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THEORY OF MECHANISMS AND MACHINES

Part 1

CLASSIFICATION AND ANALYSIS OF MECHANISMS

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The textbook (part 1) covers the basic principles of classification and analysis of mechanisms taking into account the specifics of training engineers in the field of dynamics and strength of machines. This part of the textbook sets out the objectives of the academic discipline "Theory of mechanisms and machines", its place in a number of other sciences and role in formation of the future engineers. It covers aspects of the structure and classification of mechanisms and machines, considers basic methods of kinematic analysis for various types of mechanisms, dynamic analysis of machine aggregates at transient and steady regimes, machine running control, dynamic force analysis and balancing of mechanisms. The theoretical material is accompanied by examples of its practical application for the analysis of real mechanisms and questions for self-examination.

The book is intended for students of specialty 131 "Applied Mechanics", subject areas "Dynamics and strength of machines" and "Information Systems and Technologies in aircraft construction".

Частина 1 підручника присвячена висвітленню основних принципів класифікації та аналізу механізмів з урахуванням специфіки підготовки інженерів у галузі динаміки і міцності машин. Викладено завдання дисципліни, її місце в ряду інших наук та роль у формуванні майбутніх інженерів. Висвітлено питання структури та класифікації механізмів і машин, методів дослідження кінематичних характеристик різних типів механізмів, динамічного аналізу машинних агрегатів в умовах неусталеного та усталеного режимів роботи, регулювання їх руху, силового розрахунку та урівноважування механізмів. Теоретичний матеріал супроводжується прикладами його практичного застосування для аналізу реальних механізмів, а також запитаннями для самоперевірки.

Для студентів спеціальності 131 "Прикладна механіка", що навчаються за освітніми програмами "Динаміка і міцність машин" та "Інформаційні системи і технології в авіабудуванні".

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FOREWORD

The course of the theory of mechanisms and machines prescribed in the textbook is intended for students of speciality 131 "Applied Mechanics" specializing in the field of dynamics and strength of machines. Specialists in this field of knowledge occupy a special place among mechanical engineers since their further professional activities should be related to complex calculations of strength reliability and durability of machines, constructions and their members with a given accuracy, taking into account numerous technological and operational factors. A significant amount of special disciplines of the theoretical direction, provided by the curriculum of specialization, largely approaches the mechanics and mathematics courses of classical universities. On the other hand, the engineering direction of the training of future specialists in the field of dynamics and strength of machines requires a thorough study of general engineering disciplines, in particular, the theory of mechanisms and machines, for acquiring knowledge, skills and abilities in the analysis of a load of real constructions, the impact of various dynamic factors associated with the movement of solids in the structure of machines and mechanisms, and for construction of adequate calculation schemes.

Part 1 of the textbook is devoted to the consideration of the principles of classification and the analysis of mechanisms and machines. It contains an introduction and seven sections.

Chapter 1 covers the main discipline tasks, historical background, its place and role among other general engineering disciplines.

Chapter 2 is devoted to the coverage of the basic concepts, terms and definitions, structure of mechanisms and machines, principles of their classification.

The following chapters outline the main methods of kinematics and dynamic analysis of mechanisms and machines, their power calculation. The construction of reliable calculation schemes of research objects for further evaluation of their solid reliability and resource is impossible without mastering the general methods of analysis of mechanisms, machines, devices, the correctness of their choice in solving specific engineering problems, without understanding the general principles of the realization of motion, the interaction of elements in the composition of mechanical systems that determine their kinematic and dynamic characteristics. Both analytical and graph-analytical methods of research are widely presented in the textbook. A comparative analysis of the possibilities of different methods, their advantages and disadvantages are given, attention is paid to the rational choice of certain methods of research depending on the task and the necessary accuracy of the calculations. The issues of running control of machines and mechanisms, their balancing both at the design stage and after manufacturing, to provide the most optimal dynamic characteristics are also considered.

The theoretical material is accompanied by examples of solving practical problems for different types of mechanisms. Questions for student self-testing of knowledge are given at the end of each chapter.

The second part of the textbook will be devoted to the issues of synthesis of mechanisms with lower and higher kinematic pairs, their force calculation taking into account friction in kinematic pairs, wear resistance and vibration protection during operation.

Chapter 1. INTRODUCTION TO THE SUBJECT OF STUDY

As the educational discipline, the Theory of Mechanisms and Machines plays one of the major roles among fundamental general engineering disciplines, which provide the basis for the professional training of the future mechanical engineers.

The theory of mechanisms and machines is a science, which studies the general methods of research of mechanisms and machines properties and design of their schemes [3].

The textbook describes the structure, classification and analysis of mechanisms according to the curriculum of the credit module 1 of the discipline “Theory of Mechanisms and Machines” for the speciality 131 “Applied Mechanics” (subject areas “Dynamics and strength of machines” and “Information Systems and Technologies in aircraft construction”). This credit module is devoted to the study of methods of structural, kinematic, dynamic analysis of the main types of mechanisms that are most widely used in modern machines. As a result, students acquire:

– ***Knowledge*** – to know the basic laws of kinematics and dynamics of mechanisms and their systems; principles of realization of movement with the help of mechanisms, the interplay of mechanisms in the machine, which causes kinematics and dynamic properties of a mechanical system; general methods of analysis of miscellaneous types of mechanisms;

– ***Ability*** – to know in practice how to realize systematic approaches to machine and mechanisms analysis; to discover kinematics and dynamic properties of mechanisms with the help of modern analytical and graphical methods;

– ***Experience*** – to use measuring equipment and accessory to define kinematics and dynamic operation factors of machines and mechanisms; development of algorithms of the calculation programs for operation factors on a computer, implementation of specific calculations.

The theory of mechanisms and machines is connected with such general engineering and special disciplines, as “Parts of machines”, “Vibration theory and stability of motion”, “Fundamentals of robotics”.

Historical background. The development of human society is associated with the usage and advancing of mechanisms: from the simple lever at the beginning of civilization to overspecified mechanical systems of the present.

The elementary mechanisms (leverages, gearings) were widely used in ancient Egypt and other civilizations of antiquity.

The epoch of Renaissance in Europe was marked not only by extraordinary achievements in medicine, literature, art but also in the engineering sphere. It is necessary to mention Leonardo da Vinci, who left to the descendants the designs of composite mechanisms of weaver's and woodworking machines, of flight vehicles etc. In a half of the century the famous Italian doctor and mathematician Gerolamo Cardano was investigating the operation of mechanisms of clocks and grinding mills.

Leonardo da Vinci (1452–1519)
Italian artist and scientist, inventor, writer, musician, one of the most prominent representatives of the High Renaissance, a vivid example of the "universal man" (Latin homo universalis). He laid the foundations of modern mechanics as the inventor of the original mechanisms and the experimenter who carried out the first systematic studies in the field of strength, first introduced the concept of friction coefficient.

Gerolamo Cardano (1501–1576)
Italian mathematician, engineer, philosopher, physician, astrologer. Despite the fact that Cardano practically all his life was engaged in medicine, he left a noticeable mark in many branches of science. His significant contribution to the development of algebra (in particular he proposed the solution of different types of cubic equations), probability theory. As an engineer, he described in detail various mechanisms, believed to be the inventor of cardan shaft.

In 18th century engineering started to form as a field of industry. Its further development was im possible without the studying of scientific grounds of creation of the faultless mechanisms and machines. The considerable contribution to the development of mechanics in 18th century was made by the French scientists G. Amonton and Ch. Coulomb, who studied the laws of friction. The well-known Swiss mathematician and mechanic L. Euler solved a series of problems of kinematics and dynamics, studied oscillation of elastic bodies. He offered involute profile for a gear tooth.

The theory of mechanisms and machines as a science began to form at the beginning of 19th century. Its further successes were connected with the names of such scientists as F. Grasgof and F. Relo (Germany), S. Roberts and R. Willis (England), T. Olivier (France), P. Chebyshev, M. Petrov, M. Zhukovsky, L. Assur, I. Artobolevsky (Russia).

In 20th century the paces of technological progress were extremely high. The machine industry was roughly developed. The manufactures of the 19th century changed into computer-aided productions equipped with high-precision automatic machines. The fully automated plants appeared. The epoch of robotics, highly profitable computer-controlled productions started. On the one hand, the requests to quality of created machines and mechanisms increased, but on the other hand, the huge possibilities for a solution of the most difficult problems of synthesis and analysis appeared. These problems could not be solved before. The solution methods for these problems were developed. The new knowledge accrued in the field of the theory of machines and mechanisms.

Ukrainian scientists made the considerable contribution into the development of the theory of mechanisms and machines in 20th century. The powerful schools of mechanics have arisen in Kyiv, Kharkiv, Odesa, Dnipropetrovs'k. Names of the Ukrainian scientists K. Zablonsky, S. Kozhevnikov, B. Kostetsky, F. Ivanchenko and others are known far outside of our state.

S. M. Kozhevnikov (1906–1988)
Doctor of Sciences, professor, head of the theory of machines and mechanisms department of the Institute of Mechanics of the National Academy of Sciences of Ukraine. In 1983 he was elected an honorary member of the International Federation for the Theory of Mechanisms and Machines. He laid the foundations for a fundamentally new scientific direction - non-stationary dynamics of machines with real physical properties of drive links, made a significant contribution to the development of the theory of structural analysis and synthesis of mechanisms, biomechanics, and others.

K. I. Zablonsky (1915–2006)

Doctor of Sciences, professor, Honored Scientist of Ukraine, Rector of the Odessa Polytechnic Institute. He is the author of numerous fundamental works in the field of rigidity, loading ability of gearing, as well as textbooks on the theory of mechanisms and machines, machine parts, applied mechanics.

F. K. Ivanchenko (1918–2005)

Doctor of Sciences, Professor, headed the Department of Technical Mechanics of Kyiv Polytechnic Institute. He studied the problems of the theory of machines and mechanisms, in particular, the design and calculation of lifting-and-shifting machines and equipment of metallurgical workshops. He developed the theory, bases of calculation of power-supply parameters and dynamics of heavy machines.

The basic problems and tasks. The achievements of the theory of mechanisms and machines are significant. Nevertheless, it is necessary to mark, that today the theory of mechanisms is much more developed than the theory of machines.

The theory of mechanisms studies such exploratory receptions of properties of mechanisms and circuit designing (analysis and synthesis), which are common to all or specific groups of mechanisms, irrespective of particular assigning of machines, instruments, vehicles. For example, the same mechanism, made in the form of belt or gear transmission, can be found in cars, machine tools, various devices.

The quality of created mechanisms and machines in many cases depends on entity of developing and applying of general engineering methods. The more fully the criteria of productivity, reliability, accuracy, efficiency are taken into account at the design stage, the more perfect constructions will be created.

Application skills of methods of kinematics and dynamic exploration of mechanisms and machines are essential for making the design schemes and simulation of loading conditions of constructions and their members to further strength calculations, stiffness and stability, and also for correct carrying out of an experiment.

Chapter 2. THE STRUCTURE OF THE MECHANISM

The mechanism is the basis of any machine. It can be shortly defined as:

The mechanism is a device for converting the mechanical motions of resistant bodies.

Because the resistance bodies that are used in most machines are intended to be very stiff, they will be considered to be *rigid bodies* in these discussions.

The mankind has constructed a great variety of miscellaneous mechanisms for the history. Therefore, as well as in any other natural science, principles (systems) of classification of objects of study were designed in theory of mechanisms and machines. Currently, there are several of them:

– according to the structural sign;
– according to transmission of motion;
– according to the functional sign;
– according to the nature of motions of mechanism links;
– according to the design sign or type of the mechanism.

2.1. GENERAL NOTIONS AND DEFINITIONS

2.1.1. Link, part

In order to provide the desired motions in the mechanism, it is necessary to connect or joint the bodies like links in a chain (Fig. 1.1)

Link is a rigid body, which belongs to the mechanism composition.

The conventional image of the *crank-and-slider mechanism* is shown in Fig. 2.1. It belongs to *crank-and-rod mechanisms*. It consists of following links: a *crank* 1 carries out the full turn per cycle of a mechanism motion; a *connecting rod* 2 realizes a *planar* or *two-dimensional motion*; a *piston* or *sliding block* or *slider* 3 carries out a pure translation – *back-and-forth motion*; and a *ground* as immovable link, which is rigidly fixed to a reference system.

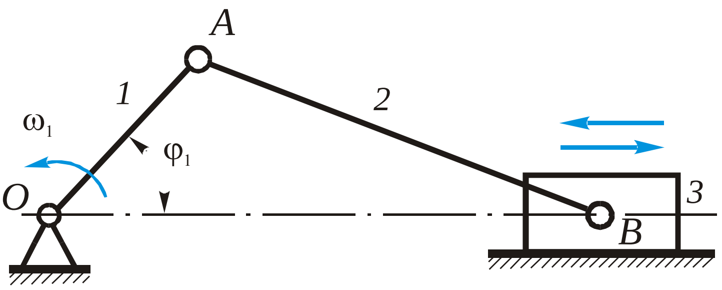


Fig. 2.1. Crank and-slider mechanism

The part is a rigid body, which belongs to a link composition.

Between parts in a link there are no relative movements.

Fig. 2.2 shows a con-rod (link 2). Here 1, 2, 3, 4 are parts of this link.

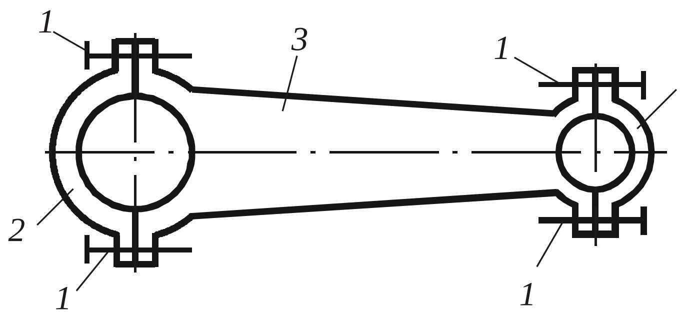


Fig. 2.2. Link and its parts

2.1.2. Kinematic pairs

The kinematic pair or kinematic joint is the moveable connection of two links, which confines their definite relative motions.

In Fig. 2.1 letters *O*, *A* and *B* indicate kinematic pairs.

There are such kinds of kinematic pairs:

– *planar* or *two-dimensional* pairs and *spatial* or *three-dimensional* pairs;

– sliding pair (Fig. 2.3, *a*) and turning pair (revolute joint or pivot or pin joint (Fig. 2.3, *b*) and spherical joint or ball joint or ball & socket (Fig. 2.3, *c*));

– lower (Fig. 2.3) and higher (Fig. 2.4) pairs.

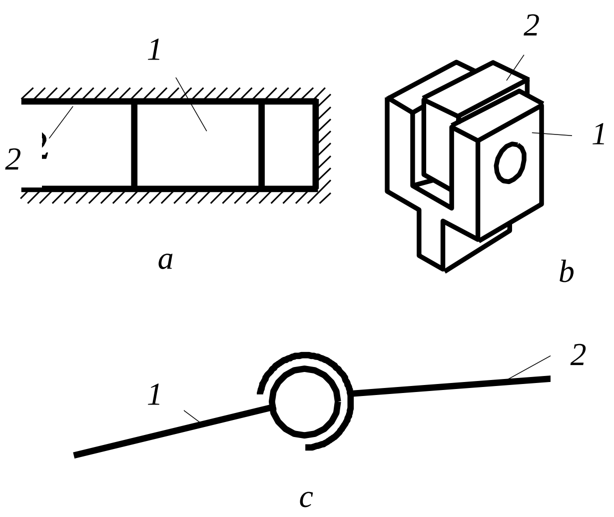


Fig. 2.3. Lower pairs: *a* – sliding pair;
b – turning pair; *c* – spherical joint

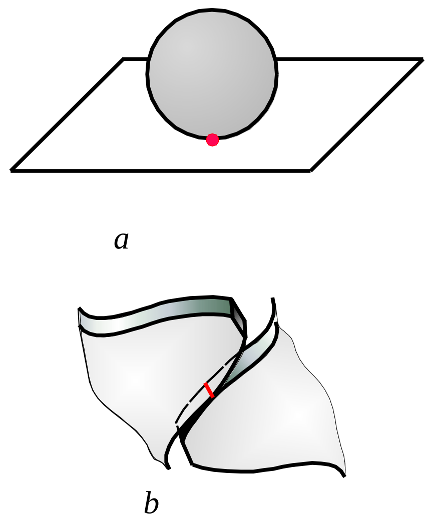


Fig. 2.4. Higher pairs: *a* – point contact;
b – line contact

In lower pairs the contact between links is held on a surface; and in higher – through line or in a point.

The higher pairs have a series of peculiarities:

– they require positive closing: force closure (Fig. 2.5, *a*) or form closure (Fig. 2.5, *b*);

– they have higher specific pressure in a contact zone than the lower kinematic pairs;

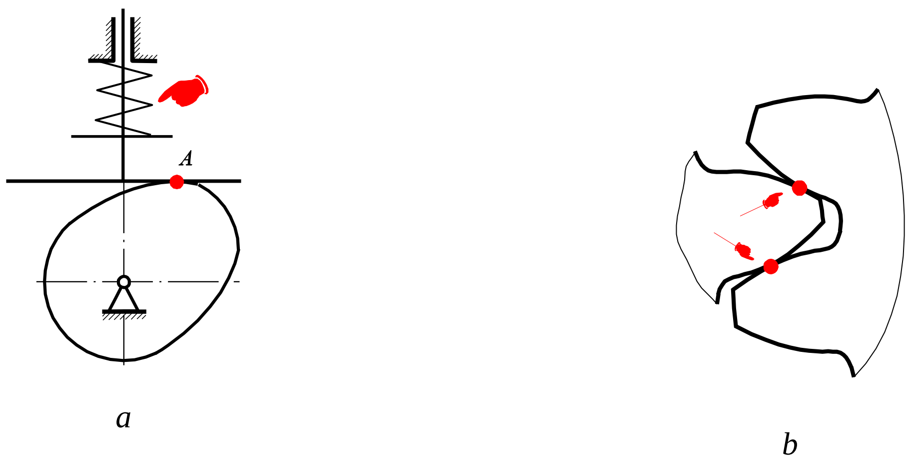


Fig. 2.5. Positive closing methods for higher pairs: *a* – force closure;
b – form closure

– they, unlike the lower kinematic pairs, have no reversibility (kinematic inversion principle), which develops in dependence of the law of link relative motion on a choice of a reference system.

In Fig. 2.6 lower kinematic pairs are shown. From a figure we can see that the relative motion of link points by identical pathways does not depend on a link with which the reference system is connected.

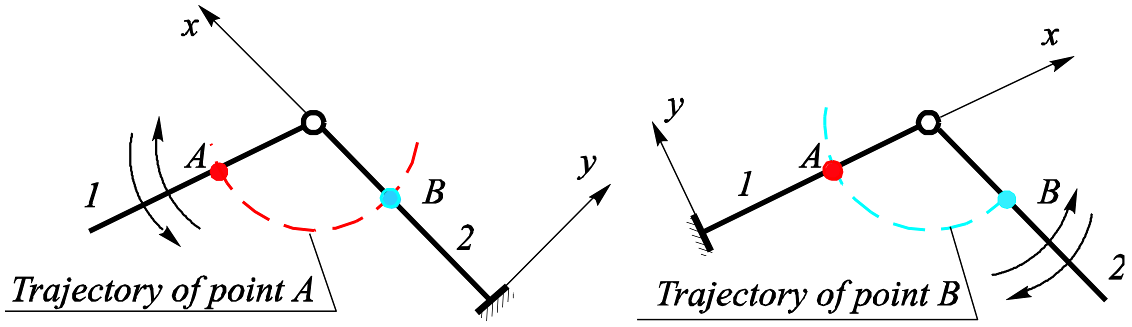


Fig. 2.6. The kinematic inversion procedure for the chain with a lower pair

In the higher pair, which is created by a circle and a straight line (Fig. 2.7), by moving a circle by a straight line (reference system is connected with a straight line) each point of a circle circumscribes a cycloid (Fig. 2.7, *a*). And if connect coordinate system with a circle, by moving a straight line by a circle, its points will circumscribe an involute (Fig. 2.7, *b*).

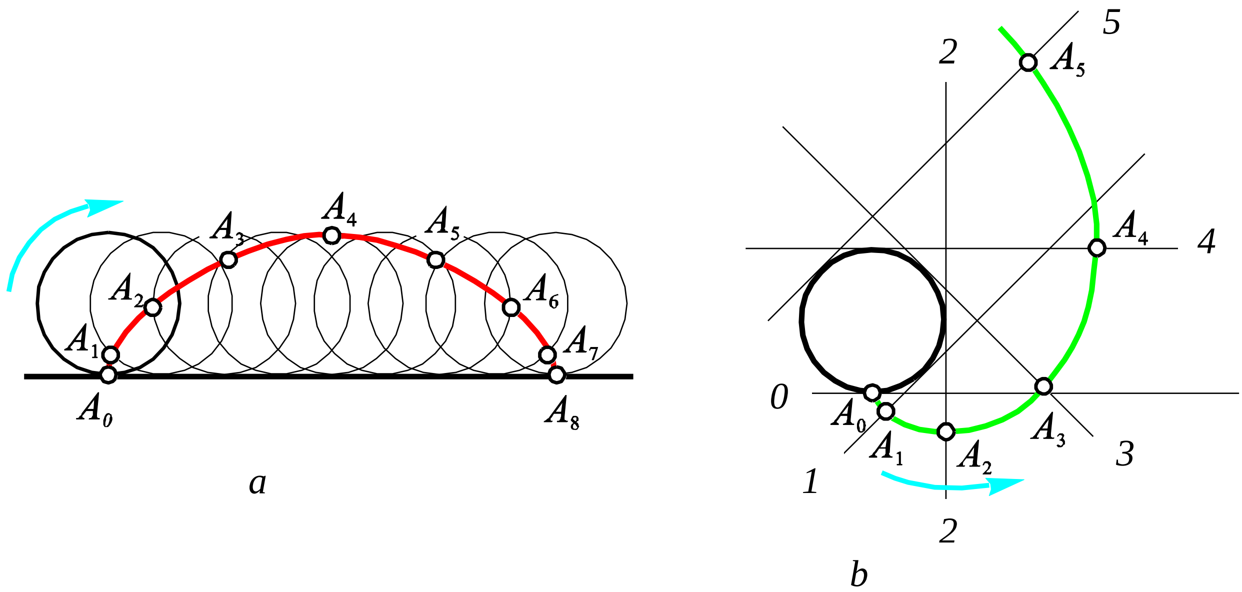


Fig. 2.7. Cycloid (*a*) and involute (*b*)

The kinematic pairs are classified according to the number of constraints, which are superimposed by their elements on relative motion of links.

In mechanics constraints are understood as limitations, which are superimposed on positions and velocities of points of a mechanical system, which are realized at any forces, acting at a system.

Coordinates of points of a mechanical system and their velocities are satisfied by an equation of constraints.

There are geometrical, differential, holonomic and non-holonomic constraints. Geometrical ideal constraints are studied in theory of mechanisms. Ideal constraints are the constraints, for which the sum of all reactions work on virtual displacements of a system equals to zero.

The class of kinematic pair is defined by the number of limitations (constraints), which the pair superimposes on relative motion of links.

In Fig. 2.8 there are examples of pairs of different classes. Here arrows indicate possible motions, which are enabled by the pair.

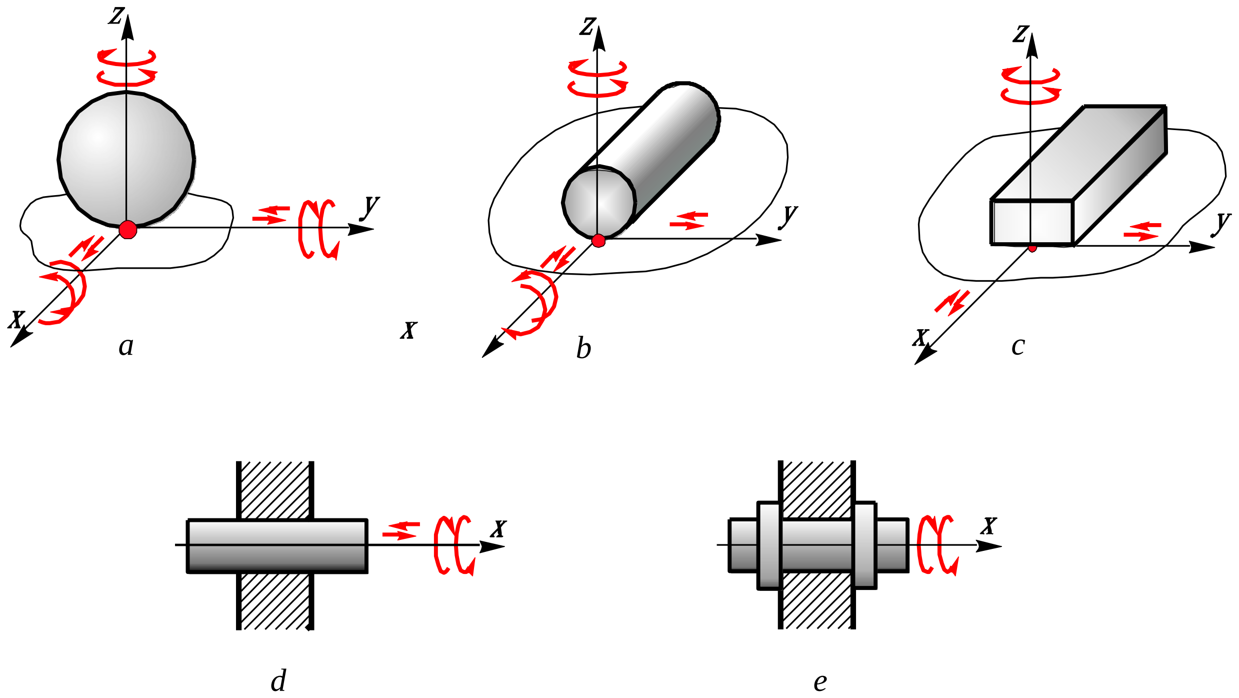


Fig. 2.8. Examples of kinematic pairs: *a* – 1-st class; *b* – 2-nd class; *c* –3-rd class; *d* – 4-th class; *e* – 5-th class;

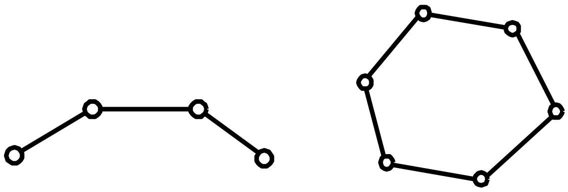
2.1.3. Kinematic chain

The kinematic chain is a system of links that form kinematic pairs.

There are different kinematic chains:

- opened or unclosed (Fig. 2.9, *a*) and closed (Fig. 2.9, *b*)
- simple (Fig. 2.9) and composite (Fig. 2.10), which contain basic link that organize more than two kinematic pairs;
- spatial and planar.

In planar chains links move in parallel planes.



a *b*

Fig. 2.9. Simple kinematic chains: opened (*a*) and unclosed (*b*)

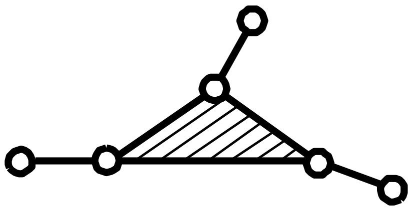


Fig. 2.10. Composite kinematic chain

The mechanism is also a kinematic chain. Fig. 2.11 shows us some mechanism schemes. All given examples are closed kinematic chains. Figures 2.11, *a* – *d* depict the four-bar linkages with only lower kinematic pairs. Pin-jointed four-bar linkage in Fig. 2.11, *a* is called *crank-and-rocker mechanism*. Crank-slider linkages are shown in Fig. 2.11, *b* – *d*: inline and offset crank-and-slider mechanisms (Fig. 2.11, *b* and *c* respectively) contain sliding pairs are created by movable links with the ground; *crank-guide* or *shaper quick-return mechanism* or *quick-return link mechanism* (Fig. 2.11, *d*) contains intermediate sliding pair when turning links (a crank and a rocking link) are grounded. All these mechanisms belong to *hinged-lever mechanisms* or *leverages*.

Figures 2.11, *e* and *f* depict the three-bar linkages with both lower and higher kinematic pairs.

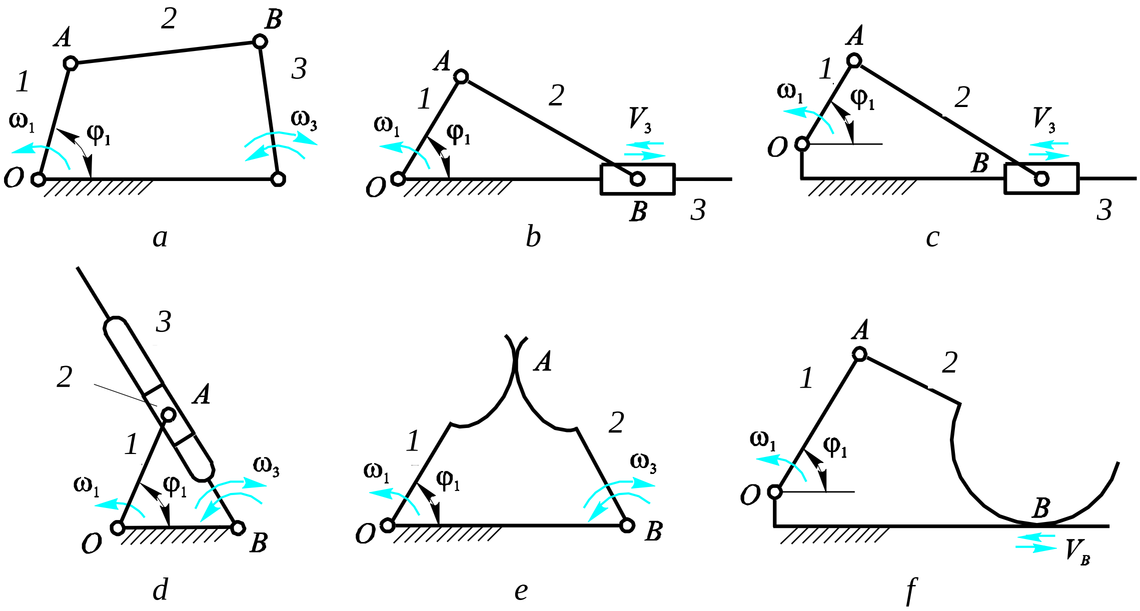


Fig. 2.11. Examples of mechanisms as closed kinematic chains: *a* – crank-and-rocker mechanism; *b* and *c* – crank-and-slider mechanisms; *d* – crank-guide; *e* and *f* – three-bar linkages with both lower and higher kinematic pairs

We can formulate the following definition.

The mechanism is a closed kinematic chain that contains a fixed link (ground) and in which, for a given motion of one or more links, the rest realize definite motions.

The input link is the link set with motions, which are converted by the mechanism. The mechanism can have several of them.

The output link is the link (or several links) that realizes motions, for which the mechanism has been created.

2.1.4. Mobility of a kinematic chain

Generalized coordinates. As it is known, the position of a body in space is determined by six independent parameters: three coordinates of the origin of a coordinate system, connected with a movable body, relative to some immovable reference system, and three Euler angles, which describe the turn of this system relatively to an immovable one.

Parameters, which uniquely describe the position of a mechanical system in space, are called generalized coordinates.

For crank-and-slider mechanism (See Fig. 2.1) a crank angle can be estimated as a generalized coordinate. There is only one generalized coordinate here.

Initial links. These are the links from which we begin to determine the mechanism positions.

The link, which has a generalized coordinate, is called an initial link of a mechanism.

Initial and input links do not necessarily coincide. In fact, any link of a mechanism can be initial, including an input link and an intermediate one, if it simplifies an analysis of a mechanism. So we choose the initial link guiding by expediency.

The amount of initial links of the mechanism equals amounts of generalized coordinates.

Number of degrees of freedom of the mechanism. Each link which is outside of a kinematic chain has six degrees of freedom. If we have *m* links, the general degree of freedom equals 6*m*, as well as 6*m* generalized coordinates.

As constraints in kinematic pairs of a chain are geometrical, they superimpose limitations only on displacements of points of links. In this case the number of degrees of freedom of a chain coincides with the number of generalized coordinates.

If to connect an immovable reference system with one of the links, so we speak about an axis of a kinematic chain concerning this link. As there is a fixed link in the mechanism – *ground*, so a reference system is connected with it, and we speak about an *axis* or a *motion freedom* of the mechanism concerning a ground.

At geometrical constraints the motion freedom can be determined behind a differential between a total number of generalized coordinates of movable links and number of equations of constraints, if these equations are independent, that is any of them cannot be a consequent of another.

Let us designate: $5p_5$ − a number of limitations, which are superimposed on mechanism links by pairs of the 5-th class (p_5 − number of pairs of the 5-th class); $4p_4$ − number of limitations superimposed by pairs of the 4-th class (p_4 − number of pairs of the 4-th class) etc.

Possible number of generalized coordinates is 6n, where $n = m - 1$ is a number of movable links of the mechanism (one link – the ground – is immovable).

Then the motion freedom of the mechanism can be found by the formula:

$$w = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1. \qquad (2.1)$$

It is the formula of Somov-Malyshev for the spatial mechanism.

For the planar mechanism, where the plain automatically superimposes three limitations on motions of links, so only two types of kinematic pairs are possible: lower pairs (the 5-th class) and higher pairs (the 4-th class). The Formula (2.1) will acquire an aspect

$$w = 3n - 2p_5 - p_4. \qquad (2.2)$$

Equations (2.1), (2.2) are called *structural equations of mechanisms*.

2.1.5. Kinematic scheme of a mechanism

Kinematic scheme of a mechanism is a conventional representation of a mechanism in the scale μ_l $\left(\frac{\text{m}}{\text{mm}}\right)$, ***where only the dimensions of links that influence the kinematics of a mechanism are kept.***

On the kinematic scheme of hinged-lever mechanisms (leverages) the graphical symbols are used. Such basic symbols you can see below in the Fig. 2.12.

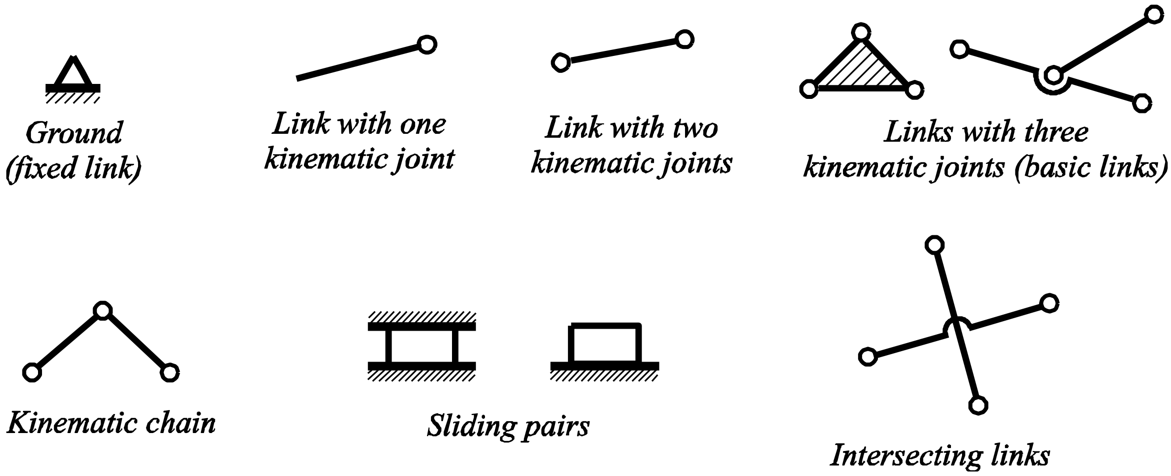


Fig. 2.12. Kinematics graphical symbols

2.1.6. Passive (redundant) constraints and links

While developing of formulas of a mechanism (2.1) and (2.2), all the equations of constraints considered to be independent. However, when creating kinematic chains, some additional constraints can be used that do not affect the motion freedom of the kinematic chains, but they transform these chains into statically indeterminate ones. If such constraints exist, the equation (2.1) becomes indefinite:

$$w = 6n - (5p_5 + 4p_4 + 3p_3 + 2p_2 + p_1 - q), \tag{2.3}$$

where q – a number of redundant constraints.

In this equation w and q are unknown. Solutions of such equations are very difficult. In practice for most planar mechanisms their motion freedom is evident. That is motion freedom w can be determined on the basis of geometric analysis and the equation (2.3) can be solved for q. While analyzing, it is important to find out correctly which constraints are redundant and to withdraw them. For example, on the Fig. 2.13 you can see the scheme of a twin-crank mechanism ($l_{OA}=l_{BC}$) of a wheel drive of a steam locomotive.

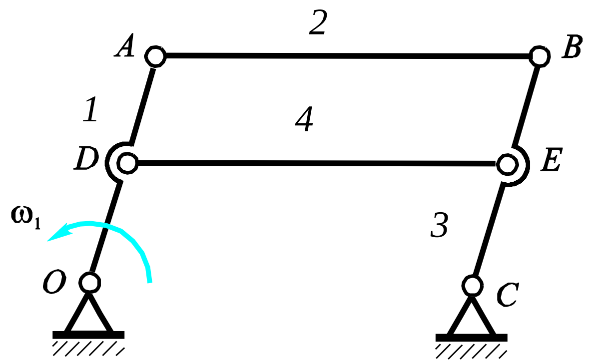


Fig. 2.13. The scheme of the twin-crank mechanism

You can calculate its motion freedom according to the formula (2.2):

$$w = 3n - 2p_5 = 0.$$

According to the structural formula, we have a fixed frame. But it is evident that this mechanism has one generalized coordinate φ_1, i.e. $w = 1$. Here the link 4 is passive. Its withdrawal will not influence the kinematics of a mechanism in any way.

Its motion freedom is calculated according to the formula (2.2):

Links and constraints, which belong to the mechanism, but do not influence its kinematics, are called passive or redundant.

And with what purpose are such links and constraints introduced to the composition of the mechanism?

Firstly, to increase the rigidity of its construction.

Secondly, to eliminate an uncertainty of the mechanism motion in some of its positions.

The position of the mechanism, in which at the prescribed motion of input links, the positions of the output links are uncertain, are called the dead positions.

Example of such position for crank-and-rocker mechanism is shown in Fig. 2.14.

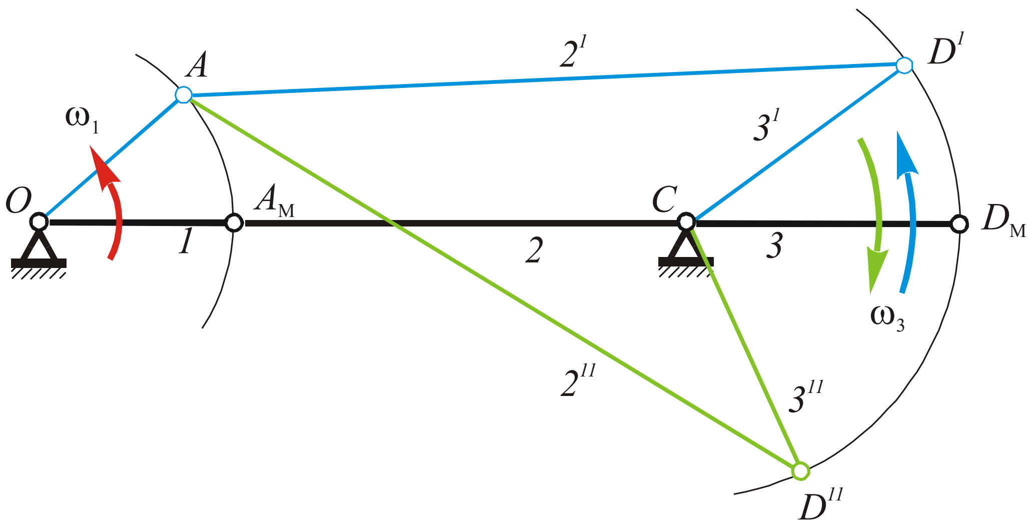


Fig. 2.14. Crank-and-rocker mechanism

Thirdly, passive links can also be inputted with the purpose of substitution of sliding friction in kinematic pairs by resistance to rolling.

According to the structural formula for a cam gear (Fig. 2.15) we get:

$$w = 3n - 2p_5 - p_4 = 3 \cdot 3 - 2 \cdot 3 - 1 = 2.$$

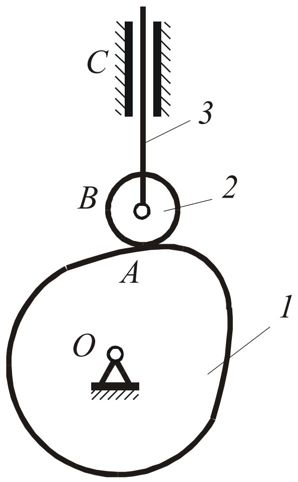


Fig. 2.15. Cam system

It appears from this that here the laws of motion of two links must be posed. But it is apparent that there is only one input link here. That is a cam 1. The roller 2 – is a passive link. If we eliminate it, we will have:

$$w = 3 \cdot 2 - 2 \cdot 2 - 1 = 1.$$

2.2. STRUCTURAL SYNTHESIS OF THE MECHANISM

2.2.1. The Assur group concept

The structural synthesis of the mechanism is the designing of its structure chart.

The <u>structure chart</u> is a conventional representation of the mechanism, which contains a fixed link, movable links and kinematic joints, on which their positional relationship without the observance of a proportion of the links' dimensions is pointed.

At the heart of the structure synthesis the method lays proposed by L. Assur. He offered to esteem each mechanism, as a chain, formed by layering of *structural groups* − so called *Assur groups*, affixed to the *primary (elementary) mechanism*.

L. V. Assur (1878–1920)

Russian mechanic and machine scientist whose works on kinematics and dynamics of the mechanisms laid the theoretical foundations of the Soviet school on the theory of mechanisms and machines. He created a rational classification of flat hinge mechanisms, developed a methodology for the formation of flat mechanisms of any complexity by the method of sequential stratification of kinematic chains. called "Assur groups", suggested the division of mechanisms into families, classes, species, orders, etc.

The <u>primary mechanism</u> is such mechanism, further partition of which on the components is impossible without breaking of its main function − transmission of the motion.

Examples of such mechanisms are shown on a Fig. 2.16: *a* − with a sliding link; *b* − with a turning link.

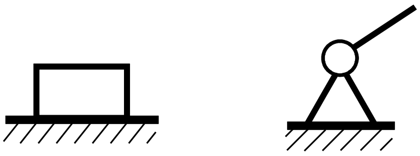


Fig. 2.16. Primary mechanisms with a sliding (*a*) and turning (*b*) links

The motion freedom of these mechanisms is $w = 3 \cdot 1 - 2 \cdot 1 = 1$.

Let us see the synthesis on an example of the pin-jointed six-bar linkage with a motion freedom $w=1$. The primary mechanism is a turning link with a ground (Fig. 2.16, *b*).

In order not to change the motion freedom (and there was only one input link), the associated structural groups with regard to the exterior pivots should have the motion freedom $w=0$. That is

$$w = 3n - 2p_5 = 0. \qquad (2.4)$$

The higher pairs do not enter to structural groups.

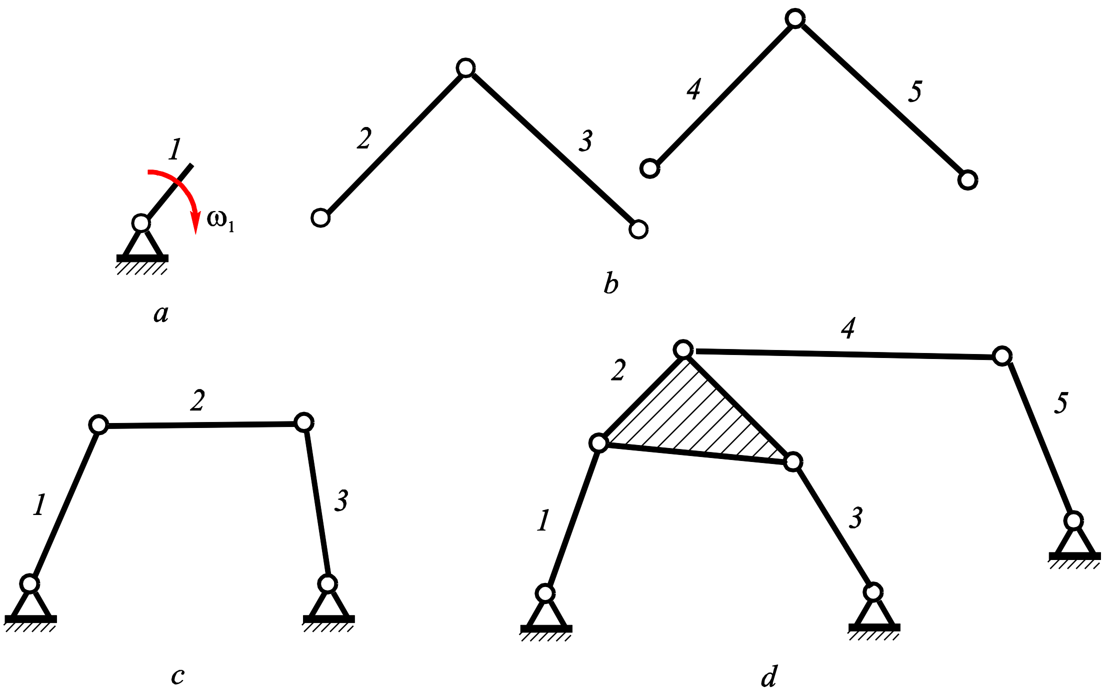


Fig. 2.17. The pin-jointed six-bar linkage synthesis: *a* – the primary mechanism; *b* – structural groups; *c* – four-bar linkage; *d* – six-bar linkage

The structural group or Assur group is called an open kinematic chain, the degree of freedom of which with regard to the elements of exterior kinematic joints is equalto zero point. Moreover, the group cannot be decomposed into simpler ones that would meet this condition.

The elementary chain contains, according to the condition (2.4), two movable links and three pivots (Fig. 2.17, *b*).

The first group is attached to the primary mechanism and to the ground (Fig. 2.17, *c*). The second group can be connected to any of movable link and to the ground. In Fig. 2.17, *d* this group is attached to the link 2, which in this case is basic.

It is easy to make sure, that the synthesized mechanism corresponds to the delivered condition – it is pin-jointed six-bar linkages with $w = 1$.

2.2.2. Classification of Assur groups

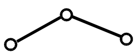
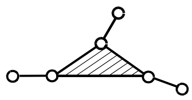
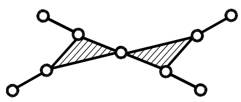
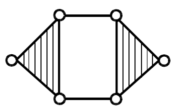
The most typical structural groups for modern mechanisms are introduced in the table 2.1.

Leonid Assur has offered to divide all groups into classes and orders.

Classification according to Assur. To groups of classes I and II such groups are referred, which do not contain variable closed loops, and differ by the fact that the groups of class II include basic links, which are attached only to other basic links (Fig. 2.18). The groups of class III contain one variable closed loop; groups of class IV contain two variable closed loops etc.

The order of group is determined by a number of driving elements – *arms* (See Table 2.1 and Fig. 2.18)

Table 2.1. **The most typical structural groups and their classification**

The type of the group	**Classification**	
	According to Assur	**According to Artobolevsky**
	class I, the 2-nd order	class II, the 2-nd order
	class I, the 3-rd order	class III, the 3-rd order
	class I, the 4-th order	class III, the 4-th order
	class III, the 0 order	class IV, the 2-nd order

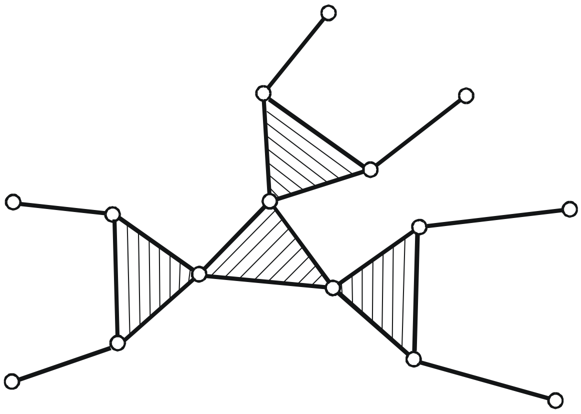
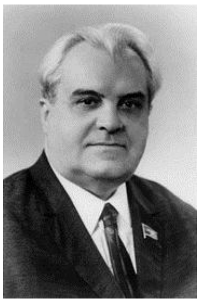


Fig. 2.18. Structural group of the class II and the 6-th order according to Assur



I. I. Artobolevsky (1905–1977)
Outstanding mechanical scientist, specialist in the field of the theory of mechanisms and machines, academician of the Academy of Sciences of the USSR. He was the head of the department of kinematics and dynamics of machines of the Mechanical Engineering Research Institute (Moscow), professor of the Moscow Aviation Institute, developed a classification of spatial mechanisms and proposed methods for their kinematic analysis. He was the founder of the scientific school of the theory of automated action machines. I. I. Artobolevsky is the author of classical textbooks on the theory of mechanisms and machines.

Classification according to Artobolevsky. Nowadays this system of a structural classification is the mainly used:

– a class of a group is defined by the number of pivots in the most difficult closed loop.
– an order of a group is defined by the number of pivots, with the help of which the group is attached to the mechanism.

I. Artobolevsky proposed to divide the groups of the class II and the 2-nd order into five kinds, the schemes of which you can see in Fig. 2.19.

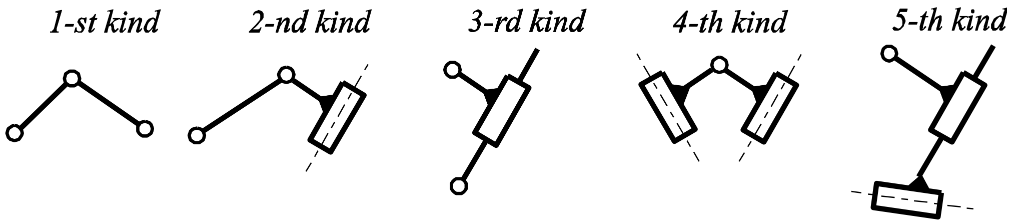
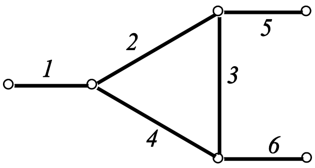
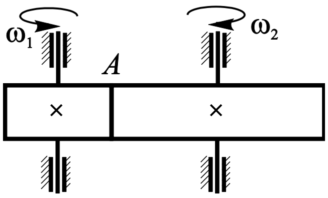
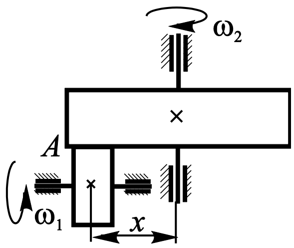


Fig. 2.19. Assur groups of the class II and the 2-nd order according to Artobolevsky

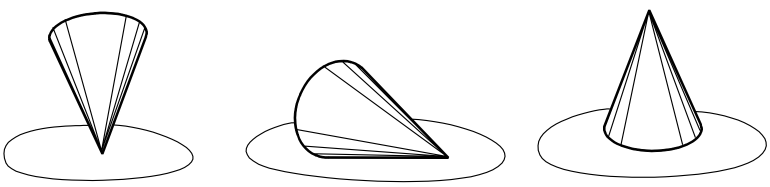
QUESTIONS FOR SELF-TESTING

1. What does the theory of mechanisms and machines study?
2. What are the main problems of the theory of mechanisms and machines?
3. What is called a mechanism?
4. What are the basic principles (signs) of mechanisms classification?
5. What is called a link and what's the part? What is the difference between these concepts?
6. 

How many links are in this chain? Which of the tagged items are only parts?
7. What is called a kinematic pair? What types of kinematic pairs do you know and what are their features?
8. What does the term "constraint" mean and what types of constraints do you know? How do they differ?
9. 

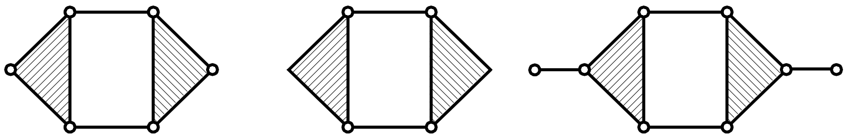


a *b*

What types of constraints are in the kinematic pairs *A* in the following schemes: geometrical or differential?
10. On which scheme (See question 9) is the constraint in the kinematic pair *A* nonholonomic? Substantiate your answer.
11. How is the class of kinematic pair determined?
12. 

a *b* *c*

What classes of pairs does the cone form with the plane in different positions?
13. What is the kinematic chain?
14. What are the types of kinematic chains? What are their special features?

15. 

a *b* *c*

Which of the following kinematic chains are not closed?

16. What classes of pairs can be formed between the links of a plane kinematic chain?
17. Define the mechanism as a kinematic chain.
18. What are the links of the mechanism called input? What are the output?
19. What is called the generalized coordinates of a mechanical system?
20. What are the names of the mechanism links, which are assigned generalized coordinates? Are these links the same as the input ones?
21. What is the difference between the notions of the degree of freedom and the motion freedom of the mechanism?
22. Write down the formulas for determining the motion freedom of spatial and planar mechanisms.
23. What is called the kinematic scheme of the mechanism?
24. What links and constraints in mechanisms are called passive? Do they affect motion freedom?
25. For what purpose can passive links and constraints be introduced into the mechanism scheme?
26. What positions of mechanism are called dead positions?
27. What is called the structure chart of the mechanism?
28. What is called the structural group or the Assur group?
29. What is the principle of constructing of mechanism schemes proposed by Assur?
30. How is the class and order of the structural group determined according to Artobolevsky?

Chapter 3. CLASSIFICATION OF MECHANISMS AND MACHINES

3.1. STRUCTURE ANALYSIS AND STRUCTURE CLASSIFICATION OF THE MECHANISM

The basic aim of the structure analysis of the mechanism is the definition of its class and order.

The class and order of the mechanism is instituted by the class and order of the most complicated structural group, which goes into its composition.

Structural analysis is carried out in the reverse order to synthesis:

– for the given mechanism it is necessary to plot the structure chart;
– to distinguish the primary mechanism;
– to divide a chain that has remained on structural groups.

Constructing of the structure chart. The structure chart is under construction on a given kinematic scheme in such succession:

– passive links and constraints are discovered and removed from the scheme of the mechanism;
– all higher pairs are substituted by lower ones;
– all sliding pairs are substituted by turning ones;
– links, which form three and more kinematic pairs, in the form of polygon with an amount of vertexes equal to the amount of joints. A fixed link is shown according to this principle.

To substitute higher pair by lower one hinges A and B are put at the centre of curvature (Fig. 3.1). They are connected by the link AB, the axis of which coincides a base tangent in the contact point C. They are also attached to hinges O and O_1 by the links OA and O_1B. The obtained four-bar linkage mechanism with lower pairs is a *kinematic equivalent* to the initial one with the higher pair. Speeds and accelerations of its points and applicable points of the source mechanism are identical that is easy to show (See [1]). Obtained kinematically similar mechanism is called *the equivalent mechanism.*

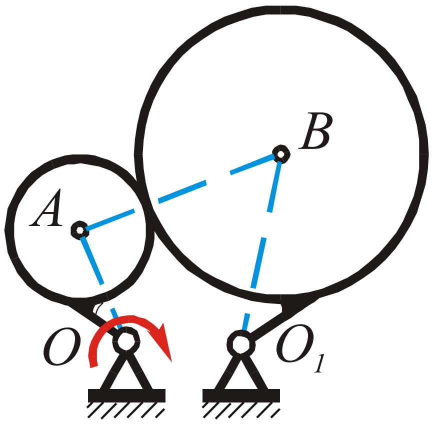


Fig. 3.1. Kinematically similar mechanisms

To substitute a sliding pair by turning one we can regard translation of a slider as an instantaneous rotation relatively a point, which is on infinity, and the slider itself is a turning element of infinite length (Fig. 3.2).

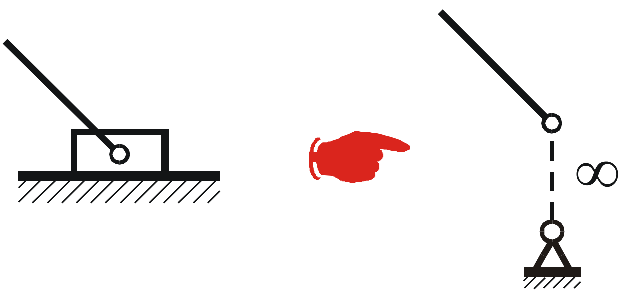


Fig. 3.2. Substitution of a sliding pair by turning one

The remark: *on the structure chart the links should not cross each other.*

Example 3.1. To determine the class and order of the mechanism (Fig. 3.3).

Historical reference. *This mechanism provides rectilinear motion of the point C without the use of guides. Its invention has allowed to solve a lot of technical problems, for example, related to the seal in the place of attachment of the piston to the rod in the piston pump to ensure tightness. The mechanism was named in honor of the inventors - the Lithuanian of Jewish origin Yomi Tov Lipman Lipkin (1846–1876), student of professor P. L. Chebyshev, and the officer of the engineering corps of the French army Charles Nicolas Poselje (1832 - 1913)*

Let us record a structure formula of the mechanism. We have movable links $n = 7$ *and lower kinematic pairs* $p_5 = 10$. *In each of the hinges A, B, D and O1 we have two kinematic pairs.*

The higher kinematic pairs in the mechanism are missed. Thus the motion freedom of the mechanism is $w = 3 \cdot 7 - 2 \cdot 10 - 0 = 1$.

Let us plot the substitute mechanism, having figured basic links, for which we accept links 1, 2 and 7, in the form of triangles (Fig. 3.3, b). And then, combining the kinematic pairs that belong to the rack into a polygon and placing them at its vertices, we build a structure chart (Fig. 3.3, c).

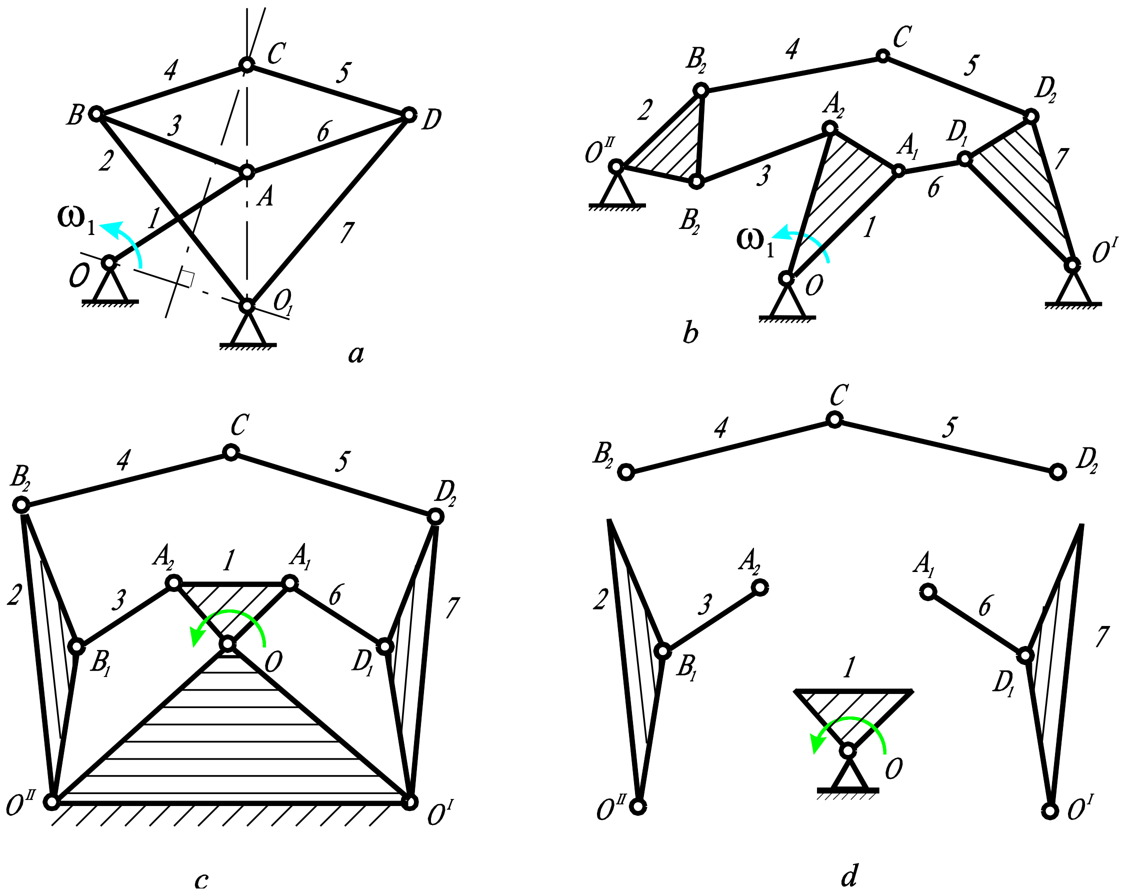


Fig. 3.3. Lipkin-Posellie mechanism: *a* – the kinematic scheme; *b* – the substitute mechanism; *c* – the structure chart; *d* – structural groups

We select the primary mechanism, which includes a fixed link and a link 1. Then an attached kinematic chain is decomposed on structural groups (Fig. 3.3, d), according to some rules:

1. *One link cannot be included into different structural groups.*
2. *The ground cannot go into structural group as a link.*
3. *It is necessary to start the analysis with the last attached structural group, concerning the primary mechanism.*

Apparently from the analysis held (Fig. 3.3, d), the examined mechanism is the mechanism of the class II and the 2-nd order.

Example 3.2. To determine the class and order of a driving mechanism of desktop of a planer (Fig. 3.4).

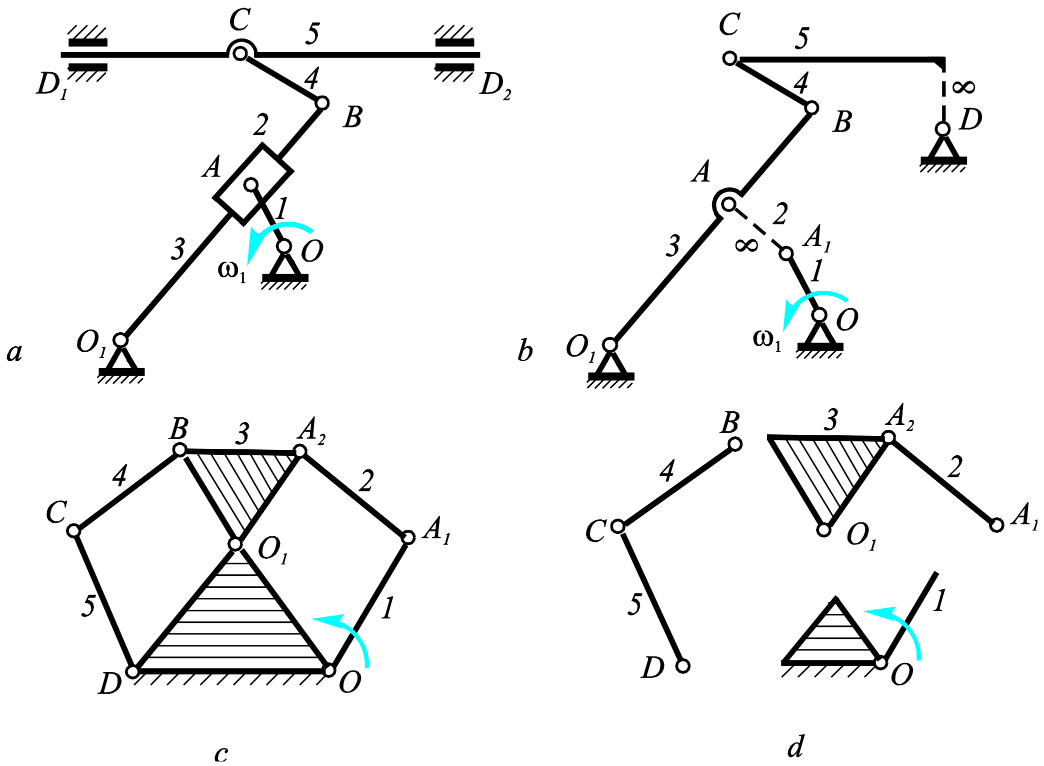


Fig. 3.4. Driving mechanism of desktop of a planer: *a* – the kinematic scheme; *b* – the substitute mechanism; *c* – structure chart; *d* – Assur groups

Let us calculate the motion freedom of the mechanism. According to the scheme the number of movable links is $n = 5$, *the number of lower kinematic pairs is* $p_5 = 8$. *There are no higher pairs. Then* $w = 3 \cdot 5 - 2 \cdot 8 = -1$, *that signifies, that it is a rigid hyperstatic structure. But it is apparent that there exists the mechanism, which has one input link − crank 1. So, in the scheme there is the passive constraint. It is the pair* D_1 *or* D_2. *We must consider them as one pair as they are formed by the same links: by a fixed link and a slider 5.*

Thus, we have kinematic pairs $p_5 = 7$. *Then* $w = 3 \cdot 5 - 2 \cdot 7 = 1$.

By substituting sliding pairs between a slide block 2 and a link 3 and between a slider (desktop) 5 and turning frame, and also showing a basic link 3 in the form of a triangle, we plot the scheme of the substitute mechanism (Fig. 3.4, b). Then we plot the structure chart (Fig. 3.4, c) and we carry out a structure analysis (Fig. 3.4, d).

We can see that our mechanism is the mechanism of the class II and the 2-nd order.

Example 3.3. To determine the class and order of a cam gear (Fig. 3.5).

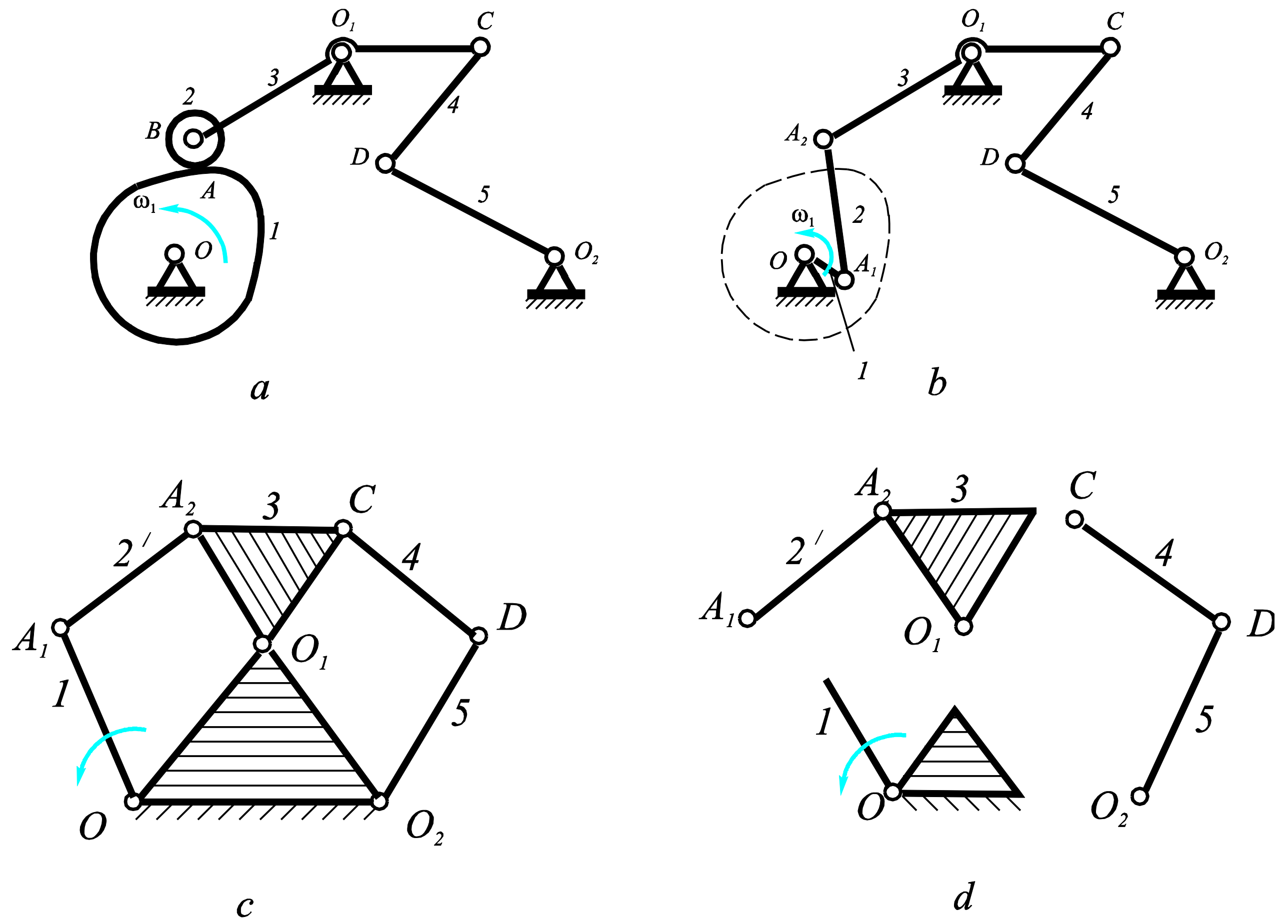


Fig. 3.5. Cam mechanism: *a* – the kinematic scheme; *b* – the substitute mechanism; *c* – structure chart; *d* – structural groups

While analyzing this kinematic scheme, we reveal in it a passive link – roller 2. We remove it and find that the number of movable links is $n = 4$, *the number of lower kinematic pairs is* $p_5 = 5$ *and the number of higher kinematic pairs is* $p_4 = 1$. *The motion freedom is* $w = 3 \cdot 4 - 2 \cdot 5 - 1 = 1$.

We plot the scheme of the substitute mechanism (Fig. 3.5, b). For this purpose the higher pair A between a cam 1 and a rocker 3 we substituted by two lower pairs A_1 *and* A_2 *and additional link 2'. Link 3 is basic. We show it in the form of a triangle.*

Three kinematic pairs belong to the ground. We show it in the form of a triangle and plot the structure chart (Fig. 3.5, c). A structure analysis (Fig. 3.5, d) demonstrates that we have the mechanism of the class II and the 2-nd order.

From the examples above it is possible to make a conclusion that the examined mechanisms are different as to their construction and functionality. They belong to the same class and order. And a driving mechanism of desktop of a planer (Fig. 3.4) and cam system (Fig. 3.5) even have identical structure charts.

Thus, classification according to the structural sign allows:

1. To join mechanisms, different as to the functional and design signs and to use for them identical methods of analysis and synthesis.

2. The structure analysis allows selecting methods of analysis depending on what link is accepted as initial. The point is that it is possible to change the class of the mechanism depending on what link will go into the composition of the primary mechanism. So, if in the six-bar linkage (Fig. 2.17, *d*) the initial link is the link 1, we get the mechanism of the class II and of the 2-nd order. And if it will be the link 5, we will get the mechanism of the class III and of the 3-rd order.

The most of the mechanisms existing are referred to the classes II, III and IV according to Artobolevsky.

Next, we consider some more examples of the construction of structural schemes and the definition of the class and order of mechanisms in various fields of engineering.

Example 3.4. To determine the class and order of the mechanism of the truck body hoist (Fig. 3.6).

Here the wheel III is a passive link and it is not taken into account in the further analysis. Then the number of movable links is $n=4$*, the number of lower kinematic pairs is* $p_5=5$ *(in unit A we have two kinematic pairs), the number of higher kinematic pairs is* $p_4=1$*. The motion freedom is* $w=3\cdot 4-2\cdot 5-1=1$.

We plot the scheme of the substitute mechanism (Fig. 3.6, b). To do this, we replace the sliding pair B between links 1 and 2 with the turning pair and replace the higher pair E between the rack and the link 4 with two lower pairs E_1 *and* E_2 *and an additional link 5.*

Link 4 is basic. We show it in the form of a triangle.

Three kinematic pairs belong to the ground. We show it in the form of a triangle and plot the structure chart (Fig. 3.6, c). A structure analysis (Fig. 3.6, d) demonstrates that we have the mechanism of the III class and the 3-rd order.

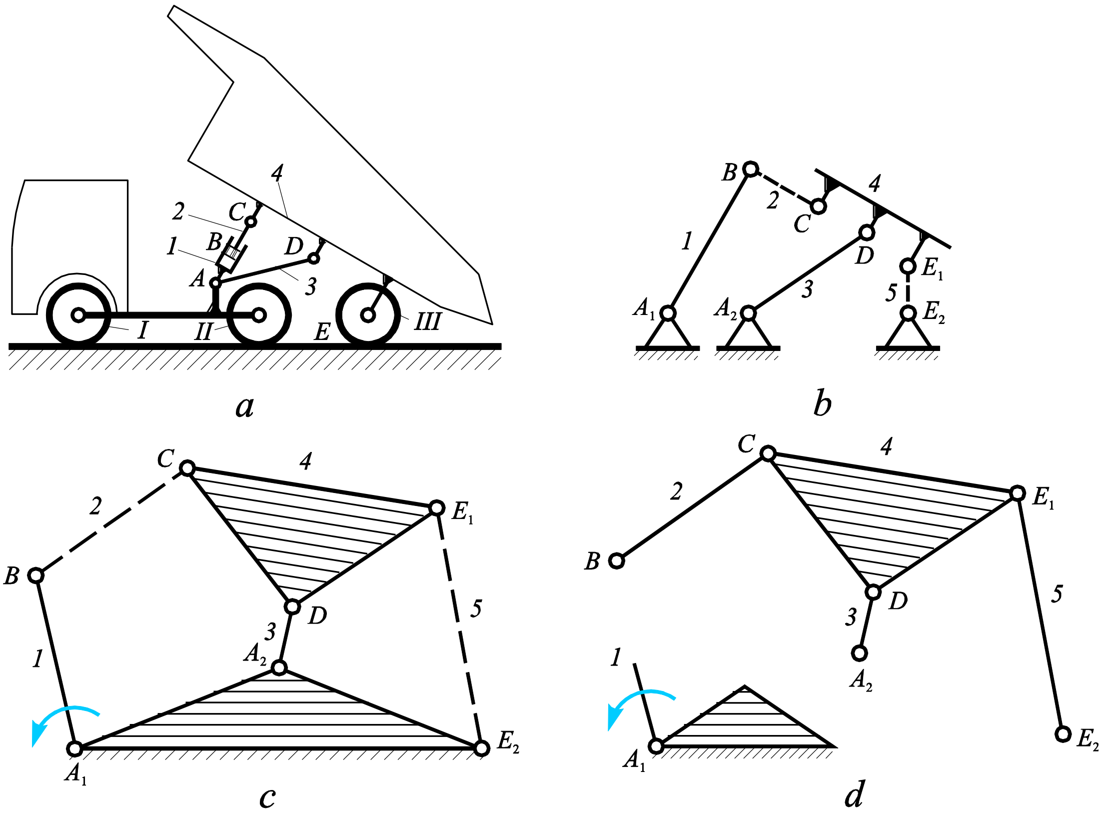


Fig. 3.6. Mechanism of the truck body hoist: *a* – the kinematic scheme; *b* – the substitute mechanism; c – structure chart; *d* – structural groups

Example 3.5. To determine the class and order of two-piston compressor mechanism (Fig. 3.7).

A preliminary analysis of the kinematic scheme of the mechanism allows us to conclude that there are two pairs among three sliding pairs M, N and P that impose passive constraints on links 5 and 8.

The choice of which of these pairs to take into account in a structural study is arbitrary. We exclude from consideration the pairs N and P, for example. Then we will have: the number of movable links is $n=8$*, the number of lower kinematic pairs is* $p_5=11$ *(these are pairs A, B, C, D, E, F, G, H, K, L i M) and one higher kinematic pair N between gear wheels 1 and 2 –* $p_4=1$*. Thus the motion freedom of this mechanism is* $w=3\cdot8-2\cdot11-1=1$.

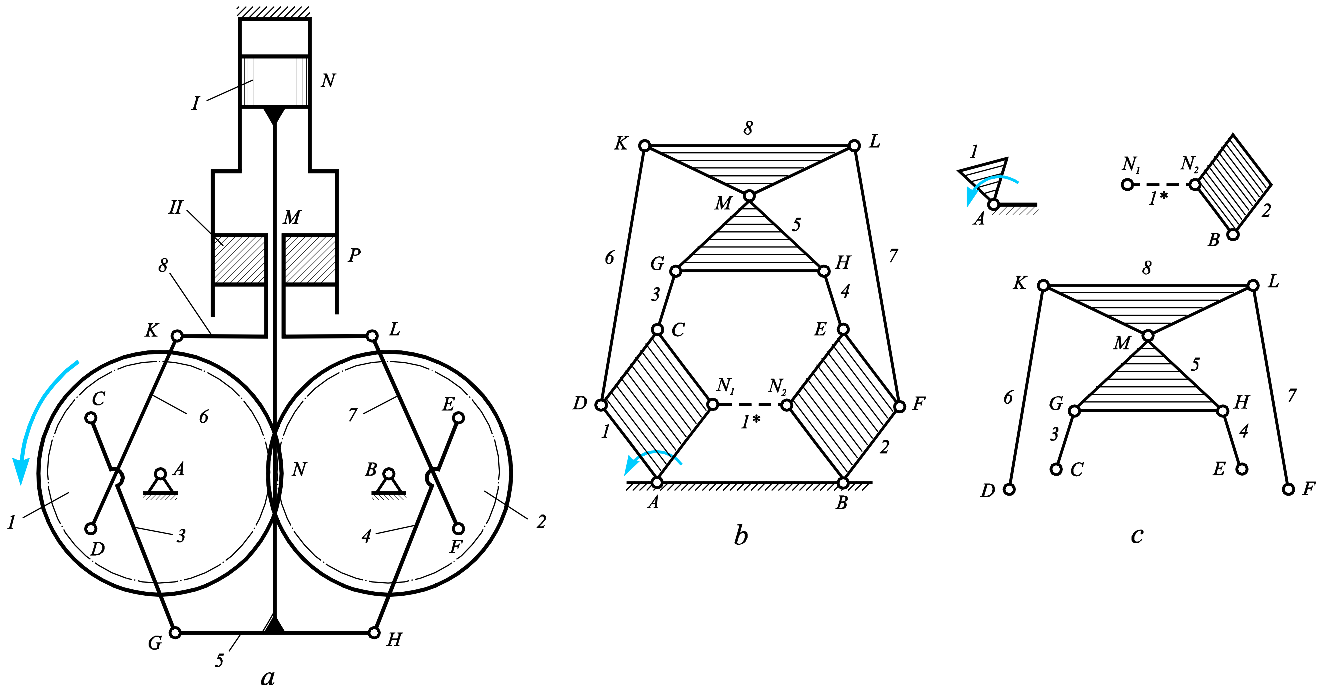


Fig. 3.7. Mechanism of two-piston compressor: *a* – the kinematic scheme; *b* – structure chart; *c* – Assur groups

The construction of substitute mechanism will not be performed in this example, but it will be compatible with the construction of the structural scheme.

We represent fixed link, which owns two kinematic pairs A and B, in Fig. 3.7, b. Links 1 and 2 are basic, as they form four kinematic pairs. We depict them in the scheme as quadrangles. The higher pair N between them is replaced by two lower pairs N_1 *and* N_2 *and the additional link 1*. The sliding pair M between links 5 and 8 is replaced by a hinge. After completing all the constructions, we check the motion freedom obtained according to the structural scheme:* $n = 9$, $p_5 = 13$, $w = 3 \cdot 9 - 2 \cdot 13 = 1$.

According to the scheme (Fig. 3.7, a), link 1 is an input link. Select the primary mechanism (Fig. 3.7, c) and decompose the attached kinematic chain into structural groups. There are two such groups: a group of the class II and the 2-nd order, formed by links 1 and 2, and a group of the class III and the 4-th order, including the remaining links of the chain.*

So we have a mechanism of the class III and the 4-th order.

It is proposed to carry out a structural study of this mechanism with a different choice of passive constraints among the sliding pairs M, N and P.

Example 3.6. To determine the class and order of multistage planetary gear mechanism (Fig. 3.8).

The number of movable links is $n = 5$, *the number of lower kinematic pairs is* $p_5 = 5$ *(kinematic pairs* A_1, A_3, *B, C, D), the number higher kinematic pairs is* $p_4 = 4$. *Hence, the motion freedom is* $w = 3 \cdot 5 - 2 \cdot 5 - 4 = 1$. *Four kinematic pairs belong to the gearbox housing:* A_1, A_3, *B i D. Links 1, 2, 3, 4 are basic links. On the structural scheme, we shall depict them in the form of polygons with the corresponding number of vertices. Higher pairs replace the additional links and hinges. The structural scheme is shown in Fig. 3.8, b.*

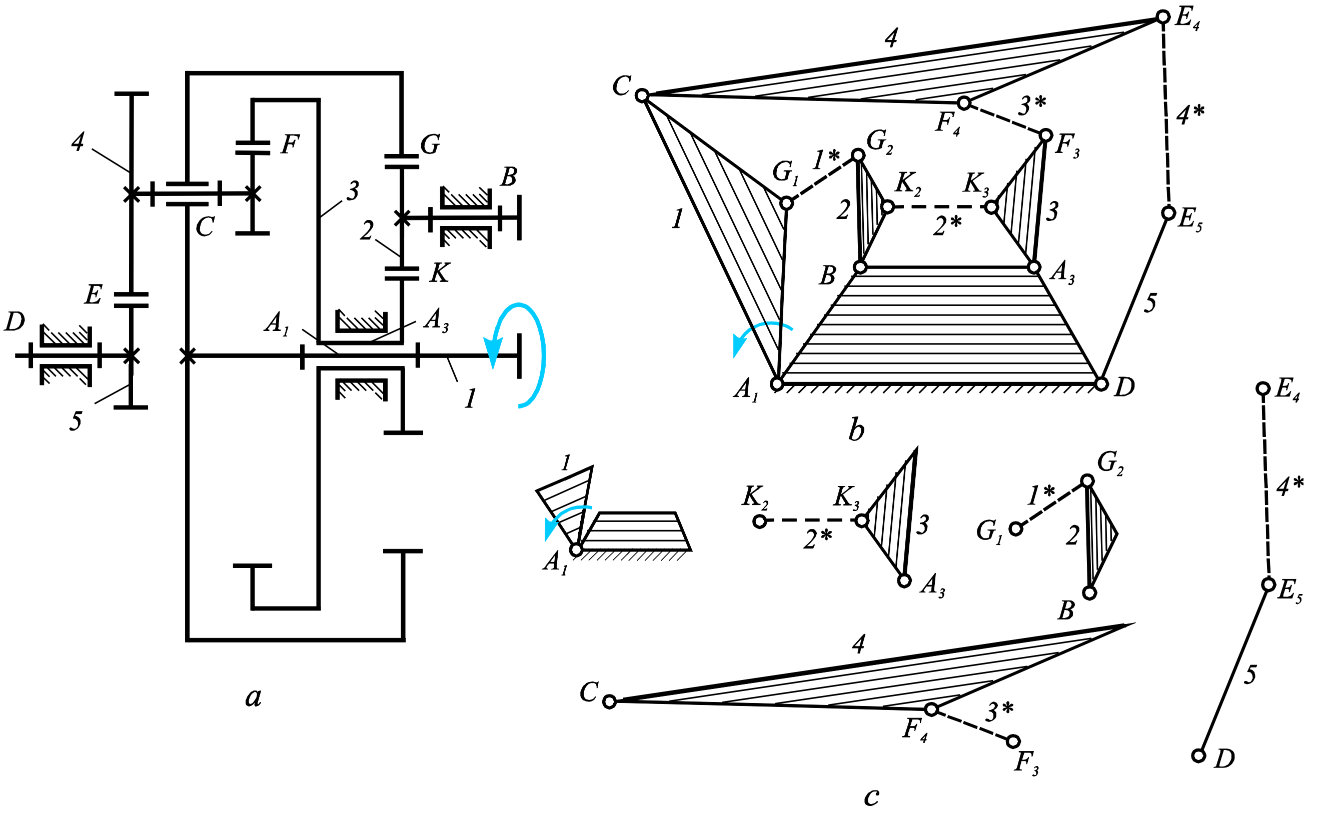


Fig. 3.8. Multistage planetary gearing mechanism: *a* – the kinematic scheme; *b* – structure chart; *c* – structural groups

Select the primary mechanism (ground with the link 1) and decompose the attached kinematic chain into structural groups. All selected groups are dyads (Fig. 3.8, c).So we have a mechanism of II class and the 2-nd order. It is proposed to establish the class and the order of this mechanism provided that the gear 5 will be an initial link.

3.2. CLASSIFICATION OF MECHANISMS ACCORDING TO THEIR FUNCTIONS

The modern economy is full of various mechanisms. According to functions of mechanisms in different production sectors, they can be grouped in such classes:

- mechanisms of engines;
- mechanisms of transmissions and actuators;
- control mechanism;
- mechanisms of self-feeder, transportation and push-up mechanisms;
- mechanisms of sorting, packing, labeling;
- calculating-solving mechanisms;
- plotting mechanisms (examples are shown on the Fig. 3.9 and 3.10).

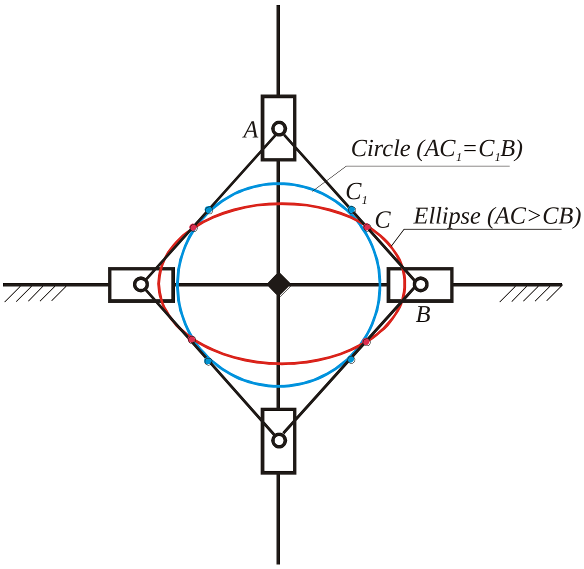


Fig. 3.9. Trammel

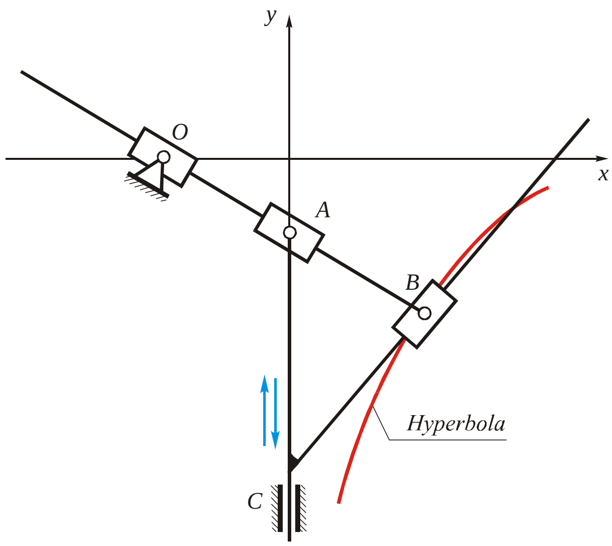


Fig. 3.10. Hyperbola plotting mechanism

3.3. CLASSIFICATION OF MECHANISMS ACCORDING TO THE TRANSMISSION MODE OF MOTION

In mechanisms, the transmission of motion from input links to output ones can be carried out with the help of such methods:

- *mechanical* – with the help of solids;
- *hydraulic* – with the help of fluid;
- *pneumatic* – with the help of gas;
- *electromagnetic* – with the help of an electromagnetic field.

3.4. CLASSIFICATION OF MECHANISMS ACCORDING TO THE NATURE OF MOTION OF THEIR LINKS

Following this sign the mechanisms are distinguished according to the motions, which are carried out by the links, which belong to its structure. The mechanisms can be:

- with sliding links;
- with turning links;
- with links, which carry out planar or two-dimensional motions;
- with links, which carry out spatial or three-dimensional motions.

3.5. CLASSIFICATION OF MECHANISMS ACCORDING TO THE DESIGN (ACCORDING TO THE TYPE OF THE MECHANISM)

Probably it is the commonly used system of classification of mechanisms in engineering practices. Such basic types of mechanisms are distinguished:

- hinged-lever;
- frictional;
- cam;
- gearings (spur and helical gearing, worm-gear, hyperbolic gearing, etc.);
- with flexible connection (belting, cable, chain);
- screw-nut;
- wedge-bar (Fig. 3.11);
- with a discontinuous motion of output links (Geneva mechanism or maltese cross mechanism (Fig. 3.12), ratchet mechanism (Fig.3.13) and others).

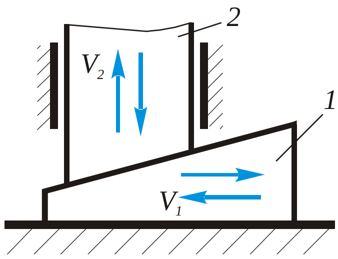


Fig. 3.11. Wedge-bar mechanism

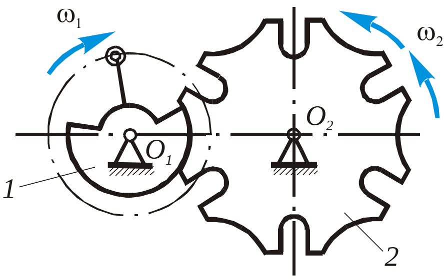


Fig. 3.12. Maltese-cross mechanism

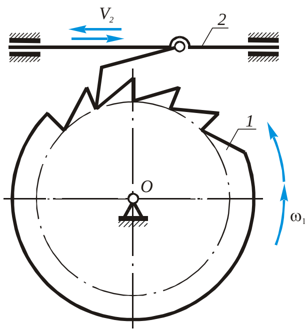


Fig. 3.13. Ratchet mechanism

3.6. MACHINE. AGGREGATE

Machine is a device, which executes mechanical motions for transforming of energy, materials and information with the purpose of substitution or facilitation of physical and intellectual labour of a person [3].

We distinguish:

- *energy machines*, intended for transforming of any kind of energy into mechanical one and on the contrary;
- *technological machines* (engineering tools, presses, swages, rolling mills, etc.);
- *transport machines* (automobiles, transporters, cranes, airplanes, escalators, etc.);
- *information-calculating machines* (mechanical and electronic calculators, computers, etc.).

Joining of the energy machine (engine) with the executive machine (technological, transport, etc.) is called the machine aggregate.

The requests to modern machines. In order to meet the world standards of quality, modern machines must have such properties.

1. Reliability augmentation at given life. In Fig. 3.14 curves of expenditures on designing and working costs are shown (Fig. 3.14, *a*) depending on life of the machine. From the diagram (Fig. 3.14, *b*) on total cost you can see that there is an optimal life when the machine will be the most economical. The life of any machine has to be adjusted with the term of its obsolescence.

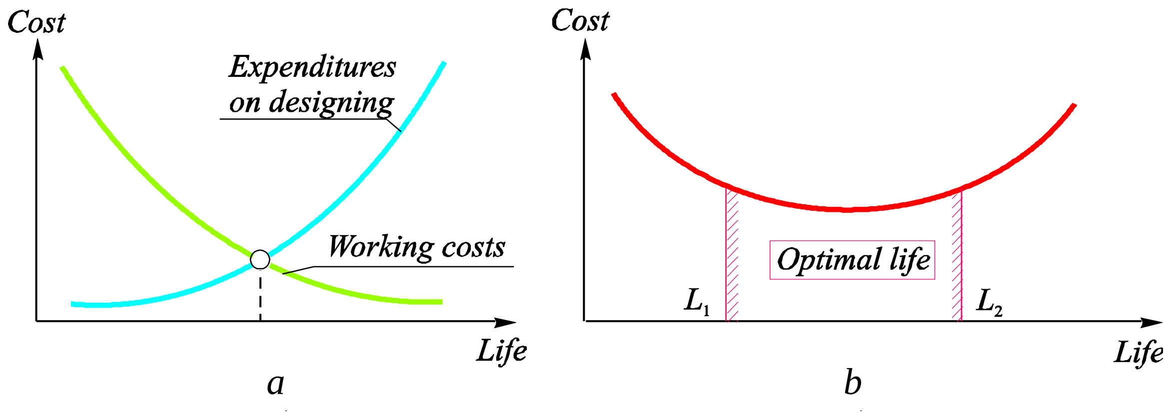


Fig. 3.14. Cost-life diagram: *a* – designing and working costs;
b – the curve of total cost;

2. High efficiency. For example, to design machines, which work by a continuous principle. Let's take a rotary overburden excavator. Its usage instead of scoop excavator under certain conditions is much more effective.

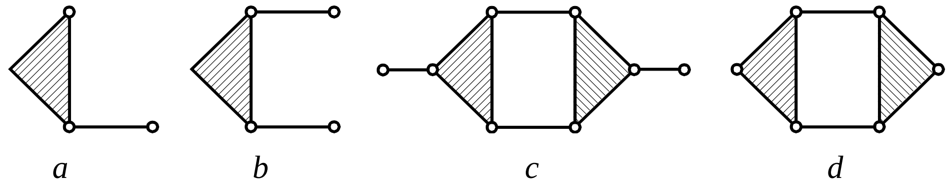
3. High quality and accuracy.

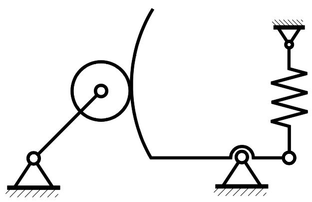
4. Economy.

5. Ecological purity.

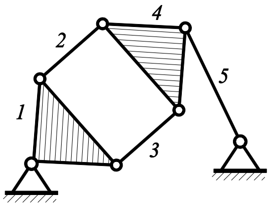
Depending on a branch range of application, this list can be continued.

QUESTIONS FOR SELF-TESTING

1. What mechanism is called the *primary (elementary) mechanism*? Give examples of such mechanisms.
2. What is called *the basic link*?
3. 

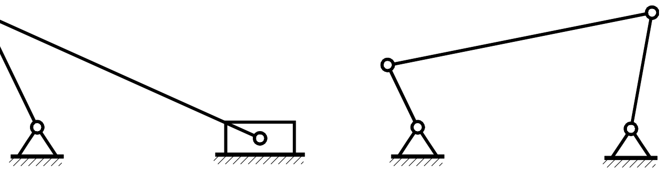
   Which of the following kinematic chains are Assur groups?
4. What is the main task of structural analysis of mechanisms?
5. Formulate the sequence in which the structural analysis of the mechanism is carried out.
6. What is the difference between the kinematic and the structural scheme of the mechanism?
7. Which pairs are called *lower*? What class do they belong in planar mechanisms?
8. What pairs are called *higher*? What class do they belong in planar mechanisms?
9. What are the main stages of construction of a structural scheme of the mechanism?
10. What mechanism is called *a substitute*?
11. Why is the substitute mechanism called *kinematically equivalent* to the given one?
12. 

    How many passive links (constraints) are in this mechanism?
13. Construct a substitute mechanism scheme for the mechanism presented in question 12.
14. How can we determine the class and the order of mechanism by its structural scheme?

15. 

What link of this mechanism should be chosen as the initial one so that its class according to Artobolevsky would be lower?

16. What features can we characterize mechanisms of engines, transmissions, actuators?

17. 

a *b*

What types of mechanisms by the design feature and the nature of motion of the links do these crank mechanisms represent?

18. Give examples of mechanisms that can be classified according to the transmission mode of motion.
19. Give the definition of "machine".
20. What are the main types of machines following the modern classification?
21. What is called a machine aggregate?
22. List the basic requirements for modern machines.

Chapter 4. KINEMATIC ANALYSIS OF MECHANISMS

***The kinematics* is a branch of mechanics, which studies the laws of motion of particles and rigid bodies, disregarding of reasons causing this motion.**

Two primal tasks are solved in kinematics: *analysis and synthesis* of the mechanism.

While analyzing, the existing mechanism is esteemed and its motion characteristics are studied. The task of synthesis is to create mechanisms with given kinematic characteristics.

The methods of analysis are the most developed today, that is, the study of already existing mechanisms. The methods of synthesis are developed deeply only for cams and gearing mechanisms.

4.1. ANALYTICAL METHODS OF KINEMATIC ANALYSIS

4.1.1. The main kinematic parameters of the mechanism

Kinematic parameters or motion characteristics of the mechanism contain coordinates and motion paths of points; generalized coordinates of links; displacements of points and links, their velocities and accelerations, and also *position function* and *transfer functions of a mechanism*.

There are three basic exploratory methods are predominantly used now − analytical methods of the kinematic diagrams, method, method of the vector diagrams (velocity and acceleration diagrams).

4.1.2. Transfer functions and velocity ratios

The position function of the mechanism. In practice, the study of kinematic parameters is convenient to conduct not as the function of time, but as a function of generalized coordinates of initial links. That is, when the position of the mechanism is known, these parameters are evaluated independently of the law of variation of generalized coordinates in time which can be unknown for us. In other words, the kinematic parameters of the mechanism, in this case, depend only on its kinematic scheme (geometry) and do not depend on time.

The position function of a mechanism is called the dependence of a coordinate of an output link on generalized coordinates of the mechanism.

For example:

$$\varphi_n = \varphi_n(\varphi_1, \varphi_2, ..., \varphi_s).$$

Here n – is the index of the output link; $\varphi_1, ..., \varphi_s$ – are generalized coordinates of the mechanism.

Let's examine the mechanism with motion freedom w=2 (Fig. 4.1), n-th link of which carries out the turning motion with the angular velocity ω_n.

The position function of the n-th link:

$$\varphi_n = \varphi_n(\varphi_1, \varphi_2).$$

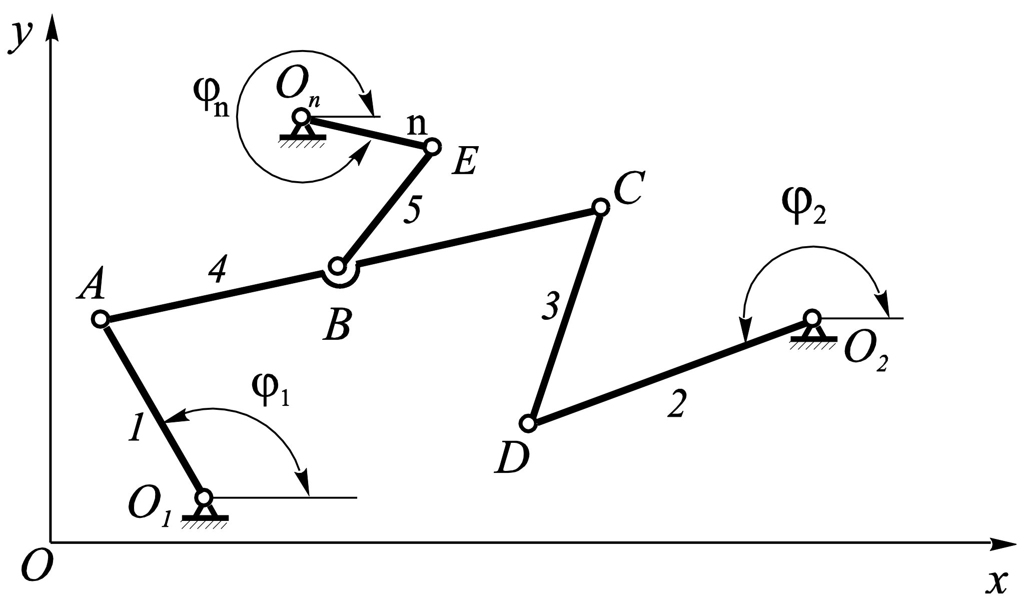


Fig. 4.1. Pin-jointed linkage with w=2

Transfer functions of the mechanism. Let's discover the angular velocity of the *n*-th link (Fig. 4.1).

$$\omega_n = \frac{\partial\varphi_n}{\partial t} = \frac{\partial\varphi_n}{\partial\varphi_1}\frac{\partial\varphi_1}{\partial t} + \frac{\partial\varphi_n}{\partial\varphi_2}\frac{\partial\varphi_2}{\partial t},$$

or

$$\omega_n = u_{n1}^{(2)} \cdot \omega_1 + u_{n2}^{(1)} \cdot \omega_2. \tag{4.1}$$

Here $u_{n1}^{(2)} = \dfrac{\partial\varphi_n}{\partial\varphi_1}$; $u_{n2}^{(1)} = \dfrac{\partial\varphi_n}{\partial\varphi_2}$ – are *the quotient velocity ratio.*

For the mechanism with the motion freedom w=1 (ω_2=0)

$$\omega_n = \frac{d\varphi_n}{d\varphi_1}\frac{d\varphi_1}{dt} = u_{n1} \cdot \omega_1, \tag{4.5}$$

where $u_{n1}^{(2)} = \dfrac{\partial\varphi_n}{\partial\varphi_1} = \dfrac{\omega_n}{\omega_1} = i_{n1}$ – the ratio of the angular velocities of links, which is called *the angular velocity ratio or velocity ratio.*

The physical content of the quotient velocity ratio is the following: $u_{n1}^{(2)}$ – velocity ratio between the links *n* and 1, when the link 2 is unmovable (Fig. 4.1); $u_{n2}^{(1)}$ – velocity ratio between the links *n* and 2, when the link 1 is unmovable.

The quotient velocity ratio is also called the *angular velocity analogue*. It is true, if in the equation (4.1) $\omega_2 = 0$, and $\omega_1 = 1$ c^{-1}, then $\omega_n = u_{n1}^{(2)}$.

Let's find the velocity of the point *E* (Fig. 4.1), which position is determined by the position vector $\overrightarrow{r_E} = \overrightarrow{r_E}(\varphi_1, \varphi_2)$:

$$\overrightarrow{V_E} = \frac{d\overrightarrow{r_E}}{dt} = \frac{\partial\overrightarrow{r_E}}{\partial\varphi_1}\frac{d\varphi_1}{dt} + \frac{\partial\overrightarrow{r_E}}{\partial\varphi_2}\frac{d\varphi_2}{dt}$$

or

$$\overrightarrow{V_E} = \frac{\partial \overrightarrow{r_E}}{\partial \varphi_1}\omega_1 + \frac{\partial \overrightarrow{r_E}}{\partial \varphi_2}\omega_2 . \qquad (4.2)$$

Where $\frac{\partial \overrightarrow{r_E}}{\partial \varphi_1}$ and $\frac{\partial \overrightarrow{r_E}}{\partial \varphi_2}$ are the *velocity analogues* of the point *E*.

For the mechanism with motion freedom *w*=1 (for example the velocity $\omega_2 = 0$) the equation (4.2) takes the form

$$\overrightarrow{V_E} = \frac{d\overrightarrow{r_E}}{d\varphi_1}\omega_1 . \qquad (4.3)$$

Let's determine the angular acceleration of the link *n*. We will confine to a case, when the motion freedom of the mechanism is *w*=1 (ω_2=0)

$$\varepsilon_n = \frac{d^2\varphi_n}{dt^2} = \frac{d}{dt}\left(\frac{d\varphi_n}{d\varphi_1}\omega_1\right) = \frac{d^2\varphi_n}{d\varphi_1^2}\frac{d\varphi_1}{dt}\omega_1 + \frac{d\varphi_n}{d\varphi_1}\frac{d\omega_1}{dt}$$

or

$$\varepsilon_n = \frac{d^2\varphi_n}{d\varphi_1^2}\omega_1^2 + \frac{d\varphi_n}{d\varphi_1}\varepsilon_1 ,$$

where $\frac{d^2\varphi_n}{d\varphi_1^2} = u'_{n1}$ is the angular acceleration analogue.

Let's determine the tangential acceleration of the point *E*, having taken a time derivative from the velocity (4.3). After some simple transformations we will get:

$$\overrightarrow{a_E^\tau} = \frac{d^2\overrightarrow{r_E}}{d\varphi_1^2}\omega_1^2 + \frac{d\overrightarrow{r_E}}{d\varphi_1}\varepsilon_1 .$$

Here $\frac{d^2 r_E}{d\varphi_1^2}$ – is the analogue of the tangential acceleration of the point.

The velocities and accelerations analogues are called the <u>transfer functions</u> of the mechanism.

4.1.3. Examples of kinematic studies of some typical mechanisms

Example 4.1. To determine the velocity and the acceleration of the point *B* of the offset crank-and-slider mechanism, kinematic scheme of which you can see in Fig. 4.2 on a scale μ_l, if ω_1=Const. Determine the velocity ratio between the connecting rod 2 and the crank 1, as well as the angular acceleration of the connecting rod.

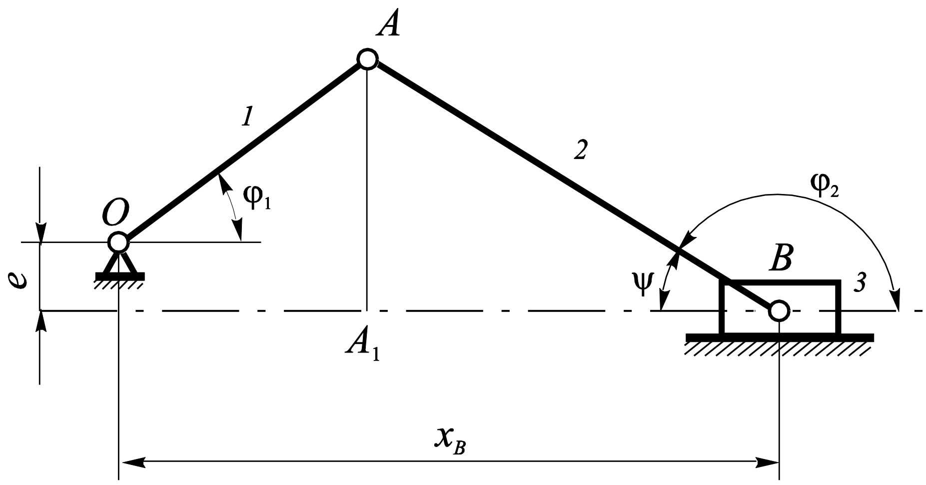


Fig. 4.2. An offset crank-and-slider mechanism

Let's determine the position function of the point B:

$$x_B = x_B(\varphi_1);$$

$$x_B = l_{OA} \cdot \cos\varphi_1 + l_{AB} \cdot \cos\psi . \qquad (4.4)$$

From the ΔABA_1 we find:

$$\sin\psi = \frac{l_{OA}\sin\varphi_1 + e}{l_{AB}} = \sin(\pi - \varphi_2).$$

Then

$$\cos\psi=\sqrt{1-\sin^2\psi}=\sqrt{1-\left(\frac{l_{OA}\sin\varphi_1+e}{l_{AB}}\right)^2}\ .$$

If we substitute the expression that we have just found in (4.4) we get

$$x_B=l_{OA}\cos\varphi_1+\sqrt{l_{AB}^2-(e+l_{OA}\sin\varphi_1)^2}\ . \tag{4.5}$$

The velocity of the point B

$$V_B=\frac{dx_B}{dt}=\frac{dx_B}{d\varphi_1}\omega_1 .$$

We find the velocity analogue of the point B if we take a derivative from its position function (4.5)

$$\frac{dx_B}{d\varphi_1}=-l_{OA}\sin\varphi_1+\frac{-2(e+l_{OA}\sin\varphi_1)l_{OA}\cos\varphi_1}{2\sqrt{l_{AB}^2-(e+l_{OA}\sin\varphi_1)^2}} .$$

Taking into account the fact, that when ω_1=Const, the angular acceleration of the crank is ε_1=0, the acceleration of the point B

$$a_B=\frac{d^2x_B}{dt^2}=\frac{d^2x_B}{d\varphi_1^2}\cdot\omega_1^2 .$$

Here the acceleration analogue

$$\frac{d^2x_B}{d\varphi_1^2}=-l_{OA}\cos\varphi_1-\frac{l_{AB}^2l_{OA}^2\cos^2\varphi_1-l_{AB}^2l_{OA}\left(e+l_{OA}\sin\varphi_1\right)\sin\varphi_1+l_{OA}\left(e+l_{OA}\sin\varphi_1\right)^3\sin\varphi_1}{\left[l_{AB}^2-\left(e+l_{OA}\sin\varphi_1\right)^2\right]^{\frac{3}{2}}} .$$

Let us determine the velocity ratio between the connecting rod and the crank:

$$u_{21}=\frac{\omega_2}{\omega_1}=\frac{d\varphi_2}{d\varphi_1}=\frac{d}{d\varphi_1}\left(\arcsin\frac{l_{OA}\sin\varphi_1+e}{l_{AB}}\right)=\frac{l_{OA}\cos\varphi_1}{l_{AB}\sqrt{1-\left(\frac{l_{OA}\sin\varphi_1+e}{l_{AB}}\right)^2}}=-\frac{l_{OA}}{l_{AB}}\cdot\frac{\cos\varphi_1}{\cos\varphi_2} .$$

Here $\sqrt{1-\left(\dfrac{l_{OA}\sin\varphi_1+e}{l_{AB}}\right)^2}=\cos\left(\pi-\varphi_2\right)=-\cos\varphi_2$.

Let us determine the transfer function of the angular acceleration of the connecting rod:

$$\frac{d^2\varphi_2}{d\varphi_1^2}=\frac{d}{d\varphi_1}\left(u_{21}\right)=\frac{d}{d\varphi_1}\left(-\frac{l_{OA}\cos\varphi_1}{l_{AB}\cos\varphi_2}\right)=\frac{l_{OA}}{l_{AB}}\cdot\left(\frac{\cos\varphi_2\cdot\sin\varphi_1-u_{21}\cos\varphi_1\cdot\sin\varphi_2}{\cos^2\varphi_2}\right).$$

The angular acceleration of the connecting rod is

$$\varepsilon_2=\omega_1^2\cdot\frac{l_{OA}}{l_{AB}}\cdot\left(\frac{\cos\varphi_2\cdot\sin\varphi_1-u_{21}\cos\varphi_1\cdot\sin\varphi_2}{\cos^2\varphi_2}\right).$$

Example 4.2. To determine position and transfer functions of a pin-jointed four-bar linkage (Fig. 4.3).

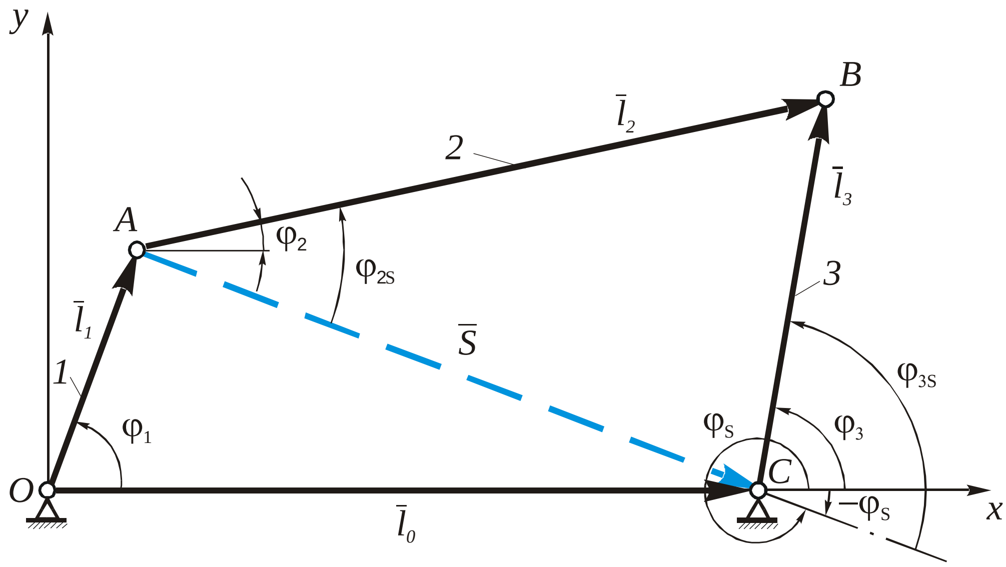


Fig. 4.3. A crank-and-rocker mechanism

It is easy to make certain of the fact that the mathematical ratio for this type of mechanisms will be rather bulky if take advantage of the approach applied in Example 3.1. Therefore, we will conduct investigations by applying <u>*a method of closed vector loops*</u>.

Defining a position function, it is convenient to fracture the closed loop OABC into two: OAC and ABC. In these closed loops, the link axes are given as vectors $\vec{l}_1,\ \vec{l}_2,\ \vec{l}_3,\ \vec{l}_0$. *Here* $\vec{S}$ *- is a changeable vector, which defines a positional relationship of points A and C.*

Then the following closure conditions are true for triangular closed loops:

- *for a closed loop OAC*

$$\vec{l}_1 + \vec{S} - \vec{l}_0 = 0\,;$$

- *for a closed loop ABC*

$$\vec{l}_2 - \vec{S} - \vec{l}_3 = 0\,.$$

We project vectors on axes x, y and get the relations

$$\begin{cases} l_1 \cos\varphi_1 + S\cos\varphi_S - l_0 = 0; \\ l_1 \sin\varphi_1 + S\sin\varphi_S = 0. \end{cases} \tag{4.6}$$

Hence

$$\mathrm{tg}\varphi_S = \frac{-l_1 \sin\varphi_1}{l_0 - l_1\cos\varphi_1}. \tag{4.7}$$

From the second equation of a system (4.6)

$$S = -\frac{l_1 \sin\varphi_1}{\sin\varphi_s}.$$

Let us esteem a closed loop ABC. We mark tilt angle of vectors $\vec{l}_2$ *and* $\vec{l}_3$ *to* $\vec{S}$ *respectively* φ_{2S} *and* φ_{3S}. *Using a cosine law, we receive such equations:*

$$l_2^2 = S^2 + l_3^2 - 2Sl_3\cos\left(\pi - \varphi_{3S}\right) = S^2 + l_3^2 + 2Sl_3\cos\varphi_{3S}\,;$$

$$l_3^2 = S^2 + l_2^2 - 2Sl_2\cos\varphi_{2S}\,.$$

Then

$$\varphi_{3S} = \arccos\frac{l_2^2 - S^2 - l_3^2}{2Sl_3}\,; \tag{4.8}$$

$$\varphi_{2S} = \arccos\frac{S^2 + l_2^2 - l_3^2}{2Sl_2}\,. \tag{4.9}$$

We note the conditions

$$\begin{cases} \varphi_{3S} - \varphi_3 = -\varphi_S\,; \\ \varphi_{2S} - \varphi_2 = -\varphi_S\,. \end{cases}$$

Hence

$$\varphi_3 = \varphi_S + \varphi_{3S}\,; \tag{4.10}$$

$$\varphi_2 = \varphi_S + \varphi_{2S}\,. \tag{4.11}$$

In an equation (4.10) and (4.11) angle φ_S *is determined through* φ_1 *according to the equation (4.7), angles* φ_{2S} *and* φ_{3S} *are determined through the given sizes of links according to the equations (4.8) and (4.9).*

According to the cosine law (closed loop OAC) we note

$$S = \sqrt{l_1^2 + l_0^2 - 2l_1 l_0 \cos\varphi_1}\,.$$

We get a position function of the link 3:

$$\varphi_3 = \operatorname{arctg}\frac{-l_1 \sin\varphi_1}{l_0 - l_1\cos\varphi_1} + \arccos\frac{l_2^2 - l_3^2 - l_0^2 - l_1^2 + 2l_1 l_0\cos\varphi_1}{2l_3\sqrt{l_1^2 + \ell_0^2 - 2l_1 l_0\cos\varphi_1}}\,.$$

Similarly, we get a position function of the link 2.

$$\varphi_2 = \operatorname{arctg}\frac{-l_1 \sin\varphi_1}{l_0 - l_1\cos\varphi_1} + \arccos\frac{l_2^2 - l_3^2 + l_1^2 + l_0^2 - 2l_1 l_0\cos\varphi_1}{2l_2\sqrt{l_1^2 + \ell_0^2 - 2l_1 l_0\cos\varphi_1}}\,.$$

Let's determine transfer functions of the mechanism (analogues of angular velocities and accelerations of links).

For this purpose, we esteem a closed vectorial loop OABC (Fig. 4.4).

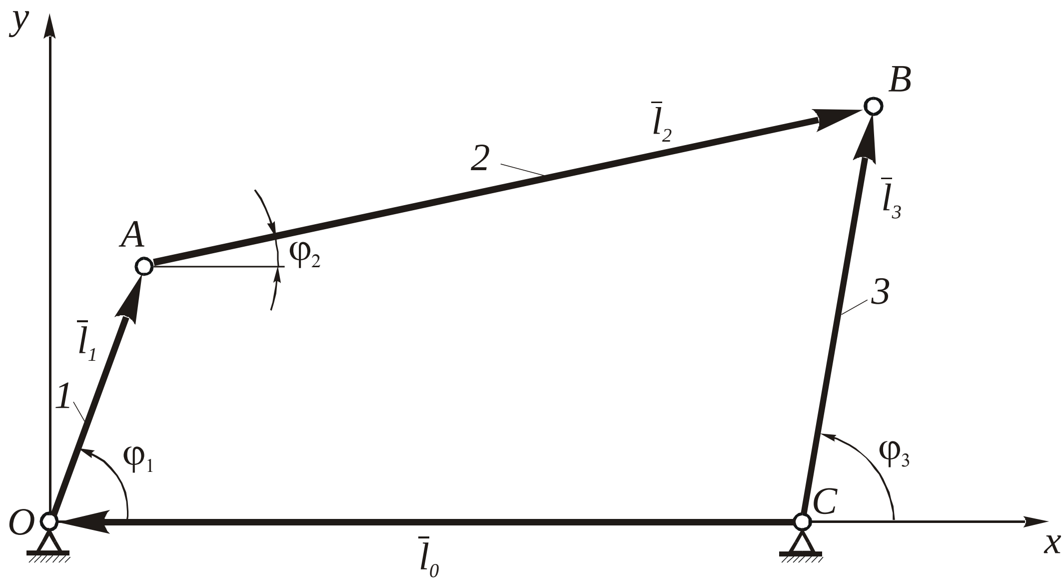


Fig. 4.4. Four-bar linkage

Condition of its closure

$$\vec{l}_0 + \vec{l}_1 + \vec{l}_2 = \vec{l}_3 .$$

In a projection to the axis x we get:

$$l_1 \cos\varphi_1 + l_2 \cos\varphi_2 - l_3 \cos\varphi_3 = l_0 .$$

We differentiate this equation on φ_1*:*

$$-l_1 \sin\varphi_1 - l_2 \sin\varphi_2 \frac{d\varphi_2}{d\varphi_1} + l_3 \sin\varphi_3 \frac{d\varphi_3}{d\varphi_1} = 0 . \qquad (4.12)$$

Here $\frac{d\varphi_2}{d\varphi_1} = u_{21}$*;* $\frac{d\varphi_3}{d\varphi_1} = u_{31}$ *are velocity ratios or angular velocity analogues of links 2 and 3 respectively.*

Let us copy equation (4.12) as:

$$l_1 \sin\varphi_1 + l_2 u_{21} \sin\varphi_2 = l_3 u_{31} \sin\varphi_3 . \qquad (4.13)$$

Now we turn axes x and y by the angle φ_2*. Then we subtract this angle in equation (4.13):*

$$l_1 \sin(\varphi_1 - \varphi_2) = l_3 u_{31} \sin(\varphi_3 - \varphi_2).$$

Hence, the velocity ratio between the first and third links is:

$$u_{31} = \frac{l_1 \sin(\varphi_1 - \varphi_2)}{l_3 \sin(\varphi_3 - \varphi_2)}.$$

Similarly

$$u_{21} = -\frac{l_1 \sin(\varphi_1 - \varphi_3)}{l_2 \sin(\varphi_2 - \varphi_3)}.$$

In order to define the angular acceleration analogues of links 2 and 3 we differentiate equations (4.13) by generalized coordinate φ_1*. We get the following equation:*

$$l_1 \cos\varphi_1 + l_2 u_{21}^2 \cos\varphi_2 + l_2 u_{21}' \sin\varphi_2 = l_3 u_{31}^2 \cos\varphi_3 + l_3 u_{31}' \sin\varphi_3.$$

Here $u_{21}' = \dfrac{d^2\varphi_2}{d\varphi_1^2}$*;* $u_{31}' = \dfrac{d^2\varphi_3}{d\varphi_1^2}$ *are angular acceleration analogues of links 2 and 3 respectively.*

Alternately, turning axes x and y by angles φ_2 *and* φ_3*, we get:*

$$u_{21}' = -\frac{l_1 \cos(\varphi_1 - \varphi_3) - l_3 u_{31}^2 + l_2 u_{21}^2 \cos(\varphi_2 - \varphi_3)}{l_2 \sin(\varphi_2 - \varphi_3)};$$

$$u_{31}' = \frac{l_1 \cos(\varphi_1 - \varphi_2) - l_2 u_{21}^2 + l_3 u_{31}^2 \cos(\varphi_3 - \varphi_2)}{l_3 \sin(\varphi_3 - \varphi_2)}.$$

If angular velocity ω_1 *and angular acceleration* ε_1 *of the input link are given, it is possible to determine angular velocities and accelerations of other links of the mechanism:*

$$\omega_2 = u_{21} \cdot \omega_1; \ \omega_3 = u_{31} \cdot \omega_1;$$

$$\varepsilon_2 = \varepsilon_1 \cdot u_{21} + \omega_1^2 \cdot u_{21}'; \ \varepsilon_3 = \varepsilon_1 \cdot u_{31} + \omega_1^2 \cdot u_{31}'.$$

4.1.4. Method of coordinate transformation

In case of investigation of planar mechanisms, this method, in comparison, for example, with a closed vector loops method, is more complex. Nevertheless, when investigating spatial mechanisms a method of coordinate transformation is very effective.

Using a matrix form of equations record, the method simplifies applying a computer.

The formulas of transforming of Cartesian coordinates x_j, y_j, z_j in axes x_i, y_i, z_i are:

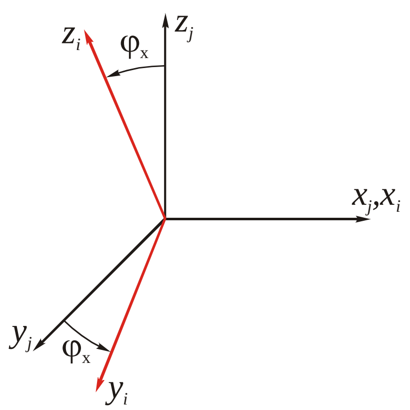
$$\begin{cases} x_i = a_{11}x_j + a_{12}y_j + a_{13}z_j; \\ y_i = a_{21}x_j + a_{22}y_j + a_{23}z_j; \\ z_i = a_{31}x_j + a_{32}y_j + a_{33}z_j. \end{cases} \tag{4.14}$$

Here $a_{11} = \cos(x_i x_j)$, $a_{12} = \cos(x_i y_j)$ etc.

The coefficients in an equation (4.14) can be given in matrix notation:

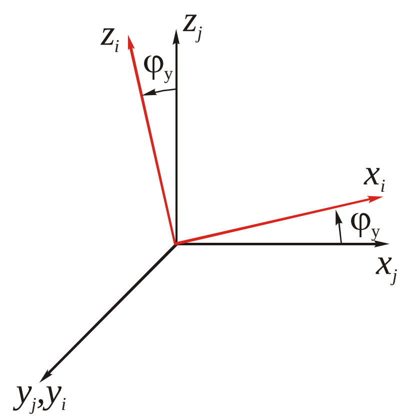
$$T_{ij} = \begin{Vmatrix} \cos(x_i x_j) & \cos(x_i y_j) & \cos(x_i y_j) \\ \cos(y_i x_j) & \cos(y_i y_j) & \cos(y_i z_j) \\ \cos(z_i x_j) & \cos(z_i y_j) & \cos(z_i z_j) \end{Vmatrix}. \tag{4.15}$$

For coordinate space matrices, which define rotation about one of the axis there are:



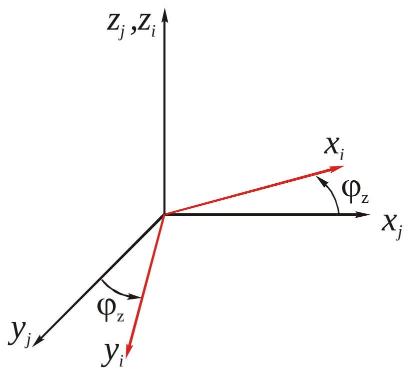
$$T_x = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_x & -\sin\varphi_x \\ 0 & \sin\varphi_x & \cos\varphi_x \end{Vmatrix}.$$

Fig. 4.5. Rotation about the *x*-axis



$$T_y = \begin{Vmatrix} \cos\varphi_y & 0 & \sin\varphi_y \\ 0 & 1 & 0 \\ -\sin\varphi_y & 0 & \cos\varphi_y \end{Vmatrix}.$$

Fig. 4.6. Rotation about the *y*-axis



$$T_z = \begin{Vmatrix} \cos\varphi_z & -\sin\varphi_z & 0 \\ \sin\varphi_z & \cos\varphi_z & 0 \\ 0 & 0 & 1 \end{Vmatrix}.$$

Fig. 4.7. Rotation about the *z*-axis

Example 3.3. To determine a position function of the point *D* of an unclosed planar kinematic chain of a manipulator arm (Fig. 4.8)

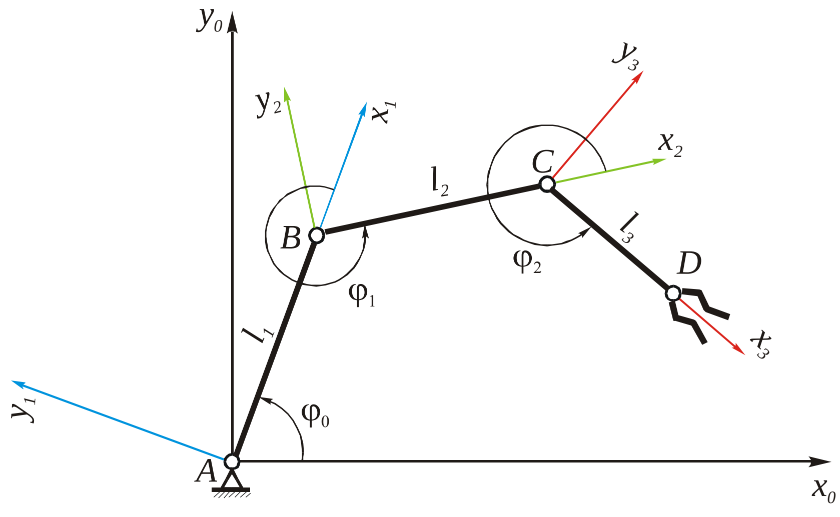


Fig. 4.8. Kinematic chain of a manipulator arm

Motion freedom of the chain is $w = 3n - 2p_5 = 3\times3 - 2\times3 = 3$. *In other words, we have three general coordinates* φ_0, φ_1', φ_2' *in the coordinate system* x_0-y_0.

First, let us connect the coordinate system to the link 3 (x_3-y_3*). In this coordinate system* $x_{D3} = l_3;\ y_{D3} = 0$.

Then we proceed to the coordinate system x_2-y_2*, which is connected to the link 2. In these axes, the coordinates of a point D are determined by expressions:*

$$\begin{cases} x_{D2} = l_2 + x_{D3}\cos\varphi_2 - y_{D3}\sin\varphi_2; \\ y_{D2} = x_{D3}\sin\varphi_2 + y_{D3}\cos\varphi_2. \end{cases} \tag{4.16}$$

Connecting the coordinate system to the link 1 (x_1-y_1*), we get:*

$$\begin{cases} x_{D1} = l_1 + x_{D2}\cos\varphi_1 - y_{D2}\sin\varphi_1; \\ y_{D1} = x_{D2}\sin\varphi_1 + y_{D2}\cos\varphi_1. \end{cases} \tag{4.17}$$

For the given coordinate system associated with the fixed link:

$$\begin{cases} x_{D0} = x_{D1}\cos\varphi_0 - y_{D1}\sin\varphi_0; \\ y_{D0} = x_{D1}\sin\varphi_0 + y_{D1}\cos\varphi_0. \end{cases} \tag{4.18}$$

Connecting the identity 1≡1 to the equation sets, we get square matrices as (4.15):

$$T_{32} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_2 & -\sin\varphi_2 \\ 0 & \sin\varphi_2 & \cos\varphi_2 \end{Vmatrix};\quad T_{21} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_1 & -\sin\varphi_1 \\ 0 & \sin\varphi_1 & \cos\varphi_1 \end{Vmatrix};\quad T_{10} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_0 & -\sin\varphi_0 \\ 0 & \sin\varphi_0 & \cos\varphi_0 \end{Vmatrix}.$$

The left parts of the equations (4.16)-(4.18), taking into the account the identity 1≡1, are:

$$r_{D3} = \begin{Vmatrix} 1 \\ x_{D3} \\ y_{D3} \end{Vmatrix};\quad r_{D2} = \begin{Vmatrix} 1 \\ x_{D2} \\ y_{D2} \end{Vmatrix};\quad r_{D1} = \begin{Vmatrix} 1 \\ x_{D1} \\ y_{D1} \end{Vmatrix};\quad r_{D0} = \begin{Vmatrix} 1 \\ x_{D0} \\ y_{D0} \end{Vmatrix}.$$

These equations can be shown as:

$$r_{D2} = T_{32} \cdot r_{D3}; \quad r_{D1} = T_{21} \cdot r_{D2}; \quad r_{D0} = T_{10} \cdot r_{D1}.$$

Finally, we get:

$$r_{D0} = T_{32} \cdot T_{21} \cdot T_{10} \cdot r_{D3}.$$

In a full size and after some transformations this equation can be shown as:

$$\begin{Vmatrix} 1 \\ x_{D0} \\ y_{D0} \end{Vmatrix} = \begin{Vmatrix} 1 \\ l_1 \cos\varphi_0 + l_2 \cos(\varphi_0 + \varphi_1) + l_3 \cos(\varphi_0 + \varphi_1 + \varphi_2) \\ l_1 \sin\varphi_0 + l_2 \sin(\varphi_0 + \varphi_1) + l_3 \sin(\varphi_0 + \varphi_1 + \varphi_2) \end{Vmatrix}.$$

4.2. KINEMATIC ANALYSIS OF MECHANISMS BY THE METHOD OF DIAGRAMS

4.2.1. Motion diagrams of points and links of mechanisms

***A kinematic diagram** is a graphic image of the law of variation for one of the kinematic parameters of a mechanism in the function of time or generalize coordinate.*

Let us construct the kinematic diagram for the position function of the crank-slider linkage $S_B = S_B(\varphi_1)$ (Fig. 4.9). We assume $\omega_1 = Const$.

It is more convenient to start the construction from the leftmost or rightmost positions of the slider, which are *the extreme positions of the mechanism*. The path of the point A is divided into n equal parts (8-12). For example, they define the displacement S_B in each 30º of the angle of the crankshaft turning.

The diagram is constructed on a scale on axes S_B and φ_1

$$\mu_{S_B} = \frac{S_{B_i} \cdot \mu_l}{0S_i}, \ \frac{\text{mm}}{\text{mm}}; \qquad \mu_{\varphi_1} = \frac{\varphi_1}{0i}, \ \frac{\text{radian}}{\text{mm}}.$$

In order to get the diagram of the velocity analogues for the point *B*, we must differentiate the obtained dependence. In practice, we use the method of graphical differentiation and integration.

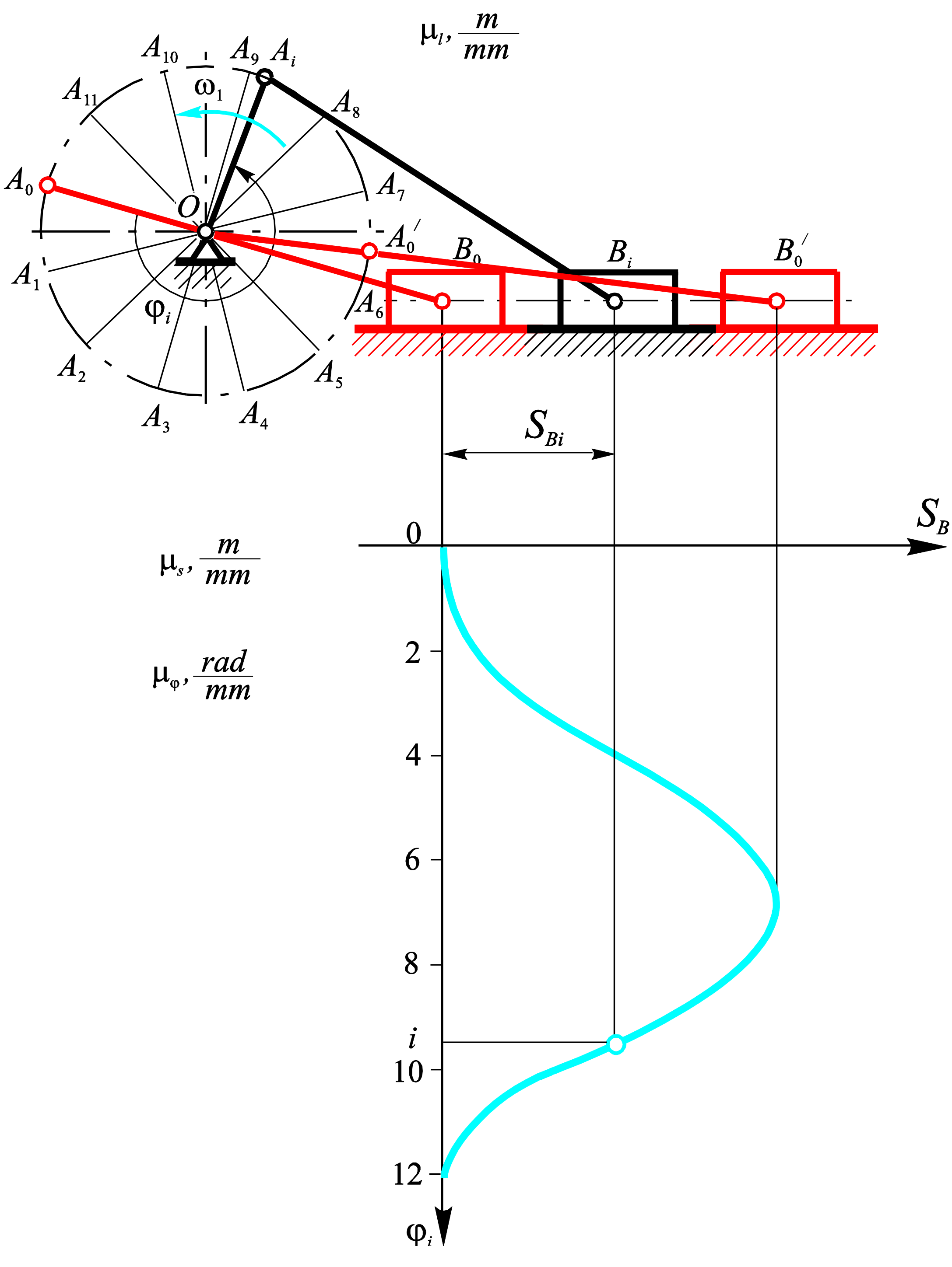


Fig. 4.9. Displacement diagram of point *B* of an offset crank-and-slider mechanism (graph of position function)

4.2.2. Graphical differentiation

In Fig. 4.10 the graph of the function $S = S(\varphi_1)$ is shown

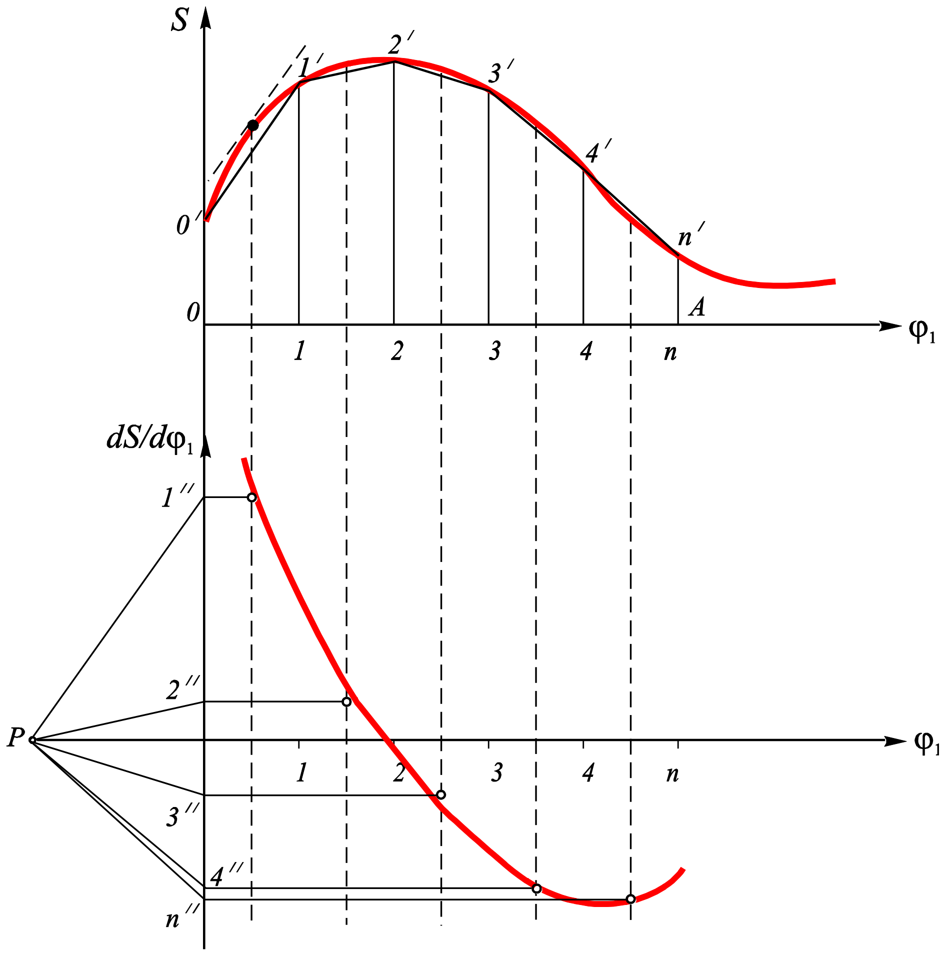


Fig. 4.10. Graphical differentiation (chord method)

Let's construct the graph of the derivative $\dfrac{dS}{d\varphi_1} = \dfrac{dS}{d\varphi_1}(\varphi_1)$ in the range *O-A*.

For this purpose:

1. We divide the segment *OA* into *n* equal parts.

2. In each range 0-1, 1-2, ... 4-*n* we define the value of the derivative of function $S = S(\varphi_1)$, which is approximate to the slope ratio of the tangent to the curve in the middle of the range. The less the range we have, the more accurate the value of the derivative is found. In practice, the *chord method* is used. For this purpose we join the points $0'$ and $1'$; $1'$ and $2'$ etc. by the chord segment. In the first approximation, they can be considered as parallel to the tangent in the middle of range.

3. From the arbitrarily chosen pole *P* (Fig. 4.10) straights parallel to the chords (these are the beams $P1''; P2''$ etc.) are drawn to the crossing with the vertical axis.

4. From points 1″; 2″ etc. we draw straight lines, parallel to the horizontal axis till their crossing with vertical lines, drawn through the middle of the intervals of the partition.

5. Through the obtained crossing points we draw the curve which is the graph of the derivative of a function $S = S(\varphi_1)$, plotted in the scale $\mu_{\frac{dS}{d\varphi_1}}$.

Now we find the scale of the graph of the derivative. For this purpose, we study the fragment *AB* of the graph of the function $S = S(\varphi_1)$ (Fig. 4.11).

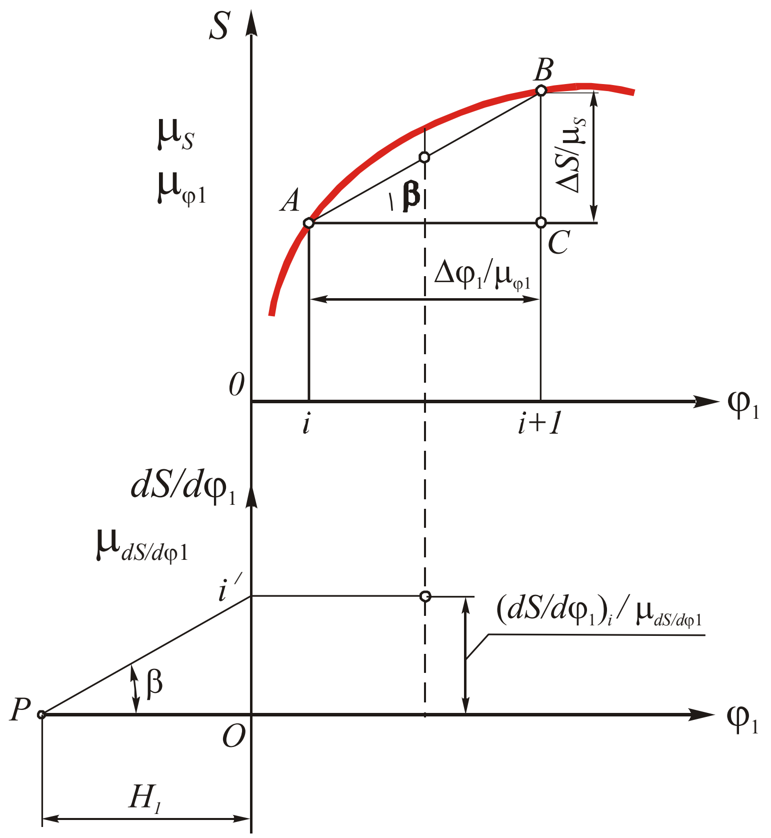


Fig. 4.11. Determination of a scale

In the triangle *ABC*, the hypotenuse of which is the chord *AB*, the legs are found as

$$BC = \frac{\Delta S}{\mu_S}; \quad AC = \frac{\Delta\varphi_1}{\mu_{\varphi_1}}.$$

Derivative function in the middle of the interval *i–i+1*:

$$\left(\frac{dS}{d\varphi_1}\right)_i = \frac{\Delta S}{\Delta\varphi_1} = \frac{BC}{AC}\frac{\mu_S}{\mu_{\varphi_1}} = \frac{\mu_S}{\mu_{\varphi_1}} tg\beta .$$

On the other side:

$$\left(\frac{dS}{d\varphi_1}\right)_i = Oi' \cdot \mu_{\frac{dS}{d\varphi_1}} = \mu_{\frac{dS}{d\varphi_1}} \cdot H_1 \cdot tg\beta .$$

Composing these two equations, we have:

$$\mu_{\frac{dS}{d\varphi_1}} = \frac{\mu_S}{\mu_{\varphi_1} \cdot H_1} . \tag{4.19}$$

4.2.3. Graphical integration

In practice, we often must define an integral of a function, which is given graphically. For example:

$$\varphi(t) = \int_{t_0}^{t_i} \omega dt .$$

In Fig. 4.12 there is a graph of the angular velocity $\omega(t)$ considering values of the scales μ_ω and μ_t .

Integration is conducted in the way inversed to the graphical differentiation:

1. We divide the interval *OA* into *n* equal parts in such a way that the motion in the ambit of the interval is considered as steady.

2. We replace curvilinear trapezoids $00'1'1$, $11''2'2$ etc. by squarely equal rectangles with sides O-1 and y_1; 1-2 and y_2 etc.

3. The ends of mean ordinates y_1, y_2 etc. are projected on the axis ω, and the points y_1', y_2' etc. are found.

4. We connect them with the arbitrarily chosen pole *P* by rays Py_1', Py_2' etc.

5. On the sought graph $\varphi(t)$ we draw lines $01'';1''2''$ etc., parallel to the rays Py'_1, Py'_2 etc. We draw the first segment from the coordinate origin to crossing with a vertical straight, which limits the interval 0-1 on the right. The second segment is drawn from the obtained point of crossing with the limit of the first interval to the crossing with the limit of the second interval 1-2 etc.

6. Replace the obtained piecewise continuous curve by a smooth curve, which is the sought graph of an integral function $\varphi(t)$. Now it is easy to show that a scale on the axis φ, according to formula (4.19), is calculated by the formula

$$\mu_\varphi = \mu_\omega \cdot \mu_t \cdot H_1.$$

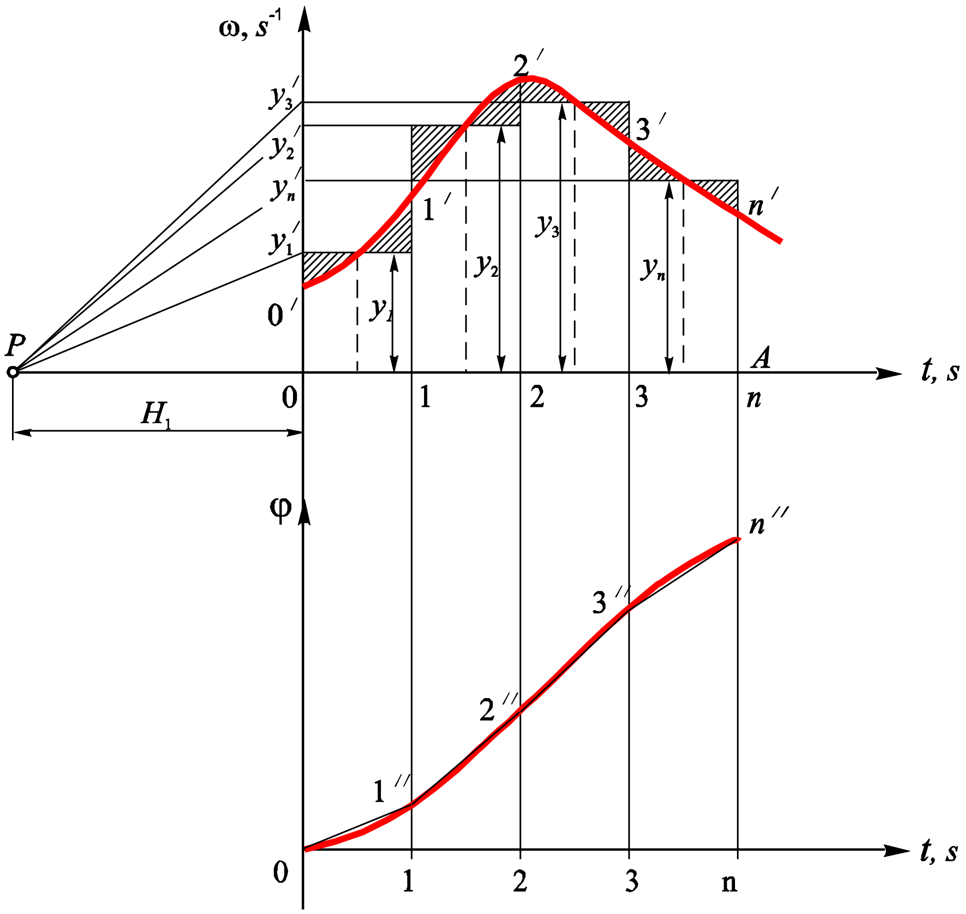


Fig. 4.12. Graphical integration

The method of graphical differentiation and integration is not exact enough. Therefore, it is used for the approximate determination of the kinematic parameters of the mechanism.

4.2.4. Validation of the executed constructions

In Fig. 4.13 there are diagrams of dimensions, velocity and acceleration analogues of the point *B* of a crank-slider mechanism (Fig. 4.13), found by the method of graphical differentiation of the curve $S_B = S_B(\varphi_1)$.

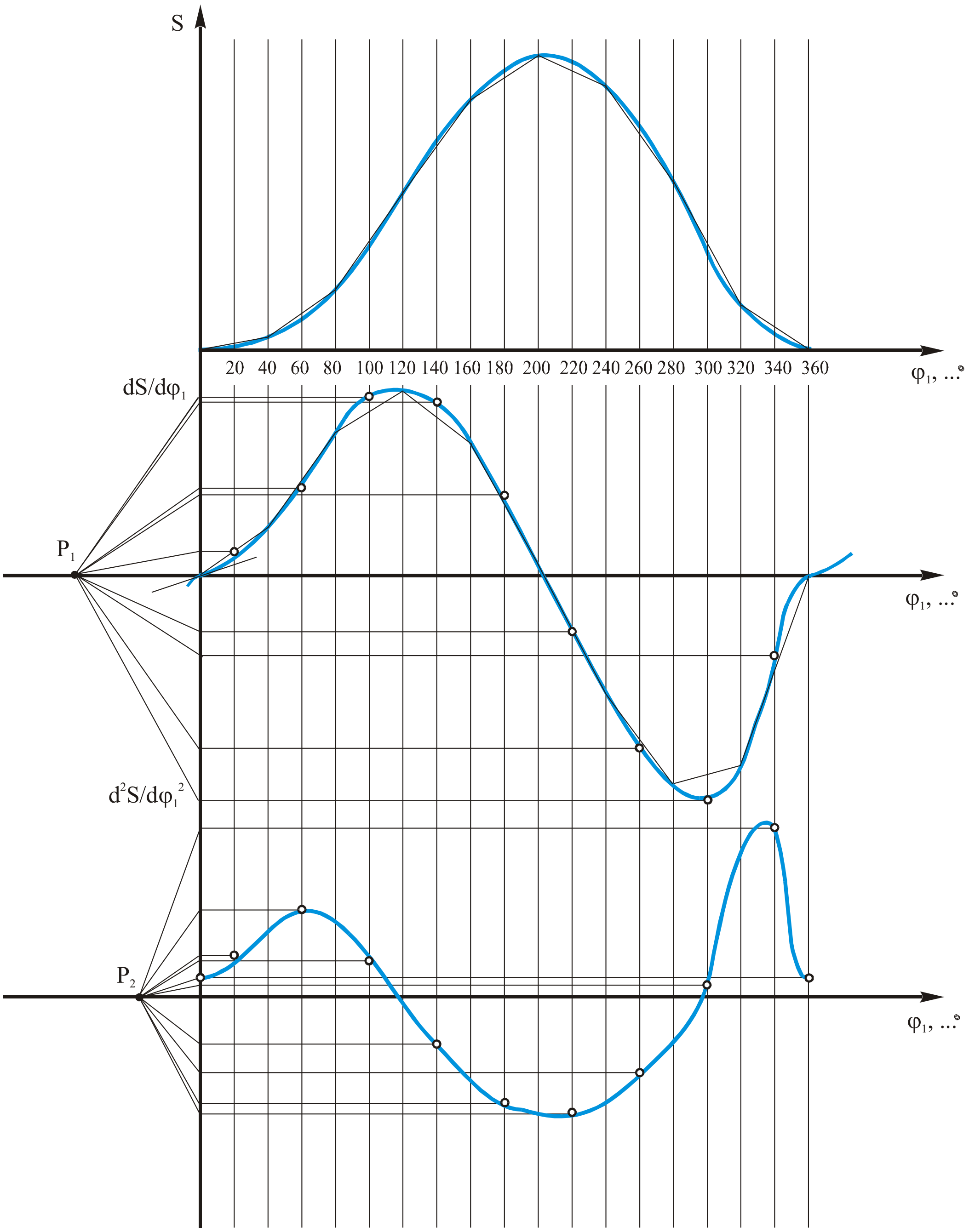


Fig. 4.13. Validation of the executed constructions

Validation of the executed constructions is conducted by the following features:

1. The extreme values of the integral curve correspond to the zero values of the differential curve.

2. Points of inflection of the integral curve correspond to the extreme values of the differential curve.

3. Segments where the integral curve increases correspond to the positive segments of the differential curve.

4. Declining segments of the integral curve correspond to the negative segments of the differential curve.

5. Tangents constructed to the curve both in the initial and end points (beginning and end of the mechanism motion cycle) must be identically directed.

6. If it is necessary to specify the graphs of curves in some interval, we make an additional division of the interval.

4.3. METHOD OF VECTOR DIAGRAMS IN KINEMATIC ANALYSIS OF MECHANISMS

4.3.1. Position diagrams of a mechanism

The image of a kinematic scheme of a mechanism in the chosen scale, which corresponds to the set position of an initial link, is called <u>the position diagram of a mechanism</u>.

In Fig. 4.9 position diagrams of a crank- slider linkage are shown in a scale μ_l .

Thus, the position diagrams of a mechanism are built by conducting the kinematic analysis, for example by the method of kinematic diagrams.

4.3.2. Velocity and acceleration diagrams

The mechanism velocity diagram is a diagram, which in the form of segments shows vectors identical in the magnitude and direction with velocities of different points of mechanism links at the present instant of time.

The velocity diagram for a mechanism is a complex of several velocity diagrams for its separate links, in which the poles of diagrams *p* are a general point – *a pole of the velocity diagram of the mechanism.*

A diagram, which in the form of segments shows vectors identical in the magnitude and direction with accelerations of different points of mechanism links at the present instant of time, is called <u>the mechanism acceleration diagram</u>.

In the method of the velocity and acceleration diagrams, we use the planar motion theorem of solid (links).

Planar (two-dimensional) motions can be considered to be combinations of two simple displacement types: translation and rotation.

Let the body shown in Fig. 4.14, *a* executes the planar motion. As it is known, its motion can be assigned by setting the laws of motions of its two points, for example, *A* and *B*.

In any moment of time, it is possible to find the point rigidly bound with a body, the velocity of which equals to zero. This is an instantaneous centre of rotation (ICR) of a body *P*. If we have a slide body, ICR lies at infinity. Relatively to this point, a body does turning motion with the angular velocity ω at the present instant of time. In other instant of time, this velocity can be other.

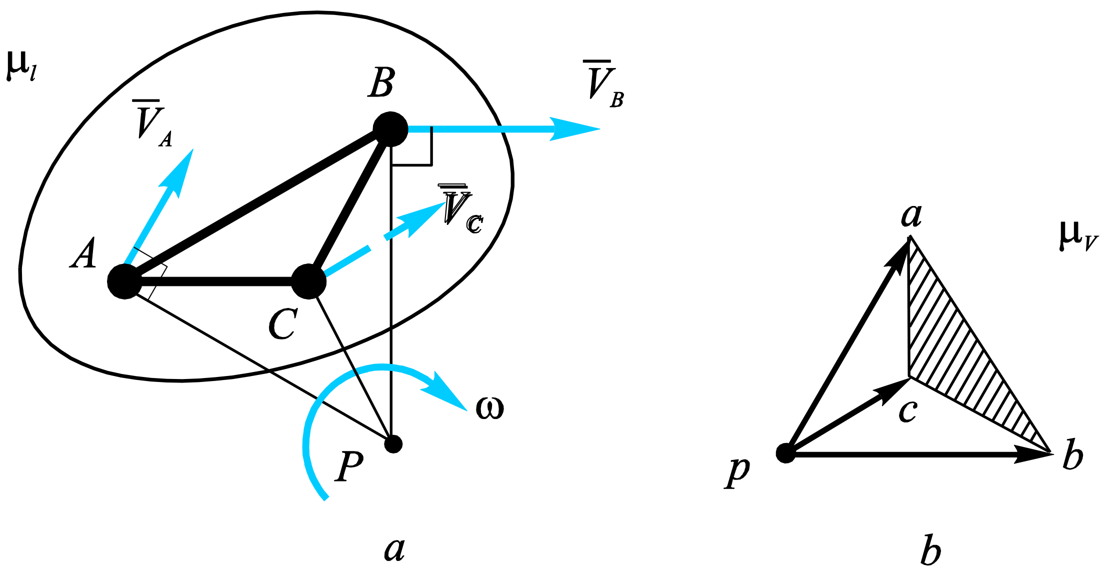


Fig. 4.14. Velocity diagram for the rigid body: *a* – the rigid body; *b* – the velocity diagram

Let us transfer the vectors of absolute velocities $\bar{V}_A$ and $\bar{V}_B$ in the point *p* (pole of the velocity diagram). We will get the velocity diagram of the segment AB (Fig. 4.14, *b*) in a scale

$$\mu_V = \frac{V_A}{pa} = \frac{V_B}{pb}, \quad \frac{m \cdot s^{-1}}{mm}$$

The velocity of the point *C*, according to the planar motion theorem

$$\bar{V}_C = \bar{V}_B + \bar{V}_{CB}$$

or

$$\bar{V}_C = \bar{V}_A + \bar{V}_{CA}.$$

Here the vectors V_{CB} and V_{CA} represent the amounts by which the velocity of point *C* differs from those of points *B* and *A*.

Let us assume that we know the velocity V_C. Then a segment on a diagram

$$p_C = \frac{V_C}{\mu_C}.$$

On the other hand

$$V_A = \omega \cdot PA; \qquad V_B = \omega \cdot PB; \quad V_C = \omega \cdot PC.$$

Then

$$\mu_V = \frac{\omega \cdot PA}{pa} = \frac{\omega \cdot PB}{pb} = \frac{\omega \cdot PC}{pc}$$

or

$$\frac{PA}{pa} = \frac{PB}{pb} = \frac{PC}{pc}.$$

This is the writing of *the similarity theorem for velocity diagrams*:

The ends of vectors of absolute velocities, are co-connected, and form a figure, similar to the link, which is situated similarly to it and turned on 90°.

In the similarly situated figures, traversal directions coincide in indexes.

Properties of velocity diagrams:

1. Vectors which go out from a pole are absolute velocities.
2. The direction of vector is always from a pole.
3. There is always a point at the end of a vector, which coincides with the point of the link or kinematic pair.
4. Vectors on the velocity diagrams, which do not pass through a pole, are relative velocities.
5. The direction of relative velocities on a diagram is always from the second index to the first one.

Let us study the body (Fig. 4.15), the motion of which is assigned by the motion of points *A* and *B*. *П* is an instantaneous centre of accelerations (ICA).

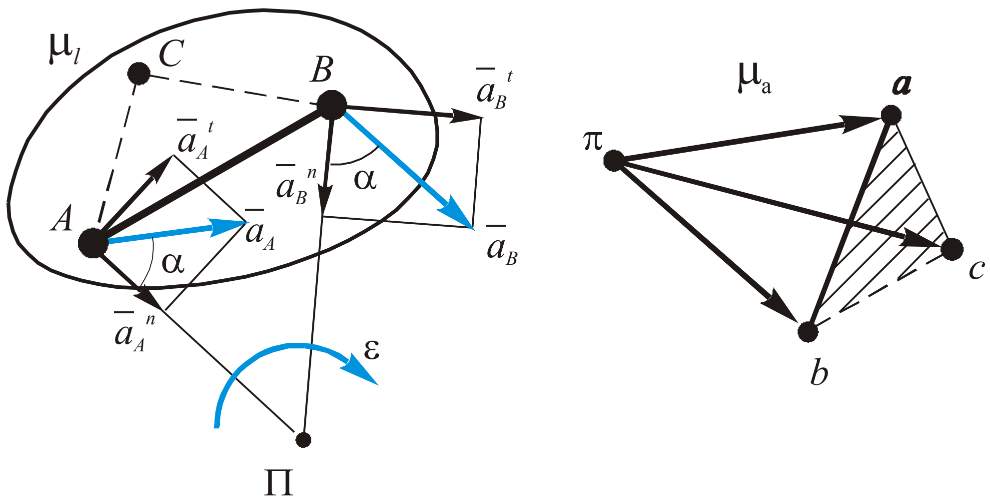


Fig. 4.15. Acceleration diagram for rigid body

Acceleration of the point *A*

$$a_A = \sqrt{\left(a_A^n\right)^2 + \left(a_A^\tau\right)^2}$$

Here $a_A^n = \omega^2 \cdot ПA$ is normal acceleration; $a_A^\tau = \varepsilon \cdot ПA$ is tangential acceleration. Then

$$a_A = ПA\sqrt{\omega^4 + \varepsilon^2}\ .$$

Analogously, for the point *B*:

$$a_B = ПB\sqrt{\omega^4 + \varepsilon^2}\ .$$

According to Fig. 4.15

$$\mathrm{tg}\alpha = \frac{a_A^\tau}{a_A^n} = \frac{a_B^\tau}{a_B^n} = \frac{\varepsilon \cdot ПA}{\omega^2 \cdot ПA} = \frac{\varepsilon \cdot ПB}{\omega^2 \cdot ПB}\ .$$

Finally, we get

$$\mathrm{tg}\alpha = \frac{\varepsilon}{\omega^2}\ .$$

We construct the acceleration diagram, replacing beginnings of total acceleration vectors into the general pole of diagram π (Fig. 4.15). A scale factor of the acceleration diagram:

$$\mu_a = \frac{a_A}{\pi a} = \frac{a_B}{\pi b},\ \frac{m \cdot s^{-2}}{mm}\ .$$

Let us formulate a *similarity theorem for acceleration diagrams*:

The ends of vectors of total accelerations, are co-connected, and form a figure similar to the link, which is situated similarly to it and turned on $\angle\left(180^\circ - \alpha\right)$**,** ***where*** $\alpha = arctg\ \varepsilon/\omega^2$ **.**

The acceleration of the point C can be found by the similarity theorem, according to which ΔABC is similar to Δabc (Fig. 4.15).

Properties of the acceleration diagrams:

Properties of the acceleration diagrams are similar to the properties of the velocity diagrams.

Let us have a closer look at the methods for construction of velocity and acceleration diagrams for different structural groups.

4.3.3. Velocity and acceleration diagrams construction for double arm groups (dyad) with three turning pairs

The diagrams construction procedure for such groups lies in the compilation of vector equations of velocities and accelerations for each of the link and their joint solution.

Let us consider a structure group consisting of links 1 and 2 (Fig. 4.16, *a*).

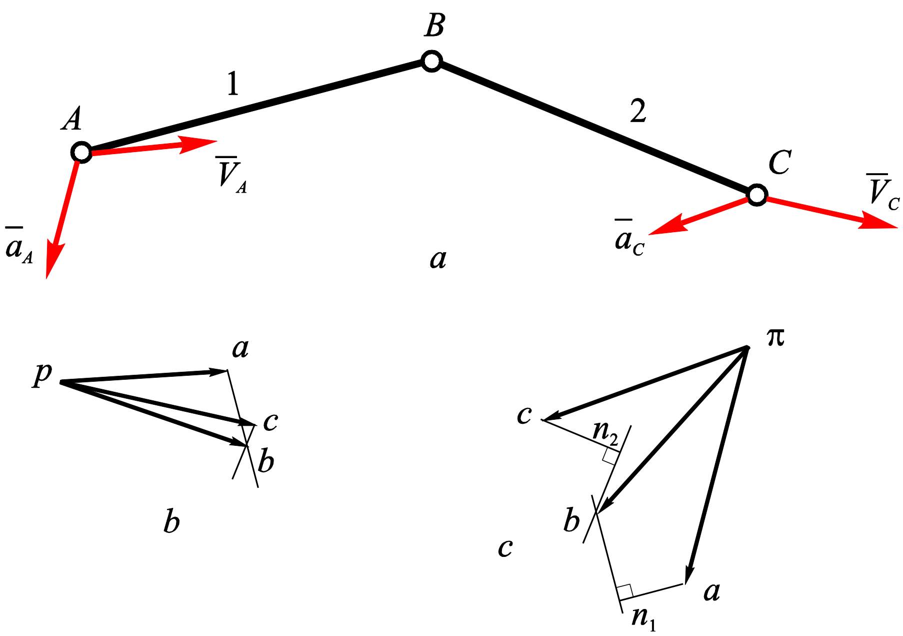


Fig. 4.16. Double arm group (dyad) with three hinges: *a* – dyad; *b* – the velocity diagram; *c* – the acceleration diagram

As there is no relative translational motion of links in the pin joint B, the condition $\bar{V}_{B1} = \bar{V}_{B2}$ is valid. Links are producing planar motions. Hence we can write down such equations for each link:

$$\begin{cases} \bar{V}_B = \bar{V}_A + \bar{V}_{BA}; \\ \bar{V}_B = \bar{V}_C + \bar{V}_{BC}. \end{cases} \tag{4.20}$$

Since the relative motions of links are turning, relative velocity directions are $\bar{V}_{BA} \perp BA$ and $\bar{V}_{BC} \perp BC$.

Here vectors $\bar{V}_A$ and $\bar{V}_C$ are known. We need to find vectors $\bar{V}_B$, $\bar{V}_{BA}$ and $\bar{V}_{BC}$.

Let us construct the velocity diagram. We choose the pole p and plot off vectors $\overline{pa}$ and $\overline{pc}$ the scale of which corresponds to the specified velocity vectors $\bar{V}_A$ and $\bar{V}_C$. The scale factor of the velocity diagram is calculated from the ratio:

$$\mu_V = \frac{V_A}{pa} = \frac{V_C}{pc}, \qquad \frac{m \cdot s^{-1}}{mm}.$$

The set of equations (4.20) is jointly solved in the vector form. This is done by drawing the $\overline{ba} \perp AB$ vector direction from the end of vector $\overline{pa}$, and $\overline{bc} \perp BC$ vector direction from the end of the $\overline{pc}$ vector. The intersection point is the solution to the set (4.20). Point b is the end point of the $\overline{pb}$ vector. Therefore, having calculated the scale factor μ_V makes it possible to find all unknown vectors (Fig. 4.16, *b*).

Now we have to calculate accelerations. According to the compound motion theorem we can write down equations for links:

$$\begin{cases} \bar{a}_B = \bar{a}_A + \bar{a}_{BA}^n + \bar{a}_{BA}^\tau; \\ \bar{a}_B = \bar{a}_C + \bar{a}_{BC}^n + \bar{a}_{BC}^\tau. \end{cases} \tag{4.21}$$

Here $\bar{a}_A$ and $\bar{a}_C$ are given accelerations of points A and C of the structure group. Normal accelerations are calculated by these formulas:

$$a_{BA}^n = \frac{V_{BA}^2}{l_{AB}} = \frac{(ba \cdot \mu_V)^2}{l_{AB}}; \qquad a_{BC}^n = \frac{V_{BC}^2}{l_{BC}} = \frac{(bc \cdot \mu_V)^2}{l_{BC}},$$

where *ba* and *bc* are segments of the velocity diagram (Fig. 4.16, *b*). The directions for these accelerations are also known: $\bar{a}_{BA}^n \parallel BA$ and directed to the point A (the centre of rotation at the relative motion of link *1*); $\bar{a}_{BC}^n \parallel BC$ and directed to the point C (the centre of rotation at the relative motion of link *2*).

We cannot calculate the value of tangential accelerations $\bar{a}_{BA}^\tau$ and $\bar{a}_{BC}^\tau$ as angular accelerations of links in the turning motion are unknown. Only their directions $\bar{a}_{BA}^\tau \perp BA$ and $\bar{a}_{BC}^\tau \perp BC$ are known.

The set (4.21) is solved for the unknown vectors $\bar{a}_B$, $\bar{a}_{BA}^\tau$, $\bar{a}_{BC}^\tau$ through the acceleration diagram. Its scale factor

$$\mu_a = \frac{a_A}{\pi a} = \frac{a_C}{\pi c}, \quad \frac{m \cdot s^{-2}}{mm}.$$

From the chosen pole of acceleration diagram π we plot off vectors $\overline{\pi a}$ and $\overline{\pi c}$ (Fig. 4.16, *c*), which in direction coincide with the given acceleration vectors of points *A* and *C*. From the ends of these vectors we draw vector $\overline{an_1}$ from the point *B* to the point *A* (Fig. 4.16, *a*), and vector $\overline{cn_2}$ from *B* to *C*. Here

$$an_1 = \frac{a_{BA}^n}{\mu_a}; \quad cn_2 = \frac{a_{BC}^n}{\mu_a}.$$

The point b is an intersection point of the directions of tangential accelerations and the end of the unknown vector $\overline{\pi b}$. The total acceleration of point B is calculated as follows

$$\overline{a}_B = \pi b \cdot \mu_a .$$

Therefore, the velocity and acceleration of the point B of a double arm structure group with three turning pairs can be found if the values and directions of the end hinges of the links that form it (points A and B) are known.

4.3.4. Velocity and acceleration diagrams construction for double arm groups (dyad) with the end sliding and turning pairs

Fig. 4.17 shows such a group. It forms a sliding pair with link 4, while the transportation motion of this link is rotational.

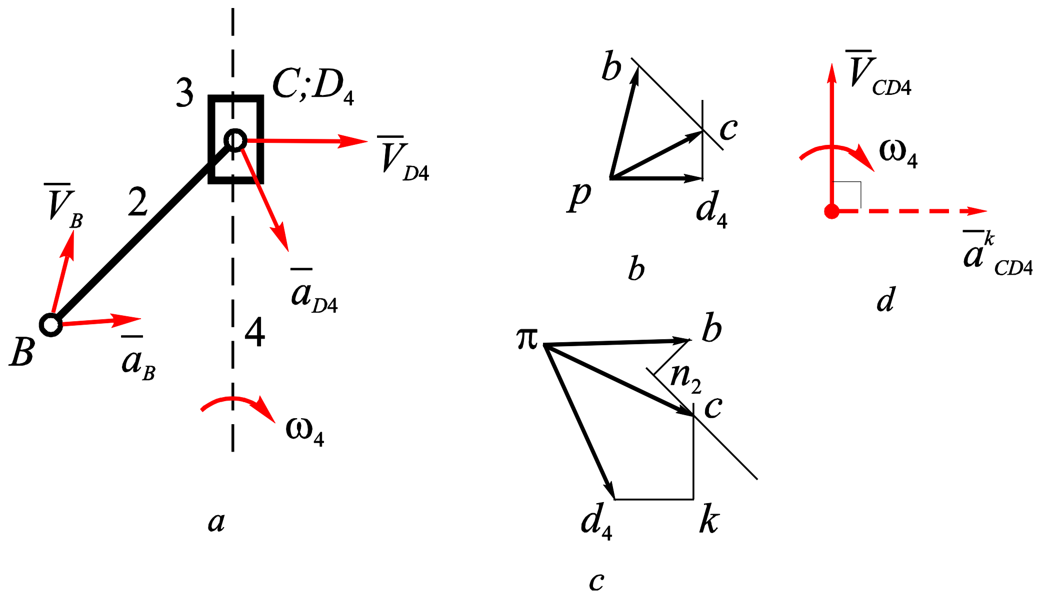


Fig. 4.17. Double arm group (dyad) with the end sliding pair:
a – dyad; *b* – the velocity diagram; *c* – the acceleration diagram;
d – Zhukovski method of Coriolis acceleration
direction determination

Let us assume that velocities and accelerations of the points B of link 2 and D_4 of link 4 are given. Angular velocity of link 4 is also given.

Let us calculate the velocity of the point *C*. As in the previous example, there is no relative sliding motion of link 2 and 3 the condition $\overline{V}_{C2} = \overline{V}_{C3} = \overline{V}_{C}$ is valid. Thus, by the planar motion theorem, the set equations for these links are as follows

$$\begin{cases} \overline{V}_{C_2} = \overline{V}_B + \overline{V}_{CB}; \\ \overline{V}_{C_3} = \overline{V}_{D_4} + \overline{V}_{CD_4}. \end{cases}$$

Here $\overline{V}_{CB} \perp CB$ as the relative motion of link 2 is rotational. The direction of the relative velocity $\overline{V}_{CD_4}$ is parallel to link 4, since the motion of link 3 relative to link 4 is translational.

Solving this set of equations jointly with the help of the velocity diagram (Fig. 4.17, *b*), enables us to find the unknown vectors $\overline{V}_C$, $\overline{V}_{CD_4}$ and $\overline{V}_{CB}$

$$V_C = pc \cdot \mu_v ;\ V_{CD_4} = cd_4 \cdot \mu_V ;\ V_{CB} = cb \cdot \mu_V .$$

The procedure of velocity diagram construction is the same as in the previous example.

Let us find the acceleration of the point *C*:

$$\begin{cases} \overline{a}_{C_2} = \overline{a}_B + \overline{a}^n_{CB} + \overline{a}^\tau_{CB}; \\ \overline{a}_{C_3} = \overline{a}_{D_4} + \overline{a}^r_{CD_4} + \overline{a}^k_{CD_4}. \end{cases}$$

Both for velocities and accelerations condition $\overline{a}_{C_2} = \overline{a}_{C_3} = \overline{a}_C$ is valid.

The second link produces the planar motion. Normal acceleration in the turning motion of this link with regards to the point *B* is as follows

$$a^n_{CB} = \frac{V^2_{CB}}{l_{CB}} = \frac{(\mu_V \cdot cb)^2}{l_{CB}} .$$

We plot off vector $\overline{bn_2}$ in the velocity diagram oriented from the point C to point B (Fig. 4.17, *a*). Then the scalar of this vector is

$$bn_2 = \frac{a^n_{CB}}{\mu_a}.$$

The direction of the tangential acceleration $\bar{a}^{\tau}_{CB} \perp CB$ is plotted off from the point n_2 in the acceleration diagram.

The relative motion of the link 3 by the link 4 is translational, characterised, from one point, by relative acceleration $\bar{a}^{r}_{CD_4}$. At the same time the transportation motion of link 4 is turning, which results in Coriolis acceleration:

$$\bar{a}^{k}_{CD_4} = 2\bar{\omega}_4 \times \bar{V}_{CD_4}.$$

Its magnitude

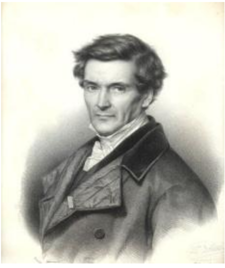
$$a^{k}_{CD_4} = 2\omega_4 \cdot V_{CD_4}.$$

The direction of the Coriolis acceleration is found by Zhukovski method (Fig. 4.17, *d*). In order to find the direction of the acceleration $\bar{a}^{k}_{CD_4}$, we need to turn the relative velocity vector of sliding motion $\bar{V}_{CD_4}$ (that is found from the velocity diagram: its direction is from the second index d_4 to the first one c) 90º towards the direction of the angular velocity of transportation motion ω_4. Thus, we draw vector $\overline{d_4 k}$ oriented towards the Coriolis acceleration, from the point d_4 in the acceleration diagram. Its magnitude

$$d_4 k = \frac{a^{k}_{CD_4}}{\mu_a}.$$

From the point k we need to draw the direction of the relative acceleration parallel to link 4. Its intersection with the direction of the tangential acceleration of link 2 the point C is found. Then by connecting it to the pole of acceleration diagram the accelerations $\bar{a}_C$, $\bar{a}^r_{CD_4}$, $\bar{a}^\tau_{CB}$ are calculated:

$$a_C = \pi c \cdot \mu_a; \quad a^r_{CD_4} = kc \cdot \mu_a; \qquad a^\tau_{CB} = n_2 c \cdot \mu_a.$$



Gustave-Gaspard Coriolis (1792–1843)

French engineer, mathematician and mechanic, professor at the Polytechnic School in Paris, member of the Paris Academy of Sciences. His most famous work was devoted to the study of "new type" inertia forces - the Coriolis effect.
He is also known for his acceleration theorem in absolute and relative motions, called the Coriolis theorem.

N. E. Zhukovski (1847–1921)

Russian mechanic, founder of hydraulic- and aeromechanics, Honorary Member of Moscow University (1916), Honored Professor of the Moscow Technical School (since 1918 – Moscow Higher Technical School) Corresponding Member of the Imperial Academy of Sciences for the category of Mathematical Sciences (1894).

Consider the examples of velocity and acceleration diagrams construction for mechanisms with structural groups of the II class and the 2-nd order of various types. It should be noted that in the next examples the specific dimensions of the links of the mechanisms and the velocities of their input links will not be specified, but only the principles of velocity and acceleration diagrams construction will be considered.

Example 4.4. Construct velocity and acceleration diagrams for the crank-and-slider mechanism (Fig. 4.18), assuming that $\omega_1 = \text{const}$.

This mechanism contains one structural group of the class II and the 2-nd order, formed by links 2 and 3, with an external translational pair (of the second type behind Artobolevsky) between the slider 3 and the ground.

1. Construction of the mechanism velocity diagram.

The analysis starts with the primary mechanism. Determine the linear velocity of point A of the crank as $V_A = \omega_1 l_{OA}$. *We draw the segment pa from the selected pole p of the velocity diagram (Fig .4.18, b), at that the scale factor of the diagram* $\mu_V = \frac{V_A}{pa}, \ \frac{m \cdot s^{-1}}{mm}$.

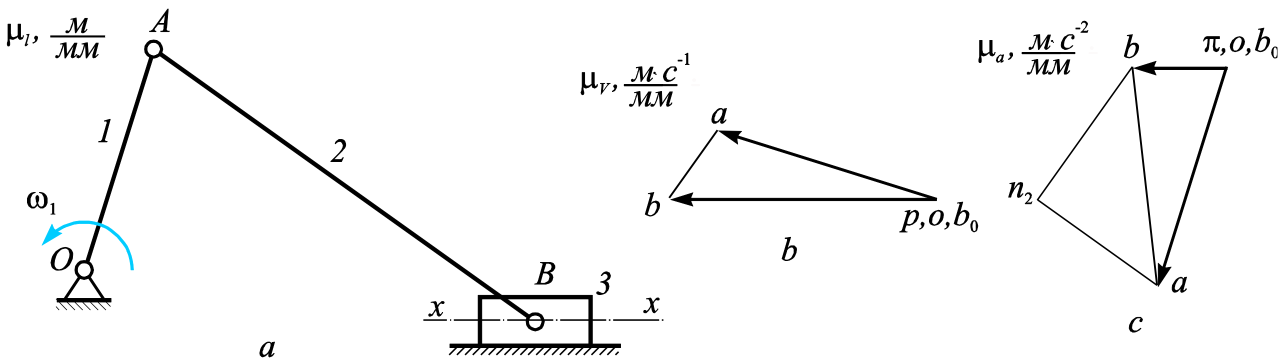


Fig. 4.18. To example 4.4: *a* - kinematic scheme of the mechanism**;**
b – the velocity diagram; *c* – the acceleration diagram

Next, we consider the structural group. The velocity of point B (hinge centre) is expressed by equations:

$$\begin{cases} \overline{V}_{B_2} = \overline{V}_A + \overline{V}_{B_2A}; \\ \overline{V}_{B_3} = \overline{V}_{B_0} + \overline{V}_{B_3B_0}. \end{cases} \tag{4.22}$$

Here, $\overline{V}_{B_2} = \overline{V}_{B_3}$, $\overline{V}_{B_0} = 0$ *(this point on the velocity diagram is at the pole).*

Further, in accordance with the written equations, we draw the directions of relative velocities: from the point a of the velocity diagram we draw a straight line perpendicular to BA ($\overline{V}_{B_2A} \perp BA$), *and from the pole we draw a straight line parallel to the direction of motion of the slider* ($\overline{V}_{B_3B_0} \parallel$ *x-x). The point b of the intersection of these lines is a solution of the set of equations. The velocity of point B is found as* $V_B = \mu_V \cdot pb$.

2. Construction of the mechanism acceleration diagram.

Since by condition $\omega_1 = \text{const}$, *only normal acceleration takes place at point A of the crank* $a_A = \omega_1^2 l_{OA}$. *From the selected pole* π *of the acceleration diagram, we draw the segment* πa *in the direction from point A to point O – the centre of rotation of the crank (Fig. 4.18, c). The scale factor of the acceleration diagram is determined from the ratio* $\mu_a = \frac{a_A}{\pi a}, \ \frac{m \cdot s^{-2}}{mm}$.

Acceleration of point B is found as a solution of the set of vector equations, given that $a_{B_2} = a_{B_3} = a_B$ *:*

$$\begin{cases} \overline{a}_B = \overline{a}_A + \overline{a}_{BA}^n + \overline{a}_{BA}^\tau; \\ \overline{a}_B = \overline{a}_{B_0} + \overline{a}_{BB_0}. \end{cases}$$

Here the acceleration $a_{BA}^n = \dfrac{V_{BA}^2}{l_{AB}} = \dfrac{(ba \cdot \mu_V)^2}{l_{AB}}$ *is parallel to AB and directed from point B to point A in the diagram (Fig. 4.18, a);* $\overline{a}_{BA}^\tau \perp BA$ *;* $a_{B_0} = 0$ *;* $\overline{a}_{BB_0} \parallel$ *x-x.*

The normal acceleration of point B relative to point A in the turning motion of link 2 is set from point a on the acceleration diagram as a segment $an_2 = \dfrac{a_{BA}^n}{\mu_a}$*, parallel to segment AB in the scheme (Fig. 4.18, a) in the direction from point B to point A (See Fig. 4.18, c).*

Next, from point n_2 *we draw the direction of tangential acceleration* $\overline{a}_{BA}^\tau \perp BA$*. From the pole* π *we draw a straight line, parallel x-x, – the direction of translation motion of the point* B_3 *of the slider relative to the housing, and hence its acceleration. The point of intersection of these two directions corresponds to point B on the acceleration diagram. From here we find the magnitude and direction of acceleration of the point B:* $a_B = \pi b \cdot \mu_a$.

Example 4.5. Construct velocity and acceleration diagrams for the mechanism (Fig. 4.19), assuming that $\omega_1 = \text{const}$.

This mechanism contains one structural group of the class II and the 2-nd order with one intermediate translational pair (of the third Artobolevsky type) between the connecting rod 2 and the sliding block 3.

1. Construction of the mechanism velocity diagram.

The velocity of point A of the crank $V_A = \omega_1 l_{OA}$. $V_A = \omega_1 l_{OA}$. *We draw the segment pa from the selected pole p of the velocity diagram (Fig. 4.19, b), at that the scale factor of the diagram* $\mu_V = \dfrac{V_A}{pa}, \ \dfrac{m \cdot s^{-1}}{mm}$.

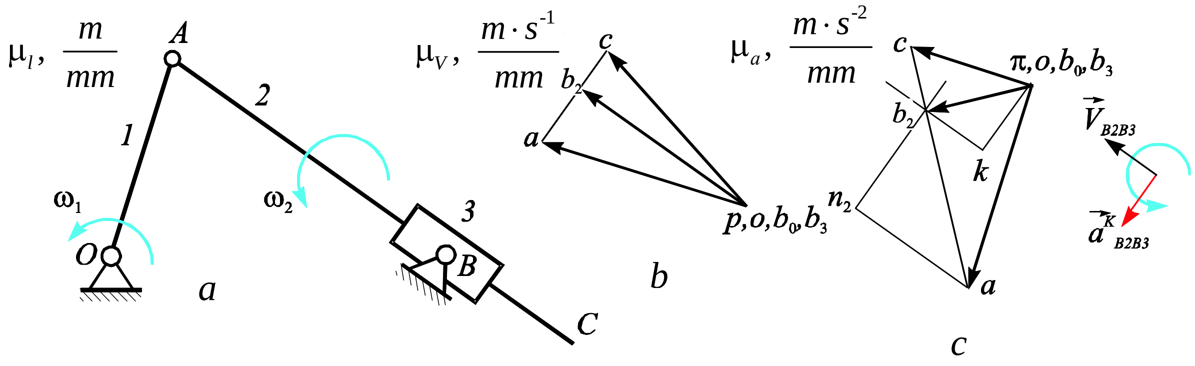


Fig. 4.19. To example 4.5: *a* - kinematic scheme of the mechanism**;**
b – the velocity diagram; *c* – the acceleration diagram

Next we consider the structural group. The connecting rod 2 performs a planar motion. The velocity of point B, belonging to this link and coinciding with the centre of the hinge, is expressed, as in the previous example, as the sum of the velocity in the transportation sliding motion together with point A and relative turning motion around point A (See equation system (4.22)). Unlike the previous example, the absolute velocities of points B_2 *and* B_3 *do not coincide, since there is a relative sliding motion between the connecting rod and the sliding block. We obtain such set of vector equations for the velocity of point B of the connecting rod:*

$$\begin{cases} \overline{V}_{B_2} = \overline{V}_A + \overline{V}_{B_2A}; \\ \overline{V}_{B_2} = \overline{V}_{B_3} + \overline{V}_{B_2B_3}. \end{cases}$$

Here $\overline{V}_{B_3} = 0$, *since the hinge B between the sliding block and the ground is fixed (this point on the velocity diagram (Fig. 4.19, b) is at the pole).*

Further, in accordance with the written equations, we draw the directions of relative velocities: from the point a of the velocity diagram we draw a straight line perpendicular to BA in the scheme ($\overline{V}_{B_2A} \perp BA$), *and from the pole p we draw a straight line parallel to the direction of motion of the connecting rod relative to the sliding block* ($\overline{V}_{B_2B_3} \parallel BA$)*. The point* b_2 *of intersection of these lines is a solution of the set of equations. The velocity of point* B_2 *is found as* $V_{B_2} = \mu_V \cdot pb_2$.

2. Construction of the mechanism acceleration diagram.

Since the condition of the problem $\omega_1 = \text{const}$, *only normal acceleration takes place at point A of the crank* $a_A = \omega_1^2 l_{OA}$. *From the selected pole* π *of the acceleration diagram, we draw the segment*

πa in the direction from point A to point O – the centre of rotation of the crank (Fig. 4.19, c). The scale factor of the acceleration diagram is determined from the ratio $\mu_a = \dfrac{a_A}{\pi a}, \ \dfrac{m \cdot s^{-2}}{mm}$.

Acceleration of point B is found as a solution to the set of vector equations:

$$\begin{cases} \overline{a}_{B_2} = \overline{a}_A + \overline{a}^n_{B_2A} + \overline{a}^\tau_{B_2A}; \\ \overline{a}_{B_2} = \overline{a}_{B_3} + \overline{a}^r_{B_2B_3} + \overline{a}^k_{B_2B_3}. \end{cases} \tag{4.23}$$

Here the acceleration $a^n_{B_2A} = \dfrac{V^2_{B_2A}}{l_{AB}} = \dfrac{(b_2a \cdot \mu_V)^2}{l_{AB}}$ *is parallel to AB and directed from point B to point A in the diagram (Fig. 4.19, a);* $\overline{a}^\tau_{B_2A} \perp BA$*; the acceleration* $a_{B_3} = 0$*; relative acceleration* $\overline{a}_{B_2B_3} \parallel AB$*.*

Since link 2 produces a planar motion, that is, there is a relative turning motion, then there is an Coriolis acceleration during the relative sliding motion of links 2 and 3 $\overline{a}^k_{B_2B_3} = 2\overline{\omega}_2 \times \overline{V}_{B_2B_3}$. *The magnitude of this acceleration is found as a module of the cross product:* $a^k_{B_2B_3} = 2\omega_2 \cdot V_{B_2B_3}$, *and the direction according to the rule of Zhukovsky (Fig. 4.19, c). Here* $\omega_2 = \dfrac{V_{B_2A}}{l_{B_2A}} = \dfrac{\mu_V \cdot b_2a}{l_{B_2A}}$. *To find the direction ω_2, the velocity vector* $\overline{V}_{B_2A}$ *(segment ab_2 in the velocity diagram) is transferred to the point B in the scheme of the mechanism, from which we find the direction of rotation of the connecting rod around point A. We remind that the direction of relative velocity on the diagram is from the second index to the first.*

The normal acceleration of the point B_2 relative to the point A in the turning motion of the link 2 is set from the point a on the acceleration diagram in the form of a segment parallel to the segment AB in the scheme (Fig. 4.19, a) in the direction from point B to point A. Further, from the point n_2 we draw the direction of tangential acceleration. $\overline{a}^\tau_{B_2A} \perp BA$.

According to the second equation of the set (4.23), the immovable point B_3 is at the pole. From the pole π we draw the segment $\pi k = \dfrac{a^k_{B_2B_3}}{\mu_a}$ *in a definite direction of the Coriolis acceleration (See Fig. 4.19, c). From the end of this segment, we draw the straight line in the direction of the relative acceleration* $\overline{a}_{B_2B_3} \parallel AB$ *to the intersection with the direction of tangential* *acceleration* $\bar{a}^{\tau}_{B_2A}$. *The point* b_2 *of the intersection of these directions we connect with the pole and find the magnitude and direction of acceleration of the point* B_2*:* $a_{B_2} = \pi b_2 \cdot \mu_a$.

We find the velocity and the acceleration of point C according to the similarity theorem. This point belongs to the link 2 and lies on the same line with the points A and B (Fig. 4.19, a). Its position on the velocity and acceleration diagrams is found from the proportion: $\dfrac{CA}{ca} = \dfrac{BA}{b_2a}$.

Example 4.6. Construct velocity and acceleration diagrams for aircraft stabilizer lift mechanism (Fig. 4.20). The velocity V_{21} of the piston relative to the hydraulic cylinder is given. Suppose that $V_{21} = \mathrm{Const}$.

We will conduct a preliminary structural study of the mechanism. We have 5 movable links and 7 lower kinematic pairs. Then $w = 3n - 2p_5 = 3 \cdot 5 - 2 \cdot 7 = 1$. *Having selected hydrocylinder 1 as the initial link, pivotally connected to the rack, it is easy to verify that the attached kinematic chain contains two structural groups of the II class and 2-nd order. The first attached group is formed by links 2 and 3 with one external sliding pair A (the 2-nd kind group according to Artobolevsky). The second attached group is formed by links 4 and 5 with turning pairs (the 1-st kind group according to Artobolevsky).*

<u>*We propose to construct a structural scheme of the mechanism and to decompose it into structural groups on one's own.*</u>

1. *Construction of the mechanism velocity diagram.*

According to the task, the velocity of the piston 2 relative to the hydraulic cylinder 1 is set. Since we have a translational motion, the law of relative motion for any two points of links 1 and 2 will be the same. So, if we connect a certain point O_2 *with the piston, which at this point in time coincides with the point* O_1 *of the hydraulic cylinder (Fig. 4.20, a), then the velocities of the relative motion of these points* $\bar{V}_{O_2O_1} = \bar{V}_{A_2A_1} = \bar{V}_{21}$. *Express the absolute velocity of the point* O_2 *through the velocity of the transportation motion of the point* O_1*:*

$$\bar{V}_{O_2} = \bar{V}_{O_1} + \bar{V}_{O_2O_1} = \bar{V}_{O_2O_1} = \bar{V}_{21}.$$

Here $\bar{V}_{O_1} = 0$, *because the point* O_1 *is fixed.*

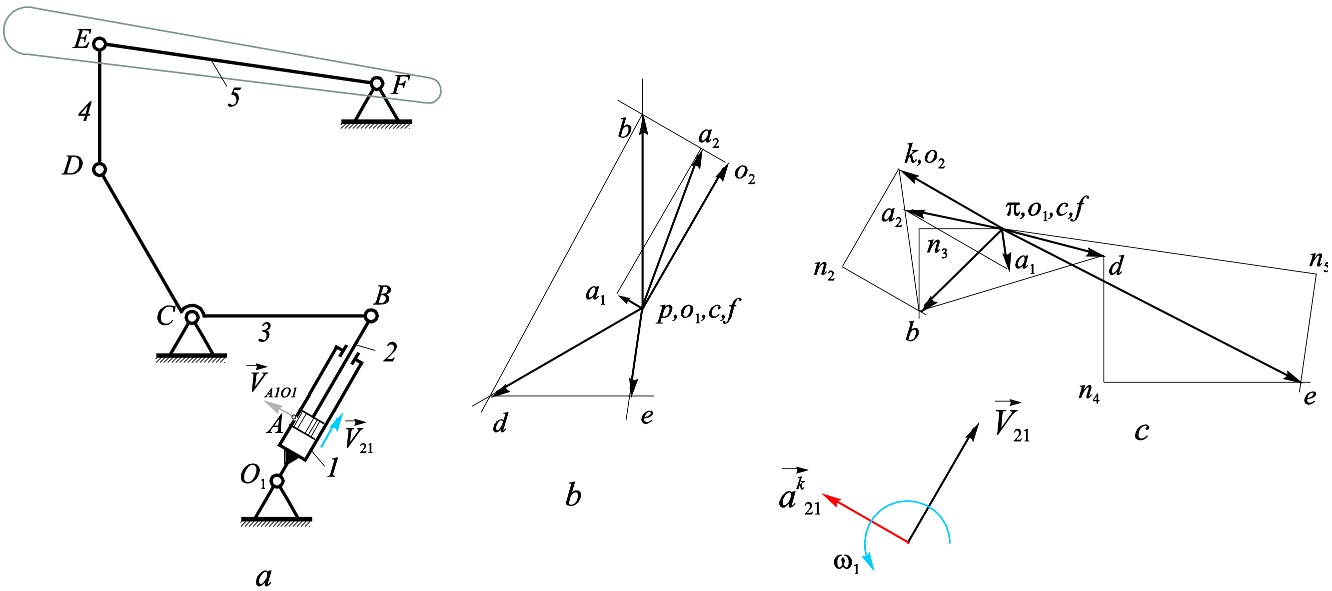


Fig. 4.20. To example 4.6: *a*- kinematic scheme of the mechanism**;**
b – the velocity diagram; *c* – the acceleration diagram

The velocity vector $\overline{V}_{O_2O_1}$ comes out of the pole on the velocity diagram (segment po$_2$ *in Fig. 4.20, b). The scale factor of the velocity diagram* $\mu_V = \dfrac{V_{21}}{po_2}, \ \dfrac{m \cdot s^{-1}}{mm}$.

Consider the first attached structural group (links 2 and 3). For the velocity of point B (intermediate hinge) we have the following set of vector equations:

$$\begin{cases} \overline{V}_B = \overline{V}_{O_2} + \overline{V}_{BO_2} = \overline{V}_{O_2O_1} + \overline{V}_{BO_2}; \\ \overline{V}_B = \overline{V}_C + \overline{V}_{BC}. \end{cases}$$

Here $\overline{V}_C = 0$*, since point C belongs to the housing.*

Then, we draw the direction of the relative velocity of turning motion $\overline{V}_{BO_2} \perp BO_1$ *on the scheme (Fig. 4.20, a) from the point o*$_2$ *(the end of the vector* $\overline{V}_{O_2O_1}$*) on the of velocity diagram, and from the pole p – the direction of the relative velocity of turning motion* $\overline{V}_{BC} \perp BC$*. Connect the intersection point B of these directions with the pole p and find the velocity* $V_B = \mu_V \cdot pb$.

The magnitude and direction of the absolute velocity of the point A$_2$ *of the piston can be found from the velocity diagram. Link 2 is represented by a segment bo*$_2$ *in the diagram. According to the similarity theorem, the position of the point a*$_2$ *is determined from the proportion:* $\dfrac{BO_1}{bo_2} = \dfrac{BA}{ba_2}$.

The velocity of the point A_1 of the cylinder is related to the velocity of the point A_2 of the piston by the vector equation: $\bar{V}_{A_2} = \bar{V}_{A_1} + \bar{V}_{A_2A_1}$. *Solve this equation for velocity* $\bar{V}_{A_1}$ *:*

$$\bar{V}_{A_1} = \bar{V}_{A_2} + \bar{V}_{A_1A_2}$$

and draw the segment $a_2a_1 = \dfrac{V_{12}}{\mu_V}$ *from point a_2 on the velocity diagram in the opposite direction to* po_2 *(here* $\bar{V}_{A_1A_2} = -\bar{V}_{A_2A_1}$ *). We connect the end of this segment - point a_1 to the pole p and find the velocity* $V_{A_1} = \mu_V \cdot pa_1$. *Obviously, the drawn segment pa_1 is perpendicular to BO_1 in the scheme.*

Consider the second attached Assur group (links 4 and 5).

Point F belongs to the hinge between link 5 and ground. Thus velocity $\bar{V}_F = 0$ *(this point is at the pole of the velocity diagram). We find the velocity of point D using a similarity theorem. To do this, we construct a triangle pbd on the segment pb of the velocity diagram, similar to the triangle CBD on the kinematic scheme of the mechanism, and is situated similarly to it (the direction of traversal by points in both triangles must be the same). The segment pd of the diagram corresponds to the absolute velocity of point D:* $V_D = \mu_V \cdot pd$.

The velocity of point E is found as a solution of the set of vector equations:

$$\begin{cases} \bar{V}_E = \bar{V}_D + \bar{V}_{ED}; \\ \bar{V}_E = \bar{V}_F + \bar{V}_{EF}. \end{cases} \tag{4.24}$$

In accordance with the written equations, we draw the directions of relative velocities: from point d of the diagram we draw a line perpendicular to ED on the scheme ($\bar{V}_{ED} \perp ED$ *), and from the pole p – a line, perpendicular to EF on the scheme (* $\bar{V}_{EF} \perp EF$ *). The point e of intersection of these lines is a solution to the system of equations (4.24). The velocity of point E is found as* $V_E = \mu_V \cdot pe$.

2. Construction of the mechanism acceleration diagram.

Express the acceleration of the point O_2, which belongs to the piston, as

$$\bar{a}_{O_2} = \bar{a}_{O_1} + \bar{a}^r_{O_2O_1} + \bar{a}^k_{O_2O_1}.$$

Here $\overline{a}_{O_1}=0$*, since the point* O_1 *belongs to the ground;* $\overline{a}^{r}_{O_2O_1}=0$*, because the velocity* $V_{21}=\text{Const}$ *on the condition of the task. Then get it*

$$\overline{a}_{O_2}=\overline{a}^{k}_{O_2O_1}\,, \qquad (4.25)$$

where $\overline{a}^{k}_{O_2O_1}$ *is the Coriolis acceleration, which occurs because the hydraulic cylinder 1 turns around the hinge* O_1*, and the piston 2 moves translationally relative to the hydraulic cylinder:* $\overline{a}^{k}_{O_2O_1}=2\overline{\omega}_1\times\overline{V}_{O_2O_1}$*. We will find the magnitude of this acceleration as* $a^{k}_{O_2O_1}=2\omega_1\cdot V_{21}$*, and direction according to Zhukovsky rule. The magnitude of the angular velocity* $\omega_1=\dfrac{V_{A_1O_1}}{l_{AO_1}}=\dfrac{\mu_V\cdot pa_1}{l_{AO_1}}$*, and its direction we find in the direction of the linear velocity of the point* A_1*. To do this, we put the beginning of the vector* $\vec{V}_{A_1O_1}$ *into point A in a scheme whose direction we determine from the velocity diagram - this is the direction from the point p to the point* a_1 *(See Fig. 4.20, b).*

Next, from the selected pole π *we draw the segment* πk *(it is the segment* πo_2 *according to (4.25)) in the found direction of Coriolis acceleration (Fig. 4.20, c).*

Now consider the first attached structural group (links 2 and 3). We know accelerations of the point O_2 *(just found) and the point C (*$a_C=0$ *since this point belongs to the ground). Acceleration of point B is determined from the set of vector equations:*

$$\begin{cases}\overline{a}_B=\overline{a}_{O_2}+\overline{a}^{n}_{BO_2}+\overline{a}^{\tau}_{BO_2};\\ \overline{a}_B=\overline{a}^{n}_{BC}+\overline{a}^{\tau}_{BC}.\end{cases}$$

Here, the acceleration $a^{n}_{BO_2}=\dfrac{V^2_{BO_2}}{l_{BO_1}}=\dfrac{(bo_2\cdot\mu_V)^2}{l_{BO_1}}$ *is parallel to* O_1B *and is directed from point B to point* O_1 *in the scheme (Fig. 4.20, a). We draw a segment* $o_2n_2=\dfrac{a^{n}_{BO_2}}{\mu_a}$ *from point* o_2 *on the diagram. The acceleration* $a^{n}_{BC}=\dfrac{V^2_{BC}}{l_{BC}}=\dfrac{(pb\cdot\mu_V)^2}{l_{BC}}$ *is parallel to BC and is directed from point B to point* O_1 *in the scheme. We draw a segment* $\pi n_3=\dfrac{a^{n}_{BC}}{\mu_a}$ *from pole* π *on the diagram. We draw a straight line from the point* n_2 *in the direction of acceleration* $\overline{a}^{\tau}_{BO_2}\perp BO_1$*, and from* *the point* n_3 *we draw a straight line in the direction of acceleration* $\bar{a}_{BC}^{\tau} \perp BC$. *Point b lies at the intersection of these two straight lines. The magnitude and direction of acceleration of point B will be found by connecting the pole* π *with point b:* $a_B = \mu_a \cdot \pi b$.

The position of points A_2 *and* A_1 *on the acceleration diagram is found by analogy with the way it was done for the velocity diagram (See Fig. 4.20, c). Link 2 is represented by a segment* bo_2 *on the diagram. The position of point* a_2 *is found from proportion:* $\frac{BO_1}{bo_2} = \frac{BA}{ba_2}$. *We connect point* a_2 *with pole* π *and find* $a_{A_2} = \mu_a \cdot \pi a_2$.

The acceleration of point A_1 of *the cylinder is connected with the acceleration of point* A_2 *of the piston by the vector relation:*

$$\bar{a}_{A_2} = \bar{a}_{A_1} + \bar{a}_{A_2A_1}^{r} + \bar{a}_{A_2A_1}^{k}. \tag{4.26}$$

Since the piston motion relative to the cylinder is uniform, then $\bar{a}_{A_2A_1}^{r} = 0$, *and the Coriolis acceleration for all points of the piston will be the same, i.e.* $\bar{a}_{A_2A_1}^{k} = \bar{a}_{O_2O_1}^{k}$. *Solving the equation (4.26) with respect* $\vec{a}_{A_1}$, *we obtain:*

$$\bar{a}_{A_1} = \bar{a}_{A_2} + \bar{a}_{A_1A_2}^{k}.$$

We draw a segment $a_2a_1 = \frac{\bar{a}_{A_2A_1}^{k}}{\mu_a} = \frac{\bar{a}_{O_2O_1}^{k}}{\mu_a}$ *from point* a_2 *on the accelerations diagram in the direction opposite to* πo_2 *(here* $\bar{a}_{A_2A_1}^{k} = -\bar{a}_{A_1A_2}^{k}$*). The end of this segment (the point* a_1*) we connect to the pole* π *and find* $a_{A_1} = \mu_a \cdot \pi a_1$.

Consider the second attached Assur group (links 4 and 5).

Acceleration of point D will be found by constructing $\Delta \pi bd$ *on the segment* πb *of the acceleration diagram similar to* ΔCBD *on the mechanism scheme and similarly situated to it. From here we find* $a_D = \mu_a \cdot \pi d$.

Acceleration $a_F = 0$, *as the point F belongs to the ground.*

Acceleration of point E will be found by solving the set of vector equations:

$$\begin{cases} \bar{a}_E = \bar{a}_D + \bar{a}_{ED}^{n} + \bar{a}_{ED}^{\tau}; \\ \bar{a}_E = \bar{a}_{EF}^{n} + \bar{a}_{EF}^{\tau}. \end{cases}$$

Acceleration $a_{ED}^{n} = \frac{V_{ED}^{2}}{l_{ED}} = \frac{(ed \cdot \mu_V)^2}{l_{ED}}$ *is parallel to ED and is directed from point E to point D in the scheme (Fig. 4.20, a). We draw a segment* $dn_4 = \frac{a_{ED}^{n}}{\mu_a}$ *from point d. Acceleration* $a_{EF}^{n} = \frac{V_{E}^{2}}{l_{EF}} = \frac{(pe \cdot \mu_V)^2}{l_{EF}}$ *is parallel to EF and is directed from point E to point F in the scheme. We draw a segment* $\pi n_5 = \frac{a_{EF}^{n}}{\mu_a}$ *from pole* π *on the diagram. From the point* n_4 *we draw a straight line in the direction of acceleration* $\overline{a}_{ED}^{\tau} \perp ED$, *and from point* n_5 *we draw a straight line in the directionof acceleration* $\overline{a}_{EF}^{\tau} \perp EF$. *The point e lies at the intersection of these two straight lines. The magnitude and direction of acceleration of point E will be found by connecting the pole* π *with point e:* $a_E = \mu_a \cdot \pi e$.

4.3.5. Velocity and acceleration diagrams for triple arm groups (triad) with turning pairs

Let us consider a structure group of the III class and 3-nd order (Fig. 4.21).

For such groups, the construction of the velocity and acceleration diagram the so-called *Assur points* are used. They are located at the intersection of any two arms and are considered as such that belong to the basic link.

Let us assume that velocities $\overline{V}_B$, $\overline{V}_F$, $\overline{V}_G$. are given. We can find the velocity vector of the Assur point *S* by equations:

$$\begin{cases} \overline{V}_S = \overline{V}_C + \overline{V}_{SC} = \overline{V}_B + \underbrace{\overline{V}_{CB} + \overline{V}_{SC}}_{\perp SB}; \\ \overline{V}_S = \overline{V}_D + \overline{V}_{SD} = \overline{V}_G + \underbrace{\overline{V}_{DG} + \overline{V}_{SD}}_{\perp SG}. \end{cases}$$

This set is solved through the velocity diagram (Fig. 4.21, *b*).

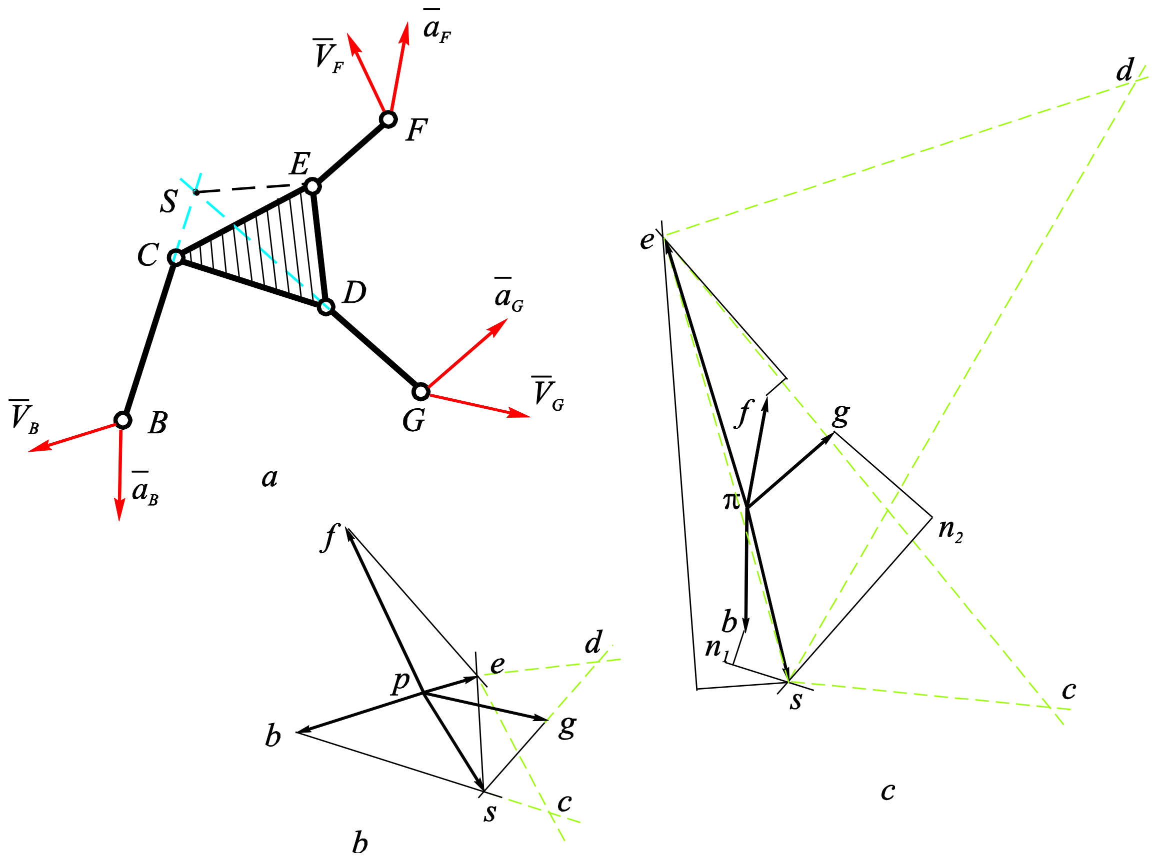


Fig. 4.21. Triple arm groups with turning pairs: *a* - scheme of the group; *b* – the velocity diagram; *c* – the acceleration diagram

In order to find a law of motion for the basic link, we need to know the law of motion for another point, besides *S*. For example, let's take point *E*:

$$\begin{cases} \overline{V}_E = \overline{V}_F + \overline{V}_{EF}^{(\perp EF)}; \\ \overline{V}_E = \overline{V}_S + \overline{V}_{ES}^{(\perp SE)}. \end{cases}$$

Velocities of other points of the link, for example *C* or *D*, are determined by the similarity theorem for velocity diagrams.

By analogy, the acceleration diagram is constructed. If accelerations for *B*, *G*, *F* are given, then for the Assur point *S* we can write down

$$\begin{cases} \overline{a}_S = \overline{a}_C + \overline{a}_{SC}^n + \overline{a}_{SC}^\tau = \overline{a}_B + \underbrace{\overline{a}_{SC}^n + \overline{a}_{CB}^n}_{\parallel SB} + \underbrace{\overline{a}_{SC}^\tau + \overline{a}_{CB}^\tau}_{\perp SB}; \\ \overline{a}_S = \overline{a}_G + \underbrace{\overline{a}_{DG}^n + \overline{a}_{SD}^n}_{\parallel SG} + \underbrace{\overline{a}_{DG}^\tau + \overline{a}_{SD}^\tau}_{\perp SG}. \end{cases}$$

To facilitate the calculation, let's assume that

$$\overline{a}_{SC}^{n}+\overline{a}_{CB}^{n}=\overline{a_{SC}^{n}+a_{CB}^{n}}=\overline{a}_{SB}^{n};$$

$$\overline{a}_{SC}^{\tau}+\overline{a}_{CB}^{\tau}=\overline{a_{SC}^{\tau}+a_{CB}^{\tau}}=\overline{a}_{SB}^{\tau};$$

$$\overline{a}_{DG}^{n}+\overline{a}_{SD}^{n}=\overline{a_{DG}^{n}+a_{SD}^{n}}=\overline{a}_{SG}^{n};$$

$$\overline{a}_{DG}^{\tau}+\overline{a}_{SD}^{\tau}=\overline{a_{DG}^{\tau}+a_{SD}^{\tau}}=\overline{a}_{SG}^{\tau}.$$

Then

$$\begin{cases}\overline{a}_{S}=\overline{a}_{B}+\overline{a}_{SB}^{n}+\overline{a}_{SB}^{\tau};\\ \overline{a}_{S}=\overline{a}_{G}+\overline{a}_{SG}^{n}+\overline{a}_{SG}^{\tau}.\end{cases}$$

The values of normal accelerations are calculated from the following equations:

$$a_{CB}^{n}=\frac{V_{CB}^{2}}{l_{CB}},\quad a_{DG}^{n}=\frac{V_{DG}^{2}}{l_{DG}},\quad a_{SC}^{n}=\frac{V_{SC}^{2}}{l_{SC}},\quad a_{SD}^{n}=\frac{V_{SD}^{2}}{l_{SD}}.$$

The acceleration of the point *E*:

$$\begin{cases}\overline{a}_{E}=\overline{a}_{S}+\overline{a}_{ES}^{n}+\overline{a}_{ES}^{\tau};\\ \overline{a}_{E}=\overline{a}_{F}+\overline{a}_{EF}^{n}+\overline{a}_{EF}^{\tau}.\end{cases}$$

Acceleration diagram is shown in Fig. 4.21, *c*.

Accelerations of other basic link points are calculated by the similarity theorem for acceleration diagram.

4.4. KINEMATIC ANALYSIS OF GEAR TRAINS

4.4.1. Determination of velocity ratios in multi-link gear trains

Compound gear train. Fig. 4.22 shows a kinematic scheme of a compound gear train.

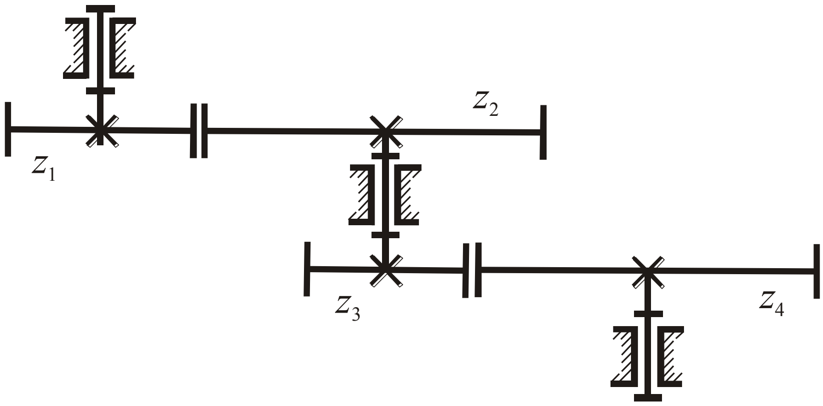


Fig. 4.22. Compound gear train

Angular velocity ratio between input and output links

$$u_{14} = \frac{\omega_1}{\omega_4}$$

or

$$u_{14} = \frac{\omega_1}{\omega_2} \cdot \frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4} = u_{12} \cdot u_{34}.$$

Here, with regards to the scheme $\omega_2 = \omega_3$.

The velocity ratio may be calculated as the ratio of teeth number of gears which form the gearing. In this case it is often called *transmission ratio*.

$$u_{14} = -\frac{z_2}{z_1}, \quad u_{34} = -\frac{z_4}{z_3}.$$

Sign „–” belongs to the external toothing.

Hence

$$u_{14} = u_{12} \cdot u_{34} = \frac{z_2}{z_1} \cdot \frac{z_4}{z_3}.$$

Velocity ratio of the compound gear train equals to the product of single velocity ratios or the ratio of a product of teeth numbers of gears with even indexes (or driven gears) to the product of teeth numbers of gears with odd indexes (or driving gears).

$$u_{1n} = u_{12} \cdot u_{34} \cdot \ldots \cdot u_{(n-1)n} = (-1)^m \frac{z_2 \cdot z_4 \cdot \ldots \cdot z_n}{z_1 \cdot z_3 \cdot \ldots \cdot z_{(n-1)}}. \qquad (4.27)$$

Here n is the number of gears; m is the number of external toothings.

Differential gear train. It is a mechanism which has a motion freedom exceeding one.

Fig. 4.23 shows a *planetary mechanism*, i.e. a mechanism that has gears with moveable axes.

The planetary mechanism, according to the figure, has four moveable links (*central* or *sun gears* 1 and 2, *planet pinion* 3, *planet carrier or arm H)*, four lower kinematic pairs, and two higher ones. Its motion freedom is $w = 3 \cdot 4 - 2 \cdot 4 - 2 = 2$.

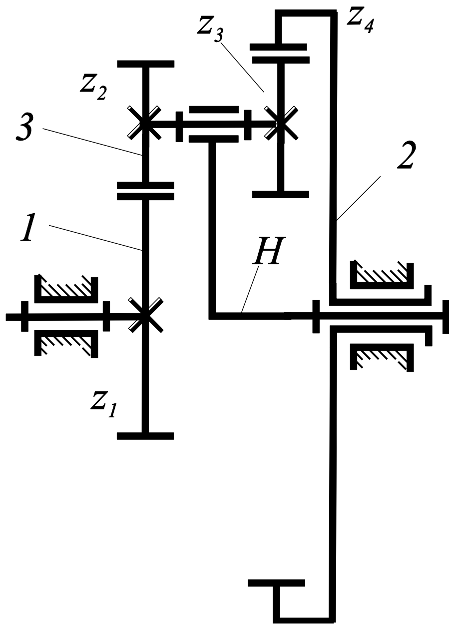


Fig. 4.23. Differential gear train

Let's find relations between the angular velocities of gears and planet carrier $(\omega_1, \omega_2, \omega_4, \omega_H)$. Here we apply kinematic inversion principle − reversibility of a motion. To perform it we assign an angular velocity $(-\omega_H)$ to the whole system. Then the planet carrier stops and the gears form common inverse compound gear train. Its angular velocities of gears are:

$$\omega_1^{(H)} = \omega_1 - \omega_H ;$$

$$\omega_2^{(H)} = \omega_3^{(H)} = \omega_2 - \omega_H ;$$

$$\omega_4^{(H)} = \omega_4 - \omega_H .$$

The upper index shows that the planet carrier *H* is conditionally still.

This approach to planetary gearings analysis was first introduced by Robert Willis.

Velocity ratio of an inverse mechanism (compound gear train), according to the formula (4.27):

$$u_{14}^{(H)} = \frac{\omega_1^{(H)}}{\omega_4^{(H)}} = \frac{\omega_1 - \omega_H}{\omega_4 - \omega_H} = -\frac{z_2}{z_1} \cdot \frac{z_4}{z_3} . \tag{4.28}$$

In this case the inverse mechanism gears create one external toothing and one internal toothing and velocity ratio is negative.

Equation (4.28) enables us to calculate each of the three angular velocities $-\omega_1$, ω_H or ω_4, if the other two velocities and gear teeth number are known.

Planetary gear train. Fig. 4.24 shows a planetary gear train. Sun gear 4 is stationary here. It's easy to prove that its motion freedom $w = 1$.

The relation between the velocities of input and output links of a planetary gear train are easily calculated by formula (4.28).

As in this case $\omega_4 = 0$, then

$$u_{14}^{(H)} = \frac{\omega_1 - \omega_H}{-\omega_H} = 1 - \frac{\omega_1}{\omega_H} = 1 - u_{1H}^{(4)}.$$

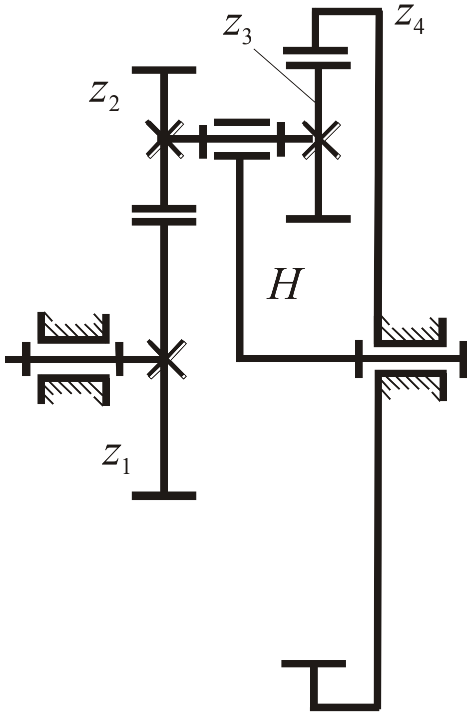


Fig. 4.24. Planetary gear train

Hence,

$$u_{1H}^{(4)} = 1 - u_{14}^{(H)}$$

or

$$u_{1H}^{(4)} = 1 + \frac{z_2 z_4}{z_1 z_3} = \frac{\omega_1}{\omega_H}. \qquad (4.29)$$

Formula (4.29) allows calculating angular velocity of the first gear (input link) or planet carrier (output link), if one of these velocities and the gear teeth number are given.

4.4.2. Velocity-distribution triangles

Let us consider a turning link (Fig. 4.25, *a*).

The velocity diagram is shown in Fig. 4.25, *b*. Here

$$V_A = \omega_1 \cdot l_{OA};\ V_B = \omega_1 \cdot l_{OB};\ V_C = \omega_1 \cdot l_{OC}.$$

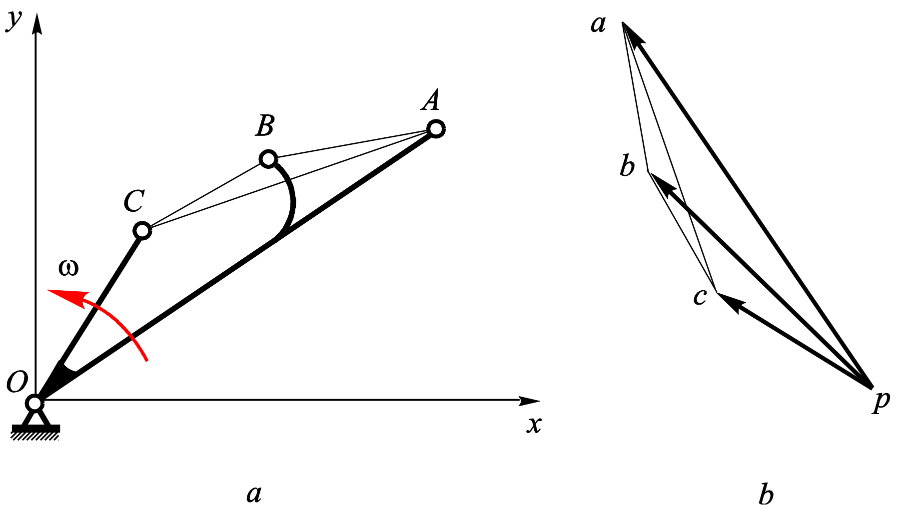


Fig. 4.25. The turning link (*a*) and its velocity diagram (*b*)

By the fact the velocity difference between two points on a rigid body is proportional to the distance between those points and is perpendicular to a line connecting them, the distribution of velocity differences for points along that line may be drawn as shown in Fig. 4.26.

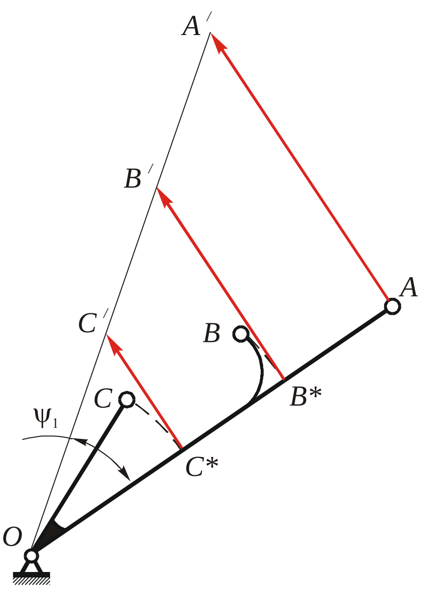


Fig. 4.26. Velocity-distribution triangle of turning link

Here

$$\overline{AA'} = \frac{\bar{V}_A}{\mu_V}; \quad \overline{B^*B'} = \frac{\bar{V}_{B^*}}{\mu_V}; \quad \overline{C^*C'} = \frac{\bar{V}_{C^*}}{\mu_V}.$$

A line connecting the tips of the velocity vectors A', B', C' is called *velocity-distribution diagram* for points on the line *OA*.

A triangle such as that formed by the vector $\overline{AA'}$, the line *OA* and a distribution line $OA',$ will be referred to as a *velocity-distribution triangle*.

The angle subtended by the distribution line OA' and the line *OA*, is found from the following equation:

$$\mathrm{tg}\psi_1 = \frac{AA'}{OA} = \frac{\mu_l \cdot V_A}{\mu_V \cdot l_{OA}} = \frac{\mu_l}{\mu_V}\omega_1.$$

Velocity of any point on the link that does not belong to the line *OA*, is easy to find. To perform it we transfer this point with a pair of compasses put in the point *O*, onto the line *OA* and then draw a perpendicular to *OA* from the obtained point, for example point C^*, until the perpendicular crosses the velocity-distribution line. If the velocity-distribution triangle (Fig. 4.26) and the velocity diagram (Fig. 4.25, *b*) have been constructed in the same scale μ_V, then segments $\overline{C^*C'} = \overline{pc}$, as $\left|\bar{V}_C\right| = \left|\bar{V}_{C^*}\right|$.

Velocity-distribution triangles are widely used to analyze gear trains, especially, compound planetary and differential gear trains.

Let us take a closer look at a planetary gear train (Fig. 4.27, *a*), where rotation from the shaft 1 is passed to the shafts of the sun gear 6 and planet carrier *H* with the help of a gear cluster – *planet pinion*.

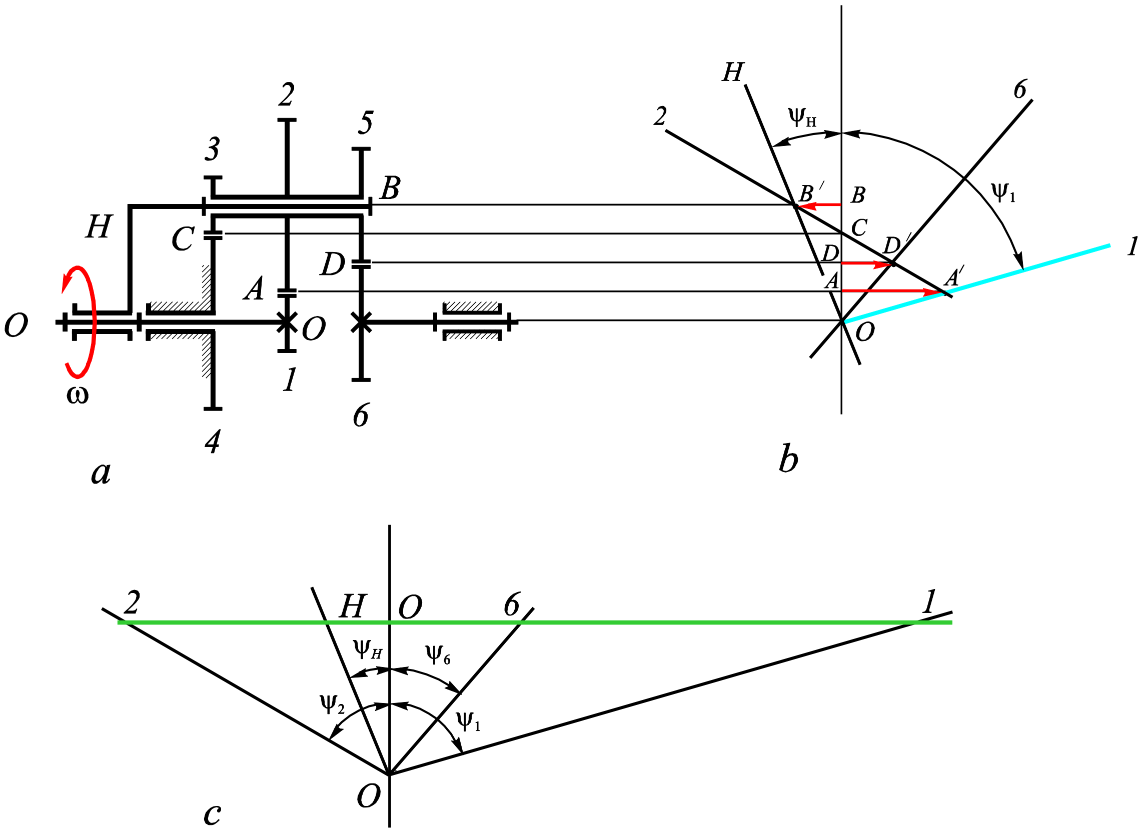


Fig. 4.27. Planetary gear train (*a*) and velocity-distribution diagrams for its links (*b*, *c*)

In Fig. 4.27, *b* straight line 1 is the velocity-distribution line of the gear 1. Velocities of the point *A* of the gear 1 and the gear 2, belonging to the planet pinion, are the same. The velocity of the point *C* equals to 0, as it belongs to the stationary sun gear 4 (point *C* – is an instantaneous centre of rotation for the planet pinion, making two-dimensional motion: revolving on the axis *B* and at the same time revolving together with the planet carrier *H* on the axis *OO*).

Line 2 is a velocity-distribution line of the planet pinion. Segment BB' characterizes the velocity of the point *B*. Then, the line *H* is a diagram of the planet carrier velocity distribution.

Then we find point D' on line 2. Segment DD' characterizes velocity of the point *D* of both the planet pinion and the gear 6. Then, straight line 6 is a velocity-distribution line of the gear *6*.

To calculate angular velocities of the gears we need to choose a point *O* (Fig. 4.27, *c*) and draw a pencil of lines parallel to the velocity-distribution lines from it (Fig. 4.27, *b*). If this pencil of lines is crossed by a perpendicular to the reference line

O-O, then the segments *OH*, *O2*, *O1*, *O6* obtained will be proportionate to the angular velocities of the respective gears:

$$\omega_1 = \frac{V_A}{r_{w1}} = \frac{AA' \cdot \mu_V}{OA \cdot \mu_l} = \frac{\mu_V}{\mu_l} tg\psi_1 = \frac{\mu_V}{\mu_l} \frac{O1}{OO} = O1 \cdot \mu_\omega .$$

That is

$$O1 = \frac{\omega_1}{\mu_\omega} .$$

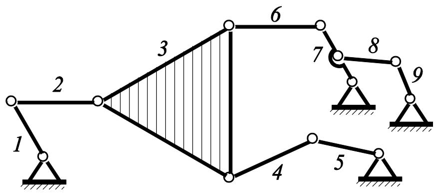
The angular velocity scale is $\mu_\omega = \dfrac{\mu_V}{\mu_l \cdot OO}, \quad \dfrac{radian \cdot s^{-1}}{mm}.$

By analogy $OH = \dfrac{\omega_H}{\mu_\omega}$, etc.

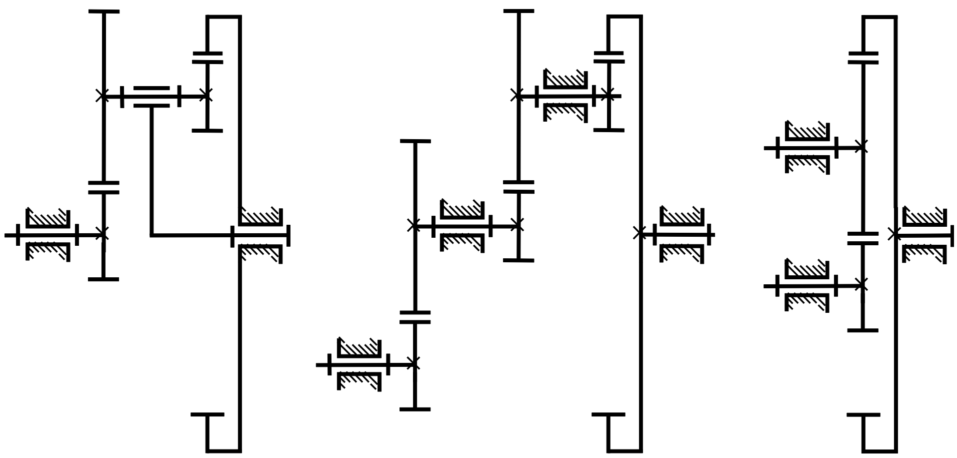
Velocity ratios are found from the following relations:

$$u_{61} = \frac{\omega_6}{\omega_1} = \frac{tg\psi_6}{tg\psi_1} = \frac{O6}{O1}; \qquad u_{H1} = \frac{\omega_H}{\omega_1} = \frac{OH}{O1} \text{ etc.}$$

QUESTIONS FOR SELF-TESTING

1. Formulate the main tasks of the kinematic analysis of mechanisms.
2. List the main kinematic characteristics of mechanisms.
3. What are the main methods of kinematic analysis?
4. What coordinates are called generalized coordinates of the mechanism?
5. What is called the position function of the mechanism?
6. 

How many generalized coordinates should be given to the mechanism shown in the figure?
7. Write down in general view the position function of the output link 9 of the mechanism (See question 6), after selecting the initial links.
8. List the main transfer functions of the mechanism.
9. Expand the physical meaning of the quotient velocity ratio between the links of the mechanism.
10. What is the difference between the concepts of “velocity” and “velocity analogue” of a mechanism point?
11. How to find the angular acceleration of the mechanism link, if angular acceleration analogue is known?
12. What is the essence of the closed vector loops method in kinematic study of hinged-lever mechanisms?
13. Write down the formulas of transforming of Cartesian coordinates when they are rotated.
14. What expression is called the transformation matrix when the axes are rotated?
15. What is called the kinematic diagrams of motion of points (links) of the mechanism?
16. What positions of the mechanism are called extreme?
17. What is the chord method in graphical differentiation?
18. What is the basis of the graphical integration method when performing a kinematic study of the mechanism?
19. How to determine the scale of the diagram obtained by the method of graphical differentiation?

20. How to determine the scale of the diagram obtained by the method of graphical integration?
21. What is called the position diagram of the mechanism?
22. What is called the velocity and acceleration diagrams of the mechanism?
23. What points of the body are called instantaneous centre of rotation and instantaneous centre of accelerations?
24. What is the velocity of mechanism point, if it is located at the pole of the velocity diagram?
25. What velocity vectors go out of the pole of the velocity diagram?
26. What acceleration vectors do not pass through the pole?
27. Formulate the similarity theorem for velocity diagrams and the similarity theorem for acceleration diagrams.
28. List the basic properties of velocity and acceleration diagram.
29. When does Coriolis acceleration appear? Write a formula to define it. How to find the direction of this acceleration for a planar mechanism according to the Zhukovsky rule?
30. What points are called the special Assur points in the kinematic study of triple arm groups?
31. 

a *b* *c*

Analyze the schemes of gear mechanisms and determine which are compound, planetary and row gear train respectively.

32. What principle underlies the stopping method of the carrier in the kinematic study of planetary mechanisms?

Chapter 5. DYNAMIC ANALYSIS OF MACHINE AGGREGATE MOTION

Two main problems exist in the dynamic analysis of mechanisms: a direct problem and an inverse one.

When solving a direct problem, we find kinematic properties of a mechanism under given loading, link masses, their dimensions and moments of inertia.

When solving an inverse problem we need to find the masses, moments of inertia, and therefore, dimensions of links, with which a mechanism under these forces makes motions in a given mode.

5.1. FORCES ACTING IN MECHANISMS AND MACHINES

Let us consider main groups of forces taken into account in the dynamic analysis of mechanisms and machines.

5.1.1. Driving forces and moments

These are forces which conduct a positive work over a time of operation or a working cycle of a mechanism.

They are applied to the input link, which in this case is called *a driving link*.

5.1.2. Resistance forces and moments

These forces conduct a negative work over a time of operation or a working cycle of a mechanism.

They are further subdivided into *forces and moments of a useful resistance* and *forces and moments of an environmental resistance*.

The forces of a useful resistance carry out the work, for performing of which the machine was created. They are applied to *driven links* (*driven members*).

The forces of an environmental resistance are forces connected to the barren backoff. They are often insignificant and are not taken into account in solving dynamical problems.

5.1.3. Gravity

It is applied in the centre of link mass.

$$G_i = m_i g \text{ .}$$

Work of these forces over a cycle equals to zero. They can conduct either positive or negative work at a certain period of time.

5.1.4. Inertia

The force appears as a result of the accelerated motion of links and can be regarded as the reaction of a mass on velocity changing.

In a general case of planar motion, inertia distributed over the volume of rigid body can be brought to the resultant vector and the moment of inertia forces, applied in the centre of a link mass.

$$\overline{F}_{a_i} = -m_i \cdot \overline{a}_{S_i} \text{ ;}$$

$$\overline{M}_{a_i} = -I_{S_i} \overline{\varepsilon}_i \text{ ,}$$

where a_{S_i} is an acceleration of the centre of mass S of a link; I_{S_i} is an inertia moment of a link mass with respect to the axis going through the centre of the link mass perpendicularly to the link’s plane of motion; ε_i is an angular acceleration of the link.

Let us have a look at some examples of finding inertia for different link movements.

Pure translation of a link. In this case accelerations at all the points of sliding link are the same. Therefore,

$$a_S = a_A = a_B \text{;} \qquad \varepsilon = 0 \text{.}$$

Then

$$\bar{F}_a = -m \cdot \bar{a}_S ;$$

$$M_a = 0 .$$

Hence, when the link is producing pure translation, inertia is brought only to the resultant vector, that is applied in the centre of link mass (Fig. 5.1).

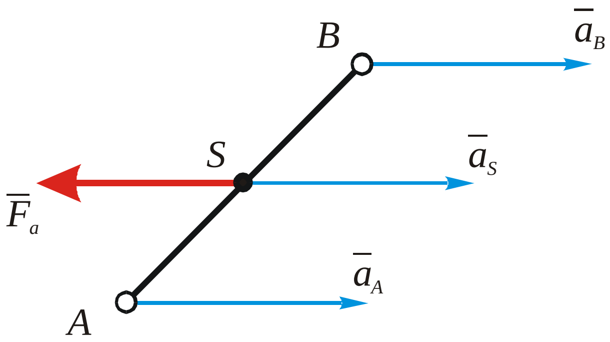


Рис. 5.1. Sliding movable link

Pure rotation of a link with regard to the centre of mass. In this case, centre of mass S of a turning link is stationary, therefore $a_S = 0$. But $\omega \neq 0, \varepsilon \neq 0$. Hence,

$$F_a = 0 ;$$

$$\bar{M}_a = -I_S \cdot \bar{\varepsilon} .$$

Therefore, when the link is turning over a stationary centre of mass, inertia itself acting on it is brought only to the moment (Fig. 5.2)

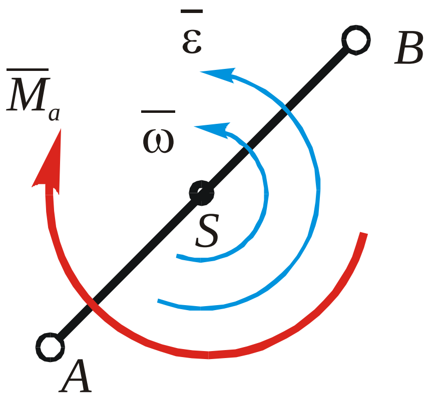


Fig. 5.2. Turning link (centre of rotation and centre of mass are coincided)

A link produces turning motion relatively to a point which does not coincide with the centre of mass. An example of a link, which carries out such motion, is shown in the Fig. 5.3.

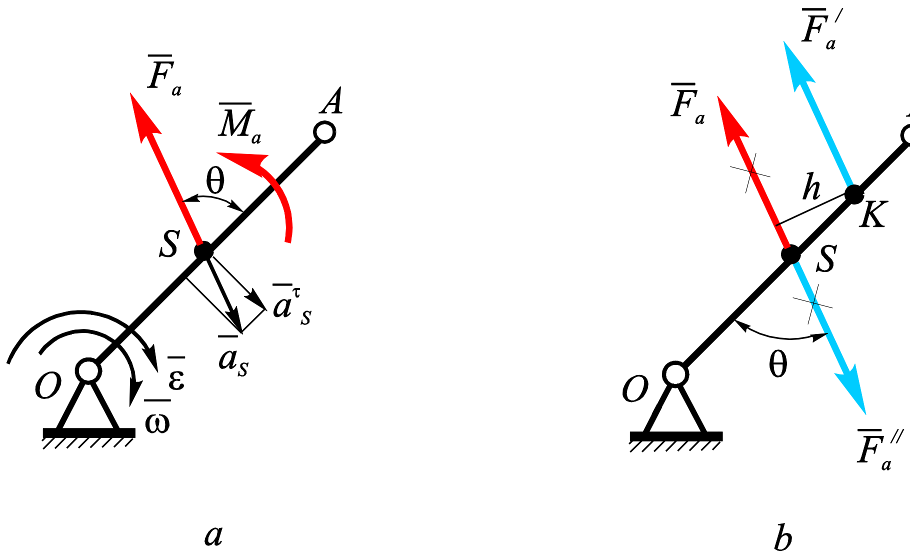


Fig. 5.3. Turning link (centre of rotation and centre of mass do not coincide):
a – initial scheme; *b* – equivalent scheme

From the diagram we can see that a resultant $\bar{F}_a$ and a moment $\bar{M}_a$ of inertia forces operate here.

We give the moment of inertia $\bar{M}_a$ as the force couple $F'_a = F''_a = F_a$ (Fig. 5.3, *b*) imposed in the arm h. As $F_a = F''_a$, so there is only one force imposed to the link. This force is equal to inertia in direction and size, but it is imposed at some distance from the centre of mass *S* – in a point *K*, which is called the centre of oscillation.

Let us define the distance *SK*, according to the Fig. 5.3, *b*.

$$h = \frac{M_a}{F_a} = \frac{I_S \cdot \varepsilon}{m \cdot a_S} = \frac{m\rho^2}{ma_S} \cdot \frac{a^t_S}{l_{OC}}.$$

Here ρ is a radius of gyration of the link; a^t_S is tangential acceleration of the centre of mass *S*.

From ΔSKL we define that $h = l_{SK} \cdot \sin\theta$. Tangential acceleration is $a_S^t = a_S \cdot \sin\theta$. Then

$$l_{SK} \sin\theta = \frac{m\rho^2 \cdot a_S \sin\theta}{ma_S \cdot l_{OS}}.$$

Hence

$$l_{SK} = \frac{\rho^2}{l_{OS}}.$$

So, the distance from the centre of rotation of the link to the centre of oscillation is

$$l_{OK} = l_{OS} + \frac{\rho^2}{l_{OS}}.$$

In practice the link centre of oscillation K is found by the method shown in the Fig. 5.4, where $\rho = \sqrt{I_S/m}$. For straight bars with the length of l radius of gyration can be defined as $\rho = l/\sqrt{12}$.

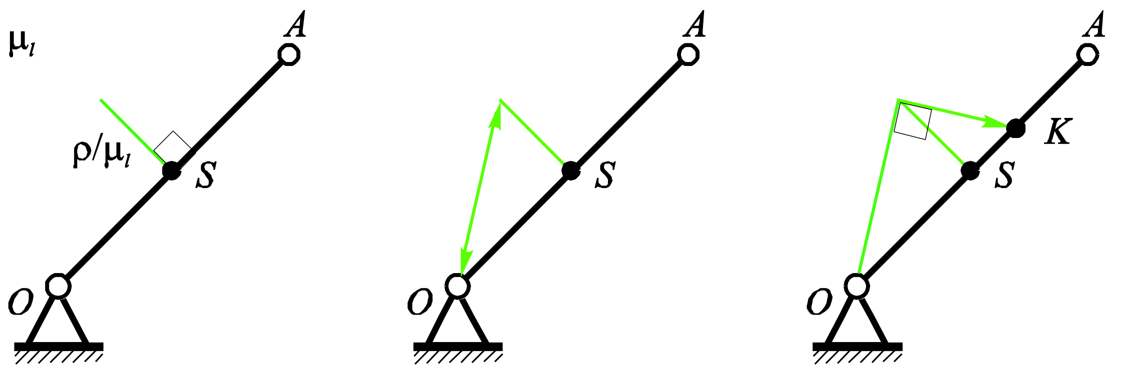


Fig. 5.4. The scheme of definition of the centre of oscillation

A link produces planar motion. Fig. 5.5 shows link and its acceleration diagram. According to the compound motion theorem

$$\bar{a}_S = \bar{a}_A + \bar{a}_{SA}. \qquad (5.1)$$

Inertia of the link is

$$\bar{F}_a = -m \cdot \bar{a}_S ,$$

or, according to (5.1)

$$\bar{F}_a = -m \cdot (\bar{a}_A + \bar{a}_{AS}) = -m \cdot \bar{a}_A - m \cdot \bar{a}_{AS} . \tag{5.2}$$

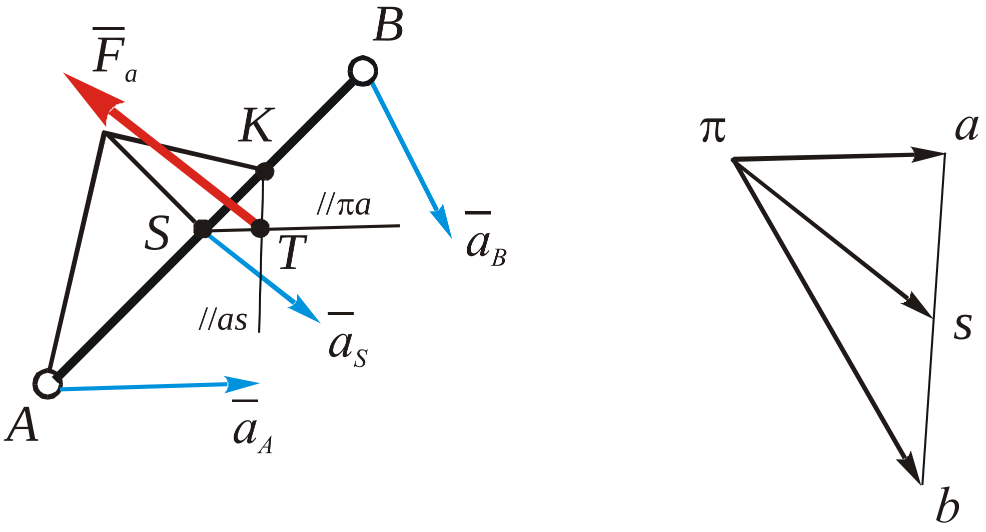


Fig. 5.5. Link produces the planar motion

The first vector is the inertia in sliding motion of a link together with a point *A* accepted as a pole; second vector is the inertia in turning motion of the link relatively to a point *A*. The first force is imposed in the centre of gravity *S*, the second one is in the centre of oscillation *K*. Action lines of these forces intersect in the point *T*, which is called *the pole of inertia*. The action line of inertia of the link $\bar{F}_a$ passes through a pole as the resulting vector of the sum of vectors (5.2).

So, in the case of planar motion of the link inertia is taken only to the resultant $\bar{F}_a$, which is imposed to the pole of inertia of the link *T*.

It is clear that the pole of inertia is instantaneous and is determined by the position of the link.

5.1.5. Forces and moments, which applied to the machine housing from outside

It is the gravity of the housing, reaction on the housing from the basis. As a housing (corps) is immobile, these forces do not carry out any work.

5.1.6. Reacting force (pressures) in kinematic pairs

These are internal forces that are the reactions on active (external) forces action, to which forces of the first four groups belong.

These forces are divided into a normal and a tangent constituents. As a rule, only normal constituents of the reacting forces are determined.

The normal constituents of reactions are called *pressures in kinematic pairs*.

Sliding pair. The distribution diagram of normal pressures is linear here (Fig. 5.6). Here it is known the direction of resultant force of a pressure, but magnitude and point of its imposing are unknown. If the length of a slider is small in comparison with the sizes of other links, so the division of pressures is permanent. Resultant force of a pressure is imposed in the centre of gravity of the slider.

Turning pair. In this pair (Fig. 5.7) we know the point of imposing of resultant (it passes through the centre of pivot), but its value and direction are unknown.

Higher pair. In this pair (Fig. 5.8) we know the point of imposing and direction of pressure (along the general normal). Its value is unknown.

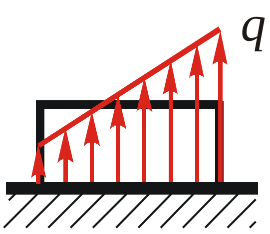


Fig. 5.6. Sliding pair

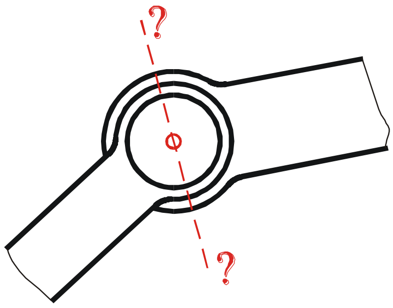


Fig. 5.7. Pin joint

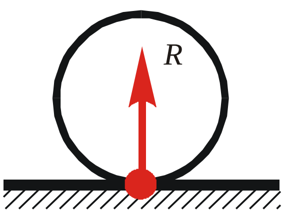


Fig. 5.8. Higher pair

Tangential components in kinematic pairs are the essence of the friction. The normal components do not execute the work, because they are perpendicular to directions of displacements of link in kinematic pairs. Forces of friction always execute the negative work.

5.2. THE PROPERTIES OF FORCES

Driving forces and resistance forces have the biggest influence on the law of mechanism motion. Their physical nature, magnitude and way of action are determined by those processes, which are in the machine during its work.

In most cases these forces change the magnitude depending on position of mechanism links and their velocities. Dependences, which show these functional relations, are called *mechanical properties of machines* and are given usually as diagrams or numbers arrays.

Mechanical properties of machines are considered assigned when solving tasks of dynamics. In future we will consider force and moment as positive, if they execute positive work at the assigned movements.

5.2.1. Properties of forces, which depend on velocity

In Fig. 5.9 there are mechanical properties of engines: *a* – asynchronous motor; *b* – DC motor. Decrease of driving moment with an increase of operating speed is characteristic for them.

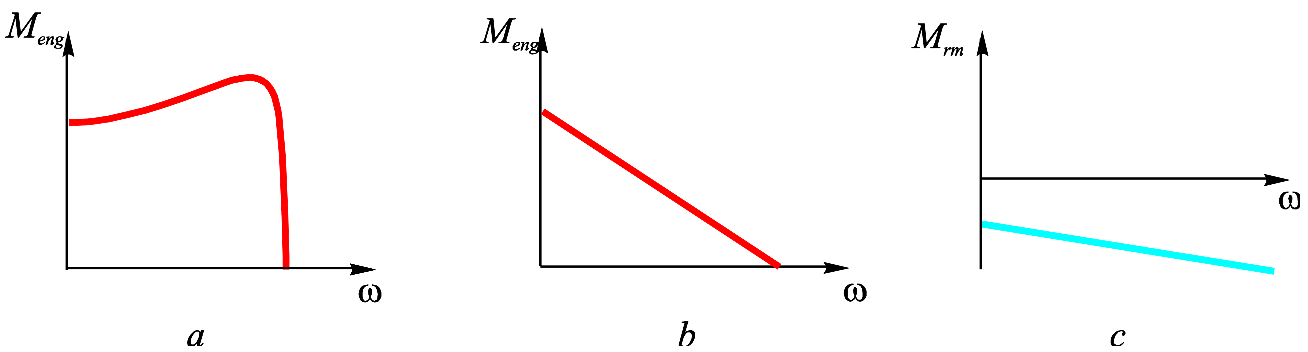


Fig. 5.9. Mechanical properties of machines, when forces depend on velocity:
a – asynchronous motor; *b* – DC motor; *c* – rotor machine

Vice versa, for rotor machines (generators, pumps, ventilators and others) increase of moment with an increase of turning velocity is characteristic (Fig. 5.9, *c*). Such combination is very useful, as it contributes to the constancy of the operations mode of aggregate electric motor - rotor machine. There is the self-regulation of the motion velocity.

5.2.2. Properties of forces, which depend on position

In Fig. 5.10 there is the mechanical property of a two-stroke internal combustion engine.

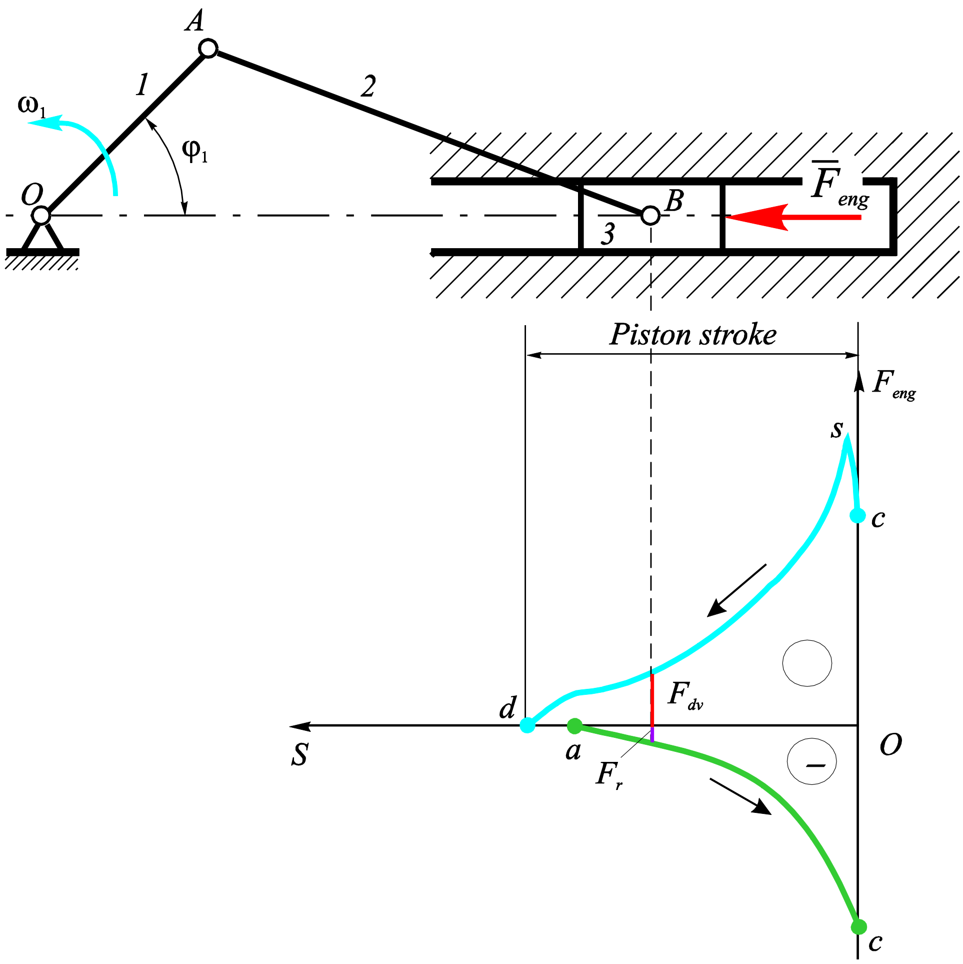


Fig. 5.10. Mechanical property of a push-pull combustion engine

Force F_{eng} in the segment *csd* carries out a positive work. This segment responds to the expansion of combustible mixture. The segment *db* responds to the piston return and exhaust. Here work is negative as the force F_{eng} is directed opposite to the motion direction.

Work is equal to the square under a curve. Fig. 5.10 shows that total work is positive (the positive square is bigger than the negative one). Consequently, F_{eng} is the driving force.

The Fig. 5.11 shows mechanical properties of electric engines (*a*) and rotor machines (*b*). As it is shown, moments do not depend on the rotor position, that is, on its angle of turn.

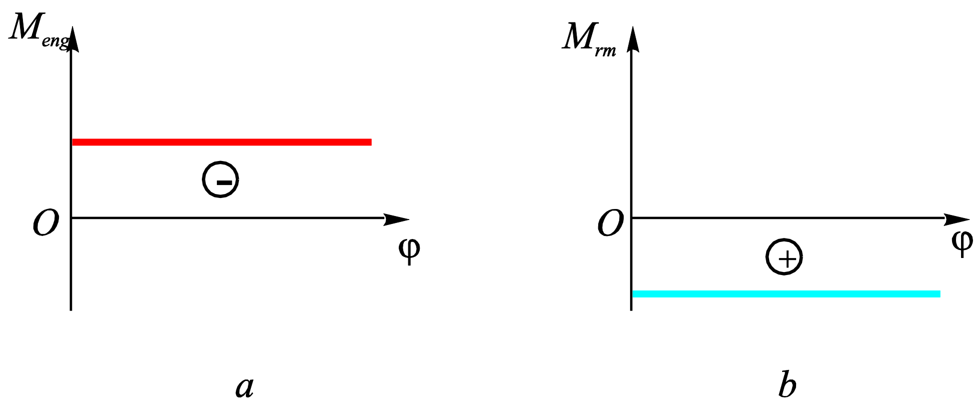


Fig. 5.11. Mechanical properties of machines, when forces depend on position: *a* – electric engines; *b* – rotor machines

5.3. THE DYNAMIC MODEL OF A MACHINE AGGREGATE. REDUCTION OF FORCES AND MASSES

Let us study the simplest machine aggregate: combustion engine and rotor machine with the intermediate gearing (Fig. 5.12).

Its motion freedom is $w = 3 \cdot 4 - 2 \cdot 5 - 1 = 1$.

At kinematic analysis it was enough to know the law of motion of any link (usually crankshaft *OB*) to explore all mechanism in the whole. This link is taken as an initial one.

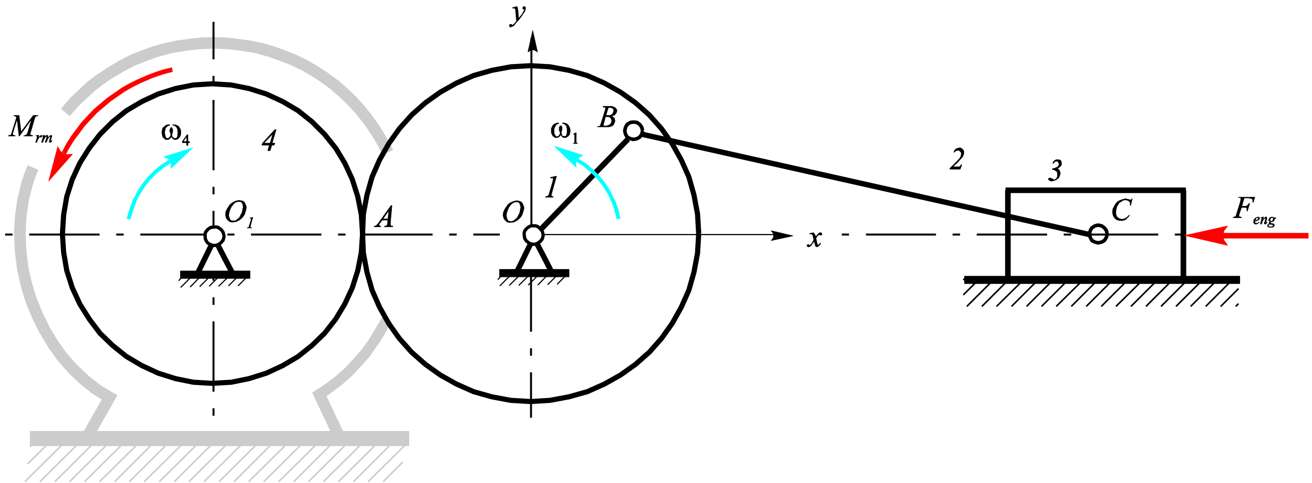


Fig. 5.12. Machine aggregate: combustion engine and rotor machine with the gearing

At the dynamic analysis the same approach is taken: the whole mechanism is substituted by the simplest model, for example by a turning link with the inertia moment I_{Σ}^{rd}, to which the moment M_{Σ}^{rd} is imposed, and at these moments the laws of motion of the model conditional link and the initial link of the real mechanism coincide:

$$\omega_1 = \omega_M . \tag{5.3}$$

Thus, when we construct the dynamic model, all forces, which act on a mechanism, are reduced to one link and replaced by some generalized force, called *total reduced moment* (or *force*). Thus the link of reduction has such *reduced mass* that its inertia is equal to inertia of the whole mechanism.

5.3.1. Reduction of forces

As it follows from Lagrange's equation (the second type), for the realization of condition (5.3), it is necessary, that the condition of equality of elementary work is maintained at the reduction of forces.

Force is called reduced to the point of the mechanism, if it, being applied at a point and directed in a motion direction of this point on tangent to the motion path, delivers the same power, as all forces acting on the mechanism.

$$P_{rd} = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \left(F_i V_i \cos\alpha_i + M_i \omega_i \right).$$

Here P_{rd} – is power, which is developed by the reduced force; α_i – is angle between the vector of force F_i and velocity vector V_i of point of attack of this force; M_i – is moment on *i-th* link, which turns with the velocity ω_i.

At a turning datum link (Fig. 5.13, *a*)

$$P_{rd} = M_{rd} \cdot \omega_1 .$$

At a sliding datum link (Fig. 5.13, *b*)

$$P_{rd} = F_{rd} \cdot V_1 .$$

Here ω_1 and V_1 are respectively angular or linear velocity of datum links.

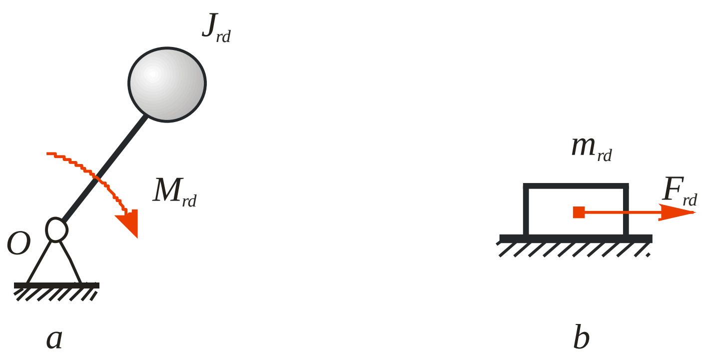


Fig. 5.13. Datum links: *a* – turning link; *b* – sliding link

So, the reduced moment acting on the datum link

$$M_{rd} = \sum_{i=1}^{n} \left(F_i \cdot \frac{V_i}{\omega_1} \cos\alpha_i + M_i \frac{\omega_i}{\omega_1} \right) \tag{5.4}$$

or reduced force

$$F_{rd} = \sum_{i=1}^{n} \left(F_i \cdot \frac{V_i}{V_1} \cos\alpha_i + M_i \frac{\omega_i}{V_1} \right). \tag{5.5}$$

5.3.2. Graphic method of definition of the reduced force (Zhukovsky lever)

According to the principle of virtual displacements in the mechanical system with nonreleasing constraints, sum of infinitesimal work of all forces, including inertia, in virtual displacements equals to zero:

$$\sum_{i=1}^{n} F_i \delta p_i = 0. \tag{5.6}$$

If we consider a mechanism as a mechanical system, constraints in which does not depend on time, so as a result of the fact that, at the assigned motion of initial links, others execute fully certain motions, the virtual displacements contain actual ones here. That is

$$\sum_{i=1}^{n} F_i dp_i = 0$$

or

$$F_1 dp_1 + F_2 dp_2 + \ldots + F_n dp_n = 0.$$

Here $dp_1, dp_2, \ldots, dp_n$ are actual displacement projections on direction of the imposed forces.

Let us study the link *AB*, to which force F_i is imposed in a point *S* (Fig. 5.14, *a*). Indeed, the infinitesimal displacement dS_i of point *S* has direction of velocity V_S.

Infinitesimal work of force F_i

$$dA_i = F_i dp_i = F_i ds_i \cdot \cos \alpha_i.$$

As $V_S = \dfrac{ds_i}{dt}$,

$$dA_i = F_i V_S \cdot \cos \alpha_i dt.$$

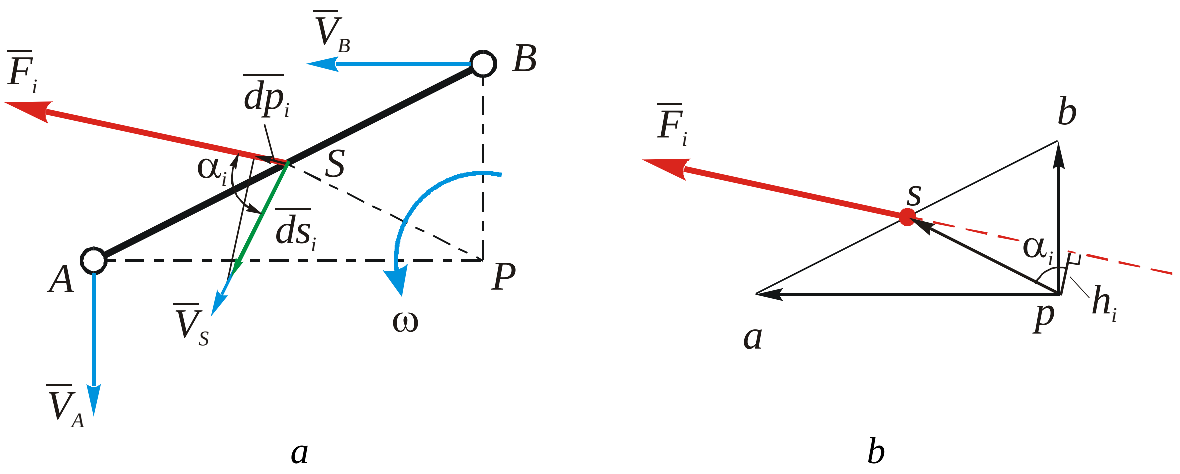


Fig. 5.14. Zhukovsky lever: *a* – link scheme; *b* – turned velocity diagram

Velocity V_S is defined from the velocity diagram. For this purpose we construct a diagram turned on 90º (Fig. 5.14, *b*) in a scale

$$\mu_V = \frac{V_A}{pa} = \frac{V_S}{ps}, \quad \left(\frac{m \cdot s^{-1}}{mm}\right).$$

From the diagram $V_S = ps \cdot \mu_V$. Then

$$dA_i = F_i \mu_V \cdot ps \cdot \cos\alpha_i \cdot dt\,. \qquad (5.7)$$

Let us replace force F_i parallel to itself on the turned diagram to the point *S*. Angle between the segment *ps* and perpendicular h_i assigned from the pole of diagram on centre line of thrust (the line of action of force) F_i, is equal to α_i. That is

$$h_i = ps \cdot \cos\alpha_i\,.$$

Then the expression (5.7) can be written as:

$$dA_i = F_i h_i \mu_V dt\,.$$

In the right part of this equation we have the expression for the moment of force F_i relatively to the pole of the velocity diagram, if it is considered as a hard lever.

Infinitesimal work of force that acts on a link of a mechanism is proportional to the moment, relatively to the pole of the velocity diagram of the same force, which is transferred to the proper point of the diagram turned on 90º.

$$dA_i = M_p\left(F_i\right)\mu_V dt .$$

As the common factor is $\mu_V dt \neq 0$, equation (5.6) can be rewritten as:

$$\sum_{i=1}^{n} M_p\left(F_i\right) = 0 .$$

If except for force F_i, the moments M_i are imposed to the mechanism, they should be replaced by the force couples imposed in some points of the link.

Example 5.1. To define the reduced moment acting on the link 1 of the pin-jointed four-bar linkage (Fig. 5.15).

According to the definition, the reduced moment, imposed to the turning link 1 is

$$M_{rd} = \sum_{i=1}^{n} M_{rd}\left(F_i\right). \tag{5.8}$$

We construct the velocity diagram of the mechanism, by turning it on 90º.

$$V_A = \omega_1 \cdot l_{OA}; \quad pa = \frac{V_A}{\mu_V}.$$

Scale of the velocity diagram is $\mu_V\left(\frac{m \cdot s^{-1}}{mm}\right)$.

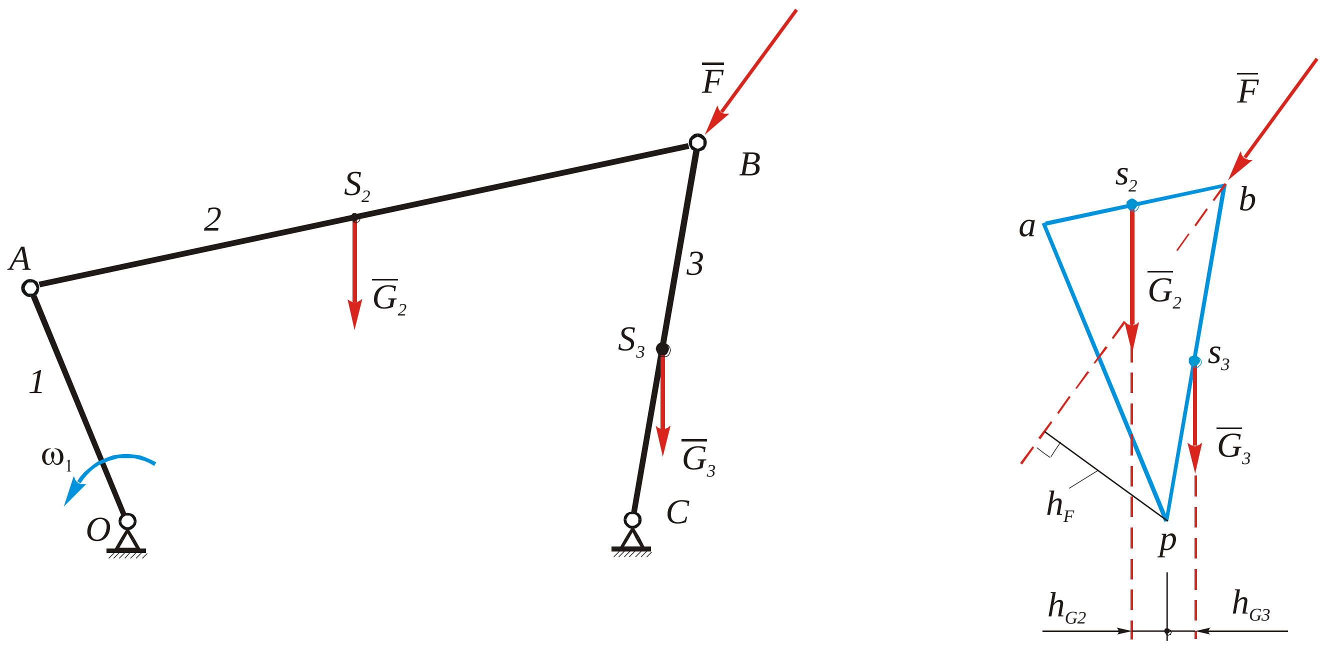


Fig. 5.15. Zhukovsky lever for rocker-and-crank mechanism

Then we transfer forces parallel to themselves to proper points on the diagram, we consider the diagram as a hard lever. According to (5.8)

$$M_{rd_p} = F \cdot h_F + G_2 \cdot h_{G_2} - G_3 \cdot h_{G_3}.$$

The reduced moment, applied to the first link of the mechanism, will be defined as

$$M_{rd} = k \cdot M_{rd_p}.$$

Here k – coefficient of a scale matching:

$$k = \frac{l_{OA}}{pa}.$$

Thus, the reduced moment is equal to the sum of moments of all assigned forces relatively to the pole of the velocity diagram.

It is not difficult to solve equations (5.4) and (5.5) analytically, if we know properties of forces, which act on the links of the mechanism and velocity analogues of links and velocity ratios (V_i/ω_1 is the velocity analogue of the *i*-th link, and ω_i/ω_1 is velocity ratio between *i*-th and first links).

5.3.3. Reduction of masses

Mass is reduced to the link of the mechanism if the link with this mass has kinetic energy which is equal to the sum of kinetic energies of all links.

$$T_{rd} = \sum T_i .$$

For a turning link kinetic energy is determined by the formula

$$T = \frac{1}{2} I_S \omega^2 ;$$

for a slider –

$$T = \frac{1}{2} m V^2 .$$

Then, if the datum link produces translational motion, we will get:

$$\frac{1}{2} m_{rd} V_1^2 = \frac{1}{2} \sum_{i,j} \left[m_i V_i^2 + I_{S\,j} \omega_j^2 \right].$$

Hence the reduced mass is

$$m_{rd} = \sum_{i,j} \left[m_i \left(\frac{V_i}{V_1} \right)^2 + I_{S\,j} \left(\frac{\omega_j}{V_1} \right)^2 \right].$$

For the datum link, which produces turning motion, the reduced inertia moment is

$$I_{S\,rd} = \sum_{i,j} \left[m_i \left(\frac{V_i}{\omega_1} \right)^2 + I_{S\,j} \left(\frac{\omega_j}{\omega_1} \right)^2 \right]. \tag{5.9}$$

Here i is the number of links, which produce translational motions, including transportation and relative translational motions of links at their two-dimensional motions; j is the number of links, which produce turning motions including transportation and relative turning motions at their two-dimensional motion.

Example 5.2. To define the reduced inertia moment of the link 1 of the geared linkage mechanism with motion freedom w=1 (Fig. 5.16).

The velocity diagram of a mechanism is represented in Fig. 5.16, b. In Fig. 5.16 c is represented the dynamic model of this mechanism.

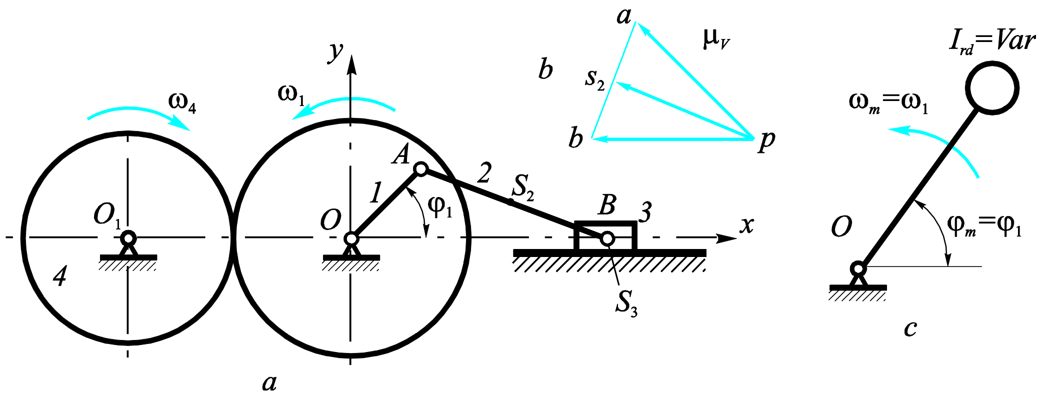


Fig. 5.16. The geared linkage mechanism: *a* – kinematic scheme;
b – the velocity diagram; *c* – the dynamic model

As the datum link produces turning motion, we will define the value of the reduced inertia moment by the formula (5.9):

$$I_{rd}=\left[I_{S_1}\left(\frac{\omega_1}{\omega_1}\right)^2+\left\{m_2\left(\frac{V_{S2}}{\omega_1}\right)^2+I_{S_2}\left(\frac{\omega_2}{\omega_1}\right)^2\right\}+m_3\left(\frac{V_B}{\omega_1}\right)^2+I_{S_4}\left(\frac{\omega_4}{\omega_1}\right)^2\right].$$

Let us convert the equation:

$$I_{rd}=I_{S_1}+\left[m_2 l_{OA}^2\left(\frac{V_{S_2}}{V_A}\right)^2+I_{S_2}\left(\frac{l_{OA}}{l_{AB}}\right)^2\left(\frac{V_{BA}}{V_A}\right)^2\right]+m_3 l_{OA}^2\left(\frac{V_B}{V_A}\right)^2+I_{S_4}u_{41}^2.$$

It is not difficult to solve this equation, when we know the sizes of the links and can determine velocities according to the velocity diagram.

It is significant that, as actual velocities in a mechanism coincide with virtual ones, it is not necessary to know the datum link motion law in Zhukovsky method when determining the reduced force, as well as determining the reduced mass or inertia moment. That is, when having the kinematic scheme of a mechanism, it is possible to construct its dynamic model, by carry-out the reduction of forces and masses, and after it we can find its motion law.

5.4. EQUATIONS OF A MECHANISM MOTION

Let us study the dynamic model of a mechanism with motion freedom w=1 (Fig. 5.17).

According to the *law of conservation of energy*, it is possible to write down:

$$T - T_0 = \sum A. \tag{5.10}$$

Here T is kinetic energy of a mechanism at the instant moment; T_0 is initial kinetic energy; $\sum A$ is a total work, which is executed by all active forces and frictions in kinematic pairs.

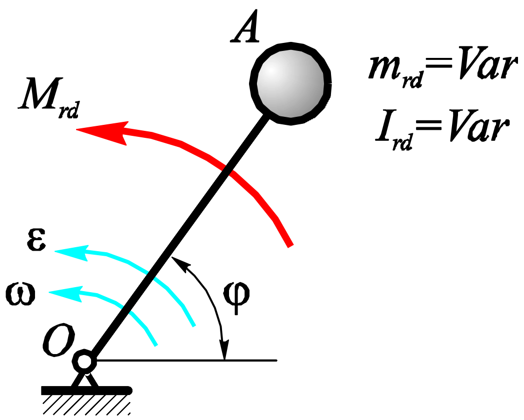


Fig. 5.17 The dynamic model of a mechanism with motion freedom w=1

5.4.1. Equation of motion in an integrated form

For a model (Fig. 5.17)

$$\sum A = \int_{\varphi_0}^{\varphi} M_{rd}\, d\varphi;$$

$$\Delta T = \frac{I_{rd}\omega^2}{2} - \frac{I_{rd\,0}\omega_0^2}{2}.$$

Thus

$$\frac{I_{rd}\omega^2}{2} - \frac{I_{rd}\omega_0^2}{2} = \int_{\varphi_o}^{\varphi} M_{зв} d\varphi \,. \tag{5.11}$$

Higher limit of integral φ is variable in general case. If forces depend only on a position of a mechanism, M_{rd} is the function of the only generalized coordinate φ. Solving this equation relatively ω, we get:

$$\omega = \sqrt{\frac{2\int_{\varphi_0}^{\varphi} M_{rd}(\varphi)\, d\varphi}{I_{rd}} + \frac{I_{rd\,0}}{I_{rd}}\omega_0^2} \,.$$

$M_{rd}(\varphi)$ should be taken with the consideration of a sign.

5.4.2. Equation of motion in a differential form

Let us differentiate an equation:

$$\frac{d}{d\varphi}\left(\frac{I_{rd}\omega^2}{2}\right) = M_{rd} \,;$$

$$\frac{d}{d\varphi}\left(\frac{I_{rd}\omega^2}{2}\right) = \frac{dI_{rd}}{d\varphi}\frac{\omega^2}{2} + \omega I_{rd}\frac{d\omega}{d\varphi}\frac{dt}{dt} = I_{rd}\frac{d\omega}{dt} + \frac{1}{2}\frac{dI_{rd}}{d\varphi}\omega^2 \,.$$

Thus,

$$I_{rd}\frac{d\omega}{dt} + \frac{1}{2}\frac{dI_{rd}}{d\varphi}\omega^2 = M_{rd} \,. \tag{5.12}$$

This is the equation of motion in a differential form, as the decision variable ω is under the derivative sign.

In this equation M_{rd} and I_{rd} should be taken with the consideration of their signs.

For mechanisms, which have $I_{rd} = Const$, for example friction mechanisms or gearings with round wheels, equation (5.12) should be simplified:

$$I_{rd} \frac{d\omega}{dt} = M_{rd} . \tag{5.13}$$

Solving the equation (5.12) relatively to ω, we can find angular acceleration of reduced link:

$$\varepsilon = \frac{M_{rd}}{I_{rd}} - \frac{\omega^2}{2I_{rd}} \frac{dI_{rd}}{d\varphi} .$$

5.5. LAW OF A MECHANISM MOTION IN THE TRANSIENT REGIME

5.5.1. The concept of transient regime

The motion process of a machine aggregate in general case consists of three phases: *start or run, a steady state mode and stop or running-out.*

Start and stop relates to *a transient regime*, which is characterized by no periodic changes of velocity of the main aggregate shaft (initial link). In the steady state mode these fluctuations are either periodic or completely absent.

In order to define the law of a machine aggregate motion in the transient regime we should know:

- kinematics of a mechanism;
- properties of geometric masses of movable links;
- mechanical properties of forces and moments;
- initial motion conditions.

Further we will study several typical examples of definition of laws of mechanism motion in the transient regime.

5.5.2. Examples of dynamic study of machine aggregates in the transient regime

Motion law of a mechanism, loaded by the forces, which depend on its position. Let us define the dependence of an angular velocity of the initial link on the angle of rotation if $I_{rd} = Var$. This is a typical example for rigs, diesel-compressors etc.

For solving the problem we use the motion equation in an integral form:

$$\omega = \sqrt{\frac{2\sum A}{J_{rd}} + \frac{J_{rd\,0}}{J_{rd}}\omega_0^2}\,. \tag{5.14}$$

The sequence of calculation can be traced on the example of a machine aggregate „internal combustion engine (ICE) – rotor machine (RM)" (Fig. 5.18) [2].

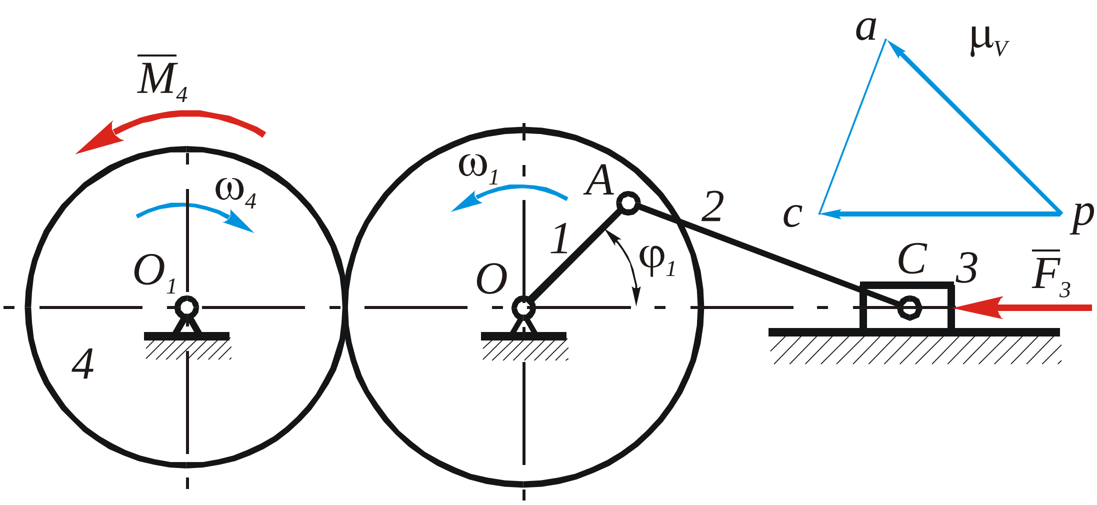


Fig. 5.18. Scheme of a machine aggregate ICE– RM

1. We reduce the masses to the initial link 1, and examine a dynamic model with motion freedom *w*=1 (Fig. 5.17). In Fig. 5.19, *a* the diagram of change I_{rd} in an interval $\varphi_0 - \varphi$ is represented. We accept $\varphi_0 = 0$.

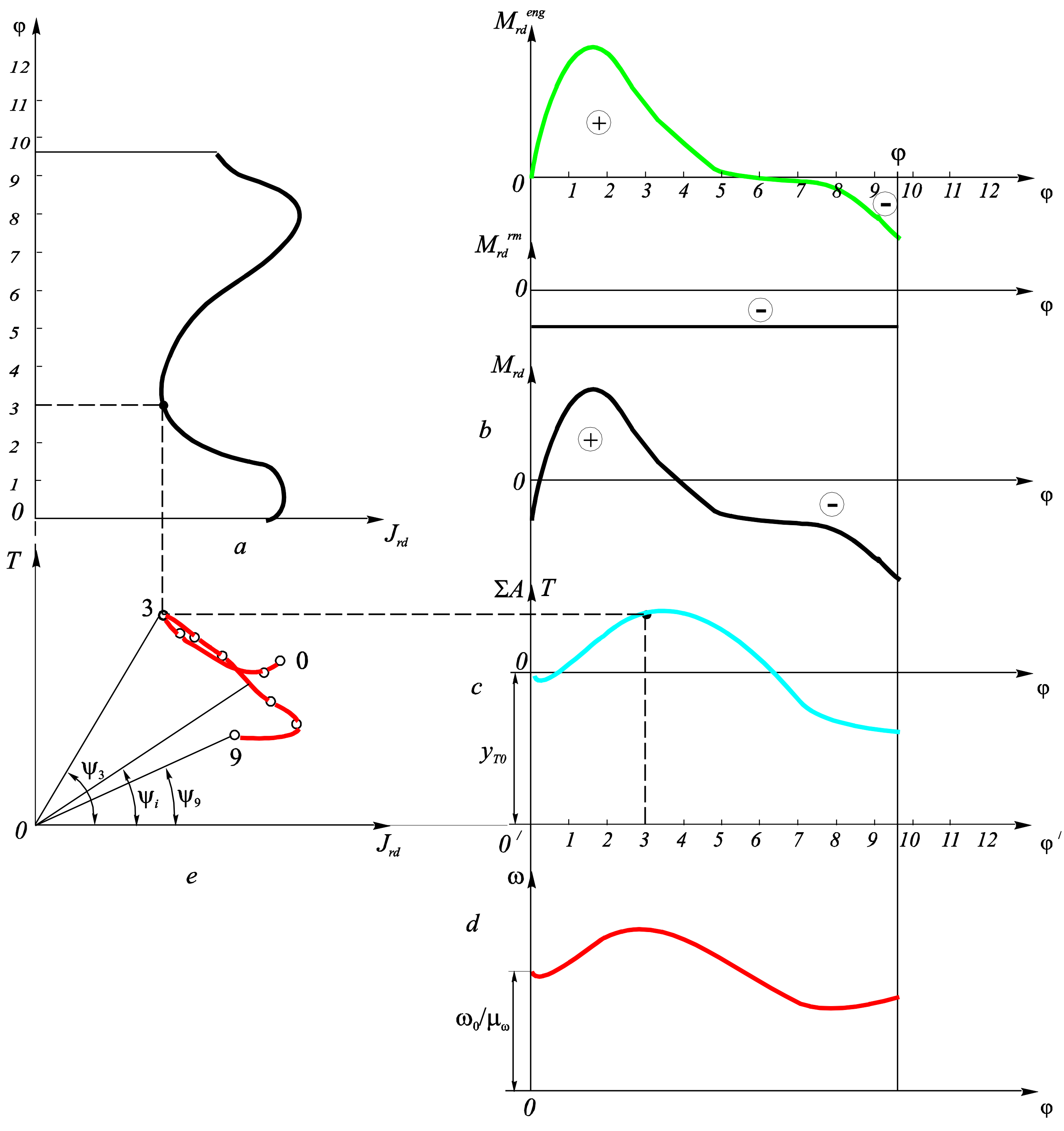


Fig. 5.19. Diagrams for the machine aggregate ICE– RM: *a* – diagram of I_{rd};
b – diagrams of moments; *c* – the diagram of total work;
d – diagram of velocity; *e* – Wittenbauer's diagram

2. According to mechanical properties we construct diagrams of a reduced moment. For this purpose, ignoring gravities, frictions and inertias, we assume that only force $\bar{F}_{eng} = \bar{F}_3$ and $\bar{M}_4 = \bar{M}_{RM}$ load on a mechanism. (It should be noted that in most cases such supposition is improper).

As the link 1 is turning, so the reduced driving moment is $M_{rd}^{driv} = M_{rd}^{eng}$:

$$M_{rd}^{eng}(\varphi) = F_3(\varphi)\frac{V_C(\varphi)}{\omega_1} = F_3(\varphi) l_{OA}\frac{V_C(\varphi)}{V_A}.$$

If $\varphi = 0$ and $\varphi = 2\pi$ $V_C = 0$.

Reduced moment of resisting forces $M_{rd}^{res} = M_{rd}^{RM}$:

$$M_{rd}^{RM}(\varphi) = M_4^{(\varphi)}\frac{\omega_4}{\omega_1}.$$

Summarized reduced moment:

$$M_{rd} = M_{rd}^{driv} + M_{rd}^{res} = M_{rd}^{eng} + M_{rd}^{RM}.$$

That means that two overhead diagrams in Fig. 5.19, *b* are added. The first graph is constructed according to the mechanical property F_{eng} of ICE, and the second one is constructed according to the mechanical property of a rotor machine.

3. Graphically we integrate the obtained curve $M_{rd} - \varphi$ and construct the diagram of total work (Fig. 5.19, *c*) $\sum A(\varphi)$.

4. We displace an axis φ on a diagram $\sum A(\varphi)$ on a size $y_{T0} = \mu_A T_0$ (Fig. 5.19, *c*), where $T_0 = I_{rd0}\omega_0^2/2$. Then ordinates which we count off from a new, displaced axis φ', will represent the current values of kinetic energy T in different positions of the mechanism (See equation (5.10)).

5. By the obtained values I_{rd} and M_{rd} for the proper values φ we determine $\omega(\varphi)$ taking into account initial conditions. $\sum A$ is put in (5.14) with its sign. Initial velocity ω_0 is set and is represented in Fig. 5.19, *d* by the segment of standing axis ω_0/μ_ω. Value I_0 is a value I_{rd} if $\varphi = \varphi_0$ (Fig. 5.19, *a*).

We can obtain an evident picture of change of angular velocity of the initial link by using the graphics for Wittenbauer's analysis. In order to obtain it, we should exclude parameter φ from diagrams $I_{rd}-\varphi$ and $T-\varphi$. For the initial link we can write down:

$$T=\frac{I_{rd}\omega_1^2}{2};$$

$$\omega_1=\sqrt{\frac{2T}{I_{rd}}}.$$

We study any position of the mechanism, for example position *3*.

$$T^{(3)}=y_T^{(3)}\cdot\mu_T; \qquad I_{rd}^3=y_I^{(3)}\cdot\mu_J.$$

Then

$$\omega_1^{(3)}=\sqrt{\frac{2y_T^{(3)}\mu_T}{y_I^{(3)}\mu_J}}=\sqrt{\frac{2\mu_T}{\mu_J}}\sqrt{\mathrm{tg}\psi_3}.$$

Thus ω_1 is proportional to $\mathrm{tg}\psi$, so passing from point to point on the graphics for Wittenbauer's analysis, we can explore the change of angular velocity in the transient regime.

Motion law of a mechanism, loaded with velocity-dependent forces. This case is typical for machine aggregates including electric engines and rotor machines [2]. Unlike the previous example, reduced inertia moment here has a constant value (all links are turning).

For the analysis it is convenient to use the motion equation in differential form (See formula (5.13)).

$$J_{rd}\frac{d\omega}{dt}=M_{rd}. \qquad (5.15)$$

Assumed that $t_0 = 0$, we will convert equation into

$$t = I_{rd} \int_{\omega_0}^{\omega} \frac{d\omega}{M_{зв}}.$$

As an example let us study the machine aggregate of a turbogenerator, which takes a run from stationary state ($\omega_0 = 0$).

In Fig. 5.20 there are mechanical properties of driving force (on turbine shaft) $M_т$ and resistance forces (on generator shaft) M_g, as well as summary reduced moment diagram M_{rd}.

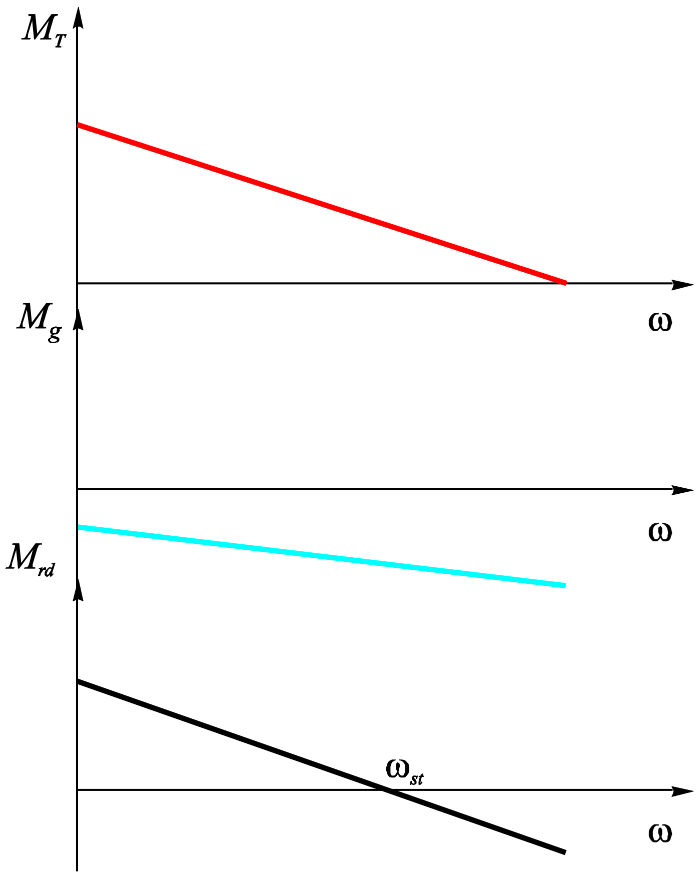


Fig. 5.20. Mechanical properties diagrams

The equation of the obtained straight is:

$$M_{rd} = M_{rd0} - B\omega. \tag{5.16}$$

Then

$$t = I_{rd} \int_0^{\omega} \frac{d\omega}{M_{rd\,0} - B\omega}. \tag{5.17}$$

Solution of the equation is:

$$\omega = \omega_0 \left(1 - e^{-\frac{t}{T}} \right). \tag{5.18}$$

Here

$$\omega_0 = \frac{M_{rd\,0}}{B}; \tag{5.19}$$

$$T = \frac{I_{rd}}{B}. \tag{5.20}$$

T is *response time of the machine aggregate*.

In Fig. 5.21 there is the diagram of the curve described by the equation (5.18). Here $ab = T$.

If $M_{rd} = Const$, than having $I_{rd} = Const$, according to the motion equation (5.15) we would have $\frac{d\omega}{dt} = Const$. Thus the mechanism motion would be uniformly accelerated. In this case we can write:

$$\omega = \frac{M_{rd}}{I_{rd}} t. \tag{5.21}$$

It is the equation of the straight $n - n$ (Fig. 5.21).

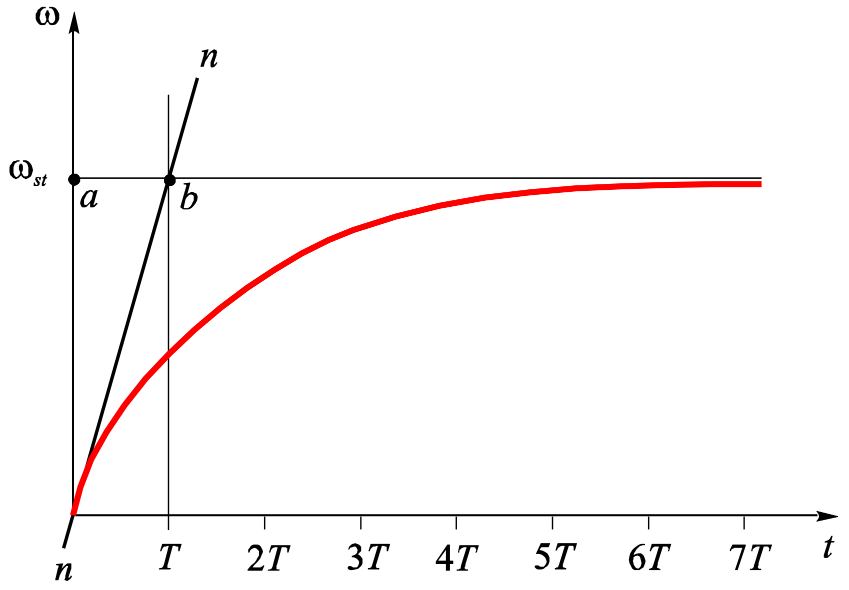


Fig. 5.21. The diagram of velocity-time dependence

Putting (5.19) into (5.20), we obtain correlation:

$$T = \frac{I_{rd}}{B} = \frac{I_{rd}\omega_0}{M_{rd}}.$$

Hence

$$\frac{M_{rd}}{I_{rd}} = \frac{\omega_0}{T}.$$

Then the equation (5.21) looks like:

$$\omega = \frac{\omega_0}{T} t,$$

that is start would continue endlessly, and ω_0 would be obtained when $t = T$. Though reduced moment $M_{rd} \neq Const$, and, according to (5.16), it decreases with the increase of velosity ω. In real turbogenerators angular velocity equals to $\omega \approx 0{,}995\omega_0$ for the time $t = 5T$.

Thus, response time T allows determining the run duration. The higher the persistence of the aggregate is, the higher T will be, according to (5.20), and so the run will last longer.

It was the solution of the direct problem. We can also solve the inverse problem. If we know necessary actuation time of the mechanism t^* (run-up time), we can find mechanism properties (inertia moments of links and their sizes).

If the aggregate consists of asynchronous engine, then it is difficult to solve this problem analytically, as it is hard to approximate the curve $M_{rd}(\omega)$ (characteristics of the asynchronous engine – see Fig. 5.9, *a*). In such case the equation (5.17) is solved graphically or by means of numerical integration on PC.

Motion law of the mechanism, loaded by the forces, dependant both on velocity and position. Such mode is typical for cutting machines, forges, aggregate "starter - combustion engine", devices with a magnetic drive (for example relay) etc.

This problem is solved by using the motion equation in the integrated form:

$$\frac{I_{rd}\omega^2}{2} - \frac{I_{rd0}\omega_0^2}{2} = \sum A.$$

When solving this equation, work of forces dependant on position separate from work of forces dependant on velocity. So the reduction of these forces is conducted separately.

In Fig. 5.22 there is the kinematic scheme of a quick-return link mechanism of a planer with a drive, which includes asynchronous engine [2].

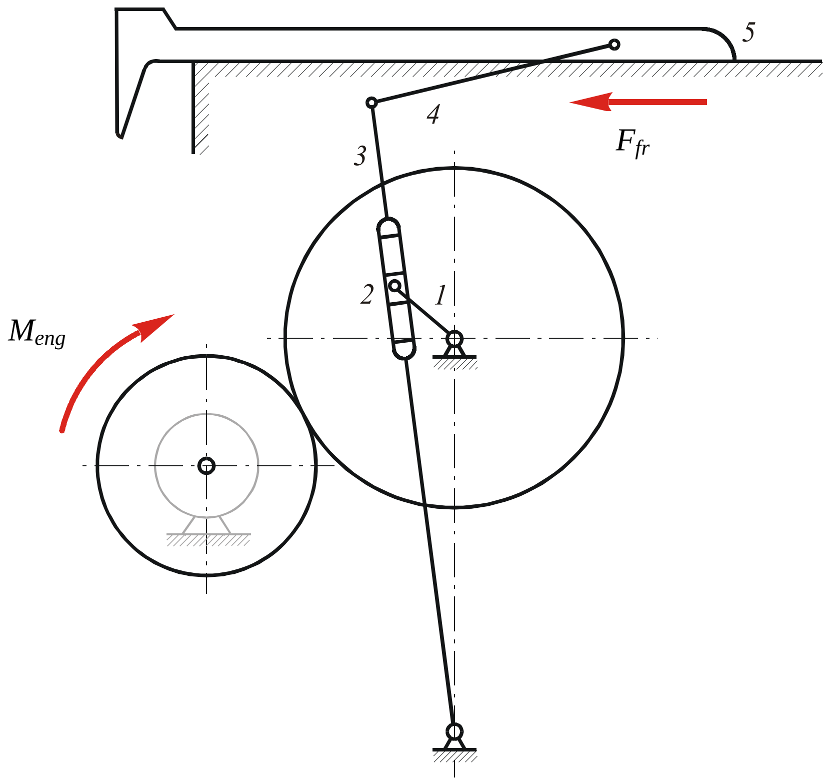


Fig. 5.22. The scheme of aggregate

The start (transient regime) is conducted in the idle mode. That is the mechanism is affected only by the friction in a slider 5 and the driving moment of the asynchronous engine. Their mechanical properties are shown in Fig. 5.23.

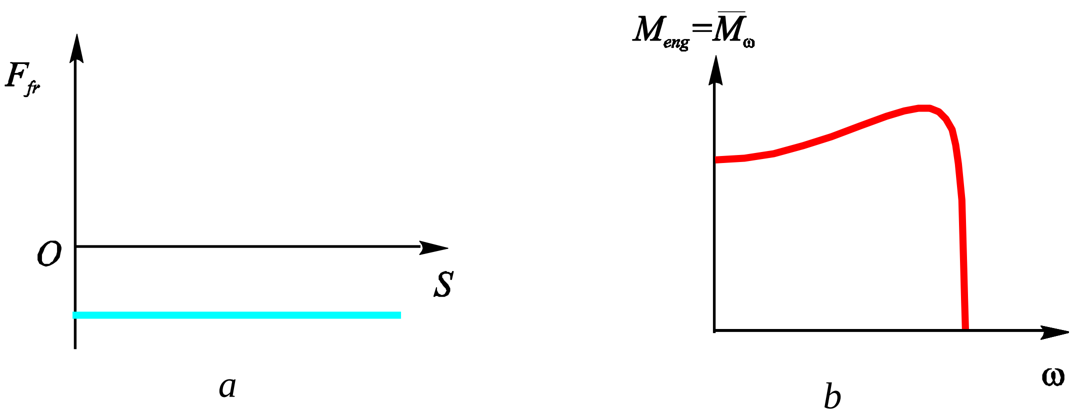


Fig. 5.23. Mechanical properties diagrams: *a* – the friction; *b* – the driving moment

Let us take the link 1 as the datum link. The diagram of the reduced inertia moment I_{rd} is shown in Fig. 5.24. In Fig. 5.25 there is the diagram of the reduced moment of the frictions M_{φ}.

Motion equation is:

$$\frac{I_{rd}\omega^2}{2} - \frac{I_{rd_0}\omega_0^2}{2} = A_{\varphi} + A_{\omega}. \qquad (5.22)$$

As at the run there are changes both in velocity and rotation angle, and we do not know how, in accordance to the rotation angle, the velocity changes, so we solve the equation (5.22), dividing the rotation angle φ into small intervals. And we solve the equation for each interval.

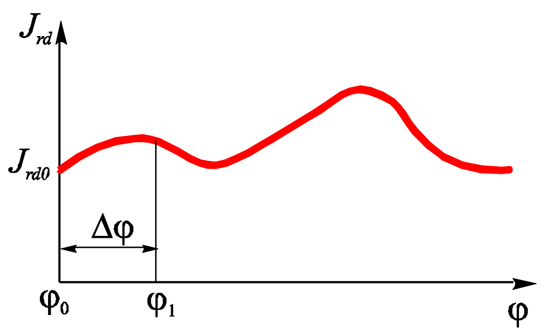


Fig. 5.24. The diagram of I_{rd}

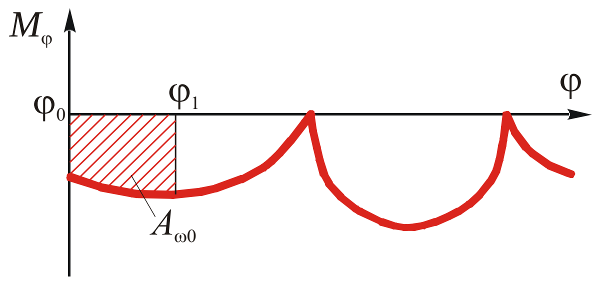


Fig. 5.25. The diagram of the reduced moment of the frictions M_φ

It is easy to find values for the zero position $M_{\omega 0}, \omega_0, I_{rd\,0}$ (See Fig. 5.23, *b* and 5.24). For the first position $\varphi_1 = \varphi_0 + \Delta\varphi$. For this interval the motion equation is:

$$\frac{I_{rd\,1}\omega_1^2}{2} - \frac{I_{rd\,0}\omega_0^2}{2} = A_{\varphi_{01}} + A_{\omega_{01}}.$$

Here I_{rd_1} will be obtained from the diagram (Fig. 5.24); $A_{\varphi 01}$ will be obtained by integrating the curve $M_\varphi(\varphi)$ in this interval (Fig. 5.25). The value $A_{\omega 01}$ cannot be obtained, as we do not know the law of changing of M_ω and ω by rotation angle. But according to the smallness of the interval, we assume that M_ω changes according to the linear law in the interval $\varphi_0 - \varphi_1$ (Fig. 5.26).

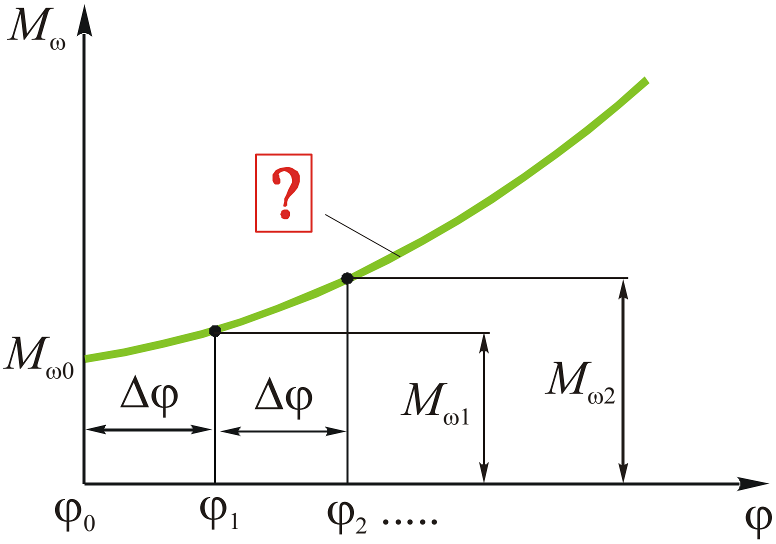


Fig. 5.26. The curve $M_\varphi(\varphi)$

Then

$$A_{\omega01} \cong \frac{M_{\omega0} + M_{\omega1}}{2} \Delta\varphi. \tag{5.23}$$

Putting (5.23) into (5.22):

$$\frac{I_{rd1}\omega_1^2}{2} - T_0 = A_{\varphi01} + \frac{M_{\omega0} + M_{\omega1}}{2} \Delta\varphi.$$

Hence

$$\frac{I_{rd1}\omega_1^2}{\Delta\varphi} - \left(\frac{2T_0}{\Delta\varphi} + M_{\omega0} + \frac{2A_{\varphi01}}{\Delta\varphi} \right) = M_{\omega1}. \tag{5.53}$$

Let us mark:

$$\frac{2T_0}{\Delta\varphi} + M_{\omega0} + \frac{2A_{\varphi01}}{\Delta\varphi} = B_{01};$$

$$\frac{I_{rd1}\omega_1^2}{\Delta\varphi} - B_{01} = M_{\omega1}. \tag{5.24}$$

In our example $A_{\varphi 0} < 0$, $T_0 = 0$, as $\omega_0 = 0$.

In the equation (5.24) $M_{\omega 1}$ and ω_1 are unknown, but they are connected by correlation (Fig. 5.23, *b*). If this correlation is given in the analytical form, than we get the system of equations relatively to $M_{\omega 1}$ and ω_1. If the diagram of mechanical properties $M_{\omega 1}$ is given (Fig. 5.23, *b*), so we construct the diagram of function $F_{01}(\omega) = \dfrac{I_{rd1}}{\Delta\varphi}\omega^2 - B_{01}$, imposing it on a mechanical property diagram (Fig. 5.27).

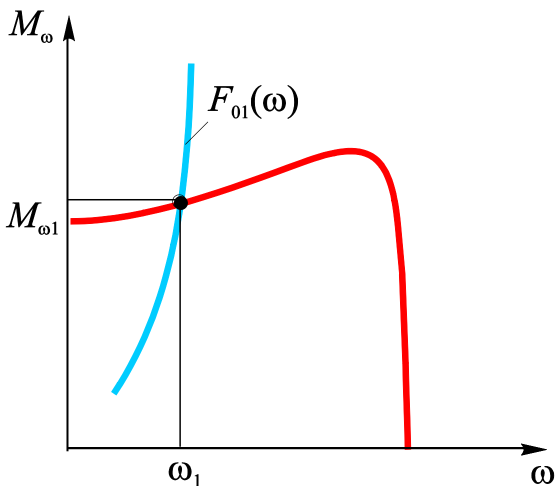


Fig. 5.27. Graphical solution in 0-1 interval

For the second interval φ_{1-2} the solving equation looks like:

$$\frac{J_{rd2}\omega_2^2}{\Delta\varphi} - B_{12} = M_{\omega 2},$$

where

$$\frac{2T_1}{\Delta\varphi} + M_{\omega 1} + \frac{2A_{\varphi 12}}{\Delta\varphi} = B_{12}.$$

A new equation is solved relatively to ω_2 using the same method as for ω_1.

Consistently passing by all intervals of angles φ etc., we obtain the diagram of law of velocity changing $\omega = \omega(\varphi)$.

5.6. STEADY REGIME OF MECHANISM MOTION

5.6.1. The non-uniform motion of a mechanism

As well as for the transient regime, we will study mechanisms with motion freedom w=1.

Steady state mode is characterized by such features:

– velocity of the initial link – periodic function of time (Fig. 5.28);
– mass characteristics and forces, applied to the mechanism, are changed periodically;
– sum of works of all forces per cycle equals to zero:

$$\sum A_C = 0$$

or

$$A_{dr}^C = \left| A_{res}^C \right| .$$

In course of cycle there is no kinetic energy increment:

$$T_0 = T_{fin} .$$

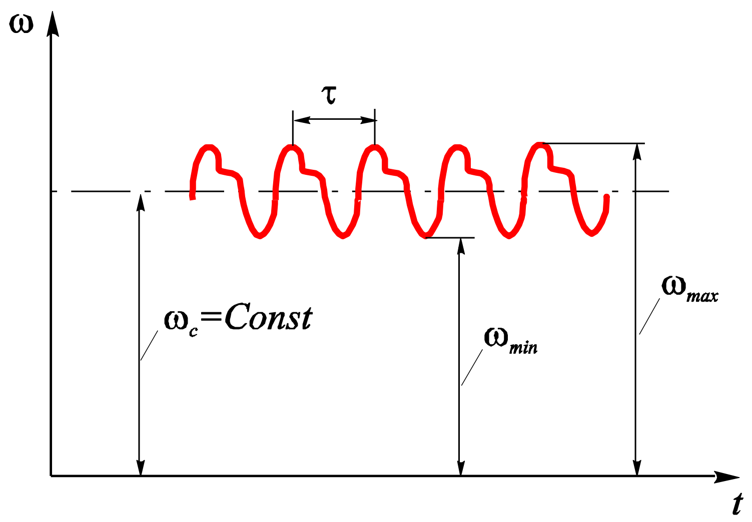


Fig. 5.28. The motion law at steady state mode of mechanism motion

So the angular velocity of the link both in the beginning and the end of the cycle is the same. The changing of ω takes place inside the cycle.

Irregularity of rotation is evaluated by the irregularity motion ratio:

$$\delta = \frac{\omega_{\max} - \omega_{\min}}{\omega_c} . \tag{5.25}$$

Here ω_c is an average velocity in cycle:

$$\omega_c \cong \frac{\omega_{\max} + \omega_{\min}}{2} . \tag{5.26}$$

The less the value δ is, the less oscillations of velocity are.

The majority of technological machines, generators of electric energy, compressors, pumps etc. work in a steady state mode.

Oscillations of velocity depend on link accelerations. They cause dynamic loading, which decreases machine lives, their kinematic accuracy etc. So the value δ should be limited:

$$\delta \le [\delta]. \tag{5.27}$$

For cutting machines $[\delta] = \frac{1}{25} - \frac{1}{50}$, diesel drives of generators $[\delta] = \frac{1}{100} - \frac{1}{200}$.

5.6.2. Determining of the irregularity motion ratio of a mechanism

For the machine aggregate let us reduce the masses of all links to the main shaft of the mechanism and divide the masses into two groups:

I group — masses with $I = Const = I_I$;

II group — masses with $I = Var = I_{II}$.

$$I_{rd} = I_I + I_{II} . \tag{5.28}$$

Irregularity of the mechanism motion takes place because of masses of the second group in it. Kinetic energy of masses of the first group:

$$T_I = \frac{1}{2} I_I \omega^2 .$$

As ω changes in the interval $\omega_{max} \leftrightarrow \omega_{min}$, kinetic energy T_I also changes:

$$\left(T_I\right)_{max} = \frac{1}{2} I_I \omega_{max}^2 ; \qquad \left(T_I\right)_{min} = \frac{1}{2} I_I \omega_{min}^2 .$$

Maximum kinetic energy difference, taking into account expressions (5.25) and (5.26), can be defined as

$$\Delta T_{I_{max}} = \frac{I_I \omega_{max}^2}{2} - \frac{I_I \omega_{min}^2}{2} = \frac{I_I}{2}\left(\omega_{max} + \omega_{min}\right)\left(\omega_{max} - \omega_{min}\right)\frac{\omega_c}{\omega_c} = \frac{I_I}{2} \delta \omega_c^2 .$$

So the condition (5.27) takes the form:

$$\delta = \frac{\Delta T_{I_{max}}}{\omega_c^2 I_I} \leq \left[\delta\right]. \qquad (5.29)$$

5.6.3. Ways of minimization of the irregularity motion ratio

There are three directions of the irregularity motion ratio minimization according to the condition (5.29):

– increase of moments of inertia of masses of the first group (increase of I_I);

– increase of an average velocity of motion ω_c;

– decrease of variation of kinetic energy of a mechanism by approaching in every moment of works of driving forces A_{dr} and resistance forces A_{res}.

Dynamic synthesis of a fly-wheel by Mertsalov's method. Using the equation (5.29), we define a moment of inertia of masses of the first group I_I (with a constant inertia moment), necessary for providing of a set value $[\delta]$:

$$I_I = \frac{\Delta T_{I_{\max}}}{\omega_c^2 [\delta]}. \tag{5.30}$$

This is dynamic synthesis equation of a fly-wheel under steady state mode. The control of I_I is provided by installation of a fly-wheel on machine drive shaft. Kinetic energy of the whole mechanism, in analogy with (5.28), can be defined as $T = T_I + T_{II}$. Hence

$$T_I = T - T_{II}.$$

Kinetic energy T is expressed from the equation $T - T_0 = \sum A$ or

$$T = \sum A + T_0.$$

Hence

$$T_I = \sum A + T_0 - T_{II}. \tag{5.31}$$

We construct the diagram $T_I(\varphi)$ for the full cycle and according to it we find ΔT_I.

Let us illustrate the method by the example. By the method known from the transient regime we construct a diagram of the work of reduced forces $\sum A = A_{dr} + A_{res}$ (Fig. 5.29, *a*).

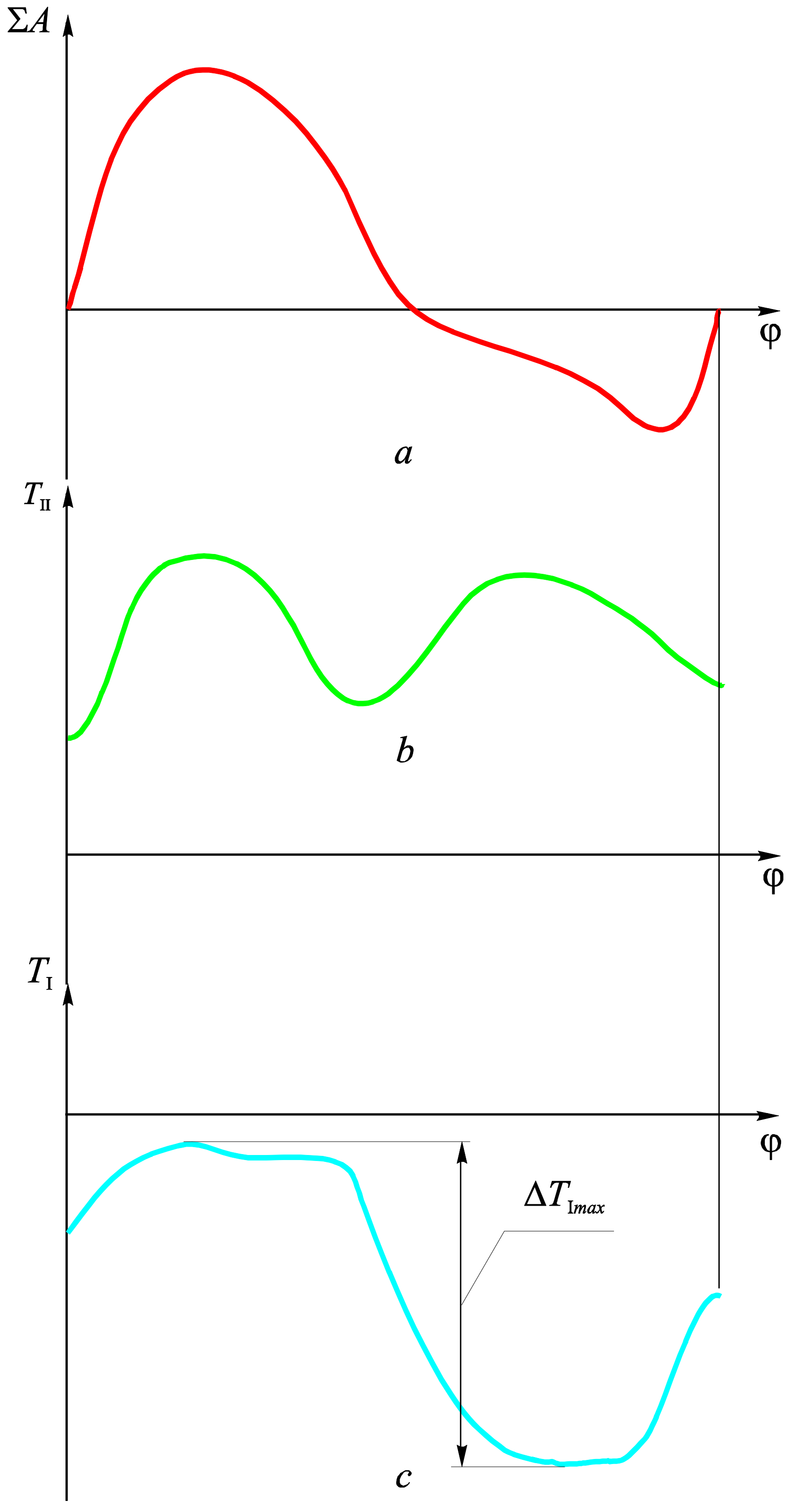


Fig. 5.29. The illustration on the Mertsalov's method realisation: *a* – the work of reduced forces; *b* – the diagram of masses kinetic energy of the second group; *c* – the diagram of kinetic energy variation for masses of the first group

The second summand in the equation (5.31) can be omitted, as it is cancelled when $\Delta T_{I_{\max}}$ defining.

Further we construct a diagram of masses kinetic energy of the second group (with a variable inertia moment) $T_{II}-\varphi$ (Fig. 5.29, *b*) by data, obtained from the equation:

$$T_{II}(\varphi) \approx \frac{1}{2} I_{II}(\varphi) \cdot \omega_c^2 .$$

Here I_{II} is inertia moment of masses of the second group.

Subtracting from the diagram of the work of reduced forces (Fig. 5.29, *a*) the diagram of masses kinetic energy of the second group (Fig. 5.29, *b*), according to the equation (5.31), we obtain the diagram of kinetic energy variation for masses of the first group (Fig. 5.29, *c*), and hence we define the maximum kinetic energy difference $\Delta T_{I_{\max}}$.

Further by the equation (5.30) we determine I_I. We should admit that necessary inertia moment of masses with the constant inertia moment I_I is much bigger than the reduced moment I_I^* of links that the mechanism consists of. So we may say that the obtained moment of inertia of masses is the moment of inertia of a fly-wheel

$$I_I \cong I_M ,$$

where I_M is the inertia moment of a fly-wheel.

What is the role of a fly-wheel in the structure of the machine aggregate? Let us consider it by an example of aggregate that consists of the combustion engine and generator. In the course of gas combustion by the engine it produces more energy than the generator uses and this energy is accumulated by the fly-wheel. At the exhaust the combustion engine takes energy, as at this stage force F conducts negative work. Energy T_I decreases, it means that the energy of the fly-wheel decreases.

So the fly-wheel either accumulates energy, when there is an excess of engine work, or gives its part. The higher an inertia moment of a fly-wheel I_M, the less a value δ.

Constructively the fly-wheel is produced as a disk (Fig. 5.30, *a*) or ring with spokes (Fig. 5.30, *b*). It is made of steel or iron.

For disk-shaped fly-wheels

$$I_M = \frac{mD_2}{8}.$$

Hence

$$D = \sqrt[4]{\frac{32 I_M}{\pi b \rho}},$$

where ρ is the specific weight.

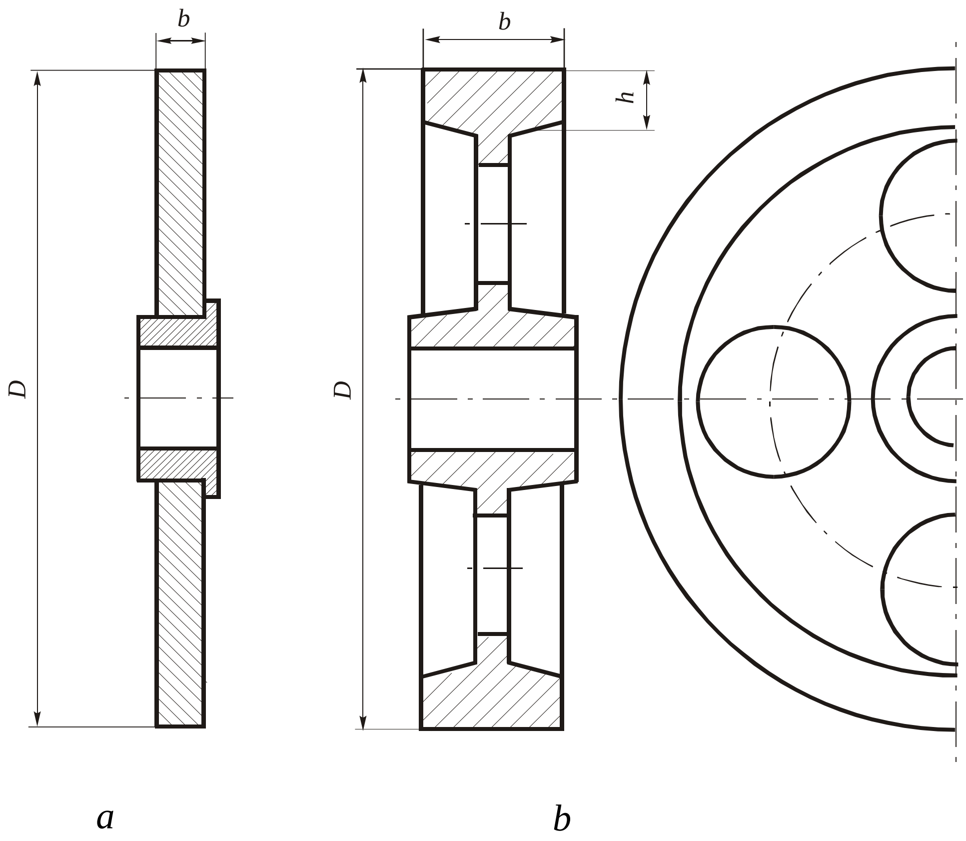


Fig. 5.30. The fly-wheel construction: *a* – disk-shaped fly-wheel; *b* – fly-wheel in the form of ring with spokes

For the fly-wheel in the form of ring with spokes

$$I_M = \frac{mD^2}{4} = \frac{\pi bh\rho D^3}{4}.$$

Marking $\lambda_b = \dfrac{b}{D}$ and $\lambda_h = \dfrac{h}{D}$, we obtain

$$D = \sqrt[5]{\frac{4I_M}{\pi\lambda_b\lambda_h}}.$$

In case of equal diameters the mass of disk-shaped fly-wheel is twice as big as the mass of the ring-shaped fly-wheel with spokes.

The fundamental matter in designing aggregates with a fly-wheel is its placement. As the moment of inertia of a fly-wheel is inverse-square-law of angular velocity ω_c (5.30), it is more rational to place it on the high speed shaft. Then the mass of the fly-wheel will be the lest. But in some cases, as for example of underrigidity of links, it may cause great oscillations in mechanisms.

Increase of the average velocity. A very effective way to increase the constancy of a given rotation speed is to increase its average level during the operation of the mechanism. As the statement is understandable, we will not study it in detail.

Machine running control by approaching of works of driving forces and resistance forces. A classic example of such control is the principle, realized in a simple gramophone, almost unknown to the present generation. The most important condition of the qualitative sound reproduction is a steady speed of rotation of a gramophone disk with a placed gramophone record on it. Soundtrack on the record is made in the form of helix (Fig. 5.31, *a*). Resistance force exerts on the pickup head with a pickup stylus, which moves along the soundtrack. The moment of this force depends on the place of pickup stylus relatively to the record, i.e. on the radius r. We have a linear function of the resistance moment. The driving moment is created by the spiral spring, which is turned by hand with the help of a special knob. So the property of the spring is also linear (Fig. 5.31, *b*), and chosen in such a way that at any moment the resistance moment is balanced by the driving moment of the spring.

In practice it is more frequent to control either value of the resistance moment, depending on value of driving moment, or value of driving moment, depending on value of resistance moment.

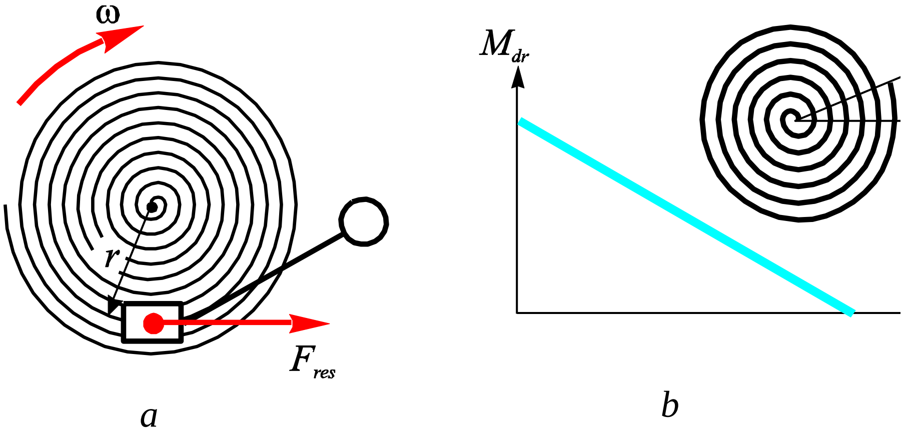


Fig. 5.31. Machine running control for gramophone drive: *a* – the scheme of loading; *b* – the mechanical property of the spring

If maintenance of a steady speed of motion is conducted by control of the resistance moment M_{res}, the unit, which provides such balancing is called governor, if it is conducted by control of the driving moment M_p, such unit is called moderator.

Let us view some schemes of governors.

1. Brake governor (Fig. 5.32).

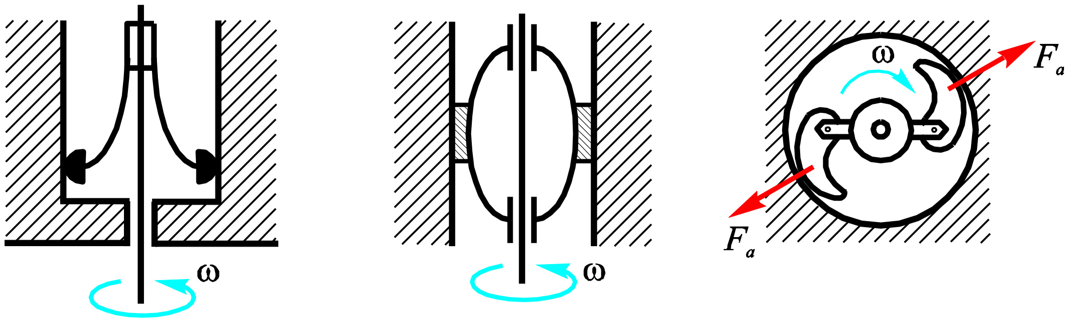


Fig. 5.32. Brake governors

2. By air resistance (Fig. 5.33).
3. Centrifugal governor (Fig. 5.34).

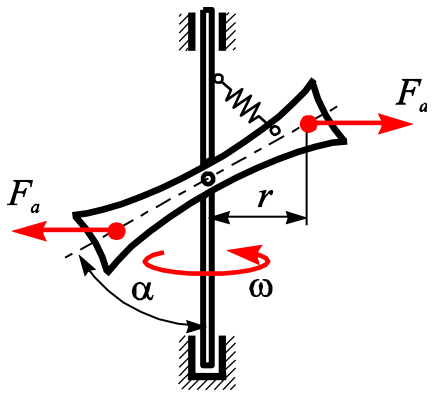


Fig. 5.33. Governor with air resistance

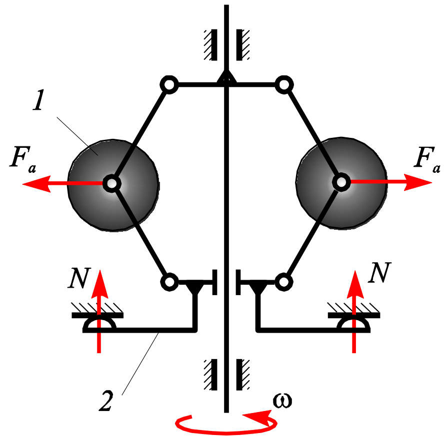


Fig. 5.34. Centrifugal governor

With increase of angular velocity driving masses 1 diverge, walking beam 2 tries to move higher, increasing the push N on the friction disk 3. In this case, the resistance moment increases, and the revolutions decrease.

The scheme of the centrifugal moderator is shown in Fig. 5.35.

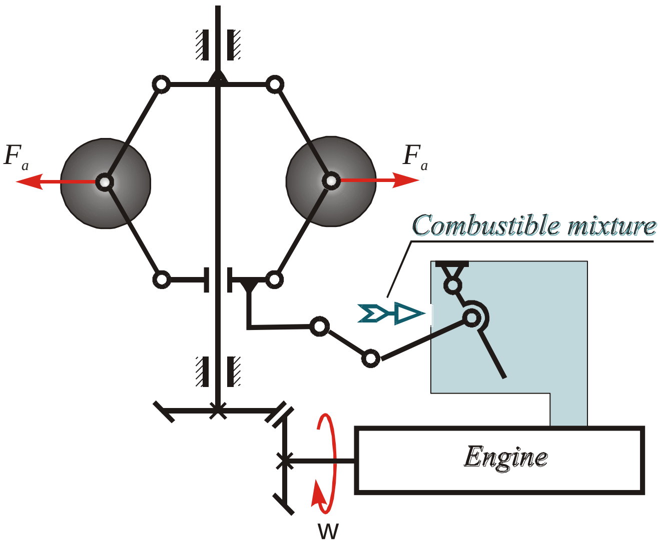
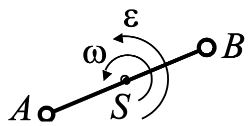


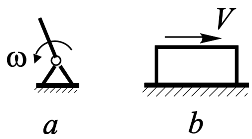
Fig. 5.35. Centrifugal moderator

Here the moderator works in the composition of internal combustion engine. The bigger the number of revolutions is, the higher the point *A* of the walking beam 1 is situated. Choke 2 closes; fuel feeding decreases and the engine decelerate.

In the car the role of moderator belongs to a driver. When the engine speed drops as a result of the growth of resistance forces, the driver gases and the desired speed is restored.

QUESTIONS FOR SELF-TESTING

1. Formulate the main tasks of the dynamic analysis of mechanisms.
2. What is the direct problem of dynamic analysis? What is the inverse problem of dynamic analysis?
3. What forces are called driving and which are resistance forces?
4. 

S – centre of masses
Where should the fixed centre of rotation of the link be located so that the resultant vector of inertia forces is zero?
5. How should the link move (See Question 4) so that the moment of inertia is zero?
6. Which point is called the centre of oscillation of the link?
7. What is called the radius of gyration of the link?
8. How to find the radius of gyration of a straight bar when its dimensions are known?
9. What is the point, associated with the link mechanism, called the pole of inertia?
10. Which dependencies are called the mechanical properties of machines?
11. What is called a dynamic model of machine aggregate?
12. What force is called reduced to the mechanism link?
13. 

To the datum link of which dynamic model should you apply a reduced force, and for what - a reduced moment? Substantiate your answer.
14. What is the method of Zhukovsky lever in determining the reduced force and on what principle is this method based?
15. What mass is called reduced to the mechanism link?
16. For what datum link (See Question 13) should you determine a reduced mass, and for which – the reduced moment of inertia when constructing a dynamic model of a machine aggregate?
17. What is determined using the equations of mechanism motion? What basic forms of these equations do you know?
18. List the main phases of machine aggregate motion.

19. What phases of machine aggregate motion pertain to transient regime?
20. What initial data should be set when determining the motion law of the machine aggregate in transient regime?
21. Give examples of machine aggregates for which the properties of forces are conveniently set as position functions?
22. What form are the equations of mechanism motion written if the mechanism consists only of turning links?
23. List the main features of the steady state mode of the machine aggregate.
24. What is called the irregularity motion ratio of the machine aggregate?
25. What are the main ways to minimize the irregularity motion ratio of the machine aggregate?
26. What is called a flywheel and what role does it play in the machine aggregate?
27. Write the dynamic synthesis equation of a fly-wheel under steady state mode.
28. What is the flywheel construction considered rational. Where is the most rational place for the flywheel in the machine aggregate?
29. For what purpose are motion governors introduced into the mechanism? What is the fundamental difference between motion governors and motion moderators?
30. What principle is the basis for the operation of brake governors?

Chapter 6. DYNAMIC FORCE ANALYSIS OF MECHANISMS

The task of dynamic force analysis of mechanisms is to define pressures in kinematic pairs as well as *balance forces* and *moments*.

At the heart of dynamic force analysis, there is kinetostatics method, which is based on the D'Alembert principle.

Active and reacting forces are balanced by the inertia forces:

$$\sum_i F_i + \sum_j F_{aj} = 0; \quad \sum_i M_i + \sum_j M_{aj} = 0.$$

Jean Le Rond D'Alembert (1717–1783)
French encyclopedic scholar, outstanding mechanic, mathematician, philosopher, Member of the Paris Academy of Sciences, French Academy and many other academies. He developed a method for solving the second-order differential equation in partial derivatives, which describes the transverse oscillations of a string (wave equation). These works, together with the later works of Euler and Bernoulli, laid the foundations of mathematical physics. He formulated the principle, bearing his name, by which the dynamics of a non-free system is reduced to static.

6.1. DYNAMIC FORCE ANALYSIS OF ASSUR GROUPS

6.1.1. Static definability condition of kinematic chain

To make the system definable, the quantity of unknowns, which should be determined, must not be more than number of equations. That is why, before solving a task on defining pressures in kinematic pairs, we should find for which chains the equality condition between a number of equations of statics (kinetostatics) and the number of unknown reacting forces in kinematic pairs is fulfilled.

For n links, on which spatial force system is acting, we can make up $6n$ equilibrium equations. For each kinematic pair a number of unknown reactions, which are to be defined from these equations, coincide with a number of constraints, which the pair superimposes. Impossibility of translation in direction of a constraint produces reaction as a force, and impossibility of rotation – as a moment.

Thus, static definability condition of space kinematic chain looks like:

$$6n = 5p_5 + 4p_4 + 3p_3 + 2p_2 + p_1, \tag{6.1}$$

where n is number of links of kinematic chain; $p_5,..., p_1$ is number of kinematic pairs of a respective class; $5p_5,...,1p_1$ is number of reactions in these pairs.

The condition (6.1) coincides with condition of equality to zero of total motion freedom of kinematic chain (2.1).

For planar kinematic chains the number of conditions of equilibrium is $3n$. Number of unknown reactions for a lower pair, according to 4.1.6, is two. In a higher pair there is one unknown reaction.

Thus, condition of equilibrium for planar kinematic chains looks like:

$$3n = 2p_5 + p_4. \tag{6.2}$$

So the condition (6.2) corresponds to zero of motion freedom of the kinematic chain. As it is known, zero of motion freedom is peculiar to Assur groups. It means that *Assur groups are statically determinate system.*

Let us look at a mechanism, the scheme of which is shown in Fig. 6.1.

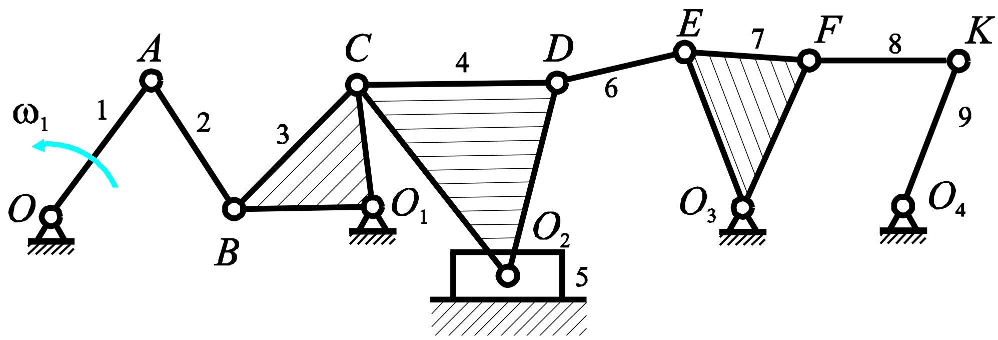


Fig. 6.1. The kinematic scheme of the mechanism

We divide kinematic chain into structural groups. Then we take any intermediate group, for example III. It consists of links 6 and 7, so $3n = 3 \cdot 2 = 6$.

Links compose four kinematic pairs of the fifth class – $2p_5 = 2 \cdot 4 = 8$. It means that number of unknown is 8, and number of conditions of equilibrium is 6.

$$3n < 2p_5 .$$

Thus, we have a hyperstatic or statically indeterminate system.

Let us take *the last attached Assur group* (links 8 and 9). Here there are two links and three hinges. The condition is true

$$3n = 2p_5 .$$

So, *unlike kinematic analysis, dynamic force analysis of a mechanism should be started from the last attached structural group.*

6.1.2. Force diagram of a structure group

In the course of dynamic force analysis we have such set parameters as size and weight of links, location of link mass centres, moments of inertia of link masses, driving and resistance forces.

It is easy to solve equations of kinetostatics graphically, by using force diagrams (closed vector loops).

This method has, on the one hand, good visualization, and on the other hand, the accuracy of simple graphical plotting is turned to be enough, since the magnitudes of loads and schemes of loading of links are usually known only very approximately.

The dynamic force analysis of structural group using the force diagrams method is performed in the following sequence:

- to define kinematic parameters of mechanism links, including accelerations of centres of link mass;
- to divide mechanism into structural groups;
- to start the dynamic force analysis from the last attached structural group;
- to define mass and inertia forces, which act on links by methods, shown in chapter 5;

- to define pressures in kinematic pairs.

  The sequence of pressure determination is:

- to make up equation of equilibrium of a structure group in a vector form;
- using method of academic Bruyevitch, to define normal and tangential components of pressures in pivots;
- to define with the help of force diagram the total forces (pressures) in pivots.

6.2. DYNAMIC FORCE ANALYSIS OF PLANAR MECHANISMS

6.2.1. Dynamic force analysis of typical mechanisms by the force diagrams method

Consider the procedure of dynamic force analysis, taking for the example crank-and-rocker mechanism.

Example 6.1. To define pressures in kinematic pairs of a pin-jointed four-bar linkage (Fig. 6.2, *a*) by method of force diagrams. To accept $\omega_1 = Const$.

1. We construct the velocity (Fig. 6.2, b) and acceleration (Fig. 6.2, c) diagrams of links for a set position of the mechanism.

For the point A of the crank the linear velocity $V_A = \omega_1 \cdot l_{OA}$, *and centripetal (normal) acceleration* $a_A = \omega_1^2 \cdot l_{OA}$.

We start to construct velocity and acceleration diagrams with segments $pa = V_A / \mu_V$ *and* $\pi a = a_A / \mu_a$.

For the point B we write down:

$$\begin{cases} \overline{V}_B \perp O_1B; \\ \overline{V}_B = \overline{V}_A + \overline{V}_{BA}; \end{cases}$$

$$\begin{cases} \overline{a}_B = \underset{\parallel O_1B}{\overline{a}_B^n} + \underset{\perp O_1B}{\overline{a}_B^\tau}\,; \\ \overline{a}_B = \overline{a}_A + \underset{\parallel AB}{\overline{a}_{BA}^n} + \underset{\perp AB}{\overline{a}_{BA}^\tau}; \end{cases} \qquad \begin{aligned} a_B^n &= \frac{V_B^2}{l_{BO_1}};\ \pi n_2 = \frac{a_B^n}{\mu_a}; \\ a_{BA}^n &= \frac{V_{BA}^2}{l_{BA}};\ an_1 = \frac{a_{BA}^n}{\mu_a}. \end{aligned}$$

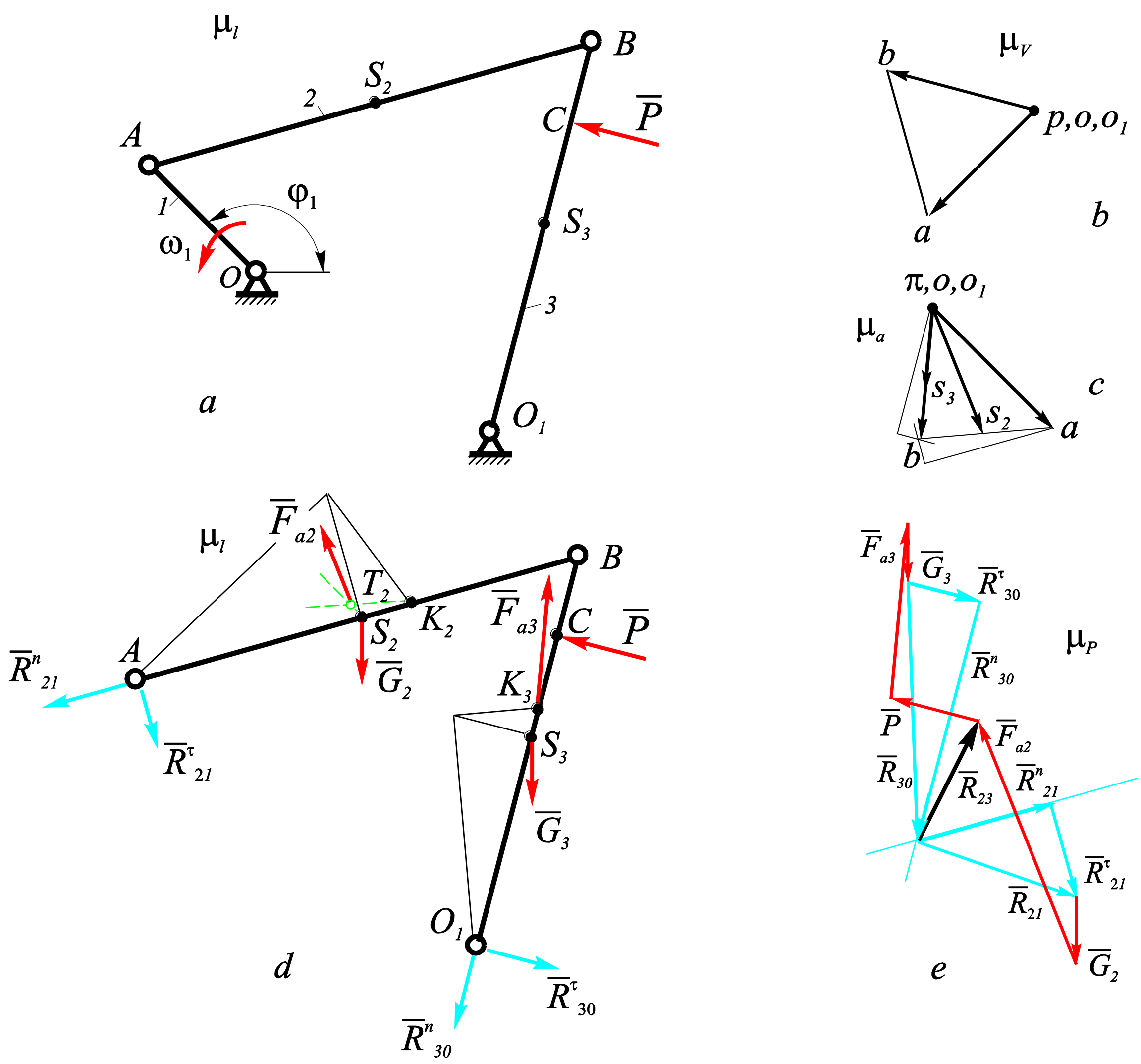


Fig. 6.2. Pin-jointed four-bar linkage force analysis: *a* – the kinematic scheme; the velocity (*b*) and acceleration (*c*) diagrams; *d*– the Assur group; *e* – the force diagram

2. *We divide the mechanism into Assur groups.*

One Assur group of II class and 2-nd order is attached to the primary mechanism (Fig. 6.2, d)

3. *As we have only one attached group, so we start analysis with it.*

We construct Assur group in the same position in scale μ_l.

4. Then we define gravities and inertia forces of links:

$$G_2 = m_2 g; \qquad G_3 = m_3 g;$$

$$\overline{F}_{a_2} = -m_2 \overline{a}_{S_2}; \quad F_{a_2} = m_2 \cdot \mu_a \cdot \pi s_2 .$$

The link 2 produces planar motion. Inertia force $\overline{F}_{a_2}$ is parallel to $\overline{a}_{S_2}$ and is imposed in a pole of inertia T_2. Considering the link 2 as a rod, we can define the value of its radius of gyration, as $\rho_2 = l_{AB} / \sqrt{12}$.

Let us study the motion of the link 2 as sum of parallel motion, for example with the point A, and turning motion around the point A. Then we define the pole of inertia T_2 at the concurrence of action lines of inertia $\overline{F}^{/}_{a2} = -m_2 \overline{a}_A$ in parallel motion with the point A, that crosses centre of mass S_2 parallel to the direction of vector $\overline{\pi a}$ of acceleration diagram, and action lines of inertia force in turning motion relatively to the point A $\overline{F}^{//}_{a2} = -m_2 \overline{a}_{S_2 A}$, which crosses the centre of oscillation of the link 2 (the point K_2) parallel to the direction of vector $\overline{s_2 a}$ of the diagram.

The link 3 produces turning motion relatively to the point O_1. Inertia $\overline{F}_{a3} = -m_3 \overline{a}_{s3}$; $(F_{a3} = m_3 \cdot \pi S_3 \cdot \mu_a)$. Centre of oscillation of the link is K_3, where this force is imposed, can be defined when finding the radius of gyration $\rho_3 = l_{BO_1} / \sqrt{12}$ and making graphical plotting.

5. Further we define pressures in kinematic pairs.

As the links 2 and 3, which belong to the Assur group, are detached from the mechanism, so in pivots O_1 and A we should impose reactions from the ground on the link 3 $\overline{R}_{30}$ and from the crank 1 on the link 2 $\overline{R}_{21}$. We impose them in the points O_1 and A correspondingly, but beforehand dividing it into normal (along the link axis) and tangential (at right angle to its axis) components, and write down the vector condition of equilibrium:

$$\overline{R}^n_{21} + \overline{R}^\tau_{21} + \overline{G}_2 + \overline{F}_{a2} + \overline{P} + \overline{F}_{a3} + \overline{G}_3 + \overline{R}^\tau_{30} + \overline{R}^n_{30} = 0 . \qquad (6.3)$$

Here are four unknowns, and as we know, from the vector equation we can find only two unknowns (2 equations in axle projections).

We find tangential components $\overline{R}_{21}^{\tau}$ *and* $\overline{R}_{30}^{\tau}$ *from the condition of equilibrium taking into account that the moment in the pivot B is equal to zero.*

For the links 2 and 3:

$$\begin{cases} \sum M_B^{(2)} = R_{21}^{\tau} \cdot l_{AB} + G_2 \cdot h_{G_2} - F_{a_2} \cdot h_{F_{a_2}} = 0; \\ \sum M_B^{(3)} = -R_{30}^{\tau} \cdot l_{O_1B} + G_3 \cdot h_{G_3} + P \cdot h_P - F_{a3} \cdot h_{F_{a_3}} = 0. \end{cases}$$

We choose the scale μ_F *(*$\frac{H}{MM}$*) and define scaling magnitudes of each unknown force:*

$$\left(R_{21}^{\tau}\right) = \frac{R_{21}^{\tau}}{\mu_F}; \qquad \left(F_{a_2}\right) = \frac{F_{a_2}}{\mu_F}. \quad etc.$$

We solve the vector equation (6.3) *geometrically by constructing the force polygon (Fig. 6.2, e), beginning and ending the construction with the unknown forces* $\overline{R}_{21}^{n}$ *and* $\overline{R}_{30}^{n}$, *constructing their directions.*

Having defined the components, we can find the values of total reacting forces:

$$R_{21} = \sqrt{\left(R_{21}^{n}\right)^2 + \left(R_{21}^{\tau}\right)^2},$$

$$R_{30} = \sqrt{\left(R_{30}^{n}\right)^2 + \left(R_{30}^{\tau}\right)^2}.$$

We should compound vectors sequentially for each link. Then directly from the diagram we can define the pressure in kinematic pair B, as the end-capping vector for the polygon of forces, imposed to the link 2 or 3. For example, from the equilibrium condition of the second link, we have:

$$\overline{R}_{21} + \underset{?}{\overline{R}_{23}} + \overline{F}_{a_2} + \overline{G}_2 = 0.$$

Here

$$\overline{R}_{23} = -\overline{R}_{32}.$$

6.2.2. Kinetostatics of the primary mechanism

Let us consider the primary mechanism, which includes the ground and the link *1* of a rocker-and-crank mechanism (See example 6.1, Fig. 6.2). We impose to the crank acting forces on it. In the point A we impose force $\overline{R}_{12} = -\overline{R}_{21}$. Here the force $\overline{R}_{12}$ is the reaction from the link *2*, attached to the primary mechanism and is the resultant of all forces imposed to the mechanism links. After the dynamic force analysis of structure groups the force $\overline{R}_{12}$ is known.

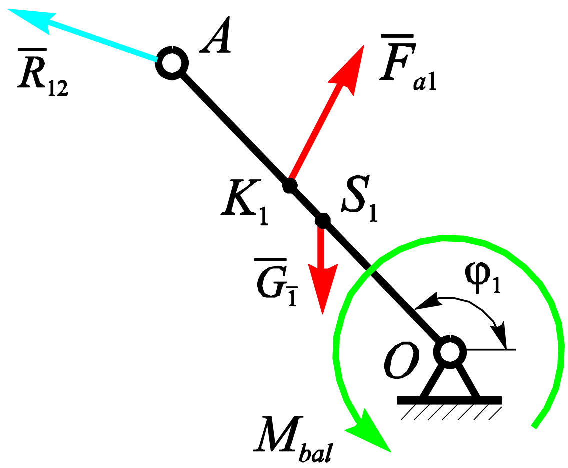


Fig. 6.3. The primary mechanism

Reactions in the pivot *O* and driving moment imposed to the initial (input, driving) link are unknown. This moment is variable, and for the point of time and set conditions it is considered as a *balance moment*. It means that in the moment of time it counteracts all forces, which act on the mechanism links, including inertia forces, but except for the reaction in crank (they are mutually balanced).

We may write down the equilibrium equation of a crank:

$$\sum M_0 = M_{bal} + M_0\left(\overline{G}_1\right) + M_0\left(\overline{F}_{a_1}\right) + M_0\left(\overline{R}_{12}\right) = 0. \qquad (6.4)$$

After defining M_{bal} for different positions within the cycle of motion of the mechanism, we construct diagram $M_{bal} - \varphi$, with help of which we find, for example, the most severe positions of the input link: $M_{bal} = M_{bal\,\max}$.

Thus, if the crank is the driving link, so M_{bal} is the driving moment; if this link is driven so it is the moment of resisting forces.

If to the driving link the moment is brought or it is taken off from it through the clutch (directly through the shaft, on which the crank is fastened), so the unknown moment M_{bal}. is the external load. If the power supply (or diversion) is carried out through friction gear or tooth gear, or brought to the slider, so balance force F_{bal} is the external load (Fig. 6.4, *a*, *b*, *d*).

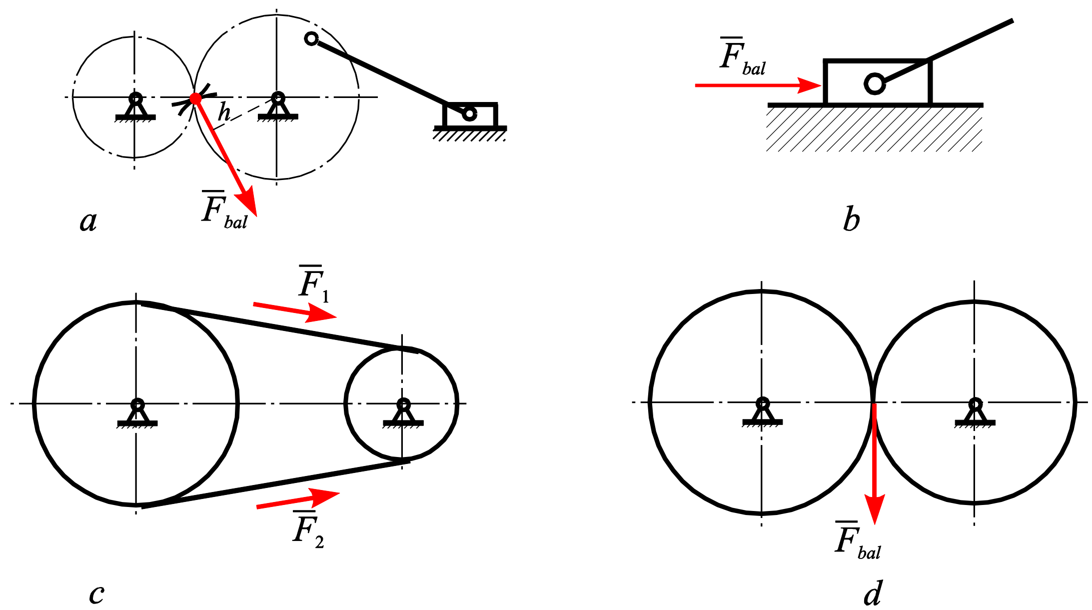


Fig. 6.4. Examples of balance forces application: *a* – the gearing; *b*– the slider; *c* – the belt drive; *d* – the friction gear;

For the belt drive (Fig. 6.4, *c*) we have two unknowns F_1 and F_2, but they are connected by the Euler theorem.

It means that in enumerated cases it is better to study balance force F_{bal}, but not balance moment M_{bal}:

$$F_{bal} = \frac{M_{bal}}{h_{bal}} .$$

Here M_{bal} is defined by the equation (6.4); h_{bal} is the actual magnitude of the arm of force, but not its scaling magnitude, taken from the kinematic scheme.

The solution of the dynamic force analysis task of the mechanism can be done by analytical method (as well as the kinematic analysis). Considering the repetition of solution within the cycle of work, this task is very laborious. For example, dynamic force analysis of the slider-crank mechanism of diesel, which works in steady state, under the change of generalized coordinate $\Delta\varphi_1 = 5°$ is turned into the solution of system of 33 combined equations, every 72 times. Laboriousness of the solution is radically taken off with the help of computer.

6.2.3. Definition of balancing forces and moments by Zhukovsky lever method

From the above mentioned it is simply to make the conclusion that the balancing moment (force) is the reduced moment or force, imposed to this link, but oppositely directed towards M_{bal} $\left(F_{bal}\right)$:

$$\bar{F}_{bal} = -\bar{F}_{rd};$$

$$\bar{M}_{bal} = -\bar{M}_{rd}.$$

The reduced moment contains all forces, imposed to the mechanism, including inertia.

Thus, we can use the same methods and design equations to define the balance moment (force), as when defining reduced moments (forces), without dynamic force analysis of the whole mechanism. (We don’t have to define pressure in kinematic pairs between input or output links and the links of structural groups attached to them). Remind that pressures in hinges are internal forces of the mechanism.

Let us use Zhukovsky lever method for the definition of M_{bal}.

Example 5.2. For the mechanism, shown in Fig. 6.5, define the balancing moment, imposed to the input link 1, by Zhukovsky lever method. To accept $\omega_1 = Const$.

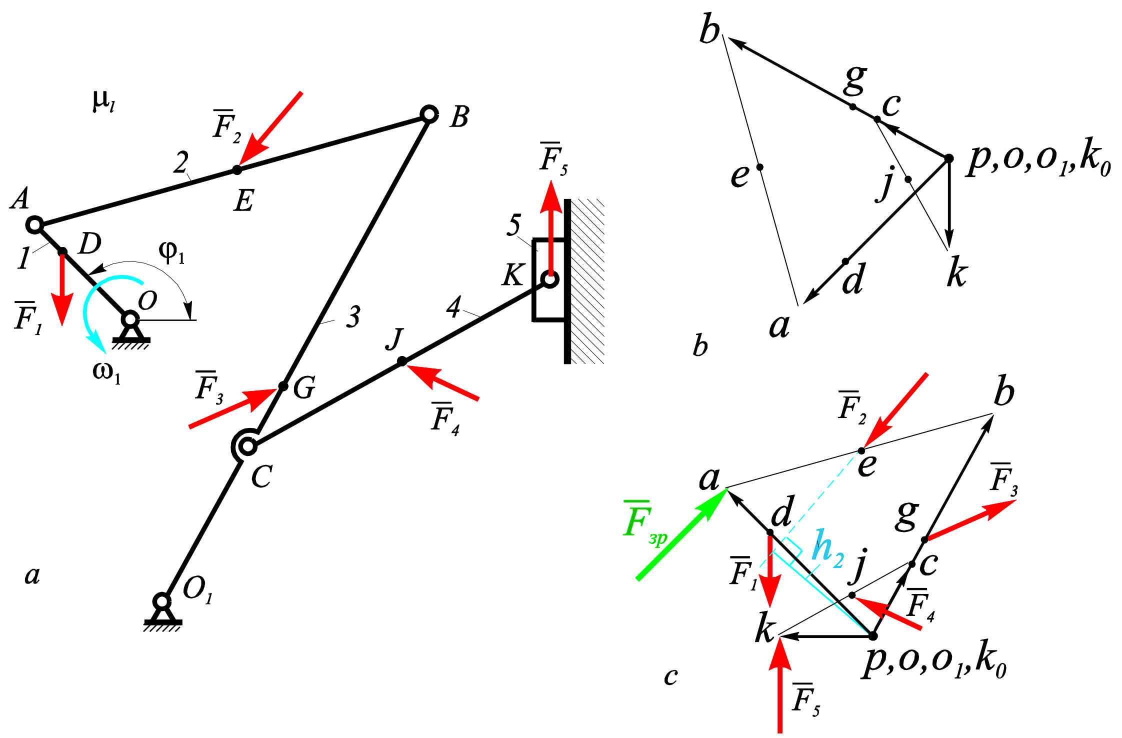


Fig. 6.5. Pin-jointed six-bar linkage force analysis: *a* – the kinematic scheme; *b* – the velocity diagram; *c* – the Zhukovsky lever

Its motion freedom is $w = 3n - 2p_5 = 3 \cdot 5 - 2 \cdot 7 = 1$. *Link 1 is input as its law of motion is set.*

The set system of external forces acts on the mechanism (See Fig. 6.5, a). In general case the mechanism as a system, which has one motion freedom, will be in out-of-balance condition. In order to balance the mechanism, we should impose in any point the balancing force $\overline{F}_{bal}$. *As the point of force application we take the point A on the link 1. We set the direction of* $\overline{F}_{bal}$, *for example perpendicular to OA.*

In order to use Zhukovsky method, we construct in arbitrary scale velocity diagram of the mechanism (Fig. 6.5, b), and turn it on 90° opposite to the direction ω₁ (Fig. 6.5, c). Let us move in corresponding diagram points all forces acting on the mechanism. To the point A in a chosen direction we impose the balancing force $\overline{F}_{bal}$. *We write down lever equilibrium condition relatively to the pole p:*

$$M_p(\overline{F}_1) + M_p(\overline{F}_2) + M_p(\overline{F}_3) + M_p(\overline{F}_4) + M_p(\overline{F}_5) + M_p(\overline{F}_{bal}) = 0.$$

Marking the arms correspondingly and considering moment's sign, we define:

$$F_{bal} = F_1 \frac{h_1}{h_{bal}} + F_2 \frac{h_2}{h_{bal}} + F_4 \frac{h_4}{h_{bal}} - F_3 \frac{h_3}{h_{bal}} - F_5 \frac{h_5}{h_{bal}}.$$

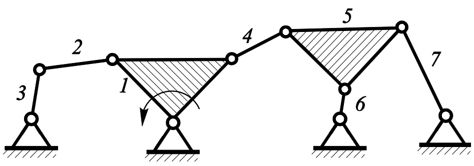
The relations of segments h_i / h_{bal} *are taken directly from the velocity diagram.*

Note: *the Fig. 6.5, c shows only the arm of the force* $\bar{F}_2$.

The method is general for all mechanisms of any class.

Under dynamic analysis and, unlike the dynamic force analysis − kinetostatics method, the forces are reduced separately to the datum link. For example, separately resisting forces, frictions, driving and other forces. Gravity is usually reduces with driving forces. The separate reduction contributes to better consideration of influence of each force on the law of motion of the mechanism.

QUESTIONS FOR SELF-TESTING

1. What are the main tasks of dynamic force analysis of the mechanism?
2. Formulate the d'Alembert principle.
3. What systems are called statically determinate?
4. Write down the condition of static definability of the kinematic chain.
5. 

In the mechanism, link 1 is the initial one. What class is this mechanism? Which of the attached Assur groups are statically determinable?
6. What parameters of the mechanism should be given in its dynamic force analysis?
7. What is called the force diagram of the mechanism?
8. In what sequence is the dynamic force analysis of the mechanism carried out using the force diagrams method?
9. What is the essence of the Bruevich method for determining pressures in kinematic pairs?
10. What is called balance force or moment?
11. If the balance moment is the moment of driving forces, then to what link of the mechanism should it be applied?
12. What is called “the Zhukovsky lever”?
13. In what sequence is the balance moment (force) determined by Zhukovsky lever method?
14. Is it possible to use the Zhukovsky lever method to perform a full dynamic force analysis of the mechanism?

Chapter 7. BALANCING OF MECHANISMS

Executing the dynamic force analysis of a mechanism, we set that in a general case in kinematic pairs, formed by turning links with a ground, reacting forces operate. At a speed-up motion of links these reactions contain dynamic components, which in a steady state mode are changed cyclically. So, they can be a source of many undesirable effects, for example vibrations, which are passed to the housing, as well as to the base. The main task is a maximum removal of this effect, which is obtained by balancing of movable masses of mechanisms.

7.1. IMBALANC OF MECHANISMS

7.1.1. Types of imbalance of a mechanism

Let us study a four-bar linkage (Fig. 7.1).

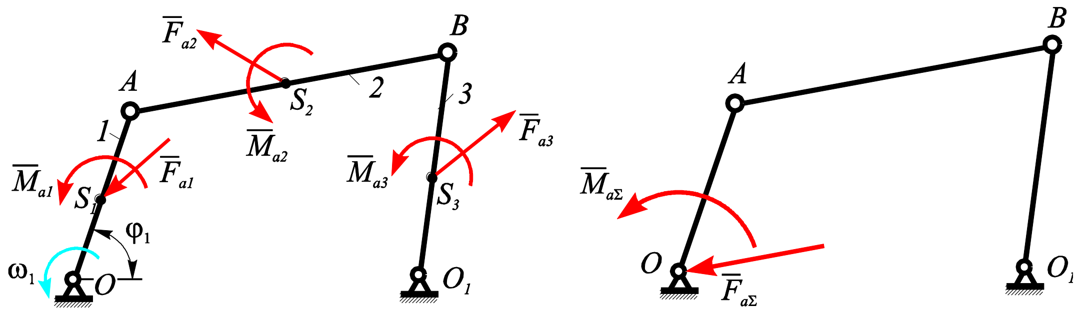


Fig. 7.1. The four-bar linkage: *a* – the initial scheme; *b* – the equivalent scheme

The law of motion of an input link is set – $\omega_1 = Const$. Other links move non-uniformly. Knowing sizes and link masses characteristics, it is possible to define their inertia forces and moments. Let us reduce all assemblage of inertia forces to the pivot *O* (Fig. 7.1, *b*) and represent as a resultant and a moment:

$$\bar{F}_{a_\Sigma} = \sum_{i=1}^{n} \bar{F}_{a_i};$$

$$\bar{M}_{a_\Sigma} = \sum_{i=2}^{n} M_0\left(F_{a_i}\right) + \sum_{i=2}^{n} M_{a_i},$$

where *n* – is quantity of movable links of a mechanism.

For the first link $\omega_1 = Const$. So $\varepsilon_1 = 0$, that means $M_{a_1} = 0$. Moreover, as the line of action of inertia of the first link $\bar{F}_{a_1}$ passes through the point *O*, the moment of this force relatively to point *O* is $M_0\left(\bar{F}_{a_1}\right) = 0$.

Loading of inertia F_{a_Σ} and the moment of inertia forces M_{a_Σ} are the reason of the dynamic loading of a ground (though these forces are engineering value and we do not consider their physical meaning). Sometimes they are called the shaking forces and shaking couples.

If the resultant $\bar{F}_{a_\Sigma}$ and the moment $\bar{M}_{a_\Sigma}$ of inertia forces are not equal to zero, such mechanism is statically unbalanced.

If the resultant of inertias of a mechanism is $\bar{F}_{a_\Sigma} = 0$, and a moment is $\bar{M}_{a_\Sigma} \neq 0$, so we have the moment imbalance of a mechanism.

7.1.2. Static and moment imbalance of a mechanism at the design stage

Static balancing of the mechanisms. In such case it is helpful when

$$\bar{F}_{a_\Sigma} = 0.$$

As a result, the dynamic effect of inertias on the base (shaking force) is excluded (but the effect of the shaking couple is remained saved).

We can write

$$\bar{F}_{a_\Sigma} = -m_\Sigma \bar{a}_S.$$

Here m_Σ is a total mass of all movable links; $\bar{a}_S$ is an acceleration of the total centre of mass of a system. So the static balancing is obtained under the condition that the total centre of mass will be immovable.

From the engineering mechanics we know, that under planar motion a body can be presented as two lumped masses (Fig. 7.2) that undertake the conditions:

$$\begin{cases} m = m_A + m_B; \\ m_A l_{AS} = m_B l_{BS}. \end{cases} \tag{7.1}$$

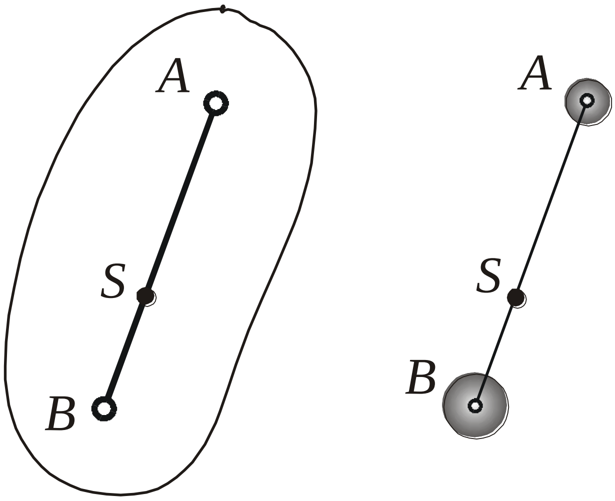


Fig. 7.2. The body and its model

The second condition indicates that the centre of mass S is situated in the same place. So it means that the resultant of inertias of a substituting system equals to the resultant of inertias of the set body. (However, the moments of inertias do not coincide, but it does not matter for the static balancing).

We carry out the static balancing of a four-bar linkage (Fig. 7.3).

We assume that the masses of links m_1, m_2, m_3 and their lengths l_1, l_2, l_3 are set. *S* is the total centre of mass of the mechanism. We change every link into two lumped mass (Fig. 7.3, *b*).

For the link *1*, using the condition (7.1), we may write:

$$\begin{cases} m_1 = m_0 + m_{A1}; \\ m_0 l_{OS_1} = m_{A1} l_{AS_1}. \end{cases}$$

Hence

$$m_1 = m_0 + m_0 \frac{l_{OS_1}}{l_{AS_1}} = m_0 \left(\frac{l_1}{l_{AS_1}} \right); \qquad m_1 = m_{A1} + m_{A1} \frac{l_{AS_1}}{l_{OS_1}} = m_{A1} \left(\frac{l_{OS_1}}{l_1} \right).$$

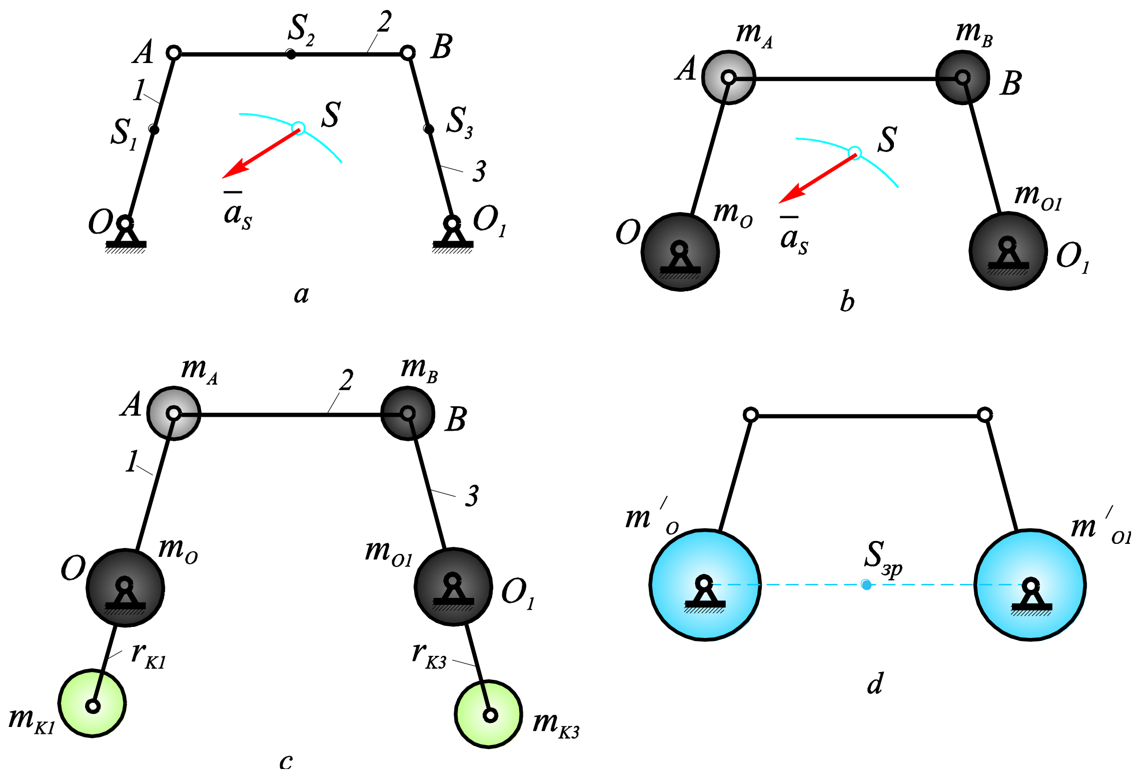


Fig. 7.3. The static balancing of a four-bar linkage: the initial scheme (*a*); the equivalent schemes (*b*, *c*, *d*)

Finally we get:

$$m_0 = m_1 \frac{l_{AS_1}}{l_1}; \qquad m_{A1} = m_1 \frac{l_{OS_1}}{l_1}.$$

By analogy:

$$m_{A2} = m_2 \frac{l_{BS_2}}{l_2}; \qquad m_{B2} = m_2 \frac{l_{AS_2}}{l_2}.$$

$$m_{B3} = m_3 \frac{l_{O_1S_3}}{l_3}; \qquad m_{O_1} = m_3 \frac{l_{BS_3}}{l_3}.$$

Thus, the mechanism is changed for four masses m_0, $m_A = m_{A1} + m_{A2}$, $m_B = m_{B1} + m_{B2}$, m_{O_1}. The total centre of mass of the mechanism is left in the same place and moves with acceleration (Fig. 7.3, *b*).

On the links 1 and 3 we put counterweights m_{K1} and m_{K3} (Fig. 7.3, *c*) in such a way that common centres of masses of two-mass systems $\left[m_A, m_{K1}\right]$ and $\left[m_B, m_{K3}\right]$ appear in points *O* and O_1 respectively. For this reason there should be such conditions:

$$\begin{cases} m_A l_1 = m_{K1} r_{K1}; \\ m_B l_3 = m_{K3} r_{K3}. \end{cases} \tag{7.2}$$

Let us combine masses on links 1 and 3:

$$m'_O = m_A + m_{K1} + m_O; \qquad m'_{O_1} = m_B + m_{K3} + m_{O_1}.$$

It means that after using the counterweights, that undergo the condition (7.2), the mechanism can be represented in the form of two unmovable masses m'_O and m'_{O_1}.

So the centre of mass of the mechanism with balancers m_{K1} and m_{K3} becomes stationary (Fig. 7.3, *d*). The masses of balancers m_{K1} and m_{K3} are chosen according to the conditions (7.2), taking radiuses r_{K1} and r_{K3}.

Moment balancing of the mechanisms. The moment balancing is conducted after the complete static balancing. The purpose of the moment balancing is to obtain the condition

$$\bar{M}_{a\Sigma} = 0.$$

Let us study a statically balanced mechanism (Fig. 7.4) [2].

The equation for the moment of inertia forces of such also includes the inertia moments of counterweight masses m_{K1} and m_{K3}.

As $\omega_1 = Const$, so $M_O\left(\bar{F}_{aK1}\right) = 0$, because the line of action of an inertia $\bar{F}_{aK1}$ in such case passes through the centre of rotation of a link – point *O*.

Thus

$$\bar{M}'_{a\Sigma} = \bar{M}_{a_2} + \bar{M}_{a_3} + \bar{M}_0\left(\bar{F}_{aK3}\right) = 0.$$

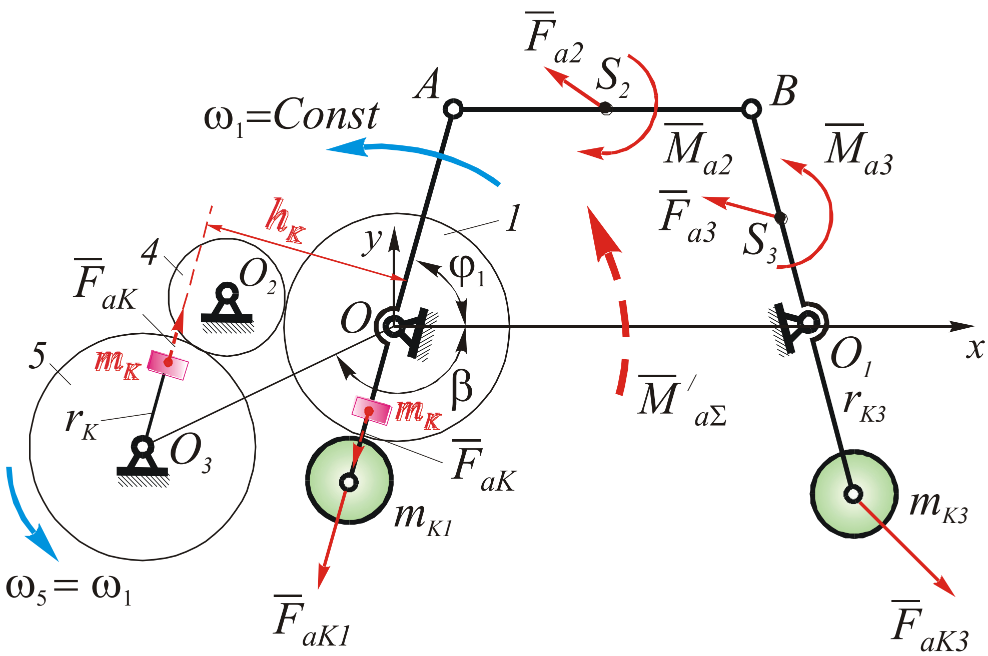


Fig. 7.4. Moment balancing of the four-bar linkage

The law of moment variation is shown in Fig. 7.5.

At the moment balancing some compensating moment M_{aK} applies to the system. The graph of its change (Fig. 7.5), summing the graph of change of the inertia moment $\bar{M}'_{a\Sigma}$, adds up to “0”. It is achieved, for example, in such a way. A crank 1 (Fig. 7.4) is rigidly attached to a gear. This gear meshes with another gear in such a way that a transmission ratio between them is equal to 1 ($u_{15} = 1$). Then $\omega_1 = \omega_5 = Const$. Directions of gear rotations are coincided.

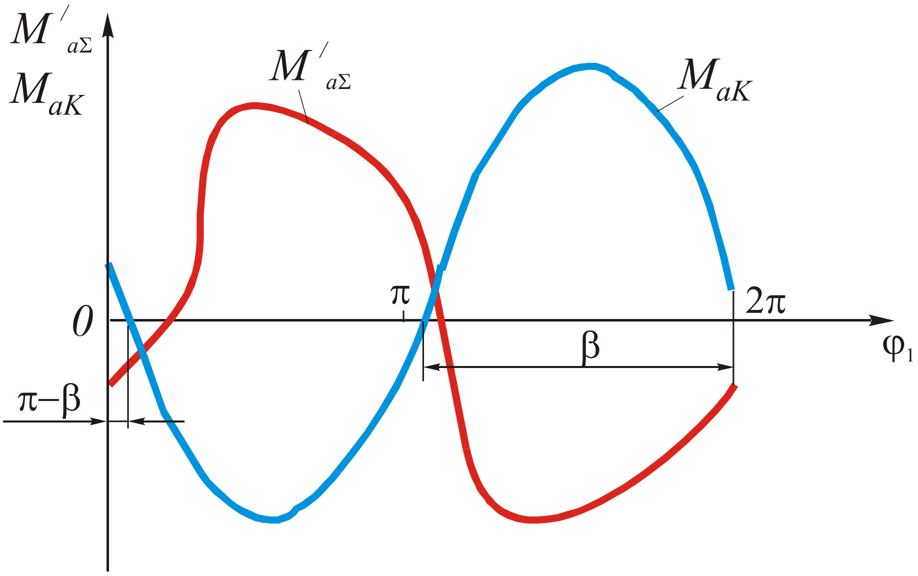


Fig. 7.5. Determination of M_{aK} magnitude

On the links 1 and 5 we place similar attachment masses m_K (Fig. 7.4). Radial coordinates r_K of these masses for links 1 and 5 coincide, and angular coordinates differ on π. Inertias $\bar{F}_{aK}$ of these masses create the force couple on the arm h_K. The magnitude of this force couple can be expressed as

$$M_{a_K} = m_K r_K \omega_1^2 h_K = m_K r_K \omega_1^2 l_{OO_3} \sin\left(\varphi_1 + \beta\right).$$

This is the equation of a sinusoid removed to an angle $\angle\beta$ to the left along the axis.

The purpose is that by varying sinusoid parameters, to maximum approach this curve to such position, when it will be in relation to the graph $\bar{M}'_{a\Sigma}$ as though in antiphase. It is better to vary value of amplitude $M_K^* = m_K r_K \omega_1^2 l_{OO_3}$ and angular β.

Choosing them in the best way, we define m_K:

$$m_K = \frac{M_K^*}{r_K \omega_1^2 l_{OO_3}}.$$

7.2. BALANCING OF ROTORS

7.2.1. Rotor imbalance and its types

Rotor can be any gyrating masses: gears, turbine wheels, crankshafts, fly-wheels etc. The problem of their balancing both on the design stage and after producing is very serious. For example turbine of a nuclear power station conducts ~400 min^{-1}, and the rotor weighs ~50 ton. Its diameter is about 2 m, and length is 20 m. Imbalance of its rotor may cause strong vibrations, or even to a catastrophe. In gas turbine engines operating speed of an engine drive shaft is about 10000 min^{-1}!

Uniform rotations of a rotor relatively to the bearing axis *z* (Fig. 7.6) may cause dynamic loads if centre of mass *S* will not be on the axis of rotation.

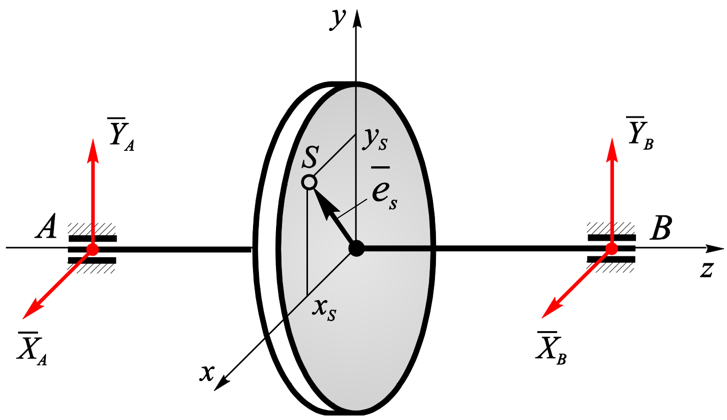


Fig. 7.6. The rotor with the eccentricity of mass

Resultant of inertia forces can be expressed as $F_a = m\omega^2\sqrt{x_S^2 + y_S^2}$. In a vector notation we will write

$$\overline{F}_a = m\omega^2\overline{e}_S.$$

Here $\overline{e}_S = \overline{r}_S$ is radius-vector of the centre of mass *S* of the rotor, and it is called *eccentricity of the rotor mass*.

We mark

$$\overline{D}_S = m\overline{e}_S.$$

It is an *resultant vector of rotor imbalance*.

Then

$$\bar{F}_a = \omega^2 \bar{D}_S .$$

Moment of inertia of the rotor is $M_a = \omega^2 \sqrt{J_{xz}^2 + J_{yz}^2}$ or $M_a = \omega^2 M_D$. Here $M_D = \sqrt{J_{xz}^2 + J_{yz}^2}$ is a *moment of rotor imbalance*. This value has a vector sense

$$\bar{M}_a = \omega^2 \bar{M}_D .$$

There are static, moment and dynamic imbalances of a rotor.

Static imbalance of a rotor occurs when the centre of mass does not lie on the axis of rotation, but the principal central axis of inertia is parallel to the rotational axis (Fig. 7.7). Such imbalance is attached, for example, to single-throw crankshaft.

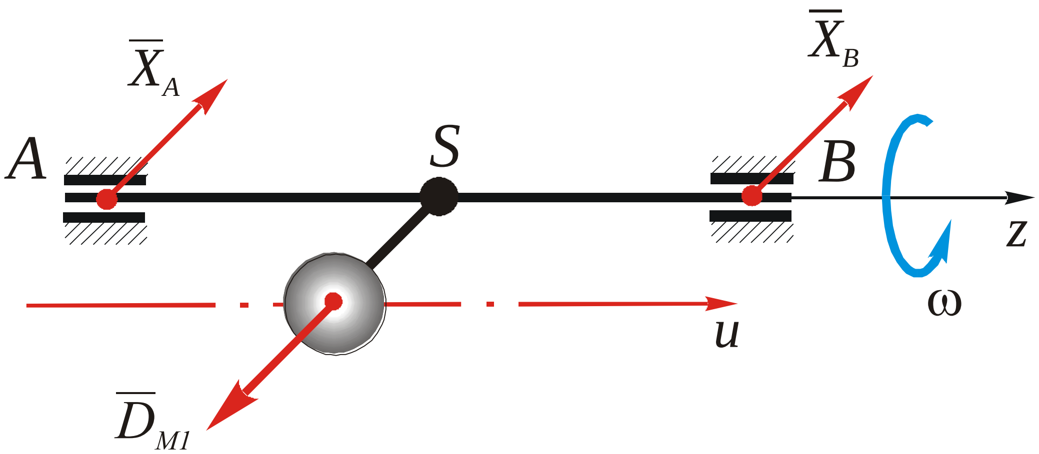


Fig. 7.7. Static imbalance of a rotor

In such case eccentricity of rotor mass is $\bar{e}_S \neq 0$; and products of inertia are $J_{xz} = J_{yz} = 0$. It means that $\bar{D}_S \neq 0$; $\bar{M}_D = 0$.

This imbalance can be easily removed, if we add to the rotor the corrective mass m_K, which provides the condition

$$\bar{D}_S = -\bar{D}_K .$$

Here $\bar{D}_K = m_K \bar{e}_K$. Though the centre of corrective mass should be placed on the line of action of vector $\bar{e}_S$ and vector of its imbalance $\bar{e}_K$ should be inversely oriented in relation to $\bar{e}_S$.

Moment imbalance of a rotor occurs when the centre of the masses lies on the rotational axis of the rotor, but its principal central axis of inertia does not coincide with rotational axis (Fig. 7.8). Such imbalance is attached, for example, to double-throw crankshaft. Here $e_S = 0$; $J_{xz} \neq 0$; $J_{yz} \neq 0$. Then $\bar{D}_S = 0$; $\bar{M}_D \neq 0$.

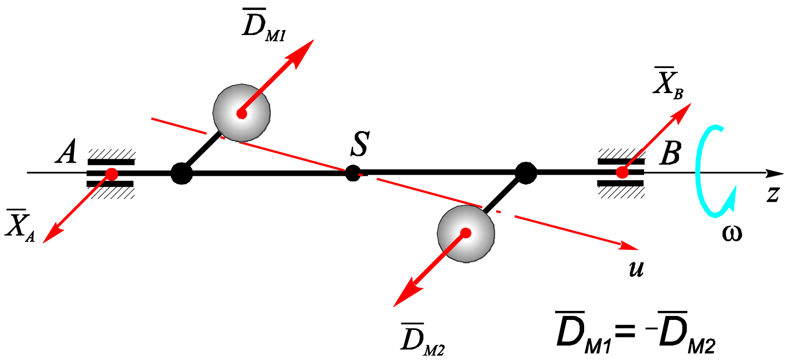


Fig. 7.8. Moment imbalance of a rotor

In order to remove imbalance we should install, as minimum, two corrective masses, for creating force couple $\bar{M}_K = -\bar{M}_D$. These masses are put in plane of changing in such a way that changing force couple appears.

Dynamic imbalance of a rotor occurs when the centre of mass is not situated on the rotational axis and this axis does not coincide with rotor principal axis ($e_S \neq 0$; $J_{xz} \neq 0$; $J_{yz} \neq 0$). It means

$$\bar{D}_S \neq 0; \quad \bar{M}_D \neq 0 .$$

In such case we have reactions at supports R_A and R_B, represented by vectors, which are crossed and rotate together with shaft (Fig. 7.9).

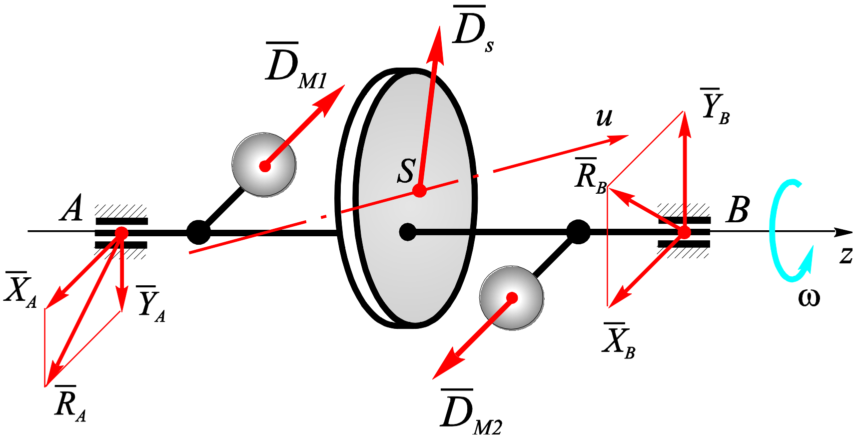


Fig. 7.9. Dynamic imbalance of a rotor

This imbalance is removed by installation relatively two masses in planes, perpendicular to the rotational axis.

7.2.2. Static and dynamic balancing of made rotors

Rotor imbalance should be removed at the design stage. But at the rotor making its imbalance arises because of deviation from design sizes, heterogeneity of material, out-of-tolerance fitting.

Static balancing of made rotor. Its purpose is matching of the centre of mass of rotor with rotational axis. It is carried out both manually with the help of knife-edge bearing (Fig. 7.10), and on special test benches (Fig. 7.11)

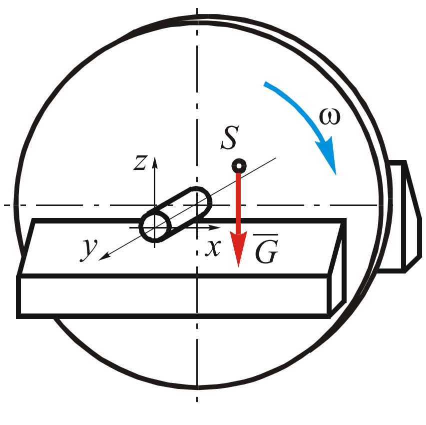


Fig. 7.10. Knife-edge bearing

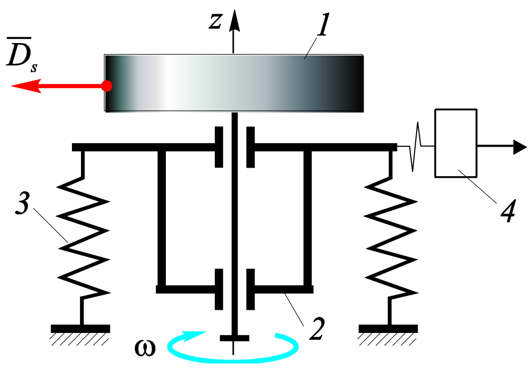


Fig. 7.11. Test bench for rotor balancing

According to the first method the rotor is installed on knife-edge bearings. That almost excludes friction in support points. If rotor is imbalanced, it will try to reach position of stable equilibrium. When rotor reaches such position, its centre of mass will be situated on the vertical straight, which belongs to the plane, passing through the point of support. After finding the place of overweight, it is either removed or added from the opposite side. The procedure is repeating as long as the rotor reaches the condition of indifferent equilibrium on knife-edge bearings, which is possible only under condition of its total balance.

In the conditions of mass production rotor balancing is carried out on special benches, one of possible drawings of which is shown in Fig. 7.11.

Rotor 1 is fastened in pivots on the plate 2. The plate is installed on elastic foundation 3, which allows high movements in space. In case of imbalance the plate executes compound motion (axis z cones). Sensor 4 oscillates. These oscillations are transformed in electrical signal, which passes to computer. After necessary calculations precise coordinates of overweight are defined.

Dynamic balancing of made rotor. It purpose is matching of principal axis of inertia of the rotor with rotational axis.

The dynamic balancing absolutely must be conducted for long rotors. It is carried out on special balancing stands. They can be of three types: with immovable rotational axis of rotor; with rotational axis, which varies in plane relatively to the second axis; with axis, which carriers out space motion.

Fig. 7.12 illustrates scheme of a balancing stand.

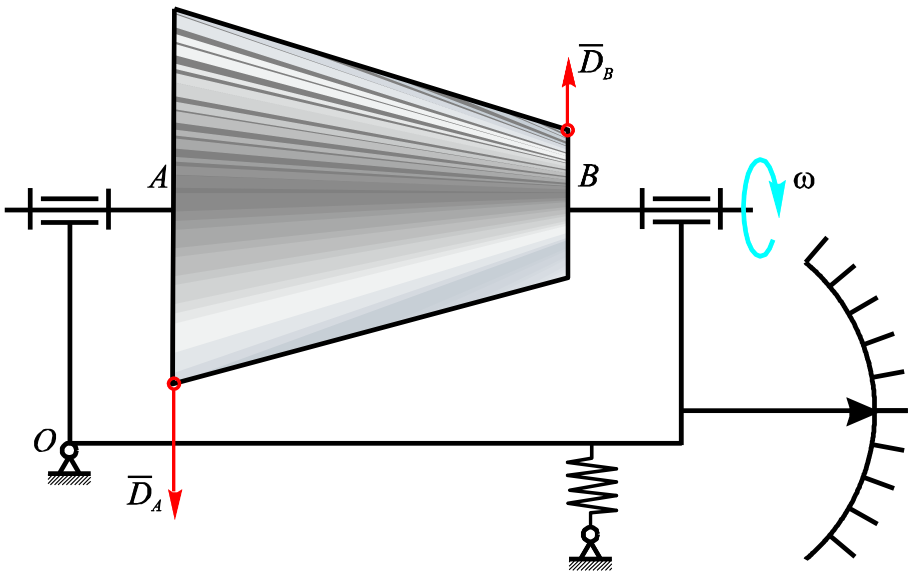


Fig. 7.12. Scheme of a dynamic balancing stand

Here A and B are planes of correction. Firstly, we eliminate imbalance in section B. For this reason in several run-ups we define value of amplitude of oscillation A_{Bi}, which is proportional to value of imbalance D_B. Thus, at the first (basic) starting the rotor takes a run in the initial state. In two trial starts the rotor takes a run with auxiliary stipulated masses in section B, which allows by the value of amplitude to define a precise value of the imbalance D_B. Then a rotor inverts for 180º and the same actions repeat for the section A.

For an accurate balancing it is carried out in vacuum. It decreases air drag and drive power, considering turbine weight.

QUESTIONS FOR SELF-TESTING

1. What is the danger of exploitation of imbalanced mechanisms?
2. Under what conditions is there static imbalance of the mechanism?
3. Under what conditions is there moment imbalance of the mechanism?
4. What condition is achieved with static balancing?
5. Write down the condition, which must correspond to the two-mass model of the body if it performs planar motion.
6. How many corrective masses should be included in the four-bar linkage for its balancing and where are they placed?
7. What conditions are achieved with moment balancing?
8. How is moment balancing of the mechanism performed?
9. What elements of machines are called rotors?
10. What is called the eccentricity of the rotor mass?
11. Write down the formula for determining the resultant vector of rotor imbalance.
12. How is the moment of the rotor imbalance determined?
13. Under what condition does a static rotor imbalance occur?
14. How many corrective masses should be used to eliminate the static imbalance of the rotor?
15. Under what condition does moment rotor imbalance occur?
16. What is the relative position of the principal central axis of inertia of the rotor and its rotational axis when there is a moment imbalance?
17. How many corrective masses should be used to eliminate the moment imbalance of the rotor?
18. Under what condition does dynamic rotor imbalance occur?
19. What is the relative position of the principal central axis of inertia of the rotor and its rotational axis when there is a dynamic imbalance?
20. How many corrective masses should be used to eliminate the dynamic imbalance of the rotor?

21. What is the main task of static balancing of made rotors?
22. For which rotors does only static balancing carry out?
23. What are the ways to eliminate static imbalance of made rotors?
24. What is the purpose of dynamic balancing of made rotors?
25. What ways is the dynamic balancing of made rotors implemented?

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ENGLISH-UKRAINIAN GLOSSARY

Термінологічний англо-український словник

A

Ability вміння.
Absolute velocity абсолютна швидкість.
Acceleration прискорення
  Accelerated motion прискорений рух.
  Acceleration analogue аналог прискорення.
  Acceleration diagram план прискорень.
  Normal acceleration нормальне прискорення.
  Tangential acceleration тангенціальне прискорення.
  Total acceleration повне прискорення.
Accrued зрослі (можливості), накопичений.
Accuracy точність.
Accurate правильний, точний.
Active force активна сила.
Achievement досягнення.
Action (*син.* **effect, impact, influence)** вплив.
Actual speed дійсна швидкість.
Actuation time (*син.* **reaction time, response time)** час спрацювання
Actuating mechanism, actuator виконавчий механізм.
Acute angle гострій кут.
Adjust ув'язувати.
Admitted region область допустимих значень.
Admissible допустимий.
Advanced передовий, прогресивний.
Advantage перевага.
Adverse conditions несприятливі умови.
Algebraic алгебраїчний.
Ambit границі, діапазон, розмах, о́кіл (точки).
Amplitude of oscillation амплітуда коливань.
Analytical method аналітичний метод.
Angular acceleration кутове прискорення.
Angular velocity кутова швидкість.
Angularly під кутом.
Antiphase протифаза.
Application практичне застосування.
Arbitrary довільний.
Arbitrarily chosen довільно вибраний.
Arbitrary scale довільний масштаб.
Arc length довжина дуги.
Arc of circle (*син.* **circular arc)** дуга кола.
Arm повідок.
Arm of force плече сили.
As few as possible (*син.* **a minimum of)** мінімальна кількість.
Aslant (*син.* **obliquely)** похило.
Aspect ratio співвідношення розмірів.
Assembling складання.
Assemblage of forces система сил.
Assign присвоювати, задавати.
Assur group група Ассура.
Assurance забезпечення.
Asynchronous motor асинхронний електродвигун.
At least хоча б, принаймні.
At right angle to … перпендикулярно до …
Attach (to) прив'язувати.
Attachment допоміжна деталь.
Attribute ознака.
Attrition (*син.* **deterioration, tearing, wear, wear-out)** знос, зношування.
Augmentation підвищення.
Auxiliary додатковий, підсобний.
Availability (*син.* **working capacity, operational integrity)** роботоспроможність.
Average speed середня швидкість.
Axes осі.
Axis (*син.* **motion freedom)** ступінь рухливості.
Axle base міжосьова відстань.

B

Back-and-forth motion зворотно-поступальний рух.
Backlash зазор, коловий зазор в зубчастій передачі.
Balance [balancing] force зрівноважувальна сила.
Balanced state рівноважний стан.
Balancer балансир.
Balancing зрівноважування, балансування.
Balancing stand балансувальний стенд.
Ball-and-socket hinge сферичний шарнір.
Barren backoff непродуктивні втрати потужності.
… be accompanied (by) супроводжуватись.
Base фундамент.
Base tangent спільна нормаль.
Basic property основна властивість.
Basic dimensions основні розміри.
Basic link базисна ланка.
Beam(*син.* **connecting rod, con-rod, coupler)** шатун.
Beam-and-crank mechanism кривошипно-шатунний механізм.
Bearing part, bearing підшипник.
Become disconnected роз'єднуватись.
Below (*син.* **from below, underneath)** знизу.
Belt пас.
Belt transmission пасова передача.
Bench стенд.
Best-case value найкраще значення.
Bound границя.
Blast вихлоп.
Brake governor гальмівний регулятор.
Break of a curve злам кривої.
By hand вручну.

C

Calculating-solving mechanism обчислювально-розв'язальний механізм.
Cam кулачок.
Cam mechanism (*син.* **cam box, cam gear)** кулачковий механізм.
Cancel відміна, закреслювання, анулювати, відміняти, скорочувати (дріб, рівняння)
Carrying capacity вантажопідйомність.
Cause причина.
Centre центр.
  Centre distance міжцентрова відстань.
  Centre line лінія центрів.
  Centre line of thrust лінія дії рівнодійної.
  Centre of curvature центр кривини.
  Centre of gravity центр ваги.
  Centre of mass центр мас.
  Centre of oscillation центр коливань (хитання).
  Centre of rotation центр обертання.
Centrifugal governor відцентровий регулятор.
Centripetal acceleration доцентрове прискорення.
Centrode (*син.* **centroid line)** центроїда.
Certain деякий.
Chain ланцюг.
  Chain loop гілка ланцюга.
Changing зміна, корекція параметрів.
Choke заслінка (карбюратора).
Chord хорда.
Chord method метод хорд.
Circle коло, круг.
Circular круговий.
  Circular arc дуга кола
Circumference довжина кола.
Circumscribe описувати.
Closed kinematic chain замкнений кінематичний ланцюг.
Closure condition умова замкненості.
Clutch муфта.
Coefficient коефіцієнт.
  Coefficient of speed fluctuation коефіцієнт нерівномірності руху.
  Coefficient of efficiency (*син.* **efficiency, coefficient of performance, COP)** коефіцієнт корисної дії .
Cog-wheel зубчасте колесо.
Coincidence збіг, суміщення.
Combustion згорання.
Combustible mixture робоча суміш (ДВС).
Common tangent спільна дотична.
Compactness компактність.
Comparative estimation порівняльне оцінювання.
Comparatively відносно.
Compensate балансувати, зрівноважувати, компенсувати, надолужувати.
Compensating компенсація, компенсуючий
  Complement доповнювати.
Compound motion складний рух.
Compound motion theorem теорема про складний рух.
Concurrence of lines перетин ліній.
Condition of equilibrium умови рівноваги.
Confine обмежувати.
Congruous відповідний.
Cone конус.
Conical (*син.* **taper)** конічний.
Connecting rod (*син.* **con-rod, coupler, beam)** шатун.
Constancy сталість.
Constant magnitude стала величина.
Constituent складова, компонента.
Constraint в'язь, обмеження, напруженість, скутість.
  Differential constraint диференціальна в'язь.
  Geometrical constraint геометрична в'язь.
  Holonomic constraint голономна в'язь.
  Ideal constraint ідеальна в'язь.
  Nonholonomic constraint неголономна в'язь.
  Nonreleasing constraint незвільнювана в'язь.
  Passive constraint пасивна в'язь.
  Redundant constraint зайва в'язь.
Construction конструкція, будівництво, побудова.
Construction unit елемент конструкції.
Contact контакт, дотик, зв'язок.
  Contact area площадка (поверхня) контакту.
  Contact line лінія контакту.
  Contact point (*син.*, **point of tangency, tangent point)** точка дотику.
  Contact zone контактна зона.
Contacting bodies контактуючі тіла.
Contacting pair контактна пара.
Continuous неперервний.
  Continuous motion неперервний рух.
Contrary assertion протилежне твердження.
Converting перетворення.
Convex camber випуклість.
Co-ordinates система координат.
Coordinate origin початок координат.
Coriolis acceleration прискорення Коріаліса.
Corrective корегуючий.
Cosine косинус, косинусоїдальний.
Coulomb friction (*син.* **dry friction)** сухе тертя.
Count рахувати.
Counteract зрівноважувати, протидіяти.
Counterweight противага.
Crank (*син.* **crankshaft)** кривошип.
  Crank mechanism кривошипний механізм
  Crank-rocker mechanism (*син.* **crank-guide, crank-and-slot mechanism, quick-return link mechanism,)** кривошипно-кулісний механізм.
  Crank-and-rod mechanism кривошипно-шатунний механізм.
  Crank-and-slider mechanism кривошипно-повзунковий механізм.
Crankshaft колінчастий вал.
Cross хрест, перекреслювати, перетинати(ся), перехрещувати(ся), переходити.
Crosscut поперечний, поперечний переріз.
Crosshead (*син.* **slider, sliding block, cylinder piston)** повзун.
Cumulative сумарний.

Curvature кривизна; вигин, згин; викривлення.
Curve крива (лінія); вигин; кривизна; закруглення; гнути, згинати; вигинати(ся).
Current поточний.
Current value поточне значення.
Current position поточне положення.
Smooth curve гладка крива.
Cutting різання.
Cutting machine металорізальний верстат.
Cycle цикл.
Cyclically циклічно.
Cycling циклічність.
Cycloid (*син.* **cyclic curve)** циклоїда.
Cylinder циліндр.
Cylinder piston (*син.* **crosshead, slider, sliding block)** повзун.
Cylindric(al) slider (*син.* **piston)** циліндричний повзун.
Cylindric(al) surface (*син.* **radial surface)** циліндрична поверхня

D

Datum дана величина, елемент даних; вихідний рівень.
Datum point точка зведення.
Datum link ланка зведення.
Datums база, базова точка (лінія, площина), початок відліку; репер; точка (лінія, площина) зведення.
DC motor двигун постійного струму.
Dead weight власна вага.
Decelerate уповільнення; зменшувати швидкість (кількість обертів).
Degree of freedom ступінь свободи.
Delineate зображати, креслити, робити начерк.
Demand вимога; вимагати, потребувати.
Dependence залежність.
Depth глибина.
Derivative of function похідна функції.
Derivative sign знак похідної.
Design креслення, ескіз, рисунок; проектувати, конструювати, робити ескіз.
Design factor розрахунковий коефіцієнт.
Design formula (*син.* **design equation)** розрахункова формула.
Design stage стадія проектування.
Design size проектний розмір.
Deterioration (*син.* **attrition, tearing, wear, wear-out)** знос, зношування.
Device прилад, пристрій, пристосування; апарат.
Devise розробляти, винаходити, придумувати.
Diesel-compressor дизель-компресор.
Diesel engine дизельний двигун, дизель.
Differential диференціал (*мат.*).
Differential gear (mechanism) диференціал (*техн.*), диференціальний механізм.
Differential gear диференціальний зубчастий механізм.
Differentiation диференціювання.
Dimension розмір, вимірність.
Dimensions габарити.
Direct current постійний струм.
Disk диск.
Dissipation розсіювання (енергії).
Dotted line (*син.* **dot line, line dotted)** пунктирна лінія, пунктир.
Line dotted (*син.* **dot line)** пунктир, пунктирна лінія.
Double подвійний, здвоєний, спарений; удвічі.
Double arm groups двоповідкова група.
Double stroke подвійний хід (поршня)
Drag опір середовища.
Drilling rig (*син.* **rig)** бурова установка.
Drive передача, привід, привідний механізм.
Drive power потужність привода.
Drive shaft привідний (головний) вал.
Drive stage ступінь передачі (механічної)
Driven ведений.
Driving ведучий, привідний.
Driving force рушійна сила.
Durability (*син.* **reliability)** надійність.
Duration (*син.* **time)** тривалість.
Dyad діада.
Dynamic динамічний, активний, діючий.
Dynamic balancing динамічне балансування.
Dynamic component динамічна складова.
Dynamic model динамічна модель.
Dynamic force analysis (*син.* **kinetostatics)** кінетостатичний аналіз, силовий розрахунок.
Dynamics динаміка, рушійні сили.
Dynamotor двигун-генераторний агрегат.

E

Eccentric ексцентрик.
Eccentricity ексцентриситет.
Economy in use економічний (про машину, обладнання).
Effect дія, вплив, явище, результат; робити, чинити; виконувати, здійснювати.
Elastic пружний, гнучкий, гума (шнур)
Elastic force пружна сила.
Elastic foundation пружна основа.
Elasticity пружність.
Eliminate усувати, виправляти.
Ellipse еліпс.
Energy gap перепад енергії.
End-capping (*син.* **locking)** замикаючий.
Engage входити в контакт, входити в зачеплення.
Engagement зачеплення.
Engine двигун, мотор.
Engine drive shaft головний вал двигуна.
Engineering машинобудування, інженерія, технічний.

Engineering costs витрати на проектування.
Engineering data технічні дані (характеристики), технічна документація.
Engineering decision технічне рішення.
Engineering factors технічні характеристики.
Engineering kinematics кінематика механізмів.
Engineered value розрахункова величина.
Engineer's system of units британська система з основними одиницями: фут, секунда, слаг.
Environment середовище, довкілля.
Environmental resistance forces сили опору середовища.
Equality рівність.
Equation рівняння.
Equation of motion рівняння руху.
Equidistant рівновіддалений, еквідистантний.
Equilibrium баланс, рівновага.
Equilibrium equation рівняння рівноваги
Equivalent mechanism замінний механізм.
Error похибка, помилка.
Estimate оцінювати.
Essential важливий, суттєвий, необхідний.
Evolvent евольвента.
Exact точний.
Excess (*син.* surplus) надлишок.
Exert діяти (про силу).
Expenditure витрати.
Expenditures on designing витрати на проектування.
External зовнішній.
External toothing зовнішнє зачеплення.
Extreme position of the mechanism крайнє положення механізму.

F

Fail невдача, зазнати невдачі, виходити з ладу, ламатися.
Feature ознака, властивість.
Fidelity точність**,** правильність, точність відтворення.
Fidelity of reproduction точність відтворення.
Field environment експлуатаційні умови.
Finite size кінцева величина.
Fit придатний; відповідний, годитися, бути придатним, постачати.
Fitted curves спряжені криві.
Fitting складання.
Fixed joint (*син.* permanent connection) нерухоме з’єднання.
Fixed link нерухома ланка.
Flank clearance бічний зазор.
Flat площина, плоска поверхня, плоский, плаский; рівний.
Fly-wheel махове колесо, маховик.
Follower (*син.* pusher) штовхач.
Foot-throttle педаль газу (акселератора).
Force сила, долати опір, змушувати.
Force closure силове замикання.
Force couple пара сил.
Force polygon многокутник сил.
Forge ковальський прес.
Form форма, обрис, формувати, складати, утворювати.
Form closure геометричне замикання.
Form of loading закон навантаження.
Form surface фасонна (криволінійна) поверхня
Frame (*син.* fixed link, housing) стояк, рама, корпус, каркас.
Friction тертя, сила тертя.
Friction(al) disk фрикційній диск.
Friction(al) force (*син.* friction) сила тертя.
Friction gear фрикційна передача.
Friction torque момент тертя.
Friction work робота сил тертя.
Kinetic friction тертя ковзання.
From the direction of … з боку.
Fuel delivery (*син.* fuel feeding) подача пального.
Full повний, наповнений, завершений; дуже, сильно, повністю.
Full angle повний кут (кут у 360°).
Full line суцільна лінія.
Full revolution per cycle повний оберт за цикл.
Full turn повний оберт.

G

Gap зазор, люфт, проміжок, щілина, інтервал, пробіл, пропуск, велика розбіжність, розрив.
Gas turbine engine газотурбінний двигун.
Gear зубчаста передача, шестірня, привід, механізм, апарат; прилад; пристрій; зчіплювати(ся) (про зубці коліс).
Gear cluster блок зубчастих коліс.
Geared linkage mechanism зубчасто-важільний механізм.
General загальний, головний, поширений, загальноприйнятий,
General form загальний вид.
Generalized coordinate узагальнена координата.
Generally (*син.* in the general case/way) в загальному випадку.
Generating ray (*син.* generating line) твірна.
Generator генератор.
Geneva mechanism (*син.* Geneva drive, Geneva stop, maltese-cross mechanism) мальтійський механізм.
Geometric(al) геометричний(а).
Geometric(al) constraint геометрична в’язь.
Geometric(al) diagram (*син.* vector diagram) векторна діаграма.
Geometrician геометр.
Geometry геометрія.

Governor регулятор.
Grade (*син.* **nature, property, quality)** якість.
Graphic діаграма, рисунок, креслення, графік; графічний, поданий як креслення або графік.
Graphical (*син.* **graphic)** графічний.
Graphical differentiation графічне диференціювання.
Graphical plotting графічна побудова.
Grapho-analytical method графоаналітичний метод.
Ground (*син.* **fixed link, housing, rack)** стояк, основа.
Guideline рекомендація, загальний курс, напрямок.
Gyrating mass обертальна маса.
Gyration обертання, обертальний рух.

H

Hatch штрихова лінія.
Hatching штриховка.
Hazard шкідливий фактор.
Heel pivot п’ята.
Helix спіраль.
High найвища точка, максимум, високо, сильно, інтенсивно.
High speed shaft швидкісний вал.
High-quality високоякісний.
Higher вищий.
Higher pair вища пара.
Hinge (*син.* **hinge pivot)** шарнір.
Hinge axis вісь шарніра.
Hinged-lever mechanisms шарнірно-важільний механізм.
Holonomic constraint голономна в’язь.
Housing (*син.* **ground, fixed link, frame)** корпус, станина, рама, стояк.
Huge possibilities величезні можливості.
Hyperbolic(al) gearing гіперболоїдна зубчаста передача.
Hyperstatic system (*син.* **redundant system, statically indeterminate system)** статично невизначувана система.
Hypotenuse гіпотенуза.

I

Idle mode режим холостого ходу.
Image образ, картина.
Imaginary уявний.
Imbalance (*син.* **unbalance)** неврівноваженість.
Imbalance resultant of a rotor головний вектор дисбалансу ротора.
Impact удар.
Impossible (*син.* **inadmissible; intolerable)** неприпустимий.
Improve вдосконалювати, покращувати.
In compliance with згідно з.
In inverse proportion обернено пропорційно.
In the general case (way) (*син.* **generally)** в загальному випадку.
In the line of вздовж, у напрямку.
In the range в межах.
In theory (*син.* **theoretically)** теоретично.
Inadmissible (*син.* **intolerable, impossible)** неприпустимий.
Incorrect (*син.* **irregular, mis-, violent, wrong)** неправильний.
Increment приріст.
Inertia інерція; сила інерці.
Inequality нерівність.
Infinite нескінченний, безмежний
The infinite (*син.* **infinity)** нескінченність, безмежність, безмежний простір.
Infinitesimal displacement елементарне переміщення.
Inhomogeneity неоднорідність.
Initial початковий, попередній.
Initial link початкова ланка.
Initial state вихідний стан.
Input вхідні дані.
Input link вхідна ланка.
Insertion включення, введення.
Inside середина; внутрішній; всередині.
Inside envelope внутрішня обвідна (лінія).
Instantaneous миттєвий, моментальний, одночасний.
Instantaneous axis миттєва вісь обертання.
Instantaneous centre of rotation миттєвий центр обертання.
Instantaneous screw axis миттєва гвинтова вісь.
Integrated form інтегральна форма.
Interaction взаємодія.
Intermediate проміжний.
Internal внутрішній.
Internal-combustion engine (*син.* **combustion engine)** двигун внутрішнього згорання.
Internal toothing внутрішнє зачеплення.
Interpenetration взаємопроникнення.
Interrelation взаємозалежність, взаємозв’язок.
Intersect перетинатись.
Intersection лінія перетину.
Intolerable (*син.* **inadmissible, impossible** неприпустимий.
Invention винахід.
Inverse обернений; зворотний; перевернутий; протилежний.
Inverse motion обернений рух.
Inverse proportion обернена пропорція, обернена пропорційність.
Inverse-square-law обернено пропорційний квадрату.
Inversely oriented обернено орієнтований (направлений).
Involute gearing евольвентна зубчаста передача.
Irrational number ірраціональне число.
Irregular (*син.* **incorrect, mis-, violent, wrong)** неправильний.
Irregularity нерівномірність.
Irregularity ratio коефіцієнт нерівномірності.
Isoline ізолінія.

J

Joint об'єднаний, спільний; точка сполучення, стик; з'єднувати, сполучати; припасовувати (частини).
Joint solution сумісний розв’язок.
Justify обґрунтувати.
Jamming заклинювання.

K

Kinematic кінематичний.
  Kinematic chain (*син.* kinematics) кінематичний ланцюг
  Kinematic diagram кінематична діаграма.
  Kinematic pair кінематична пара.
  Kinematic scheme кінематична схема.
Kinematics кінематика.
Kinetic кінетичний.
  Kinetic energy кінетична енергія.
  Kinetic friction тертя ковзання.
Kinetostatics кінетостатика.
Knife-edge bearing ножова опора.
Knob ручка, маховичок.

L

Labeling mechanism механізм маркування.
Laborious трудомісткий.
Labouriousness трудомісткість.
Law закон.
  Law of motion закон руху.
Lay off відкладати (відрізок).
Layer шар, нашарування.
Leave out виключати, не брати до уваги.
Leftmost position крайнє ліве положення.
Leg (side) сторона (трикутника).
Lever важіль; рукоятка, плече важеля.
Leverage система важелів, важільний механізм.
Liable можливий, ймовірний.
Limit границя, межа; граничний розмір, допуск; інтервал значень; обмежувати.
  Limit(ing) point гранична точка.
  Limit state граничний стан.
  Limit(ing) value граничне значення.
Limitation обмеження.
Line лінія, риска, штрих; спосіб дій; напрям.
  Line dotted (*син.* dot line, dotted line) пунктир, пунктирна лінія.
  Line of action of a force лінія дії сили.
Linear лінійний, витягнутий в лінію.
  Linear depentanizer лінійна залежність.
  Linear displacement лінійне переміщення.
  Linear function лінійна функція.
  Linear law лінійний закон.
  Linear velocity лінійна швидкість.
Link ланка, куліса; зв’язувати, з’єднувати, змикати (together, to), зчіпляти (тж. link up).
  Input link вхідна ланка.
  Output link вихідна ланка.
Linkage (*син.* leverage, link mechanism) важільний механізм.
Linking складання.
Load навантаження, вантаж.
Loading навантажування, навантаженість.
  Loading diagram (*син.* design model) розрахункова схема.
  Loading condition характер навантаження.
Locus (*син.* locus of points, point curve) геометричне місце точок.
Longitudinal force осьова сила.
Lopsided нахилений, перекошений.
Lose contact (with) відриватись.
Lower pair нижча пара.
Lowering зниження, зменшення.
Lumped mass зосереджена маса.

M

Magnetic drive електромагнітний привод.
Magnitude величина, розмір.
  Magnitude of vector абсолютна величина вектора
  Magnitude of vector magnitude модуль вектора.
Machine машина; піддавати механічній обробці; обробляти на верстаті.
  Machine aggregate машинний агрегат.
  Machine running control регулювання ходу машини.
Manufacturable (*син.* practically feasible) технологічний.
Matched відповідний, узгоджений.
Maximum possible максимально можливий.**Measure** міра, одиниця виміру, масштаб, мірило, критерій; дільник (*мат.*); міряти, вимірювати, відміряти; мати розміри.
Measurement визначення розміру, вимірювання.
Mean level середній рівень.
Mechanism механізм, апарат, конструкція, пристрій; техніка (виконання).
  Actuating mechanism (*син.* actuator) виконавчий механізм.
  Beam-and-crank mechanism кривошипно-шатунний механізм.
  Calculating-solving mechanisms розрахунково-вирішальні механізми.

Cam mechanism (*син.* **cam box, cam gear)** кулачковий механізм.
Crank mechanism кривошипний механізм.
Crank-and-rod mechanism кривошипно-шатунний механізм.
Crank-and-slider mechanism кривошипно-повзунковий механізм.
Crank-rocker mechanism (*син.* **crank-guide, quick-return link mechanism, crank-and-slot mechanism)** кривошипно-кулісний механізм.
Differential mechanism (gear) диференціал (*техн.*), диференціальний механізм.
Equivalent mechanism замінний механізм.
Geared linkage mechanism зубчасто-важільний механізм.
Geneva cross mechanism (*син.* **maltese-cross mechanism)** мальтійський механізм.
Hinged-lever mechanisms шарнірно-важільний механізм.
Link mechanism (*син.* **leverage, linkage**) важільний механізм.
Packing mechanism механізм пакування.
Planar mechanism плоский механізм
Planetary mechanism планетарний механізм.
Plotting mechanism графопобудовний механізм.
Primary (elementary) mechanism початковий механізм.
Push-up mechanism механізм подачі.
Ratchet mechanism храповий механізм.
Wedge-bar mechanism клиновий механізм.
Medium середина; середнє число; середовище; засіб, спосіб, шлях; середній, проміжний.
Mentioned вказаний.
Method метод; спосіб, система, порядок.
Method of co-ordinates transformation метод перетворення координат.
Microasperity мікронерівність.
Minimal найменший
Minimal boundary мінімальна границя.
Minimization мінімізація.
Mismatch не збігатися.
Mode спосіб.
Moderator модератор.
Modulus абсолютне значення, абсолютна величина.
Moment момент (сили), момент (проміжок часу), мить.
Motion рух, хід (машини), приводити в рух.
Motion distance величина переміщення.
Motion equation рівняння руху.
Motion freedom (*син.* **axis)** ступінь рухливості.
Motion link спрямовуюча.
Motion path траєкторія руху.
Motion transmission передача руху.
Transportation motion переносний рух.
Moveable connection рухоме з’єднання.
Moveable link рухома ланка.
Multiple багаторазовий; багатократний; численний; складний, складений; кратний (*мат.*), кратне число (*мат.*).
Multiply збільшувати(ся); множити (*мат.*).
Multiply by помножити на.
Multiple gearing багатоступінчаста зубчаста передача.
Multitude безліч.
Mutual взаємний.
Mutually balanced взаємно зрівноважений.

N

Nonholonomic constraint неголономна в’язь.
Nonreleasing constraint незвільнювана в’язь.
No-load conditions (*син.* **idling)** режим холостого ходу.
Non-uniform нерівномірний.
Normal acceleration нормальне прискорення.
Normal line (*син.* **normal, perpendicular)** нормаль.
Nuclear power station атомна електростанція.
Numbers arrays масив чисел.
Number of revolutions швидкість обертання.

O

Obliquely (*син.* **aslant)** похило.
Obliquity нахил.
Obsolescence моральне старіння.
On/to/from the right (of) справа.
Opening відкривання.
Operating condition умови експлуатації.
Operating mode робочий режим.
Operating speed робоча швидкість, частота обертання (*ел.*).
Operational integrity роботоздатність.
Optimal оптимальний.
Optimal choice оптимальний вибір (добір).
Ordinate ордината.
Oscillation (*син.* **vibration)** вібрація, коливання.
Out of (*син.* **beyond, outside)** за, зовні, вище, поза.
Out-of-balance condition неврівноважений стан.
Out-of-tolerance неточний; поза допуском.
Outermost найвіддаленіший.
Output продуктивність.
Output link вихідна ланка.
Outside зовнішня сторона (частина, поверхня); зовнішній; сторонній, що знаходиться зовні; поза, за межами, за межі; крім, за винятком
Overweight надлишкова вага.

P

Packing mechanism механізм пакування.
Pair пара (два однакових предмети).
Pair of compasses циркуль.
Pair-wise interaction попарна взаємодія.
Pairing element елемент кінематичної пари.
Parallel паралельний.
Parallel motion поступальний рух.
Parallel motion link поступально рухома ланка.
Parallel-plane movement плоско-паралельний рух.
Part деталь, частина; частково (*син.* partly); відділятись, від’єднуватись.
Particle матеріальна точка.
Pass рухатися вперед; проходити; перетинати; переходити; перевищувати, виходити за межі; витримати, пройти (випробування) ; відповідати (вимогам); зникати; припинятися.
Passing проходження; побіжний, випадковий.
Passive constraint пасивна в’язь.
Peculiarity особливість, специфічність.
Pedal педаль; натискати на педалі.
Permanent connection (*син.* fixed joint) нерухоме з’єднання.
Permissible дозволений, припустимий
Permit дозволяти, давати дозвіл; надавати можливість; допускати.
Persistence сталість, інерційність.
Phonograph патефон.
Pickup head головка звукознімача.
Pickup stylus голка звукознімача
Piecewise кусочно-лінійний.
Pilot analysis пілотний (передній) аналіз.
Pin joint шарнірне з’єднання, шарнір.
Piston поршень.
    Piston stroke хід поршня.
Pivot шарнір, цапфа; точка опори; точка обертання; стержень; вісь; крутитися, обертатися; надівати на стержень.
Pivot(al) point точка [вісь] повороту.
Plane площина; проекція; рівень (розвитку, знань тощо); крило (літака); плаский, плоский; площинний.
Plane of changing площина корекції.
Plane pinion сателіт.
Planer стругальний верстат.
Planetary gearing (*син.* planetary gear mechanism) планетарний зубчастий механізм.
Plotting machine графобудівник.
Plotting mechanism графопобудовний механізм.
Point точка; момент (часу); крапка (в десяткових дробах); поділка шкали; мета, намір; вістря, гострий кінець; кінчик; наконечник.
    Point curve (*син.* locus, locus of points) геометричне місце точок;
    Point of force application точка прикладання сили.
    Point of inflection точка перегину.
    Point of attack точка прикладання.
    Point of tangency (*син.* contact point, tangent point) точка дотику.
Pole полюс; полюсний.
    Pole of acceleration diagram полюс плану прискорень.
    Pole of inertia полюс інерції.
    Pole of velocity diagram полюс плану швидкостей.
    to be poles apart бути діаметрально протилежним.
Polished відполірований**.**
Position положення; звичайне (правильне) місце; ставити; розташовувати.
    Leftmost position крайнє ліве положення.
    Position of extremum точка екстремуму.
    Position of load application точка прикладання навантаження.
    Position vector (*син.* radius-vector) радіус-вектор.
    Rightmost position крайнє праве положення.
Position function of a mechanism функція положення механізму.
Positional relationship взаємне розташування.
Positive позитивний, реальний, точний; додатний (*мат.*); примусовий (про рух).
Positive closing примусове замикання.
Power transmission механічна передача
Precision точність, чіткість, акуратність; точний.
Precision instrument точний прилад.
Prescribed заданий (закон).
Pressure тиск; стискання; пресування.
    Specific pressure (*син.* unit pressure) питомий тиск.
    Pressure distribution розподіл тиску
Prevalent (*син.* widespread) розповсюджений.
Prevailing (*син.* widespread) широко розповсюджений.
Primary (elementary) mechanism початковий механізм.
Primitive примітивний.
Principal головний, основний.
    Principal central axis of inertia головна центральна вісь інерції.
Principle принцип, правило, закон; першопричина; причина, джерело.
    The principle of virtual displacements принцип можливих переміщень.
Procedure процедура, методика.
Process технологічний процес.
Processing обробляння; обробка даних (*комп.*).
Processing equipment технологічне обладнання.
Product продукція, продукт, виріб; добуток (*мат.*); результат, наслідок.
Product of inertia відцентровий момент інерції.
Proportion співвідношення, пропорція.
Proportional пропорційний.
Push-pull двотактний.
Push натискати; тиск, натискання.
Pusher штовхач.
Push-up mechanism механізм подачі.

Q

Quotient velocity ratio частинне передатне відношення.
Quantitative assessment кількісна оцінка.
Quantity кількість; величина (*мат.*).
Quantity of rotations per second кількість обертів за секунду.
Quick швидкий; швидко, скоро.
Quick-return link mechanism кривошипно-кулісний механізм.
Quickly швидко.
Quality якість; властивість; особливість; характерна риса.
Quality coefficient показник якості.
Quality metrics показники якості.
Quality rating оцінка якості.

R

Rack (*син.* **fixed link, ground)** стояк (у структурній схемі).
Radius (*син.* **spoke)** шпиця.
Radius-vector (*син.* **position vector)** радіус-вектор.
Radius of curvature радіус кривини.
Radius of gyration радіус інерції.
Random довільний.
Range інтервал; класифікувати; коливатися в певних межах.
Ratchet mechanism храповий механізм.
Rate відповідна частина; пропорція; коефіцієнт; ступінь; відсоток; частка; визначати; встановлювати.
Ratio відношення, пропорція; коефіцієнт; співвідношення; передатне число.
Ratio of side dimensions співвідношення розмірів сторін.
Rational number раціональне число.
Ray промінь, напівпряма.
Reaction (*син.* **reacting force)** реакція (сила протидії).
Reaction time (*син.* **response time)** стала часу.
Reasonable раціональний, коректний, прийнятний.
Rectangle прямокутник.
Rectilinear (*син.* **rectilineal)** прямолінійний.
Rectilinear generator прямолінійна твірна.
Reduced inertia moment зведений момент інерції.
Reduced (equivalent) mass зведена маса.
Reduced forces зведена сила.
Reduction of masses зведення мас.
Reduction of forces зведення сил.
Reduction ratio (*син.* **transmission ratio)** передатне число.
Refem line лінія відліку.
Reference frame (*син.* **reference system)** система відліку.
Redundant constraint зайва в'язь.
Redundant system (*син.* **hyperstatic system, statically indeterminate system)** статично невизначувана система.
Relation відношення; залежність; зв'язок.
Relationship співвідношення, залежність.
Relative displacement відносне зміщення.
Relative motion відносний рух.
Relaxation релаксація.
Reliability (*син.* **durability)** надійність.
Repetition багаторазовість; повторення.
Represent зображати; втілювати, уособлювати, являти собою.
Reproduction відтворення.
Reset скидати (оберти).
Resistance force сила опору.
Resistance to rolling тертя кочення.
Response time (*син.* **reaction time)** стала часу, час спрацювання.
Responsibility відповідальність.
Rest (upon) опиратись (на).
Resultant рівнодійна, головний вектор.
Resultant vector головний вектор.
Return повернення.
Reversal реверсування, зміна напрямку на протилежний.
Reversibility властивість оберненості.
Reversing movement зміна напрямку руху.
Right-angled triangle прямокутний трикутник.
Rightmost position крайнє праве положення.
Rigid body абсолютно тверде тіло.
Rigidity жорсткість.
Rigidly bound жорстко зв'язаний.
Ring кільце, обід.
Rocker коромисло, балансир, шатун, хитний важіль, куліса.
Rocker-and-crank mechanism кривошипно-коромисловий механізм.
Rocker-and-slider mechanism коромислово-повзунковий механізм.
Rod стержень, брус; шток, тяга.
Roll обертання; хитання; вал, барабан, циліндр; вальці; крутити(ся); обертати(ся); перекочувати(ся); повертати(ся); прокатувати; вальцювати, плющити (метал)
Roller ролик.
Rolling friction тертя кочення.
Rolling mill прокатний стан.
Rotary overburden excavator роторний екскаватор.
Rotary (rotor) machine роторна машина.
Rotating sense напрямок обертання.
Rotation обертання; періодичне повторення; чергування.
Rotation angle кут повороту.
Rotational axis вісь обертання.
Rotor ротор.

Routine problem типова задача.
Run біг; пробіг (локомотива, вагона), відрізок шляху, прогін (залізниці); політ, переліт, рейс, відстань, яку пролітає літак; хід, робота, дія (машини, двигуна); триваючий, неперервний; бігти, крутитись, обертатись; працювати, функціонувати; рухати, переміщати.
Run down зупинятися (про машину).
Run-up пуск.
Run-up of engine збільшення кількості обертів двигуна.
Run-up time час розгону.
Running хід, робочий хід, робочий стан (машини); неперервний, послідовний

S

Saltatory variation стрибкоподібна зміна.
Scale масштаб.
Scale factor масштабний коефіцієнт.
Scaling magnitude масштабне значення.
Scoop excavator ковшовий екскаватор.
Screw-nut gear передача гвинт-гайка.
Section переріз; відрізок; сегмент, частина; параграф; розділ книги; ділити на частини.
Self-feeder механізм живлення.
Separation відділення.
Sequence послідовність; порядок, ряд; наслідок, результат.
Sequential послідовний.
Series серія, ряд, група, послідовне з’єднання.
Series coupling (*син.* **series connection)** послідовне з’єднання.
Set-point заданий.
Several кілька; кожний, окремий; свій, власний, індивідуальний.
Severe несприятливий.
Shaft вал.
Shaded заштрихований.
Similar подібний, адекватний.
Similarity подібність.
Similarity theorem теорема про подібність.
Simultaneously одночасно.
Single gearing одноступінчаста зубчаста передача.
Single-throw crankshaft одноколінний колінчастий вал.
Sinusoid синусоїда.
Skew перекошуватись, нахилятись, перехрещуватись (про осі)
Skill майстерність, уміння; вправність.
Slide gauge (*син.* **trammel)** еліпсограф, штангенциркуль.
Slider (*син.* **crosshead, cylinder piston, sliding block)** повзун.
Slider-crank mechanism кривошипно-повзунковий механізм.
Sliding ковзання, проковзування.
Sliding block (*син.* **crosshead, cylinder piston, slider)** повзун.
Sliding friction тертя ковзання.
Sliding pair поступальна пара.
Slideway спрямовуюча (верстата).
Slight displacement мале переміщення.
Slope ratio (*син.* **slope)** тангенс кута нахилу.
Smooth curve гладка крива.
Smoothed (*син.* **stepless)** плавний.
Sound reproduction відтворення звуку.
Spatial force system просторова система сил.
Specific спеціальний, особливий, конкретний; певний, точний; характерний; питомий.
Specific pressure (*син.* **unit pressure)** питомий тиск.
Specific weight густина (матеріалу).
Specifically зокрема.
Speed of rotation частота обертання.
Sphere сфера.
Spheric сферичний.
Spring пружина, ресора; пружність, еластичність; відскік, випрямлення.
Spiral spring спіральна пружина.
Spoke (*син.* **radius)** шпиця.
Stable equilibrium стійка рівновага.
Standard стандартний.
Standing axis вертикальна вісь.
Start починати; братися (за щось); пускати (машину); пуск в хід; рушання з місця.
Start (*син.* **run)** розбіг, розгін (машини, агрегату).
Starter стартер.
Static(al) статичний, стаціонарний, нерухомий**.**
Statically indeterminate system (*син.* **hyperstatic system, redundant system)** статично невизначувана система.
Statics статика.
Stationary нерухомий.
Stationary state нерухомий стан.
Steady speed постійна швидкість.
Steady regime (*син.* **steady run, steady state mode)** усталений режим.
Stepless (*син.* **smoothed)** плавний, безступінчастий.
Stipulated фіксований, обумовлений.
Stop зупиняти(ся); припиняти(ся); закінчувати(ся); зупинка, затримка, припинення; кінець; пауза, перерва; обмежник, стопор.
Stop (*син.* **running-out)** вибіг (машини, агрегату).
Store накопичувати.
Structure структура; будова; будівля, споруда.
Structure chart структурна схема.
Structural group (*син.* **Assur group, structure group)** структурна група, група Ассура.
Structure synthesis структурний синтез.
Successive values послідовні значення.

Summand доданок.
Sun gear сонячне колесо.
Superpose суміщати, накладати.
Superposed суміщений.
Support reaction опорна реакція.
Support point точка опори.
Surface поверхня; обробляти поверхню, покривати поверхню (with - чим-небудь.).
Surface abrasion абразивний знос поверхні.
Surface bed поверхневий шар.
Surface capacity поверхнева міцність.
Surface condition чистота поверхні, якість поверхні.
Surface deterioration знос поверхні.
Swage штампувальний молот.
Swing коливати(ся), гойдати(ся); хитати(ся); гойдання, хитання; коливання; амплітуда коливань.
Swinging гойдання, хитання, коливання; поворотний.
Swing link хитний важіль.
Swinging arm хитний важіль, куліса.
Swinging movement зворотно-обертальний рух.

T

Take a run розганятись.
Tangent дотична; тангенс; дотичний
Tangent to curve дотична до кривої.
Tangent point (*син.* **point of tangency, contact point)** точка дотику.
Tangential acceleration тангенціальне прискорення.
Tearing (*син.* **attrition, deterioration, wear, wear-out)** знос, зношування.
Teflon фторопласт, тефлон.
Tend to infinity прямувати до нескінченності.
Tilt angle кут нахилу.
Tilted нахилений.
Time derivative похідна по часу.
To decompose a force розкласти силу.
To meet in a point перетинатись в точці.
To prove a theorem довести теорему.
Tool інструмент.
Tooth gear зубчаста передача.
Tooth profile (*син.* **tooth form)** профіль зубця.
Toothed зубчастий.
Toothing зачеплення.
Top speed гранична швидкість.
Touch дотикатись.
Total acceleration повне прискорення.
Total work сумарна робота.
Trammel еліпсограф, штангенциркуль; заважати перешкоджати, утруднювати.
Transfer function of a mechanism передатна функція механізму
Transient regime неусталений режим.
Transmission ratio (*син.* **reduction ratio)** передатне число.
Transportation motion переносний рух.
Trapezoids (*син.* **trapezium)** трапеція.
Traverse speed швидкість переміщення.
Trigonometric transformation тригонометричне перетворення.
Triple arm group (*син.* **triad)** триповідкова група (тріада).
Truck body hoist підйомник кузова вантажівки.
Turbine турбіна.
Turbine wheel робоче колесо турбіни.
Turn крутити(ся); повертати(ся), обертати(ся); оберт (колеса); поворот; зміна напряму.
Turning pair (*син.* **hinge)** обертальна пара.
Turning speed частота обертання.
Twin crank спарений кривошип.
Two-mass system двомасова система.
Two-stroke internal combustion engine двотактний двигун внутрішнього згорання.

U

Unclosed kinematic chain незамкнений кінематичний ланцюг.
Unbalance (*син.* **imbalance)** неврівноваженість.
Uniform одноманітний; однорідний; постійний, сталий.
Uniform acceleration рівномірне прискорення, рівномірно прискорений рух.
Uniform motion рівномірний рух.
Uniform rotation рівномірне обертання.
Uniformly accelerated motion рівноприскорений рух.
Unique однозначний; однозначно визначуваний (*мат.*); унікальний.
Uniquely defined (*син.* **unique)** однозначно визначуваний.
Unit агрегат, секція; вузол; елемент; одиниця; ціле; одиниця вимірювання.
Unit pressure (*син.* **specific pressure)** питомий тиск.
Unlocking розмикання.
Unreliable ненадійний.
Usage використання, користування.

V

Validation перевірка достовірності (правильності).
Variable змінний, змінна величина.
Vector вектор; векторний
Vector equation векторне рівняння
Vector polygon векторний многокутник.
Vector form векторна форма.
Velocity швидкість.
Velocity analogues аналог швидкості.
Velocity diagram план швидкостей.
Vertex angle кут при вершині.
Vertex of triangle вершина трикутника.
Vibration вібрація, коливання.
Vice versa навпаки, протилежно.
Visualization наочність; візуалізація.

W

Walking beam рухома траверса.
Wear (*син.* wear-out, attrition, deterioration, tearing) знос, зношування.
Wedge-bar mechanism клиновий механізм.
Wheelwork зубчастий механізм (зубчаста передача).
Width ширина.
Working costs експлуатаційні витрати.
Work of driving force робота рушійних сил.
Work of resistance forces робота сил опору.
Worm (*син.* worm screw) черв'як, шнек, черв'ячний гвинт.
Worm shaft черв'ячний вал, вал шнека.
Worm toothing черв'ячне зачеплення
Worm-gear (*син.* worm drive, worm gear set, worm gearing) черв'ячна передача.

ABSTRACT IN UKRAINIAN

Викладений в підручнику курс теорії механізмів і машин призначений для студентів спеціальності 131 “Прикладна механіка” спеціалізації “Динаміка і міцність машин” та “Інформаційні системи та технології в авіабудуванні”. Фахівці цієї галузі займають специфічне місце серед інженерів-механіків, оскільки їх подальша професійна діяльність має бути пов’язана із проведенням складних розрахунків міцнісної надійності та довговічності машин, споруд та їх елементівіз заданою точністю та з урахуванням численних технологічних та експлуатаційних факторів. Значний обсяг спеціальних дисциплін теоретичного спрямування, передбачених навчальним планом спеціалізації, значною мірою наближається до механіко-математичних курсів класичних університетів. З іншого боку інженерне спрямування підготовки майбутніх спеціалістів у галузі динаміки і міцності машин потребує ґрунтовного вивчення загальноінженерних дисциплін, зокрема і теорії механізмів і машин, для набуття знань, умінь та навичок у проведенні аналізу навантаженості реальних конструкцій, впливу різноманітних динамічних факторів, пов’язаних із рухом твердих тіл у складі машин і механізмів, та побудови адекватних розрахункових схем.

Частина 1 підручника присвячена розгляду принципів класифікації та аналізу механізмів і машин. Вона містить вступ і сім розділів.

У розділі 1 висвітлюються основні задачі дисципліни, історична довідка, її місце та роль серед інших загальноінженерних дисциплін.

Розділ 2 присвячений висвітленню основних понять, термінів та означень, структури механізмів і машин, принципів їх класифікації.

У наступних розділах викладені основні методи кінематичного і динамічного аналізу механізмів і машин, їх силового розрахунку. Без оволодіння загальними методами аналізу механізмів, машин, приладів, правильності їх вибору при розв’язанні конкретних інженерних задач, без розуміння загальних принципів реалізації руху, взаємодії елементів у складі механічних систем, що визначають їх кінематичні та динамічні характеристики, неможлива побудова достовірних розрахункових схем об'єктів дослідження для подальшої оцінки їх міцнісної надійності та ресурсу. В підручнику широко представлені як аналітичні, так і графоаналітичні методи дослідження. Дається порівняльний аналіз можливостей різних методів, їх переваг та недоліків, акцентується увага на раціональному виборі тих чи інших методів дослідження залежно від поставленої задачі та необхідної точності проведення розрахунків. Також розглядаються питання регулювання руху машин і механізмів, їх урівноважування як на стадії проектування, так і після виготовлення, для забезпечення найоптимальніших динамічних характеристик.

Теоретичний матеріал супроводжується прикладами розв'язання практичних задач для різних типів механізмів. У кінці кожного розділу наведено запитання
для самоперевірки студентами знань в процесі вивчення дисципліни.

Другу частину підручника буде присвячено висвітленню питань синтезу механізмів з нижчими і вищими кінематичними парами, їх силового розрахунку з урахуванням тертя в кінематичних парах, зносостійкості та віброзахисту в процесі експлуатації.