

# **Estimation of safety performance functions for urban intersections using various functional forms of the negative binomial regression model and a generalized Poisson regression model**

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## Highlights

- Multiple safety performance functions (SPFs) by crash severity are developed for urban intersections
- Various functional forms of the negative binomial (NB) regression and a generalized Poisson (GP) regression model are applied to develop the SPFs
- All the NB models and a GP model show promising results when estimating the SPFs
- On the basis of goodness of fit and predictive performance measures, the developed models are compared to choose a better model
- The performance of the NB-P model is better than the competing models for signalized intersections while the GP model outperforms other models for unsignalized intersections

# **Estimation of safety performance functions for urban intersections using various functional forms of the negative binomial regression model and a generalized Poisson regression model**

## **ABSTRACT**

Intersections are established dangerous entities of a highway system due to the challenging and unsafe roadway environment they are characterized with for drivers and other road users. In efforts to improve safety, an enormous interest has been shown in developing statistical models for intersection crash prediction and explanation. The advantage of statistical models is that they unveil important relationships between the intersection characteristics and intersection related crashes. Accurate estimates of crash frequency and identification of crash contributing factors guide safe design and help us implement policy interventions aiming for safety improvement. In this regard, the selection of the most adequate form of crash prediction model is of great importance for the accurate estimation of crash frequency and the correct identification of contributing factors. Using a six-year crash data, road infrastructure and geometric design data, and traffic flow data of urban intersections, we applied three different functional forms of negative binomial models (NB-1, NB-2, NB-P) and a generalized Poisson (GP) model to develop safety performance functions by crash severity for signalized and unsignalized intersections. This paper presents the relationships found between the explanatory variables and the expected crash frequency and reports the comparison of different models for total, injury & fatal, and property damage only crashes to obtain those with the maximum estimation accuracy for each severity level. The comparison of models was based on both the goodness of fit and the prediction performance measures.

The fitted models showed that the traffic flow and several variables related to road infrastructure and geometric design have a significant influence on the intersection crash frequency. Further, the goodness of fit and the prediction performance measures revealed that the NB-P model outperformed other models for most of the crash severity levels in the case of signalized intersections. For the unsignalized intersections, the GP model was the best performing model. Our findings suggest a potential significant improvement in the estimation accuracy of crashes on urban intersections by applying the NB-P and GP models. Improved estimation accuracy lead to a better understanding of crash occurrence which facilitate informed decisions, effective selection and design of the countermeasures, and improve safety.

### *Keywords:*

Urban intersections, Crash frequency, Crash severity, Negative binomial models, Safety performance functions, Geometric design

## 1 **1. Introduction**

2 Drivers encounter multiple interactions with turning and crossing vehicles, pedestrians, and  
3 cyclists at intersections. A plethora of information (e.g. the presence of road signs, street signs and name  
4 tags, traffic lights, channelization and road markings, conflicting, crossing and adjacent traffic  
5 movements, dedicated lanes for left and right turning vehicles, billboards and advert screens, etc.) at  
6 intersections produce an unsafe environment, which poses an enormous challenge for drivers to operate  
7 safely. The demand for instant decision making, complex urban design, dense and rigorous land use,  
8 congestion, heavy traffic, vulnerable road users, and many on-and-off-vehicle distractions overload the  
9 attentional resources of the driver. This in turn leads to poor judgment of the traffic situation, confusion,  
10 inadequate decision, and ultimately a crash. Hence, it is not surprising to note that intersections  
11 constitute the highest proportion of total crashes on the roads. Tay (2015) has provided some statistics  
12 from around the world to highlight this safety concern. In the past, the operational aspects of urban  
13 intersections, such as optimization of the traffic signals and/or reduction of vehicular and pedestrian  
14 traffic delays, travel time and congestion have received significant coverage in the literature (Dong et  
15 al., 2014; Roshandeh et al., 2014; Nesheli et al., 2009). However, these operational improvements do  
16 not account for the overall performance-based benefits (Roshandeh et al., 2016). The overall  
17 performance of the roadway network requires consideration of additional aspects like safety, comfort,  
18 cost, availability, accessibility, etc. In this paper, we have focused on the safety of intersections in urban  
19 areas.

20 The safety of intersections can be improved by understanding the factors that contribute to the  
21 occurrence of crashes and thereby, proposing appropriate countermeasures. Concerning this, an  
22 intersection safety analysis is typically suggested. One of the tools to measure the safety performance  
23 of intersections is by developing crash prediction models (CPMs). The CPMs are mathematical  
24 equations obtained through the statistical modeling of crash data and a series of explanatory variables,  
25 and are used to estimate the expected average crash frequency of roadway facilities over a specified  
26 period. They are also known as safety performance functions (SPFs) or collision prediction models  
27 (CPMs). The SPFs are applied to evaluate the safety of intersections and road segments, identify

28 hazardous locations, assess the safety of applied solutions, and compare and prioritize the best  
29 alternative designs (AASHTO, 2010). To address safety issues, the SPFs have been developed for many  
30 years now across the globe for numerous highway facilities (Elvik et al., 2019; Abdel-Aty et al., 2016;  
31 Janstrup, 2016; Cafiso et al., 2012; Persaud et al., 2012; Vieira Gomes et al., 2012; Srinivasan and  
32 Carter, 2011; Wong et al., 2007; Greibe, 2003). Leaving aside the applicability of those models, the  
33 development of the SPFs is a critical process in which a modeler makes crucial decisions. To emphasize,  
34 Hauer and Bamfo (1997) argued, “*In the course of modeling, the modeler will make two major*  
35 *decisions: (a) What explanatory variables to include in the model equation; and, (b) What should be its*  
36 *functional form*”. Factors, such as the purpose of the SPF, the availability, quality, and quantity of the  
37 data, required expertise, etc. affect those decisions.

38 American Association of State Highways and Transportation Officials (AASHTO) published  
39 the Highway Safety Manual (HSM), first in 2010 (AASHTO, 2010), and then in 2014 with a few  
40 supplements (AASHTO, 2014). The HSM offers the SPFs for prediction of intersection and road  
41 segment crashes on several highway facility types, e.g., rural two-lane and multilane highways, urban  
42 and suburban arterial and freeway ramp terminals (AASHTO, 2014; AASHTO, 2010). The predictive  
43 models in the HSM were developed using data from a small number of States. Because of the possible  
44 differences in the travel behavior, traffic conditions and road characteristics across different  
45 geographical regions, it has been highlighted that the crash relationships in these states may not be  
46 necessarily representative of those in the other states. Regarding this, the HSM guidelines recommend  
47 (i) the calibration of the HSM base models for applications in other jurisdictions or (ii) the estimation  
48 of new SPFs for the regions where a sufficient good quality local data is available. Several states in the  
49 US and other countries have thus developed their own SPFs. The SPFs given in the HSM for  
50 intersections estimate only total crashes that might not be an ideal approach since crashes vary by type  
51 and severity across intersections (Wang et al., 2019; Zhao et al., 2018; Wang et al., 2017). Some  
52 intersection might be crowded by fatal crashes only and others might experience injury or property  
53 damage only (PDO) crashes. Similarly, some intersections could have a higher proportion of a different  
54 particular type of crash compared with other intersections. Differences in the distribution of crash

55 severity and/or crash type could be attributed to the variation in the geometric design and traffic  
56 characteristics between intersections. In order to consider those variations, studies estimate predictive  
57 models for intersections by crash type (Wang et al., 2019; Gates et al., 2018; Liu and Sharma, 2018;  
58 Wu et al., 2018; Dixon et al., 2015; Geedipally and Lord, 2010), and/or by severity level (Liu and  
59 Sharma, 2018; Wang et al., 2017; Wu et al., 2013; Oh et al., 2010).

60         Regarding the statistical methodologies, the crash prediction modeling has come a long way. In  
61 the beginning, researchers used linear regression models for the estimation of crashes and determining  
62 the relationships between crash frequency and explanatory variables (Joshua and Garber, 1990;  
63 Okamoto and Koshi, 1989). However, with new research, it was soon realized that linear regression  
64 models have certain limitations in treating the non-negative and discrete nature crash data (Lord and  
65 Mannering, 2010; Miaou and Lum, 1993). This led to the adoption of count data models in crash  
66 prediction. Naturally, the first choice of researchers was the Poisson regression model which assumes  
67 that the variance of the data is equal to the mean of the data. On the other hand, the crash data is  
68 frequently characterized by over-dispersion, that is, the variance of the crash data is greater than its  
69 mean. To overcome the over-dispersion issue, the negative binomial (NB) regression models were used  
70 (Abdel-Aty & Radwan, 2000; Miaou, 1994). With the progress in statistical methods and improved  
71 computing power, more advanced techniques have been applied recently to model the crash data. Lord  
72 and Mannering (2010), and Mannering and Bhat (2014) have provided detailed accounts of the existing  
73 trends in the crash prediction and future directions. Despite all the intricacy, the traditional NB models  
74 still enjoy great popularity due to their inherent simplicity of estimation and a relatively better  
75 performance.

76         Several parameterizations of the NB models are available in the literature. Nonetheless, the NB-  
77 1 and NB-2 (Cameron and Trivedi, 1986) have been commonly used to model the count data (Wang et  
78 al., 2019; Giuffrè et al., 2014; Ismail and Zamani, 2013; Hilbe, 2011; Winkelmann, 2008; Chang and  
79 Xiang, 2003; Miaou and Lord, 2003). The two models necessarily differentiate on the basis of the  
80 relationship between the variance of the data and the mean of the data. The NB-1 assumes a linear  
81 relationship between the variance and the mean, while the NB-2 assumes a quadratic relationship.

82 Detailed estimation procedures of the two alternative forms are given in Hardin (2018), Lord and Park  
83 (2015), and Hilbe (2011). In traffic safety, the NB-2 has been frequently used to estimate the SPFs  
84 while the NB-1 has also found a few applications. For instance, Chang and Xiang (2003) created SPFs  
85 using both the NB-1 and NB-2 models to study the relationship between crashes and congestion levels  
86 on freeways. The authors found that both models showed consistent results for the relationship between  
87 crashes and traffic volume, the number of through lanes, and median. Giuffrè et al. (2014) applied the  
88 NB-1 and NB-2 models to develop the SPFs for urban unsignalized intersections. They found that the  
89 NB-1 fits the data better than the NB-2. Wang et al. (2019) also used the NB-1 and NB-2 along with  
90 standard Poisson regression and an NB-P model for estimation of the SPFs for rural two-lane  
91 intersections.

92 The applications of the NB-1 and NB-2 models, however, come with a few compromises. For  
93 instance, the NB-1 and NB-2 models both restrict the variance structure in the estimation of the SPFs  
94 (Park, 2010), that is, the mean-variance relationship of the crash data is constrained to either a linear or  
95 quadratic for the NB-1 and NB-2 models, respectively. The restricted variance structure may result in  
96 the biased estimates of model parameters and ultimately the incorrect crash forecasts (Wang et al.,  
97 2019). Furthermore, both the NB-1 and NB-2 are non-nested models and an appropriate statistical test  
98 to determine a better model of the two cannot be carried out directly (Wang et al., 2019; Greene, 2008).  
99 To account for that, Greene (2008) introduced a new functional form of the NB regression called an  
100 NB-P that nests both the NB-1 and NB-2 models. The NB-P is essentially the extension of the traditional  
101 NB models to address the restricted variance structure problem. The NB-P reduces to NB-1 when  $P=1$   
102 and to NB-2 when  $P=2$ . Since the NB-P model parametrically nests both the NB-1 and NB-2 models,  
103 it allows analysts to test the two NB functional forms (NB-1, NB-2) against a more general alternative  
104 (NB-P) for a better model (Greene, 2008; Ismail and Zamani, 2013; Hilbe, 2011). The NB-P model has  
105 been used in a few studies dealing with count data. For example, Greene (2008) applied the NB-P along  
106 with the NB-1 and NB-2 models to the German health care data. It was found that the NB-P  
107 outperformed the other two models based on the goodness of fit measures. Ismail and Zamani (2013)  
108 used the NB-1, NB-2, and NB-P models to study the Malaysian private car own damage claim counts.

109 They also reported that the NB-P model was the best performing model. In traffic safety, Wang (2019)  
110 used the NB-P along with Poisson, NB-1, and NB-2 models to study the safety performance of rural  
111 two-lane intersections by crash type and intersection type. They developed traffic only models. Their  
112 findings revealed that the NB-P model performed better than the Poisson model, NB-1, and NB-2  
113 models for most crash types and intersection types. The authors concluded that the flexible variance  
114 structure of the NB-P model significantly improves the estimation accuracy.

115 The literature review shows that the applications of the NB-P model, despite the obvious  
116 improvement compared to the traditional NB models, are still not common in traffic safety and crash  
117 prediction. To the authors' knowledge, no study has used the NB-P model to estimate SPFs for urban  
118 roads. Moreover, there has been no evidence that the NB-P model is used in the estimation of fully  
119 specified SPFs. Given that the applications of the NB-P model in road safety are rare, its potential to  
120 improve the estimation accuracy by offering a flexible variance structure, and the fact that it allows to  
121 statistically test the NB-1 and NB-2 against a general alternative, are motivations behind this work.  
122 Besides, the HSM recommendation of developing local SPFs for locations with enough data was  
123 another driving force. In this paper, we applied different functional forms of the NB regression model  
124 (NB-1, NB-2 and NB-P) and compare the results with the Generalized Poisson (GP) regression model,  
125 also a popular count data modeling technique, in the pursuit of obtaining the best model for the  
126 estimation of intersection SPFs in the urban areas. The GP model, discussed in section 2.4 in details, is  
127 an extension of the Generalized NB models (Ismail and Zamani, 2013). In the past, the GP models have  
128 been applied to study road crashes (Famoye et al., 2004), shipping damage incidents (Ismail and Jemain,  
129 2007), vehicle insurance claims (Ismail and Zamani, 2013), etc. The rationale for choosing the GP  
130 model for comparison with the NB models was that it can also accommodate the over-dispersed data  
131 equally well, has relatively less applications in the SPF estimation and the fact that it is sometimes  
132 regarded as a potential competitor to the NB models for treatment of over-dispersed count data  
133 (Melliana et al., 2013). The contribution of the current study to traffic safety literature is that it applies  
134 the functional form NB-P of the NB regression, along with the NB-1, NB-2 and a GP model for the  
135 estimation of intersection SPFs in the urban areas. A unique combination of the new approach for the



136 SPFs estimation and the use of not only the traffic flow but also other explanatory variables adds to the  
137 novelty of this work. To the best of our knowledge, no micro-level SPFs have been developed for the  
138 urban intersections in Belgium, results of this study could potentially serve the local research  
139 community involved in traffic safety as well as the industry in planning level safety assessment of new  
140 road infrastructure projects.

## 141 **2. Methodology**

142 The count data models have been widely applied to estimate crashes at the road segments and  
143 intersections in a non-negative, discrete, and random fashion (Washington et al., 2010). Since the  
144 Poisson regression models are usually not fit for modeling the crash data due to their inability to  
145 accommodate overdispersion, three different functional form of the NB model and a GP model were  
146 applied to estimate the SPFs for urban intersections in this study.

### 147 *2.1 Negative binomial model-type 2 (NB-2)*

148 The negative binomial regression is the derivative of the standard Poisson regression. It  
149 redefines the conditional mean of the standard Poisson model (equi-dispersion; variance of the data  
150 equals its mean) and incorporates a latent heterogeneity term to account for over-dispersion in data. The  
151 expected crash frequency " $\mu_i$ " at the intersection " $i$ " obtained by applying the NB model as in  
152 Washington et al. (2010) is given by:

$$\mu_i = \exp(\beta X_i + \varepsilon_i) \quad (1)$$

153 where " $X_i$ " is the vector of explanatory variables, " $\beta$ " is the vector of estimable coefficients and  
154 " $\exp(\varepsilon_i)$ " is the latent heterogeneity term, also known as an error term. When the " $\exp(\varepsilon_i)$ " follows  
155 gamma distribution with mean 1 and variance  $1/\sigma = k$  where " $k$ " represents an over-dispersion  
156 parameter, a traditional NB model, called the NB-2 model, is derived.

157 For the interest of readers, an equation 1 according to the standard Poisson regression model  
158 would have been:

$$\mu_i = \exp(\beta X_i) \quad (2)$$

159 Clearly, this lacks the term " $\exp(\varepsilon_i)$ " to account for over-dispersion.

160 The probability density function of the NB-2 model for estimation of the SPFs as in Washington  
 161 et al. (2010):

$$Prob[y_i|\mu_i] = \frac{\Gamma[(\sigma) + y_i]}{\Gamma(\sigma)y_i!} \left[ \frac{\sigma}{(\sigma) + \mu_i} \right]^\sigma \left[ \frac{\mu_i}{(\sigma) + \mu_i} \right]^{y_i} \quad (3)$$

162 where  $\Gamma$  is a gamma function. The mean and the variance of the NB-2 regression model are  
 163 equal to  $E(y_i) = \mu_i$ , and  $Var(y_i) = \mu_i + k\mu_i^2 = \mu_i(1 + k\mu_i)$ , respectively. When  $1/\sigma = k$ , the marginal  
 164 distribution function of the NB-2 model can be reproduced:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}\right) + y_i\right]}{\Gamma\left(\frac{1}{k}\right)y_i!} \left[ \frac{\frac{1}{k}}{\left(\frac{1}{k}\right) + \mu_i} \right]^{\frac{1}{k}} \left[ \frac{\mu_i}{\left(\frac{1}{k}\right) + \mu_i} \right]^{y_i} \quad (4)$$

## 165 2.2 Negative binomial model-type 1 (NB-1)

166 A re-parameterization of the variance structure of the NB model by replacing  $\frac{1}{k}$  in the NB-2  
 167 (equation 4) with  $\frac{1}{k}\mu_i$  allows for another functional form, called the NB-1 (Wang et al., 2019; Hilbe,  
 168 2011; Greene, 2008; Cameron & Trivedi, 1986). The marginal distribution function of the NB-1 is given  
 169 by:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}\mu_i\right) + y_i\right]}{\Gamma\left(\frac{1}{k}\mu_i\right)y_i!} \left[ \frac{\frac{1}{k}\mu_i}{\left(\frac{1}{k}\mu_i\right) + \mu_i} \right]^{\frac{1}{k}\mu_i} \left[ \frac{\mu_i}{\left(\frac{1}{k}\mu_i\right) + \mu_i} \right]^{y_i} \quad (5)$$

170 The mean of the NB-1 is  $E(y_i) = \mu_i$  and the variance of the NB-1 is  $Var(y_i) = \mu_i + k\mu_i$ .

## 171 2.3 Negative binomial model-type P (NB-P)

172 Greene (2008) proposed a new form of the NB regression that uses the parameter ‘‘P’’ to  
 173 represent the mean-variance relationship. It is known as the NB-P model. The NB-P model is obtained  
 174 by replacing  $\frac{1}{k}$  in the NB-2 model (equation 4) with  $\frac{1}{k}\mu_i^{2-P}$ . The marginal distribution function of the  
 175 NB-P model is given by:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}\mu_i^{2-P}\right) + y_i\right]}{\Gamma\left(\frac{1}{k}\mu_i^{2-P}\right)y_i!} \left[ \frac{\frac{1}{k}\mu_i^{2-P}}{\left(\frac{1}{k}\mu_i^{2-P}\right) + \mu_i} \right]^{\frac{1}{k}\mu_i^{2-P}} \left[ \frac{\mu_i}{\left(\frac{1}{k}\mu_i^{2-P}\right) + \mu_i} \right]^{y_i} \quad (6)$$

176 where mean and variance of the NB-P are  $E(y_i) = \mu_i$  and  $Var(y_i) = \mu_i + k\mu_i^P$ , respectively.  
177 “P” represents the functional parameter of the NB-P model.

178 All the NB models used maximum likelihood estimation (MLE) approach to estimate the  
179 parameter coefficients.

#### 180 2.4 Generalized Poisson model (GP)

181 The generalized Poisson (GP) regression is another popular approach to model count data. As  
182 an alternative to the NB regression, the GP models have the advantage of modeling both over-dispersed  
183 and under-dispersed data. Like the NB regression, the GP model has an extra parameter, called a scale  
184 or dispersion parameter. A distinctive feature of the GP dispersion parameter is that it can take both  
185 positive and negative values for over-dispersed and under-dispersed data, respectively. The probability  
186 mass function (p.m.f.) of the GP distribution given as in Yang et al. (2009):

$$Prob[Y_i|y_i] = \frac{\theta(\theta+ky_i)^{y_i-1} \exp(-\theta-ky_i)}{y_i!}, \quad y_i = 0,1,2, \dots, \quad (7)$$

187 where  $\theta > 0$ , and  $0 \leq k < 1$ . From Joe and Zhu (2005), the mean of the GP regression is  
188  $E(Y_i) = \mu = (1 - k)^{-1}\theta$ , and the variance of the GP regression is  $Var(Y_i) = (1 - k)^{-3}\theta =$   
189  $(1 - k)^{-2}\mu = \phi \cdot \mu$ . The term  $\phi = (1 - k)^{-2}$  is a dispersion factor, and it is used in the GP mass  
190 function where “k” is a dispersion parameter. It can be seen that when  $k = 0$ , a standard Poisson model  
191 is obtained. For  $k < 0$ , under-dispersion is assumed while  $k > 0$  represents over-dispersion. Since crash  
192 data normally exhibits over-dispersion, this study will assume  $k > 0$  condition. There are other  
193 parametrizations of the GP but their applications are left for future studies.

#### 194 2.5 Model structure

195 The literature offers several ways to model the relationships between intersection crash  
196 frequency and explanatory variables (Barbosa et al., 2014; Park and Lord, 2009; Nambuusi et al., 2008;  
197 Miaou and Lord, 2003). They are differentiated on the basis of the type of variables, the number of  
198 variables, the form that the variables take during the modeling process and the transformation applied  
199 to the variables (Oh et al., 2003). In this study, the following model structure was used to estimate the  
200 expected crash frequency “ $\mu_i$ ” of the intersection “i”:

$$\mu_i = \exp(\beta_0 + \beta_1 \ln(AADT_{major}) + \beta_2 \ln(AADT_{minor}) + \sum_{m=3}^n \beta_m X_m) \quad (8)$$

201 where  $\beta_0$  represents the intercept,  $AADT_{major}$  is the major approach average annual daily  
 202 traffic (AADT),  $\beta_1$  represents the coefficient estimate of the major approach AADT,  $AADT_{minor}$   
 203 represents the minor approach AADT,  $\beta_2$  represents the coefficient estimate of the minor approach  
 204 AADT,  $\beta_m$  is the vector of the coefficient estimates of explanatory variables and “ $X_m$ ” denotes the  
 205 vector of explanatory variables. For the NB models (NB-1, NB-2, and NB-P) and the GP model, the  
 206 coefficients denoted by  $\beta_m$  and a dispersion parameter denoted by “ $k$ ” were estimated but for the NB-  
 207 P, an additional parameter “P”, called a functional parameter, was also estimated.

## 208 2.6 Model comparison

209 For model comparison, both the likelihood-based and the predictive ability-based measures  
 210 were used. The likelihood-based measures consisted of the likelihood ratio test (LRT), the Akaike  
 211 Information Criteria (AIC) and the Bayesian Information Criteria (BIC). The LRT was used only when  
 212 comparing the hierarchically nested models (Greene, 2008; Wang et al., 2019). The AIC and the BIC  
 213 were used for comparing the non-nested models (Ismail and Jemain, 2007).

214 The predictive ability-based measures compared all developed models for predictive  
 215 performance using the validation data. Those included in the study were; mean prediction bias (MPB),  
 216 mean absolute deviation (MAD), and mean squared prediction error (MSPE) as in Oh et al. (2003), and  
 217 % CURE deviation and a validation factor (Hauer, 2015; Wang et al., 2019).

## 218 3. Data

219 The data used for modelling was obtained for urban intersections of Antwerp, Belgium. A  
 220 dataset consisting of crash data of six years (2010-2015), road geometric data, and traffic flow data was  
 221 created for the estimation of the SPFs. An online database of the regional government called the  
 222 Flanders road register was consulted for the intersection data. A total of 760 intersections were used for  
 223 analysis, of which 198 were signalized and 562 were unsignalized. Around 470 were three-legged  
 224 intersections and the remaining 290 were four-legged intersections. Because the skewness of  
 225 intersection has been reported to have an impact on its safety (Nightingale et al., 2017; Haleem and

226 Abdel-Aty, 2010), it was decided to include skewness as a potential explanatory variable. The smallest  
 227 angle between the two adjacent approaches of intersection, known as an intersection angle (Nightingale  
 228 et al., 2017), was used as a surrogate to define the level of skewness. A 75 degrees intersection angle  
 229 used by Haleem and Abdel-Aty (2010) was chosen as a threshold to define the levels of skewness. An  
 230 intersection angle less than or equal 75 degrees represented skewness level 1 while an intersection angle  
 231 greater than 75 degrees represented skewness level 2. A total of 217 intersections had a skewness level  
 232 1 and 543 intersections had a skewness level 2. **Table 1** provides the description of variables employed  
 233 in this study for urban signalized and unsignalized intersections.

234 Table 1 Variables description for urban intersections of Antwerp

Variable Description	Variable levels
AADT on the major approach	-
AADT on the minor approach	-
Skewness	1: Intersection angle is less than/equal to 75-degrees 2: Intersection angle is greater than 75-degrees
Legs/approaches of the intersection	1: For 4 legged intersections 0: For 3 legged intersections
Existence of stop sign on the minor approach	1: Stop sign is present on at least one minor approach 0: No stop sign on the minor approaches
Existence of stop line on the minor approach	1: Stop line is present on at least one minor approach 0: No stop line on the minor approaches
Number of left turn lane on the major approach	2: At least one left turn lane exists on each direction of the major approach 1: At least one left turn lane exists on only one direction of the major approach 0: No left turn lane exists
Number of right turn lane on the major approach	2: At least one right turn lane exists on each direction of the major approach 1: At least one right turn lane exists on only one direction of the major approach 0: No right turn lane exists
Number of through lanes of the minor approach	4 or 4+: Four and more through lanes of the minor approach 1-3: One to three through lanes of the minor approach 0: No through lane of the minor approach
Left turn (LT) movements on the minor approach	2: LT movement on each minor approach 1: LT movement on only one minor approach 0: No LT movement on the minor approach

Existence of crosswalk on minor approach	2: Crosswalk on each minor approach 1: Crosswalk on only one minor approach 0: No crosswalk
Existence of crosswalk on major approach	2: Crosswalk on each major approach 1: Crosswalk on only one major approach 0: No crosswalk
Size of the intersection <sup>a</sup>	4: for 5*4, 5*8, 6*4, 6*6, 6*8, 8*4, 8*6, 8*8, 8*10, 10*8, 10*10 3: for 3*2, 3*4, 3*6, 4*2, 4*4, 4*6 2: for 2*2, 2*3, 2*4, 2*6 1: for 1*2, 1*3, 1*4

<sup>a</sup> The first number is the total number of approach lanes for a minor approach, and the second number is the total number of through lanes for a major approach (as per, Abdel-Aty and Haleem 2011)

235           The crash data was provided by the police of Antwerp. The crash records featured the severity  
236 level of a crash, coordinates of a crash location, time and date of a crash, number of the vehicles involved  
237 and their type, maneuver of the involved vehicles at the time of the crash, data about the involved  
238 drivers, and road and pavement conditions. Only intersection and intersection-related crashes were used  
239 in the analysis. Because of the inconstancy in the definition of the influence area to classify a crash as  
240 intersection-related (Wang et al., 2008), we chose to use the HSM guidelines to differentiate the  
241 intersection and intersection-related crashes from the segment crashes. According to the HSM  
242 (AASHTO, 2014, 2010);

- 243           - An intersection crash is the one that has occurred within the physical boundaries of  
244 an intersection area
- 245           - An intersection related crash is the one that has occurred on the road segment but  
246 the presence of the intersection was the cause of that crash and it falls within its  
247 influence area

248           Using the above definition, 5128 intersection and intersection related crashes were identified  
249 for analysis. To account for the potential variation in the SPFs by crash severity, those crashes were  
250 divided into total crashes, injury & fatal crashes and property damage only (PDO) crashes.

251           The traffic data was acquired from Lantis, a mobility management company based in Antwerp.  
252 Lantis also provides its services to the Mobiliteit en Parkeren Antwerpen Ag, an office for parking and  
253 mobility services of Antwerp city. The data was received in two sets, actual counts and traffic model

254 estimates. The actual counts were collected using either manual counting techniques or loop detectors  
 255 installed at the random locations on the roads in the study network. The traffic model estimates were  
 256 generated using a microsimulation traffic model called Dynamisch Model Kernstad Antwerpen  
 257 (DMKA). It is important to note that the model was calibrated for the years 2010-2015, a period during  
 258 which the crash data was recorded. Results from several runs of the simulation model were obtained  
 259 and averaged to get a better convergence towards the actual counts. Actual counts and model generated  
 260 counts were compared at locations where both were available to check for the residuals. An absolute  
 261 difference of not greater than 5% between the simulation counts and actual counts was reported for the  
 262 majority of locations. The outliers were discarded. The authors agreed to use a combination of actual  
 263 counts and traffic model estimates to ensure as many intersections included in the SPFs estimation as  
 264 possible with a maximum degree of accuracy. **Table 2** provides the descriptive statistics of crash data  
 265 (by severity) and traffic data for signalized and unsignalized intersections used to develop the SPFs.

266 Table 2 Descriptive statistics of crash data (by severity) and traffic flow data for signalized and unsignalized intersections

Variables	Signalized Intersections				Unsignalized Intersections			
	Min.	Max.	Aver.	Std. Dev.	Min.	Max.	Aver.	Std. Dev.
<b>Total Crashes</b>	0	87	13.899	13.848	0	51	4.347	5.223
<b>PDO Crashes</b>	0	50	6.979	7.760	0	49	2.540	3.671
<b>Injury &amp; Fatal Crashes</b>	0	39	6.919	7.224	0	25	1.806	2.557
<b>Ln (AADT)<sub>major</sub></b>	183	41915	14559	9424.8	13	30648	3511	2884.1
<b>Ln (AADT)<sub>minor</sub></b>	31	26837	5225	4905.8	9	7595	1001	815.2

## 267 4. Results

268 **Table 3** and **Table 4** present the parameter estimates ( $\beta$ ) of the NB-1, NB-2, NB-P, and GP  
 269 models developed by crash severity (total crashes, PDO crashes, and injury & fatal crashes) for  
 270 signalized and unsignalized intersections, respectively. The numbers enclosed within the parenthesis  
 271 correspond to their p-values. The SPFs show that the signs of estimated parameters are similar across  
 272 different models developed for the same severity level. This indicates that given the same severity level,  
 273 the potential impact of explanatory variables on the expected crash frequency obtained from different  
 274 models is similar. The estimated parameters, however, vary slightly across different severity levels  
 275 which could be one of the reasons that imply the need to develop separate models for each crash severity

276 level. Using a 90% confidence level as in Vieira Gomes et al. (2012) for similar data, we found that  
277 five variables were significant in case of signalized intersections and four variables in case of  
278 unsignalized intersections. The significant variables included the traffic flow, the intersection skewness,  
279 the existence of crosswalk on a minor approach, the number of through lanes on a minor approach, and  
280 the number of approaches. To our surprise, the presence of exclusive left and right turn lanes were not  
281 significant in any model. The intersection size and the crosswalk on the major approaches were other  
282 insignificant explanatory variables.

#### 283 *4.1 SPFs of signalized intersections*

284 **Table 3** provides the SPF estimation results for signalized intersection. It shows that there was  
285 a statistically significant increase in the crash frequency with an increase in the natural logarithm of  
286 AADTs (which necessarily indicates an increase in traffic flow) of the major and the minor approaches  
287 of intersection. The crosswalk on a minor approach was significant only when it existed on both  
288 approaches of a signalized intersection across all developed models and all severity levels. However,  
289 there was an exception in case of the NB-2 and NB-P models of total crashes, for which, in addition to  
290 a crosswalk on each minor approach, a crosswalk variable was also significant when present on only  
291 one of the minor approaches of an intersection. The estimated coefficients in the former case were  
292 approximately double than that of the later. This was not true for other crash severity levels (the PDO,  
293 and injury & fatal crashes) and model types. The intersection skewness was significant only for total  
294 crashes (all the NB models only), and injury & fatal crashes (all models). The coefficient estimates were  
295 negative in the developed models. Since the higher skewness level was a base case, the negative sign  
296 indicates that no skewness or lower skewness (i.e., intersection angle greater than 75 degrees, please  
297 see the data section for details) results in a reduced crash frequency. In other words, intersections with  
298 no or lower skewness were safer than the intersections with higher skewness. This is a straight forward  
299 result since the presence of skewness causes larger intersection areas, obstructs views and affects sight  
300 distances. An important observation from the results was that the absence of skewness causes a greater  
301 decrease in the injury & fatal crashes than the total crash frequency.



Variables	NB-1	NB-2	NB-P	GP
	$\beta$ (p-value)	$\beta$ (p-value)	$\beta$ (p-value)	$\beta$ (p-value)
<b>TOTAL CRASHES</b>				
Intercept	-3.9067 (0.0000)	-4.2775 (0.0000)	-4.2761 (0.0000)	-3.7402 (0.0000)
AADT <sub>Major</sub>	0.4058 (0.0000)	0.3623 (0.0000)	0.3621 (0.0000)	0.3934 (0.0000)
AADT <sub>Minor</sub>	0.2450 (0.0000)	0.3002 (0.0000)	0.3003 (0.0000)	0.2425 (0.0000)
No crosswalk: 0 (Base)				
Crosswalk on one of the minor approaches: 1	0.2747 (0.3870)	0.5547 (0.0690)	0.5551 (0.0690)	0.2578 (0.3920)
Crosswalk on each of the minor approach: 2	0.8867 (0.0050)	1.2151 (0.0000)	1.2155 (0.0000)	0.8549 (0.0040)
Skewness: 1 (Base)				
Skewness: 2	-0.1572 (0.0970)	-0.2180 (0.0360)	-0.2181 (0.0360)	-0.1486 (0.1190)
Over-dispersion	4.1062	0.2977	0.2953	0.5778
<i>P</i>	1.000 (0.0000)	2.000 (0.0000)	2.0031 (0.0000)	
Log L <sup>a</sup>	-653.03	-640.65	-640.65	-651.79
<i>AIC</i>	1320.06	1295.31	1297.31	1317.58
<i>BIC</i>	1343.07	1318.33	1323.62	1340.60
<b>PDO CRASHES</b>				
Intercept	-4.4085 (0.0000)	-4.8088 (0.0000)	-4.8899 (0.0000)	-4.2727 (0.0000)
AADT <sub>Major</sub>	0.3396 (0.0100)	0.2992 (0.0010)	0.3153 (0.0010)	0.3269 (0.0010)
AADT <sub>Minor</sub>	0.2954 (0.0000)	0.3367 (0.0000)	0.3357 (0.0000)	0.2942 (0.0000)
No crosswalk: 0 (Base)				
Crosswalk on one of the minor approaches: 1	0.2377 (0.5320)	0.6326 (0.1050)	0.5671 (0.1810)	0.2382 (0.5190)
Crosswalk on each of the minor approaches: 2	0.9062 (0.0150)	1.3397 (0.0010)	1.2820 (0.0020)	0.9008 (0.0130)
Over-dispersion	2.7650	0.3840	0.6319	0.5022
<i>P</i>	1.000 (0.0000)	2.000 (0.0000)	1.7530 (0.0000)	
Log L	-538.89	-533.32	-532.91	-538.16
<i>AIC</i>	1091.79	1080.65	1081.81	1090.33
<i>BIC</i>	1114.80	1103.66	1108.12	1113.35
<b>INJURY &amp; FATAL CRASHES</b>				
Intercept	-4.9921 (0.0000)	-5.6066 (0.0000)	-5.6210 (0.0000)	-4.9458 (0.0000)
AADT <sub>Major</sub>	0.4963 (0.0000)	0.4797 (0.0000)	0.4835 (0.0000)	0.4952 (0.0000)
AADT <sub>Minor</sub>	0.1917 (0.0030)	0.2586 (0.0000)	0.2563 (0.0000)	0.1879 (0.0040)
No crosswalk: 0 (Base)				
Crosswalk on one of the minor approaches: 1	0.2618 (0.4690)	0.4785 (0.2020)	0.4738 (0.2110)	0.2631 (0.0040)
Crosswalk on each of the minor approaches: 2	0.8633 (0.0150)	1.0876 (0.0030)	1.0856 (0.0040)	0.8576 (0.0140)
Skewness: 1 (Base)				
Skewness: 2	-0.2056 (0.0620)	-0.3258 (0.0070)	-0.3213 (0.0090)	-0.1984 (0.0740)
Over-dispersion	2.3591	0.3324	0.3679	0.4727
<i>P</i>	1.000	2.000	1.9500	

	(0.0000)	(0.0000)	(0.0000)	
Log L	-531.95	524.40	-524.39	-530.87
AIC	1077.91	1062.81	1064.77	1075.74
BIC	1100.93	1085.83	1091.08	1098.76

303 Notes: <sup>a</sup> Log L: Log Likelihood

304 4.2 SPFs of unsignalized intersections

305 **Table 4** presents the coefficient estimates of the SPFs for unsignalized intersections. The traffic  
306 flows of major and minor approaches were significantly associated with crash frequency except for  
307 injury and fatal crashes where the AADT of the minor approach was found insignificant. The presence  
308 of a crosswalk on the minor approach was only significant for total crashes, and injury and fatal crashes  
309 across all developed models. Unlike signalized intersections, the crosswalk was significant when it was  
310 present on only one of the minor approaches of unsignalized intersections. The presence of crosswalk  
311 on one or both approaches was, however, significant only in case of injury and fatal crashes as can be  
312 seen in the NB-2 and NB-P models. The number of approaches/legs of an intersection was a significant  
313 predictor of total and PDO crashes at unsignalized intersections at a 90% confidence level. Intersections  
314 with three approaches/legs as a base, the positive signs of the estimated coefficients indicate higher  
315 expected crash frequency on intersections with four approaches compared to intersections with three  
316 approaches. Another statistically significant variable was the number of through lanes of the minor  
317 approaches of unsignalized intersection. A positive association was found between crash frequency and  
318 the number of through lanes of its minor approach for the total crashes, and injury & fatal crashes.  
319 While the first level of this variable was not significant, the second level, which represents four or  
320 greater number of through lanes of minor approaches was significant for total crashes. For injury &  
321 fatal crashes, all levels of the number of through lanes were significant. This means that a significant  
322 increase can be expected in total crashes, and injury & fatal crashes with an increase in the number of  
323 through lanes of the minor approach of an unsignalized intersection. It is noteworthy that this result can  
324 be generalized only to four-legged unsignalized intersections because through lanes were reported only  
325 for such facility type in this study.

Variables	NB-1	NB-2	NB-P	GP
	$\beta$ (p-value)	$\beta$ (p-value)	$\beta$ (p-value)	$\beta$ (p-value)
<b>TOTAL CRASHES</b>				
Intercept	-1.2095 (0.0000)	-1.4860 (0.0000)	-1.4082 (0.0000)	-1.1683 (0.0000)
AADT <sub>Major</sub>	0.1948 (0.0000)	0.2155 (0.0000)	0.2113 (0.0000)	0.1883 (0.0000)
AADT <sub>Minor</sub>	0.1262 (0.0010)	0.1539 (0.0010)	0.1379 (0.0010)	0.1266 (0.0010)
No crosswalk: 0 (Base)				
Crosswalk on one of the minor approaches: 1	0.2668 (0.0010)	0.1709 (0.0500)	0.2485 (0.0040)	0.2728 (0.0010)
Crosswalk on each of the minor approaches: 2	0.1609 (0.1690)	0.1787 (0.1970)	0.1743 (0.1650)	0.1728 (0.1380)
No. of approaches: 3 (Base)				
No. of approaches: 4	0.3878 (0.0010)	0.2369 (0.0950)	0.3547 (0.0070)	0.3994 (0.0010)
No. of through lanes on the minor approaches: 0 (Base)				
No. of through lanes on the minor approach: 1-3	0.0330 (0.7880)	0.1080 (0.4730)	0.0578 (0.6690)	0.0260 (0.8310)
No. of through lanes on the minor approach: 4 & 4+	0.8029 (0.0000)	0.8797 (0.0120)	0.8176 (0.0010)	0.7760 (0.0000)
Over-dispersion	2.3705	0.5680	1.4209	0.4708
<i>P</i>	1.0000 (0.0000)	2.0000 (0.0000)	1.3598 (0.0000)	
Log L <sup>a</sup>	-1369.80	-1372.45	-1368.53	-1362.96
<i>AIC</i>	2757.61	2762.89	2757.06	2743.93
<i>BIC</i>	2796.61	2801.89	2800.39	2782.93
<b>PDO CRASHES</b>				
Intercept	-1.039 (0.0020)	-1.2663 (0.0000)	-1.2223 (0.0010)	-1.0004 (0.0030)
AADT <sub>Major</sub>	0.1011 (0.0370)	0.0989 (0.0600)	0.1067 (0.038)	0.0987 (0.0420)
AADT <sub>Minor</sub>	0.1619 (0.0010)	0.2040 (0.0000)	0.1842 (0.0010)	0.1583 (0.0010)
No. of approaches: 3 (Base)				
No. of approaches: 4	0.3189 (0.0000)	0.2122 (0.0280)	0.2912 (0.0030)	0.3291 (0.0000)
Over-dispersion	1.7930	0.7114	1.1867	0.4167
<i>P</i>	1.0000 (0.0000)	2.0000 (0.0000)	1.4507 (0.0000)	
Log L	-1149.44	-1149.70	-1148.76	-1140.98
<i>AIC</i>	2308.87	2309.41	2309.51	2291.96
<i>BIC</i>	2330.54	2331.07	2335.51	2313.63
<b>INJURY &amp; FATAL CRASHES</b>				
Intercept	-3.6679 (0.0000)	-4.0729 (0.0000)	-4.0075 (0.0000)	-3.6730 (0.0000)
AADT <sub>Major</sub>	0.4225 (0.0000)	0.4507 (0.0000)	0.4489 (0.0000)	0.4229 (0.0000)
AADT <sub>Minor</sub>	0.0727 (0.1510)	0.0912 (0.1060)	0.0857 (0.1170)	0.0727 (0.1530)
No crosswalk: 0 (Base)				
Crosswalk on one of the minor approaches: 1	0.3155 (0.0030)	0.3582 (0.0020)	0.3439 (0.0020)	0.3136 (0.0030)
Crosswalk on each of the minor approaches: 2	0.2379 (0.1080)	0.3740 (0.0290)	0.3091 (0.0570)	0.2425 (0.1020)
No. of through lanes on the minor approaches: 0 (Base)				
No. of through lanes on the minor approaches: 1-3	0.4208	0.5060	0.4678	0.4167

	(0.0100)	(0.0080)	(0.0090)	(0.0110)
No. of through lanes on the minor approaches: 4 & 4+	1.2610 (0.0000)	1.2376 (0.0020)	1.2515 (0.0000)	1.2484 (0.0000)
Over-dispersion	1.2707	0.5680	0.9755	0.3489
<i>P</i>	1.0000 (0.0000)	2.0000 (0.0000)	1.4601 (0.0000)	
Log L	-931.85	-933.04	-929.64	-931.01
AIC	1881.70	1884.07	1879.28	1880.02
BIC	1920.70	1923.07	1922.61	1919.02

327 Notes: <sup>a</sup> Log L: Log Likelihood

328 4.3 Comparison and performance evaluation of the developed SPFs

329 The likelihood ratio test (LRT) was used for the comparison of either the NB-1 with the NB-P  
330 model or the NB-2 with the NB-P model since both the NB-1 and NB-2 are parametrically nested by  
331 the NB-P (Greene, 2008). The LTR was, however, not applied to compare the non-nested models, i.e.,  
332 the NB-1 model against the NB-2 model, or the NB models against the GP model. Instead, the AIC and  
333 BIC were used as in Ismail and Jemain (2007).

334 Table 5 Likelihood ratio (NB-1 vs NB-P and NB-2 vs NB-P) for signalized and unsignalized intersections

	Signalized Intersections		Unsignalized Intersections	
<b>TOTAL CRASHES</b>				
<b>Test/Criteria</b>	<b>NB-1</b>	<b>NB-P</b>	<b>NB-1</b>	<b>NB-P</b>
Log L <sup>a</sup>	-653.028	-640.655	-1369.804	-1368.529
Likelihood ratio ( $\chi^2$ )		<b>24.75</b> (0.0000) <sup>b</sup>		2.55 (0.1104)
<b>Test/Criteria</b>	<b>NB-2</b>	<b>NB-P</b>	<b>NB-2</b>	<b>NB-P</b>
Log L	-640.655	-640.6552	-1372.4456	-1368.529
Likelihood ratio ( $\chi^2$ )		0.0002 (0.9893)		<b>7.83 (0.0051)</b>
<b>PDO CRASHES</b>				
<b>Test/Criteria</b>	<b>NB-1</b>	<b>NB-P</b>	<b>NB-1</b>	<b>NB-P</b>
Log L	-538.894	-532.906	-1149.436	-1148.756
Likelihood ratio ( $\chi^2$ )		<b>11.98</b> (0.0005)		1.36 (0.2436)
<b>Test/Criteria</b>	<b>NB-2</b>	<b>NB-P</b>	<b>NB-2</b>	<b>NB-P</b>
Log L	-533.324	-532.906	-1149.705	-1148.756
Likelihood ratio ( $\chi^2$ )		0.84 (0.3606)		1.90 (0.1682)
<b>INJURY &amp; FATAL CRASHES</b>				
<b>Test/Criteria</b>	<b>NB-1</b>	<b>NB-P</b>	<b>NB-1</b>	<b>NB-P</b>
Log L	-531.954	-524.388	-931.851	-929.6407
Likelihood ratio ( $\chi^2$ )		<b>15.13</b> (0.0001)		<b>4.42</b> (0.0355)
<b>Test/Criteria</b>	<b>NB-2</b>	<b>NB-P</b>	<b>NB-2</b>	<b>NB-P</b>
Log L	524.404	-524.388	-933.036	-929.6407
Likelihood ratio ( $\chi^2$ )		0.03 (0.8569)		<b>6.79</b> (0.0092)

335 Notes: Bold values indicate statistically significant results of the LRT

336 <sup>a</sup> Log L: Log Likelihood

337 <sup>b</sup> Values in parenthesis indicate the p-value when the likelihood ratio ( $\chi^2$ ) was computed

338           The LRT indicated that the NB-P model performed better than the NB-1 model for total crashes,  
339 PDO crashes, and injury & fatal crashes in case of signalized intersections (**Table 5**). The result of the  
340 LTR test was, however, inconclusive when the NB-P and NB-2 were compared and, hence, it cannot  
341 be said with certainty which of the two was a better model. Based on the other measures, i.e., log-  
342 likelihood, the AIC, and the BIC (**Table 3**), it can be seen that NB-P and NB-2 performed relatively  
343 closely but both performed better than the NB-1 models and the GP models for crash severities. The  
344 functional parameter “P” of the estimated NB-P models was statistically significant across all severity  
345 levels. The estimated value of the functional parameter “P” of the NB-P models for total crashes, and  
346 injury & fatal crashes was close to 2 while for the PDO crashes it was significantly different from either  
347 1 or 2 (**Table 3**). Although this does not completely verify the assumption that the restricted variance  
348 structure of the NB-1 or NB-2 models may lead to biased estimates of model parameters, it does not  
349 entirely reject the possibility either, as indicated by the PDO crashes on signalized intersections and the  
350 result for the NB-1 models.

351           The LRT for unsignalized intersections showed that the NB-P and NB-1 models performed  
352 equally closely in case of total crashes and the PDO crashes and we cannot say that the difference in  
353 the NB-P and NB-1 estimates was significant but for injury & fatal crashes, the results were in the favor  
354 of the NB-P models. The NB-P model, on the other hand, outperformed the NB-2 model for total  
355 crashes, and injury & fatal crash but there was no significant difference in the estimates of the NB-2  
356 and NB-P models for the PDO crashes. Based on the AIC and BIC values, the NB-1 models performed  
357 better than the NB-2 models (non-nested models comparison, **Table 4**) for total crashes and injury &  
358 fatal crashes while results for the PDO crashes were fairly close for the two traditional NB models. The  
359 AIC and BIC, however, showed better model fit for the GP models in all crash severity levels. So, it  
360 will be safe to say that the GP model outperformed all the NB models in the case of un-signalized  
361 intersections. The functional parameter “P” of variance structure was significant for the NB-P models  
362 across all severity levels and it was not close to either 1 or 2. This verifies the assumption that the  
363 restricted variance structure of the NB-1 and NB-2 models might lead to the biased estimates of model  
364 parameters for unsignalized intersection, and, hence the NB-P that takes into account the flexible

365 variance structure would be more reliable in the accurate estimation of model parameters when there is  
 366 no GP model considered.

367 Besides the likelihood-based criteria, predictive ability-based measures were also computed to  
 368 validate the developed models and examine their predictive performance. It is important to note that  
 369 randomly selected 80% data were used for the estimation of models while the remaining 20% were used  
 370 for validation of the developed models. We compute the MPB, MAD, MSPE, % CURE deviation and  
 371 a validation factor. According to Oh et al. (2003), smaller the absolute values of the MPB, MAD, and  
 372 MSPE, better is the performance of the developed models. The % CURE deviation, which denotes the  
 373 percentage of the data points falling outside the two standard deviation limits of the Cumulative  
 374 Residual (CURE) (Hauer, 2015), shows a good fit when its values are small (Wang et al., 2019). Finally,  
 375 a factor, that we called a validation factor, was calculated as the ratio of the total predicted crashes to  
 376 the total observed crashes using the validation data. A value close to one indicated a better model (Wang  
 377 et al., 2019). Wang et al., (2019) called it a calibration factor.

378 Table 6 Predictive performance evaluation and validation of estimated models of signalized and unsignalized intersections

Crash Severity	Criteria	Signalized Intersections (198)				Unsignalized Intersections (562)			
		NB-1	NB-2	NB-P	GP	NB-1	NB-2	NB-P	GP
<b>TOTAL CRASHES</b>	<b>MPB</b>	-0.233	-0.268	-0.268	-0.237	-0.035	-0.034	-0.035	-0.034
	<b>MAD</b>	1.083	1.082	1.082	1.088	0.509	0.510	0.507	0.509
	<b>MSPE</b>	2.998	2.932	2.932	3.042	0.472	0.473	0.470	0.471
	<b>CURE Deviation (%)</b>	26	4	4	36	0	1	0	0
	<b>Validation Factor (VF)</b>	1.094	1.110	1.110	1.096	0.954	0.952	0.953	0.955
<b>PDO CRASHES</b>	<b>MPB</b>	0.043	0.031	0.040	0.041	-0.025	-0.027	-0.026	-0.027
	<b>MAD</b>	0.688	0.693	0.693	0.691	0.340	0.337	0.337	0.337
	<b>MSPE</b>	1.100	1.106	1.097	1.111	0.419	0.420	0.419	0.418
	<b>CURE Deviation (%)</b>	19	5	6	21	0	0	0	1
	<b>Validation Factor (VF)</b>	1.033	1.024	1.031	1.032	0.946	0.943	0.944	0.944
<b>INJURY &amp; FATAL CRASHES</b>	<b>MPB</b>	0.063	0.039	0.040	0.064	-0.002	0.002	-0.001	-0.002
	<b>MAD</b>	0.629	0.623	0.624	0.629	0.248	0.246	0.247	0.246
	<b>MSPE</b>	0.930	0.911	0.913	0.930	0.119	0.121	0.121	0.119
	<b>CURE Deviation (%)</b>	7	2	2	6	1	1	1	1

**Validation Factor  
(VF)**

1.058 1.036 1.037 1.059 0.996 1.006 0.998 0.997

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*Notes: MPB: Mean Prediction Bias, MAD: Mean Absolute Deviation, MSPE: Mean Squared Prediction Error*

379 In the case of signalized intersections, the predictive performance of the NB-2 and NB-P  
380 models, based on the measures in **table 6**, was better than the NB-1 model for all crash severity levels.  
381 Similarly, the NB-2 and NB-P models also outperformed the GP model for total crashes, the PDO  
382 crashes, and injury & fatal crashes. Of particular interest was the percentage CURE Deviation, the  
383 values of which were very high for all crash severity levels on the signalized intersections in case of the  
384 NB-1 and the GP models which clearly shows poor prediction performance. For the unsignalized  
385 intersections, the difference among the predictive performance measures across the developed SPFs  
386 was extremely small and somewhat inconsequential. However, a close observation shows that the  
387 performance of the NB-1, NB-P, and GP models was almost similar and slightly better than the NB-2  
388 model. This finding is in a line with the results of the AIC and BIC (**Table 4**) and the LRT (**Table 5**).  
389 To put things into perspective, the GP regression was the best among all four models for each severity  
390 level while the NB-P and NB-1 both performed closely and relatively better than the NB-2 model when  
391 only the NB models were considered.

392 **5. Discussion**

393 The current study investigated the application of the NB-1, NB-2, NB-P, and a GP model in the  
394 development of the SPFs with an aim to find a statistical model capable of improved estimation  
395 accuracy. For this purpose, a number of goodness of fit (likelihood-based) and predictive performance  
396 measures were calculated and compared. Because several variables were used in the development of  
397 the SPFs, a few of them were found to have a significant relationship with the crash occurrence on  
398 intersections.

399 *5.1 Predictor variables of crashes on urban intersections*

400 A positive association between the crash frequency and traffic volume of major and minor  
401 approaches of intersections was found for almost every severity level and every intersection type  
402 considered. This results was in accordance with our expectations. When the number of vehicles entering  
403 and/or leaving the intersection increases, it induces new turning and crossing maneuvers, that results in

404 an increased risk of new conflicts, and those additional conflicts, in some cases, are translated into  
405 actual crashes. Similar results have been reported by many studies (Wang et al., 2019; Barbosa et al.,  
406 2014; Ferreira and Couto, 2013; Vieira Gomes et al., 2012; Miaou and Lord, 2003; Greibe, 2003). It is  
407 important to note that, of all the models developed across all severity levels for both intersection types,  
408 only the traffic flow of a minor approach of unsignalized intersections in the models for injury & fatal  
409 crash was not significant. Since majority of unsignalized intersections were located on local streets  
410 where the traffic volume of minor approaches and the corresponding speed limits were relatively very  
411 low, it is possible that those factors might have contributed to reduced number of fatal and injury crashes  
412 and hence resulted in the insignificance of traffic volume of the minor approaches in models for fatal  
413 and injury crashes.

414         The presence of crosswalk on the minor approaches although significant gave somewhat mixed  
415 results for signalized and unsignalized intersections. In the case of signalized intersections, the  
416 crosswalk on the minor approaches had a significant positive association with crash frequency only  
417 when it was present on both approaches. The crosswalk, however, was not significant when existed on  
418 one of the minor approaches. Further, the estimated coefficients were often more than the double for  
419 intersections with crosswalks on both minor approaches than intersections with crosswalks on only one  
420 approach, although not significant in the later case. A possible explanation could be that, at signalized  
421 intersections with crosswalks on both minor approaches, an existing and/or entering or turning traffic  
422 will have two possible vehicle-pedestrian interactions and thus the chances of involvement in crashes  
423 will be greater, while intersection with a crosswalk on only one minor approach will have one possible  
424 vehicle-pedestrian interaction and hence lower risk of a crash. In the case of unsignalized intersection,  
425 a crosswalk on a minor approach was significant across all developed models when it was present on  
426 only one of the minor approaches. As we know, two crosswalks on the minor approaches were only  
427 present on four-legged intersections but in unsignalized category, majority of intersections were three-  
428 legged which could accommodate only one crosswalk on its minor approach at a time. Thus, the  
429 majority of three legged intersections, and the consequent presence of only one crosswalk on minor  
430 approach possibly contributed significantly to crashes on unsignalized intersections.



431           The intersection skewness was statistically significant in the models for total crashes, and injury  
432 & fatal crashes in case of signalized intersections. The association found indicates that more crashes  
433 were expected on the intersections with higher skewness level than those with no or lower skewness  
434 level. For the recollection of reader, an intersection angle of less than or equal to 75 degrees was higher  
435 skewness level and an intersection angle of greater than 75 degrees was lower skewness or no skewness.  
436 Nightingale et al. (2017) and Harkey (2013) have reported similar results when studying the influence  
437 of the skew angle on the intersection crash frequency. However, Nightingale et al. (2017) studied the  
438 rural intersection. The significance of skewness in the SPFs for signalized intersection could be  
439 attributed to the fact that drivers tend to have greater perception of safety in the case of signalized  
440 intersections than the unsignalized intersections, which reduces an amount decision-making necessary  
441 for safe driving. When such drivers encounter a skewed intersection, this potentially lead to confusion,  
442 which when reinforced by other undesirable characteristics (obstructed views, distorted sight distances,  
443 large intersection area, large turns, etc.) of skewed intersection may result in a crash.

444           The number of approaches of the intersection was found to influence the expected crash  
445 frequency at unsignalized intersections only. This was particularly true in the case of PDO crashes and  
446 total crashes. The intersections with four or more legs were expected to experience more crashes than  
447 the intersections with three legs. This was expected outcome. An increase in the number of  
448 legs/approaches increases the intersection complexity and it invites additional traffic, which could be  
449 related to an increased risk of involvement in a crash.

450           Another significant predictor of crashes at un-signalized intersections was the number of  
451 through lanes of the minor approach. The association between the number of through lanes and the  
452 expected crash frequency was positive, which means more crashes with an increased number of through  
453 lanes. Abdel-Aty and Nawathe (2006) found similar results for urban intersection but their study was  
454 focused on signalized intersections. Zhao et al. (2018) and Kamrani et al. (2017) also reported a  
455 significant positive association between crash frequency and the number of through lanes for  
456 intersections. The number of through lanes on a minor approach also indirectly informs about the size  
457 of an intersection and, thus, correspondingly higher traffic volumes. An intersection with a high number

458 of through lanes on a minor approach could have a higher expected crash frequency because of its large  
459 size that carries more traffic. This result can be generalized only to four-legged unsignalized  
460 intersections since through lanes were only reported for such facility type.

461 We also found some unexpected results, especially the insignificance of the exclusive left and  
462 right turn lanes in the developed models. It was rather opposite to the results of some studies (Al-Kaisy  
463 and Roefaro, 2012; Abdel-Aty and Haleem, 2011; Zhou et al., 2010). The reason may be that the number  
464 of intersections with exclusive left and right turn lanes in the study data was not enough to be significant  
465 in the final models. The influence of the intersection size on crash frequency was also insignificant as  
466 a predictor variable. This might be because other variables, like, the number of through lanes on a minor  
467 approaches and the number of legs/approaches of the intersection, could have acted as proxies for  
468 intersection size in the modelling process.

## 469 *5.2 The appropriate model(s) for crash estimation on urban intersections*

470 In case of signalized intersections, the comparison of likelihood-based and predictive ability-  
471 based measures both revealed that the NB-P and NB-2 models performed better than the NB-1 and GP  
472 models. Similarly when compared for the un-signalized intersections, the GP model was a winner based  
473 on the goodness of fit (likelihood-based measures), however, the performance of the GP model for  
474 predictive ability-based measures was only marginally better than the NB-1 and NB-P models. When  
475 the comparison was made among the NB models only for unsignalized intersections, the NB-P and NB-  
476 1 performed better than the NB-2. Generally, in situations where only the NB models were considered,  
477 the flexible variance structure allowed the NB-P model to outperform the traditional forms, either the  
478 NB-1 or NB. Another observation was that for one type of facility (un-signalized intersections), the  
479 better performing model was the NB-1, while for the other type of facility (signalized intersection), the  
480 better performing model the NB-2 when only the traditional NB models were compared. This finding  
481 suggests that it is necessary to check for an appropriate model form in advance.

482 The use of several functional forms of the NB regression and the equally powerful but a relative  
483 less used GP model in our study revealed that the accurate estimation of crash frequency is subjected  
484 to the selection of the appropriate functional form and model type. The flexible variance structure of

485 the NB model has the ability to improve the estimation accuracy. Further, the study results showed that  
486 it is possible that a model functional form appropriate for one sub-type of the same infrastructure might  
487 not be appropriate for another sub-type of that infrastructure.

## 488 **6. Conclusions**

489 In this study, we developed multiple SPFs by crash severity for urban intersections using the  
490 NB-1, NB-2, NB-P and GP regression in an attempt to obtain a model with a higher estimation accuracy.  
491 The data was obtained for the intersections of Antwerp, Belgium. Only those intersections were  
492 included in modeling for which a sufficient good quality data was available. Major and minor approach  
493 AADT and several other variables related to road infrastructure and geometry were used as the  
494 explanatory variables. Traffic volume was a significant predictor of crash frequency for almost all  
495 developed models and all crash severity levels. Other significant variables include the presence of a  
496 crosswalk on the minor approach and the intersection skewness in the case of signalized intersections.  
497 For unsignalized intersection, the presence of a crosswalk on the minor approach, the number of through  
498 lanes of the minor approach, and the number of legs were significant.

499 For model comparison, two sets of measures were computed. The likelihood-based measures  
500 including the LRT, the AIC and BIC were used for the checking goodness of fit of the models while  
501 the predictive ability-based measures were used for the predictive performance and validation of the  
502 models. The likelihood-based measures showed that the NB-P and NB-2 models performed better than  
503 the NB-1 and GP models in case of the signalized intersections for all crash severity levels. For  
504 unsignalized intersections, however, the GP model was relatively better than the NB models. A  
505 comparison among the NB models showed that the NB-P and NB-1 outperformed the NB-2. The  
506 predictive ability-based measures also confirmed similar results by indicating an improvement in  
507 prediction accuracy in case of the NB-P model and the GP model for signalized and unsignalized  
508 intersections, respectively.

509 The findings of this study showed that all functional forms of the NB model and the GP model  
510 were promising in the estimation of the SPFs for intersections. The developed models irrespective of

511 the functional form or type showed similar results for the influence of the explanatory variables on crash  
512 occurrence. Further, it was shown that the use of the flexible variance structure of the NB-P model  
513 and/or an entirely different GP model could bring an improvement in the estimation accuracy as  
514 indicated by the comparison of the goodness of fit and later verification by the predictive performance  
515 measures applied to the validation dataset.

516 Finally, it is hoped that the outcome of this study add to the knowledge of the SPF estimation  
517 with regard to the selection of the functional form and improvement in the accuracy and reliability of  
518 the crash estimates. Nonetheless, a future research efforts can focus on investigating the applications of  
519 the NB-P model to several other facility types or using the NB-P model in conjunction with other  
520 techniques, for instance, exploring the functional forms of the GP model of which a traditional form  
521 called GP-1 has already been used in this study.

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## **References**

- 526 AASHTO, 2014. Highway Safety Manual (HSM), First Edition, 2014 Supplement, American  
527 Association of State and Highway Transportation Officials, Washington DC.
- 528 AASHTO, 2010. Highway Safety Manual (HSM), First Edition, American Association of State and  
529 Highway Transportation Officials, Washington DC.
- 530 Abdel-Aty, M., Haleem, K., 2011. Analyzing angle crashes at unsignalized intersections using machine  
531 learning techniques. *Accid. Anal. Prev.* 43 1 , 461–470. doi:10.1016/j.aap.2010.10.002
- 532 Abdel-Aty, M., Nawathe, P., 2006. A Novel Approach for Signalized Intersection Crash Classification  
533 and Prediction. *Adv. Transp. Stud.* 9.
- 534 Abdel-Aty, M., Radwan, A.E., 2000. Modeling traffic accident occurrence and involvement. *Accid.*  
535 *Anal. Prev.* 32 5 , 633–642. doi:10.1016/S0001-4575(99)00094-9
- 536 Abdel-Aty, M.A., Lee, J., Eluru, N., Cai, Q., Al Amili, S., Alarifi, S., 2016. Enhancing and Generalizing  
537 the Two-Level Screening Approach Incorporating the Highway Safety Manual (HSM)  
538 Methods, Phase 2 (No. BDV-24-977-06). Florida Department of Transportation.
- 539 Al-Kaisy, A., Roefaro, S., 2012. Channelized right-turn lanes at signalized intersections: the U.S.  
540 experience 13.
- 541 Barbosa, H., Cunto, F., Bezerra, B., Nodari, C., Jacques, M.A., 2014. Safety performance models for  
542 urban intersections in Brazil. *Accid. Anal. Prev.* 70, 258–266. doi:10.1016/j.aap.2014.04.008
- 543 Cafiso, S., Di Silvestro, G., Di Guardo, G., 2012. Application of highway safety manual to Italian  
544 divided multilane highways. *Procedia - Soc. Behav. Sci., SIIV-5th International Congress -*  
545 *Sustainability of Road Infrastructures 2012* 53, 910–919. doi:10.1016/j.sbspro.2012.09.940

546 Cameron, A.C., Trivedi, P.K., 1986. Econometric models based on count data: Comparisons and  
547 applications of some estimators and tests. *J. Appl. Econom.* 1 1 , 29–53.

548 Chang, G.-L., Xiang, H., 2003. The Relationship between Congestion Levels and Accidents. Maryland  
549 State Highway Administration.

550 Dixon, K., Monsere, C., Avelar, R., Barnett, J., Escobar, P., Kothuri, S., Wang, Y., 2015. Improved  
551 safety performance functions for signalized intersections (No. FHWA-OR-RD-16-03). Oregon  
552 Department of Transportation.

553 Dong, C., Richards, S.H., Clarke, D.B., Zhou, X., Ma, Z., 2014. Examining signalized intersection crash  
554 frequency using multivariate zero-inflated Poisson regression. *Saf. Sci.* 70, 63–69.  
555 doi:10.1016/j.ssci.2014.05.006

556 Elvik, R., Sagberg, F., Langeland, P.A., 2019. An analysis of factors influencing accidents on road  
557 bridges in Norway. *Accid. Anal. Prev.* 129, 1–6. doi:10.1016/j.aap.2019.05.002

558 Famoye, F., Singh, K.P., Wulu, J.T., 2004. On the generalized Poisson regression model with an  
559 application to accident data. *J. Data Sci.* 2, 287–295.

560 Ferreira, S., Couto, A., 2013. Traffic flow-accidents relationship for urban intersections on the basis of  
561 the translog function. *Saf. Sci.* 60, 115–122. doi:10.1016/j.ssci.2013.07.007

562 Gates, T., Savolainen, P., Avelar, R., Geedipally, S., Lord, D., Anthony, I., Stapleton, S., 2018. Safety  
563 performance functions for rural road segments and rural intersections in Michigan | Blurbs New  
564 | Blurbs | Main (No. SPR-1645). Michigan Department of Transportation.

565 Geedipally, S.R., Lord, D., 2010. Investigating the effect of modeling single-vehicle and multi-vehicle  
566 crashes separately on confidence intervals of Poisson–gamma models. *Accid. Anal. Prev.* 42 4  
567 , 1273–1282. doi:10.1016/j.aap.2010.02.004

568 Giuffrè, O., Granà, A., Giuffrè, T., Marino, R., Marino, S., 2014. Estimating the safety performance  
569 function for urban unsignalized four-legged one-way intersections in Palermo, Italy. *Arch. Civ.  
570 Eng.* 60 1 , 41–54. doi:10.2478/ace-2014-0002

571 Greene, W., 2008. Functional forms for the negative binomial model for count data. *Econ. Lett.* 99 3 ,  
572 585–590. doi:10.1016/j.econlet.2007.10.015

573 Greibe, P., 2003. Accident prediction models for urban roads. *Accid. Anal. Prev.* 35 2 , 273–285.  
574 doi:10.1016/S0001-4575(02)00005-2

575 Haleem, K., Abdel-Aty, M., 2010. Examining traffic crash injury severity at unsignalized intersections.  
576 *J. Safety Res.* 41 4 , 347–357. doi:10.1016/j.jsr.2010.04.006

577 Hardin, J.W., Hilbe, J.M., 2018. *Generalized Linear Models and Extensions*, Second Edition. Stata  
578 Press.

579 Harkey, D.L., 2013. Effect of Intersection Angle on the Safety of Intersections. North Carolina State  
580 University.

581 Hauer, E., 2015. *The Art of Regression Modeling in Road Safety*. Springer International Publishing.  
582 doi:10.1007/978-3-319-12529-9

583 Hauer, E., Bamfo, J., 1997. Two tools for finding what function links the dependent variable to the  
584 explanatory variables., in: *ICTCT 1997 Conference*. Lund, p. 18.

585 Hilbe, J.M., 2011. *Negative Binomial Regression*, Second. ed. Cambridge university press, New York.  
586 doi:10.1017/CBO9780511973420

587 Ismail, N., Jemain, A.A., 2007. Handling overdispersion with negative binomial and generalized  
588 Poisson regression models. *Casualty Actuar. Soc. Forum Citeseer*.

589 Ismail, N., Zamani, H., 2013. Estimation of claim count data using negative binomial, generalized  
590 Poisson, zero-inflated negative binomial and zero-inflated generalized Poisson regression  
591 models. *Casualty Actuar. Soc. E-Forum* 41 20 , 1–28.

592 Janstrup, K.H., 2016. Statistical modelling of the frequency and severity of road accidents. DTU  
593 Transport.

594 Joe, H., Zhu, R., 2005. Generalized Poisson distribution: the property of mixture of Poisson and  
595 comparison with negative binomial distribution. *Biom. J. Biom. Z.* 47 2 , 219–229.  
596 doi:10.1002/bimj.200410102

597 Joshua, S.C., Garber, N.J., 1990. Estimating truck accident rate and involvements using linear and  
598 Poisson regression models. *Transp. Plan. Technol.* 15 1 , 41–58.  
599 doi:10.1080/03081069008717439

600 Kamrani, M., Wali, B., Khattak, A.J., 2017. Can data generated by connected vehicles enhance safety?:  
601 Proactive approach to intersection safety management. *Transp. Res. Rec.* 2659 1 , 80–90.  
602 doi:10.3141/2659-09

603 Liu, C., Sharma, A., 2018. Using the multivariate spatio-temporal Bayesian model to analyze traffic  
604 crashes by severity. *Anal. Methods Accid. Res.* 17, 14–31. doi:10.1016/j.amar.2018.02.001

605 Lord, D., Mannering, F., 2010. The statistical analysis of crash-frequency data: A review and  
606 assessment of methodological alternatives. *Transp. Res. Part Policy Pract.* 44 5 , 291–305.  
607 doi:10.1016/j.tra.2010.02.001

608 Lord, D., Park, B.-J., 2015. Appendix D: Negative Binomial Regression Models and Estimation  
609 Methods.

610 Mannering, F.L., Bhat, C.R., 2014. Analytic methods in accident research: Methodological frontier and  
611 future directions. *Anal. Methods Accid. Res.* 1 0 .

612 Melliana, A., Setyorini, Y., Eko, H., Rosi, S., Puhadi, 2013. The comparison of generalized Poisson  
613 regression and negative binomial regression methods in overcoming overdispersion. URL  
614 /paper/The-Comparison-Of-Generalized-Poisson-Regression-In-Melliana-  
615 Setyorini/9e679999bb7585e9965b271dc5afc462b8572114 (accessed 8.21.20).

616 Miaou, S.-P., 1994. The relationship between truck accidents and geometric design of road sections:  
617 Poisson versus negative binomial regressions. *Accid. Anal. Prev.* 26 4 , 471–482.  
618 doi:10.1016/0001-4575(94)90038-8

619 Miaou, S.-P., Lord, D., 2003. Modeling Traffic Crash-Flow Relationships for Intersections: Dispersion  
620 Parameter, Functional Form, and Bayes Versus Empirical Bayes Methods: *Transp. Res. Rec.*  
621 doi:10.3141/1840-04

622 Miaou, S.-P., Lum, H., 1993. Modeling vehicle accidents and highway geometric design relationships.  
623 *Accid. Anal. Prev.* 25 6 , 689–709. doi:10.1016/0001-4575(93)90034-T

624 Nambuusi, B.B., Brijs, T., Hermans, E., 2008. A review of accident prediction models for road  
625 intersections. UHasselt.

626 Nesheli, M.M., Che Puan, O., Roshandeh, A.M., 2009. Evaluation of effect of traffic signal coordination  
627 system on congestion. *WSEAS Trans. Adv. Eng. Educ.* 6 7 , 203–212.

628 Nightingale, E., Parvin, N., Seiberlich, C., Savolainen, P.T., Pawlovich, M., 2017. Investigation of skew  
629 angle and other factors influencing crash frequency at high-speed rural intersections. *Transp.*  
630 *Res. Rec.* 2636 1 , 9–14. doi:10.3141/2636-02

631 Oh, J., Kim, E., Kim, M., Choo, S., 2010. Development of conflict techniques for left-turn and cross-  
632 traffic at protected left-turn signalized intersections. *Saf. Sci.* 48 4 , 460–468.  
633 doi:10.1016/j.ssci.2009.12.011

634 Oh, J., Lyon, C., Washington, S., Persaud, B., Bared, J., 2003. Validation of FHWA Crash Models for  
635 Rural Intersections: Lessons Learned. *Transp. Res. Rec.* 1840 1 , 41–49. doi:10.3141/1840-05

636 Okamoto, H., Koshi, M., 1989. A method to cope with the random errors of observed accident rates in  
637 regression analysis. *Accid. Anal. Prev.* 21 4 , 317–332. doi:10.1016/0001-4575(89)90023-7

638 Park, B.-J., 2010. Application of finite mixture models for vehicle crash data analysis. Texas A&M  
639 University.

640 Park, B.-J., Lord, D., 2009. Application of finite mixture models for vehicle crash data analysis. *Accid.*  
641 *Anal. Prev.* 41 4 , 683–691. doi:10.1016/j.aap.2009.03.007

642 Persaud, B., Saleem, T., Faisal, S., Lyon, C., Chen, Y., Sabbaghi, A., 2012. Adoption of Highway Safety  
643 Manual Predictive Technologies for Canadian Highways, in: 2012 Conference of the  
644 Transportation Association of Canada. Fredericton, New Brunswick.

645 Roshandeh, A.M., Agbelie, B.R.D.K., Lee, Y., 2016. Statistical modeling of total crash frequency at  
646 highway intersections. *J. Traffic Transp. Eng. Engl. Ed.* 3 2 , 166–171.  
647 doi:10.1016/j.jtte.2016.03.003

648 Roshandeh, A.M., Levinson, H.S., Li, Z., Patel, H., Zhou, B., 2014. New methodology for intersection  
649 signal timing optimization to simultaneously minimize vehicle and pedestrian delays. *J. Transp.*  
650 *Eng.* 140 5 , 04014009. doi:10.1061/(ASCE)TE.1943-5436.0000658

651 Srinivasan, R., Carter, D., 2011. Development of Safety Performance Functions for North Carolina (No.  
652 FHWA/NC/2010-09). North Carolina Department of Transportation.

653 Tay, R., 2015. A random parameters probit model of urban and rural intersection crashes. *Accid. Anal.*  
654 *Prev.* 84, 38–40. doi:10.1016/j.aap.2015.07.013

655 Vieira Gomes, S., Geedipally, S.R., Lord, D., 2012. Estimating the safety performance of urban  
656 intersections in Lisbon, Portugal. *Saf. Sci.* 50 9 , 1732–1739. doi:10.1016/j.ssci.2012.03.022

657 Wang, K., Ivan, J.N., Ravishanker, N., Jackson, E., 2017. Multivariate poisson lognormal modeling of  
658 crashes by type and severity on rural two lane highways. *Accid. Anal. Prev.* 99, 6–19.  
659 doi:10.1016/j.aap.2016.11.006

660 Wang, K., Zhao, S., Jackson, E., 2019. Functional forms of the negative binomial models in safety  
661 performance functions for rural two-lane intersections. *Accid. Anal. Prev.* 124, 193–201.  
662 doi:10.1016/j.aap.2019.01.015

663 Wang, X., Abdel-Aty, M., Nevarez, A., Santos, J.B., 2008. Investigation of safety influence area for  
664 four-legged signalized intersections: Nationwide survey and empirical inquiry. *Transp. Res.*  
665 *Rec.* doi:10.3141/2083-10

666 Washington, S., Karlaftis, M.G., Mannering, F., Anastasopoulos, P.Ch., 2010. *Statistical and*  
667 *Econometric Methods for Transportation Data Analysis*, Second. ed. Chapman and Hall/CRC.

668 Winkelmann, R., 2008. *Econometric Analysis of Count Data*, 5th ed. Springer-Verlag, Berlin  
669 Heidelberg. doi:10.1007/978-3-540-78389-3

670 Wong, S.C., Sze, N.N., Li, Y.C., 2007. Contributory factors to traffic crashes at signalized intersections  
671 in Hong Kong. *Accid. Anal. Prev.* 39 6 , 1107–1113. doi:10.1016/j.aap.2007.02.009

672 Wu, Y., Abdel-Aty, M., Cai, Q., Lee, J., Park, J., 2018. Developing an algorithm to assess the rear-end  
673 collision risk under fog conditions using real-time data. *Transp. Res. Part C Emerg. Technol.*  
674 87, 11–25. doi:10.1016/j.trc.2017.12.012

675 Wu, Z., Sharma, A., Mannering, F.L., Wang, S., 2013. Safety impacts of signal-warning flashers and  
676 speed control at high-speed signalized intersections. *Accid. Anal. Prev.* 54, 90–98.  
677 doi:10.1016/j.aap.2013.01.016

678 Yang, Z., Hardin, J.W., Addy, C.L., 2009. A score test for overdispersion in Poisson regression based  
679 on the generalized Poisson-2 model. *J. Stat. Plan. Inference* 139 4 , 1514–1521.  
680 doi:10.1016/j.jspi.2008.08.018

681 Zhao, M., Liu, C., Li, W., Sharma, A., 2018. Multivariate Poisson-lognormal model for analysis of  
682 crashes on urban signalized intersections approach. *J. Transp. Saf. Secur.* 10 3 , 251–265.  
683 doi:10.1080/19439962.2017.1323059

684 Zhou, H., Ivan, J.N., Sadek, A.W., Ravishanker, N., 2010. Safety effects of exclusive left-turn lanes at  
685 unsignalized intersections and driveways. *J. Transp. Saf. Secur.* 2 3 , 221–238.  
686 doi:10.1080/19439962.2010.502613