

FAST AND AUTOMATIC INVERSION OF LWD RESISTIVITY MEASUREMENTS FOR PETROPHYSICAL INTERPRETATION

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it for routine petrophysical interpretation and appraisal of complex LWD and wireline resistivity measurements acquired under general geometrical and geological constraints.

ABSTRACT

This paper describes an extension of a recently developed fast inversion method (Pardo and Torres-Verdin (2015)) for estimating a layer-by-layer electric resistivity distribution from logging-while-drilling (LWD) electromagnetic induction measurements. The well trajectory is arbitrary and the developed method is suitable for any commercial logging device with known antennae configurations, including tri-axial instruments.

There are two key novel contributions in this work: First, the three-dimensional (3D) transversely isotropic (TI) formation is now approximated by a sequence of various “stitched” one-dimensional (1D) sections rather than by a single 1D reduced model. This provides added flexibility in order to approximate complex 3D formations. Second, we introduce the concept of “negative apparent resistivities” in the inversion method. By using the values of attenuation and phase differences that correspond to a “negative” resistivity in a homogeneous formation, the amount of data lost when converting magnetic fields into apparent resistivities is minimized, thus leading to a more robust inversion method that also converges faster.

The developed inversion method can be used to interpret LWD resistivity measurements and to adjust the well trajectory in real (logging) time. Numerical inversion results of challenging synthetic and actual field measurements confirm the high stability and superior approximation properties of the developed inversion algorithm. Because of the efficiency, flexibility, and stability of the inversion algorithm, formation-evaluation specialists can readily employ

INTRODUCTION

LWD resistivity measurements acquired in high-angle and horizontal wells are difficult to interpret. In presence of thin beds, anisotropy, and variable borehole trajectory, we often observe apparent resistivity logs that cannot be interpreted without corrections. In particular, infinite values of apparent resistivities frequently appear in the logs, the so-called “horns”. Separation of multi-resolution resistivity curves is another common situation that challenges the correct interpretation of the recorded measurements. Indeed, without proper numerical simulation and inversion software, it becomes impossible to detect, quantify, and reduce those effects to reliably assess existing hydrocarbon reserves.

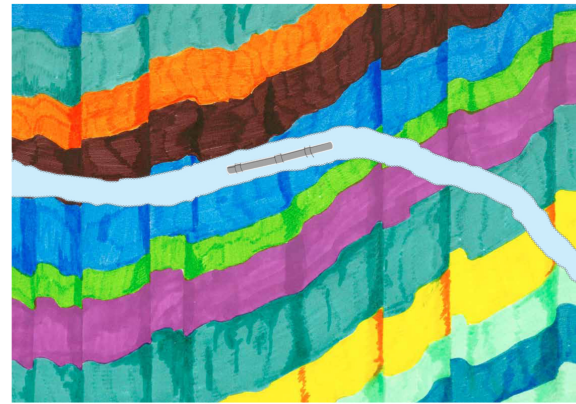
There exist multiple works on accurate numerical modeling of 3D resistivity borehole measurements since the mid-1990s (see, for example, Zhang et al., 1995; Druskin et al., 1999; Wang and Fang, 2001; Avdeev et al., 2002; Newman and Alumbaugh, 2002; Davydycheva et al., 2003; Wang and Signorelli, 2004; Pardo et al., 2007, 2008b, 2008a; Nam et al., 2008, 2010). These simulators have been successfully applied to model and interpret different physical effects occurring in 3D geometries, and have enabled formation-evaluation specialists to design and develop highly sophisticated logging instruments. Despite such recent advances, these methods have failed to provide accurate results in a limited amount of time due to the high complexity associated with 3D simulations in arbitrary geometries. Specifically, for mesh-based methods such as finite elements, finite differences, and boundary elements, the size of the system of linear equations becomes excessively large to be solved in

real (logging) time, preventing their efficient inversion in the context of LWD devices.

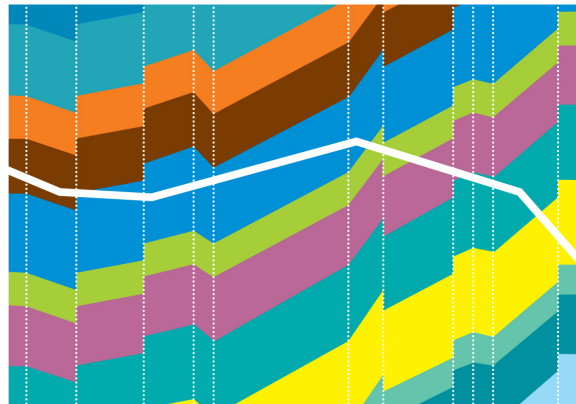
Thus, many companies working on this problem base some of their inversion processes on a reduced model composed of a planar TI layered media with unknown piecewise constant resistivities. For such a simplified model, semi-analytical solutions can be easily computed in a fraction of a second (see Kong, 1972; Chew and Chen, 2003; Løseth and Ursin, 2007; Zhong et al., 2008; Streich and Becken, 2011), enabling the fast simulation of borehole resistivity measurements. The use of a reduced model for inversion of resistivity LWD measurements was already proposed a few decades ago by Meyer (1993), Merchant et al. (1996), and Anderson (2001). Further developments in the area were recently advanced by Ijasan et al. (2014), where they assumed that bed boundaries were first determined from nuclear measurements, followed by a subsequent estimation of electrical properties within each layer based on resistivity measurements. This latter work was limited to the use of one specific LWD resistivity device, and no tri-axial induction instruments were considered in the interpretation. More recently, Pardo and Torres-Verdín (2015) extended this work by enabling the possibility of employing different commercial logging instruments with known antennae configurations, including tri-axial measurements.

In our current work, we further extend our inversion method to the case of LWD measurements acquired in rather general 3D geological structures, such as the one displayed in Fig. 1 (a). To that end, we approximate the original model with a piecewise 1D model (Fig. 1 (b)), which is subsequently approximated for each logging position by a single 1D model, as described in Fig. 2.

Additionally, we introduce the concept of “negative apparent resistivities”. They are needed to properly interpret attenuation and phase difference values obtained in highly deviated wells in heterogeneous formations that, when considered in a homogeneous formation, correspond to a negative apparent resistivity. Currently, such values are interpreted as infinite apparent resistivities, giving rise to the so-called horns, which suppress valuable information about subsurface properties. With the proposed interpretation of such attenuation and phase different values as negative apparent resistivities, we infer further information about the formation, leading to a faster and more robust inversion algorithm.



(a) Original model



(b) Approximated piecewise 1D model

Fig. 1 (a) Original 3D geometrical/geological model and (b) simplified piecewise 1D model used for inversion.

The proposed method is also compatible in combination with tri-axial, directional, and azimuthal logging instruments. Measurements acquired in multiple wells and possibly with various logging instruments can be simultaneously inverted using the proposed approach.

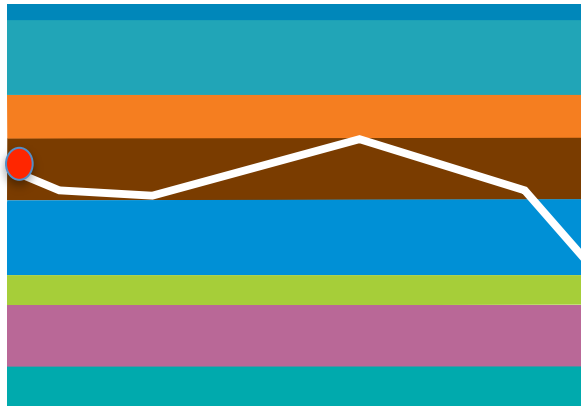
In our numerical simulations, similar conclusions are obtained when considering noise in the measurements, but this introduces an additional level of complexity. Therefore, to better illustrate and focus on the aforementioned main contributions of the paper, we avoid dealing with noisy measurements and error bars in what follows.

FORWARD METHOD

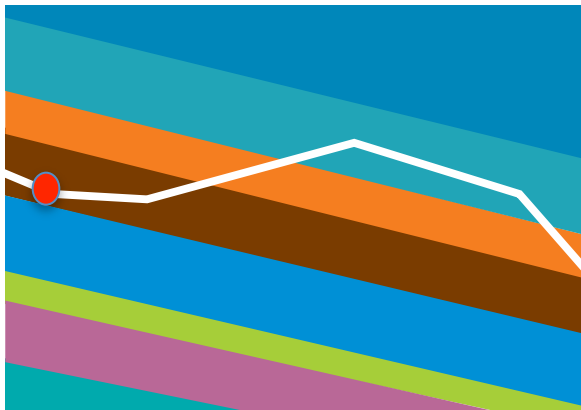
Forward simulations are computed semi-analytically for a sequence of 1D reduced models arising from a single piecewise 1D model, where both borehole and mandrel effects are assumed negligible.

PIECEWISE 1D MODEL

A piecewise 1D model such as the one shown in Fig. 1(b) is approximated by a sequence of 1D models that depend upon the logging position, as illustrated in Fig. 2. This sequence of 1D geological models has to be simulated in real time.



(a) 1D Model for first logging positions (Section 1)



(b) 1D Model for Section 2

Fig. 2 First two 1D geological models in the sequence of 1D models considered to approximate the piecewise 1D model of Fig. 1(b).

1.5D SEMI-ANALYTICAL SOLUTIONS

Once a 1D model has been determined for each logging position, a semi-analytical solution

expressed in terms of the magnetic field radiated by a magnetic dipole in a planarly layered media can be easily computed, both for isotropic (Wait, 1951) and transversely isotropic (TI) (Kong, 1972) formations. To obtain such solution, we first perform a Hankel transform in the horizontal plane. This is equivalent to the use of two Fourier transforms, reducing the original partial differential wave equation into a Helmholtz type ordinary differential equation (ODE) that depends only upon the vertical direction. The advantage of using a single Hankel transform instead of two Fourier transforms is that the inverse Hankel transform only requires a 1D integration rather than a 2D integration that would be needed if one would consider Fourier transforms. The resulting ODE can be easily solved analytically by expressing the solution over each layer as a linear combination of the two existing fundamental solutions, and finding their coefficients by imposing the continuity of the solution and its flux across different materials. The resulting coefficients enforcing proper interface conditions among neighboring layers can be interpreted as transmission and reflection coefficients; recursive formulas exist to compute them efficiently (see, e.g., Kong, 1972). The 3D magnetic field is computed by numerically evaluating the inverse Hankel transform of the above spectral 1D solution.

APPARENT RESISTIVITIES

In conventional LWD instruments, the components of the magnetic field \mathbf{H} are further post-processed to obtain apparent resistivities. The main ideas used to perform this operation are delineated in the works of (Bonner, 1995) and (Clark, 1988). First, we compute at two different receivers RX_1 and RX_2 the z -component of the magnetic field excited by a z -oriented dipole. We denote these quantities as $H_{zz}^{RX_1}$ and $H_{zz}^{RX_2}$, respectively. Receiver one RX_1 is always the one closest to the transmitter. Then, we compute the logarithm of its ratio, which can be expressed using basic properties of complex numbers as

$$\log \frac{H_{zz}^{RX_1}}{H_{zz}^{RX_2}} = \log \left(\frac{|H_{zz}^{RX_1}|}{|H_{zz}^{RX_2}|} \right) + i[ph(H_{zz}^{RX_1}) - ph(H_{zz}^{RX_2})] \quad (1)$$

where the first term in the right hand side is referred to as the “attenuation”, while the second one is the “phase difference”. In the above formula, ph designates the phase (angle) of a complex number.

Then, for the case of a homogeneous formation, we can compute numerically the relation between the above numbers (attenuation and phase difference) with the formation resistivity, as illustrated in Fig. 3. This transformation is recorded for each combination of one transmitter and two receivers, and applied to all measurements, including those acquired in heterogeneous formations. The result of applying this transformation to non-homogeneous formations is called “apparent resistivity”, where the name “apparent” indicates that the above transformation is indeed only exact for homogeneous formations. For measurements acquired in a vertical well in a heterogeneous formation, the “apparent resistivity” can be interpreted as an average of the actual resistivity distribution of the formation. However, in certain cases where measurements are acquired in highly deviated wells in a heterogeneous formation, the apparent resistivity has nothing to do with the actual formation resistivity surrounding the logging instrument, and assuming that the apparent resistivity is a good approximation of the actual formation resistivity is simply incorrect. Numerical inversion is needed in these situations.

A prominent example where the actual and apparent resistivities greatly differ can be observed in the so-called horns, which often appear in resistivity LWD logs acquired in highly deviated wells. They frequently correspond to a situation in which the measured attenuation and/or phase difference cannot be reproduced in a homogeneous formation. For example, the value of attenuation in a homogeneous formation cannot be negative, given that the magnetic field amplitude cannot increase as one moves away from the transmitter, which implies

$$\frac{|H_{ZZ}^{RX_1}|}{|H_{ZZ}^{RX_2}|} > 1 \rightarrow \log\left(\frac{|H_{ZZ}^{RX_1}|}{|H_{ZZ}^{RX_2}|}\right) > 0. \quad (2)$$

However, in heterogeneous formations, the attenuation may be significantly smaller than zero. When performing the transformation dictated by Fig. 3 for a negative attenuation value, one transforms it into an infinite value of resistivity (or some pre-determined cut-off value, e.g., 5000 Ohm-m), producing a horn in the log. This postprocessing technique has two major drawbacks. First, it disables the possibility of interpreting apparent resistivities as valid approximations of actual resistivities, as emphasized above. Second, valuable measurement information is lost when transforming attenuations

and phase differences into apparent resistivities. For instance, two different attenuations equal to -0.2 and -0.5 will both be converted into the same apparent resistivity value: infinity. This loss of information has far reaching adverse consequences, as described in the section below entitled “Numerical Results”.

To partially alleviate the second problem, we propose to consider values of attenuations and phase differences corresponding to negative resistivities in a homogeneous formation. In other words, we extend the transformation of Fig. 3 such that it becomes as bijective as possible by using numbers corresponding to negative resistivities in a homogeneous formation (i.e., negative apparent resistivities). Thus, we prevent a significant loss of information during the postprocessing step, i.e., while converting magnetic fields into apparent resistivities.

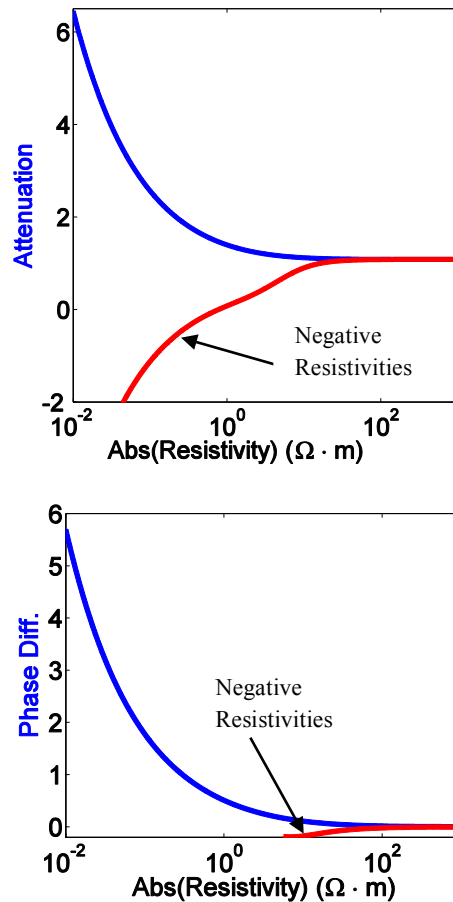


Fig. 3 Relationship between the resistivity of the homogeneous formation and the attenuation (top panel) and phase difference (bottom panel) for a particular logging instrument. The blue line identifies positive resistivities, while the red line designates negative ones.

INVERSION METHOD

The inversion method requires as input one or various apparent resistivity logs, a description of the logging instrument, the well trajectory, and the bed boundaries; it delivers: (a) a layer-by-layer resistivity distribution that minimizes the misfit between the measurements and their numerical simulations, and (b) the corresponding uncertainty bars expressing the degree of uncertainty of the inversion results.

INVERSION ALGORITHM

The objective of the inversion algorithm is to estimate the resistivity distribution, ρ , by minimizing the following cost functional:

$$C(s) = \|H(s) - M\|^2 + \alpha \|s - s_0\|^2 \quad (3)$$

Here, $s = s(\rho)$ is a variable that depends upon the resistivity, for example, $s = \frac{1}{\rho}$, $s = \log \rho$, or $s = \rho$, $H(s)$ is the set of simulated measurements for a given s (either some component of the magnetic field or apparent resistivities), M is the set of noisy measurements, α is a regularization (or stabilization) parameter, and s_0 is an a priori distribution of s , in our case, the resistivity distribution of the previous iteration.

For the minimization problem, we use the following deterministic iterative method

$$s^{(n+1)} = s^{(n)} + \delta s^{(n)}, \quad (4)$$

where $s^{(n)} = s(\rho^{(n)})$ and $\delta s^{(n)}$ is the n^{th} increment of the solution. To determine $\delta s^{(n)}$, we first derive a Taylor series expansion of $H(s^{n+1})$

$$H(s^{(n+1)}) = H(s^{(n)}) + J_s^{(n)} \delta s^{(n)}, \quad (5)$$

where $J_s^{(n)} = \frac{\partial H(s^{(n)})}{\partial s}$ is the Jacobian matrix. Then, we use Gauss-Newton's method to solve for the updated increment $\delta s^{(n)}$ of the solution.

JACOBIAN MATRIX

To compute the Jacobian (sensitivity) matrix $J_s^{(n)}$, we employ the chain rule

$$J_s^{(n)} = \frac{\partial H(s^{(n)})}{\partial s} = \frac{\partial H(s^{(n)})}{\partial \rho} \frac{\partial \rho}{\partial s}, \quad (6)$$

where the second factor on the right side $\frac{\partial \rho}{\partial s}$ is computed analytically, while the first term is numerically calculated from finite-difference approximations of partial derivatives.

REGULARIZATION TERM

To minimize the ill-posedness of the inverse problem, we add a regularization term to the cost functional. This term constitutes a bias to the solution toward the a priori solution s_0 . A compromise between stability and accuracy is thus needed when adding the regularization term. In this paper, we select the regularization term to account for a given percentage of the total cost functional, for example, 10%. To achieve the above goal, we employ the Taylor's series expansion of $H(s^{(n+1)})$ to approximate:

$$C(s) \approx \left\| H(s^{(n)}) + J_s^{(n)} \delta s^{(n)} - M \right\|^2 + \alpha^n \|s^{(n)} + \delta s^{(n)} - s_0\|^2 \quad (7)$$

Then, we select

$$\alpha^n = 0.1 \frac{\left\| H(s^{(n)}) + J_s^{(n)} \delta s^{(n)} - M \right\|^2}{\|s^{(n)} + \delta s^{(n)} - s_0\|^2} \quad (8)$$

To obtain the value of α^n , we perform a fixed-point iteration.

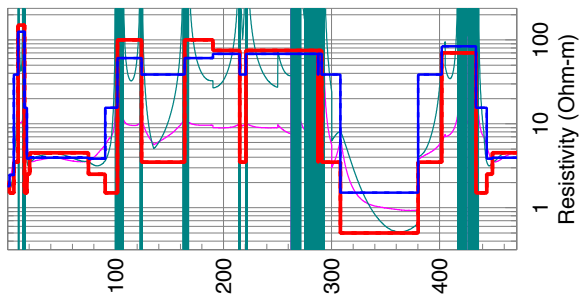
NUMERICAL RESULTS

For the numerical results obtained in this section, we consider a commercial LWD instrument equipped with two transmitters and two receivers. The number of frequencies of operation is also two. Logging measurements are acquired every half a foot along the well trajectory.

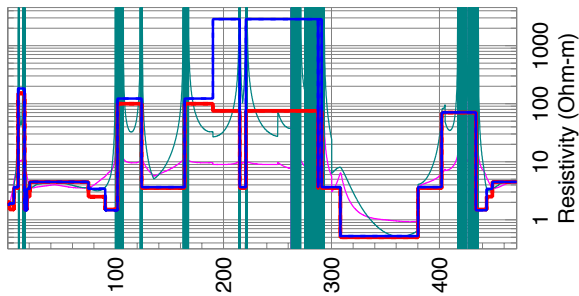
We consider a realistic example containing layers of various thicknesses and resistivity contrasts. The model problem and well trajectory are described in Fig. A-1 in the appendix. The thicknesses of the thinnest and thickest layers are 0.38 m, and 1.52 m, respectively, while the resistivity of the formation ranges from 0.5 $\Omega.m$ to 150 $\Omega.m$. Dip angle is between 82 and 91 degrees with respect to the

vertical direction. This synthetic example is intended to appraise the effectiveness of the inversion method in a highly deviated well with a realistic geological formation.

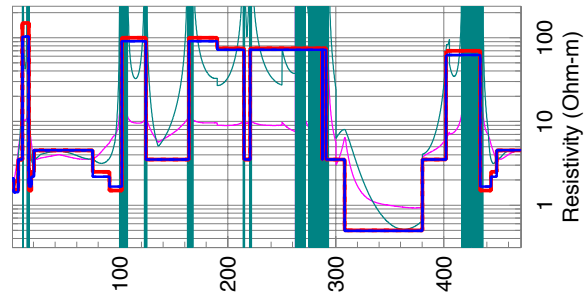
Fig. 4 displays the results obtained from the inversion method using traditional apparent resistivities for an isotropic formation. First, we observe that logs are discontinuous (see, for example, the logs around a horizontal depth equal to 250m). Discontinuities correspond to variations from a given 1D geological model to a different one in the sequence described in Fig. 2. A detailed analysis of such approximation errors will be the subject of a future study. We also observe an elevated number of invalid measurements (vertical green and pink lines), which correspond to attenuation and phase values that cannot be reproduced with a homogeneous formation, as explained in the subsection “Apparent Resistivities”. The loss of information resulting from those invalid measurements produces some minor discrepancies on inverted results with respect to the exact ones (see, for instance, the first layer at a horizontal depth of approx. 20m).



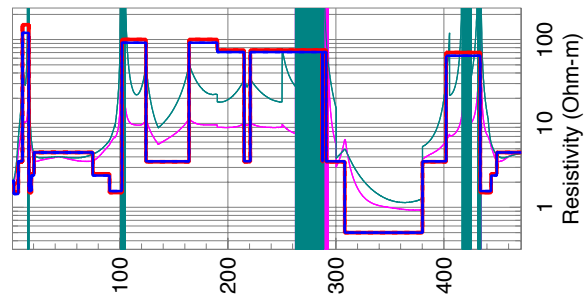
(a) Iter 0



(b) Iter 2



(c) Iter 4



(d) Iter 8

Fig. 4 Synthetic (isotropic) example. Inversion results at iterations 0 (average of the apparent resistivities over each layer), 2, 4, and 8 in terms of horizontal depth (m). Red and green curves are the apparent resistivity logs for the attenuation and phase, respectively. Invalid measurements, i.e., those delivering an infinite apparent resistivity are identified with a green (or pink) vertical line. A thick red line designates the actual resistivity, while the blue line is the result of inversion.

When upgrading the method for computing apparent resistivities by introducing the aforementioned “negative apparent resistivities” (see Fig. 5), the number of invalid measurements drastically decreases. As a result, convergence speed of the method increases (in this case, from eight iterations to four), and inverted results improve (red and blue curves coincide).

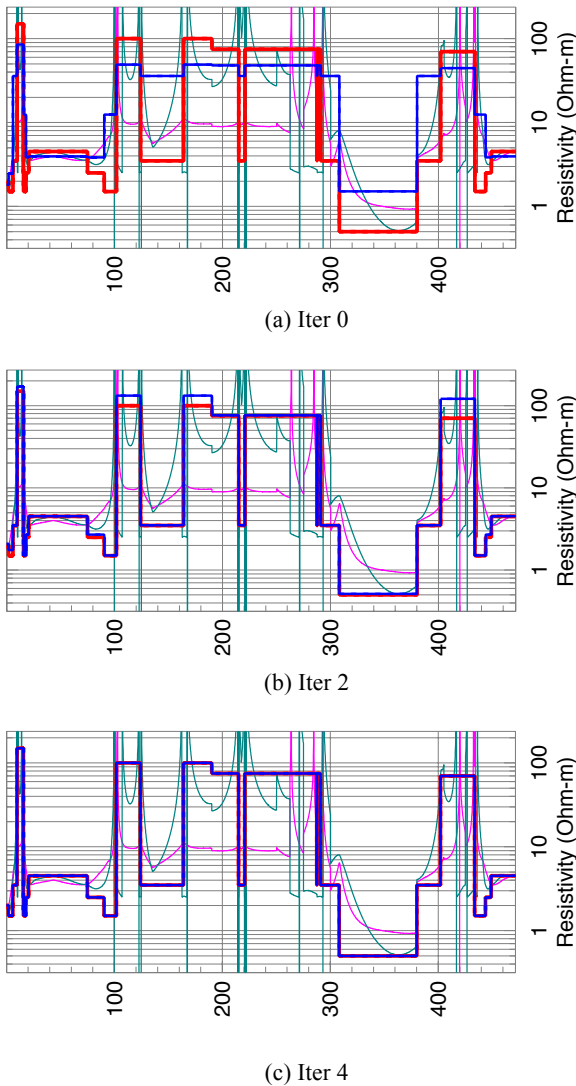


Fig. 5 Synthetic (isotropic) example when considering negative apparent resistivities. Inversion results at iteration 0 (average of the apparent resistivities over each layer), 2, and 4 in terms of horizontal depth (m). Red and green curves are the absolute values of the apparent resistivity logs for the attenuation and phase, respectively. Invalid measurements, i.e., those delivering an apparent resistivity above the cut-off value of 5000 Ohm-m are denoted with a green (or pink) vertical line. A thick red line identifies the actual resistivity, while the blue line describes the result of inversion.

When considering a TI formation, the number of unknown conductivities increases, as does the non-uniqueness of the inverse problem. For this case, the loss of information generated by invalid apparent resistivities produces a more severe deterioration of the inverted results, as shown in Fig. 6 (a). Again, not only the results of the inversion are significantly

more accurate when considering negative resistivities (see Fig. 6 (b)), but also the convergence of the inversion algorithm improves by a factor of two.

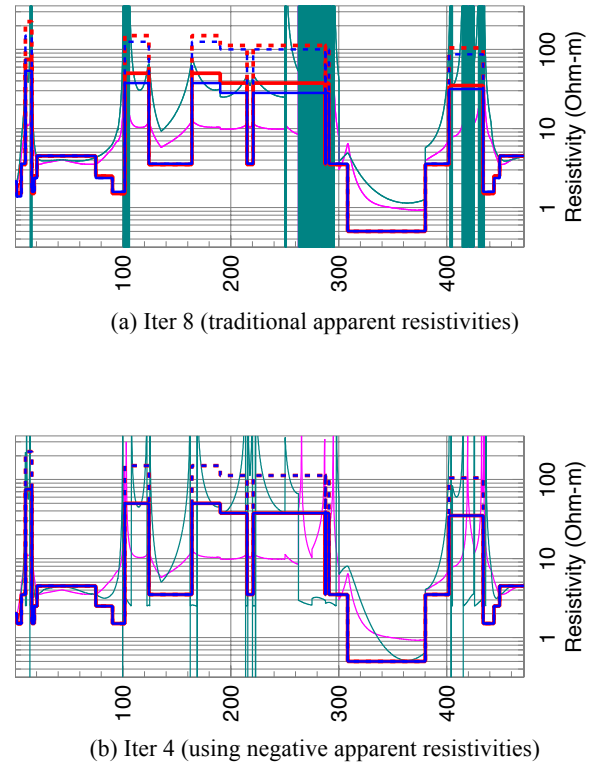


Fig. 6 Synthetic anisotropic example. Inversion results using (a) traditional apparent resistivities after the eighth iteration, and (b) extended apparent resistivities that can take negative values after the fourth iteration. Red and green curves identify the absolute values of the apparent resistivity logs for the attenuation and phase, respectively. Invalid measurements, i.e., those delivering an apparent resistivity above the cut-off value of 5000 Ohm-m are identified with a green (or pink) vertical line. A thick red line describes the actual resistivity, while the blue line designates the result of inversion. Solid lines describe horizontal resistivities, and dashed lines vertical ones.

SUMMARY

We developed an inversion method for the fast 1D interpretation of LWD resistivity measurements. It assumes a piecewise layered 1D geological formation and it ignores borehole and invasion effects. The well trajectory is arbitrary, and it enables interpretation of commercial logging instruments as long as the antennae configurations are available.

A prominent feature of this method is the use of

“negative apparent resistivities” to prevent the loss of information occurring when transforming attenuations and phase differences into apparent resistivities. Numerical results confirm the advantages of employing such upgraded transformation. Specifically, the method converges faster and the final results are more accurate.

Inversion results confirm the robustness, efficiency, and stability of the proposed method.

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“ABOUT THE AUTHORS”

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(SPWLA) and is currently Assistant Editor for *Geophysics* and chair of the editorial board of *The Leading Edge* (SEG). Dr. Torres-Verdín is recipient of the 2014 Gold Medal for Technical Achievement from the SPWLA, of the 2006 Distinguished Technical Achievement Award from the SPWLA, of the 2008 Formation Evaluation Award from the SPE (Society of Petroleum Engineers), of the 2003, 2004, 2006, and 2007 Best Paper Awards in *Petrophysics* by the SPWLA, of the 2006 and 2014 Best Presentation Awards and the 2007 Best Poster Award by the SPWLA.

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APPENDIX

Figure of the model problem and well trajectory used in the numerical results.

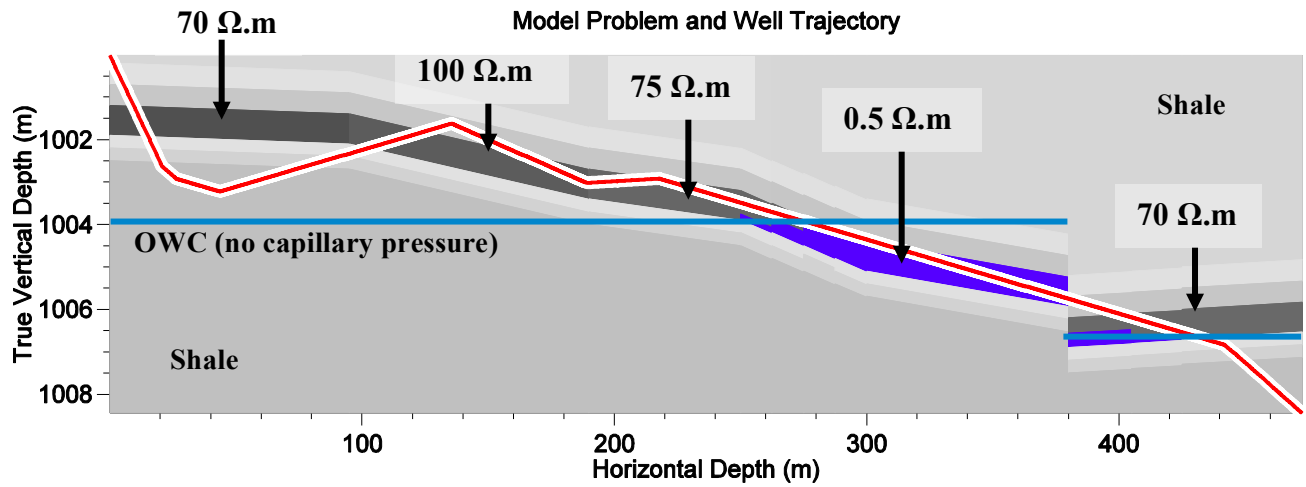


Fig. A-1. Model problem and well trajectory.