Optimal labor income taxation and social security programs when private savings are unobservable*

Takao Kataoka† **and Yoshihiro Takamatsu**‡

Abstract: We investigate the optimal labor income taxation in a two-period economy with indivisible labor, stochastic preference/productivity shocks and hidden savings. We calculate the optimal allocation of an example economy and show that it is not implementable under the ordinary labor income tax system. Given this limitation, we propose two social security programs that can supplement the ordinary labor income tax system: one is a compulsory pension insurance system and the other is the provision of a limited times benefit to low-income households. We find that combination of taxation based on current labor income and either of these two supplemental programs makes the optimal allocation implementable in our example economies.

Keywords: Dynamic optimal income taxation, Hidden saving, Indivisible labor, Labor income tax, Social security programs

1. Introduction

By extending Mirrlees' (1971) static framework to dynamic settings, recent optimal income taxation studies have considered a class of economy in which each household's production skill is privately informed and stochastically evolves over time. It is often

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† Professor of Faculty of Commerce, Waseda University

‡ Associate Professor of Faculty of Economics, Meiji Gakuin University

assumed that households' labor income and asset trade are publicly observable (this implies the observability of consumption as well). In such environments, optimal allocations satisfy the so-called reciprocal Euler condition rather than the standard Euler condition (Golosov et al. 2003; Kocherlakota 2005) and are implementable in a market-based economy by imposing taxes on labor and capital.[1] The required income taxation is history-dependent; in other words, the tax rate may depend on the taxpayer's whole history of labor income.[2]

The assumption of observable consumptions/savings, however, may have limited empirical relevance in modern economies. There are a number of ways that a household can secretly transfer purchasing power to the future, such as accumulating durables and foreign assets. (Hereafter, we refer to these as hidden savings.) Thus, we investigate an economy in which households can make hidden savings. In this economy, any allocation that does not satisfy the Euler condition is not implementable and the government can not impose capital taxation. We show that the optimal allocation can be implemented in a market economy with a history-dependent labor income tax.

This history-dependent nature of labor income taxation is, however, rather complicated and rare in practice.[3] It is questionable whether governments in the real world can adopt such full history dependent income tax rules. Recently, Weinzierl (2011) examines partial tax reforms to age-dependent labor income taxation, and shows that, under the assumption of observable consumptions/savings, there are small welfare losses from implementing not full optimal history-dependent income taxation, but age-dependent income taxation. Although we are also interested in practical policy recommendations, we concentrate on policies to implement the optimum and not partial

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[1] Golosov et al. (2007) and Kocherlakota (2010) review the literature in this regard. Kocherlakota (2005) also shows that capital taxation is regressive and that the average tax rate is zero, thereby raising no revenue when preferences are time-separable. Recently, however, Grochulski and Kocherlakota (2010) show that average capital tax rates can be negative when preferences are intertemporally non-separable.

[2] Albanesi and Sleet (2006) show that the history-dependent taxes can be replaced with taxes on current labor and wealth in each period. Their conclusion relies critically on the assumption that the skill shock process is i.i.d., however.

[3] There are some exceptions include the tax-favoring treatment of retirement savings/income and mortgage loan deductions. Overall, however, income taxes make little use of historical-based structures.

reforms.

In this paper, we pay attentions to a more practical income tax system which is solely based on current labor income. The optimal allocation is unimplementable under this restricted labor income tax rule if no other policy instruments are available. Thus we introduce two supplemental social security programs, which are similar to actual Social Security systems in developed countries. The two supplements are a pension insurance system and a limited times benefit to low-income households.

The pension system is compulsory for young workers and benefits participants if they do not work in old age. The benefit to low-income households is age-dependent and the number of times a household can receive it throughout its lifetime is limited. We show that the combination of tax on current labor income and either of these two supplemental programs makes the optimum implementable.

Grochulski and Kocherlakota (2010) show that, under the assumption of observable consumptions/savings, the optimal allocation can be implemented through a combination of three policy instruments that they term social security systems, namely a flat tax on current labor income until retirement, a retrospective capital income tax only at retirement that is dependent on labor income history, and a history-contingent payment after retirement. Although our paper relates to theirs, we allow for hidden savings.

Because of the difficulty of characterizing optimal policy in our economy, most of our analysis relies on numerical simulations of a rather simple two-period economy with indivisible labor.[4] Diamond and Mirrlees (1978) and Golosov and Tsyvinski (2006, 2007) use similar models in order to investigate disability insurance.[5] These papers assume that disability is an absorbing state; in other words, once disabled, labor status is no longer a matter of choice. Consequently, the possibility of disability benefits

[4] Indivisible labor means that a household's labor decision is discrete, namely either to work the given hours or to not work at all. Diamond (1980) develops this labor supply model in the optimal income taxation literature.

[5] Diamond and Mirrlees (1978) conclude that the optimal social insurance plan needs to be supplemented by an interest income tax to discourage private saving. Golosov and Tsyvinski (2006, 2007) propose introducing an asset-tested disability insurance system in which a transfer is paid only if an agent has assets below a specified maximum because those who anticipate making false claims would accumulate assets to help smooth consumption.

discouraging recovered individuals from leaving disability rolls is not considered.

By contrast, the "disability" or unproductive status in our economy may not be permanent (i.e., there may be some probability of recovery). In our example economy, the old households who were productive and worked when young should work if and only if they are currently productive at the optimal incentive-compatible allocation. This feature, however, may not apply to the old households who did not work when young. In fact, such old households must work irrespective of their current productivities at the optimal incentive-compatible allocation of our example economy. This lack of sensitivity of labor supply to current productivity status has a negative welfare effect from the viewpoint of production efficiency but eases the incentive compatibility (IC) constraints. This asymmetric treatment between older groups who worked when young and those who did not explains why the two supplemental programs discussed herein help. For example, older groups who worked and joined the pension system when young are protected against productivity shocks, meaning that they can quit their jobs if they become unproductive in old age. Older groups who did not work when young, on the other hand, must work since they did not join the pension system.

The remainder of the paper is organized as follows. In section 2, we describe the model setup and define the optimal assignment under the direct mechanism where the government assigns labor supply and disposable income conditional on the reported type history. We compare the optimal assignment and market equilibrium without taxation in an example economy. In section 3, we show the government can implement this optimal assignment in a market economy with labor income taxation based on labor income history. Then, we show that the optimal assignment is unimplementable under taxation based on current labor income. In section 4, we describe the two supplemental programs and show that a combination of tax based on current labor income and either of these two programs makes the optimal assignment implementable. We conclude in section 5.

2. Model

We consider a two-period dynamic economy populated by one unit of continuous households. Two nondurable commodities, a consumption good and effective labor (labor for short), exist in each period $t \in \{0,1\}$. Labor is indivisible, meaning that each household's labor supply is 0 or 1, while the consumption good is perfectly divisible. Production technology that transforms one unit of labor into one unit of consumption good in the same period is available. There is an external financial market in which risk-free bonds are anonymously traded at a fixed interest rate r.

Households are identical except for the preference/productivity types that affects the disutility associated with labor supply. There are two possible types, namely H and L, in each period. Let $c_t \in \mathbb{R}_{+}$, $y_t \in \{0,1\}$, and $\theta_t \in \{H,L\}$ denote consumption, labor supply, and type of a household in period t, respectively. We assume that the ex-post utility is an additively separable function over consumption and labor supply in both periods,

$$U(c_0) - V(y_0,\theta_0) + \beta(U(c_1) - V(y_1,\theta_1)),$$

where $\beta \in (0,1)$, $U : \mathbb{R}_{+} \to \mathbb{R}$ and $V : \{0,1\} \times \{H,L\} \to \mathbb{R}$ represent the discount factor, the utility associated with consumption, and the disutility associated with labor supply status, respectively. We assume U is continuously differentiable, strictly increasing, strictly concave, and that it satisfies the boundary condition $\lim_{c \to 0} U'(c) = \infty$. Disutilities of "not-working," $V(0,H)$ and $V(0,L)$, are normalized to zero. We further assume that disutilities of "working," $V(1,L)$ and $V(1,H)$, are positive and $V(1,L)$ is strictly larger than $V(1,H)$, that is, type L households are "unproductive" in the sense that they suffer more from one unit of labor supply than do type H households.

The preference type stochastically evolves over time, and the households privately learn their type at the beginning of each period. Hereafter, we refer to period 0 households with type $\theta_0 \in \{H,L\}$ as young θ_0, and period 1 households with type history $(\theta_0,\theta_1) \in \{H,L\}^2$ as old (θ_0,θ_1). We denote $\pi_0(\theta_0) \in (0,1)$ as the population size of young θ_0 (where $\pi_0(H) + \pi_0(L) = 1$). Young θ_0 becomes old (θ_0,θ_1) in the next period with transition probability $\pi_1(\theta_0,\theta_1) \in [0,1]$ (where $\sum_{\theta_1} \pi_1(\theta_0,\theta_1)$ is 1 for $\theta_0 = H,L$).

The anonymity of financial trade implies that the government can not directly assign individual consumption level. This section considers a direct mechanism in which the government assigns a combination of disposable income and labor supply to the households conditional on reported type history.

A disposable income assignment is represented by $D = (D_0, D_1)$ where $D_0 : \{H,L\} \to \mathbb{R}$ and $D_1 : \{H,L\}^2 \to \mathbb{R}$ denote disposable income of young and old, respectively. Let D^{θ_0} denote the restriction of D to the households who report θ_0 at young. For example, D^H is the pair of $D_0^H = D_0(H) \in \mathbb{R}$ and $D_1^H : \{H,L\} \to \mathbb{R}$ defined by $D_1^H(\theta_1) = D_1(H,\theta_1)$. We call D^{θ_0} a disposable income schedule for young θ_0 and we may write $D = (D^H, D^L)$. Similarly, a labor supply assignment is represented by $Y = (Y_0 : \{H,L\} \to \{0,1\}, Y_1 : \{H,L\}^2 \to \{0,1\})$, and we may write $Y = (Y^H, Y^L)$ where Y^{θ_0} denotes a labor supply schedule for young θ_0. Note that if we repeat this economy in the manner of overlapping generations models with no population growth, then

$$\sum_{\theta_0}\pi_0(\theta_0)\left(Y_0(\theta_0)+\sum_{\theta_1}\pi_1(\theta_0,\theta_1)Y_1(\theta_0,\theta_1)\right),$$

represents GDP.

A pair of D and Y is called an assignment. An assignment must satisfy the following resource feasibility constraint,

$$\sum_{\theta_0}\pi_0(\theta_0)\left(Y_0(\theta_0)-D_0(\theta_0)+\frac{1}{1+r}\sum_{\theta_1}\pi_1(\theta_0,\theta_1)(Y_1(\theta_0,\theta_1)-D_1(\theta_0,\theta_1))\right)\geq 0. \tag{1}$$

Suppose a household does not make false reports for a while. Given D^{θ_0}, a young θ_0 chooses its private saving s to maximize

$$U(D_0(\theta_0)-s)+\beta\sum_{\theta_1}\pi_1(\theta_0,\theta_1)U(D_1(\theta_0,\theta_1)+(1+r)s).$$

This is a strictly concave problem and the first-order condition uniquely determines the optimal saving. Let $EU(D^{\theta_0},\theta_0)$ denote the maximized expected utility of consumption. Given Y^{θ_0}, young θ_0's expected disutility of labor, $EV(Y^{\theta_0},\theta_0)$, is

$$EV(Y^{\theta_0},\theta_0)\equiv V(Y_0(\theta_0),\theta_0)+\beta\sum_{\theta_1}\pi_1(\theta_0,\theta_1)V(Y_1(\theta_0,\theta_1),\theta_1).$$

Since the individual types are private information, assignment (D,Y) must have no incentive to make a false report. Let $\Delta=(\Delta_0,\Delta_1)$ be a reporting strategy of a young household after observing its true type, where $\Delta_0\in\{H,L\}$ is the reporting type at period 0 and $\Delta_1:\{H,L\}\to\{H,L\}$ is the plan of reporting types in period 1 conditional on the true type observed at the period.

Suppose young θ_0 chooses a reporting strategy Δ, then, its effective disposable income is $D_0(\Delta_0)$ at young and $D_1(\Delta_0,\Delta_1(\theta_1))$ at old. Let D^{Δ} denote this effective disposable income schedule. Similarly, its effective labor supply schedule is Y^{Δ}. Now the incentive compatibility requires the following conditions,

$$EU(D^{\theta_0},\theta_0)-EV(Y^{\theta_0},\theta_0)\geq EU(D^{\Delta},\theta_0)-EV(Y^{\Delta},\theta_0), \tag{2}$$

for any θ_0 and any non-truth telling reporting strategy Δ. There are 14 incentive compatibility conditions in general.[6] If, however, a labor assignment for some old does

[6] There are 2 strategies for Δ_0. At old, there are 4 conditional strategies for Δ_1. Thus, There are 8 strategies for each young θ_0 and one of which is the truth telling strategy. As a result, there are 7 false reporting strategies for each young θ_0.

not depend on the reporting type at old, that is, $Y_1(\theta_0, H) = Y_1(\theta_0, L)$ for some θ_0, then the incentive compatibility conditions for young θ_0 and three non-truth telling reporting strategies satisfying $\Delta_0 = \theta_0$ are summarized by

$$D_1(\theta_0, H) = D_1(\theta_0, L).$$

We refer to an assignment satisfying (1) and (2) as a feasible assignment.

An assignment is said to be optimal if the government maximizes the unconditional expected utility,

$$\sum_{\theta_0} \pi_0(\theta_0)(EU(D^{\theta_0}, \theta_0) - EV(Y^{\theta_0}, \theta_0)),$$

subject to (1) and (2).

Note that the optimal assignment is not unique for the government because of the Ricardian equivalence. To see this, let (D, Y) be a feasible assignment and consider the following disposable income assignment D' defined by

$$D_0'(\theta_0) = D_0(\theta_0) + q(\theta_0), \quad D_1'(\theta_0, \theta_1) = D_1(\theta_0, \theta_1) - (1+r)q(\theta_0), \quad \text{for all } \theta_0, \theta_1,$$

where q is an arbitrary function from $\{H, L\}$ to $\mathbb{R}$. It is obvious that assignment (D', Y) is also feasible, because this arrangement can be undone by adjusting anonymous private savings. This means, any feasible assignment can be normalized such that no household makes non-trivial saving. A normalized feasible assignment (D, Y) must satisfy the following Euler (no saving) conditions:

$$U'(D_0(\theta_0)) = \beta(1+r)\sum_{\theta_1} \pi_1(\theta_0, \theta_1)U'(D_1(\theta_0, \theta_1)), \quad \text{for all } \theta_0. \tag{3}$$

Now, we define that (D, Y) is an optimal normalized assignment if it maximizes the unconditional expected utility subject to (1), (2), and (3). Let $(\tilde{D}, \tilde{Y})$ denote the optimal normalized assignment. Given $(\tilde{D}, \tilde{Y})$, general optimal assignment $(D, \tilde{Y})$ satisfies the following condition for some (s^H, s^L),

$$\begin{cases} D_0(\theta_0) = \tilde{D}_0(\theta_0) + s^{\theta_0}, & \text{for all } \theta_0 \\ D_1(\theta_0, \theta_1) = \tilde{D}_1(\theta_0, \theta_1) - (1+r)s^{\theta_0}, & \text{for all } \theta_0, \theta_1. \end{cases} \tag{4}$$

The number of control variables and constraints makes the analysis of the optimal normalized assignment difficult while this is a standard Kuhn-Tucker problem. In the following sections, thus, we specify the utility function and stochastic process, and study numerically obtained $(\tilde{D}, \tilde{Y})$.[7]

2.1. An example economy

We consider the following example economy. In period 0, 80% of households are productive, that is, $\pi_0(H)=0.8$, $\pi_0(L)=0.2$, and the transition probability matrix of the types is

$$\begin{pmatrix} \pi_1(H,H) & \pi_1(H,L) \\ \pi_1(L,H) & \pi_1(L,L) \end{pmatrix} = \begin{pmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{pmatrix}.$$

With these parameters, 40% of old households are unproductive. We assume the logarithmic utility function for consumption, $U(c)=\ln(c)$, and that the disutility of work is $(V(1,H),V(1,L))=(-\ln(1/2),-\ln(1/4))$. If a productive household works and consumes all of its own products, utility in the period is $\ln(1)-V(1,H)=\ln(1/2)$. This amounts to the utility when it does not work and receives a half of the worker's product. Thus, the disutility of work for productive households amounts to a half of their products. Similarly, the disutility of work for unproductive households amounts to three-quarters of their products. Moreover, one period corresponds to 25 years and both of the real interest rate and the annual discount rate are 2%, that is, $\beta=0.98^{25}\approx 0.60$ and $1+r=0.98^{-25}\approx 1.66$.

Table 1 compares the market equilibrium without tax, the first best allocation, and the optimal assignment. The table has four parts and the first part shows the parameter specifications. The second column is the benchmark case and the followings have one of parameters different from the benchmark. The second part shows the market equilibrium without tax. In the table, $C_0(\theta_0)$ and $C_1(\theta_0,\theta_1)$ denote the consumption of young θ_0 and old (θ_0,θ_1), respectively. Y^L, $C_0(L)$, $C_1(L,H)$ and $C_1(L,L)$ at the benchmark case here, for example, claims that young L works and consumes 0.655 at young, works and consumes 1.572 at old if productive, and does not work and consumes 0.572 at old if unproductive. η^{θ_0} denotes the ratio of the value of consumption schedule to the value of labor supply schedule of young θ_0, that is,

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[7] Labor indivisibility makes the choice set of Y finite and simplifies the numerical procedure. Given a choice of Y, the problem to find the best D becomes a maximization problem with 6 control variables, three equality constraints (budget constraint and two Euler conditions), and 14 non-negative conditions of incentive compatibility. We solve it for each choice of Y and compare the solutions to find the optimal normalized assignment.

Table 1. Market equilibrium without tax, the first best allocation, and the optimal normalized assignment

Parameter specifications						
$V(1,H)$	$-\ln(1/2)$	$-\ln(2/3)$	$-\ln(1/2)$	$-\ln(1/2)$	$-\ln(1/2)$	$-\ln(1/2)$
$V(1,L)$	$-\ln(1/4)$	$-\ln(1/4)$	$-\ln(1/3)$	$-\ln(1/4)$	$-\ln(1/4)$	$-\ln(1/4)$
$\pi_0(H)$	0.8	0.8	0.8	0.9	0.8	0.8
$\pi_1(H,H)$	0.7	0.7	0.7	0.7	0.8	0.7
$\pi_1(L,L)$	0.8	0.8	0.8	0.8	0.8	0.9
Market equilibrium without tax						
Y^H	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)
$C_0(H)$	1	1	1	1	1	1
$C_1(H,H)$	1	1	1	1	1	1
$C_1(H,L)$	1	1	1	1	1	1
η^H	1.0	1.0	1.0	1.0	1.0	1.0
Y^L	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,0)	(1,1,0)
$C_0(L)$	0.655	0.655	1	0.655	0.655	0.639
$C_1(L,H)$	1.572	1.572	1	1.572	1.572	1.599
$C_1(L,L)$	0.572	0.572	1	0.572	0.572	0.599
η^L	1.0	1.0	1.0	1.0	1.0	1.0
GDP (z^e)	1.84	1.84	2.0	1.92	1.84	1.82
Welfare (w^e)	−1.411	−1.077	−1.29	−1.324	−1.378	−1.415
First best allocation						
Y^H	(1,1,0)	(1,1,0)	(1,1,0)	(1,1,0)	(1,1,0)	(1,1,0)
$C_0(H)$	0.725	0.725	0.91	0.806	0.755	0.717
$C_1(H,H)$	0.725	0.725	0.91	0.806	0.755	0.717
$C_1(H,L)$	0.725	0.725	0.91	0.806	0.755	0.717
η^H	0.817	0.817	1.025	0.908	0.816	0.808
Y^L	(0,1,0)	(0,1,0)	(1,1,1)	(0,1,0)	(0,1,0)	(0,1,0)
$C_0(L)$	0.725	0.725	0.91	0.806	0.755	0.717
$C_1(L,H)$	0.725	0.725	0.91	0.806	0.755	0.717
$C_1(L,L)$	0.725	0.725	0.91	0.806	0.755	0.717
η^L	9.628	9.628	0.91	10.707	10.028	19.057
GDP (z^f)	1.4	1.4	1.76	1.55	1.48	1.38
z^f/z^e	0.761	0.761	0.88	0.807	0.804	0.758
Welfare (w^f)	−1.322	−0.987	−1.283	−1.242	−1.29	−1.33
Δw^f	0.057	0.057	0.005	0.053	0.056	0.054
Optimal normalized assignment						
$Y^H=\tilde{Y}^H$	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,0)	(1,1,0)
$C_0(H)=\tilde{D}_0(H)$	0.787	0.788	1	0.816	0.83	0.787
$C_1(H,H)=\tilde{D}_1(H,H)$	1.072	0.92	1	1.111	1.042	1.072
$C_1(H,L)=\tilde{D}_1(H,L)$	0.485	0.591	1	0.503	0.457	0.485
η^H	0.933	0.903	1.0	0.967	0.936	0.933
$Y^L=\tilde{Y}^L$	(0,1,1)	(0,1,1)	(1,1,1)	(0,1,1)	(0,1,1)	(0,1,1)
$C_0(L)=\tilde{D}_0(L)$	0.613	0.722	1	0.636	0.612	0.613
$C_1(L,H)=\tilde{D}_1(L,H)$	0.613	0.722	1	0.636	0.612	0.613
$C_1(L,L)=\tilde{D}_1(L,L)$	0.613	0.722	1	0.636	0.612	0.613
η^L	1.63	1.918	1.0	1.69	1.627	1.63
GDP$(\tilde{z})$	1.56	1.56	2.0	1.63	1.64	1.56
$\tilde{z}/z^e$	0.848	0.848	1.0	0.849	0.891	0.857
Welfare$(\tilde{w})$	−1.369	−1.004	−1.29	−1.29	−1.339	−1.378
$\Delta\tilde{w}$	0.027	0.046	0	0.021	0.025	0.024

$$\eta^{\theta_0} = \frac{C_0(\theta_0) + \sum_{\theta_1} \frac{\pi_1(\theta_0,\theta_1) C_1(\theta_0,\theta_1)}{1+r}}{Y_0(\theta_0) + \sum_{\theta_1} \frac{\pi_1(\theta_0,\theta_1) Y_1(\theta_0,\theta_1)}{1+r}}.$$

Since no income redistribution takes place in the market equilibrium without tax, both of η^H and η^L are unity here. This absence of redistribution leads, in the most cases, all the households except for old (L,L) to work. Only old (L,L) does not work because young L expects to stay unproductive in old age with a high probability, and thus saves more in order to prepare for the future low productivity.

The third part of the table shows the first best allocation which maximizes the unconditional expected utility subject to (1) alone. The first best consumption assignment does not depend on preference type because of the consumption smoothing. Productive households tend to work more due to the production efficiency. In fact, households work if and only if they are productive in all the cases except for the fourth column. This fact and the consumption smoothing make the first best allocation incentive incompatible. The first best allocation has a higher welfare but lower GDP comparing to the market equilibrium without tax. The welfare difference, $0.099 = -1.322 + 1.411$, corresponds to 5.7% increase of labor productivity and Δw^f shows this number.[8] GDP at the first best allocation is 76.1% of the GDP at the market equilibrium without tax in the benchmark case.

The last part of the table shows the optimal normalized assignment. The incentive compatibility condition makes the welfare level between the market equilibrium and the first best. At the benchmark case, the welfare difference between the market equilibrium and the optimal assignment amounts to 2.7% change of the labor productivity (see the row of $\Delta\tilde{w}$). Consumption schedule is not smooth here. For example, consumption of old (H,H), 1.072 at the benchmark, is higher than that of old (H,L), 0.485, to prevent the former from pretending the latter. In all the cases except for the fourth column, the government assigns labor supply to unproductive old (L,L) as well as all the productive households. Recall that the first best assigns labor to the productive households only in these cases. If the government chooses this first best labor assignment, it has to set the disposable income of old (L,H) to be sufficiently higher than that of old (L,L) in order to prevent a false report by old (L,H). The welfare cost created by this consumption

[8] Increase of labor productivity by 5.7% at the market equilibrium causes the proportional increase of consumption schedule and increases the welfare by $(1+\beta)\ln(1.057) = 0.099$.

uncertainty dominates the benefit of the first best labor assignment in these cases. Thus, the optimal normalized assignment offers the same consumption-labor combination to old (L,H) and old (L,L). The same argument does not apply to the income-labor combination of old (H,H) and old (H,L) since young H works more and enjoy larger consumption in old age, which reduces the curvature of U function and welfare gain from the consumption smoothing.

These basic properties of the optimal assignment apply to all the cases except for the fourth column, where the difference of $V(1,H)$ and $V(1,L)$ is small. In this exceptional case, the market equilibrium coincides the optimal assignment so that there is no room for welfare improvement by income redistribution.

One may think that the numbers in Table 1 depends on the parameter specifications. The optimal labor assignment, however, is fairly insensitive to parameter changes. In fact, we calculated the optimal normalized assignment and market equilibrium without tax for all the combinations of parameters $\pi_0(H) \in \{0.7, 0.8, 0.9, 0.99\}$, $\pi_1(H,H) \in \{0.6, 0.7, 0.8, 0.9\}$, $\pi_1(L,L) \in \{0.7, 0.8, 0.9, 0.99\}$, and $(V(1,H), V(1,L))$ is $(-\ln(1/2), -\ln(1/4))$, $(-\ln(1/2), -\ln(1/3))$, $(-\ln(2/3), -\ln(1/4))$, $(-\ln(2/3), -\ln(1/3))$, or $(-\ln(2/3)), -\ln(1/2))$. We found that 160 cases out of 320 reveal features similar to the the fourth column of the table, that is, the optimal labor assignment is equivalent to the market equilibrium labor supply so that there is no role for taxation, while the optimal labor assignment is the following, the same as the benchmark case, in 84 of the other 160 cases.

$$\begin{cases} \tilde{Y}_0(H) = 1, & \tilde{Y}_1(H,H) = 1, \quad \tilde{Y}_1(H,L) = 0 \\ \tilde{Y}_0(L) = 0, & \tilde{Y}_1(L,H) = \tilde{Y}_1(L,L) = 1 \end{cases}. \tag{5}$$

We assume the optimal labor assignment satisfies (5) in the following sections.

3. Implementation of the optimal assignment by history dependent labor income taxation

Section 2 investigated the direct mechanism by which the government assigns a combination of disposable income and labor supply conditional on the reported type history. This section examines how we can implement the optimal assignment in a market-based economy by labor income tax. Labor income taxation in this section may depend not only on labor income at the period but also on labor income history, and is represented by the disposable income policy $X = (X_0, X_1)$ where $X_0 : \{0,1\} \to \mathbb{R}$ and $X_1 : \{0,1\}^2 \to \mathbb{R}$ are disposable income of young and old, respectively. Given X and young θ_0's labor supply schedule Y^{θ_0}, let $X(Y^{\theta_0}) \in \mathbb{R}^3$ denote the implied disposable income schedule of young θ_0, that is, $X(Y^{\theta_0}) = (X_0(Y_0(\theta_0)), X_1(Y_0(\theta_0), Y_1(\theta_0, H)), X_1(Y_0(\theta_0), Y_1(\theta_0, L)))$. Given X, young θ_0 chooses Y^{θ_0} to maximize

$$EU(X(Y^{\theta_0}), \theta_0) - EV(Y^{\theta_0}, \theta_0). \tag{6}$$

If Y^{θ_0} maximizes the above for all θ_0, and the implied assignment, that is, the pair of $D = (X(Y^H), X(Y^L))$ and $Y = (Y^H, Y^L)$, satisfies the budget constraint (1), then (X, Y) is called a market equilibrium and we say this history dependent disposable income policy X implements assignment (D, Y). If the equilibrium attains the highest unconditional expected utility among all market equilibria, we say X is optimal. The optimal disposable income policy is, by the similar argument of the previous section, not unique because of the Ricardian equivalence.

Notice that, any assignment implemented by an equilibrium (X, Y) must be incentive compatible under the direct mechanism owing to the revelation principle. This means X is optimal if it implements the optimal normalized assignment $(\tilde{D}, \tilde{Y})$.

Suppose X implements the optimal normalized assignment satisfying (5). By the definition of equilibrium, $\tilde{Y}^{\theta_0}$ maximize (6) for all θ_0 and

$$\begin{cases} X_0(1) &= \tilde{D}_0(H) \\ X_1(1,1) &= \tilde{D}_1(H,H) \\ X_1(1,0) &= \tilde{D}_1(H,L) \\ X_0(0) &= \tilde{D}_0(L) \\ X_1(0,1) &= \tilde{D}_1(L,H) = \tilde{D}_1(L,L) \end{cases} . \tag{7}$$

Conditions (7) determine X except for $X_1(0,0)$. The optimal assignment does not allow households to experience the ex-post labor supply history $(y_0 = y_1 = 0)$, thus the above condition does not specify the disposable income for such a never working old. In this market-based economy, by the contrast, the government must announce disposable incomes for all possible labor supply histories including $X_1(0,0)$. To implement the optimal assignment, $X_1(0,0)$ must be set low enough so that no household choose the situation.

The equilibrium conditions of $(X, \tilde{Y})$ and (7) require

$$EU(\tilde{D}^{\theta_0}, \theta_0) - EV(\tilde{Y}^{\theta_0}, \theta_0) \geq EU(X(Y^{\theta_0}), \theta_0) - EV(Y^{\theta_0}, \theta_0),$$

for any θ_0 and any Y^{θ_0}. If Y^{θ_0} has no possibility of never-working, $(y_0 = y_1 = 0)$, expost, then the incentive compatibility condition of the optimal assignment implies the above condition. If Y^{θ_0} has positive probability of never-working expost, on the other hand, the right hand side is strictly increasing in $X_1(0,0)$. Thus, the above condition determines the upper bound of $X_1(0,0)$, denoted by $\overline{e}$. In the benchmark case of our example economy, the upper bound $\overline{e}$ is –0.022. Now, X is an optimal history dependent labor income tax if,

$$\begin{cases} X_0(1) & = \tilde{D}_0(H) + s^H \\ X_1(1,1) & = \tilde{D}_1(H,H) - (1+r)s^H \\ X_1(1,0) & = \tilde{D}_1(H,L) - (1+r)s^H \\ X_0(0) & = \tilde{D}_0(L) + s^L \\ X_1(0,1) & = \tilde{D}_1(L,H) - (1+r)s^L \\ X_1(0,0) & \le \overline{e} - (1+r)s^L \end{cases}, \tag{8}$$

for some (s^H, s^L).

4. History independent labor income taxation

The previous section showed that the optimal assignment can be implemented in a market based economy if the government can impose tax based on labor income history. This history dependent nature, however, is uncommon in real-life labor income tax systems. This section examines a simpler and more realistic tax system which is solely based on current labor income. Taxation here is represented by a function $T:\{0,1\} \to \mathbb{R}$ and the implied disposable income policy E is

$$\begin{cases} X_0(0) = X_1(1,0) = X_1(0,0) = -T(0) \\ X_0(1) = X_1(1,1) = X_1(0,1) = 1 - T(1) \end{cases}. \tag{9}$$

Suppose an assignment (D, Y) is implemented by X satisfying (9), then we say the assignment is implemented by history independent labor income taxation T. If no other assignment implemented by an history independent labor income taxation attains higher unconditional expected utility, T is said to be optimal.

(8) and (9) must hold if a history independent labor income taxation T implements one of the optimal assignments. Eliminating X, s^H, and s^L, we must have,

$$1 - T(1) + \frac{1 - T(1)}{1+r} = \tilde{D}_0(H) + \frac{\tilde{D}_1(H,H)}{1+r},$$

$$1 - T(1) - \frac{T(0)}{1+r} = \tilde{D}_0(H) + \frac{\tilde{D}_1(H,L)}{1+r},$$

$$-T(0) + \frac{1 - T(1)}{1+r} = \tilde{D}_0(L) + \frac{\tilde{D}_1(L,H)}{1+r},$$

It is not possible for $T(0)$ and $T(1)$ to satisfy the three conditions in general. Therefore, the optimal assignment is not implementable here.

Table 2 shows the optimal history independent labor income tax and the implemented assignment for the example economy.[9] The labor assignment here is the same as the optimal labor assignment of Table 1 except for the case of the fourth column while the attained unconditional expected utility is a little short. The row of Δw^i shows the difference of unconditional expected utility here and no-tax market equilibrium in terms of labor productivity. In the benchmark case, this number, 1.7%, is a little short of the number of the optimal assignment, $\Delta\tilde{w} = 2.7\%$. Thus, the history independence of labor income tax costs $\Delta\tilde{w} - \Delta w^i = 1\%$ of labor productivity or GDP in the benchmark case. The numbers of the row of $\Delta\tilde{w} - \Delta w^i$, differs across the columns, but they are between 0.5% and 1% except for the fourth column.

Table 2. Optimal history independent tax

Parameter specifications						
$V(1,H)$	$-\ln(1/2)$	$-\ln(2/3)$	$-\ln(1/2)$	$-\ln(1/2)$	$-\ln(1/2)$	$-\ln(1/2)$
$V(1,L)$	$-\ln(1/4)$	$-\ln(1/4)$	$-\ln(1/3)$	$-\ln(1/4)$	$-\ln(1/4)$	$-\ln(1/4)$
$\pi_0(H)$	0.8	0.8	0.8	0.9	0.8	0.8
$\pi_1(H,H)$	0.7	0.7	0.7	0.7	0.8	0.7
$\pi_1(L,L)$	0.8	0.8	0.8	0.8	0.8	0.9
Optimal allocation with history independent tax						
$T(0)$	−0.308	−0.49	0	−0.316	−0.33	−0.308
$T(1)$	0.084	0.186	0	0.062	0.075	0.084
Y^H	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,0)	(1,1,0)
$C_0(H)$	0.804	0.765	1	0.824	0.844	0.804
$C_1(H,H)$	1.101	0.895	1	1.128	1.06	1.101
$C_1(H,L)$	0.493	0.571	1	0.505	0.465	0.493
η^H	0.955	0.876	1	0.978	0.952	0.955
Y^L	(0,1,1)	(0,1,0)	(1,1,1)	(0,1,1)	(0,1,1)	(0,1,1)
$C_0(L)$	0.537	0.506	1	0.55	0.554	0.537
$C_1(L,H)$	0.537	0.788	1	0.55	0.554	0.537
$C_1(L,L)$	0.537	0.464	1	0.55	0.554	0.537
η^L	1.426	6.834	1	1.461	1.472	1.426
GDP (z^i)	1.56	1.4	2	1.63	1.64	1.56
z^i / z^e	0.848	0.761	1	0.849	0.891	0.857
Welfare (w^i)	−1.383	−1.02	−1.29	−1.299	−1.35	−1.392
Δw^i	0.017	0.036	0	0.016	0.018	0.014
$\Delta\tilde{w}-\Delta w^i$	0.01	0.01	0	0.005	0.007	0.01

5. Two social security programs that supplement the history independent labor income taxation

Section 3 showed that the optimal assignment is implementable if labor income tax is allowed to depend on personal labor income history. It may be difficult for the government to levy such a complex tax in actual economy, however. This section proposes two programs that supplement the history independent tax system to make the optimal assignment implementable. These two supplements are termed a "compulsory pension insurance system" and a "limited times age-dependent benefit to low-income households".

5.1. Compulsory pension insurance system

Consider a market economy with history independent labor income tax T and a compulsory public pension insurance system. A working young here must join the pension system and pay premium $p_0 \in \mathbb{R}$. It can receive pension benefit $p_1 \in \mathbb{R}$ if it does not work in old age. Not-working young is not asked to join the pension system (that is, need not pay p_0 when young, but they therefore cannot receive the benefit in old age). Given T and (p_0, p_1), the implied disposable income policy X is

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[9] We can calculate the optimal T according to the following procedure. Let us choose an arbitrary number $h \in (0,1)$ and set the tentative tax T by $T(0) = -h$ and $T(1) = 0$. Given this, we solve the choice of labor supply schedules by the households, (Y^H, Y^L), then calculate the ratio of the present value of the tentative disposable income and the one of the before tax labor income,

$$\alpha \equiv \frac{\sum_{\theta_0} \pi_0(\theta_0)\left(Y_0(\theta_0) - T(Y_0(\theta_0)) + \frac{1}{1+r} \sum_{\theta_1} \pi_1(\theta_0, \theta_1)(Y_1(\theta_0, \theta_1) - T(Y_1(\theta_0, \theta_1))) \right)}{\sum_{\theta_0} \pi_0(\theta_0)\left(Y_0(\theta_0) + \frac{1}{1+r} \sum_{\theta_1} \pi_1(\theta_0, \theta_1) Y_1(\theta_0, \theta_1) \right)}.$$

Since the implied assignment of the tentative tax T may not satisfy the budget constraint, we apply the proportional adjustment to the disposable income by the factor of α. In other words, we adjust T such that $T(0) = \alpha h$ and $T(1) = 1 - \alpha$. Since the utility function is logarithmic, this adjustment does not affect the choice of labor supply and makes the implied assignment satisfy the budget constraint. Thus, the adjusted T determines an implemented assignment. The grid research with respect to h yields the optimal history independent labor income taxation T.

$$\begin{cases} 1-T(1)-p_0 & = X_0(1) \\ -T(0) & = X_0(0) \\ 1-T(1) & = X_1(1,1) \\ -T(0)+p_1 & = X_1(1,0) \\ 1-T(1) & = X_1(0,1) \\ -T(0) & = X_1(0,0) \end{cases} . \tag{10}$$

If X implements one of optimal (not necessarily normalized) assignments, X must satisfy (8). Thus, given e not larger than $\overline{e}$, (8) and (10) determine a combination of history independent labor income tax T and compulsory public pension insurance system (p_0, p_1),[10] that is,

$$\begin{pmatrix} T(0) \\ -1+T(1) \\ p_0 \\ p_1 \\ s^H \\ s^L \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1+r & 0 \\ -1 & 0 & 0 & 1 & 1+r & 0 \\ 0 & -1 & 0 & 0 & 0 & 1+r \\ -1 & 0 & 0 & 0 & 0 & 1+r \end{pmatrix}^{-1} \begin{pmatrix} \tilde{D}_0(H) \\ \tilde{D}_0(L) \\ \tilde{D}_1(H,H) \\ \tilde{D}_1(H,L) \\ \tilde{D}_1(L,H) \\ e \end{pmatrix}.$$

Calculating the last column of the inverse matrix, we have,

$$\frac{\partial T(0)}{\partial e} = -\frac{1}{2+r} < 0,$$

$$\frac{\partial T(1)}{\partial e} = \frac{1+r}{2+r} > 0,$$

$$\frac{\partial p_0}{\partial e} = -1,$$

$$\frac{\partial p_1}{\partial e} = -1.$$

The last two equations claim that the size of pension insurance system is minimized when e is set at the upper bound $\overline{e}$. The next table shows the optimal combination of the

[10] The following equation determines private savings, s^θ, as well as the policy instruments, (T, p_0, p_1). If the government chooses the optimal policy, then young θ chooses $\tilde{Y}^\theta$ and the corresponding saving s^θ to realizes the optimal assignment. Similar arguments also hold in the next subsection.

history independent labor income tax and the supplementary pension system at $e=\overline{e}$.

The first part of table 3, the parameter specifications, is the same as the previous tables except for the fourth column of Table 1, the case having no room for welfare improvement by taxation. The middle part shows the optimal combination of history independent labor income tax and the minimized pension insurance system. The net incomes for young households, which take into account labor income tax/transfers as well as pension premium/benefits, are $0.815\,(=1+0.01-0.186)$ for workers and 0.374 for not-workers, in the benchmark case. Joining the pension insurance system must be compulsory since its return p_1/p_0 is lower than the market rate $1+r=1.67$. The income tax system runs a deficit (both $T(1)$ and $T(0)$ are negative) but it is financed by the surplus created by the pension system.

Because young working H are protected by the pension benefit in old age, they do not hesitate to retire when they become unproductive. By contrast, young L households do not join the pension system, and thus they have to work in old age even when they are unproductive. This asymmetric protection against preference/productivity shocks in old

Table 3. Two supplemental policies

Parameter specifications					
$V(1,H)$	−ln (1/2)	−ln (2/3)	−ln (1/2)	−ln (1/2)	−ln (1/2)
$V(1,L)$	−ln (1/4)	−ln (1/4)	−ln (1/4)	−ln (1/4)	−ln (1/4)
$\pi_0(H)$	0.8	0.8	0.9	0.8	0.8
$\pi_1(H,H)$	0.7	0.7	0.7	0.8	0.7
$\pi_1(L,L)$	0.8	0.8	0.8	0.8	0.9
$\overline{e}$	−0.66	−0.564	−0.685	−0.642	−0.655
Compulsory pension insurance system					
$T(0)$	−0.374	−0.44	−0.388	−0.374	−0.369
$T(1)$	−0.01	−0.188	−0.047	−0.008	−0.018
p_0	0.186	0.562	0.192	0.157	0.199
p_1	0.049	0.42	0.051	0.05	0.063
s^H	0.037	−0.162	0.039	0.021	0.032
s^L	−0.239	−0.282	−0.248	−0.239	−0.244
Limited times age dependent benefit to low income households					
$T(0)$	−0.259	−0.09	−0.268	−0.275	−0.245
$T(1)$	0.106	0.162	0.073	0.09	0.106
b_0	0.186	0.562	0.192	0.157	0.199
b_1	0.049	0.42	0.051	0.05	0.063
s^H	0.107	0.049	0.111	0.08	0.107
s^L	−0.169	−0.07	−0.176	−0.18	−0.169

age makes the asymmetric labor supply plan of the optimal assignment implementable.

This compulsory pension insurance system shares a similarity with the pension insurance systems in developed countries. In Japan, for example, workers must join a public earnings-related pension systems (*Kousei Nenkin*) in addition to the basic public pension (*Kokumin Nenkin*). Older workers (over 65 years) are able to receive pension benefits provided their total income (from earnings and pension benefits) does not exceed a certain amount (470,000 yen a month in 2020).[11] Above this limit, half of the excess is reduced from the full pension payment but the basic pension is still paid in full. By contrast, not-working youngs cannot join the employee pension scheme and only benefit from the basic pension in which the pension premium and benefits are fixed.[12] This earnings-related property of pension insurance has been adopted in many countries.

5.2. Limited times age-dependent benefit to low-income households

Another policy provides asymmetric protection against preference/productivity shocks in old age. Suppose the government provides benefits to not-working households. The benefit is age-dependent and "one time" in the sense that a household cannot receive the benefit more than once in its lifetime. Let b_t denote the benefit to not-working households of age $t \in \{0,1\}$. Now the implied disposable income policy X is

$$\begin{cases} 1-T(1) & = X_0(1) \\ -T(0)+b_0 & = X_0(0) \\ 1-T(1) & = X_1(1,1) \\ -T(0)+b_1 & = X_1(1,0) \\ 1-T(1) & = X_1(0,1) \\ -T(0) & = X_1(0,0) \end{cases}.$$

Setting X by (8) for some e not larger than $\bar{e}$, we can solve the combination of history independent labor income tax and the benefit system that implements an optimal assignment by

[11] For workers 60 to 64 years old, the amount is 280,000 yen a month in 2020. The amount will be raised to 470,000 yen a month in 2022.

[12] It is possible to make optional contributions in order to receive a higher pension after retirement. For details on Japan and other developed countries, see OECD (2015).

$$\begin{pmatrix} T(0) \\ -1+T(1) \\ b_0 \\ b_1 \\ s^H \\ s^L \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1+r & 0 \\ -1 & 0 & 0 & 1 & 1+r & 0 \\ 0 & -1 & 0 & 0 & 0 & 1+r \\ -1 & 0 & 0 & 0 & 0 & 1+r \end{pmatrix}^{-1} \begin{pmatrix} \tilde{D}_0(H) \\ \tilde{D}_0(L) \\ \tilde{D}_1(H,H) \\ \tilde{D}_1(H,L) \\ \tilde{D}_1(L,H) \\ e \end{pmatrix},$$

where e is an arbitrary number smaller than the upper bound $\overline{e}$. Solving the last column of the inverse matrix again,

$$\frac{\partial T(0)}{\partial e} = -1 < 0,$$

$$\frac{\partial T(1)}{\partial e} = 0,$$

$$\frac{\partial b_0}{\partial e} = -1,$$

$$\frac{\partial b_1}{\partial e} = -1.$$

Thus, setting e to the upper bound yields the optimal policy combination with minimum subsidy system. The last part of table 3 shows the results. The net incomes of young households are 0.894 $(= 1 - 0.106)$ for workers and 0.445 $(= 0.259 + 0.186)$ for non-workers. Older groups are protected against preference/productivity shocks only if they were productive when young.

This benefit system shares a similarity with the U.S. TANF (Temporary Assistance for Needy Families) program, a block grant that supports low-income families with children. The U.S. federal government imposes time limits on TANF recipients.[13] In general, families cannot receive assistance for more than five years (note that states have the flexibility to extend assistance beyond this five-year limit for up to 20% of their monthly cases). The federal government also requires welfare recipients to work after receiving at most 24 months of TANF benefits, although, again, states can choose shorter deadlines.

[13] For detailed information on TANF, see U.S. House of Representatives Committee on Ways and Means (2018, chap. 7).

6. Conclusions

In the present paper, we investigated optimal labor income tax policies in a two-period open economy with a consumption good and indivisible labor in each period. Households are identical except for stochastic and discrete preference/productivity types, namely productive and unproductive, which affect the disutility of effective labor supply. We assumed that individual types and savings (and consumptions) are private information. These assumptions imply that the government cannot impose capital taxation and that the realized consumption plan must satisfy the Euler condition. We define the optimal allocation under the direct mechanism, where the government assigns the disposable income-labor supply combination conditional on the history of types reported by households.

Because of the difficulty of characterizing the optimal allocation, most of our investigation is based on numerical simulation of an example economy. We calculated the optimal allocations under various parameter specifications and found they share a common property, namely labor supply is sensitive to the preference/productivity shock for the old who worked in young while the old who did not work in young must work regardless of the shock, for a wide range of parameter specifications.

We also showed that the optimal allocation is implementable under the market economy by using appropriate labor income taxation if the tax rate for old groups depends not only on their current labor income but also on their history of labor income. The optimal allocation is unimplementable, however, under commonly observed tax systems that are solely based on current labor income, because the optimal allocation requires the difference in the disposable incomes of the working and not-working old populations to be history-dependent.

In the next step, we calculated the second-best optimal taxation under the restriction that the tax rate depends solely on current labor income. In this respect, the welfare loss due to the restriction is likely between 0.5% and 1% of GDP.

Given these findings, we proposed two social security policies to supplement the typical tax systems based on current labor income. One was a compulsory pension insurance system for young workers that benefits participants if they do not work in old age. This system is a standard instrument all around the world. The other was a limited times age-dependent benefit provided to low-income households. This program is less conventional, but has been established in some countries. We could therefore provide a rationale for these social security programs. These policies could be designed to provide asymmetric protection against preference/productivity shocks to older groups who worked when young and did not, and therefore encourage households to work at least once. Most of our arguments are based on a numerical example. Many of the claims, however, are robust to parameter changes.

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