

Compensation effect analysis in DIE method for through-casing measuring formation resistivity

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Abstract: Double-Injection-Electrodes (DIE) model and its compensation arithmetic method has been proven to be very useful for eliminating the errors caused by electrode-scale mechanical tolerances in formation resistivity measurement through metal case. In this paper, we found that even minor casing joint or casing corrosion may deteriorate the measurement accuracy. Based on theoretical analysis and self-adaptive goal oriented *hp*-Finite Element (FE) simulations, the compensation effects of DIE model were estimated. The calculated results from DIE model are always close to the real formation resistivity no matter the metal casing is ideal or not. Meanwhile, large errors occur in Single-Injection-Electrode (SIE) model, where the calculated formation resistivity may provide negative numbers when casing joint or casing corrosion exists. The Double-Injection-Electrode (DIE) model is predicted to have good compensation effects to many non-ideal situations with uneven metal casing besides electrode-scale mechanical tolerances.

I Introduction

Acquisition of through-casing resistivity (TCR) measurement was first proposed by Alpin in 1939 [1]. He stated that when current is injected into a casing, the voltage differences on the casing well are highly influenced by the formation resistivities. Thus, formation resistivity could be inferred by measuring the voltage differences.

Due to the weak voltage signals below 1 μV obtained in TCR measurements and the limited technology existing at the time for measuring such a low voltage signals, Alpin's method could not be implemented at that time.

Several decades later, recent advances in weak signal processing and measurement technologies have allowed Alpin's method to be applied. The report on the field test of the prototype of the tool (Vail et al., 1995) was another important step towards the development of the technology. During the last two decades, the interest in electrical logging through casing has grown considerably, since resistivity estimations in cased wells is as topic of great importance for the logging industry.

Several studies have been performed concerning TCR measurements. In particular, vertical resolution of the measurement, effects caused by the cement sheath, casing inhomogeneities, and the finite length of the casing have been studied by Schenkel (1990), Kaufman and Wightman (1993), Schenkel and Morrison (1994), Tabarovsky et al.

(1994), Zinger et al. (1994), and Singer et al. (1995 and 1998) [2]-[10]. Based on Kaufman measurement mode, various attempts have been made to build systems for logging formation resistivity in cased boreholes [11]-[12].

Kaufman's theory has always been seemed as the technology basis of through casing resistivity measurement. However, Kaufman's Single-Injection-Electrode (SIE) model and relative arithmetic may present some disadvantages when applied to complex TCR models. In reference [13], Chen and Pardo (2010) found that the measurement errors are very sensitive to the mechanical tolerances. In order to reduce the errors, a Double-Injection-Electrode (DIE) model was presented to correct the SIE model. While most analytical methods cannot be applied to complex geometries, simulation of TCR measurements via numerical methods is rather challenging due to the high electrical conductivity contrast and small thickness of casing [14]-[21]. Utilizing a 2-D axially symmetric numerical method based on a self-adaptive goal oriented *hp*-finite-element method (FEM), Chen and Pardo proved that, when considering the SIE model, it is still not certain that the error can be controlled below 5% even if the mechanical tolerance is below 0.01%. However, when using the DIE model accompanied with compensation arithmetic method, even a 10% mechanical tolerance has very little impact on the results (nearly 2%).

In this paper, we extend the analysis and study of the DIE method to the case of casing imperfections. In particular, the errors caused by casing joint and casing corrosion besides mechanical tolerance are calculated through theoretical analysis and self-adaptive goal oriented *hp*-finite-element method (FEM) simulations, through which the compensation effects of symmetrical arithmetic method derived from DIE model are estimated correctly. Moreover, it is predicted that DIE model will decrease the errors caused by other non-ideal situations of uneven metal casing besides casing joint and casing corrosion.

II SIE and DIE models

A. SIE model

For the SIE model, the casing is assumed to be a uniform and highly conductive steel pipe with an infinite length. The formation is assumed to be a homogeneous medium around the casing. The leakage current is known to be perpendicular to the casing. As shown in Fig. 1, the current is injected into the casing from electrode A, and the

formation apparent resistivity around point D, namely ρ_{SIE} , is estimated as:

$$\rho_{SIE} = (\Delta z)^2 \frac{U_D}{\Delta^2 U} r_c \quad (1)$$

$$\Delta^2 U = U_{CD} - U_{DE}$$

where r_c is the casing resistance per meter, Δz is the distance between two consecutive electrodes, and U_D , U_{CD} , and U_{DE} are the voltage difference signals described in Fig.1. Notice that the computed result ρ_a obtained from equation (1) is not the real resistivity of formation, but the formation resistance of the neighboring horizontal layer. The conversion factor from ρ_a to the real formation resistivity depends upon the mechanical characteristics of the casing.

In order to estimate ρ_a , we need to calculate r_c and measure ($U_{CD} - U_{DE}$), Δz , and U_D . The quality of the approximation of ρ_a is highly dependent upon the accuracy of the estimated steel casing resistance r_c . There exist two ways to calculate r_c .

One is a theoretical calculation method. Assuming known values of ρ_c (steel casing resistivity), a and Δa (radius of casing and the thickness of casing, respectively), r_c can be represented as:

$$r_c = \rho_c \frac{1}{2\pi a \Delta a} \quad (2)$$

In equation (2), ρ_c is sensitive to temperature change, and 100 degrees temperature variation will bring an excursion over 20% to ρ_c . Thus, in equation (2), the calculated r_c can not reflect the real steel casing resistance, and the theoretical calculation method will inevitably produce a large error in resistivity measurement.

The second method to estimate r_c is based on a practical measurement that is immune to temperature variations. In this method, the operation program becomes more complex, which lowers the logging efficiency. As shown in Fig. 2, the current I is injected from electrode A and collected at electrode B. Then, r_c can be inferred from the following equation:

$$r_c = \frac{U'_{CD}}{I' \Delta z} = \frac{U'_{DE}}{I' \Delta z} = \frac{U'_{CD} + U'_{DE}}{2I' \Delta z} \quad (3)$$

Thus, the apparent formation resistivity in SIE model becomes:

$$\rho_{SIE} = \frac{\Delta z U_D (U'_{CD} + U'_{DE})}{2I' (U_{CD} - U_{DE})} \quad (4)$$

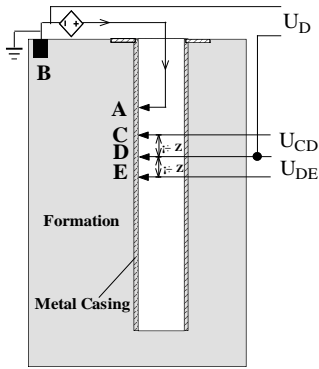


Fig.1 SIE model

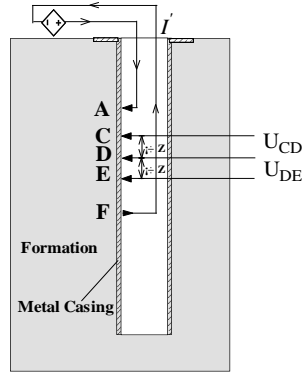


Fig.2 r_c measurement in SIE model

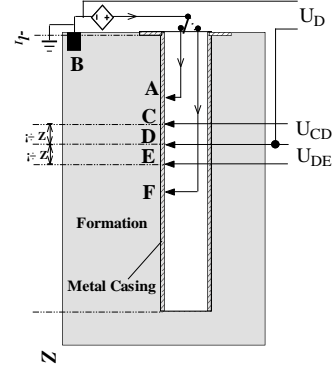


Fig.3 DIE model

B. DIE model

As shown in Fig. 3, the Double-Injection-Electrodes (DIE) model differs from that of SIE in that an additional current injection electrode F is used, and point D is assumed to be the midpoint of I_{AF} . The current is injected from electrode A and electrode F alternately, and the injected currents are I_A and I_F , respectively. When the current I_A is injected from electrode A, the voltage difference between electrode C and D is now denoted as U_{A-CD} in order to distinguish it from U_{F-CD} , the voltage difference between electrodes C and D when the current I_F is injected from electrode F. The remaining symbols can be understood in the same way, as described in equation (5).

$$\begin{aligned} U_{A-CD} &= U_{A-C} - U_{A-D} \\ U_{F-CD} &= U_{F-C} - U_{F-D} \\ U_{A-DE} &= U_{A-D} - U_{A-E} \\ U_{F-DE} &= U_{F-D} - U_{F-E} \\ U_{A-CE} &= U_{A-CD} + U_{A-DE} \\ U_{F-CE} &= U_{F-CD} + U_{F-DE} \\ \Delta^2 U_A &= U_{A-CD} - U_{A-DE} \\ \Delta^2 U_F &= U_{F-CD} - U_{F-DE} \end{aligned} \quad (5)$$

Without applying the connection method used in SIE model as shown in Fig. 2, now r_c can be obtained directly:

$$r_c = \frac{\Delta^2 U_A + U_{A-CE}}{2I_A \Delta z} + \frac{\Delta^2 U_F - U_{F-CE}}{2I_F \Delta z} \quad (6)$$

In the above equation, $\Delta^2 U_A$ and $\Delta^2 U_F$ provide the apparent compensations for the real steel casing resistance, through which the influence of the parallel formation resistance is eliminated, especially when the formation resistance is low.

In the innovative DIE model, the other two important parameters, U_D and $\Delta^2 U$, presented in equation (1) are defined as:

$$U_D = \frac{U_{A-CE} U_{F-D} - U_{F-CE} U_{A-D}}{U_{A-CE} - U_{F-CE}} \quad (7)$$

$$\Delta^2 U = \frac{U_{A-CE} \Delta^2 U_F - U_{F-CE} \Delta^2 U_A}{U_{A-CE} - U_{F-CE}}$$

Thus, the final formation apparent resistivity in DIE model becomes:

$$\rho_{DIE} = \frac{\Delta z}{2} \left(\frac{\Delta^2 U_A + U_{A-CE}}{I_A} + \frac{\Delta^2 U_F - U_{F-CE}}{I_F} \right) \cdot \frac{U_{A-CE} U_{F-D} - U_{F-CE} U_{A-D}}{U_{A-CE} \Delta^2 U_F - U_{F-CE} \Delta^2 U_A} \quad (8)$$

III Theoretical analysis

The object of the analysis is to determine the errors caused by casing joint and casing corrosion in SIE and DIE models. As shown in Fig.4, the metal casing is assumed to have an overall length h , an average radius a and a thickness Δa . The injected current I is located at electrode A or F. The leaking current distributes along the casing, and the total leaking current values of casing segments h_1 , h_2 and h_3 are respectively ΔI_1 , ΔI_2 and ΔI_3 . It is assumed that the conductivity of metal casing is more than 10^9 times of formation conductivity, so the leaking current ΔI_2 is considered distributed uniformly along the casing segment h_2 .

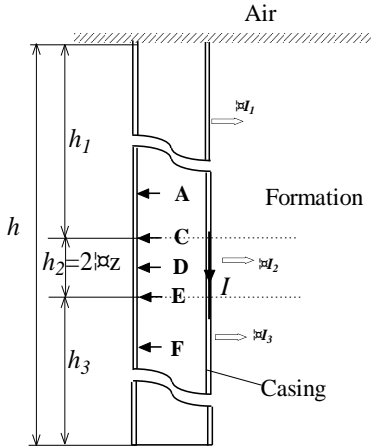


Fig.4 Currents distributed along the casing

When the current is injected from electrode A, the current flowing along the casing vertically at points C, D and E can be described as

$$I_C = \Delta I_2 + \Delta I_3$$

$$I_D = \frac{1}{2} \Delta I_2 + \Delta I_3$$

$$I_E = \Delta I_3 \quad (9)$$

$$U_{CD} = r_{CD} \left(\frac{3}{4} \Delta I_2 + \Delta I_3 \right)$$

$$U_{DE} = r_{DE} \left(\frac{1}{4} \Delta I_2 + \Delta I_3 \right)$$

If we assume:

$$h_2 \ll h$$

$$\Delta I_2 \ll \Delta I_1 + \Delta I_2 \quad (10)$$

Then, U_D , ΔI_1 , ΔI_2 and ΔI_3 are independent of the electrode where the current is injected (A or F). When the current is injected from electrode F, the vertical currents along the casing are:

$$I_C = -\Delta I_1$$

$$I_D = -\Delta I_1 - \frac{1}{2} \Delta I_2$$

$$I_E = -\Delta I_1 - \Delta I_2 \quad (11)$$

$$U_{CD} = -r_{CD} \left(\Delta I_1 + \frac{1}{4} \Delta I_2 \right)$$

$$U_{DE} = -r_{DE} \left(\Delta I_1 + \frac{3}{4} \Delta I_2 \right)$$

In absence of casing joints or corrosion, r_{CD} (the casing resistance between electrode C and electrode D) is equal to r_{DE} (the casing resistance between D and electrode E), that is:

$$r_{CD} = r_{DE} = \Delta z \cdot r_c \quad (12)$$

In SIE model, the formation resistivity ρ_{SIE} can be calculated by substituting equations (9) and (12) into equation (1).

$$\rho_{SIE} = 2 \Delta z \frac{U_D}{\Delta I_2} \quad (13)$$

Substituting equations (9), (11), and (12) into equation (4), ρ_{DIE} (the formation resistivity of DIE model) becomes:

$$\rho_{DIE} = 2 \Delta z \frac{U_D}{\Delta I_2} \quad (14)$$

There is no doubt that casing joints or corrosion will bring variations to r_{CD} and r_{DE} .

If

$$r_{CD} = K_1 \cdot \Delta z \cdot r_c$$

$$r_{DE} = K_2 \cdot \Delta z \cdot r_c \quad (15)$$

then, the formation resistivities in SIE model and DIE model can be calculated as:

$$\rho_{SIE}^* = 2\Delta z \frac{(K_1 + K_2)U_D}{(3K_1 - K_2)\Delta I_2 + (K_1 - K_2)\Delta I_3} \quad (16)$$

$$\rho_{DIE}^* = \Delta z \frac{(K_1 + K_2)^2 U_D}{2K_1 K_2 \Delta I_2}$$

The length of metal casing is more than 1000 times that of the casing radius, so the leaking current can be assumed to distribute nearly uniformly along the casing, that is to say ΔI_1 , ΔI_2 and ΔI_3 are proportional to h_1 , h_2 and h_3 respectively. From equations (13), (14) and (16), we derive the relative errors caused by casing resistance variations in SIE and DIE models:

$$e_{SIE} = \frac{\rho_{SIE}^* - \rho_{SIE}}{\rho_{SIE}} = \frac{(K_2 - K_1)(2h_2 + h_3)}{(3K_1 - K_2)h_2 + (K_1 - K_2)h_3} \quad (17)$$

$$e_{DIE} = \frac{\rho_{DIE}^* - \rho_{DIE}}{\rho_{DIE}} = \frac{(K_1 - K_2)^2}{4K_1 K_2}$$

Equation (17) indicates that the errors caused by casing joint or corrosion in SIE model are much more sensitive to the coefficients K_1 and K_2 than those in DIE model. Moreover, because h_3 is always much longer than h_2 , it is not certain that e_{SIE} will be limited below 5% even if the difference between K_1 and K_2 is less than 0.1%.

IV *hp*-FEM Simulations

In this section, we simulate SIE model and DIE model at DC. Substituting the simulation results to equations (4) and (8), we can estimate the formation resistivity. By comparing those numerical results against the real formation resistivity assumed before simulations, the performance of SIE and DIE arithmetic methods are estimated.

Simulation of TCR measurements via numerical methods is rather challenging due to the high electrical conductivity contrast and small thickness of casing [13]-[21]. Here, we utilize a 2-D axially symmetric numerical method based on a self-adaptive goal oriented *hp*-finite-element method (FEM) that accurately simulates such logging measurements. This method constructs automatically an optimal grid with varying element sizes h and polynomial orders of approximation p throughout the computational grid, and it produces high-accuracy solutions that we employ to compare the performance of the SIE model vs. the DIE model..

A. Ideal casing

For SIE model, there are two steps to estimate the formation resistivity:

(1) one is to measure the casing resistance r_c , as shown in Fig.2. The relative parameters are described as:

$$\begin{aligned} h &= 100m \\ h_D &= 50m \\ l_{AD} &= l_{DF} = 1.5m \\ \Delta z &= 0.5m \\ a &= 0.15m \\ \Delta a &= 0.01m \end{aligned} \quad (18)$$

In the above Equation: 1.) h , a and Δa are the length, average radius, and thickness of the casing; 2.) h_D is the distance from the ground to electrode D; 3.) l_{AD} and l_{DB} are the distances between A and D and between D and F; 4.) Δz is the length unit of electrode-scale and it is equal to 0.5m. The casing resistivity ρ_c , the borehole resistivity ρ_b and the formation resistivity ρ are respectively assumed to be $1 \times 10^{-6} \Omega \cdot m$, $1 \Omega \cdot m$ and $100 \Omega \cdot m$. The injected current I' is assumed to be 100 A, and the potential differences U'_{CD} and U'_{DE} are calculated through *hp* FEM simulations:

$$\begin{aligned} I' &= 100 \text{ A} \\ U'_{CD} &= 0.002858 \text{ V} \\ U'_{DE} &= 0.002849 \text{ V} \end{aligned} \quad (19)$$

(2) the second step is to compute the potential U_D and second potential difference $\Delta^2 U$, as shown in Fig.1. All the parameters are the same as those described above. When a current I with 100 A is injected from electrode A, the relative potentials are:

$$\begin{aligned} I &= 100 \text{ A} \\ U_C &= 1.232315 \text{ V} \\ U_D &= 1.229457 \text{ V} \\ U_E &= 1.226630 \text{ V} \end{aligned} \quad (20)$$

Substituting the results into equation (4), ρ_1 , the calculated formation resistivity of SIE model is obtained, as shown in TABLE I.

In DIE model, all the parameters can be measured in one step. As shown in Fig.3, a current with value of 100 A is injected from electrode A and F alternatively, and all the relative potentials can be calculated through simulations:

$$\begin{aligned} I_A &= I_F = 100 \text{ A} \\ U_{A-C} &= 1.232315 \text{ V} \\ U_{A-D} &= 1.229457 \text{ V} \\ U_{A-E} &= 1.226630 \text{ V} \\ U_{F-C} &= 1.226133 \text{ V} \\ U_{F-D} &= 1.228712 \text{ V} \\ U_{F-E} &= 1.231321 \text{ V} \end{aligned} \quad (21)$$

Substituting the results into equation (8), ρ_2 , the calculated formation resistivity of DIE model is obtained, as shown in TABLE I.

B. Presence of casing joints

Casing joint is a popular method that accounts for possible imperfections on a metallic casing. We consider a casing joint that has wider thickness compared with the main part of casing, as shown in Fig. 5. Taking no account of the contact resistance, the joint located between electrode C and electrode D is considered to have a resistivity equal to the casing resistivity $1 \times 10^{-6} \Omega \cdot m$.

According to the simulation procedures described above, the potentials at the measurement electrodes are computed.

For SIE model, the relative potentials are:

$$\begin{aligned} I' &= 100 \text{ A} \\ U'_{CD} &= 0.002607 \text{ V} \\ U'_{DE} &= 0.002831 \text{ V} \\ I &= 100 \text{ A} \\ U_C &= 1.232211 \text{ V} \\ U_D &= 1.229604 \text{ V} \\ U_E &= 1.226776 \text{ V} \end{aligned} \quad (22)$$

For DIE model, the relative potentials are:

$$\begin{aligned} I_A &= I_F = 100 \text{ A} \\ U_{A-C} &= 1.232211 \text{ V} \\ U_{A-D} &= 1.229046 \text{ V} \\ U_{A-E} &= 1.226776 \text{ V} \\ U_{F-C} &= 1.226280 \text{ V} \\ U_{F-D} &= 1.228634 \text{ V} \\ U_{F-E} &= 1.231244 \text{ V} \end{aligned} \quad (23)$$

Based on equations (4) and (8), ρ_3 and ρ_4 , formation resistivities for the case of a casing joint from SIE and DIE models, respectively can be easily calculated, as shown in TABLE I.

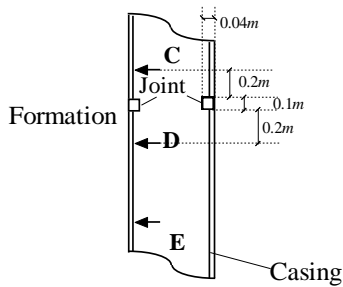


Fig.4 Casing with joint

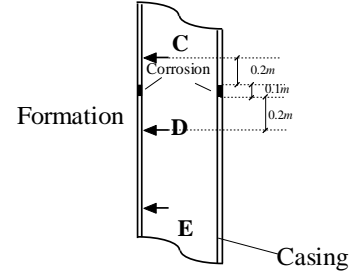


Fig.5 Casing with corrosion

C Presence of casing corrosion

Typically, due to corrosion and damage, a casing is not uniform in resistivity along the borehole and, hence, it contains zones with different conductivities. Fig. 6 shows that there is a corrosion with resistivity $5 \times 10^{-6} \Omega \cdot m$ between electrode C and electrode D.

According to the simulation procedures described above, the potentials at the measurement electrodes are computed.

For SIE model, the relative potentials are:

$$\begin{aligned} I' &= 100 \text{ A} \\ U'_{CD} &= 0.003418 \text{ V} \\ U'_{DE} &= 0.002829 \text{ V} \\ I &= 100 \text{ A} \\ U_C &= 1.232704 \text{ V} \\ U_D &= 1.229286 \text{ V} \\ U_E &= 1.226460 \text{ V} \end{aligned} \quad (24)$$

For DIE model, the relative potentials are:

$$\begin{aligned} I_A &= I_F = 100 \text{ A} \\ U_{A-C} &= 1.232704 \text{ V} \\ U_{A-D} &= 1.229286 \text{ V} \\ U_{A-E} &= 1.226460 \text{ V} \\ U_{F-C} &= 1.225964 \text{ V} \\ U_{F-D} &= 1.229049 \text{ V} \\ U_{F-E} &= 1.231658 \text{ V} \end{aligned} \quad (25)$$

Based on equations (4) and (8), ρ_5 and ρ_6 , formation resistivities for the case of a casing joint from SIE and DIE models, respectively can be easily calculated, as shown in TABLE I.

TABLE I
hp FEM simulation results

Formation Resistivity	SIE Model			DIE Model		
	$\rho_1(\Omega \cdot m)$	$\rho_3(\Omega \cdot m)$	$\rho_5(\Omega \cdot m)$	$\rho_2(\Omega \cdot m)$	$\rho_4(\Omega \cdot m)$	$\rho_6(\Omega \cdot m)$

1	1.08832	-0.15268	0.056982	1.102353	1.132804	1.109076
10	11.7206	-1.38802	0.530107	11.54158	12.09743	11.86932
100	120.8839	-13.7254	5.255496	119.039	124.3616	119.905

TABLE II
Theoretical analysis results

Formation Resistivity	SIE Model			DIE Model		
	$\rho(\Omega \cdot m)$	$\rho_1(\Omega \cdot m)$	$\rho_3(\Omega \cdot m)$	$\rho_5(\Omega \cdot m)$	$\rho_2(\Omega \cdot m)$	$\rho_4(\Omega \cdot m)$
1	1	-0.38	0.06	1	1	1.09
10	10	-3.8	0.6	10	10	10.9
100	100	-38	6	100	100	109

D. Compensation effect of DIE model

In DIE model, the calculated formation resistivities ρ_2 , ρ_4 and ρ_6 are very close to the assumed real formation resistivity. However, ρ_3 and ρ_5 calculated from SIE model have great errors compared with the real formation resistivity. A clear conclusion can be drawn: minor case joint or corrosion will bring great errors to SIE model, but the DIE model is insensitive to the uneven metal casing conditions such as case joints and case corrosion.

In Section “Theoretical Analysis”, the errors caused by resistance variations were proved to have relationship with coefficients K_1 , K_2 , h_2 and h_3 . In the specific conditions described above, all the coefficients were known, and the relative formation resistivities (ρ_1 to ρ_6) could be estimated according to equation (17). The estimation results are shown in TABLE II. Comparing the data listed in TABLE I and TABLE II, it can be found that the *hp*-FEM simulation results have good accuracy and agrees well with the theoretical results.

To show the robustness of the DIE method, we have considered three different formation resistivities. The relative apparent resistivities we obtain for the SIE and DIE models are shown in TABLE I. In all cases we observe a superior accuracy of the DIE method with respect to the SIE method.

V Conclusions

In this contribution, the working principles of SIE model and DIE model were presented. Based on theoretical analysis and self-adaptive goal oriented *hp*-FEM simulations, we provide quantitative estimates of the errors in apparent resistivity readings caused by casing joints and casing corrosion, which are typical field situations with uneven metal casing. Comparing the calculated results with the assumed real formation resistivities, we conclude that even minor casing joint or corrosion will bring great errors to SIE model, while DIE model is always immune to the affecting factors. Moreover, the results of theoretical analysis are backed up by those of numerical simulations.

Acknowledgments

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