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Day-Ahead Energy and Reserve Dispatch Problem under Non-Probabilistic Uncertainty

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Abstract: The current energy transition and the underlying growth in variable and uncertain renewable-based energy generation challenge the proper operation of power systems. Classical probabilistic uncertainty models, e.g., stochastic programming or robust optimisation, have been used widely to solve problems such as the day-ahead energy and reserve dispatch problem to enhance the day-ahead decisions with a probabilistic insight of renewable energy generation in real-time. By doing so, the scheduling of the power system becomes, production and consumption of electric power, more reliable (i.e., more robust because of potential deviations) while minimising the social costs given potential balancing actions. Nevertheless, these classical models are not valid when the uncertainty is imprecise, meaning that the system operator may not rely on a unique distribution function to describe the uncertainty. Given the Distributionally Robust Optimisation method, our approach can be implemented for any non-probabilistic, e.g., interval models rather than only sets of distribution functions (ambiguity set of probability distributions). In this paper, the aim is to apply two advanced non-probabilistic uncertainty models: Interval and ϵ-contamination, where the imprecision and in-determinism in the uncertainty (uncertain parameters) are considered. We propose two kinds of theoretical solutions under two decision criteria—Maximinity and Maximality. For an illustration of our solutions, we apply our proposed approach to a case study inspired by the 24-node IEEE reliability test system.

Keywords: energy and reserve dispatch; imprecise uncertainty; maximinity and maximality; optimal decision

1. Introduction

One of the important points in our life is to deal with uncertainties. The uncertainty is present because of lack of information or data. Philosophically, it is about 'known unknowns', e.g., deterministic/classical uncertainty or 'unknown unknowns', e.g., in-deterministic—imprecise (advanced) uncertainty [1]. One of the famous uncertainty models is a probabilistic (data-driven or analytical) model. These models' intentions are to represent for instance agent's belief (agents like humans, machines, or robots) about the domain they are operating in, and which describe and even determine the actions they will take in a diversity of situations or realisations. Probability theory provides a normative system for reasoning and decision making in the face of uncertainty. Bayesian or precise probability models have the property that they are completely decisive, i.e., a Bayesian agent always has a best choice when faced with several alternatives, whatever their state of information

is. While many may view this as an advantage, it is not always realistic. There are two problems, Gilboa [2] offerstheirtorical surveys, with (precise) probabilities as a model to describe uncertainty: (i) The interpretation is not clear or at least, the consequences in the real world are not clear. Therefore, we want an operational and behavioural model. (ii) The model is too precise meaning the model is unique and static (while in reality, the uncertainty is changing dynamically). In any decision problem under precise uncertainty, there is always an optimal solution. One can behold some degenerate cases, for instance in gambling, decide between two actions: to buy or sell a gamble. The assumption whether there is a fair price or is not, either to accept or reject a gamble (like a utility), is not that crucial, merely the acknowledgement of the possibility of indecision is what counts. Imprecise probability models deal with said problems explicitly allowing for indecision while retaining the normative and coherent stance of the Bayesian approach. We follow the terminology and school of thought of Walley [1,3] who follows the tradition of Frank Ramsey [4], Bruno de Finetti [5,6] and Peter Williams [7,8] in trying to establish a rational model for a subject's beliefs and reasoning for modelling imprecise uncertainty.

Interpretation of Lower and Upper Previsions—Walley's Integration

A unification of many of the above-mentioned imprecise probability theories was proposed by Walley [1]. In terms of probability interpretations, Walley's formulation of imprecise probabilities is based on the subjective variant of the Bayesian interpretation of probability. Walley defines upper and lower probabilities as particular cases of upper and lower previsions and the gambling framework advanced by Bruno de Finetti [9]. In simple terms, a decision maker's lower prevision $\underline{P}(\cdot)$ is the highest price α at which the decision-maker is sure he or she would buy a gamble f such as utility (reward or loss) function,

$$\underline{P}(f) := \sup\{\alpha : f - \alpha \geq 0\},$$

and the upper prevision $\overline{P}(\cdot)$ is the lowest price β at which the decision-maker is sure he or she would buy the opposite of the gamble (which is equivalent to selling the original gamble) f,

$$\overline{P}(f) := \inf\{\beta : \beta - f \geq 0\}.$$

If the upper and lower previsions are equal, they jointly represent the decision maker's fair price for the gamble f, the price at which the decision-maker is willing to take either side of the gamble. The existence of a fair price leads to precise (classical) probabilities. The allowance for imprecision, indecision, or a gap between a decision maker's upper and lower previsions, is the primary difference between precise and imprecise probability theories—shown in Figure 1. Such gaps arise naturally, for instance, in betting markets which happen to be financially illiquid due to asymmetric information. Henry Kyburg [10] also gives this gap repeatedly fortheir interval probabilities, though he and Isaac Levi [11] give other reasons for intervals representing states of belief as well.
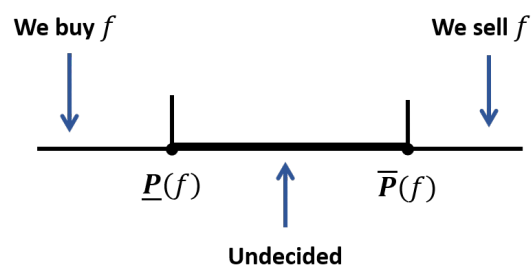

Figure 1. Highest buying price and lowest selling price for a gamble f—imprecise models allow for indecision.

Our case study here is an especial case of linear programming problem under uncertainty. There are many applications for linear programming under uncertainty problems,

some of them are given by Dantzig in [12] (Example 2), which is about finding the minimum expected cost diet in a Nutrition problem. These are some examples of a broader application on optimisation under uncertainty: generation of electrical power, operation of reservoirs, inventory management, portfolio selection, facility planning, pollution control, stabilisation of mechanisms, path planning problem [13], epistemic uncertainty in AI and machine learning [14], and analysis of biological systems (another classical way to solve Linear Programming (LP) problems under interval uncertainty is interval arithmetic [15]) see, e.g., [16].

In the electricity generation mix, the continuous growth in renewable energy generation is incentivised by governments as well as intergovernmental climate change agreements. This motivation brings challenges into the safe operation of power systems. One of these challenges is related to the uncertainty and variability stemming from weather-dependent energy generation which may jeopardise the day-ahead scheduling of power systems [17]. In practice, the generators are dispatched within the day-ahead energy dispatch for the next day, while reserve capacities are booked within the day-ahead reserve dispatch. These reserves may be activated in real-time if needed, aiming at compensating the potential deviations from day-ahead forecasts. In the current European regulatory policy, energy and reserve are dispatched separately and sequentially whereas a simultaneous dispatch is considered in the U.S. liberalised framework.

Traditionally, the energy and reserve dispatch problems are solved with a deterministic insight of uncertainty (i.e., considering a single-point forecast of renewable-based generators). However, the growing uncertainty stemming from weather-dependent generators requires uncertainty-aware day-ahead energy and reserve dispatch. Considering the U.S. simultaneous energy and reserve dispatch (in the European Union, the reserve and energy markets are cleared sequentially, while they are cleared jointly in the U.S.), this paper focuses on the incorporation of the uncertainty into the energy and reserve dispatch solution with two advanced uncertainty models which account for imprecision and in-determinism, i.e., the erroneous modelling of uncertainty via a unique distribution function. Classical uncertainty modelling techniques have been widely used in recent literature. Stochastic programming considers the availability of a discrete set of scenarios assigned with an occurrence probability, which approximates the ideal true distribution function [18]. This optimisation framework is known to provide poor out-of-sample performances unless the number of scenarios increases, which in turn increases the computational burden to solve the problem [19]. Differently, robust optimisation minimises the energy and reserve dispatch decision costs under the worst-case realisation of uncertainty within an uncertainty set (i.e., the set of plausible realisations of uncertainty). However, the focus on worst-case realisation is known to provide over-conservative solutions [20].

Differently, distributionally robust optimisation (DRO) [21,22] considers a set of potential distributions, called ambiguity set, to hedge against the inevitable error made by relying on a unique distribution which is never exact. The decisions are next selected to be optimal for the worst-case distribution inside the ambiguity set, i.e., the one that mostly affects the objective function. The ongoing research suggests that DRO may perform similarly to scenario-based programming or robust optimisation by finely tuning the size of the ambiguity set and therefore DRO may even perform better in some particular cases [23]. However, our approach does not require a set of distribution functions. The two uncertainty models, given in this paper, are non-probabilistic, i.e., pure vacuous interval model and contamination of an interval model with a single probability distribution function. In both models, we do not need to make an ambiguity set (which requires a lot of data to build, anyhow).

These classical probabilistic modelling frameworks (e.g., stochastic and robust optimisation) are not valid when the uncertainty is imprecise, meaning that the naive reliance on a unique distribution describing the uncertainty may result in suboptimal energy and reserve dispatch. Imprecision or indeterminism exists when the uncertainty is changing. For instance, (i) it is not unique and varies (ii) the uncertain model is a non-probabilistic

indeterministic model such as interval (iii) there are missing data which is vague and conflicting or (iv) the data deal with a belief that may be subjective such as rare event (like weather condition). Hence, those said reasons do not allow us to build stochastic or have a single (true) distribution about the uncertainty.

The paper is organised as follows. In Section 2, the model and theoretical results are discussed. Section 3 talks about numerical results regarding day-ahead energy and reverse dispatch problem and illustrates the results on a 24-node IEEE reliability test system. Section 5 discusses the conclusion and the future works.

2. Model

2.1. Recap

This section is a quick overview of the most important concepts of imprecise decision theory (for more details, we propose consulting [1,24,25]) and uncertain linear programming (LP) problem. Since the Day-Ahead energy and reserve dispatch problem is an LP problem we first discuss general LP problem under uncertainty in the next Section 2.1.1.

2.1.1. Linear Programming under Uncertainty

Many applications for LP under uncertainty (LPUU) problems exist, few are addressed by Dantzig in [12] (Example 2), which is habitually about obtaining the minimum expected cost, e.g., most inexpensive diet in a Nutrition problem. Here is some fascinating more comprehensive application of optimisation under uncertainty: Optimisation under uncertainty in Artificial Intelligence, Generation of electrical power, Operation of reservoirs, Inventory management, Facility planning, Optimal portfolio selection, Pollution control, Stabilisation of mechanisms, and Analysis of biological systems [16]. An LPUU problem is a generalisation of the LP problem where at least one of the coefficients of the LP problem is uncertain (meaning they are not deterministic, we do not know the exact values or the values are not known, precisely). For instance, the only information about the coefficient is lower or upper values in an interval or some unique probabilistic information. The generic (standard) linear programming problem under uncertainty is defined as follows,

$$
\begin{aligned}
\text{maximise} \quad & U^T x \\
\text{such that} \quad & Yx \leq Z, \ x \geq 0
\end{aligned}
\tag{1}
$$

where $x \in \mathbb{R}^n$ is an optimisation vector of variables x_j, U is a random vector taking values $u \in \mathbb{R}^n$, the matrices Y and Z are random matrices taking values $y \in \mathbb{R}^{m \times n}$ and $z \in \mathbb{R}^m$, respectively (we assumed that y_{ij}, z_i and u_j the elements of Y, Z and U are independent. In this paper, we worked with the maximisation operator. Since $\min U^T x = -\max \ -U^T x$, therefore, all results and proves can be applied and held for the minimisation operator as well).

2.1.2. Reformulating LP Problem as a Decision Problem

Our approach is to reformulate the LP problem (1) to a well-posed decision problem. In a decision problem, first we need to define a gain (loss) function G_x for each decision $x \geq 0$ in problem (1), as follows:

$$
G_x := (U^T x - L) I_{Yx \leq Z} + L
\tag{2}
$$

where $I_{Yx \leq Z}(v)$ is an indicator function which is equal to one if x is in the feasibility space and is zero when x is infeasible for any realisations $v = (y, z)$ that the random variable $V = (Y, Z)$ assumes. Maximising G_x is equivalent to solve problem (1). Because, for each decision $x \geq 0$ and any outcome or realisation (y, z) when x is feasible (or equivalently $I_{Yx \leq Z}(v) = 1$) then we have a reward equal to $U^T x$ otherwise we have to be punished with real value L (or equivalently $I_{Yx \leq Z}(v) = 0$). $L \in \mathbb{R}$ is small enough and is interpreted as a penalty/punishment value for violating the constraints. Second, we need to define

decision criteria. In this paper, two decision criteria are used: maximinity and maximality, for more details, please see [24,25].

Maximality Criterion

Consider a case that a decision maker seeks decisions x—so called maximal decisions/ solutions—that are undominated in pairwise comparison with all other decisions (partial order), i.e., no decision z is consider better than x:

$$
\begin{aligned}
x \text{ is maximal} &\Leftrightarrow \nexists z \in \mathcal{X},\, z \succ x \\
&\Leftrightarrow \forall z \in \mathcal{X},\, z \nsucc x \\
&\Leftrightarrow \forall z \in \mathcal{X},\, \overline{P}(G_x - G_z) \geq 0 \\
&\Leftrightarrow \inf_{z \in \mathcal{X}} \overline{P}(G_x - G_z) \geq 0.
\end{aligned}
\tag{3}
$$

By applying the maximality criterion in (3) to the gain function which is defined in (2), we have $x \geq 0$ is maximal if and only if,

$$
\inf_{w \geq 0} \overline{P}\big((U^T x - L) I_{Yx \leq Z} - (U^T w - L) I_{Yw \leq Z}\big) \geq 0.
\tag{4}
$$

Maximinity (Γ-Maximin) Criterion

Maximin solutions derive from worst-case reasoning (worst-case scenario), i.e., they are the decisions that have the highest lower expected utility (similarly, maximaxity solutions—best-case reasoning/scenario—can be found by just replacing the lower prevision $\underline{P}$ to the upper prevision $\overline{P}$ in (5)),

$$
x \text{ is maximin or gamma maximin} \Leftrightarrow x \in \operatorname*{argmax}_{z \in \mathcal{X}} \underline{P}(G_z)
\tag{5}
$$

Note that, any maximin solutions are also in maximal solutions set [25]. In both, maximinity and maximality criteria, $\operatorname{argmax}_{z \in \mathcal{X}} \underline{P}(G_z)$ and $\inf_{z \in \mathcal{X}} \overline{P}(G_x - G_z)$ are functions of x in $\mathcal{X}$, therefore, we need to calculate and find that, (i) in maximinity: for which $z \in \mathcal{X}$ the function—$\underline{P}(G_z)$—has the highest value, and (ii) in maximality: the function—$\inf_{z \in \mathcal{X}} \overline{P}(G_x - G_z)$—is positive or zero. For further information and details in decision making with imprecise probabilities, we refer to [26].

By applying the maximinity criterion in (5) to the gain/loss function defined in (2), we find the set of maximin solutions which are given by,

$$
\operatorname*{argmax}_{x \geq 0} \underline{P}(G_x) = \operatorname*{argmax}_{x \geq 0} \underline{P}\big((U^T x - L) I_{Yx \leq Z} + L\big)
\tag{6}
$$

In the next two sections, we will give maximin and maximal solutions to problem (1) in two separate uncertainty models—intervals and ϵ-contamination.

2.1.3. Interval Model
Maximin Solutions in Interval Case

By combining these interval prevision definitions in [25] with Equation (6), the maximin solutions become a classical linear programming problem is an inner feasibility space which can be written as:

$$
\begin{aligned}
&\max_{x \in \mathbb{R}^n} \quad \underline{U}^T x \\
&\text{such that} \quad \overline{Y}x \leq \underline{Z},\, x \geq 0.
\end{aligned}
\tag{7}
$$

Maximal Solutions in Interval Case

We arrange the decision $x \geq 0$ in an (partial) order so that is not dominated by any other decisions $w \geq 0$. The maximal solutions become a classical feasibility problem:

$$\left\{ x \in \mathbb{R}_{\geq 0}^n : x \in \overline{\mathbb{A}} \text{ and } \underline{u}^T x \geq \overline{u}^T x_m \right\} \equiv \left\{ x \in \mathbb{R}_{\geq 0}^n : \underline{Y} x \leq \overline{Z} \text{ and } \underline{u}^T x \geq \overline{u}^T x_m \right\}. \quad (8)$$

where, $\overline{u} = (\overline{u}_1, \cdots, \overline{u}_n)$, x_m is the maximin solution and $\overline{\mathbb{A}}$ is an outer feasibility space defined as follows,

$$\overline{\mathbb{A}} := \bigcup_{(y,z) \in A \times B} \{ x \geq 0 : yx \leq z \} := \left\{ x \in \mathbb{R}_{\geq 0}^n : \underline{Y} x \leq \overline{Z} \right\}$$

One of the interesting properties in these results is that the solutions in both criteria—maximinity and maximality—do not depend on L. For more details, see [27,28],

2.1.4. ϵ-Contamination Model
Maximin and Maximal Solutions in Contamination Case

By combining the general maximin solution in (6) and the general maximal solutions in (4) with the lower prevision definition in [25], the maximin solution $x_M^\epsilon \in X_M^\epsilon$ can be found by solving the linear optimisation problem which are defined as follows ($0 < \epsilon < 1$),

$$X_M^\epsilon := \left\{ \underset{x \geq 0}{\operatorname{argmax}} \, P\big((U^T x - L) I_{Y x \leq Z} \big) \epsilon \right\} \cup \left\{ \underset{\{x \in \mathbb{R}_{\geq 0}^n : \overline{y} x \leq \underline{z}\}}{\operatorname{argmax}} \big((1 - \epsilon) \underline{u}^T x \big) \right\}. \quad (9)$$

The maximal solutions are given as a convex set and can be found by vertex enumeration or more advance Lagrange Duality method (see, e.g., [29])

$$\left\{ \underset{x \geq 0}{\operatorname{argmax}} \, P\big((U^T x - L) I_{Y x \leq Z} \big) \epsilon \right\} \cup \left\{ (1 - \epsilon) x \in \mathbb{O}_\epsilon \right\} \quad (10)$$

where $\mathbb{O}_\epsilon$ is the outer feasibility space (which turns out to be convex) and is defined as follows,

$$\mathbb{O}_\epsilon := \bigcup_{(y,z,u) \in \mathcal{B}} \left\{ x \in \mathbb{R}_{\geq 0}^n : yx \leq z \wedge u^T x \geq u^T x_M^\epsilon \right\}$$

$$= \left\{ x \geq 0 : \underline{y} x \leq \overline{z} \wedge \overline{u}^T x \geq \underline{u}^T x_M^\epsilon \right\}.$$

2.2. Day-Ahead Energy and Reserve Dispatch Problem

The Day-Ahead (DA) energy and reserve dispatch problem (11) aims at minimising the social electricity costs (which is equivalent to maximising the social welfare when the load is considered inelastic) by properly scheduling the generators. These costs are composed of (i) the cost of energy generation $c^T p$ where $c \in \mathbb{R}^{|\mathcal{G}|}$ represents the generation costs in (\$/MWh) and $p \in \mathbb{R}^{|\mathcal{G}|}$ represents the energy generation from the generators in (MWh), (ii) the cost of procuring upward/downward reserve capacity $\overline{c}^T \overline{r} + \underline{c}^T \underline{r}$ where $\overline{c}, \underline{c} \in \mathbb{R}^{|\mathcal{G}|}$, respectively represent the upward/downward reserve procurement costs in \$/MW and $\overline{r}, \underline{r} \in \mathbb{R}^{|\mathcal{G}|}$ represents the reserve procurement of the generators in MW and, (iii) the real-time cost of balancing actions $c^T Y \xi$ where $Y \in \mathbb{R}_{\leq 0}^{|\mathcal{G}| \times |\mathcal{W}|}$ is a matrix containing the participation factor of each generator to the per-unit uncertain renewable energy deviation from day-ahead forecast $\xi \in \mathbb{R}^{|\mathcal{W}|}$, such that $Y \xi$ represents the balancing action in real-time of the generators in MW. The set of decision variables is given by $\{ p, \overline{r}, \underline{r}, Y \}$ whereas the uncertainty stems from the deviation in renewable-based energy generation from day-

ahead forecast ξ. The mathematical model of the joint DA-energy and reserve dispatch problem is,

$$\min_{p,\bar{r},\underline{r},Y} \quad c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \xi \tag{11}$$

$$\text{s.t.} \quad p + \bar{r} \le p^{\max} \tag{12}$$

$$p - \underline{r} \ge p^{\min} \tag{13}$$

$$0 \le \bar{r}, \underline{r} \le r^{\max} \tag{14}$$

$$e_{|\mathcal{G}|}^T p + e_{|\mathcal{W}|}^T W \mu - e_{|\mathcal{D}|}^T d = 0 \tag{15}$$

$$e_{|\mathcal{G}|}^T Y + e_{|\mathcal{W}|}^T W = \mathbf{0} \tag{16}$$

$$-\underline{r} \le Y \xi \tag{17}$$

$$Y \xi \le \bar{r}. \tag{18}$$

From our definition of variables Y (MW) and ξ (p.u.), the mathematical interpretation of the cost function is still consistent concerning the units. Because, in this paper, the energy and reserve dispatch problem solved for a single period of one hour. This means that power and energy have the same numerical value (E (MWh) $= P$ (MW) $\times \Delta t$ (h)), where $\Delta t = 1$ (h). For clarity and unit consistency, we may multiply the real-time costs $c^T Y \xi$ by a constant $\Delta t = 1$ (h) but this will not affect the final results. The constraints (12) and (13) verifies that the schedule respects the generator upper and lower capacity limits, i.e., $p^{\max} \in \mathbb{R}^{|\mathcal{G}|}$ and $\mathbf{0} \in \mathbb{R}^{|\mathcal{G}|}$. Equation (14) imposes the maximum allowed capacity for reserve procurement of each generator as $r^{\max} \in \mathbb{R}^{|\mathcal{G}|}$. The day-ahead energy balance is required by Equation (15) where e is a vector of ones which dimension is given by the index, $W \in \mathbb{R}^{|\mathcal{W}| \times |\mathcal{W}|}$ is a diagonal matrix containing the renewable generator capacity and $\mu \in \mathbb{R}^{|\mathcal{W}|}$ represents the day-ahead renewable generation forecast such that $e_{|\mathcal{W}|}^T W \mu$ is equal to the scheduled sum of renewable power generation; $d \in \mathbb{R}^{|\mathcal{D}|}$ corresponds to the vector of demands. The real-time balance is required by (16) where the participation factors are selected in such a way that any deviation ξ will be compensated by the generators. Constraints (17) and (18), respectively imposes that the downward/upward reserve activation in real-time does not exceed the booked reserve capacity in day-ahead.

The problem (11) is a linear programming problem under uncertainty, where the uncertainty is in the goal function as well as the constraints. There are 84 optimisation variables and 161 constraints (which is consist of 84 sign constraints, 72 inequality, and 5 equality constraints). In this paper, we use the imprecise decision theory (for more info and deeper insight concerning the theory of imprecise probability and imprecise decision theory, we refer the reader to [24,25]) to solve the problem. Some of the theories and definitions, which are used in this paper, are explained in Section 2.1. In the next section, first, we explain about the use case on DAERD problem, then we will talk about the theoretical results for the problem (11) under two imprecise uncertainty models: Interval and ϵ-contamination, wherein each model, we propose the solutions under two decision criteria—Maximinity and Maximality.

2.3. Use Case—Data and DA-Problem Modelling

As our case study, we use an adapted version of the IEEE 24-node reliability test system [30], which is composed of 12 conventional generators, 4 wind farms and 17 loads, shown in Figure 2 as follows.

The total generation capacity is equal to 2835.1 (MW), in which 1600 (MW) correspond to the sum of renewable generators capacity. Reserves can be booked up to 798 (MW) out of the maximum conventional generator capacity. The total load, which is assumed inelastic, equals to 2207 (MW). The costs and capacity parameters are given in Table 1 for each (individual) actor connected to the grid, as follows,
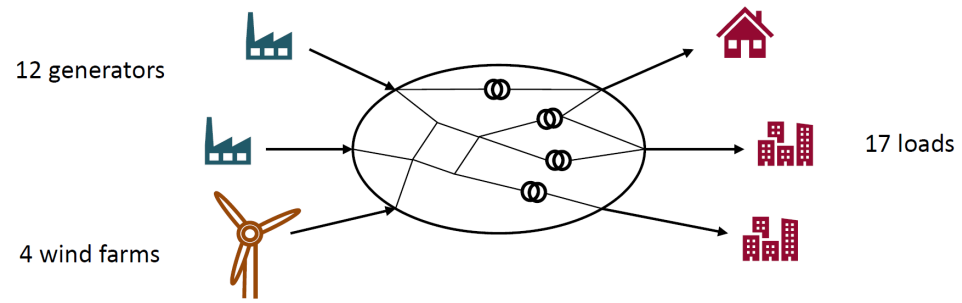
Figure 2. DA-Energy and Reserve Dispatch problem—Adapted IEEE 24-node reliability test system.

Table 1. Network parameters.

Generators	1	2	3	4	5	6	7	8	9	10	11	12	
c_g ($/MWh)	13.32	13.32	20.7	20.93	26.11	10.52	10.52	6.02	5.47	7	10.52	10.9	
$\bar{c}_g$ ($/MW)	1.68	1.68	3.3	4.07	1.89	5.48	5.48	4.98	5.53	8	3.45	5.11	
$\underline{c}_g$ ($/MW)	2.32	2.32	4.67	3.93	3.11	3.52	3.52	5.02	4.97	6.00	2.52	2.9	
$P_g^{\max}$ (MW)	127.7	127.7	294	496.5	50.4	130.2	130.2	336	336	252	260.4	294	
$P_g^{\min}$ (MW)	0	0	0	0	0	0	0	0	0	0	0	0	
$r^{\max}$ (MW)	48	48	84	216	42	36	36	60	60	48	72	48	
Wind Farms	**1**	**2**	**3**	**4**									
W (MW)	500	500	300	300									
Loads	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13–17**
d (MW)	84 ...245	75 77	139 258	58 141	55 100	106	97	132	135	150	205	150	...

2.4. Interval Model—DA-Energy and Reserve Dispatch Problem

We assume ξs are given by interval models and μs are mean values in each interval,

$$
\begin{aligned}
\min_{p,\bar{r},\underline{r},Y} \quad & c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \xi \\
\text{s.t.} \quad & 0 \le p + \bar{r} \le p^{\max}, \quad p - \underline{r} \ge p^{\min} \\
& 0 \le \underline{r}, \bar{r} \le r^{\max} \\
& e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
& e_1^T Y + e_2^T W = \mathbf{0} \\
& -\underline{r} \le Y\xi \quad \& \quad Y\xi \le \bar{r} \\
& \xi_i := [\underline{\xi}_i, \bar{\xi}_i]
\end{aligned}
\tag{19}
$$

where,

$e_1 = \mathbf{1}_{|\mathcal{G}|}$, $e_2 = \mathbf{1}_{|\mathcal{W}|}$, $e_3 = \mathbf{1}_{|\mathcal{D}|}$ are unit metrics,

$p, \underline{r}, \bar{r} \in \mathbb{R}_{\ge 0}^{|\mathcal{G}|}$, and $Y \in \mathbb{R}_{\le 0}^{|\mathcal{G}| \times |\mathcal{W}|}$ are the optimisation variables,

$W, c, \underline{c}, \bar{c}, p^{\max}, p^{\min}, r^{\max}, d$ are given via Table 1,

$g \in \mathcal{G}$, $W \in \mathcal{W}$, $d \in \mathcal{D}$ represent the set of generators, wind farms, and demands, respectively.

This problem—because of the term $c^T Y \xi$ in the goal function—is an uncertain Symbolic Linear Programming (S-LP).

2.4.1. Maximin Solutions in Interval Case

According to the theoretical maximin solution given by (7), the worst-case scenario theoretical solution is formulated as a classical LP problem under the smallest (**inner**) feasibility space.

$$
\begin{aligned}
\min_{p,\bar{r},\underline{r},Y} \quad & c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\varsigma} \\
\text{s.t.} \quad & 0 \leq p + \bar{r} \leq p^{\max}, \quad p - \underline{r} \geq p^{\min} \\
& 0 \leq \underline{r}, \bar{r} \leq r^{\max} \\
& e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
& e_1^T Y + e_2^T W = \mathbf{0} \\
& -\underline{r} \leq Y \bar{\varsigma} \quad \& \quad Y \underline{\varsigma} \leq \bar{r}
\end{aligned}
\tag{20}
$$

where, $\xi_i := [\underline{\xi}_i, \bar{\xi}_i]$ and this is a classical S-LP problem.

2.4.2. Maximal Solutions in Interval Case

According to the theoretical maximal solutions which are given by (8) in Section 2.1, we consider a bigger feasibility space (i.e., the uncertainty space is shrunk) to compute a set of feasible solutions (which may or may not be optimal). This problem can be expressed as a classical feasibility problem as follows,

$$
\forall X := (p, \bar{r}, \underline{r}, Y) \in
\left\{
\begin{array}{l}
c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\varsigma} \leq k \\
0 \leq p + \bar{r} \leq p^{\max}, \quad p - \underline{r} \geq p^{\min} \\
0 \leq \underline{r}, \bar{r} \leq r^{\max} \\
e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
e_1^T Y + e_2^T W = \mathbf{0} \\
-\underline{r} \leq Y \underline{\varsigma} \quad \& \quad Y \bar{\varsigma} \leq \bar{r}
\end{array}
\right\}
\tag{21}
$$

where, $\xi_i := [\underline{\xi}_i, \bar{\xi}_i]$ and k is the maximin solution given by (20). Equation (21) thereby represents a set potential solution to the initial problem (11), that perform better than the maximin (worst-case) solution in terms of the optimal objective function, but maybe less restrictive on the consideration of uncertainty. Since this feasibility space is convex, we need to find the vertices (i.e., forming the convex hull). Therefore, the complexity of the solution is not NP-hard and represents the potential of our approach.

2.5. ϵ-Contamination Case—DA-Energy and Reserve Dispatch Problem

Definition 1. *An ϵ-contamination model is a convex combination of two uncertainty models. (i) Probabilistic model, e.g., probability measure P, (ii) non-probabilistic (imprecise) model, e.g., interval model (in this paper) Q, which is defined as follows:*

$$
\underline{P}(G_x) = (1 - \epsilon) P(G_x) + \epsilon \underline{Q}(G_x)
\tag{22}
$$

The uncertainty about ξ is given by an ϵ-contamination model, which are the probabilistic model, for instance, normal distribution function ξ_i^P and interval model ξ_i^Q as follows,

$$
\begin{aligned}
\min_{p,\bar{r},\underline{r},Y} \quad & c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \xi \\
\text{s.t.} \quad & 0 \le p + \bar{r} \le p^{\max}, \quad p - \underline{r} \ge p^{\min} \\
& 0 \le \underline{r}, \bar{r} \le r^{\max} \\
& e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
& e_1^T Y + e_2^T W = \mathbf{0} \\
& -\underline{r} \le Y\xi \quad \& \quad Y\xi \le \bar{r} \\
& \xi_i^Q := [\underline{\xi}_i^Q, \overline{\xi}_i^Q] \quad \& \quad \xi_i^P \sim \mathcal{N}_i(\mu_\xi, \sigma_\xi)
\end{aligned}
\tag{23}
$$

This problem—because of the term $c^T Y \xi$ in the goal function—is an uncertain (S-LP) problem and we have two independent probabilistic (ξ_i^P) and non-probabilistic (ξ_i^Q) uncertain models for each ξ_i. The ϵ-contamination model is more advanced that considers both precise and imprecise models. A precise model, for instance, a probabilistic model such as probability distributions. An imprecise model, for instance, a non-probabilistic model such as intervals. One technique to construct these two independent models is via two different tests—Robustness test and Reliability/Sensitivity test.

2.5.1. Maximin Solutions in ϵ-Contamination Case

From the theoretical solution (9), the maximin solution is a classical symbolic programming problem as follows,

$$
\begin{aligned}
&\epsilon\left(\operatorname*{argmin}_{p,\bar{r},\underline{r},Y} \ c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \xi^P\right) \ \bigcup \ \bar{\epsilon}\left(\operatorname*{argmin}_{p,\bar{r},\underline{r},Y} \ c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\xi}^Q\right) \\
&\text{s.t.} \quad 0 \le p + \bar{r} \le p^{\max}, \ p - \underline{r} \ge p^{\min} \qquad \text{s.t.} \quad 0 \le p + \bar{r} \le p^{\max}, \ p - \underline{r} \ge p^{\min} \\
&\qquad 0 \le \underline{r}, \bar{r} \le r^{\max} \qquad\qquad\qquad\qquad\qquad 0 \le \underline{r}, \bar{r} \le r^{\max} \\
&\qquad e_1^T p + e_2^T W \mu - e_3^T d = 0 \qquad\qquad\qquad\quad e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
&\qquad e_1^T Y + e_2^T W = \mathbf{0} \qquad\qquad\qquad\qquad\qquad e_1^T Y + e_2^T W = \mathbf{0} \\
&\qquad -\underline{r} \le Y\xi^P \quad \& \quad Y\xi^P \le \bar{r} \qquad\qquad\qquad -\underline{r} \le Y\overline{\xi}^Q \quad \& \quad Y\underline{\xi}^Q \le \bar{r} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (24)
\end{aligned}
$$

where, $0 < \epsilon < 1$ (for the spacing, we assume $\bar{\epsilon} := 1 - \epsilon$), the elements of ξ^P are given by the normal distribution functions $\mathcal{N}_i(\mu_\xi, \sigma_\xi)$, and the elements of $\underline{\xi}^Q, \overline{\xi}^Q$ are the lower and upper bounds of given interval models: $[\underline{\xi}_i^Q, \overline{\xi}_i^Q]$.

2.5.2. Maximal Solutions in ϵ-Contamination Case

Considering the theoretical maximal solutions (10), the less conservative maximal scheduling parameter for the problem (11) is given as a convex combination of linear optimisation problem and a classical convex hull problem as follows,

$$
\left(\underset{p,\bar{r},\underline{r},Y}{\mathrm{argmin}} \ c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \bar{\zeta} \right) \epsilon \ \bigcup \ \bar{\epsilon} \left\{ \begin{array}{l} c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\bar{\zeta}} \leq k_\epsilon \\ 0 \leq p + \bar{r} \leq p^{\max} \\ p - \underline{r} \geq p^{\min} \\ 0 \leq \underline{r}, \bar{r} \leq r^{\max} \\ e_1^T p + e_2^T W \mu - e_3^T d = 0 \\ e_1^T Y + e_2^T W = \mathbf{0} \\ -\underline{r} \leq Y \underline{\zeta} \quad \& \quad Y \underline{\bar{\zeta}} \leq \bar{r} \end{array} \right.
$$

with constraints:

$$
\begin{array}{l}
\text{s.t.} \quad 0 \leq p + \bar{r} \leq p^{\max}, \ p - \underline{r} \geq p^{\min} \\
\qquad 0 \leq \underline{r}, \bar{r} \leq r^{\max} \\
\qquad e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
\qquad e_1^T Y + e_2^T W = \mathbf{0} \\
\qquad -\underline{r} \leq Y \bar{\zeta} \quad \& \quad Y \bar{\zeta} \leq \bar{r}
\end{array}
\tag{25}
$$

where,

$0 < \epsilon < 1$ (for the spacing, we assume $\bar{\epsilon} := 1 - \epsilon$),

k_ϵ is the maximin solution,

$\bar{\zeta}_i := [\underline{\zeta}_i, \bar{\zeta}_i]$, and

$\bar{\zeta}$ is the mean value from the given probabilistic model.

3. Numerical Maximin and Maximal Solutions for the DA-Energy and Reserve Dispatch Problem

In our past works [25], we have done theoretical and numerical studies about the generic LPUU problem. In this section, we present numerical solutions. We discuss the potential benefits of our methods compare to the classical uncertainty modelling techniques. To describe the uncertainty stemming from the deviation $\bar{\zeta}$ in wind power generation from a day-ahead forecast μ, we use a dataset [31] which is composed of 1000 historical observations of wind power generation for four wind farms. We calculate the expected value over this dataset and use it as forecasted wind power generation μ; i.e., the deviations are computed by retrieving μ to each sample in the dataset, such that the resulting distribution has a zero-mean. These new samples are used in Sections 3.1 and 3.2 to to define intervals and Gaussian distributions to model the uncertainty, respectively.

3.1. Interval Case

We assume $\bar{\zeta}$s are given as four intervals for four farms and μs are mean values in each interval,

$$
\begin{array}{l}
\underset{p,\bar{r},\underline{r},Y}{\min} \quad c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \bar{\zeta} \\[4pt]
\quad \text{s.t.} \quad 0 \leq p + \bar{r} \leq p^{\max}, \quad p - \underline{r} \geq p^{\min} \\
\qquad\quad 0 \leq \underline{r}, \bar{r} \leq r^{\max} \\
\qquad\quad e_1^T p + e_2^T W \mu - e_3^T d = 0 \\
\qquad\quad e_1^T Y + e_2^T W = \mathbf{0} \\
\qquad\quad -\underline{r} \leq Y \bar{\zeta} \quad \& \quad Y \bar{\zeta} \leq \bar{r} \\
\qquad\quad \bar{\zeta}_i \in \big\{ [-0.2313, 0.3999], [-0.2305, 0.4007], [-0.1705, 0.2719], [-0.1110, 0.3166] \big\} \\
\qquad\quad \mu_i \in \{0.08437, 0.085105, 0.050705, 0.10279\}
\end{array}
\tag{26}
$$

where,

$e_1 = \mathbf{1}_{1 \times 12}$, $e_2 = \mathbf{1}_{1 \times 4}$, $e_3 = \mathbf{1}_{1 \times 17}$,

$p, \underline{r}, \bar{r} \in \mathbb{R}^{|\mathcal{G}|=12}_{\geq 0}$, $Y \in \mathbb{R}^{|\mathcal{G}| \times |\mathcal{W}|=12 \times 4}_{\leq 0}$ are the optimisation variables,

W, c, $\underline{c}$, $\bar{c}$, $p^{\max}$, $p^{\min}$, $r^{\max}$, d are given via Table 1, and

$g \in \mathcal{G}$, $W \in \mathcal{W}$, $d \in \mathcal{D}$ represent the set of generators, wind farms, and demands, respectively.

This problem—because of the term $c^T Y \bar{\zeta}$ in the goal function—is a (S-LP) problem.

3.1.1. Numerical Maximin Solution in Interval Case

Considering maximin solution (20), the worst case scenario solution for the problem (19) is given as a classical LP problem under the smallest (**inner**) feasibility space:

$$\min_{p,\bar{r},\underline{r},Y} \quad c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\xi}$$

$$\text{s.t.} \quad 0 \le p + \bar{r} \le p^{\max}, \quad p - \underline{r} \ge p^{\min}$$

$$0 \le \underline{r}, \bar{r} \le r^{\max}$$

$$e_1^T p + e_2^T W\mu - e_3^T d = 0$$

$$e_1^T Y + e_2^T W = \mathbf{0}$$

$$-\underline{r} \le Y\bar{\xi} \quad \& \quad Y\underline{\xi} \le \bar{r} \tag{27}$$

where,

$$\mu_i \in \{0.08437, \ 0.085105, \ 0.050705, \ 0.10279\},$$
$$\underline{\xi}_i \in \{-0.2313, -0.2305, -0.1705, -0.1110\}, \text{ and}$$
$$\bar{\xi}_i \in \{0.3999, 0.4007, 0.2719, 0.3166\}.$$

We solve the S-LP via Julia tool [32] with Gurobi optimiser [33]. Julia tool uses Gurobi optimiser which, is very powerful to solve symbolic optimisation problems. For a detailed comparison to the other tools see [34]. The numerical solutions are as follows. The objective value is 29184.94 := k,

$$p = [98.28, \ 97.58, \ 84, \ 168.11, \ 0, \ 117.58, \ 117.58, \ 314.77, \ 336, \ 252, \ 218.76, \ 271.55],$$

$$\bar{r} = [29.40, \ 30.10, \ 48.59, \ 96.71, \ 0, \ 12.62, \ 12.62, \ 21.23, \ 0, \ 0, \ 41.64, \ 22.45],$$

$$\underline{r} = [48, \ 48, \ 84, \ 168.11, \ 0, \ 36, \ 36, \ 36.74, \ 0, \ 0, \ 72, \ 48],$$

$$Y^T = - \begin{bmatrix} 36.1 & 0 & 210.1 & 0 & 0 & 0 & 0 & 73.8 & 0 & 0 & 180.1 & 0 \\ 0 & 0 & 0 & 419.6 & 0 & 0 & 0 & 18.01 & 0 & 0 & 0 & 62.44 \\ 123.5 & 176.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 113.7 & 113.7 & 0 & 0 & 0 & 0 & 72.6 \end{bmatrix}$$

Under the worst-case scenario, the decisions are immunised against a higher set of potential realisation of uncertainty in real-time. This intuition suggests that the solution of (27) may be less risky (i.e., more conservative). This insight can be confirmed by the high total amount of booked reserves (required to compensate for the potential high deviations when the uncertainty is high). This is worth $\sum \bar{r} = 315.36 \ [MW]$ and $\sum \underline{r} = 576.85 \ [MW]$, respectively for upward and downward reserves [35].

3.1.2. Numerical Maximal Solutions in Interval Case

Considering maximal solutions (21), the less conservative theoretical solution for problem (19) is given as a classical convex hull problem which is the largest (**outer**) feasibility space:

$$\forall X := \{p, \bar{r}, \underline{r}, Y\} \in \left\{ \begin{array}{l} c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\xi} \le k = 29184.94 \\ 0 \le p + \bar{r} \le p^{\max}, \quad p - \underline{r} \ge p^{\min} \\ 0 \le \underline{r}, \bar{r} \le r^{\max} \\ e_1^T p + e_2^T W\mu - e_3^T d = 0 \\ e_1^T Y + e_2^T W = \mathbf{0} \\ -\underline{r} \le Y\underline{\xi} \quad \& \quad Y\bar{\xi} \le \bar{r} \end{array} \right\} \tag{28}$$

where,

$$\mu_i \in \{0.08437, \ 0.085105, \ 0.050705, \ 0.10279\},$$
$$\underline{\xi}_i \in \{-0.2313, -0.2305, -0.1705, -0.1110\},$$
$$\bar{\xi}_i \in \{0.3999, 0.4007, 0.2719, 0.3166\}.$$

The calculation is also done via Julia tool to find the vertices of the convex hull. The vertices are saved as an output file (because of the big dimension for this problem).

Maximin solution is about the worst-case scenario solution. In other words, working with the worst-case solution gives the least risks that a decision-maker could make. We take the worst-case (maximin) solution as the lowest risky solution. Since a final decision maker may not always need to take the lowest risky solution, the maximal solutions could be better candidates (optimal decision) by accepting some risks. One way to calculate these risks is to easily measure the distance between the maximal solutions and the worst-case solution (for the objective values). We use L^1 norm to calculate the distances (risks) and normalise the result for comparison. Meaning, for every maximal solution, we measure the distance between those solutions and the maximin solution using the L^1 norm. Figure 3 shows the optimality versus (accepted) risk.
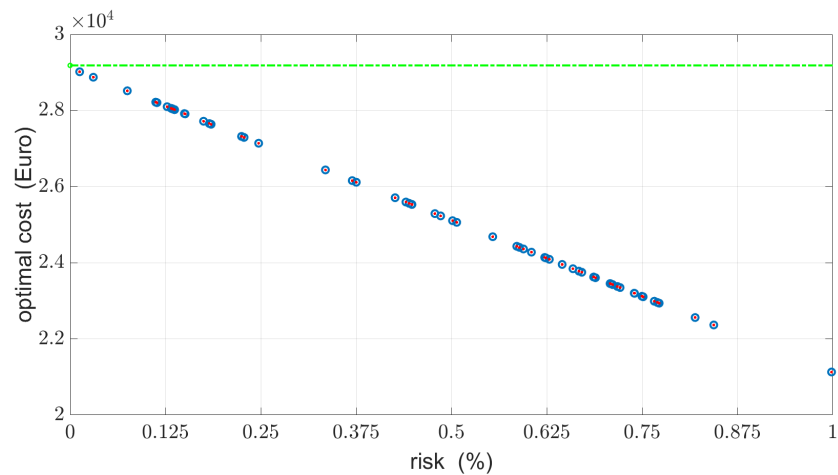

Figure 3. Optimality versus Risk (distance).

We observe that the costs decrease when the distance between the decision set and the benchmark solution set (which is taken as the less risky set) increases. This means that lower operating costs may be achieved by the system operator via reducing the conservativeness of the scheduling decisions. However, this may result in penalties related to the higher reliability risks endorsed by the decision-maker (e.g., load shedding costs if the demand is not satisfied in real-time which are not considered in this study).

3.2. ϵ-Contamination Case

In this case, we assume that ζs are given as four intervals and four independent Normal Distribution Functions, which are estimated from the data in Table 1. As shown in Figures 4 and 5. For four farms, μs are mean values of each distribution functions.
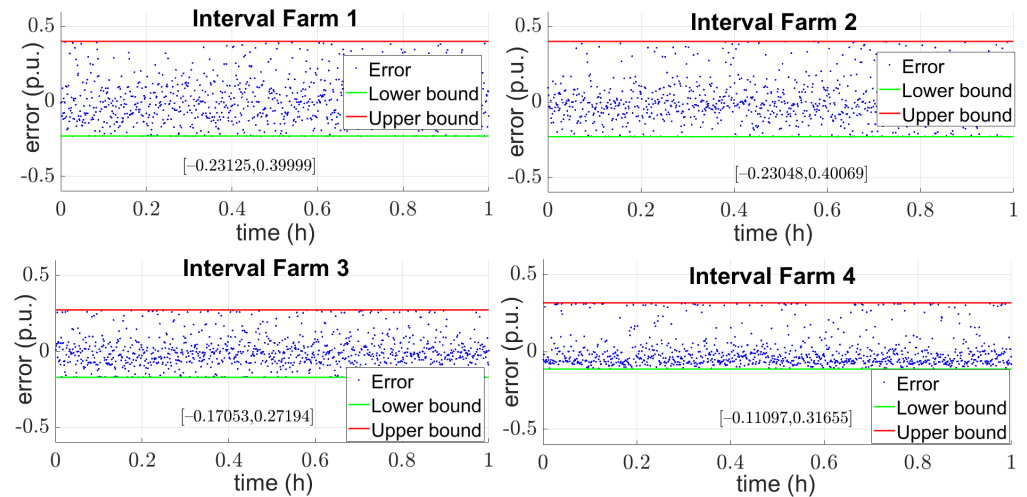

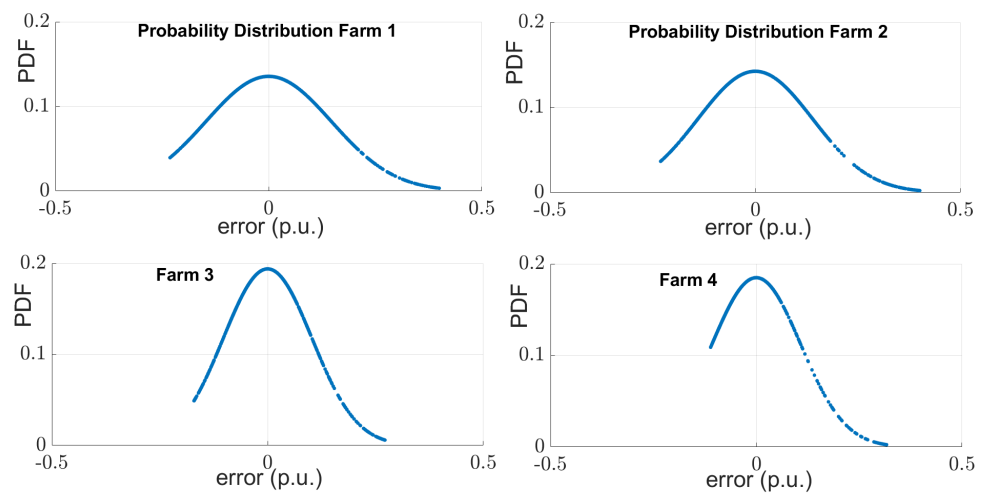**Figure 4.** The first hour measured power error variations.

Figure 5. The first hour measured power error distributions.

3.2.1. Numerical Maximin Solutions in ϵ-Contamination Case

Considering the theoretical solution (24), the numerical solution for the problem (23) is a classical symbolic programming problem as follows,

$$
\epsilon \left(\operatorname*{argmin}_{p,\bar{r},\underline{r},Y} c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \bar{\zeta} \right) \bigcup \bar{\epsilon} \left(\operatorname*{argmin}_{p,\bar{r},\underline{r},Y} c^T p + \bar{c}^T \bar{r} + \underline{c}^T \underline{r} + c^T Y \underline{\zeta} \right)
$$

$$
\text{s.t.}\ 0 \leq p + \bar{r} \leq p^{\max},\ p - \underline{r} \geq p^{\min} \quad \text{s.t.}\ 0 \leq p + \bar{r} \leq p^{\max},\ p - \underline{r} \geq p^{\min}
$$

$$
0 \leq \underline{r}, \bar{r} \leq r^{\max} \qquad\qquad\qquad 0 \leq \underline{r}, \bar{r} \leq r^{\max}
$$

$$
e_1^T p + e_2^T W \mu - e_3^T d = 0 \qquad\qquad e_1^T p + e_2^T W \mu - e_3^T d = 0
$$

$$
e_1^T Y + e_2^T W = \mathbf{0} \qquad\qquad\qquad e_1^T Y + e_2^T W = \mathbf{0}
$$

$$
-\underline{r} \leq Y \bar{\zeta} \ \ \& \ \ Y \bar{\zeta} \leq \bar{r} \qquad\qquad -\underline{r} \leq Y \bar{\underline{\zeta}} \ \ \& \ \ Y \underline{\zeta} \leq \bar{r} \qquad (29)
$$

where,

$$
0 < \epsilon < 1\ (\bar{\epsilon} := 1 - \epsilon),
$$

$$\mu_i \in \{0.08437, 0.085105, 0.050705, 0.10279\},$$
$$\underline{\zeta}_i \in \{-0.2313, -0.2305, -0.1705, -0.1110\},$$
$$\overline{\zeta}_i \in \{0.3999, 0.4007, 0.2719, 0.3166\}, \text{ and}$$
$$\zeta \in \{0.2367, 0.2325, 0.1774, 0.1321\}.$$

The numerical solution for the probabilistic-part—left side in (24)—is calculated by Julia tool (with Gurobi optimiser) as follows. The objective value: 15535.715,

$$p = \begin{bmatrix} 48, & 57.96, & 0, & 189.45, & 42, & 130.2, & 130.2, & 336, & 336, & 252, & 260.4, & 294 \end{bmatrix},$$

$$\overline{r} = \begin{bmatrix} 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix},$$

$$\underline{r} = \begin{bmatrix} 48, & 48, & 0, & 189.45, & 42, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}, \text{ and}$$

$$Y^T = \begin{bmatrix} -35.36 & 0 & 0 & -309.25 & -155.39 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -270.58 & 0 & 0 & -29.43 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -300 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (*).$$

For all $0 < \epsilon < 1$ (and $\overline{\epsilon} := 1 - \epsilon$) the numerical maximin solutions are: The objective value: $15535.715\epsilon + 29184.94\overline{\epsilon}$,

$$p = \begin{bmatrix} 48\epsilon + \overline{\epsilon}98.3, & 58\epsilon + \overline{\epsilon}97.6, & \overline{\epsilon}84, & 189.5\epsilon + \overline{\epsilon}168.1, & 42\epsilon, & 130.2\epsilon + \overline{\epsilon}117.6 + \ldots \\ \ldots 130.2\epsilon, & \overline{\epsilon}117.6, & 336\epsilon + \overline{\epsilon}314.8, & 336, & 252, & 260.4\epsilon + \overline{\epsilon}218.8, & 294\epsilon + \overline{\epsilon}271.6 \end{bmatrix},$$

$$\overline{r} = \begin{bmatrix} 29.40, & 30.10, & 48.59, & 96.71, & 0, & 12.62, & 12.62, & 21.23, & 0, & 0, & 41.64, & 22.45 \end{bmatrix}\overline{\epsilon},$$

$$\underline{r} = \begin{bmatrix} 48, & 48, & \overline{\epsilon}84, & 189.5\epsilon + \overline{\epsilon}168.1, & 42\epsilon, & \overline{\epsilon}36, & \overline{\epsilon}36, & \overline{\epsilon}36.8, & 0, & 0, & \overline{\epsilon}72, & \overline{\epsilon}48 \end{bmatrix}, \text{ and}$$

$$Y^T = -\begin{bmatrix} 35 + \overline{\epsilon}36 & 0 & \overline{\epsilon}210 & \overline{\epsilon}309 & \overline{\epsilon}155 & 0 & 0 & \overline{\epsilon}73 & 0 & 0 & \overline{\epsilon}180 & 0 \\ 0 & 0 & 0 & 500\epsilon + \overline{\epsilon}420 & 0 & 0 & 0 & \overline{\epsilon}18 & 0 & 0 & 0 & \overline{\epsilon}62 \\ \overline{\epsilon}124 & 271\epsilon + \overline{\epsilon}177 & 0 & 0 & 29\epsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 300\epsilon & 0 & 0 & 0 & 0 & \overline{\epsilon}114 & \overline{\epsilon}114 & 0 & 0 & 0 & 0 & \overline{\epsilon}73 \end{bmatrix} \quad (+)$$

3.2.2. Numerical Maximal Solutions in ϵ-Contamination Case

Considering the theoretical maximal solutions (25), the less conservative maximal scheduling parameter for the problem (23) is given as a convex combination of linear optimisation problem and a classical convex hull problem as follows,

$$\left(\underset{p,\overline{r},\underline{r},Y}{\operatorname{argmin}} \ c^T p + \overline{c}^T \overline{r} + \underline{c}^T \underline{r} + c^T Y \zeta \right)\epsilon \ \bigcup \ \begin{cases} c^T p + \overline{c}^T \overline{r} + \underline{c}^T \underline{r} + c^T Y \overline{\zeta} \leq 29184.94 \\ 0 \leq p + \overline{r} \leq p^{\max} \\ p - \underline{r} \geq p^{\min} \\ 0 \leq \underline{r}, \overline{r} \leq r^{\max} \\ e_1^T p + e_2^T W \mu - e_3^T d = 0 \\ e_1^T Y + e_2^T W = \mathbf{0} \\ -\underline{r} \leq Y\underline{\zeta} \quad \& \quad Y\overline{\zeta} \leq \overline{r} \end{cases}$$

subject to
$$\begin{aligned} \text{s.t.} \quad & 0 \leq p + \overline{r} \leq p^{\max}, \ p - \underline{r} \geq p^{\min} \\ & 0 \leq \underline{r}, \overline{r} \leq r^{\max} \\ & e_1^T p + e_2^T W \mu - e_3^T d = 0 \\ & e_1^T Y + e_2^T W = \mathbf{0} \\ & -\underline{r} \leq Y\zeta \quad \& \quad Y\zeta \leq \overline{r} \end{aligned} \qquad \overline{\epsilon} \qquad (30)$$

where,

$$0 < \epsilon < 1 \ (\overline{\epsilon} := 1 - \epsilon),$$
$$\mu_i \in \{0.08437, 0.085105, 0.050705, 0.10279\},$$
$$\underline{\zeta}_i \in \{-0.2313, -0.2305, -0.1705, -0.1110\},$$
$$\overline{\zeta}_i \in \{0.3999, 0.4007, 0.2719, 0.3166\}, \text{ and}$$
$$\zeta \in \{0.2367, 0.2325, 0.1774, 0.1321\}.$$

The numerical results are calculated by Julia tool (with Gurobi optimiser) and the output file contains convex combination of the vertices.

4. Discussion

4.1. Numerical Results

To better show the high potential outcome of the interval and the contamination numerical results, we focus on three cases.

Case (i) where the epsilon is very close to zero. In this case, the contamination solution coincides with the purely probabilistic uncertain model (normal distribution) solution. When $\epsilon = 1$ then the imprecise part of contamination model is zero and we have a pure precise case. In this case, the maximin and maximal solutions are the same as $(*)$ in the solutions proposed in Section 3.2.1. In this case, the cost is equal to 15535.72 which is almost half of the interval case solution 29184.94 discussed in Section 3.1.1. Case (ii) where the epsilon is very close to zero. In this case, the contamination solution coincides with the interval solution. When $\epsilon = 0$ then the precise part of the contamination model is zero and we have a pure imprecise case. In this case, the maximin and maximal solutions are equal to the interval solutions discussed in Sections 3.1.1 and 3.1.2. We have discussed that maximal solutions are more optimal and riskier than the maximin solution. The most optimal solution has the highest risk, and the cost value is 21130, while the lowest risky solution has the cost value of 29184.94.

Case (iii) where the epsilon is between zero and one. In this case the contamination maximin (maximal) solution(s) given in Section 3.2.1, is defined based on the selected ϵ value. The cost value in this case is between 15535.72 and 29184.94. For instance, if a decision maker chooses $\epsilon \in (0, 0.5)$, e.g., $\epsilon = 0.35$, then he trusts less (%35) to the probabilistic model and %65 to the imprecise model. The cost value with $\epsilon = 0.35$ is equal to $15535.72 \times 0.35 + 29184.94 \times 0.65 = 18970.2 + 5437.5 = 24407.7$ which is lower than the interval case and higher than the pure probabilistic case.

In other words, the decision-maker has more freedom and measures (for risk and optimality) to decide either less (more) risky solutions or less (more) optimal solutions based on the other preferences/conditions such as economic situation, weather status, safety measure, social status, and many more.

4.2. Comparison

In this paper, we focus on providing two sets of solutions based on two decision criteria under the imprecise models (imprecise decision approach). These imprecise models which are presented in this paper are rather new in the DA-Energy and Reserve Dispatch Problems. To measure and quantify the imprecise uncertainty we use (coherent) lower and upper previsions which is one of the advanced measures to deal with the imprecision. For reasoning we have implemented the imprecise decision approach. As a comparison to other methods dealing with uncertainty models for instance combined hybrid uncertainty model proposed in [36] we have the following arguments. The approach in [36] is based on propagation of uncertainty (the sensitivity analysis) and mainly focuses on the classical interval arithmetic rather than providing direct (less/more conservative) solutions, see Equation (15) in [36]. Although, the Contamination model is another Hybrid or even called Mixed Model to contaminate non-probabilistic with probabilistic models via choosing proper tuning factor (ϵ).

Contamination model is a probabilistic model when the epsilon is one which is out-scope because $0 < \epsilon < 1$. In general, from the definition, the Contamination model is convex combination of precise and imprecise uncertainty models which in this paper we assumed a normal distribution for precise case and the interval for imprecise case. One of our future work will be using other distributions in the Contamination model, e.g., Weibull model, or more advanced and more informative Probability Box model for the imprecise part of the contamination model.

Another argument is to the interval rough number which is more commonly used in decision theory [37,38] rather than the pure interval variables proposed in this paper. Generally, the interval rough number is based on the interval arithmetic which usually

used for propagating the uncertainty, while in our approach we use the imprecise decision approach [24,25] to quantify and measure the imprecise uncertainty. We use two imprecise decision criteria for reasoning about the solutions (is not propagating of the uncertainty). More specifically, the problem which we have focused in this paper is a 161×84 linear system with a symbolic linear cost function where for the propagation method/s would be too hard problem to be considered.

5. Conclusions

In this paper, we envisage the use of non-probabilistic uncertainty modelling techniques such as interval model and more advanced, ϵ-contamination model (a mixture of non-probabilistic and probabilistic models) for solving the day-ahead energy and reserve dispatch problem under renewable generation uncertainty. We provide the novel theoretical background required for solving this problem under two criteria, i.e., maximum and maximal criteria via more generic advanced theory—imprecise decision theory. We solve the problem and discuss the potential trade-off between reliability risks and optimal costs of day-ahead scheduling decisions. We observe that the decision-maker can achieve lower costs by endorsing a riskier attitude. The risk is defined by the distance from the worst-case solution via L^1 norm. Furthermore, by using these advanced models we consider imprecision in the uncertainty rather than a unique distribution from the classical uncertainty models. An additional interesting result is to help the decision-maker to select the optimal decision set within the available solutions. This method can be related to a given risk threshold, and therefore improve the applicability of the proposed approach.

As prospect to the current work, Probability Box (P-BOX) uncertainty modelling technique [39], which is also an advanced and more informative model, may allow the decision-maker to account for the uncertainty in the input distribution function to hedge against erroneous distributions. Since we are focusing here on more advanced models like Contamination which we need a probabilistic (non-probabilistic) model(s) we used the simple Normal Distribution case. However, we will be applying more advanced uncertainty using, e.g., Weibull Distribution for our future work.

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Data Availability Statement: We applied our proposed approach to the case study inspired by the 24-node IEEE reliability test system data. We used the adapted version of the IEEE 24-node reliability test system data, which the data can be found in this link https://drive.google.com/file/d/0Bzq9B9 vW0gM0enphc05kWXBtWHc/edit?usp=sharing [30].

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Abbreviations

The following abbreviations and symbols are used in this manuscript:

DA	day-ahead
LP	linear programming
S-LP	symbolic linear programming
LPUU	LP under uncertainty
sup	supremum
inf	infimum
DRO	distributionally robust optimisation
$\mathcal{G}(n)$	set of generators (connected to bus n)

$g \in \mathcal{G}$	generator
$\mathcal{D}(n)$	set of loads (connected to bus n)
$d \in \mathcal{D}$	load
$\mathcal{W}(n)$	set of wind farms (connected to bus n)
$w \in \mathcal{W}$	wind farm
c_g	production cost of generator g [€/MWh]
$c^T p$	cost of energy generation
$c \in \mathbb{R}^{|\mathcal{G}|}$	the generation costs in \$/MWh
$p \in \mathbb{R}^{|\mathcal{G}|}$	the energy generation from the generators in MWh
$\bar{c}^T \bar{r} + \underline{c}^T \underline{r}$	the cost of procuring upward/downward reserve capacity
$\bar{c}, \underline{c} \in \mathbb{R}^{|\mathcal{G}|}$	the upward/downward reserve procurement costs in \$/MW
$\bar{r}, \underline{r} \in \mathbb{R}^{|\mathcal{G}|}$	the reserve procurement of the generators in MW
$c^T Y \xi$	the real-time cost of balancing actions
$Y \in \mathbb{R}^{|\mathcal{G}| \times |\mathcal{W}|}$	participation factor of each generator per-unit uncertain renewable energy deviation
$\xi \in \mathbb{R}^{|\mathcal{W}|}$	day-ahead forecast
$Y \xi$	the balancing action in real-time of the generators in MW
$X := \{p, \bar{r}, \underline{r}, Y\}$	scheduling decision parameter
$\{p, \bar{r}, \underline{r}, Y\}$	the set of decision variables

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