIC		5.7923		

Table 4. Multivariate (M)GARCH models.

	CCC-MGARCH			DCC-MGARCH			VCC-MGARCH		
	Coef.	Std. Error	z	Coef.	Std. Error	z	Coef.	Std. Error	z
BTC									
ARCH L ₁	0.1157219	0.242308	4.78 **	0.1179887	0.243649	4.84 **	0.1154838	0.242022	4.77 **
GARCH L ₁	0.8579689	0.277337	30.94 **	0.8558714	0.276567	30.95 **	0.8584539	0.277092	30.98 **
cons	0.7202004	0.2369002	30.4 **	0.7228328	0.2353516	30.7 **	0.7145526	0.2358767	30.3 **
RIOT									
ARCH L ₁	0.618105	0.138402	4.47 **	0.615976	0.136573	4.51 **	0.620191	0.139318	4.45 **
GARCH L ₁	0.9315316	0.130029	71.64 **	0.9323184	0.127162	73.32 **	0.9315534	0.130415	71.43 **
cons	0.3609495	0.1241317	2.91 **	0.3533077	0.1220651	2.89 **	0.36499	0.1257836	2.90 **
VISA									
ARCH L ₁	0.2196341	0.455961	4.82 **	0.2412854	0.455877	5.29 **	0.2123614	0.457027	4.65 **
GARCH L ₁	0.2783623	0.88319	3.15 **	0.2479007	0.738845	3.36 **	0.3088241	0.1003738	30.8 **
cons	0.4284671	0.676298	6.34 **	0.4447715	0.600111	7.41 **	0.4074951	0.751599	5.42 **
MAST									
ARCH L ₁	0.3144456	0.591334	5.32 **	0.3315486	0.602443	5.50 **	0.3104572	0.600185	5.17 **
GARCH L ₁	0.1140479	0.496595	2.30 **	0.1119515	0.462801	2.42 **	0.131731	0.595819	2.21 **
cons	0.6212404	0.59697	10.41 **	0.6232054	0.584486	10.66 **	0.6099208	0.651673	9.36 **
NVD									
ARCH L ₁	0.4120287	0.1044437	3.94 **	0.4164346	0.1016261	4.10 **	0.751948	0.377091	1.99 **
GARCH L ₁	−0.304007	0.156859	−1.94*	−0.269155	0.172217	−1.56*	0.6948316	0.13216	5.26 **
cons	3.545683	0.270191	13.12 **	3.4860.43	0.2649787	13.16 **	10.76436	0.4879063	2.21 **

Table 4. Cont.

	CCC-MGARCH			DCC-MGARCH			VCC-MGARCH		
	Coef.	Std. Error	z	Coef.	Std. Error	z	Coef.	Std. Error	z
KBR									
ARCH L ₁	0.084833	0.038641	2.20 **	0.08407	0.040724	20.6 **	0.084996	0.038168	2.23 **
GARCH L ₁	−0.9208836	0.456257	−20.18 **	−0.9213761	0.459383	−20.6 **	−0.9220857	0.443027	−20.81 **
cons	5.710.864	0.3275781	17.43 **	5.743225	0.3296634	17.42 **	5.716813	0.3262242	17.52 **
Adjustment	λ_1			0.164511	0.063529	2.59 **	0.055529	0.033387	1.66 **
	λ_2			0.8429994	0.538045	15.67 **	0.9058289	0.389035	23.28 **
AIC	18015.13			18005.96			18021.19		
BIC	18167.86			18167.94			18183.17		

* significance level $\alpha = 0.1$; ** significance level $\alpha = 0.5$.

In the CCC and VCC variants, they are all significant. In DCC, Bitcoin returns are not significantly correlated with the MASTERCARD variable. As mentioned in the previous paragraph, this variable has been included to improve the quality of the model. In summary, in view of the results and observing the fit of the model (λ), which is significant in DCC and CCC and the AIC and BIC scores although similar, it can be determined that, theoretically, the model that best fits the data is the dynamic model.

In addition, Table 5 shows the comparison between the quality of the predictions of the MGARCH models and the GARCH(1,1) model. The forecast precision statistics for the developed models do not present notable differences, the smallest difference being the DCC-MGARCH model. The results of the DM test indicate that a greater difference in the precision of the prediction of the GARCH(1,1) model respects each of the variants of the MGARCH approach. Focusing on the MGARCH, the dynamic correlation models—DCC and VCC—improve the test of the constant correlation—CCC—model.

Table 5. Diebold–Mariano test.

Model	Test Statistics	Difference	p-Value	
CCC vs. DCC	0.7051	0.02441	0.4807	
CCC vs. VCC	1.102	0.07035	0.2706	
DCC vs. VCC	0.7097	0.04594	0.4779	
CCC vs. GARCH	0.7233	0.02033	0.9423	
DCC vs. GARCH	0.157	0.04473	0.8752	
VCC vs. GARCH	0.3099	0.09067	0.7566	
	CCC-MGARCH	DCC-MGARCH	VCC-MGARCH	GARCH(1,1)
MAE %	3.2900344	3.2901131	3.2899625	3.2899992
MAPE %	0.18185568	0.18077208	0.18045294	0.18531575
RMSE	4.6804338	4.6805512	4.680403	4.6812991

5. Discussion

In the framework of the academic discussion on the nature of Bitcoin, this work has attempted to explain the relationship between the volatility of Bitcoin and those of several key financial variables. For the development of the models, the study has focused on the Conditional Correlation MGARCH approach. However, like authors such as [40–44], GARCH(1,1) has been taken as a starting point. In this way, the work coincides with previous studies by defining the GARCH standard model as a suitable alternative for Bitcoin modeling, although it does not reach the efficiency presented by the CCC, DCC, and VCC MGARCH models. Therefore, based on our results, the application of CC-MGARCH models is presented as a suitable alternative to model the volatility of Bitcoin.

The models developed have the ability to predict the volatility of Bitcoin, though the adjustment of each of the financial environment variables tested offers disparate outcomes regarding the significance of its correlation with the cryptocurrency. Thus, in the correlation analysis, it is observed that the Bitcoin volatility is inversely correlated with that of USD/EUR in each of the multivariate models examined. The commodities considered in the study, gold and oil, show a nonsignificant correlation. The volatility of the stocks of companies linked to blockchain technology—RIOT, NVDA, and KBR—and those of payments methods—VISA and MASTERCARD—present a clear and significant conditional correlation with that of the cryptocurrency. Connecting these variables with Bitcoin models can help build more accurate models.

In the comparative study between the three variants of Multivariate GARCH models, that is, CCC, DCC, and VCC, it is observed that the BIC and AIC scores of each of them are very similar, although the DCC suggests the best fit. When the models are evaluated in a practical way through the RMSE, MAE, MAPE, and Diebold–Mariano test, two conclusions are reached. In multivariate models, the forecast accuracy statistics are very similar, although the lowest error between the real and the estimated variance of DCC and VCC models is evidenced.

In the context of Bitcoin, the finding of this research confirms the most realistic hypothesis of time-varying conditional correlation over versus constant-correlation. In any case, the still insufficient maturity of the time-series of this asset means that the study of Bitcoin modeling remains open.

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Appendix A

Table A1. Correlations coefficients.

	BTC	SP500	GOLD	WTI	RIOT	NIKKEI	USD/EUR	KBR	VISA	MAST	NVDA
BTC	1.000										
SP500	−0.0539	1.000									
GOLD	0.0105	0.0049	1.000								
WTI	0.0282	−0.1040	0.0033	1.000							
RIOT	0.0689 *	−0.1962	−0.0246	0.0470	1.000						
NIKKEI	−0.0262	−0.1066	−0.0955	0.1180	0.0282	1.000					
USD/EUR	−0.0113	−0.0793	0.2145	0.0607	0.0553	−0.0239	1.000				
KBR	0.0783 **	−0.0375	−0.0549	0.0078	0.1158	0.0576	0.0323	1.000			
VISA	0.0831 **	−0.5938	−0.0162	0.0337	0.1544	0.0946	0.0404	−0.0060	1.000		
MAST	0.0578	−0.5660	−0.0030	0.0566	0.1911	0.0994	0.0519	−0.0204	0.8842	1.000	
NVDA	0.0977 **	−0.4458	−0.0356	0.0179	0.1445	0.0607	−0.0725	−0.0038	0.5121	0.5121	1.000

* significance level $\alpha = 0.1$; ** significance level $\alpha = 0.5$.

References

1. Nakamoto, S. Bitcoin: A Peer-to-Peer Electronic Cash System. *Bitcoin* **2009**, *4*, 1–9.
2. Weber, B. Bitcoin and the legitimacy crisis of money. *Camb. J. Econ.* **2015**, *40*, 17–41. [CrossRef]
3. Bouri, E.; Gupta, R.; Tiwari, A.K.; Roubaud, D. Does Bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions. *Financ. Res. Lett.* **2017**, *23*, 87–95. [CrossRef]
4. Statista. Market Capitalization of Cryptocurrencies from 2013 to 2019. Available online: <https://www.statista.com/statistics/730876/cryptocurrency-market-value/> (accessed on 12 October 2020).
5. Statista. Bitcoin Price Index from July 2012 to July 2020. Available online: <https://www.statista.com/statistics/326707/bitcoin-price-index/> (accessed on 12 October 2020).
6. Papadimitriou, T.; Gogas, P.; Gkatzoglou, F. The evolution of the cryptocurrencies market: A complex networks approach. *J. Comput. Appl. Math.* **2020**, *376*, 112831. [CrossRef]
7. Baur, D.G.; Hong, K.H.; Lee, A.D. Bitcoin: Medium of exchange or speculative assets? *J. Int. Financ. Mark. Inst. Money* **2018**, *54*, 177–189. [CrossRef]
8. Brauneis, A.; Mestel, R. Price discovery of cryptocurrencies: Bitcoin and beyond. *Econ. Lett.* **2018**, *165*, 58–61. [CrossRef]
9. Swan, M. *Blockchain: Blueprint for a New Economy*; O'Reilly Media, Inc.: Newton, MA, USA, 2015; ISBN 978-1-4919-2044-2.
10. Underwood, S. Blockchain beyond bitcoin. *Commun. ACM* **2016**, *59*, 15–17. [CrossRef]
11. CoinMarketCap. Available online: <https://coinmarketcap.com> (accessed on 15 January 2021).
12. Bitcoin Is Fiat Money, Too. Available online: <https://www.economist.com/free-exchange/2017/09/22/bitcoin-is-fiat-money-too> (accessed on 12 October 2020).
13. Dorfman, J. Bitcoin Is An Asset, Not a Currency. Available online: <https://www.forbes.com/sites/jeffreydorfman/2017/05/17/bitcoin-is-an-asset-not-a-currency/> (accessed on 12 October 2020).
14. Fernández-Villaverde, J.; Sanches, D.R. *On the Economics of Digital Currencies*; Working Paper No. 18-7; Federal Reserve Bank of Philadelphia: Philadelphia, PA, USA, 2018. [CrossRef]
15. Hazlett, P.K.; Luther, W.J. Is bitcoin money? And what that means. *Q. Rev. Econ. Financ.* **2020**, *77*, 144–149. [CrossRef]
16. Glaser, F.; Zimmermann, K.; Haferkorn, M.; Weber, M.C.; Siering, M. Bitcoin—Asset or currency? Revealing users' hidden intentions. In Proceedings of the ECIS 2014 Proceedings—22nd Europe Conference Information Systems, Tel Aviv, Israel, 9–11 June 2014.
17. Yermack, D. Is Bitcoin a Real Currency? An Economic Appraisal. In *Handbook of Digital Currency: Bitcoin, Innovation, Financial Instruments, and Big Data*; Elsevier: Amsterdam, The Netherlands, 2015; pp. 31–43. ISBN 9780128023518. [CrossRef]
18. Baek, C.; Elbeck, M. Bitcoins as an investment or speculative vehicle? A first look. *Appl. Econ. Lett.* **2015**, *22*, 30–34. [CrossRef]
19. White, R.; Marinakis, Y.; Islam, N.; Walsh, S. Is Bitcoin a currency, a technology-based product, or something else? *Technol. Forecast. Soc. Chang.* **2020**, *151*, 119877. [CrossRef]
20. Bouri, E.; Shahzad, S.J.H.; Roubaud, D.; Kristoufek, L.; Lucey, B. Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. *Q. Rev. Econ. Financ.* **2020**, *77*, 156–164. [CrossRef]
21. Dyhrberg, A.H.; Foley, S.; Svec, J. How investible is Bitcoin? Analyzing the liquidity and transaction costs of Bitcoin markets. *Econ. Lett.* **2018**, *171*, 140–143. [CrossRef]
22. Koutmos, D. Liquidity uncertainty and Bitcoin's market microstructure. *Econ. Lett.* **2018**, *172*, 97–101. [CrossRef]
23. Scharnowski, S. Understanding Bitcoin liquidity. *Financ. Res. Lett.* **2020**, 101477. [CrossRef]
24. Urquhart, A. The inefficiency of Bitcoin. *Econ. Lett.* **2016**, *148*, 80–82. [CrossRef]
25. Nadarajah, S.; Chu, J. On the inefficiency of Bitcoin. *Econ. Lett.* **2017**, *150*, 6–9. [CrossRef]
26. Tiwari, A.K.; Jana, R.K.; Das, D.; Roubaud, D. Informational efficiency of Bitcoin—An extension. *Econ. Lett.* **2018**, *163*, 106–109. [CrossRef]
27. Cheah, E.T.; Mishra, T.; Parhi, M.; Zhang, Z. Long Memory Interdependency and Inefficiency in Bitcoin Markets. *Econ. Lett.* **2018**, *167*, 18–25. [CrossRef]
28. Kristoufek, L. On Bitcoin markets (in)efficiency and its evolution. *Phys. A Stat. Mech. Its Appl.* **2018**, *503*, 257–262. [CrossRef]
29. Nan, Z.; Kaizoji, T. Market efficiency of the bitcoin exchange rate: Weak and semi-strong form tests with the spot, futures and forward foreign exchange rates. *Int. Rev. Financ. Anal.* **2019**, *64*, 273–281. [CrossRef]
30. Yu, M. Forecasting Bitcoin volatility: The role of leverage effect and uncertainty. *Phys. A Stat. Mech. Its Appl.* **2019**, *533*, 120707. [CrossRef]
31. Zhang, H.; Li, S. Forecasting volatility in financial markets. *Key Eng. Mater.* **2010**, *439–440*, 679–682. [CrossRef]
32. McAleer, M.; Medeiros, M.C. Realized volatility: A review. *Econom. Rev.* **2008**, *27*, 10–45. [CrossRef]
33. Balcilar, M.; Bouri, E.; Gupta, R.; Roubaud, D. Can volume predict Bitcoin returns and volatility? A quantiles-based approach. *Econ. Model.* **2017**, *64*, 74–81. [CrossRef]
34. Bariviera, A.F. The inefficiency of bitcoin revisited: A dynamic approach. *arXiv* **2017**, arXiv:1709.08090. [CrossRef]
35. Katsiampa, P. Volatility estimation for Bitcoin: A comparison of GARCH models. *Econ. Lett.* **2017**, *158*, 3–6. [CrossRef]
36. Bollerslev, T. Generalized autoregressive conditional heteroskedasticity. *J. Econom.* **1986**, *31*, 307–327. [CrossRef]
37. Engle, R.F. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* **1982**, *50*, 987. [CrossRef]
38. Dyhrberg, A.H. Bitcoin, gold and the dollar—A GARCH volatility analysis. *Financ. Res. Lett.* **2016**, *16*, 85–92. [CrossRef]

39. Manganelli, S.; Engle, R. Value at Risk Models in Finance. Available online: <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp075.pdf> (accessed on 18 November 2020).
40. Ardia, D.; Bluteau, K.; Rüede, M. Regime changes in Bitcoin GARCH volatility dynamics. *Financ. Res. Lett.* **2019**, *29*, 266–271. [[CrossRef](#)]
41. Gronwald, M. Is Bitcoin a Commodity? On price jumps, demand shocks, and certainty of supply. *J. Int. Money Financ.* **2019**, *97*, 86–92. [[CrossRef](#)]
42. Hung, J.C.; Liu, H.C.; Yang, J.J. Improving the realized GARCH's volatility forecast for Bitcoin with jump-robust estimators. *N. Am. J. Econ. Financ.* **2020**, *52*, 101165. [[CrossRef](#)]
43. Trucíos, C. Forecasting Bitcoin risk measures: A robust approach. *Int. J. Forecast.* **2019**, *35*, 836–847. [[CrossRef](#)]
44. López-Cabarcos, M.Á.; Pérez-Pico, A.M.; Piñeiro-Chousa, J.; Šević, A. Bitcoin volatility, stock market and investor sentiment. Are they connected? *Financ. Res. Lett.* **2020**, *38*, 101399. [[CrossRef](#)]
45. Chan, W.H.; Le, M.; Wu, Y.W. Holding Bitcoin longer: The dynamic hedging abilities of Bitcoin. *Q. Rev. Econ. Financ.* **2019**, *71*, 107–113. [[CrossRef](#)]
46. Kumar, A.S.; Anandarao, S. Volatility spillover in crypto-currency markets: Some evidences from GARCH and wavelet analysis. *Phys. A Stat. Mech. Its Appl.* **2019**, *524*, 448–458. [[CrossRef](#)]
47. Guesmi, K.; Saadi, S.; Abid, I.; Ftiti, Z. Portfolio diversification with virtual currency: Evidence from bitcoin. *Int. Rev. Financ. Anal.* **2019**, *63*, 431–437. [[CrossRef](#)]
48. Kyriazis, N.A.; Daskalou, K.; Arampatzis, M.; Prassa, P.; Papaioannou, E. Estimating the volatility of cryptocurrencies during bearish markets by employing GARCH models. *Heliyon* **2019**, *5*, e02239. [[CrossRef](#)]
49. Engle, R.F.; Lilien, D.M.; Robins, R.P. Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model. *Econometrica* **1987**, *55*, 391. [[CrossRef](#)]
50. Engle, R. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *J. Bus. Econ. Stat.* **2002**, *20*, 339–350. [[CrossRef](#)]
51. Tse, Y.K.; Tsui, A.K.C. A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *J. Bus. Econ. Stat.* **2002**, *20*, 351–362. [[CrossRef](#)]
52. Ali, O.; Ally, M.; Clutterbuck; Dwivedi, Y. The state of play of blockchain technology in the financial services sector: A systematic literature review. *Int. J. Inf. Manag.* **2020**, *54*, 102199. [[CrossRef](#)]
53. Ma, J.; Gans, J.S.; Tourky, R. *Market Structure in Bitcoin Mining*; National Bureau of Economic Research: Cambridge, MA, USA, 2018. [[CrossRef](#)]
54. Luther, W.J.; Salter, A.W. Bitcoin and the bailout. *Q. Rev. Econ. Financ.* **2017**, *66*, 50–56. [[CrossRef](#)]
55. Hui, C.H.; Lo, C.F.; Chau, P.H.; Wong, A. Does Bitcoin behave as a currency?: A standard monetary model approach. *Int. Rev. Financ. Anal.* **2020**, *70*, 101518. [[CrossRef](#)]
56. IFRS-IFRIC. Holdings of Cryptocurrencies—June 2019. Available online: <https://cdn.ifrs.org/-/media/feature/supporting-implementation/agenda-decisions/holdings-of-cryptocurrencies-june-2019.pdf> (accessed on 18 November 2020).
57. Hussain Shahzad, S.J.; Bouri, E.; Roubaud, D.; Kristoufek, L. Safe haven, hedge and diversification for G7 stock markets: Gold versus bitcoin. *Econ. Model.* **2020**, *87*, 212–224. [[CrossRef](#)]
58. Selgin, G. Synthetic commodity money. *J. Financ. Stab.* **2015**, *17*, 92–99. [[CrossRef](#)]
59. Smales, L.A. Bitcoin as a safe haven: Is it even worth considering? *Financ. Res. Lett.* **2019**, *30*, 385–393. [[CrossRef](#)]
60. Baur, D.G.; Lucey, B.M. Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financ. Rev.* **2010**, *45*, 217–229. [[CrossRef](#)]
61. Urquhart, A.; Zhang, H. Is Bitcoin a hedge or safe haven for currencies? An intraday analysis. *Int. Rev. Financ. Anal.* **2019**, *63*, 49–57. [[CrossRef](#)]
62. Wang, G.; Tang, Y.; Xie, C.; Chen, S. Is bitcoin a safe haven or a hedging asset? Evidence from China. *J. Manag. Sci. Eng.* **2019**, *4*, 173–188. [[CrossRef](#)]
63. Dyhrberg, A.H. Hedging capabilities of bitcoin. Is it the virtual gold? *Financ. Res. Lett.* **2016**, *16*, 139–144. [[CrossRef](#)]
64. Selmi, R.; Mensi, W.; Hammoudeh, S.; Bouoiyour, J. Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold. *Energy Econ.* **2018**, *74*, 787–801. [[CrossRef](#)]
65. Brière, M.; Oosterlinck, K.; Szafarz, A. Virtual currency, tangible return: Portfolio diversification with bitcoin. *J. Asset Manag.* **2015**, *16*, 365–373. [[CrossRef](#)]
66. Bouri, E.; Molnár, P.; Azzi, G.; Roubaud, D.; Hagfors, L.I. On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Financ. Res. Lett.* **2017**, *20*, 192–198. [[CrossRef](#)]
67. Bouri, E.; Jalkh, N.; Molnár, P.; Roubaud, D. Bitcoin for energy commodities before and after the December 2013 crash: Diversifier, hedge or safe haven? *Appl. Econ.* **2017**, *49*, 5063–5073. [[CrossRef](#)]
68. Conlon, T.; Corbet, S.; McGee, R.J. Are cryptocurrencies a safe haven for equity markets? An international perspective from the COVID-19 pandemic. *Res. Int. Bus. Financ.* **2020**, *54*, 101248. [[CrossRef](#)]
69. Ghabri, Y.; Guesmi, K.; Zantour, A. Bitcoin and liquidity risk diversification. *Financ. Res. Lett.* **2020**, 101679. [[CrossRef](#)]
70. Yang, R.; Yu, L.; Zhao, Y.; Yu, H.; Xu, G.; Wu, Y.; Liu, Z. Big data analytics for financial Market volatility forecast based on support vector machine. *Int. J. Inf. Manag.* **2020**, *50*, 452–462. [[CrossRef](#)]

71. Andersen, T.G.; Bollerslev, T. Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts. *Int. Econ. Rev. (Philadelphia)* **1998**, *39*, 885. [[CrossRef](#)]
72. Gao, C.T.; Zhou, X.H. Forecasting VaR and ES using dynamic conditional score models and skew Student distribution. *Econ. Model.* **2016**, *53*, 216–223. [[CrossRef](#)]
73. Conrad, C.; Custovic, A.; Ghysels, E. Long- and Short-Term Cryptocurrency Volatility Components: A GARCH-MIDAS Analysis. *J. Risk Financ. Manag.* **2018**, *11*, 23. [[CrossRef](#)]
74. Bollerslev, T. Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized Arch Model. *Rev. Econ. Stat.* **1990**, *72*, 498. [[CrossRef](#)]
75. Chan, F.; Lim, C.; McAleer, M. Modelling multivariate international tourism demand and volatility. *Tour. Manag.* **2005**, *26*, 459–471. [[CrossRef](#)]
76. Gardebroek, C.; Hernandez, M.A. Do energy prices stimulate food price volatility? Examining volatility transmission between US oil, ethanol and corn markets. *Energy Econ.* **2013**, *40*, 119–129. [[CrossRef](#)]
77. Garcia-Jorcano, L.; Benito, S. Studying the properties of the Bitcoin as a diversifying and hedging asset through a copula analysis: Constant and time-varying. *Res. Int. Bus. Financ.* **2020**, *54*, 101300. [[CrossRef](#)]
78. Kwon, J.H. Tail behavior of Bitcoin, the dollar, gold and the stock market index. *J. Int. Financ. Mark. Inst. Money* **2020**, *67*, 101202. [[CrossRef](#)]
79. Engle, R. *Estimates of the Variance of US Inflation Base on the ARCH Model*; University of California, San Diego Discussion Paper; University of California: San Diego, CA, USA, 1980; pp. 80–114.
80. De Almeida, D.; Hotta, L.K.; Ruiz, E. MGARCH models: Trade-off between feasibility and flexibility. *Int. J. Forecast.* **2018**, *34*, 45–63. [[CrossRef](#)]
81. Orskaug, E. Multivariate DCC-GARCH Model -With Various Error Distributions. Master's Thesis, Institutt for Matematiske Fag, Norges Teknisk-Naturvitenskapelige Universitet, Trondheim, Norway, June 2009.
82. Jarjour, R.; Chan, K.S. Dynamic conditional angular correlation. *J. Econom.* **2020**, *216*, 137–150. [[CrossRef](#)]
83. Lanza, A.; Manera, M.; McAleer, M. Modeling dynamic conditional correlations in WTI oil forward and futures returns. *Financ. Res. Lett.* **2006**, *3*, 114–132. [[CrossRef](#)]
84. Diebold, F.X.; Mariano, R.S. Comparing predictive accuracy. *J. Bus. Econ. Stat.* **2002**, *20*, 134–144. [[CrossRef](#)]