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How to explain the cross-section of equity returns through Common Principal Components

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Abstract

In this paper we propose a procedure to obtain and test multifactor models based on statistical and financial factors. In order to select the factors included in the model, as well as the construction of the portfolios, we use a multivariate technique called Common Principal Components. A block-bootstrap methodology is developed to assess the validity of the model and the significance of the parameters involved. Data come from Reuters, correspond to nearly 1250 EU companies, and span from October 2009 to October 2019. We also compare our bootstrap-based inferential results with those obtained via classical testing proposals. Methods under assessment are time-series regression and cross-sectional regression. The main findings indicate that the multifactor model proposed improves the Capital Asset Pricing Model with regard to the adjusted-R^2 in the time-series regressions. Cross-section regression results reveal that Market and a factor related to Momentum and mean of stocks' returns have positive risk premia for the analysed period. Finally, we also observe that tests based on block-bootstrap statistics are more conservative with the null than classical procedures.

Keywords: Asset Pricing, Bootstrap, Common Principal Component Analysis, Cross-Sectional Regression, Factor Models, Time Series

1 Introduction

Traditionally, finance theory has relied upon the risk-return relationship, i.e. the higher the risk (usually measured through the standard deviation of returns), the higher the return. This concept is at the core of the Capital Asset Pricing Model (CAPM), see Sharpe (1964); Lintner (1965); Mossin (1966), which represents the expected profitability of the i-th stock, $E(R_i)$, in the following manner:

$$E(R_i) = r_f + \beta_i(E(r_m) - r_f),$$

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where r_f is the risk-free rate, $E(r_m) - r_f$ is the Market Risk Premium, and β_i the sensitivity of expected excess asset's return associated with the i-th asset.
However, several authors have reported some breaches in this theory. For example, less volatile stocks seem to have higher returns, see Frazzini and Pedersen (2014), while Lintner (1965) and Miller and Scholes (1972) obtained certain inconsistencies when testing the model with NYSE stocks. Academia has pointed to the existence of several other factors that, beyond volatility, affect the returns of assets (basically, investors obtain a reward for bearing risks different from volatility). Some of these factors, relying on financial measures, are already considered as classical and have been tested in different Markets, see Fama and French (1992). Others, incorporating also macro or industry-related measures, such as Interest Rates levels (Viale et al. (2009)), or Oil price (Ramos et al. (2017)), have been less studied or remain undiscovered. Another part of the literature focuses on higher order statistical moments of returns, see Elyasiani et al. (2020); Lemperiere et al. (2017). Lately, such measures are combined with others involving psychological factors, such as Momentum, in order to build more sophisticated models, see Carhart (1997). What appears to be clear is that multifactor models should explain the behaviour of assets' returns better than CAPM. Typically, these models take the form of the following equation:

$$E(R_i) = \alpha_i + \beta_i' \lambda.$$

The expected return for the i-th portfolio has a linear relationship with a set of factors λ. Usually this set includes CAPM's Market factor and other additional factors as independent variables. New debates arise today regarding which factors should be included in asset pricing models (see Fama and French (2015) regarding the redundancy of the value factor) and whether certain factors are not working anymore.
When studying these multifactor models, two main difficulties may appear. First, the procedures to test the significance of the factors rely on the construction of portfolios. We use portfolios instead of stocks because the errors of α and β are higher for individual stocks as their volatility is higher and portfolios have more stable characteristics and are less prone to missing data than stocks. However, the question on which factors to select in order to build the portfolios remains unanswered. Firstly, there is an issue of dimensionality: on one hand, as we increase the number of portfolios to account for the various factors, the number of companies per portfolio decreases and this could be relevant for analysis of Markets not as developed as the US or the Euro-Zone; on the other hand, there might also be a loss in efficiency in using too few portfolios as the model could fail to explain the cross-section of individual assets. Secondly, as Feng et al. (2020) suggest, selecting a few portfolios based on some characteristics could bias the results in favor of these factors. In this paper, we propose to build the portfolios using Common Principal Components, a multivariate technique developed by Flury (1984). This technique can be regarded as an extension of classical Principal Components when the available information is organized in more than one dataset. In our case, we have several factors measured along a time period for a large group of companies. The idea is to search for a common set of orthogonal axes that capture a high percentage of the variability of the factors observed in all the companies.

Second, traditional inferential procedures about multifactor models, like Fama and French (1993) and Fama and MacBeth (1973), strongly rely on assumptions regarding the data: for instance, factors being uncorrelated over time, normally distributed errors which are i.i.d. over time and independent of the factors, etc. When these assumptions are not satisfied, classical estimators may be biased. Efron (1972) developed the Bootstrap methodology as a resampling technique to approximate the distribution of test statistics. In the Asset Pricing literature, Cueto et al. (2020) proposed a block-bootstrap procedure that accounts for time dependency in order to test the validity of the model and the significance of the parameters involved. This procedure grows, among others, on that of Chou and Zhou (2006), who bootstrap a Wald test for the case where residuals and asset returns are jointly i.i.d and use a block-bootstrap for a Wald-type GMM test in the non-i.i.d case. Grané and Veiga (2008) explored the block bootstrap for computing the unconditional distribution of returns and reported huge differences in the minimum capital risk requirement estimates when using conditional approaches (such as GARCH-type models and stochastic volatility models) specially for long positions and larger investment horizons.

The objectives of this paper are: (1) to propose a multifactor model based on statistical and financial factors using Common Principal Components (CPC) to reduce its number of dimensions, (2) to develop non-parametric resampling techniques that account for time dependency in order to test the validity of the model and the significance of the parameters involved, and (3) to compare bootstrap-based inferential techniques with the classical proposals.

In particular, the financial and statistical factors considered are: Market Capitalization and Total Assets (measures of size), Price to Book ratio (measure of cheapness), Return on Assets and Return on Equity (measures of profitability), Momentum, and four statistical measures (mean, standard deviation, kurtosis and skewness). The multifactor model with four CPC-factors is able to explain 90% of the variability of the data. The first CPC-factor is a linear combination of mean and Momentum returns; the second and third CPC-factors are linear combinations of skewness and kurtosis returns and finally, the fourth CPC-factor is the standard deviation of the return. Interestingly, none of them include the financial ratios. A possible explanation is that these ratios do not add enough variability compared to statistical factors.

The main findings are that CAPM cannot explain by itself the return of the portfolios as β for Market is higher for portfolios with high standard deviation (CPC4) and α is higher (and positive) for portfolios with high Momentum and mean (CPC1). In fact, when we incorporate additional factors, we notice that Momentum and mean (CPC1-factor), despite being correlated with the Market factor, and standard deviation (CPC4-factor) help explaining the cross-section of European stocks during the period considered. With regard to the time-series regressions, apart from the calculation of βs and αs, which seem to be quite robust despite the relaxation of the assumptions of the model, GRS p-value is much higher for the bootstrap. Finally, two factors present risk premia different from zero in the cross-section, which are Market and the factor based on mean and Momentum (CPC1-factor). These findings lead us to conclude that the multifactor model based on CPC-factors is a good model

with regard to the adjusted-R^2, able to explain excess returns, although in the analysed time period only one of the CPC-factors presents positive risk premia.
The remainder of this paper is organized as follows. Section 2 contains a description of the data and methodology; specifically, in Section 2.1 we present and describe our data, while in Section 2.2 we explain the methodology used to construct the portfolios and to test the validity the multifactor model, that is, time-series and cross-sectional classical methodologies and the block-bootstrap procedure. Section 3 contains the results of the analysis and the comparison of the application of classical and bootstrap inferential procedures to the data, while the final conclusions are discussed in Section 4.

2 Materials and Methods

2.1 Data and factors

2.1.1 Data description

We start with 2393 European companies that were selected from all EU countries. As is usual in the literature on factor models, we excluded Financials as they usually have high leverage ratios, affecting several financial ratios. The database comprises monthly data from Oct-2009 to Oct-2019 and includes prices and several financial ratios, such as Market capitalization, Price to Book and Price to Earnings ratios, number of shares outstanding, Return on Assets and Return on Equity, number of shares traded, and Total Assets. We apply several filters to the Data: first, we delete all companies with less than 30 months of complete data (which leads to 1305 remaining companies); second, we apply a transaction filter, excluding all companies that do not show transactions for a whole semester; third, we exclude companies with no Market Cap info for more than 2 consecutive years; finally, we exclude companies with non-positive Equity at the end of any year. After passing all these filters, we end up with 1230 companies. Next, we calculate monthly returns for all the companies and estimate the Risk Free Rate (r_f) with the 2 year German Bond yield and Market (r_m) through the STOXX 600. Monthly returns were calculated with the natural logarithms of the market price divided by the market price in the previous month.
Table 1 contains some descriptive statistics by year. In particular, it provides information on the number of firms, the number of months and summary statistics on prices by year. Figure 1 contains a graphical description of the data.
We can observe that average price decreases until 2012 (from 34.70 in 2009 to 26.55 in 2012) and then increases until 2018 (47.43). This behaviour is related to the 2008 Global Financial Crisis and can be seen more clearly in Figure 1. In terms of standard deviation, we can observe a similar pattern, although the slope in the decrease is more abrupt than for the mean values.

2.1.2 Factors under study

Cueto et al. (2020) introduce three new factors based on statistical measurements on stock

Year	Companies	Months	Mean	SD	Min	Median	Max
2009	1230	3	34.70	455.92	0.05	7.61	18043.85
2010	1230	12	33.76	363.74	0.03	8.15	15513.50
2011	1230	12	29.23	176.98	0.03	8.57	6515.83
2012	1230	12	26.55	132.78	0.02	7.82	4486.71
2013	1230	12	28.28	116.58	0.03	9.01	3430.00
2014	1230	12	31.64	127.10	0.01	10.95	3800.00
2015	1230	12	35.24	138.22	0.01	12.16	4000.00
2016	1230	12	37.46	166.02	0.01	12.29	5999.00
2017	1230	12	45.42	194.01	0.01	14.95	5999.99
2018	1230	12	47.43	211.38	0.01	14.70	6600.00
2019	1230	10	46.17	241.41	0.01	13.34	9200.00

Table 1: Descriptive statistics of the data by year

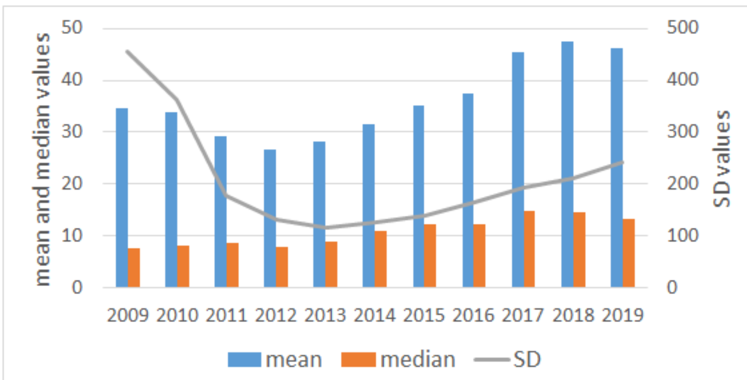

Figure 1: Summary statistics of the data by year

prices. Here we consider such factors calculated on stocks' returns together with a Momentum variable, which is equal to the 12-month logarithmic return of prices, Market Capitalization and Total Assets (measures of size), Price to Book ratio (measure of cheapness), Return on Assets, and Return on Equity (measures of profitability). For all the financial ratios, we take the value appearing 6 months in advance except for Total Assets, which corresponds to the increment of this measure over the previous 12 months. This way, we incorporate all the factors included in the 5-factor model by Fama and French (2015).
Regarding the statistical measures for all the stocks in the sample, we applied a rolling window taking into account the previous 12 observations. These measures are:

- Mean of returns for each year and company:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

- Standard Deviation (SD) of returns for each year and company:

$$s_n = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- Excess Kurtosis (Kurt) of returns for each year and company:

$$Kurt = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{s_n^4} - 3$$

measures how fat are the tails of a distribution in comparison of those of a normal random variable. Positive (negative) values indicate heavier (thinner) tails than those of a normal random variable.

- Skewness (Skew) of returns for each year and company:

$$Skew = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{s_n^3}$$

Positive values indicate that the distribution is positively skewed, that is, the right tail is longer than the left one, while the contrary occurs for negative values. When both tails are similar, the skewness is roughly 0.

As can be seen in Table 2, Market is positively correlated with Momentum, mean, skewness, and Total Assets, while it is negatively correlated with the remaining factors. This could be useful for investors willing to invest in uncorrelated portfolios (uncorrelated in terms of factors, not assets). Notice further that correlations are mainly low, except for Momentum and mean, which are highly correlated. Such an issue could lead to multicolineality in models built with these factors and, thus, to greater standard errors and larger confidence intervals for the model coefficients.

	Market	**Mom.**	**Mean**	**SD**	**Kurt**	**Skew**	**Market Cap**	**P/B**	**ROA**	**ROE**	**Total Assets**
Market	1.000	0.042	0.025	-0.002	-0.001	0.010	-0.006	-0.005	-0.016	-0.005	0.001
Momentum	0.042	1.000	0.959	-0.066	-0.062	0.231	0.008	0.049	0.049	0.199	0.022
Mean	0.025	0.959	1.000	-0.070	-0.062	0.243	0.009	0.054	0.059	0.207	0.023
SD	-0.002	-0.066	-0.070	1.000	0.200	0.036	-0.117	0.010	-0.020	-0.115	-0.009
Kurt	-0.001	-0.062	-0.062	0.200	1.000	0.068	-0.059	0.003	-0.011	-0.029	-0.009
Skew	0.010	0.231	0.243	0.036	0.068	1.000	-0.088	0.008	-0.014	0.012	-0.003
Market Cap	-0.006	0.008	0.009	-0.117	-0.059	-0.088	1.000	0.001	0.081	0.047	0.010
P/B	-0.005	0.049	0.054	0.010	0.003	0.008	0.001	1.000	0.015	0.065	0.008
ROA	-0.016	0.049	0.059	-0.020	-0.011	-0.014	0.081	0.015	1.000	0.187	0.197
ROE	-0.005	0.199	0.207	-0.115	-0.029	0.012	0.047	0.065	0.187	1.000	0.051
Total Assets	0.001	0.022	0.023	-0.009	-0.009	-0.003	0.010	0.008	0.197	0.051	1.000

Table 2: Correlations among the factors

2.2 Methodology

2.2.1 Common Principal Components

Before analyzing whether certain factors manage to explain the expected return of a set of portfolios, we must first form the portfolios. In order to do so, we use the Common Principal Components technique, introduced by Flury (1984). The underlying idea of CPC is to represent in the same common orthogonal axes several groups of individuals/objects (of possibly different sample sizes) for which the same number of variables/measurements have been observed. In our case, the variables are the ten factors under consideration and groups correspond to the 1230 companies. That is, each group is formed by a particular company and the dataset is composed by the observations of the factors for this particular company along the time period under consideration.

As in classical Principal Component Analysis, the goal is to determine a number of uncorrelated linear combinations of the variables that maximize their variance for each company. In this case, however, despite the linear combinations will be the same for all companies, the associated variances to each component may change among them, which results in a reduction of the number of parameters to estimate when maximizing the variance explained by the model.

In the setup of the problem, we have f variables $x_1, ..., x_f$ (factors) observed on p companies along time periods of size $(n_1, ..., n_p)$. Given $\mathbf{S}_1, ..., \mathbf{S}_p$, the variance-covariance matrices of the f variables for each company, we would like to find an orthogonal matrix $\mathbf{U}$ and p diagonal matrices $\mathbf{\Lambda}_1, ..., \mathbf{\Lambda}_p$ such that:

$$\mathbf{S}_\ell = \mathbf{U}\,\mathbf{\Lambda}_\ell\,\mathbf{U}', \quad \ell = 1, \ldots, p,$$

where matrix $\mathbf{U}$ if formed by the common eigenvectors of $\mathbf{S}_1, ..., \mathbf{S}_p$ and diagonal matrix $\mathbf{\Lambda}_\ell$ contains the eigenvalues of each $\mathbf{S}_\ell$ in descending order.

Note that this model may not exist since, given two positive definite symmetric matrices, their eigenvectors are equal if and only if these matrices fulfill the commutative property. So, in general, matrices $\mathbf{S}_\ell$ will not have the same eigenvectors, unless they fulfill the commutative property.

Nevertheless, this problem is solved numerically, and the idea is to find a matrix $\mathbf{U}$ and p matrices $\mathbf{\Lambda}_\ell$, such that each $\mathbf{U}\,\mathbf{\Lambda}_\ell\,\mathbf{U}'$ is as similar as possible to $\mathbf{S}_\ell$. Thus, the CPC-model can be viewed as a rotation yielding variables that are "as uncorrelated as possible" simultaneously in p groups.

To determine how similar they are, Flury and Gaustchi (1986) propose a numerical algorithm that minimizes the following discrepancy measure of "simultaneous diagonalizability":

$$F(\mathbf{U}) = \prod_{\ell=1}^{p} \left[\frac{\det(\mathrm{diag}(\mathbf{U}'\,\mathbf{S}_\ell\,\mathbf{U}))}{\det(\mathbf{U}'\,\mathbf{S}_\ell\,\mathbf{U})} \right]^{n_\ell}, \tag{1}$$

which naturally arises in the context of maximum likelihood estimation in principal component analysis of several groups under the assumption of multivariate normality, see Flury (1988).

Proposition 2.1. *Let $\mathbf{S}_1, ..., \mathbf{S}_p$ be positive definite symmetric matrices of dimension $f \times f$. Then expression* (1) *satisfies that $F(\mathbf{U}) \geq 1$ and $F(\mathbf{U}) = 1$ if $\mathbf{S}_\ell = \mathbf{U}\mathbf{\Lambda}_\ell\mathbf{U}'$, for $\ell = 1, \ldots, p$.*

The second part of Proposition 2.1 is straightforward, while the upper bound for $F(\mathbf{U})$ follows from Hadamard's inequality, that we present in Lemma 2.3 after two preliminary results.

Lemma 2.1 (Inequality between geometric and arithmetic means)**.** *For any $x_1, \ldots, x_n > 0$, we have*

$$\left(\prod_{i=1}^{n} x_i\right)^{1/n} \leq \frac{1}{n}\sum_{i=1}^{n} x_i.$$

Proof. After the convexity of function $\phi(x) = -\log(x)$ in $\mathbb{R}^+$ and Jensen's inequality,

$$\begin{aligned}
\phi\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) &\leq \frac{1}{n}\sum_{i=1}^{n}\phi(x_i) \\
-\log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) &\leq -\frac{1}{n}\sum_{i=1}^{n}\log(x_i) = -\log\left(\prod_{i=1}^{n} x_i\right)^{1/n} \\
\left(\prod_{i=1}^{n} x_i\right)^{1/n} &\leq \frac{1}{n}\sum_{i=1}^{n} x_i.
\end{aligned}$$

□

Lemma 2.2. *For any $f \times f$ matrix of correlations $\mathbf{R}$, we have $\det(\mathbf{R}) \leq 1$.*

Proof. Let $\lambda_1, \ldots, \lambda_f$ be the eigenvalues of $\mathbf{R}$. Applying the inequality between the geometric and arithmetic means given in Lemma 2.1, we have that

$$(\det(\mathbf{R}))^{1/f} = \left(\prod_{i=1}^{f}\lambda_i\right)^{1/f} \leq \frac{1}{f}\sum_{i=1}^{f}\lambda_i = \frac{1}{f}\mathrm{tr}(\mathbf{R}) = 1.$$

□

Lemma 2.3 (Hadamard's inequality)**.** *For any $f \times f$ positive definite symmetric matrix $\mathbf{S}$, we have $\det(\mathbf{S}) \leq \prod_{i=1}^{f} s_{ii}$, where $(s_{11}, \ldots, s_{ff}) = \mathrm{diag}(\mathbf{S})$.*

Proof. Let $\mathbf{S}$ be an $f \times f$ positive definite symmetric matrix and $\mathbf{D} = \mathrm{diag}(\mathbf{S})$, then $\mathbf{R} = \mathbf{D}^{-1/2}\,\mathbf{S}\,\mathbf{D}^{-1/2}$ is an $f \times f$ matrix of correlations, from where we can obtain $\mathbf{S}$ and compute its determinant as follows:

$$\det(\mathbf{S}) = \det(\mathbf{D})\ \det(\mathbf{R}) = \prod_{i=1}^{f} s_{ii}\ \det(\mathbf{R}) \leq \prod_{i=1}^{f} s_{ii},$$

since after Lemma 2.2, $\det(\mathbf{R}) \leq 1$. □

Preliminary setup of the algorithm to compute the CPC-model Flury and Gaustchi (1986) propose an algorithm to solve a system of equations that leads to the minimizer of function $F(\cdot)$ in (1) among $f \times f$ orthogonal matrices and for any given $f \times f$ positive definite symmetric matrices $\mathbf{S}_1, ..., \mathbf{S}_p$.
In first place, observe that the denominator in (1) is constant since

$$\det(\mathbf{U}'\,\mathbf{S}_\ell\,\mathbf{U}) = \det(\mathbf{U})\det(\mathbf{S}_\ell)\det(\mathbf{U}) = \det(\mathbf{S}_\ell).$$

As a consequence, any minimizer of $F(\mathbf{U})$ would also be a minimizer of its numerator, which we will denote by $G(\mathbf{U})$. Taking logarithms, we have that

$$\log G(\mathbf{U}) = \sum_{\ell=1}^{p} n_\ell \log(\det(\mathrm{diag}(\mathbf{U}'\mathbf{S}_\ell\mathbf{U}))).$$

The diagonal elements of $\mathbf{U}'\mathbf{S}_\ell\mathbf{U}$ are $(\mathbf{U}'\mathbf{S}_\ell\mathbf{U})_{ii} = \mathbf{u}_i'\mathbf{S}_\ell\mathbf{u}_i$, where $\mathbf{u}_i$ is the i-th column of $\mathbf{U}$. Using this notation, we have that

$$\log G(\mathbf{U}) = \sum_{\ell=1}^{p} n_\ell \sum_{i=1}^{f} \log(\mathbf{u}_i'\mathbf{S}_\ell\mathbf{u}_i).$$

Next, the extreme values of $\log G(\mathbf{U})$ are computed, with the restriction that $\mathbf{U}$ is an orthogonal matrix:

$$\Phi = \log G(\mathbf{U}) - \sum_{i=1}^{f} \mu_i(\mathbf{u}_i'\mathbf{u}_i - 1) - \sum_{i=1}^{f}\sum_{j=1}^{f} \mu_{ij}\mathbf{u}_i'\mathbf{u}_j, \tag{2}$$

where μ_i, μ_{ij}, $1 \le i, j \le p$, $i \ne j$ are Lagrange multipliers. It can be assumed that $\mu_{ij} = \mu_{ji}$ since $\mathbf{u}_i'\mathbf{u}_j = \mathbf{u}_j'\mathbf{u}_i$. Differentiating (2) with respect to $\mathbf{u}_k$, we have that

$$\frac{\partial \Phi}{\partial \mathbf{u}_k} = \sum_{\ell=1}^{p} n_\ell 2\mathbf{u}_k'\mathbf{S}_\ell/\lambda_{\ell k} - \mu_k 2\mathbf{u}_k' - \sum_{i=1, i\ne k}^{f} \mu_{ik}\mathbf{u}_i' = 0,$$

where $\lambda_{\ell k}$ is the k-th eigenvalue of $\mathbf{S}_\ell$. Post-multiplying the previous expression by $\mathbf{u}_j$, $j \ne k$, we have that:

$$\sum_{\ell=1}^{p} n_\ell \mathbf{u}_k'\mathbf{S}_\ell\mathbf{u}_j/\lambda_{\ell k} = \mu_{jk}.$$

Analogously, differentiating (2) with respect to $\mathbf{u}_j$ and post-multiplying the resulting expression by $\mathbf{u}_k$, $k \ne j$, one obtains that

$$\sum_{\ell=1}^{p} n_\ell \mathbf{u}_j'\mathbf{S}_\ell\mathbf{u}_k/\lambda_{\ell j} = \mu_{kj}.$$

Subtracting both expressions leads to the following system of equations:

$$\sum_{\ell=1}^{p} n_\ell \left(\frac{\mathbf{u}_k' \mathbf{S}_\ell \mathbf{u}_j}{\lambda_{\ell k}} - \frac{\mathbf{u}_j' \mathbf{S}_\ell \mathbf{u}_k}{\lambda_{\ell j}} \right) = 0,$$

whose solutions are the columns of matrix $\mathbf{U}$, from which it is possible to obtain matrices $\mathbf{\Lambda}_\ell$. Finally, the system of equations is efficiently solved by means of the aforementioned algorithm of Flury and Gaustchi (1986), which is a generalization of the well-known Jacobi method for computing eigenvectors and eigenvalues of a single symmetric matrix.
The CPC algorithm is implemented in the `R` package `multigroup`, designed to study multi-group data, where the same set of variables are measured on different groups of individuals. Within this package, we specifically use the function `FCPCA` to perform the CPC calculation.

2.2.2 Portfolio construction

In order to build the portfolios in the more rational manner, we first take the proposed financial and statistical measures and standardize them to zero mean and unit variance. We recall that these measures are: Market Capitalization and Total Assets (measures of size), Price to Book ratio (measure of cheapness), Return on Assets and Return on Equity (measures of profitability), Momentum and the statistical measures already mentioned.
Then we seek for a common pattern in all companies attending to all measures or factors. Thus, we compute the CPC-model with the aim of obtaining a few uncorrelated components that explain as much as possible the ten measures included in the analysis for all companies. We selected the first four principal components since the average percentage of variability explained by them was 90%. Additionally, to check the robustness of the CPC loadings, we estimated them by a bagging procedure, see (Breiman, 1996). Specifically, we selected groups of 100 companies (without replacement) for which we calculated the CPC-model. After repeating this procedure 5 times we present the results and compare them with the CPC-model computed with all the data in Table 3.

	Dim 1	**Dim 1 (bagging)**	**Dim 2**	**Dim 2 (bagging)**	**Dim 3**	**Dim 3 (bagging)**	**Dim 4**	**Dim 4 (bagging)**
Momentum	70.92	70.77	-0.06	2.15	-4.92	-5.61	2.12	2.32
Mean	70.12	69.82	0.60	2.76	-4.08	-4.62	1.93	2.14
SD	-3.31	-3.78	4.66	4.60	-7.11	-6.04	99.58	99.52
Kurt	-4.40	-7.29	72.27	72.81	-68.46	-67.39	-8.42	-7.81
Skew	4.30	3.02	68.95	68.02	72.26	72.95	2.08	1.27
Market Cap	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
P/B	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
ROA	0.25	0.32	-0.05	-0.11	-0.08	-0.04	-0.65	-0.85
ROE	2.23	2.33	-0.04	0.08	-0.27	-0.21	-0.69	-1.06
Total Assets	0.02	0.01	-0.03	0.00	-0.04	-0.06	-0.05	-0.10

Table 3: Loadings computed with all the data vs. loadings computed by bagging

Regarding the bagging results of Table 3, the CPC-loadings changed signs in some iterations and exchanged CPC2 for CPC3 in others (the percentage of variability explained by these

two components is very similar – around 22%). Such exchange in some of the components is a well-known problem and is related to the Flury-Gautschi algorithm used to solve (1), see (Flury, 1988). We proceeded to change signs accordingly in order to present consistent results.
Observing the CPC-loadings, we see that the first CPC is a linear combination of mean and Momentum returns, the second and third CPCs are linear combinations of skewness and kurtosis returns and, finally, the fourth CPC is the standard deviation return. In what follows, we call these components CPC1 to CPC4. Interestingly, none of these CPCs include the financial ratios.

Portfolio setup with the CPC model Next, we get the representation of each company in the CPC model (multiplying the loadings by the standardized variables), compute percentiles for each of them and assign to portfolios accordingly following Fama and French (1992).

- Stocks with low CPC1 are included in portfolios 1-b-c-d, while stocks with high CPC1 are included in portfolios 2-b-c-d.
- Stocks with low CPC2 are included in portfolios a-1-c-d, while stocks with high CPC2 are included in portfolios a-2-c-d.
- Stocks with low CPC3 are included in portfolios a-b-1-d, while stocks with high CPC3 are included in portfolios a-b-2-d.
- Stocks with low CPC4 are included in portfolios a-b-c-1, while stocks with high CPC4 are included in portfolios a-b-c-2.

The resulting portfolios, which are summarized in Table 4, are updated monthly based on the previous month measurements. For each portfolio, we calculate the average monthly returns. Additionally, we calculate each factor as the excess return of the higher portfolio in each category minus the return of the lower portfolio. All returns are calculated for equally-weighted portfolios at $t + 1$. In Figure 2, we have plotted the cumulative returns of all the portfolios.
As we commented before, it is interesting that none of the CPCs include financial measures, which might occur because these ratios do not add enough variability compared to statistical factors. However, in a way, statistical measures could be also capturing some of the financial characteristics of the stocks as we see in Table 5, which includes average factors for each of the portfolios considered and standard deviation, kurtosis and skewness of the returns. We notice, for example, that portfolios with high CPC1 present in average higher PB ratio, higher ROA and ROE, lower kurtosis, and positive skew. Portfolios with high CPC4 show in average lower PB ratio, lower ROA and ROE, lower kurtosis and higher skew.
The best portfolios are those with high mean and Momentum (CPC1), except for portfolio 2-2-1-2, while the three portfolios with the poorest performances are 1-1-2-2, 1-2-2-2 and 1-1-1-2. They share the feature of having low mean and Momentum (CPC1) and high standard deviation (CPC4).

Portfolio number	Portfolio description	Portfolio composition CPC1	CPC2	CPC3	CPC4
1	1-1-1-1	Low	Low	Low	Low
2	1-1-1-2	Low	Low	Low	High
3	1-1-2-1	Low	Low	High	Low
4	1-1-2-2	Low	Low	High	High
5	1-2-1-1	Low	High	Low	Low
6	1-2-1-2	Low	High	Low	High
7	1-2-2-1	Low	High	High	Low
8	1-2-2-2	Low	High	High	High
9	2-1-1-1	High	Low	Low	Low
10	2-1-1-2	High	Low	Low	High
11	2-1-2-1	High	Low	High	Low
12	2-1-2-2	High	Low	High	High
13	2-2-1-1	High	High	Low	Low
14	2-2-1-2	High	High	Low	High
15	2-2-2-1	High	High	High	Low
16	2-2-2-2	High	High	High	High

Table 4: Description of portfolios

Portfolio number	Portfolio description	Portfolio composition (mean values) M. Cap	T. assets	PB	ROA	ROE	MOM	mean	sd	kurt	skew
1	1-1-1-1	4922.44	8816.40	1.94	4.57	8.47	-0.086	-0.008	0.057	2.935	-0.555
2	1-1-1-2	2072.12	6275.54	1.64	1.80	-0.28	-0.233	-0.021	0.116	2.865	-0.580
3	1-1-2-1	4063.73	6734.97	1.88	4.48	7.93	-0.094	-0.009	0.055	2.161	0.025
4	1-1-2-2	1839.13	5504.61	1.56	1.84	-4.27	-0.233	-0.021	0.108	2.079	0.000
5	1-2-1-1	3263.27	6024.39	1.84	4.22	7.74	-0.075	-0.007	0.059	4.308	-0.189
6	1-2-1-2	1044.30	3209.33	1.46	1.37	-14.23	-0.241	-0.022	0.146	4.263	-0.227
7	1-2-2-1	2795.40	4915.16	1.75	4.05	5.53	-0.080	-0.007	0.055	2.961	0.575
8	1-2-2-2	1025.77	3036.45	1.62	1.41	-9.23	-0.208	-0.019	0.118	2.853	0.558
9	2-1-1-1	6655.51	8231.53	2.65	7.33	14.65	0.186	0.017	0.058	2.729	-0.494
10	2-1-1-2	2654.52	4986.17	2.50	5.76	11.55	0.306	0.028	0.106	2.614	-0.440
11	2-1-2-1	6033.27	7107.41	2.52	7.26	14.15	0.172	0.016	0.055	2.089	0.023
12	2-1-2-2	2570.95	4606.20	2.42	5.76	10.76	0.284	0.026	0.099	1.999	0.068
13	2-2-1-1	2946.72	3876.48	2.33	6.85	12.89	0.169	0.016	0.060	4.068	0.222
14	2-2-1-2	1218.32	2307.18	2.29	5.23	10.51	0.369	0.034	0.140	4.121	0.502
15	2-2-2-1	3016.33	3701.31	2.32	7.13	13.15	0.172	0.016	0.056	2.943	0.600
16	2-2-2-2	1031.39	2519.19	2.15	6.31	8.45	0.333	0.031	0.111	2.877	0.672

Table 5: Characteristics of portfolios

2.2.3 Classical methodologies

Once we have defined the N portfolios that we are about to analyse, we perform a time-series regression for each of them:

$$R_t^i - r_f = \alpha_i + \beta_i' f_t + \epsilon_t^i, \quad t = 1, 2, \ldots, T.$$

To assess the ability of a model to explain excess returns, we use the GRS test proposed by Gibbons et al. (1989) whose null hypothesis is $H_0 : \alpha_1 = \ldots = \alpha_N = 0$. Under the

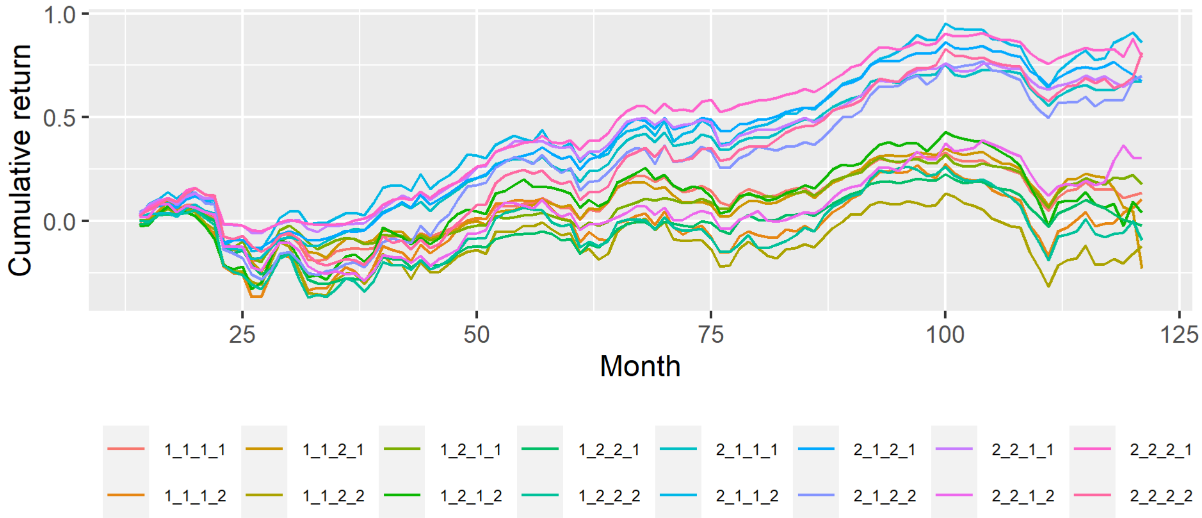

Figure 2: Portfolios' cumulative returns

assumption that ϵs are normally distributed, the test statistic is:

$$\frac{T-N-K}{N}[1+\overline{f}'\Sigma_f^{-1}\overline{f}]^{-1}\hat{\alpha}'\Sigma^{-1}\hat{\alpha} \sim F_{N,T-N-K}, \tag{3}$$

where K is the number of factors, Σ_f the covariance matrix of the factors, and Σ the residual covariance matrix. The purpose of the test, whose statistic takes also into account the sampling error of estimates in β_i, is to determine if the α_i are jointly zero assuming that the distribution of returns and factors is multivariate normal. As suggested by Fama (2015), the GRS test is against an unspecified alternative, both for portfolios and factors. On the one hand, the model may pass the test for a set of portfolios, but fail for another. On the other hand, we do not specify additional factors that could produce a violation of the model (however, we get some intuition on which factors should be included or excluded from the model by observing the number of portfolios whose coefficients are significantly different from zero).
Then we run a cross-sectional regression to estimate the vector of risk premia λ with the model obtained taking expectations in the time-series regression equation:

$$E(R^i - r_f) = \hat{\alpha}_i + \hat{\beta}_i'\lambda, \quad \text{for } i = 1, \ldots, N.$$

where the estimates for $\hat{\beta}_i$ are those obtained in the TS regression at the first step. In conclusion $\hat{\lambda}$ represents the slope coefficients in the cross-sectional regression which is run without intercept, while $\hat{\alpha}$ are the residuals in the cross-sectional regression. If the estimated βs are important determinants of average returns, then the risk premium, λ_i, should be statistically significant. We have used the covariance of the residuals of the TS regression to calculate the standard error of λ_i to take into account the correlation across assets. Additionally, the error terms for λ_i must include the error of estimating β (see Shanken, 1992, for the so-called Shanken correction), although the difference may be very small in practice. Finally, we use t-statistics to test the significance of each of the λ_i.

2.2.4 Resampling techniques: Bootstrap

As in Cueto et al. (2020), we follow a block-bootstrapping pairs scheme to preserve the time correlation of the data of B bootstrap samples of block size $b = 2$ months. We describe briefly the procedure and refer to the original paper for further details.

- **Step 1** Estimate benchmark regression models, one for each portfolio. For each portfolio, we save α, β, their corresponding t-statistics, residuals, the estimates of risk factors and the GRS statistic.
- **Step 2** Produce a set of simulation runs which is the same for every portfolio to preserve the cross-correlation of the returns.
- **Step 3** Use the simulated time indices to build a new series of α-free portfolio returns.
- **Step 4** Run the time-series factor model regression on the artificially constructed returns. Obtain $\hat{\alpha}$, $\hat{\beta}$ and the corresponding confidence intervals. Finally, generate samples of the GRS Statistic, calculate different percentiles from the bootstrapped distribution and compare them to the original GRS statistic.

  In this paper, we improve the methodology proposed in our previous work. We have noticed that, given the construction of the GRS statistic, when we face portfolios with heteroskedasticity, the estimation of the variance through the residual covariance matrix may not be appropriate. Thus, we propose a new statistic, $Q(\alpha) = \hat{\alpha}'\Sigma^{-1}\hat{\alpha}$, for which we will calculate the covariance matrix of αs, Σ, through a nested bootstrap. Then, a new step appears:

- **Step 5** Run the time-series factor model regression on the artificially constructed returns. Obtain $\hat{\alpha}$, $\hat{\beta}$ and the corresponding confidence intervals. Finally, generate bootstrap samples of the $Q(\alpha)$ statistic that makes use of the covariance matrix of the αs which is approximated by means of a nested bootstrap and compare the bootstrapped statistics with the original $Q(\alpha)$ statistic.

  Regarding cross-sectional regression, we have used βs and average returns from each bootstrapped sample to determine the significance of the risk premia estimates (λs). Given that βs are estimated, the estimates of the λs might present substantial bias and we use a reverse bootstrap percentile interval to determine the significance of the factors. To be consistent, we also present this type of bootstrap interval for all the estimated parameters.

3 Results

3.1 Time-Series regressions

In this section, we present the results for the time-series regression of two models. The first one is the CAPM model and the second one is a multifactor model including Market and four factors determined by CPC1 to CPC4. The dependent variables are the returns of the 16 portfolios.

3.1.1 Model 1: CAPM

When we analyze a 1-factor model, only taking into consideration the Market factor, we notice that the p-value of the GRS statistic is 1.93%, thus rejecting that all αs are jointly zero at 5% significance level. This could be an indication that other factors are missing because the portfolios are sorted to have greater cross-section differences, but, given that this is a well studied factor, we would like to understand if this rejection may be due to an anomalous period of time and/or to a breach in certain assumptions of the model.
First, we split the 108 months in two periods: from 1 to 80 (first subperiod) and from 30 to 108 (second subperiod), both with approximately the same number of months. We find that the GRS p-value for the first subperiod is 3.43%, while the p-value for the second subperiod is 0.24%. This could indicate that the CAPM may have not behaved correctly during the second period, while global Central Banks have been injecting huge amounts of liquidity in the system.
Additionally, in order to confirm if the OLS hypotheses are fulfilled (normality, homoscedasticity and independence), we performed several analysis following Soumaré et al. (2013). The results are presented in Table 6, where we show the different statistics analyzed with their p-values in parentheses and the number of portfolios for which we reject the null at a 5% significance level for the different tests.

	Statistics (p-value)			**Number of portfolios**		
	GRS	**GRS boot.**	$Q(\alpha)$	**non-norm.**	**heteros.**	**autocorr.**
Whole period	2.024 (1.93%)	2.024 (17.4%)	42.57 (29.5%)	6	4	5
Months 1–80	1.924 (3.43%)	1.924 (8.2%)	38.62 (27.5%)	3	5	4
Months 30–108	2.736 (0.24%)	2.736 (17.8%)	61.35 (30.6%)	3	1	8

Table 6: Metrics for CAPM with different time periods

A Shapiro-Wilk test was run on each set of residuals, suggesting that, in general, if we consider both subperiods individually, the residuals of the time-series regression are normally distributed. This changes when we consider the whole period, where 6 portfolios have a p-value lower than 5% in the test. Generally, these are portfolios with high mean and Momentum (CPC1) and low standard deviation (CPC4).
Additionally, we performed a Breusch-Pagan test on the residuals of the regressions, showing that the homoscedasticity assumption is violated for portfolios 9, 10, 11 and 13 (all of

them share the characteristic of having a high mean and Momentum (CPC1) and, generally, present low levels of the remaining CPCs). Interestingly, during the second subperiod, we reject the homocedasticity only for portfolio 16.

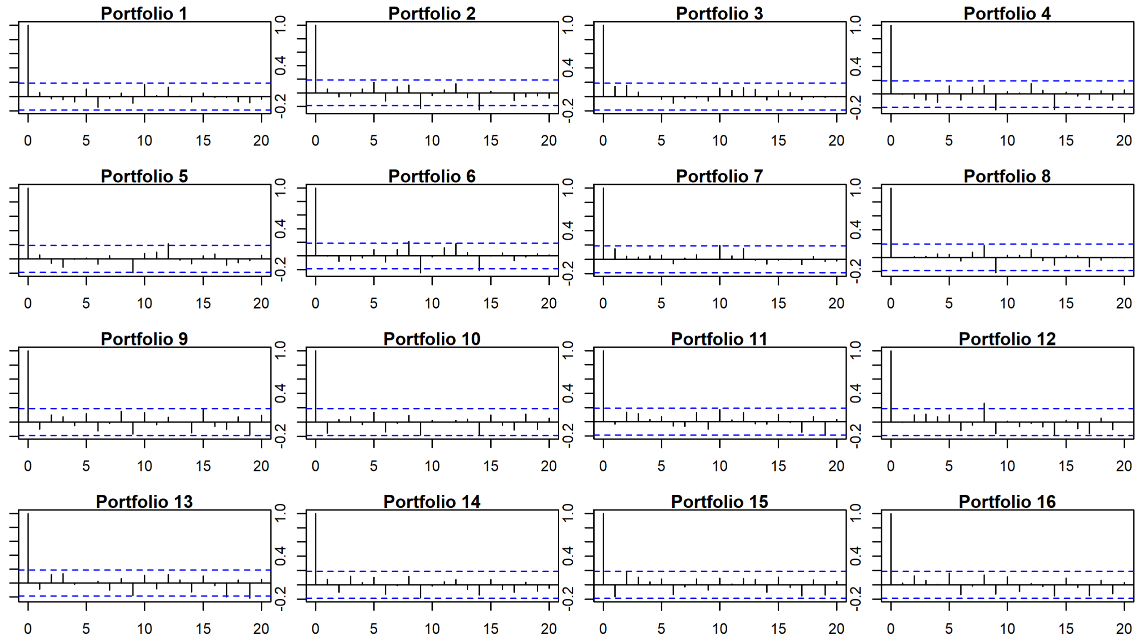

Figure 3: Autocorrelation charts for errors in TS regression for CAPM for the whole period

In Figure 3 we show the autocorrelation charts for errors in time-series regression for CAPM for the whole period. The charts indicate that residuals are especially correlated for lags equal to or higher than 9. The Durbin-Watson test also suggests that errors might be autocorrelated for 5 portfolios for lag equal 9 when considering the whole period (generally, those with low mean and Momentum (CPC1) and high standard deviation (CPC4)), 4 for the first subperiod and 8 for the second one (p-values lower than 5%).

These three facts suggest that the bootstrap procedure developed in Cueto et al. (2020) might be useful to approximate the distribution of the GRS statistic. Applying this procedure (see table 6), we obtained a bootstrap p-value of 17.4% for the whole period, so we cannot reject that the αs are jointly zero. Something similar happens when we consider the first subperiod (GRS p-value of 8.2%) and the second subperiod (GRS p-value of 17.8%). However, we do not feel comfortable with the results as the p-values of the Bootstrap are far away from the p-values of the traditional methodology; remember that the GRS test estimates Σ through the residual covariance matrix. We suspect that the existence of heteroscedasticity in some of the portfolios could make this estimation inaccurate. Thus, we propose a new statistic, $Q(\alpha) = \hat{\alpha}'\Sigma^{-1}\hat{\alpha}$, for which we will calculate the covariance matrix of αs, Σ, through a nested bootstrap. The results obtained with the $Q(\alpha)$ statistic reinforce the conclusions reached with the GRS bootstrap, since the p-values of this new statistic are even higher than those of the former (between 1.6 and 3.3 times). Thus, we do not reject that the αs are jointly zero at a 5% significance level.

	Estimates		t-statistics		Bootstrap CI $(2.5\%, 97.5\%)$		
Portfolio	α	**Market**	α	**Market**	α	**Market**	R^2
1	-0.001	0.685	-0.546	10.856	-0.007, 0.002	0.533, 0.845	0.526
2	-0.003	0.954	-0.695	9.109	-0.013, 0.002	0.746, 1.161	0.439
3	-0.004	0.543	-1.068	4.982	-0.016, -0.002	0.324, 0.736	0.190
4	-0.005	0.946	-1.235	8.853	-0.018, -0.002	0.722, 1.172	0.425
5	0.000	0.485	-0.041	7.141	-0.006, 0.004	0.317, 0.649	0.325
6	-0.003	0.823	-0.680	7.380	-0.014, 0.002	0.604, 1.024	0.339
7	-0.002	0.542	-0.905	7.948	-0.010, 0.001	0.389, 0.716	0.373
8	-0.004	0.831	-1.032	7.702	-0.016, -0.001	0.577, 1.078	0.359
9	0.004	0.632	1.655	9.304	0.003, 0.012	0.432, 0.840	0.450
10	0.005	0.865	1.398	8.898	0.003, 0.016	0.622, 1.097	0.428
11	0.004	0.619	1.647	8.967	0.003, 0.013	0.398, 0.827	0.431
12	0.003	0.852	0.960	8.613	0.000, 0.014	0.602, 1.112	0.412
13	0.004	0.532	1.936	8.128	0.004, 0.013	0.362, 0.725	0.384
14	0.000	0.657	0.128	6.932	-0.006, 0.008	0.472, 0.847	0.312
15	0.006	0.454	2.300	6.467	0.007, 0.015	0.264, 0.628	0.283
16	0.005	0.729	1.459	7.672	0.003, 0.016	0.500, 0.991	0.357

Notes. GRS: 2.024, P-value: 0.0193, P-value Boot.: 0.174, P-value $Q(\alpha)$: 0.295

Table 7: Results for Time-Series estimation of CAPM

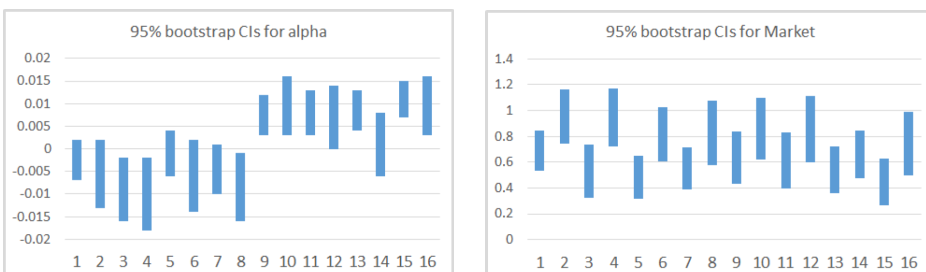

Figure 4: Graphical comparison of 95% Bootstrap CIs for Model 1

In Table 7 we present the results for the classical and the bootstrap methodologies for Model 1. Columns "Estimates" contain the estimates for α and β for Market. The classical t-statistics for these estimates are in columns 4 and 5. Columns 6 and 7 correspond to the basic bootstrap confidence intervals and R^2 is reported in the final column. A graphical comparison is given in Figure 4. Regarding the estimates of the model, we find that the coefficient for Market is always positive and statistically significant for classical methodology and for the resampling technique. The estimate varies between 0.95 and 0.45, but it is, in general, higher for portfolios with high standard deviation (CPC4). This confirms the traditional relationship between expected return and β explained in the CAPM. However, as we will see later and as different studies have suggested, this relationship does not hold once we include new factors and review the cross-section regression. The coefficients of determination indicate that this model explains between 19.0% and 52.6% of the variability of the returns of the portfolios. Additionally, under the classical inferential methodology, we find that α is not statistically significant (except for portfolio 15) which is positive for portfolios with high mean and Momentum (CPC1) and negative otherwise. The resampling technique finds more portfolios where α is statistically significant. As we have already discussed, the GRS test rejects, at a 5% significance level, that all pricing errors are equal to zero which might indicate that other factors are missing.

3.1.2 Model 2: Market and factors CPC1 to CPC4

When we analyze the 5-factor model, we notice that the p-value of the GRS statistic is 0.90%, lower than in the previous model and also rejecting, at a 5% significance level, that all αs are jointly zero. This could indicate that the model does not reflect correctly the variability of the returns of the portfolios.
As before, we split the 108 months in two periods: from 1 to 80 (first subperiod) and from 30 to 108 (second subperiod). We find that the GRS p-value for the first subperiod is 5.19%, while the p-value for the second subperiod is 0.38%. This could indicate that this model has not behaved correctly during the second subperiod, as we discussed for the previous model. Results for GRS p-values (both for classical and resampling methodology), $Q(\alpha)$ p-value and normality, heteroscedasticity and autocorrelation tests are presented in Table 8.

	Statistics (p-value)			**Number of portfolios**		
	GRS	**GRS boot.**	$Q(\alpha)$	**non-norm.**	**heteros.**	**autocorr.**
Whole period	2.242 (0.90%)	2.242 (17.1%)	56.59 (22.6%)	10	1	12
Months 1-80	1.806 (5.19%)	1.806 (22%)	44.92 (32.8%)	1	1	7
Months 30-108	2.625 (0.38%)	2.625 (13%)	56.19 (27.8%)	8	1	7

Table 8: Metrics for 5-factors model with different time periods

For this second model, the Shapiro-Wilk test run on each set of residuals suggests that the number of non-normal portfolios is higher than in the previous model. In fact, when we consider the whole period, 10 portfolios have a p-value lower than 5% in the test. If we consider both subperiods individually, we can only reject the null for one portfolio in subperiod 1 (portfolio 5), while there are 8 portfolios with this same characteristic in subperiod 2.
We also ran a Breusch-Pagan test on the residuals of the regressions, showing that the homoscedasticity assumption is violated for portfolio 11 during subperiod 1 and portfolio 14 during the second subperiod and the whole period. In this model, the heteroscedasticity problem seems to be (at least, partially) solved.
In Figure 5 we show the autocorrelation charts for errors in time-series regression for the 5-factor model for the whole period. According to the charts, residuals are correlated for lag equal to 10 and beyond. The Durbin-Watson test also suggests that errors may be autocorrelated for 12 portfolios for lag equal 10 when considering the whole period, 7 for

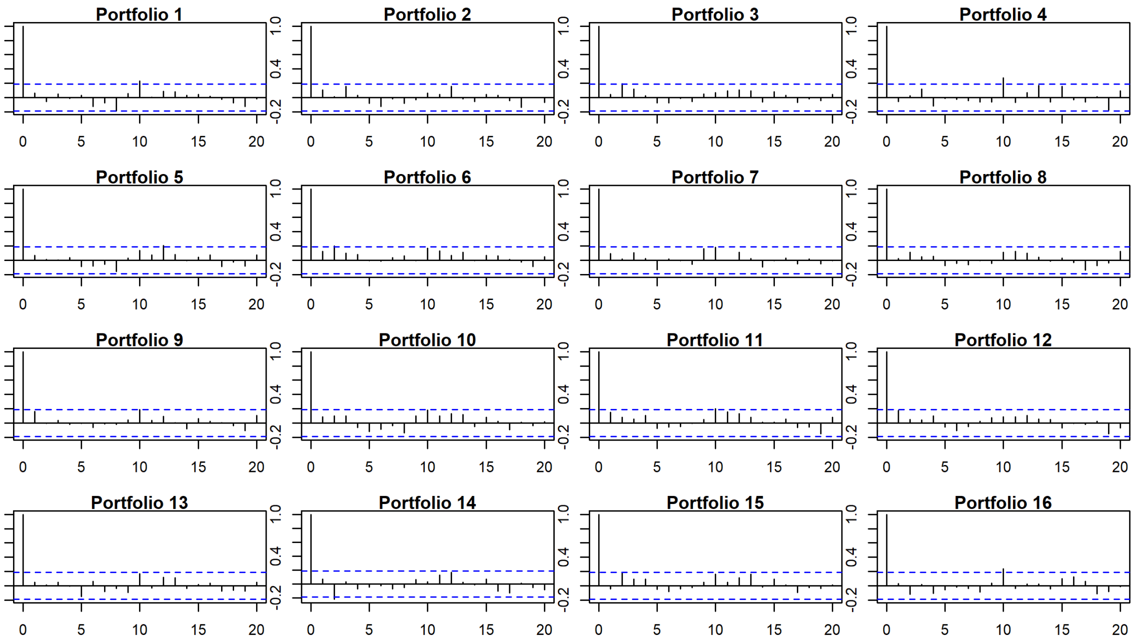

Figure 5: Autocorrelation charts for errors in TS regression for 5-factor model for the whole period

the first subperiod and 7 for the second subperiod (p-values lower than 5%). In this case, incorporating the CPC-factors results in an increase in the number of portfolios that present autocorrelation.

Again, given that the three previous assumptions of the model are breached, resampling might be useful to approximate the distribution of the GRS statistic. When using the bootstrap, we find that 17.1% of the values of the sample are higher than the GRS statistic, so we cannot reject the hypothesis that all the αs are jointly zero. As in the previous case, the results obtained with the new statistic $Q(\alpha)$ reinforces our decision of not rejecting the hypothesis that all the αs are jointly zero at a 5% significance level.

In Table 9 we present the results for the estimates and t-statistics for the classical methodology for Model 2. When we observe the coefficients (α and β) generated for the 16 portfolios we notice that: (1) we cannot reject that each of the α are zero; (2) none of the Market β can be considered statistically zero and the coefficients have stabilized around 0.35 (as commented before, once we control for additional factors, and specifically for a factor linked to volatility, the Market β effect that appeared in the previous model disappears); (3) βs for CPC1-factor and CPC4-factor seem to be significantly different from 0 as only 1 coefficient in each of them presents a t-statistic whose absolute value does not exceed 1.96 and (4) the average adjusted-R^2 increases to 63.5% (this model explains between 26.3% and 85.3% of the variation in the returns of the portfolios). CPC1-factor estimates are negative for portfolios with low mean and Momentum (CPC1), while the contrary happens in portfolios with high mean and Momentum (CPC1). Estimates for CPC2-factor are negative except for portfo-

Portfolio	Estimates						t-statistics						Adj-R^2
	α	Market	CPC1	CPC2	CPC3	CPC4	α	Market	CPC1	CPC2	CPC3	CPC4	
1	0.001	0.436	-0.292	-0.381	0.626	0.687	0.731	6.674	-2.891	-1.503	3.029	6.199	0.688
2	0.002	0.286	-0.315	-1.601	0.497	1.607	0.693	3.819	-2.713	-5.497	2.092	12.621	0.825
3	0.000	0.290	-0.503	-1.257	0.719	0.288	-0.078	2.189	-2.453	-2.444	1.712	1.280	0.263
4	0.001	0.316	-0.547	-1.001	1.195	1.734	0.384	4.574	-5.127	-3.738	5.471	14.813	0.853
5	0.003	0.208	-0.311	-0.546	0.073	0.643	1.314	2.883	-2.791	-1.954	0.318	5.260	0.533
6	0.003	0.386	-0.592	0.358	-0.170	1.440	0.972	4.113	-4.072	0.981	-0.570	9.030	0.712
7	0.000	0.325	-0.303	-0.218	0.976	0.682	0.174	4.473	-2.690	-0.771	4.234	5.525	0.562
8	-0.001	0.306	-0.271	-0.598	1.428	1.654	-0.212	3.303	-1.896	-1.664	4.872	10.531	0.710
9	0.002	0.345	0.406	-0.879	0.625	0.832	1.129	5.021	3.821	-3.296	2.873	7.138	0.654
10	0.003	0.312	0.561	-1.874	0.442	1.368	1.225	3.784	4.404	-5.861	1.695	9.784	0.747
11	0.002	0.378	0.376	-1.018	1.040	0.659	1.084	5.254	3.376	-3.649	4.568	5.396	0.620
12	0.000	0.362	0.709	-0.871	0.948	1.670	0.195	4.622	5.854	-2.867	3.825	12.567	0.774
13	0.002	0.404	0.475	-0.025	0.478	0.640	0.885	5.457	4.150	-0.085	2.039	5.094	0.517
14	-0.001	0.259	0.370	-0.668	0.576	1.280	-0.220	2.679	2.469	-1.779	1.880	7.792	0.558
15	0.003	0.279	0.470	-0.302	0.414	0.679	1.416	3.489	3.802	-0.973	1.636	5.002	0.430
16	0.003	0.401	0.471	0.264	1.637	1.585	1.283	5.005	3.800	0.847	6.447	11.649	0.719

Notes. GRS: 2.242, P-value: 0.009, P-value Boot.: 0.171, P-value Q(α): 0.226

Table 9: Results for Time-Series estimation of Model 2

lios 6 and 16 and, in general, they are only significant for portfolios with low skewness and kurtosis (CPC2). Estimates for CPC3-factor are all positive except for portfolio 6. Finally, estimates for CPC4-factor are always positive and significant for all portfolios except for portfolio 3. Additionally, βs for CPC4-factor are higher for portfolios with high standard deviation (CPC4).

Portfolio	Bootstrap CI (2.5%,97.5%)					
	α	Market	CPC1	CPC2	CPC3	CPC4
1	-0.840, 0.435	0.299, 0.605	-0.528, -0.044	-0.966, 0.179	0.105, 1.153	0.376, 0.945
2	-0.851, 0.452	0.145, 0.435	-0.582, -0.085	-2.288, -1.054	-0.129, 1.333	1.188, 1.888
3	-1.288, 0.666	0.038, 0.538	-0.869, -0.165	-2.966, -0.060	-0.236, 1.505	-0.453, 0.758
4	-1.410, 0.684	0.157, 0.496	-0.762, -0.315	-1.611, -0.462	0.680, 1.619	1.403, 1.978
5	-0.343, 0.473	0.065, 0.363	-0.567, -0.071	-1.242, -0.086	-0.461, 0.659	0.279, 0.876
6	-0.161, 0.797	0.147, 0.628	-0.930, -0.242	-0.916, 1.272	-0.900, 0.427	0.881, 1.785
7	-1.271, 0.413	0.190, 0.483	-0.518, -0.074	-0.894, 0.365	0.365, 1.622	0.363, 0.894
8	-1.770, 0.425	0.119, 0.465	-0.526, -0.006	-1.532, 0.035	0.732, 1.973	1.197, 1.938
9	-0.892, 0.008	0.209, 0.494	0.149, 0.623	-1.446, -0.413	0.091, 1.232	0.483, 1.067
10	-0.912, 0.201	0.137, 0.490	0.170, 0.879	-2.840, -1.137	-0.292, 1.313	0.809, 1.694
11	-1.329, 0.008	0.213, 0.554	0.052, 0.680	-1.854, -0.372	0.445, 1.737	0.212, 0.968
12	-1.255, 0.005	0.177, 0.566	0.453, 0.980	-1.680, -0.145	0.332, 1.513	1.316, 1.931
13	-0.759, 0.008	0.216, 0.596	0.221, 0.720	-0.708, 0.548	-0.082, 1.022	0.274, 0.907
14	-1.154, 0.646	0.114, 0.424	-0.166, 0.753	-2.052, 0.300	-0.491, 2.086	0.581, 1.698
15	-0.821, 0.010	0.087, 0.458	0.260, 0.706	-1.298, 0.412	-0.343, 0.966	0.351, 0.910
16	-2.012, 0.010	0.265, 0.589	0.210, 0.751	-0.252, 0.906	0.876, 2.468	1.254, 1.916

Table 10: Results for Bootstrap estimation of Model 2

Table 10 contains the results for the inference based on resampling techniques. A graphical comparison is shown in Figure 6. The results of the basic bootstrap confidence intervals are consistent with what we have already commented. Despite having a lower GRS p-value than Model 1, we can conclude that this model is better as (1) resampling techniques show that

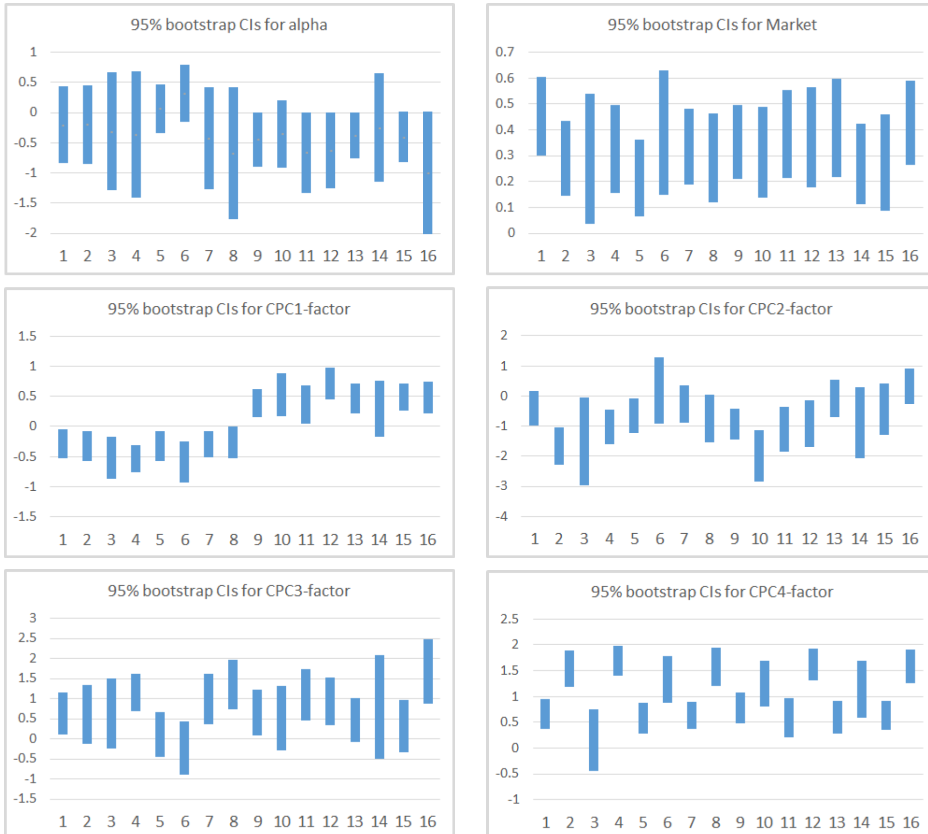

Figure 6: Graphical comparison of 95% Bootstrap CIs for Model 2

we cannot reject that αs are jointly zero; (2) the average adjusted-R^2 increases; (3) Market βs change and stabilize around 0.35 and (4) factors based on mean and Momentum (CPC1-factor) and standard deviation (CPC4-factor) only present 1 non-significant portfolio.
Next, we will proceed to analyse the significance of the different risk premia for both models.

3.2 Cross-Sectional regression

In this section we present the results for the cross-section regression of the two models under study: CAPM (Model 1) and the five-factor model including Market and factors CPC1 to CPC4 (Model 2).

3.2.1 Model 1: CAPM

The GRS bootstrap test for Model 1 suggests that we cannot reject that all αs are jointly zero and further it registers an explained variability between 0.19 and 0.526. Market seems to be statistically significant in all the portfolios.
We use the βs estimated in Table 7 to examine if the factor is priced on the cross-section of returns. We must take into account that despite the fact that we are working with portfolios,

risk premia will be estimated from the estimated coefficients, which can lead to an important bias, especially if the model is not well specified.
Market's risk premium is 0.0043 per month and, as a consequence, returns depend positively on the Market. The reported t-statistic of 0.874, which in this case includes the Shanken correction, see Shanken (1992), suggests that this factor is not statistically significant at a 5% significance level. The correction seems to be minor in this case, as the t-statistic for the simple regression (Fama-Macbeth approach) is 0.8776. The basic bootstrap confidence interval $[-0.0078, 0.0015]$ confirms that this factor is not statistically significant during the considered period. The statistical non-significance of the Market factor despite its wide confirmation in the financial literature could be related to the fact that we are not controlling for other factors, as the results from Model 2 suggest.

3.2.2 Model 2: Market and factors CPC1 to CPC4

Again, we use the estimated βs for Model 2 (see Table 9) to examine if the different factors are priced on the cross-section of returns.

		***t*-statistic**		**Bootstrap**
	Estimate	**FM**	**CS**	95% **CI**
Market	0.0108	2.054	1.5447	0.0091, 0.0254
CPC1	0.0071	9.625	3.5836	0.0017, 0.0181
CPC2	-0.0001	-0.085	-0.0502	-0.0126, 0.0038
CPC3	-0.0012	-2.241	-1.1303	-0.0150, 0.0014
CPC4	0.0002	0.445	0.1139	-0.0120, 0.0043

Table 11: Results for Cross-Sectional estimation of Model 2

The highest risk premia are those of Market, which is 0.0108, and the one associated with the factor based on mean and Momentum (CPC1-factor), which is 0.0071. In both cases, returns depend positively on them. The rest of the risk premia are much lower and non-significant at 5% significance levels both for traditional methods and resampling techniques. The reported t-statistics, which in the case of CS include the Shanken correction, suggest that the only statistically significant factor at conventional significance levels is that based on mean and Momentum (CPC1-factor). However, bootstrap intervals indicate that both Market and CPC1-factor λs are significantly different from zero (at 5% level) contributing to the model. This is consistent with the 4-factor model by Carhart (Carhart, 1997). Additionally, once we control for a volatility factor (CPC4), we notice that expected returns, at least during the period considered, do not depend on standard deviation. See Blitz et al. (2014) for potential explanations and a historical review of the volatility effect.

3.3 Comparison between classical methodologies and bootstrap methods

As we have seen, the existence of non-normality, heteroscedasticity and autocorrelation in the residuals of the regressions and, additionally, multicollinearity (high correlation among the different factors used in the analysis), may affect the estimates of α, β_i and GRS computed through classical methodologies.

In Table 12, we compare the number of portfolios where $\beta_i = 0$ for both methodologies: in the models presented without a sub-index significance tests have been run according to the classical methodologies, while in the models with a sub-index b, inferential bootstrap techniques have been used. Indeed, when we observe the results presented in this table, we notice that the p-values of the bootstrap GRS tests are always greater than those obtained from the classical methodologies, showing that the former are more conservative (we fail to reject the null hypothesis that the α are jointly 0 more often). We also observe that the number of portfolios where $\beta_i = 0$ varies depending on the methodology used.

Model	**Factors**	**Number of Portfolios where** $\beta_m = 0$	$\beta_{CPC1} = 0$	$\beta_{CPC2} = 0$	$\beta_{CPC3} = 0$	$\beta_{CPC4} = 0$	**GRS p-value**	**Adj-R^2**
Model 1	Market	0	-	-	-	-	0.019	0.190-0.526
Model 1b	Market	0	-	-	-	-	0.174	
Model 2	Market & CPCs	0	1	9	6	1	0.009	0.263-0.853
Model 2b	Market & CPCs	0	1	8	8	1	0.171	

Table 12: Comparison of classical methodologies and bootstrap methods

4 Conclusions

We propose a procedure to obtain and test multifactor models based on statistical and financial factors and illustrate it on a large dataset corresponding to nearly 1250 EU companies and spanning from October 2009 to October 2019. However, the procedure is general enough to be extended to other factors, companies or time period.

The first methodological contribution relies on using Common Principal Components to build the portfolios and summarize factors' information by capturing a high percentage of the variability of the datasets. In this paper, we considered factors like Market Capitalization and Total Assets (measures of size), Price to Book ratio (measure of cheapness), Return on Assets and Return on Equity (measures of profitability), Momentum, and four statistical measures, such as mean, standard deviation, kurtosis, and skewness. The second methodological contribution is the development of a block-bootstrap procedure to assess the validity of the model and the significance of the parameters involved.

The main findings indicate that the multifactor model proposed improves the Capital Asset Pricing Model with regard to the adjusted-R^2 in the time-series regressions. Cross-section regression results reveal that Market and a factor related to Momentum and mean of stocks' returns have positive risk premia for the analysed period. Finally, we also observe that tests based on block-bootstrap statistics are more conservative with the null than classical procedures.

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