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## Article

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# Indirect measurement of flow of liquid pumped with pump packages 

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#### Abstract

This paper discusses some research results for indirect measurement methods for pump packages based on their head-capacity curves. The paper shows the results of experimental research into indirect measurement by approximation of vertical lift performance of a pump package with algebraic polynomials of 2nd and 3rd order illustrating their applicability to solution of tasks pertaining to retrospective estimation of polynomial coefficients for the vertical lift performance approximation. Values of relative errors for indirect measurement of transferred liquid flow with such polynomials are given.


Keywords: flow meter; pump package; indirect measurement; polynomial.

## 1. Introduction

Knowledge of flow $Q$ of liquid being pumped through a certain section of a pipeline in a certain period is one of prerequisites for efficient operation of modern motor pump-operated pipelines, including oil pipelines. Such knowledge allows specifying and solving a set of problems requiring mathematical and numerical modeling of pipeline processes and equipment, as well as control of such processes and equipment in real time among other tasks.

However, prohibitively high cost of flow meters prevent operators from installation of such instruments in sufficient numbers. Thus, flow meters are installed only on certain parts of a pipeline and at a large distance from each other. Such flow meter arrangement leads to scarcity of information on movement of transferred oil and petroleum products through different parts of the pipeline. Besides their prohibitive costs modern flow meters are also characterized by relatively low reliability and high complexity of operation. These disadvantages stipulate the critical task to invent and study methods of indirect measurement of transferred liquid flow and development of hardware and software for their implementation. Development of such hardware and providing different pipelines with them will allow to increase reliability and efficiency of the pipeline operation.

This paper discusses some research results for indirect measurement methods for pump packages based on their head-capacity curves. Currently pump packages are provided with relatively reliable, inexpensive and simple to use pressure gages and other pressure-reading instruments which allow measuring developed pressure with high precision. Together with available possibilities to plot package's rating curve which is adequate enough in its description of relations between the pressure $P$ developed by a pump package and flow $Q$ of the transferred liquid they open ample opportunities for practical implementation of the transferred flow indirect measurement method under consideration in this paper.

This method may be employed at modern oil and gas pipelines, utility water pipelines and others as an estimating method for transferred fluid flow ${ }^{[1]}$. The proposed method allows to
make reasonable considerations for aging, wear, and changes in the flow curve of the fluid being transferred by the pump package.

## 2. Substantial and mathematical content of the indirect measurement of flow of liquid transferred by pump packages and other parts of the pipeline

Theory and practice of engineering measurements show ${ }^{[2]}$, that indirect measurement of a certain physical, chemical, economical or other variable is its measurement (determination of its numerical value) based on use of a known functional dependency between this variable and a value of a certain other variable which can be measured directly. At that, the functional dependency between the indirectly and directly measured variables is assumed to be an unambiguous correspondence, where each value of the directly measured variable may be mapped to the one and only value of the indirectly measured variable and vice versa. In our case directly and indirectly measured variables are respectively pressure $P$, developed by the pump package and flow $Q$ of transferred liquid, while the functional dependency used is socalled vertical lift performance of the pump package $(Q, P)$ - a characteristic of the pump package that links the developed pressure $P$ and flow $Q$ of the transferred liquid. Thus, the substantial content of the indirect measurement task for transferred liquid flow from the pump package vertical lift performance consists of determination of an unknown numerical value of flow $Q$ of the liquid transferred by the pump package in correspondence to the measured and thus known value of the pump package developed pressure $P$, using the vertical lift performance.

From the analytic point of view the vertical lift performance of the pump package is just a function defined by an equation in the form
$P=P(Q)$.
At that, this function may be presented in tabular, graphed or analytical form, as a formula linking values of $P$ and $Q$. It is obvious, that independently of the form in which the vertical lift performance $(Q, P)$ characteristic is given, it links pressure $P$, developed by the pump package to the flow $Q$ of transferred liquid. It is also obvious that from the mathematical point of view the task of indirect measurement of flow $Q$ reduces to calculation of numerical value Qo of the flow $Q$, corresponding to a given value $P_{0}$ of pump package pressure $P$. From (1), one may say that calculation of $Q_{0}$, reduces to solution of the following equation:
$P_{0}=P\left(Q_{0}\right)$.
To simplify the following investigation and its results hereinafter we will consider a concrete type of pump package, namely D2500-62 which is widely used in different Russian pipelines ${ }^{[3]}$.

Graphical representations of its vertical


Figure 1. Graphic representation of lifting power and other characteristics of D2500-62 type pump package lift performance and other characteristics are given in Fig. 1, all the values were obtained during factory tests. ( $Q, P$ )-characteristic in this picture is not shown explicit, but may be easily obtained from formula $P_{i}=H_{i}{ }^{*} \rho^{*} g$, where $H$ is head, $\rho$ is transferred liquid density and $g$ is gravity acceleration; other dependencies in the figure are efficiency $\eta$, positive suction head $\Delta h$ and lifting power N.As our analysis shows, the ( $Q, P$ ) graphs of other Russian pump packages employed in Russian pipelines have similar characteristics. Thus, the results below are universal enough and may be easily extended to other types of pump packages.

Analysis of the graph shows that the $(Q, P)$-characteristic is a function which is continuous, quite smooth and monotonically decreasing with rising $Q$; these features are common
throughout the whole interval $I_{Q}$ of the transferred liquid flow values $Q$. From the point of view of calculus and approximation theory of real functions of one argument ${ }^{[4]}$ these features of ( $Q, P$ )-characteristic mean that it may be arbitrarily closely approximated with widely known and well-studied elementary functions as is common for tabulated or graphically-defined functions.

The most popular of such functions are algebraic and trigonometric polynomials, as well as exponential polynomials, fractional rational functions, etc. As it is known from calculus [4], the simplest of these functions and consequently the easiest for calculations and analytical studies as well as for different practical applications are algebraic polynomials. Our numerical experiments show that use of second degree algebraic polynomials allows for acceptable accuracy of $(Q, P)$ characteristic approximation. Thus, this characteristic of the pump package may be reasonably represented with the following equation:
$P(Q)=c_{0}+c_{1}{ }^{*} Q+c_{2}{ }^{*} Q^{2}$,
where $c_{0}, c_{1}$ and $c_{2}$ are constant coefficients which are independent from flow $Q$ of the liquid transferred with the pump package.

Some quantitative estimations of error and accuracy for approximation of ( $Q, P$ ) characteristic for D2500-62 pump package with algebraic polynomials of the form (3), supporting this conclusions are given in section 4.

## 3. Analysis of possible approaches to the task of indirect measurement of transferred liquid flow from pump package ( $Q, P$ ) characteristic

Let us consider two approaches to this task based on different methods of definition of $(Q, P)$ characteristic.

### 3.1. An approach based on definition of $(Q, P)$ with a Table 1

Table 1. Vertical lift performance of a pump package

| $j$ | $Q_{j}$ | $P_{j}$ |
| :--- | :--- | :--- |
| 1 | $Q_{1}$ | $P_{1}$ |
| 2 | $Q_{2}$ | $P_{2}$ |
| $\ldots$ | $\cdots$ | $\cdots$ |
| $T N$ | $Q_{m}$ | $P_{T}$ |

The following symbols are used in this table: $j$ are so-called value ranks; $Q_{j}$ of flow $Q$, in ascending order, or in other words, index numbers of values $Q_{j}$ of flow $Q$, given according to ascending order of transferred liquid flow $Q_{i} ; \tau$ is a number of available values of flow $Q_{j}$ on the pump package which satisfies the inequalities $3<\tau<\infty$; $Q_{j}$ is the value of flow $Q$, which takes $j$-th rank in the set of the flow values $Q$ sorted in ascending order; $P_{j}$ is the value of pressure $P$, developed by the pump package corresponding to $j$-th value $Q_{j}$ of the flow $Q$.

Search for $Q_{0}$ of the flow $Q$, which corresponds to $P_{0}$ of the pressure $P$, developed by the pump package may be performed with many currently known methods employed for solution of such problems, for example: 1) dichotomy method; 2) Fibonacci number method; 3) golden section technique; 4) stochastic approximation technique and others. Such wide variety of available techniques is an obvious advantage of this method.

Its disadvantage is high memory requirements to store the Table 1. The table consists of real numbers, so it takes from 4 to 8 bytes per cell and the table as a whole will take up about $42^{*} T$ or $8{ }^{*} 2{ }^{*} T$ bytes of memory. With large $m$ the memory requirements become quite significant. Modern microprocessors (MP) and micro controllers (MC) have large memory capacity and allow storing and processing of large amounts of data. However, use of modern MPs and MCs significantly raises the cost of the instrument designed for their use, thus limiting or excluding their application to this problem. An additional point is that real applications usually require storage of not one, but tens and hundreds of such tables, thus many MPs and MCs reach their memory limits pretty quickly. This problem, of course may be solved with additional peripheral storage, such as FLASH cards or HDDs. However, use of peripheral devices will obviously
impair dynamic characteristics, first of all speed, increases costs and lowers reliability of hardware and software solution for indirect measurement of liquid flow delivered by a pump package.

### 3.2. An approach based on analytical definition of $(Q, P)$.

As was mentioned above, vertical lift performance of a pump package may be approximated with arbitrary precision employing many currently known polynomials and other relatively simple functions. However, to provide the indirect measurement algorithm with the highest efficiency in both amount of computation and memory use, the most rational technique would be to use algebraic polynomials of the form (3), using some known method of estimation [5] and Table 2 to determine coefficients $c_{0}, c_{1}$ and $c_{2}$. It is explicit from (3), that use of any polynomials to find values of $Q_{0}$, corresponding to a given value of $P_{0}$ may be reduced to solution of a second order algebraic equation whose roots are easy to obtain analytically.

## 4. Some results of numerical experiments

The aim of the experiments was to approximate the ( $Q, P$ ) characteristic of a D2500-62 pump package with algebraic polynomials of different orders and estimation of its accuracy.

Computation of polynomial coefficients was performed by pseudoinverse matrix method ${ }^{[5-6]}$ commonly used to estimate unknown variables. At that, numeric values of flow $Q$, and produced pressure $P$, obtained from digitizing the graphs of the $(Q, P)$ characteristic of the pump package (see Fig. 1) were used as a required input of the method. Digitizing was performed with a sampling rate of $\Delta Q=30 \mathrm{~m}^{3} / \mathrm{h}$. As a resulted 100 points ( $Q_{j}, P_{i}$ ) were obtained, $i=\overline{1,100}$.

Numeric values of polynomial coefficients were obtained as a solution to the approximation of numerical values of $Q_{j}$ and $P_{j}$ with algebraic polynomials of first, second and third order, they are given in Table 2. The same table shows values of RMS error $S$ and average relative error $\delta$.
Table 2. Results of numerical experiments

| Order of the polynomial | Polynomial coefficients |  |  |  | $S, \mathrm{~Pa}$ | $\delta, \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Co | $C_{1}$ | $C_{2}$ | C3 |  |  |
| 1 | 8,038e+5 | -81.157 | 0,0 | 0,0 | 3,257e+5 | 4.299 |
| 2 | 7,337e+5 | 60.408 | -0.0472 | 0,0 | 3,271e+4 | 0.391 |
| 3 | 7,389e+5 | 39.015 | -0.0293 | -3,973e-6 | 2,511e+4 | 0.314 |

Here $S$ is the RMS error, $\delta$ is the average relative error of $(Q, P)$ characteristic approximation.

It is obvious, that first order polynomial does not describe the vertical lift performance with practically acceptable accuracy. It is not rational to use such a polynomial for solution of the task to indirectly measure flow $Q$ of the liquid transferred with a pump package because the calculated value $Q_{0}$ of the flow $Q$, significantly differs from the true value $Q_{\text {true }}$ of the flow $Q$.

Results from approximation of the pump package vertical lift performance with a third order polynomial are shown graphically in Figure 2, analogous results (not shown here) were obtained for a second order polynomials. Analysis of the results shows that polynomials of second and third order can approximate the pump package vertical lift performance with acceptable enough accuracy. Table 2 shows that average relative error of approximation is under $0.5 \%$.

Let us compute the value $Q_{0}$ of the flow $Q$, using polynomials of second and third order obtained in approximation and shown in Table 2. This experiment considered the part of the D2500-62 pump package vertical lift performance from 2000 to $3000 \mathrm{~m}^{3} / \mathrm{h}$. Relative error does not exceed $2.5 \%$ when a second order polynomial is used. Figure 3 shows a graph illustrating dependency of of relative error $\delta$ on the true value $Q$ true of flow $Q$, obtained on a single sample with a third order polynomial.


Fig.2. The initial function and its approximating third order polynomial.


Fig. 3. Dependency of relative error on the true value of flow while using third order polynomials
As the results of our computations show, determination of the value of $Q$, with third order polynomials gives relative error not exceeding 1.3\%, which is shown in Fig. 3.

## 5. Conclusion

Let us give a resume of previous sections and register three principal results.

1. A method for indirect measurement of the transferred liquid flow based on approximation of $(Q, P)$ characteristic of the pump package with algebraic polynomials of first, second and third order is proposed and studied.
2. The proposed method of approximation allows to approximate the vertical lift performance of the pump package with acceptable accuracy, which in its turn allows formulating and solving not only tasks of indirect measurement of liquid flow, but tasks of mathematical and numerical modeling of pipeline processes and equipment.
3. The results of numerical modeling clearly illustrate its viability and applicability to the task in retrospect mode, thus opening great prospects to its application in design of automated pipeline control systems.

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