Some proposals for the solution of the Carnap-Popper discussion on 'inductive logic' (*)

1. The explicata (1).

The explicata for 'degree of confirmation (corroboration)' proposed by Carnap and Popper may be described as follows:

a. Carnap's **c**-function is, as he defines himself, the relative logical probability of a hypothesis *h*, given an evidence *e*.

$$\mathbf{c}\ (h,\ e) = \mathbf{m}\ (h,\ e)$$

or, more generally

$$\mathbf{c}\ (h,\ e) = \mathbf{P}\ (h,\ e)$$

b. Popper's **C**-functions (as his **E**-functions) in their not-relativised formulation are relations between the absolute and the relative logical probability of the hypothesis *h*, given an evidence *e*. Perhaps one wonders at this. Indeed, Popper is continually defining his **C**- and **E**-functions in terms of the absolute and the relative logical probability of an evidence *e*, given a hypothesis *h*. However from his definitions

$$\mathbf{E}\ (h,\ e) = \frac{\mathbf{P}\ (e,\ h) - \mathbf{P}\ (e)}{\mathbf{P}\ (e,h) + \mathbf{P}\ (e)}$$

$$\mathbf{C}\ (h,\ e) = \mathbf{E}\ (h,\ e)\ \ [\,1 + \mathbf{P}\ (h)\ \mathbf{P}\ (h,\ e)]$$

$$\mathbf{C}'\ (h,\ e) = \frac{\mathbf{P}\ (e,\ h) - \mathbf{P}\ (e)}{\mathbf{P}\ (e,\ h) + \mathbf{P}\ (e) - \mathbf{P}\ (e.\,h)}$$

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(1) The most systematic explanation of the **c**-function can be found in Carnap's *Probability*, Carnap's actual views however do no more completely coincide with the contents of this book. For Popper's **E**- and **C**-functions see e. g. his *Logic.*

it is easily to prove that

$$\mathbf{E}\ (h,\ e) \quad = \quad \frac{\mathbf{P}\ (h,\ e) - \mathbf{P}\ (h)}{\mathbf{P}\ (h,\ e) + \mathbf{P}\ (h)}$$

$$\mathbf{C}\ (h,\ e) \quad = \quad \frac{[\mathbf{P}\ (h,\ e) - \mathbf{P}\ (h)]\ [1 + \mathbf{P}\ (h)\ \mathbf{P}\ (h,\ e)]}{[\mathbf{P}\ (h,\ e) + \mathbf{P}\ (h)]}$$

$$\mathbf{C}'\ (h,\ e) \quad = \quad \frac{\mathbf{P}\ (h,\ e) - \mathbf{P}\ (h)}{\mathbf{P}\ (h,\ e) + \mathbf{P}\ (h) - \mathbf{P}\ (h,\ e)\ \mathbf{P}\ (h)}$$

It is even more clear that the alternative **E**-function

$$\mathbf{E}_1\ (h,\ e) \quad = \quad \frac{\mathbf{P}\ (e,\ h)}{\mathbf{P}\ (e)}$$

can be written as

$$\mathbf{E}_1\ (h,\ e) \quad = \quad \frac{\mathbf{P}\ (h,\ e)}{\mathbf{P}\ (h)}$$

This **E**-function will be no more examined in this article, because this **E**-function does not seem to allow to define an elegant $\mathbf{C}_1$ (h, e) ([2]).

2. Disadventages of the explicata.

Both explicata have two striking groups of 'disadvantages'.

a. Carnap's **c**-function leads to the paradoxical implication that a hypothesis h may be disconfirmed by a given evidence e, while an other hypothesis h' may be confirmed by the same e, and that nevertheless $\mathbf{c}\ (h', e) > \mathbf{c}\ (h, e)$. This disadvantage is noted by Popper ([3]) and proceeds from the fact, that the absolute logical probability of a hypothesis may have decisive influence on his relative logical probability, and hence on his **c**-value. The second group of disadvantages of Carnap's explicatum consists of a set of consequences from the properties of almost-L-true and almost-L-false sentences.

b. The first group of disadvantages of Popper's **C**-functions is a counterpart of the first group of Carnap's, and proceeds from the fact, that the **C**-values of hypotheses are in many cases not influenced at all by the absolute probability of these hypotheses. So it seems intuitively desirable to accept that a disjunction of two well-corroborated hypotheses is higher corroborated than each of these hypotheses taken apart. A gambler will pre-

(2) The function $\mathbf{C}_1\ (h, e) = \mathbf{E}_1\ (h, e)\ [\ 1 + \mathbf{P}\ (h)\ \mathbf{P}\ (h, e)]$ which may be defined from the last **E**-function, is ranging from 0 to $+\ \infty$, and provides the value $1 + \mathbf{P}^2\ (h)$ for neutral evidence ($\mathbf{P}\ (h) = \mathbf{P}\ (h, e)$).

(3) Popper: *Logic*, pp. 390 ff.

fer to bet at a given ratio on two possible outcomes rather than to bet at the same ratio on one of them only, and a doctor will prefer to count with as many hypotheses as possible in curing a dying patient. If they followed Popper's **C**-functions, they would find an other result, because *in most cases* (see below) $\mathbf{C}(h, e) > \mathbf{C}(h \vee h', e) < \mathbf{C}(h', e)$. The second group of disadvantages of the **C**-function rises from the property:

$\mathbf{C}(h, h) = 1 - \mathbf{P}(h)$.

This leads to the unacceptable result that a confirmed hypothesis is always corroborated to a higher degree than a lot of less general, but verified (and hence true) hypotheses.

In this article will be argued:

a. that two different explicanda are confounded in the intuitive 'degree of confirmation', and that the first group of disadvantages of the **c**-function as well as the first group of disadvantages of the **C**-functions are necessary properties of the different explicanda, which the explicata are trying to seize.
b. that the second group of disadvantages of the **c**-function are disadvantages that may not be repaired by the (qualified-) instance-confirmation nor by Hintikka's α-parameter, but that are perfectly repaired by Kemeny's proposal (asymptotic values).
c. that the second group of disadvantages of the **C**-function are genuine disadvantages which are confusion-bearing and which may not be repaired at all. To overcome this, an alternative function (the **K**-function will be proposed.

3. Rejection of deductivism.

The comparison of Carnap's and Popper's proposals becomes much simpler, if deductivism is rejected and if all factual knowledge up to a given time is used as evidence in calculating **C**-values. As there are indeed arguments against deductivism, it appears preferable to start with these.

3.1 In Popper's proposal (4) to calculate **C**-values, aprioristic arguments are involved.

Indeed the calculation of values of $\mathbf{P}(e)$ cannot be based on merely deductive arguments, but is only possible by a (necessary aprioristic) choice of a distribution of absolute logical probabilities.

3.2 Popper's metric is not general enough to calculate **C**-values.

(4) Popper: *Logic*, pp. 410 ff.

In Popper's text only a method for calculating **E**-values is given. Indeed there is not mentioned how **P** (h) and **P** (h, e) may be determined, and to determine them is necessary for arriving at **C**-values. Here in turn an aprioristic factor has to be introduced.

Let it be mentioned, that even the calculation of **P** (e, h), which Popper seems to present as determined by the merely analytic probability calculus, is not always possible in this way. If for example the hypothesis is a disjunction of statistical hypotheses (5), then the absolute probabilities of the single hypotheses that compose the disjunction, must be determined. These problems rise *a fortiori*, if different hypotheses are compared.

3.3 Popper's metric for absolute probabilities partly coincides with Carnap's **m***.

Popper takes a 'Laplacian distribution' for determining **P** (e). The calculation of **P** (e, h) for him is purely deductive (6).

It is clear that, starting from Carnap's system of state-descriptions, a statistical hypothesis, or a statistical report about observed facts, turns out to be a *structure-description* (or a disjunction of structure-descriptions). If **m*** (7) is choosen, then we arrive at the same result as Popper with his metric. Indeed **m*** (8) gives equal weights to structure-descriptions.

3.4 Popper's argument based on Bernouilli's 'law of great numbers' leads to results opposed to his own theory.

Popper is using neither the evidence e as such nor a statistical report of it, say e', but a disjunction of statistical hypotheses, say e^*. This however is contrary to Popper's very valuable requirements concerning *precision* (9).

The following elements play a part in the argument where Popper uses Bernouilli's law.

a. $\mathbf{P}\ (e_1^*, h) = p_1$

The p_1 must be nearly to 1, in order to make $[\mathbf{P}\ (e_1^*, h_1) - \mathbf{P}\ (e_1^*)]$ great.

(5) Such hypotheses are :

$(x)\ [\ \mathbf{p}\ (R\ (x)) \geqslant r]$

$(x)\ (y)\ [\ R\ (x) \supset [\ r \leqslant \mathbf{p}\ (S\ (x)) \leqslant s.]]$ wherein $\mathbf{p}$ denotes an objective probability.

(6) As explained above this cannot be always the case.

(7) To-day **m*** is no more defended by Carnap, because it is clearly inadequate for Carnap's ends, cfr. Carnap & Stegmüller : *Inductive*, pp. 251-252.

(8) **m*** attributes the same probability to *structure-descriptions*.

(9) If a sample has a width n, m elements have the property A and $m/n = r$, then the statistic report is : nr elements out of the sample (with width n) are A. Popper however does not use nr but $n\ (r \pm k)$, in order to apply Bernouilli's law.

b. e_1^* : n $(r + k)$ individuals of the sample (with size n) have a given property A. The lower the value of k, the lower that of **P** (e_1^*) and hence, the higher that of [**P** (e_1^*, h_1) — **P** (e_1^*)].

Suppose:

h_1 : r (< 1) of the individuals of the universe are A.
e' (the *statistical* report) : nr individuals of the sample are A.
Clearly h_1 is the hypothesis which is most 'confirmed' or 'corroborated' by e'.

Now suppose:

h_2 : s (< 1) of the individuals of the universe are A.
e_2^* : n $(s \pm l)$ individuals of the sample (with size n) are A.
$(s + l) > r > (s - l)$

Clearly, it is possible to choose such a s and a l, that:

$$\mathbf{P}\ (e_2^*, h_2) = p_2 = p_1$$
$$l = k$$

and consequently:

$$[\mathbf{P}\ (e_1^*, h_1) - \mathbf{P}\ (e_1^*)] = [\mathbf{P}\ (e_2^*, h_2) - \mathbf{P}\ (e_2^*)]$$

This means that, if neither e nor e' but a e^* is used, then it depends on the particular choice of the e^*, which hypothesis may reach the highest **C**-value.

There is no logical reason to choose a disjunction of statistical reports rather than another. Indeed

$$n\ (r + k) > nr > n\ (r - k)$$
$$n\ (s + l) > nr > n\ (s - l)$$

and hence

$$e' \supset e_1^*$$
$$e' \supset e_2^*$$

As there is no logical reason to prefer a disjunction of reports, rather than another such disjunction, there is no reason to prefer one of the (in principle infinite number of) hypotheses which may reach the highest **C**-value. This leads to the acceptance of a *disjunction* of hypotheses instead of to the acceptance of a *more precise* hypothesis.

On the other hand, if Popper uses the report as such or even the report in its statistical form, it can be *proved* that, even for the most ideal e', **P** (e', h) will decrease with the increasing width of the sample.

This conclusion is not catastrophic for the **C**-function because, if the report itself or if the statistical form is used, **P** (e, h) or **P** (e', h) will decrease, but much slower than **P** (e) or **P** (e') will do. Hence the **E**- and **C**-values may increase, if the size of the sample grows. This however underlines the importance of the choice of an *a priori* distribution.

3.5 The argument concerning dependence, which Popper uses against $\mathbf{m}^*$ makes defect.

As Popper notes, the laws of the abstract probability-calculus must stay in inductive logic. He argues, that the independence of single events does not hold, if Carnap's $\mathbf{m}^*$-function is accepted. This arises from a confusion.

What is meant by the logical independence in the abstract calculus is that, given a definite distribution, the fact that an event took place does not influence the probability of an other event. Now this holds very well in Carnap's system for all **m**-functions of the λ-continuum.

Suppose we are tossing a die. To say that a definite outcome does not influence the probability of the outcome of the following throw, means that, *given the true distribution* (depending on properties of the die), the *objective* probability of some outcome or other of the following throw, stays the same, and namely, the *objective* probability determined by the distribution. If in Carnap's system the distribution (a structure-description or a disjunction of structure-descriptions) is given, say d, only then the logical probability-value **c** (h, d) corresponds with the *objective* probability of h, and *is independent of the forgoing outcomes.* If however one does not know the true objective distribution, but has only a report e of outcomes of preceding tosses, then **c** (h, e) has *only* to determine a *logical* probability. This *logical* probability may depend on preceding outcomes, even if the *objective* probability does not.

3.6 Hume's argument holds against $\mathbf{m}^+$ (10) as against $\mathbf{m}^*$.

Each regular **m**-function has the property set forth in 3.5. On the other hand, each such **m**-function states a definite dependence (one of them being independence).

However there seems not to be any logical reason for choosing one of them, and hence every particular choice is aprioristic. Consequently Hume's argument may be used alike against each particular choice.

3.7 An inductive **m**-function is needed in order to reach adequate **C**-functions.

Let h be a hypothesis about only future facts. If $\mathbf{m}^+$ is choosen, then always:

$$\mathbf{P}\ (h) = \mathbf{P}\ (h, e)$$

(10) $\mathbf{m}^+$ attributes the same probabilities to all *state-descriptions.*

consequently

$$\mathbf{C}\ (h,\ e) = 0$$

This means that the hypothesis is not 'corroborated' at all by e, whatever it may be. Hence the **C**-function leads *not* to acceptable results, if $\mathbf{m}^+$ is choosen.

An analogous inacceptable result holds even for hypotheses describing the whole universe (and not only the not observed part). Take a lot of hypotheses which assert that what took place, had to be taken place, and which assert further whatever about the future. All of these hypotheses, which have a same absolute probability, have also a same **C**-value, and the **C**-values differ only in as much as the absolute probabilities differ.

So for example, let e be : 'the hundred up to now observed individuals had property R', let h_1 be : 'all individuals have the property R', and let h_2 be : 'the first hundred individuals have the property R, all the others the property $\overline{\mathrm{R}}$'.

Clearly :

$$\mathbf{m}^+\ (h_1) = \mathbf{m}^+\ (h_2)$$

$$\mathbf{c}^+\ (h_1, e) = \mathbf{c}^+\ (h_2,\ e)$$

hence it follows

$$\mathbf{C}\ (h_1,\ e) = \mathbf{C}\ (h_2,\ e)$$

This example also demonstrates that an inductive **m**-function is needed to arrive at adequate **C**-values.

Popper may argue, that such hypotheses may not be choosen. This rule is not deductively founded and hence, even if $\mathbf{m}^+$ was merely deductive, what it is not, Popper would need aprioristic rules in order to reach adequate **C**-values.

3.8 What was criticised before, was the more 'logical' aspect of deductivism, i. e. the thesis that science proceeds by a merely deductive method of testing. Another aspect of deductivism is the requirement that only observations, which took place after the formulation of the hypothesis, may be used as evidence. However once the logical aspect is rejected, no serious arguments in favour of the second aspect seem to stay.

Furthermore, the second aspect may be criticised on his own. So for example, it does not seem acceptable at all, that a hypothesis should have a lower value, only because it was formulated later. Such arguments are in favour of the supposition, that Popper defends the second aspect of deductivism only in order to safe the 'logical' one.

Whatever may be, Popper's **C**-functions and many of his opinions may be very usefull, even if deductivism is rejected. This last point will become clearer below.

4. The intuitive explicanda : $conf_1$ and $conf_2$

The two explicanda that are involved in the intuitive concept of 'degree of confirmation' would be the following.

a. $conf_1$: an explicandum concerning *degrees of certainty*. Suppose one is asserting that a definite well-limited sociological hypothesis, which is supported by a set of factual materials, is *better confirmed* than an other sociological hypothesis which is also supported by factual materials, but which is so general, that the evidence is surely not to be regarded as conclusive. Here the intuitive 'better confirmed' stands for an expression about certainty.

   However, there are other cases where such expressions can hardly be translated in terms of confirmation. So it seems paradoxical to speak about '*a priori* degrees of certainty' as about degrees of confirmation. E. g. it sounds contra-intuitive to say that, if we know nothing about the planet Mars, except the fact that it exists, then the sentence 'there are living beings on Mars' should be *confirmed* to a degree of 1/5. The same holds, ascribing degrees of confirmation to predictions about singular events ; e.g. : the sentence 'the outcome of the following throw with this die will be a six' is confirmed to a degree of 1/7. The reason that these and analogous expressions sound contra-intuitive depends on the way one's intuition is confounding the two confirmation concepts. In areas where both have the same properties the two are used at the same time, in other areas only one of them is used. Consequently the contradictory word-usage does not become shown, before exact explicata are constructed.

b. $conf_2$. This explicandum is dominated by a classifying viewpoint : the fact that a hypothesis may be confirmed, neutrally confirmed or disconfirmed. Starting from this, the concept is made comparative in this way, that one hypothesis is said to be *more confirmed* than another, if it is 'more supported by facts', *whatever the a priori degrees of certainty of these hypothesis may be.* Consequently, disconfirmed hypotheses always have a lower $conf_2$-value than confirmed ones, and a hypothesis, whose low (actual) $conf_1$-value is caused by its low *a priori* $conf_1$-value, may have an higher $conf_2$-value than another hypothesis with higher $conf_1$-value.

This confirmation-concept is viewed if one says, that (universal and very general) physical theories to-day are more *confirmed* than most (very narrow) psychological ones, even if these have only a numerically limited application field. Here also, the confusion with the other explicandum produces a number of paradoxical results, e.g. concerning the $conf_2$-value of disjunctions of hypotheses.

Now probabilities will be examined. The first explicandum can clearly be reconstructed as a (relative) logical probability. An explicatum for $conf_2$ has to be relation between the absolute logical probability of hypotheses and their relative ones, given an evidence. That $conf_2$ of h_1, given e_1, is greater than $conf_2$ of h_2, given e_2, means indeed that the probability of h_1 is *more increased* by e_1 than the probability of h_2 is by e_2. 'More increased' means that the 'distance' between $\mathbf{P}$ (h_1) and $\mathbf{P}$ (h_1, e_1) is greater than that between $\mathbf{P}$ (h_2) and $\mathbf{P}$ (h_2, e_2).

It is not difficult to understand why one's intuition confounds both explicanda. If the number of observed instances, which are permitted by a hypothesis, increases, the values of both $conf_1$ and $conf_2$ increase also. If hypotheses with the same '*a priori* $conf_1$-value' are compared, both explicanda are introducing the same order. Misled by such cases, one's intuition identifies two concepts, which lead to very different results in other areas.

As the reader has already understood, the author of this article is holding that Carnap's **c**-function is an explicatum for $conf_1$ and Popper' **C**-functions explicata for $conf_2$. Indeed Carnap's **c**-function is defined as a *relative probability* function (interpreted as a logical relative probability), while Popper's **C**-functions can, as was noted in a previous section, be defined as *relations between absolute and relative (logical) probabilities.*

Now there is much material which is worked out formerly and which can be used in favour of the value of Carnap's function as explicatum for the intuitive concept $conf_1$; e. g. what is worked out concerning rational betting-quotients. The situation is different in case of Popper's **C**-functions. There are many possible definable relations between absolute and relative probabilities and it is not immediately clear which of them may be a good explicatum for $conf_2$ or, which of them is a good explicatum for $conf_2$ and at the same time has a value as an instrument for scientific work.

5. About the possible usefulness of the explicata.

The last point of the preceeding section is very important. Just because of the confusion in one's intuition, intuitive explicanda can only be used as inspirative sources in constructing more or less useful, exact explicata. Consequently, a more important problem is, to answer what instruments are needed in science. Both Carnap and Popper will be in accord, that only this answer can be decisive in evaluating their proposals ; both indeed agree, and in recent times they formulated this very explicitly, on the only relative power of intuitive argumentations and evaluations.

The thesis to be set forth here is, roughly said, the following. Carnap is searching for an evaluating system which will be useful to decision making in *applied* science (as medicine, engineering, a. s. o.). Let us call such systems *PC*-systems. Popper, on the other hand, is searching for the analogous for *theoretical* science. Let us call such systems *TC*-systems.

For practical decisions (where science is applied) one needs knowledge about the *certainty* of hypotheses. On the other hand, for deciding on the choice of a hypothesis in theoretical science, one needs knowledge about the *content* of hypotheses and about the *certainty* we have, *with respect to that content* (to the absolute probabilities of the hypotheses).

Once we accept an *a priori* distribution for logical probabilities (a **m**-function), there can be little doubt that Carnap's **c**-function determines to what extent we may be *certain* about a hypothesis. In other words there may be little doubt that the **c**-function determines what we are justified to bet on the hypothesis. Surely it is clear that hypotheses, which have a high absolute probability, even if they are disconfirmed (in both senses) may nevertheless have a higher *PC*-value (relative probability) than confirmed hypotheses with lower absolute probability. This even has to be the case since a fair betting function is intended.

What is needed in applied science is just this. Surely also problems of utility play a part here, but they depend for a great deal on probabilities or degrees of certainty. Everywhere, the practical scientist is in search of increasing certainty as far as possible. In planning a bridge, the engineer calculates it so, that theoretically his construction should be able to withstand a higher weight than necessary. The same holds in cosmonautics etc. From this point of view the first group of disadvantages of the **c**-function disappears.

The relation between $conf_1$, *PC*-functions and Carnap's **c**-function does not seem to need much discussion ; below one will find more evidence to assume the **c**-function as a good *PC*-function.

Popper's **C**-functions on the other hand can clearly not be used as *PC*-functions ; **C**-values of confirmed hypotheses are always higher than those of disconfirmed ones, irrespective of the relative probability of the hypotheses. If **C**-functions were nevertheless interpreted as *PC*-functions, this would lead to the paradoxes described in section 2. as the first group of 'disadvantages' of Popper's **C**-functions.

Now *TC*-functions will be considered. The theoretical scientist likes to formulate hypotheses which are very general, i. e. which have a high *content*, and hence low absolute logical probability. There is no doubt that this old thesis of Popper's is right. It should however not be forgotten, that science likes hypotheses, which have not only high content, but which are also well supported by factual evidence. Thus, in evaluating hypotheses from a theoretical point of view, a scientist should look both at content and at relative probability; if an adequate explicatum is possible here, it must be defined as a *relation between the absolute probability* of hypotheses and their *relative probability*, with respect to an evidence.

As noted in section 1., Popper's **C**-function is such a relation. There must however be taken care; there are different kinds of relations between absolute and relative probabilities, which may have importance for decisions in theoretical science. In the light of what is worked out in this article two kinds of relations seem important.

Let the first kind be denoted as *TCa*-functions. These are functions which express how worthy a hypothesis is to be accepted, given its content and its relative probability. Such functions must have at least the following properties:

a. with respect to tautological evidence, hypotheses with a higher content must receive a higher value,
b. the same must hold for verified hypotheses,
c. the value of a hypothesis must increase, if the relative probability of the hypothesis increases, and decrease, if the relative probability decreases.

It is clear, that the **C**-functions cannot be *TCa*-functions; indeed they have the property

$\mathbf{C}(h, t) = 0 \qquad (t = \text{tautology})$

and so, they do not fulfill condition a. In section 7. other properties of the **C**-function will be found that are contradictory with desiderata for *TCa*-functions.

Let *TCb* functions be a second kind of relations between absolute and relative probabilities. These functions only denote 'confirmation'-aspects. They denote what is the value of hypotheses with respect to their being supported by facts, i. e. with respect to the measure in which their logical probability is increased or lowered. *TCa*-functions were at the same time judging the content and the 'confirmation'-value. *TCb*-functions on the other hand only express how valuable a hypothesis is made by observed facts, which have altered the relative probability of the hypothesis. *TCb*-functions say nothing about the content of the hypothesis in a direct way.

Suppose a scientist says : 'this hypothesis is very well supported by facts but it has little scientific value, while it is so narrow'. The first part of this sentence is typically a *TCb*-expression, meaning that the probability of the hypothesis is much increased by facts. The second part says, that nevertheless the hypothesis has an absolute probability which is too high, and hence a content which is too low to be a scientifically important hypothesis.

TCb-expressions only take into account the question whether the probability of a hypothesis is incrased or lowered. However, in doing this, the content factor reappears, but only indirectly. Indeed, the sentence 'the probability of a hypothesis h_1 is more increased than that of an other one h_2', means that the 'distance' between $\mathbf{P}\ (h_1)$ and $\mathbf{P}\ (h_1, e)$ is greater than the distance between $\mathbf{P}\ (h_2)$ and $\mathbf{P}\ (h_2, e)$; the distance however cannot be measured adequately by merily distracting one probability from the other, but must necessarily be a *relative* distance, and namely relative to the value of the absolute (or of the relative) probabilities. Consequently *TCb*-functions build a well-limited class of relations between absolute and relative probabilities.

The *TCb*-functions must have property c. of the *TCa*-functions, but for all hypotheses the *a priori* values (i. e. the values with respect to tautological evidence) must be identical ; the analogous must hold for the values of verified hypotheses. Indeed, the fact that these values should depend on the content of the hypotheses, would not only be superfluous, but also very misleading. It should be remarked that all these properties fit very well with $conf_2$, whereas the properties of *TCa*-functions clearly do not.

Popper's **C**-functions make defect as *TCb*-functions because of

$$\mathbf{C}\ (h, h) = \mathbf{P}\ (\bar{h})$$

At the same time they have the property

$$\mathbf{C}\ (h, t) = 0$$

and a lot of other ones (see below), which are necessary or at least compatible with their being *TCb*-functions ; so for example, the first group of 'disadvantages' of the **C**-functions are necessary properties of *TCb*-functions.

It needs not be remarked that Carnap's **c**-function cannot be a *TCb*-function (nor evidently a *TCa*-function). In defending Carnap's **c**-function against Popper's attacks, Bar-Hillel argued (11), that the scientist (clearly the theoretician) is searching theories with high **c**-values (relative to the evidence of that time), *and* with low absolute logical probability.

(11) Bar-Hillel : *Comments*, p. 156.

Against this can be objected, that it is always possible to choose a h_2 such that

$$\mathbf{c}\ (h_1 \vee h_2,\ e) > \mathbf{c}\ (h_1,\ e)$$

and thus Bar-Hillel's argument is missing a point. This indeed leads to the search for an adequate *TCa*-function, which Carnap's is not. There. should however also be noted, that the **c**-function is not a *TCb*-function Theoretical scientists search for theories whose relative probability is much increased with respect to their absolute probability (hence theories with high *TCb*-values), and not theories that have high **c**-values (all hypotheses of e. g. physics have indeed the **c**-value zero). In this respect Popper's criticism is well founded.

This objection should be made against Bar-Hillel as well as against Kemeny, where he writes (12), that Carnap is interested in " ... the determination of whether we are scientifically justified to accept the hypotheses on the given evidence ". Perhaps Kemeny sees very well the difficulty that a *TCb*-function is needed, because he writes : " ... after investigating the *content* of this formula, Einstein decided that the available evidence made it sufficiently *probable* to accept it" (13). Indeed Kemeny can hardly not have seen that the probability in question differs only asymptotically (14) from zero.

If however the above interpretation of Kemeny's text is right, then one could wonder why Kemeny does not arrive at the conclusion that Carnap's theory fails at this point, a point which is of the greatest importance for theoretical science. Indeed if the question on *TCb*-functions is not resolved, Carnap's theory can serve the theoretician only in comparing hypotheses with identical absolute probability, and in a few other too limited cases.

6. The K-function, a proposal for an adequate *TCb*-function.

From the results of the preceding section it seems interesting to define a new explicatum for 'degree of confirmation' which (a) has not the disadvantages of the **C**-function, (b) has all properties for being a good explicatum of $conf_2$, and (c) has to be an adequate *TCb*-function.

(12) Kemeny in : Schilpp : *Carnap*, pp. 711-712.

(13) ibid. p. 712 (italics ours).

(14) Kemeny introduced asymptotic values for probabilities, in cases where Carnap's limit-procedure leads to zero (see below).

The author of this article has searched for such a function. The following proposal will be referred to as **K**-function and may in its most intuitive form be defined as

$$\mathbf{K}\ (h,\ e) =_{\text{Def.}} \frac{\mathbf{P}\ (e,\ h)}{\mathbf{P}\ (e,\ h) + \mathbf{P}\ (e,\ \bar{h})}$$

It can be proved that

$$\mathbf{K}\ (h,\ e) = \cfrac{1}{1 + \cfrac{\cfrac{1}{\mathbf{P}\ (h,\ e)} - 1}{\cfrac{1}{\mathbf{P}\ (h)} - 1}}$$

This function is thus, as well as the **C**-functions, a relation between the absolute and the relative probability of a hypothesis. It ranges from 1 to 0 and reaches the value of 1/2 in cases where $\mathbf{P}\ (h,\ e)$ equals $\mathbf{P}\ (h)$, i. e. in cases where the hypothesis is neutrally confirmed. In the following section its properties will appear more clearly.

7. Inquiry concerning some more technical problems.

7.1 The three explicata may be brought in relation with som non-ambiguous intuitive notions [15].
It is easy to prove that, if

$\mathbf{P}\ (h) \neq 0$
$\mathbf{P}\ (h) \neq 1$
$\mathbf{P}\ (e) \neq 0$

then

a. verification :

$$\mathbf{c}\ (h,\ e) = 1 \equiv \mathbf{C}\ (h,\ e) = \mathbf{P}\ (\bar{h}) \equiv \mathbf{K}\ (h,\ e) = 1$$

b. confirmation :

$$1 > \mathbf{c}\ (h,\ e) > \mathbf{P}\ (h) \equiv \mathbf{P}\ (\bar{h}) > \mathbf{C}\ (h,\ e) > 0 \equiv 1 > \mathbf{K}\ (h,\ e) > 1/2$$

c. neutral confirmation :

$$\mathbf{c}\ (h,\ e) = \mathbf{P}\ (h) \equiv \mathbf{C}\ (h,\ e) = 0 \equiv \mathbf{K}\ (h,\ e) = 1/2$$

d. disconfirmation :

$$\mathbf{P}\ (h) > \mathbf{c}\ (h,\ e) > 0 \equiv 0 > \mathbf{C}\ (h,\ e) > -1 \equiv 1/2 > \mathbf{K}\ (h,\ e) > 0$$

(15) For reasons explained above, deductivism is disregarded here. Consequently the c- and C-functions have the same arguments *h and e*.

e. falsification :

$$\mathbf{c}\ (h,\ e) = 0 \ \equiv\ \mathbf{C}\ (h,\ e) = -1 \ \equiv\ \mathbf{K}\ (h,\ e) = 0$$

Hence, if it is remarked thar verification means $\mathbf{C}\ (h,\ h) = \mathbf{P}\ (\bar{h})$, it may be said that the three functions lead in these cases to analogous results.

The cases where $\mathbf{P}\ (e) = 0$ or where h is logically true or false, are not important enough to be considered here. In case where h is almost-L-true or almost-L-false, the **K**- and the **C**-functions do not become undefinite but lead to values, if a limit is introduced as a limit of **C**- or **K**-values and not as a limit of $\mathbf{P}\ (h_1)$ and $\mathbf{P}\ (h_1,\ e)$ separetely, or if $\mathbf{P}\ (h)$ and $\mathbf{P}\ (h,\ e)$ are expressed as asymptotic values.

The above given scheme stays holding, if it is a little modified. In the case of verification e. g., $\mathbf{c}\ (h,\ e)$ must be really 1 and may not differ asymptotically from this value ; in the case of confirmation, it suffices that

$$\lim.\ \frac{\mathbf{c}\ (h,\ e)}{\mathfrak{m}\ (h)} > 1$$

which may be the case, even if both tend to zero ; a. s. o. In general the values of the **c**-function must be calculated as asymptotic values instead of using the limit-procedure.

The above constations are important for the following points :

a. the scheme helps to clarify why $conf_1$ and $conf_2$ are so easily confounded in one's intuition.

b. the **C**-functions give with respect to $conf_2$ and *TCb*-functions inadequate values in the case of verification ; they give inadequate values with respect to *TCa*-functions in the case of neutral confirmation.

c. the **K**-function may be a good explicatum for $conf_2$ and an adequate member of the class of *TCb*-functions.

7.2 Now some other similarities of the **c**-, **C**- and **K**-functions will be regarded.

In case of

$$\mathbf{P}\ (h_1) = \mathbf{P}\ (h_2)$$

it can be proved that

$$\mathbf{c}\ (h_1,\ e_1) \gtreqless \mathbf{c}\ (h_2,\ e_2)$$

is equivalent with

$$\mathbf{C}\ (h_1,\ e_1) \gtreqless \mathbf{C}\ (h_2,\ e_2)$$

and with

$$\mathbf{K}\ (h_1,\ e_1) \gtreqless \mathbf{K}\ (h_2,\ e_2)$$

This theorem means that, if two hypotheses with equal **m**-value are compared, then the three explicata lead to the same order, and this even in the case each hypothesis is related to a different evidence. The same holds *a fortiori*, if the values of only one hypothesis are compared with respect to different evidences. Consequently if a hypothesis reaches a higher degree of certainty, then it also receives a higher confirmation-value from a theoretical point of view (at least if a **C**- or the **K**-function is an adequate *TCb*-function). This sets forth an important relation between both groups of explicata.

Comparing the values that the three explicata attribute to structure-descriptions (as statistical hypotheses), given one or other evidence, one will see that in most cases the same order is introduced by the explicata. Furthermore in many cases an analogous order is introduced by Fisher's likelihood-function (**c** (e, h)). This may explain why acceptable results may be reached in statistical practice.

7.3 There are some remarks to be made about $\mathbf{P}(h) = 0$ and $\mathbf{P}(h) = 1$. Indeed there reappears the old problem for the **c**-function concerning almost-L-true and almost-L-false sentences. If the **c**-function is an adequate explicatum for $conf_1$ and an adequate *PC*-function, then it must have the properties of a rational betting-quotient.

This is a reason to reject the (qualified-) instance-confirmation [16] and Hintikka's introduction of the α-parameter [17]. Indeed none of them permits to arrive at fair betting-quotients [18].

Furthermore, it must be objected against the (qualified-) instance-confirmation :

a. it can be used only in case of general hypotheses and not in case of statistical ones.
b. it can be used only in case of hypotheses in L_∞ ; the same hypotheses in all L_N, with N great enough, will have the same value.
c. it can be used only in case of not falsified hypotheses ; otherwise these could receive a value near to 1.

Hintikka's solution also is hit by the second disadvantage. Furthermore it cannot be applied to statistical (non-general) hypotheses that are too complex [19].

(16) cfr. Carnap : *Probability*, pp. 573 ff.

(17) Hintikka : in : Hintikka & Suppes : *Aspects*, p. 133 ff.

(18) This does not mean that they may not have other uses.

(19) Too complex are those hypotheses (a) that have as denominator a number greater than the N of the L_N determined by α, and (b) with, at the same time, a denominator indivisible by the numerator.

However there is another solution, which seems to be a true one, namely that of Kemeny, introducing asymptotic values in the cases of almost-L-true and almost-L-false sentences [20]. This solution does not seem to have the disadvantages of the preceding ones and leads to fair betting-quotients. This is a solution for the second group of 'disadvantages' of the **c**-function.

7.4 Very clarifying is the examination of *quantitative generality*.
Suppose we have a hypothesis h_1 in L_N of the form

$(x)\ (A\ (x))$

which is the most confirmed one (in both senses) of the set of hypotheses [21] :

$(x)\ (\mathbf{p}\ (A\ (x)) = n)$ $\qquad (0 \leqslant n \leqslant 1)$

Suppose further :

h_2 : the same hypothesis in L_{N+M} $\qquad (M > 0)$

In this case holds

a. $\mathbf{P}\ (e, h_1) = \mathbf{P}\ (e, h_2) = 1$
b. $\mathbf{P}\ (h_1) > \mathbf{P}\ (h_2)$

and hence

c. $\mathbf{P}\ (h_1.e) = \mathbf{P}\ (h_1)$ — from (a)
d. $\mathbf{P}\ (h_2.\ e) = \mathbf{P}\ (h_2)$ — from (a)
e. $\mathbf{P}\ (h_1.\ e) > \mathbf{P}\ (h_2.\ e)$ — from (b), (c) and (d)
f. $\mathbf{P}\ (h_1, e) > \mathbf{P}\ (h_2, e)$ — from (e)
g. $\mathbf{c}\ (h_1, e) > \mathbf{c}\ (h_2, e)$ — from the definition

This is in accord with the concept of *PC*-function.
Furthermore :

h. $\mathbf{P}\ (e, h_1) - \mathbf{P}\ (e) = \mathbf{P}\ (e, h_2) - \mathbf{P}\ (e)$ — from (a)
i. $\mathbf{P}\ (e, h_1) + \mathbf{P}\ (e) = \mathbf{P}\ (e, h_2) + \mathbf{P}\ (e)$
j. $\mathbf{E}\ (h_1, e) = \mathbf{E}\ (h_2, e)$ — from (h) and (i)
k. $\mathbf{P}\ (h_1, e)\ \mathbf{P}\ (h_1) > \mathbf{P}\ (h_2, e)\ \mathbf{P}\ (h_2)$ — from (b) and (f)
l. $\mathbf{C}\ (h_1, e) > \mathbf{C}\ (h_2, e)$ — from (j), (k) and definition

m. $$\frac{\mathbf{P}\ (e, h_1) - \mathbf{P}\ (e)}{\mathbf{P}\ (e, h_1) + \mathbf{P}\ (e) - \mathbf{P}\ (e.\ h_1)} > \frac{\mathbf{P}\ (e, h_2) - \mathbf{P}\ (e)}{\mathbf{P}\ (e, h_2) + \mathbf{P}\ (e) - \mathbf{P}\ (e.\ h_2)}$$
from (h), (i) and (e)

n. $\mathbf{C}'\ (h_1, e) > \mathbf{C}'\ (h_2, e)$ — from (m) and definition,

(20) Kemeny : *Measure,* pp. 290-293.
(21) **p** denotes an objective probability.

Hence, according to both Popper's **C**-functions, the most confirmed hypothesis of a set (if it has the above specified form), reaches a lower value according as the number of individuals of the language has increased.

The **K**-function also has the same property.

o. $\dfrac{1}{\mathbf{P}\,(h_1, e)} - 1 < \dfrac{1}{\mathbf{P}\,(h_2, e)} - 1$ from (f)

p. $\dfrac{1}{\mathbf{P}\,(h_1)} - 1 < \dfrac{1}{\mathbf{P}\,(h_2)} - 1$ from (b)

Both h_1 and h_2 are confirmed. Hence their relative probabilities are higher than the absolute probabilities; consequently:

q. $$\frac{\dfrac{1}{\mathbf{P}\,(h_1, e)} - 1}{\dfrac{1}{\mathbf{P}\,(h_1)} - 1} < \frac{\dfrac{1}{\mathbf{P}\,(h_2, e)} - 1}{\dfrac{1}{\mathbf{P}\,(h_2)} - 1}$$

r. $$\frac{1}{1 + \dfrac{\dfrac{1}{\mathbf{P}\,(h_1, e)} - 1}{\dfrac{1}{\mathbf{P}\,(h_1)} - 1}} > \frac{1}{1 + \dfrac{\dfrac{1}{\mathbf{P}\,(h_2, e)} - 1}{\dfrac{1}{\mathbf{P}\,(h_2)} - 1}}$$ from (q)

s. $\mathbf{K}\,(h_1, e) > \mathbf{K}\,(h_2, e)$ from (r) and definition

These proofs, which may be supplied by other more general ones, learn that the **K**- and **C**-functions attribute lower values to (a lot of confirmed) hypotheses, according as they speak about numerically greater universes. This is a further argument demonstrating, that the **C**- and **K**-functions are not *TCa*-functions. Furthermore, it demonstrates that, from this point of view the desideratum $\mathbf{C}\,(h, h) = \mathbf{P}\,(\bar{h})$ does not help, that quantitatively more general hypotheses may reach lower values than others, and stay reaching lower values to the moment wherein the less-general hypothesis is verified, i. e. to the moment that as many individuals are observed as the less-general hypothesis speaks about. The results have also other consequences for some claims of Popper's. However this property of the **C** and **K**-functions is compatible with $conf_2$ and with the necessary properties of *TCb*-functions.

7.5 At least as important as the problem of *quantitative generality* is that of *qualitative generality*. This problem may be taken more generally in this sense that, within one single language L_N, all hypotheses may be studied with respect to their content. It is a well-known property of the probability-calculus that

$\mathbf{P}\ (h_1,\ e) \leqslant \mathbf{P}\ (h_1 \vee h_2,\ e)$

In most cases hypotheses with higher content (more-general hypotheses) have a lower **c**-value than hypotheses with higher absolute probability. This is a necessary consequence of the concept of rational betting-quotient.

In most cases **C**- and **K**-values become *lower*, if one passes from well-confirmed hypotheses with high content to other ones with lower content. This is in striking dissimilarity with the case of quantitative generality.

There are however exceptions to this general trend. Suppose e. g.

$\mathbf{P}\ (e) = 1/2$
$\mathbf{P}\ (h_1) = 1/8$
$\mathbf{P}\ (h_1,\ e) = 1/4$
$\mathbf{P}\ (h_2) = 1/8$
$\mathbf{P}\ (h_2,\ e) = 19/80$
$\vdash\ \overline{(h_1.h_2)}$

in this case [22]:

$\mathbf{C'}\ (h_1,\ e) = 4/11$
$\mathbf{C'}\ (h_2,\ e) = 24/71\ (< 4/11)$
$\mathbf{C'}\ (h_1 \vee h_2,\ e) = 76/\ 197\ (> 4/11)$

hence

$\mathbf{C'}\ (h_1 \vee h_2, e) > \mathbf{C'}\ (h_1,\ e) > \mathbf{C'}\ (h_2,\ e)$

The same holds for the **K**-values.
This is an example of the rule that, if two hypotheses have equal or only slightly different (positive) **C**-values, then the disjunction of the hypotheses has a higher **C**-value than each of them. The same holds for the **K**-function.

The fact that, in most cases, well-confirmed hypotheses that are more general, reach higher values, seems for $conf_2$ at least acceptable, and furthermore seems for *TCb*-functions very desirable. Indeed suppose a hypothesis that predicts that a set of facts (S_1) will take place and another one that predicts that the same set or a definite other one ($S_1 \vee S_2$) will take place. Now, if (S_1) is the case, it seems reasonable to say that the first hypothesis has to reach a higher *TCb*-value than the latter.

The cases wherein a disjunction of hypotheses reaches a higher value than each of both arguments of the disjunction, could be reconciled with the

(22) the same holds for the **C**-function.

concept of *TCb*-functions as follows. If two hypotheses have equally or nearly equally the same *TCb*-value, then it seems better not to conclude in favour of one of the two. The fact that the disjunction reaches a higher value could be seen as an indication of such cases.

This explanation may have some plausible aspects. However it must be noted, that a very arbitrary factor is involved here, namely the choice of a particular *TCb*-function. Comparing **C**- and **K**-values, one can note that the cases wherein the mechanism takes place, are not identical for both. Even if the **C**-function is rejected, it is clear that other *TCb*-functions than the **K**-function may exist, and so the problem remains.

7.6 A lot of theorems about relations between **C**- and **K**-functions can be constructed. So for example if:

$$\frac{\mathbf{P}\,(h_1 .\, e_1)}{\mathbf{P}\,(h_1)\,\mathbf{P}\,(e_1)} = \frac{\mathbf{P}\,(h_2 .\, e_2)}{\mathbf{P}\,(h_2)\,\mathbf{P}\,(e_2)}$$

then

$$\mathbf{K}\,(h_1, e_1) \gtreqless \mathbf{K}\,(h_2, e_2) \equiv \mathbf{C}\,(h_1, e_1) \gtreqless \mathbf{C}\,(h_2, e_2)$$

Where the implicans is an unequality, the implicatum is substituted by some implications. The author of this article did not detect theorems wherein the **C**-function became preferable to the **K**-function, nor theorems that might justify the criticised desideratum about maximum-**C**-values.

8. Conclusion

From what precedes it may be concluded:

a. that Popper's deductivism has to be rejected; the following conclusions however remain, even if deductivism is not rejected.
b. that Carnap's **c**-function may be a good *PC*-function, but that it is not a *TC*-function.
c. that Popper's **C**-functions are neither *PC*-functions nor *TCa*-functions, and that the **K**-function is to be preferred as *TCb*-function.
d. that it is difficult to evaluate a *TCb*-function, if the problem of the *TCa*-functions is not resolved.

A reader, who is not familiar with 'inductive logic' (in the widest sense), may perhaps get the impression that Carnap's contribution is not very important and that, on the other hand, Popper's contribution is very defective. He should realise however that the intention of this article was not to make a judgment about these contributions, but only to discuss some problems. Carnap's contribution lays on another level and is as such very important, *a fortiori* while *TC*-functions cannot at all be applied or even judged, if a *PC*-function is not worked out. On the other hand, it must be remarked that Popper's major thesis against the **c**-function, namely that it cannot be a 'confirmation'-function, is a right one, if he means that it cannot be a *TC*-function.

The author of this article can only hope to have cleared up some points in order to help to solve the Carnap-Popper discussion and to open some perspectives to the great lot of work that has to be done in inductive logic.

Diderik BATENS

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The italics denote the word that is used as reference in the text.

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