

## Direct Robust Active-Reactive Power Control of a Doubly-Fed Induction Machine

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### Introduction

A vector controlled Doubly-Fed Induction Machine is an attractive solution for high performance restricted speed-range electric drives and energy generation applications [1]. Fig.1 reports the typical connection scheme of this machine. This solution is suitable for all of the applications where limited speed variations around the synchronous speed are present. Since the power handled by the rotor side (slip power) is proportional to slip, an energy conversion is possible using a rotor-side power converter, which handles only a small fraction of the overall system power. In variable speed drives, during motor operating conditions, the slip power is regenerated by rotor side converter to the line grid, resulting in efficient energy conversion. Variable-speed energy generation systems have several advantages when compared with fixed-speed synchronous and induction generation [2].

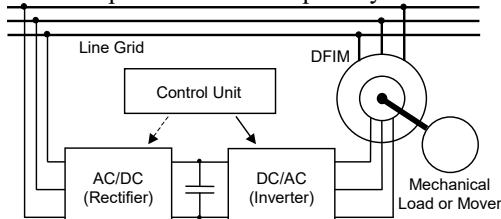


Fig. 1 – Typical connection scheme of a DFIM

An important feature of the vector controlled DFIM reported in [1] and [3] is the possibility to achieve de-coupled control of the stator-side active and reactive power in both motor and generator applications. Moreover, if suitable controlled AC/AC converter is used to supply the rotor side, the power components of the overall system can be controlled with low current harmonic distortion in the stator and rotor sides.

The fundamentals of DFIM vector control are presented in [1] and [3]. Different strategies were proposed to solve the DFIM control problem. The most important results are reported in [1], [3] - [4]. All of them are based on the classical concept of field orientation (stator or air-gap flux) used as torque-flux de-coupling technique for induction motor control. The classical approach for DFIM vector control [1] requires measurements of stator, rotor currents and rotor position. In order to achieve synchronization with line voltage vector for soft connection to the line-grid the information about line voltages is additionally needed. Exact knowledge of induction machine inductances (including saturation effect) is required to compute fluxes from currents. The necessity of high-precision rotor position measurement has been addressed in [4]. In [5] and [6] authors proposed an alternative approach for the design of DFIM active-reactive power control. Both controllers are developed in a line-voltage oriented reference frame. Since the line voltage vector can be easily measured with negligible errors, this reference frame is DFIM parameter invariant in contrast to the field oriented one. Moreover, information about line voltage is typically needed in order to perform the soft connection of the DFIM to the line grid during preliminary excitation-synchronization stage.

The aim of this paper is to introduce a new robust output feedback control algorithm of the DFIM active and reactive power. It is shown that direct closed loop control of active and reactive power guarantees global asymptotic regulation of output variables, under condition of measurable stator currents, voltages and rotor position and speed. The proposed nonlinear output feedback controller demonstrates strong robustness properties with respect to stator and rotor resistances/inductances variation and rotor position measurement errors. In addition, owing to the closed loop structure with true-stator-current feedback signals the controller compensates for non-idealities of electric machine magnetic structure, delivering improved stator current waveforms.

### I. DFIM model and control objectives

Under assumption of linear magnetic circuits and balanced operating conditions, the equivalent two-phase model of the symmetrical DFIM, represented in an arbitrary rotating (d-q) reference frame is:

$$\begin{aligned}
 \dot{\varepsilon} &= \omega \\
 \dot{\omega} &= \frac{1}{J} [T_g - T], \quad T_g = \mu (\psi_d i_q - \psi_q i_d) \\
 \dot{i}_d &= -\gamma i_d + \omega_0 i_q + \alpha \beta \psi_d + \beta \omega \psi_q + \frac{1}{\sigma} u_d - \beta u_{2d} \\
 \dot{i}_q &= -\omega_0 i_d - \gamma i_q - \beta \omega \psi_d + \alpha \beta \psi_q + \frac{1}{\sigma} u_q - \beta u_{2q} \\
 \dot{\psi}_d &= -\alpha \psi_d + (\omega_0 - \omega) \psi_q + \alpha L_m i_d + u_{2d} \\
 \dot{\psi}_q &= -(\omega_0 - \omega) \psi_d - \alpha \psi_q + \alpha L_m i_q + u_{2q} \\
 \dot{\varepsilon}_0 &= \omega_0
 \end{aligned} \tag{1}$$

where:  $i_d, i_q, \psi_d, \psi_q$  are the components of the stator current and rotor flux vectors;  $u_{2d}, u_{2q}$  are the components of the rotor voltage vector, while  $u_d, u_q$  represent the line voltage components (stator windings are directly connected to the line grid);  $\varepsilon$  and  $\omega$  are rotor angular position and speed;  $T$  is the external torque applied to the mechanical system of the DFIM;  $T_g$  is the torque produced by the electrical machine;  $J$  is the total rotor inertia;  $\varepsilon_0$  and  $\omega_0$  are angular position and speed of the (d-q) reference frame with respect to the a-axis of the fixed stator reference frame (a-b).

Variables expressed in (d-q) reference frame are given by:

$$\begin{aligned} \mathbf{x}_{1dq} &= \mathbf{e}^{-J\varepsilon_0} \mathbf{x}_{1ab} \\ \mathbf{x}_{2dq} &= \mathbf{e}^{-J(\varepsilon_0 - \varepsilon)} \mathbf{x}_{2uv} \end{aligned} \quad \text{where } \mathbf{e}^{-J\xi} = \begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix}, \mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad (2)$$

where:  $\mathbf{x}_{yz}$  stands for two dimensional vectors in the generic (y-z) reference frame; subscript '1' stands for stator variables while '2' for rotor variables; (u-v) indicates rotor reference frame and  $\varepsilon$  is the rotor angle (i.e. the angle between the u-axes and the a-axes).

Positive constants in (1), related to electrical and mechanical parameters of DFIM, are defined as follows

$$\sigma = L_1 \left( 1 - \frac{L_m^2}{L_1 L_2} \right), \beta = \frac{L_m}{\sigma L_2}, \mu = \frac{3 L_m}{2 L_2}, \alpha = \frac{R_2}{L_2}, \gamma = \left( \frac{R_1}{\sigma} + \alpha L_m \beta \right),$$

where  $R_1, R_2, L_1, L_2$  are stator/rotor resistances and inductances respectively,  $L_m$  is magnetizing inductance. One pole-pair is assumed without loss of generality. When DFIM is used as electric generator,  $T$  is the torque produced by a controlled primary mover. Torque  $T_g$  produced by the DFIM is a perturbation for speed control system of the primary mover. Without loss of generality, it is assumed that the mechanical dynamics of the DFIM is properly stabilized by the primary mover speed-controller.

The main control objective considered is the regulation of DFIM stator-side active and reactive powers, given by:

$$P_a = \frac{3}{2} (u_d i_d + u_q i_q), \quad P_r = \frac{3}{2} (u_q i_d - u_d i_q). \quad (3)$$

In order to reduce the effect of the above inaccuracies in the reference frame generation and in vector transformations, a *line (stator) voltage vector* reference frame (d-q) has been adopted (the d-axis is aligned with the line voltage vector). This reference frame is independent of machine parameters and position sensor resolution; only information from two voltage sensors and rotor position sensor are needed, instead of four current sensors.

Using the line voltage vector reference frame, a simple and smooth connection of the stator windings to the line grid can be performed during start-up procedure of the DFIM-based system.

Measured line (stator) voltages in two-phase presentation are equal to

$$u_a = U \cos(\omega_0 t + \varphi_0), \quad u_b = U \sin(\omega_0 t + \varphi_0) \quad (4)$$

where  $U$  and  $\omega_0$  are the line voltage amplitude and the angular frequency,  $\varphi_0$  is the angular position of the line voltage vector.

The synchronous stator voltage oriented reference frame is defined setting in (1) and (2)

$$\cos(\varepsilon_0) = \frac{u_a}{U}, \quad \sin(\varepsilon_0) = \frac{u_b}{U} \quad (5)$$

Under such transformation  $u_d=U$  and  $u_q=0$  in DFIM model (1). In addition, currents  $i_d$  and  $i_q$ , in line-voltage oriented reference frame, represent the active and reactive components of the stator current vector. The expressions of active and reactive powers (3) can be presented as:

$$P_a = \frac{3}{2} U i_d, \quad P_r = -\frac{3}{2} U i_q \quad (6)$$

From (6), it follows that active-reactive power control objective is equivalent to active-reactive stator currents control. Let  $P_a^*$  and  $P_r^*$  are the references for the power components at stator side for the DFIM. Using (6), references for the components of the stator current, in the voltage-oriented reference frame, are given by

$$i_d^* = \frac{2 P_a^*}{3 U}, \quad i_q^* = -\frac{2 P_r^*}{3 U} \quad (7)$$

The control problem of DFIM *generator* is formulated in terms of stator active-reactive current regulation as follows.

**Proposition.** Consider DFIM model (1) under coordinate transformation (2), (5). Let assume that:

A.1. The stator voltage amplitude and frequency are constants (stator windings are directly connected to the line-grid).

A.2. References for active and reactive stator currents are constant and bounded, or represent ramp signals with bounded first derivative and bounded amplitude.

A.3. Under the assumption of a properly controlled primary mover the rotor speed is time varying, measurable and bounded together with its first time-derivative.

A.4. Stator currents and voltages as well as rotor position and speed are available from measurements.

Under these conditions there exists a dynamic output feedback controller in the form

$$\begin{pmatrix} \mathbf{u}_{2d} \\ \mathbf{u}_{2q} \end{pmatrix} = \mathbf{f}(\mathbf{i}_d^*, \mathbf{i}_q^*, \dot{\mathbf{i}}_d^*, \dot{\mathbf{i}}_q^*, \mathbf{i}_d, \mathbf{i}_q, \mathbf{U}, \omega_0, \omega, \mathbf{z}), \quad \dot{\mathbf{z}} = \varphi(\omega_0, \mathbf{i}_d^*, \mathbf{i}_q^*, \dot{\mathbf{i}}_d^*, \dot{\mathbf{i}}_q^*), \quad (8)$$

which guarantees:

O.1. Asymptotic active-reactive stator current regulation independently of speed variations; i.e.

$$\lim_{t \rightarrow \infty} (\tilde{\mathbf{i}}_d) = 0, \quad \lim_{t \rightarrow \infty} (\tilde{\mathbf{i}}_q) = 0 \quad (9)$$

where  $\tilde{\mathbf{i}}_d = \mathbf{i}_d - \mathbf{i}_d^*$ ,  $\tilde{\mathbf{i}}_q = \mathbf{i}_q - \mathbf{i}_q^*$  are current errors.

O.2. Boundness of all internal signals and synchronization of the control actions with the stator voltage vector.

O.3. Robustness with respect to stator and rotor resistance variations and constant error in rotor position measurement.

The proof of O.1. and O.2. of the proposition is given by the controller design and the stability analysis presented in the next Section. The validity of O.3. is confirmed by experimental tests reported in section IV, as well as structure of the control algorithm.

## II. Design of the Direct Robust Control Algorithm for DFIM

The design procedure is performed in two steps: a flux control is designed first, to achieve flux regulation, then the current control algorithm is developed.

Let define the flux regulation errors as:

$$\tilde{\Psi}_d = \Psi_d - \Psi_d^*, \quad \tilde{\Psi}_q = \Psi_q - \Psi_q^*, \quad (10)$$

where flux references,  $\Psi_d^*$  and  $\Psi_q^*$ , will be defined later according to stator current control objectives.

Using definition (10), the last two equations of the DFIM model (1) can be rewritten in ‘error form’ as

$$\begin{aligned} \dot{\tilde{\Psi}}_d &= -\alpha(\tilde{\Psi}_d + \Psi_d^*) + \omega_2(\tilde{\Psi}_q + \Psi_q^*) + \alpha L_m(\tilde{\mathbf{i}}_d + \mathbf{i}_d^*) + \mathbf{u}_{2d} - \dot{\Psi}_d^* \\ \dot{\tilde{\Psi}}_q &= -\alpha(\tilde{\Psi}_q + \Psi_q^*) - \omega_2(\tilde{\Psi}_d + \Psi_d^*) + \alpha L_m(\tilde{\mathbf{i}}_q + \mathbf{i}_q^*) + \mathbf{u}_{2q} - \dot{\Psi}_q^* \end{aligned} \quad (11)$$

where  $\omega_2 = \omega_0 - \omega$  is the slip angular frequency.

Constructing the flux control algorithm as

$$\begin{aligned} \mathbf{u}_{2d} &= \alpha \dot{\Psi}_d^* - \omega_2 \Psi_q^* - \alpha L_m \dot{\mathbf{i}}_d^* + \dot{\tilde{\Psi}}_d + \mathbf{v}_d \\ \mathbf{u}_{2q} &= \alpha \dot{\Psi}_q^* + \omega_2 \Psi_d^* - \alpha L_m \dot{\mathbf{i}}_q^* + \dot{\tilde{\Psi}}_q + \mathbf{v}_q, \end{aligned} \quad (12)$$

the flux error dynamics becomes

$$\begin{aligned} \dot{\tilde{\Psi}}_d &= -\alpha \tilde{\Psi}_d + \omega_2 \tilde{\Psi}_q + \alpha L_m \tilde{\mathbf{i}}_d + \mathbf{v}_d \\ \dot{\tilde{\Psi}}_q &= -\alpha \tilde{\Psi}_q - \omega_2 \tilde{\Psi}_d + \alpha L_m \tilde{\mathbf{i}}_q + \mathbf{v}_q, \end{aligned} \quad (13)$$

where  $\mathbf{v}_d, \mathbf{v}_q$  will be defined later.

Applying the control algorithm (12), the current error dynamics from the first two equation of (1) can be rewritten as

$$\begin{aligned} \dot{\tilde{\mathbf{i}}}_d &= -\gamma \tilde{\mathbf{i}}_d + \omega_0 \tilde{\mathbf{i}}_q + \alpha \beta \tilde{\Psi}_d + \beta \omega \tilde{\Psi}_q - \beta \mathbf{v}_d - \dot{\mathbf{i}}_d^* - \beta \dot{\tilde{\Psi}}_d - \frac{R_1}{\sigma} \mathbf{i}_d^* + \omega_0 \mathbf{i}_q^* + \frac{1}{\sigma} \mathbf{U} + \beta \omega_0 \Psi_q^* \\ \dot{\tilde{\mathbf{i}}}_q &= -\gamma \tilde{\mathbf{i}}_q - \omega_0 \tilde{\mathbf{i}}_d + \alpha \beta \tilde{\Psi}_q - \beta \omega \tilde{\Psi}_d - \beta \mathbf{v}_q - \dot{\mathbf{i}}_q^* - \beta \dot{\tilde{\Psi}}_q - \frac{R_1}{\sigma} \mathbf{i}_q^* - \omega_0 \mathbf{i}_d^* - \beta \omega_0 \Psi_d^*. \end{aligned} \quad (14)$$

From equations (14) it follows that flux references should satisfy the following differential equations

$$\begin{aligned} \dot{\Psi}_d^* &= \frac{1}{\beta} \left( \beta \omega_0 \Psi_q^* + \frac{1}{\sigma} \mathbf{U} - \frac{R_1}{\sigma} \mathbf{i}_d^* + \omega_0 \mathbf{i}_q^* - \dot{\mathbf{i}}_d^* \right) \\ \dot{\Psi}_q^* &= \frac{1}{\beta} \left( -\beta \omega_0 \Psi_d^* - \frac{R_1}{\sigma} \mathbf{i}_q^* - \omega_0 \mathbf{i}_d^* - \dot{\mathbf{i}}_q^* \right) \end{aligned} \quad (15)$$

Substituting (15) into (14) the resulting current-flux error dynamics becomes

$$\begin{aligned} \dot{\tilde{\mathbf{i}}}_d &= -\gamma \tilde{\mathbf{i}}_d + \omega_0 \tilde{\mathbf{i}}_q + \alpha \beta \tilde{\Psi}_d + \beta \omega \tilde{\Psi}_q - \beta \mathbf{v}_d \\ \dot{\tilde{\mathbf{i}}}_q &= -\gamma \tilde{\mathbf{i}}_q - \omega_0 \tilde{\mathbf{i}}_d + \alpha \beta \tilde{\Psi}_q - \beta \omega \tilde{\Psi}_d - \beta \mathbf{v}_q \\ \dot{\tilde{\Psi}}_d &= -\alpha \tilde{\Psi}_d + \omega_2 \tilde{\Psi}_q + \alpha L_m \tilde{\mathbf{i}}_d + \mathbf{v}_d \\ \dot{\tilde{\Psi}}_q &= -\alpha \tilde{\Psi}_q - \omega_2 \tilde{\Psi}_d + \alpha L_m \tilde{\mathbf{i}}_q + \mathbf{v}_q \end{aligned} \quad (16)$$

According to (15), the flux references are given by a linear differential equation which is not really implementable, since it is stable but not asymptotically (i.e. it is a non-autonomous harmonic oscillator). A particular solution of (15), where oscillating terms are avoided, is given by



to guarantee that  $\mathbf{P} > \mathbf{0}$ .

The time derivative of  $V$  along the trajectories of (21) is negative semidefinite and equal to

$$\dot{V} \leq 0, \quad (25)$$

with

$$\gamma_1 = \left[ 1 + \frac{\sigma}{R_1} (2\alpha + \alpha L_m \beta + k_i) + \frac{1}{\gamma_2 \omega_0^2} \right] \quad (26)$$

Note that conditions (24) and (26) give freedom in the selection of current controller parameters  $k_i$  and  $k_{ii}$ , in fact for each  $k_i > 0$  and  $k_{ii} > 0$  it is possible to find  $\gamma_1$  and  $\gamma_2$  in order to satisfy the above conditions.

From (21), (22) and (25) it follows that standard Lyapunov stability analysis based on the Barbalat's Lemma (see [7]), can be applied. From the stability analysis we conclude that

$$\lim_{t \rightarrow \infty} (\tilde{i}_d, \tilde{i}_q, z_d, z_q)^T = \mathbf{0} \quad (27)$$

$$\lim_{t \rightarrow \infty} (y_d, y_q)^T = \mathbf{0}. \quad (28)$$

The control objectives O.1 and O.2 are globally achieved according to stability analysis.

The complete equations of the nonlinear current controller are given by (12), (18), (19),

with the actual rotor side voltages

$$\begin{bmatrix} \mathbf{u}_{2u} \\ \mathbf{u}_{2v} \end{bmatrix} = \mathbf{e}^{\mathbf{J}(\epsilon_0 - \epsilon)} \begin{bmatrix} \mathbf{u}_{2d} \\ \mathbf{u}_{2q} \end{bmatrix} \quad (29)$$

### III. Experimental tests

The experimental tests have been performed using a small (5kW) wound-rotor induction machine whose rated data are: nominal voltage 380 V<sub>RMS</sub> (Y-connected), nominal power 5 kW, nominal speed 100 rad/s, stator resistance ( $R_1$ ) 0.95  $\Omega$ , rotor resistance ( $R_2$ ) 1.8  $\Omega$ , stator inductance ( $L_1$ ) 0.094 H, nominal frequency 50 Hz, nominal torque 50 Nm, pole pairs 3, rotor inductance ( $L_2$ ) 0.088 H, magnetizing inductance ( $M$ ) 0.082 H, rotor inertia ( $J$ ) 0.1 kgm<sup>2</sup>.

The following operating conditions have been considered:

1. The initial time interval 0 – 0.95 s is used to start the primary mover, to perform the excitation and synchronization of the DFIM with the line voltage vector and to connect stator to the line grid.
2. At time  $t = 0.95$  s the active current reference trajectory is applied, starting from zero initial value and reaching the 90% of rated value (corresponding to 45Nm of produced torque with reactive current equal to zero).
3. At time  $t = 1.3$  s the reference trajectory for reactive component of the stator current is applied. The stator current references are shown in Fig 2.

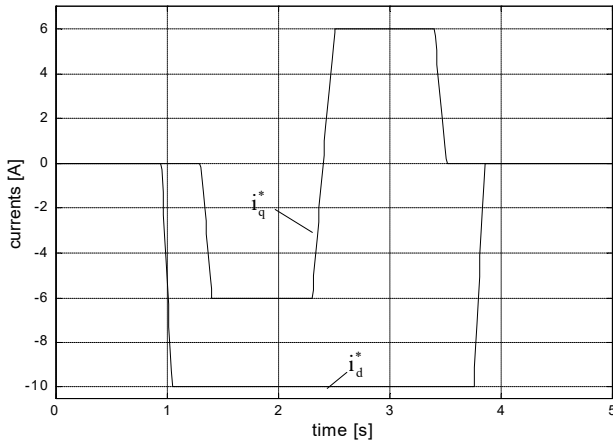


Fig. 2 Stator current references

The controller gains during all of the tests are set at  $k_i = 200$ ;  $k_{ii} = 10000$ .

In Figs. 3, 4 experimental results are reported. The proposed controller has been discretized using simple backward-derivative method and implemented on a DSP-based control board with a sampling time of 200 $\mu$ s. The PWM frequency of the adopted inverter is 5 kHz. The resolution of the encoder mounted on the rotor shaft is 1024 ppr. The aim of this test is to verify the performance of the proposed active-reactive current controller. In Fig. 3 the behavior of the electric variables is reported. The initial condition of the results reported in Fig. 3 is characterized by stator windings connected to the three-phase line grid and stator currents regulated to zero by the proposed controller. Some transient can be noted in the stator current errors when the reference is variable, null

error is guaranteed in steady state conditions.

In Fig.4 the real stator phase voltage and current in a fixed stator reference frame are reported when the reference for the reactive current reference is set to 0, while the active current one is equal to 10A.

### IV. Conclusions

The new direct active – reactive power controller for DFIM provides global asymptotic regulation in presence of induction machine parameters variation and rotor position measurements errors. In addition, it delivers an improved stator current waveform compensating for non-idealities of the induction machine electromagnetic circuit. The experimental tests confirm the high dynamic performance and robustness of the proposed controller. The proposed controller is suitable for energy generation applications, where restricted variations of the speed around the synchronous velocity are present.

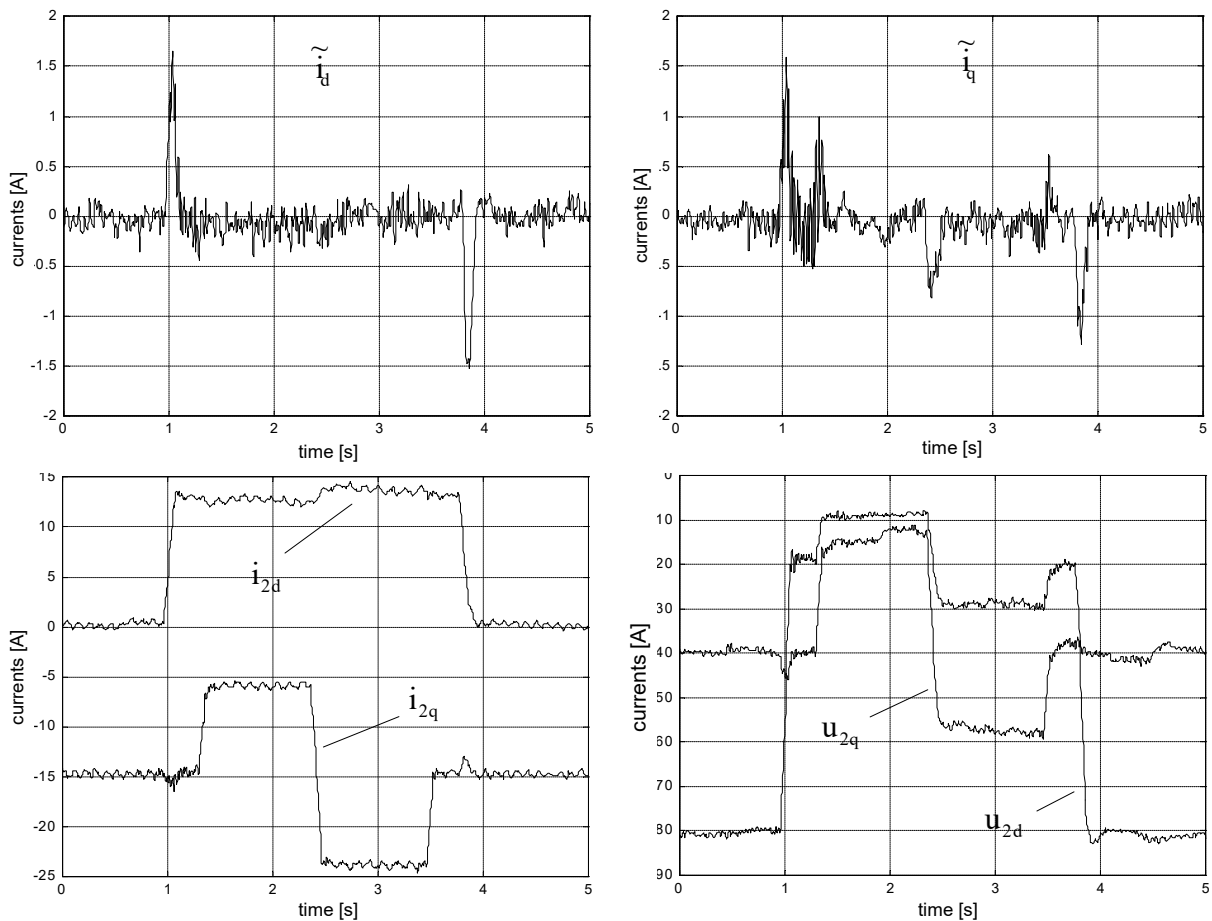


Fig. 3 Experimental results with active-reactive current regulation

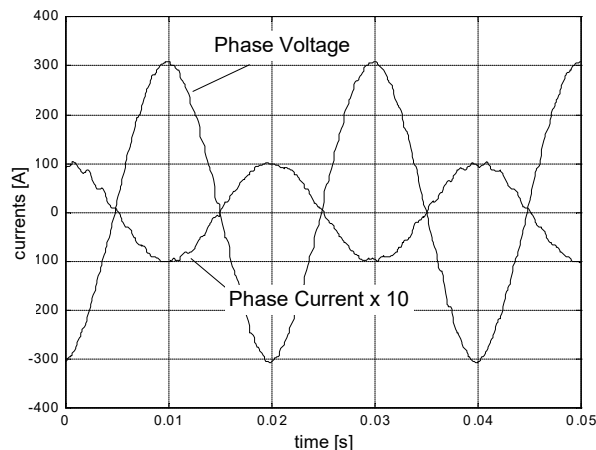


Fig. 4 Experimental results: stator phase voltage and current with active-reactive current regulation

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