# International Conference on Dynamic Control and Optimization

DCO 2021

Book of Abstracts

University of Aveiro — Portugal February 3 to 5, 2021



# International Conference on Dynamic Control and Optimization

DCO 2021

Book of Abstracts

International Conference on occasion of 65th birthday of Andrey V. Sarychev

University of Aveiro — Portugal February 3 to 5, 2021

The conference was supported by The Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

### Title

International Conference on Dynamic Control and Optimization – DCO 2021: Book of Abstracts

### **Editors**

João Pedro Cruz Tatiana Tchemisova

### Publisher

UA Editora Universidade de Aveiro

## ISBN

978-972-789-675-2

#### DOI

https://doi.org/10.34624/vf5g-vv39

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# **Sponsors**



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Center for Research & Development in Mathematics and Applications



#### **CEMAPRE**

Center for Mathematics Applied to Forecasting and Economic Decision



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Department of Mathematics, University of Aveiro



#### UA

University of Aveiro



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Foundation for Science and Technology



#### CIM

International Center for Mathematics

# International Conference on Dynamic Control and Optimization

#### Welcome

Dear participants.

On behalf of the Organizing Committee of the international conference on Dynamic Control and Optimization, DCO 2021, we are pleased to welcome you (virtually) to the Department of Mathematics, DMat, of the University of Aveiro, Portugal.

The project of organizing this conference was inspired by the fact that Professor Andrey Sarychev, who during nine years, from 1993 to 2002, was our teacher, colleague and friend at DMat, is celebrating his 65th birthday on February 2, 2021.

Unfortunately, the pandemic, which occurred in 2020 and has not stopped until now, did not allow us to organize the conference in the format in which it was planned: a pleasant meeting in a narrow circle of old and later scientists and friends of Andrey, his colleagues, co-authors and students. We entered 2021 cautiously and still keep our distance from each other. But we can not stop the time, our dear Andrey deserves to celebrate his anniversary in a special way this year, and we will do so, as fortunately, there are different ways to meet even being far apart from each other and from him. So we decided to continue our project and organize our conference in a virtual form, which, of course, is a less warm, but safer form of communication.

The main topics of the conference are closely connected with the main areas of interest of Andrey Sarychev. We are very happy that all our eleven invited spearers, prominent international researchers and professors who in certain periods of their life have worked more or less close with Andrey, have kindly accepted our invitation and will share with all participants their knowledge, experience and scientific results.

More than forty abstracts have been submitted and about seventy participants from nineteen countries have registered for our conference and this is the result of the hard work of the International Scientific Committee to whose members we are very grateful.

The scientific program of our conference consists of eleven plenary talks and eleven contributed sessions divided into three streams: Dynamic Control, Optimization, and Applications of Control and Optimization.

We are very thankful to the sessions' organizers and all the participants. Our special gratitude is directed to the Invited Speakers: **Andrei Agrachev**, SISSA (Scuola Internazionale Superiore di Studi Avanzati), Italy, **Yuliy Baryshnikov**, University of Illinois at Urbana-Champaign, USA, **Ugo Boscain**, C-MAP, École Polytechnique, France, **Jean-Baptiste Caillau**, Université Côte d' Azur, France, **Alexey Davydov**, Vladimir State University and Moscow

State University, Russia, **Manuel Guerra**, ISEG (Instituto Superior de Economia e Gestão), Portugal, **Fátima Silva Leite**, University of Coimbra, Portugal, **Maria do Rosário de Pinho**, University of Porto, Portugal, **Laura Poggiolini**, University of Florence, Italy, **Yuri Sachkov**, Program Systems Institute of RAS, Perslavl-Zalessky, Russia, and **Sergei Tabachnikov**, Pennsylvania State University, USA.

The organizers and scientific committee of the DCO 2021 conference are deeply grateful to the editors of Optimization, the Journal of Mathematical Programming and Operations Research, and its editor-in-chief, Christiane Tammer, for the opportunity to publish some of the revised works of the conference participants in a Special Issue, as well as to the Springer Nature editor, Aliaksandr Birukou, and the Springer Editorial Team who dedicated a separate volume of the Springer Proceedings in Mathematics & Statistics to the papers presented at our conference.

The conference was organized by the Department of Mathematics of the University of Aveiro, the Center for Research and Development in Mathematics and Applications (CIDMA, University of Aveiro) and the Center for Applied Mathematics and Economics (CEMAPRE, University of Lisbon) with the support of CIM (https://www.cim.pt/) International Center of Mathematics, Portugal and FLAD (https://www.flad.pt/) Luso-American Foundation, Portugal.

We hope that all the efforts of the Program and Organizing committees will result in a reach, productive, dynamic and interesting scientific forum.

Co-chairs: Tatiana Tchemisova, Delfim Torres and Alexandre Plakhov.

# Laudatio for Andrey V. Sarychev



Andrey V. Sarychev was born in Moscow, ex-USSR, in 1956. He received MSc in 1977 and PhD in Mathematics in 1980, at the Moscow Institute of Physics and Technology, and received Habilitation in 1998, at the University of Aveiro, Portugal. He held several visiting research positions at various scientific centers of Germany, Italy, Netherlands, and France. From 1993 to 2002 he was associate and then full professor at the University of Aveiro. Since 2002 he is full professor at the University of Florence, Italy.

Andrey Sarychev's research interests lie in the areas of nonlinear dynamical control systems, control of evolution PDE, optimal control, sub-Riemannian geometry, ordinary differential equations, calculus of variations, and propagation of acoustic waves in elastic media.

He edited several books and published more than 60 peer-reviewed articles in scientific journals and monographs. The list of his co-authors comprises over 30 scientists from nine countries.

As a member of program and organizing committees, he co-organized various scientific events. He is a longstanding member of the editorial board of Journal of Dynamical and Control Systems, and served as an Associate Editor for SIAM Journal of Control and Optimization in 2002-2010.

He coordinated three and participated in eight research projects of Portugal, Italy and Germany.

Andrey was the scientific adviser of seven PhD students and three young researchers from Portugal and France. He also supervised several Master students in Russia and Portugal.

#### Major scientific achievements

In his 1980 PhD Thesis, Andrey studied the index of the second variation of nonlinear control systems in the neighbourhood of extremals. This started

a fruitful line of research, leading to further results in the same direction and important results on the optimality of extremals, higher order optimality conditions, controllability, reachability and other fundamental properties of control systems.

He made important contributions to Lie-theoretic methods in control problems, invariants of control systems, and nilpotent approximations. He also studied relaxed and generalized controls and their relation to the Lie-algebraic structure of control systems.

His interests range further than control theory, applying his ideas and skills to several adjacent fields where he made significant contributions. These include calculus of variations, sub-Riemannian geometry, ordinary differential equations, functional analysis, and game theory, among others.

At the same time, he worked on problems of elastic waves, and become interested in control of various types of partial differential and evolution equations. He obtained important results concerning the controllability of such infinite dimensional systems by finite dimensional controls.

We wish Andrey excellent health, long and fruitful scientific life plenty of brilliant discoveries!

On behalf of the organizers

Manuel Guerra

#### General Information

### **Invited Speakers**

- Andrei Agrachev, SISSA (Scuola Internazionale Superiore di Studi Avanzati), Italy
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- Jean-Baptiste Caillau, Université Côte d'Azur, France (caillau@unice.fr)
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- Yuri Sachkov, Program Systems Institute of RAS, Perslavl-Zalessky, Russia (sachkov@sys.botik.ru)
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# Social Program

# Wednesday, February 3, 9:15

Opening Session with presence of Prof. Andrey V. Sarychev, University of Florence; Prof. Manuel António, Director of the Department of Mathematics, University of Aveiro; Prof. Luís Castro, Vice-Rector of the University of Aveiro, Prof. Manuel Guerra, ISEG, Lisbon and Prof. Domingos Cardoso, Department of Mathematics, University of Aveiro.

## Wednesday, February 3, 17:15

Prof. Andrey V. Sarychev's Tribute Session

# Friday, February 5, 13:15

Closing Session

# Conference Program and Sessions

# Dynamic Control and Optimization, DCO 2021 Final Program

	Program	
	riogiani	
	3.02.2021 - <b>Wednesday</b>	
09:00 - 9:15	Registration	
09:15-9:45	Opening session, Main Room	
09:45-10:00	Break	
10:00-10:50	Plenary 1, Main Room	Invited speaker 1: Andrey Agrachev Chair: Manuel Guerra
10:50-11:00	Break	
10.00 11.00	Diedit .	Invited speaker 2: Ugo Boscain
11.00-11:50	Plenary 2, Main Room	Chair: Lev Lokutsievskiy
11:50-13:00	Break	,
13:00-14:40	Contributed sessions, Rooms 1 and 2	WA1 and WA2
14:40-15:00	Break	
15.00-15:50	Plenary 3, Main Room	Invited speaker 3: <b>Yuliy Baryshnikov</b> Chair: Andrey Sarychev
15:50-16:00	Break	
16:00-16:50	Plenary 4, Main Room	Invited speaker 4: <b>Sergei Tabachnikov</b> Chair: Alexander Plakhov
16:50-17:00	Break	
17:00-18:15	Contributed sessions, Rooms 1 and 2	WB1 and WB2
18:15-19:00 or later	Prof. Andrey V. Sarychev's Tribute Session	We invite all participants to bring a glass of a favorite drink to make a toast to Andrey Sarychev ③
	Main Room	

	04.02.2024	
	04.02.2021 - <b>Thursday</b>	
09:00-10:40	Contributed sessions, Rooms 1 and 2	TA1 and TA2
10:40-11:00	Break	
		Invited speaker 5: Laura Poggiolini
11:00-11:50	Plenary 5, Main Room	Chair: Andrey Agrachev
11:50-13:00	Break	
		Invited speaker 6: Aleksey Davydov
13:00-13:50	Plenary 6, Main Room	Chair: Alexander Plakhov
13:50-14:00	Break	
14:00-15:15	Contributed sessions Rooms 1 and 2	TB1 and TB2
15:15-15:30	Break	
		Invited speaker 7: Fátima Leite
15:30-16:20	Plenary 7, Main Room	Chair: Laura Poggiolini
16:20-16:30	Break	
		Invited speaker 8: Manuel Guerra
16:30-17:20	Plenary 8, Main Room	Chair: Alexey Davydov

	05.02.2021 - Friday	
		Invited speaker 9: Jean-Baptiste Caillau
09:00-09:50	Plenary 9, Main Room	Chair: Yuri Sachkov
09:50-10:00	Break	
		Invited speaker 10: M. do Rosario de Pinho
10:00-10:50	Plenary 10, Main Room	Chair: Paula Rocha
10:50-11:00	Break	
		Invited speaker 11: Yuri Sachkov
11:00-11:50	Plenary 11, Main Room	Chair: Jean-Baptiste Caillau
11:50-12:00	Break	
12:00-13:15	Contributed sessions, Rooms 1, 2, and 3	FA1 , FA2, and FA3
13:15-13:30	Closing session, Main Room	

Note: The timetable is presented in the time of Lisbon:

Central European Time (CET) minus 1h

# Abstracts

### A constraint modeling approach for forest management

Eduardo Eloy<sup>1</sup> (m47215@alunos.uevora.pt), Vladimir Bushenkov<sup>2</sup> (bushen@uevora.pt), Salvador Abreu<sup>3</sup> (spa@uevora.pt)

 $^1$  University of Évora, Portugal;  $^2$  CIMA, University of Évora, Portugal;  $^3$  NOVA-LINCS/University of Évora, Portugal

In this article we present a novel modeling for Forest Management planning, based on a finite-domain constraint programming approach. We compare our model to the hitherto used IP implementation and argue that the new formulation is both more end-user friendly, as it is expressed in terms closer to the direct problem domain and more scalable, performance-wise, because of the augmented ability to use parallel computing resources stemming from Constraint Programming.

**Keywords:** Constraint Programming, Combinatorial Optimization Metaheuristics.

The research was carried out in the frame of the project POCI-01-0145-FEDER-030391 (PTDC/ASP-SIL/30391/2017) (BIOECOSYS) funded by the FCT.

#### A directional curvature formula for convex bodies in $\mathbb{R}^n$

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For a compact convex subset F of  $\mathbb{R}^n$ , with the origin in its interior, we presente a formula to compute the curvature at a fixed point on its boundary, in the direction of any tangente vector. For this we assume that the boundary of F is given by an implicit equation near the fixed point. That formula is equivalent to the existing ones (namely in [1] and [2]), but it is more elegant and easier to apply.

**Keywords:** Convex Set, curvature, Implic Function Theorem, tangent vector. **References:** 

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### A new class of variational problems of Herglotz type

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The main goal of this talk is to generalize the variational problem of Herglotz considering the case where the Lagrangian depends not only on the independent variable, an unknown function x and its derivative, and an unknown functional z, but also on the end points conditions and a real parameter. Herglotz's problems of calculus of variations of this type cannot be solved using the standard theory. Main results of this talk are necessary optimality condition of Euler- Lagrange type, natural boundary conditions and the DuBois-Reymond condition for our non-standard variational problem of Herglotz type. Since the variational problem of Herglotz is a generalization of the classical problem of the calculus of variations, from our main results a number of important corollaries are obtained.

**Keywords:** Calculus of variations, variational problems of Herglotz type, Euler-Lagrange equation, natural boundary conditions, Dubois-Reymond equation.

This work was supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

# Abnormal trajectories and abnormal set for the nilpotent (2,3,5,8) distribution

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We consider the left-invariant sub-Riemannian structure on the free nilpotent Lie group of rank 2 and step 4, this structure has growth vector (2,3,5,8). We describe abnormal trajectories and study the abnormal set, i.e., the set of points filled by abnormal trajectories starting at the identity. In particular, we show that this set is sub-analytic of dimension 5. Moreover, this set is not closed, not smooth, and not semi-analytic. We discuss optimality of abnormal trajectories. Finally, we present some open questions.

Keywords: free nilpotent Lie group, abnormal trajectories, abnormal sets

# An optimal control for a backward doubly stochastic differential equation via Malliavin calculus

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In this paper, we consider a stochastic control problem for a backward doubly stochastic differential equation governed by standard and fractional Brownian motions, with Hurst parameter  $\frac{1}{2} < H < 1$ . The set of the control domain is convex. A stochastic maximum principle is derived using two famous approaches: the Doss-Sussmann transformation and the Malliavin calculus. The criterion to be minimized is in the general form, with initial cost.

**Keywords:** Fractional Brownian motion, standard Wiener motion, Doss-Sussmann transformation, Malliavin derivative, maximum principle, stochastic optimal control, variational inequality

# Assessing the traffic-related impacts of automated vehicles introduction

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It is expected that in the next few decades there will be automated vehicles (AVs) sharing the roads with conventional vehicles (CVs), and this imply that public policymakers need to analyse the benefits/consequences of such introduction, namely, regarding network impacts in terms of travel time, pollutant emissions and safety. The aim of this research is to provide a study focusing on a multi-criteria traffic assignment model based on minimizing travel time, distance travelled (fuel consumption) and pollutant emissions considering different AVs scenarios in a road network, and exploring the associated safety issues under different driving behaviours. Results show AVs introduction can in fact have a contribution in reducing the pollutant emissions provided their behaviour is a combination in its majority of cautious AV. Results also suggest that replacing 20% of the AV aggressive fleet by cautious AVs yield worst results in terms of emissions, even when compared to the 100% aggressive AVs. These studies are relevant for decision making, particularly for strategic policy making and planning, and can therefore help local and public authorities to contribute for achieving their sustainable mobility goals, specially to anticipate the near future possible impacts.

**Keywords:** Automated vehicles, pollutant emissions, road safety, dynamic traffic assignment.

# Decentralized control for deployment of multi-agent dynamical systems

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The paper examines the deployment strategy for the multi-agent system (MAS) based on a Voronoi diagram which is dynamically updated. By a multi-agent system we mean a (finite) number of homogenous dynamical objects, so-called agents, communicating and cooperating with each other. We deliver sufficient conditions to steer the system into a stable static configuration, i.e. constant over time, which ensures optimal coverage of the working environment, in both cases: with and without delays. The control strategy is based on the tracking of the center of each Voronoi cell (in this work defined by Chebyshev centers, i.e. centers of the largest possible circles that can be inscribed in the

corresponding Voronoi cells). A series of theoretical results on the convergence of the MAS are presented along with the illustrations by numerical simulations.

Keywords: Multi-agent system, Voronoi diagram, deployment strategy.

# Decomposition approach for solving multi-criteria integer programming problems: the case of forestry management in Portugal

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When managing forestry in one region, it is necessary to take into account the evolution of the forest over decades. Dynamic simulation models make it possible to formulate these control problems in the form of large-dimensional linear models with integer variables. In the management process, it is necessary to take into account various, often contradictory economic and environmental aspects, and therefore an approach based on multi-criteria optimization becomes adequate and useful. One of the widely used methods of multi-criteria decision-making is the Goal Programming. Knowing the Pareto frontier in the criterion space helps a lot in choosing a reasonable reference point. However, the construction of the Pareto frontier for large-size models is computationally expensive, even in the case of linear programming models. In the presentation, a decomposition approach to constructing a Pareto frontier in the case of largesize models with integer variables is described. The whole region is considered as a set of sub-regions, for which convex approximations of the Pareto frontier are constructed. Then, at the next stage, the Pareto frontier for the entire region is approximated on the base of Pareto frontiers for sub-regions. The proposed approach was used to develop a decision support system for forest management in the Vale do Sousa region, Portugal. The obtained results are given.

**Keywords:** Multi-objective optimization, goal programming, integer programming, Pareto frontier, decomposition, forest management.

The research was carried out in the frame of the project POCI-01-0145-FEDER-030391 (PTDC/ASP-SIL/30391/2017) (BIOECOSYS) funded by the FCT.

### Dynamics of distributed populations and its optimization

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Analysis of population dynamics and changes in these dynamics under anthropogenic impact, for example, changes in the parameters of its habitat or exploitation of the population itself in any form, are in the focus of investigation of various research. In recent decades, the tasks of rational nature management, preservation of the environment and biodiversity have increased the demand for results in this area and increased the attention of scientists to this topic.

We consider an exploited population distributed in a periodic environment with independent point wise dynamics like the Verhulst model [8] or with the same dynamics taking into account diffusion, which is already described by an equation of the Kolmogorov-Piskunov-Petrovsky-Fisher type [6, 7]. Under natural asumptions on the model parameters and average time quality criteria that characterize income from the exploitation in the long term, there is proved the existence of an optimal strategy of exploitation.

For example, consider a population on n-dimensional torus with dynamic described by equation

$$p_t = (\alpha(x)p_x)_x + [a(x) - u(x)]p - b(x)p^2, \tag{1}$$

where p = p(t, x) is the population density at the point x of the torus at the time t, and the positive definite matrix  $\alpha$  and the functions a and b characterize the diffusion of the population, its growth and intraspecific competition, respectively. The functions a and b are measurable and bounded and besides the last one is positive and separated from zero (that is  $b_0 \leq b$  for some positive constant  $b_0$ ). With respect to the matrix  $\alpha$  it is additionally assumed that the derivatives of its entries satisfy the Hölder condition with some positive exponent. The last condition was imposed on in [2], some of the results from which we use.

The measurable function u is a control, which satisfies the constraints

$$U_1 \leq u \leq U_2$$

with some bounded measurable functions  $U_1, U_2$ . Such a control is called admissible. In fact, the control can also reflect the care of the population, for example, one could increase its growth rate by taking its negative values or subtract a part of the population in order to reduce intraspecific competition in case of the positive ones. Such a model can be viewed as close to the one studied in [2], where n = 1, but diffusion was absent, and exploitation was impulsive.

The management objective is to obtain the better time averaged income

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int_{0}^{\tau} \int_{T^n} u(t, x) p(t, x) dx dt \tag{2}$$

over all admissible controls. Selection of such a functional has to provide optimal exploitation of the population in the long run. Such type functionals were considered in the optimization of processes of various nature and are well known in the mathematical theory of control [1, 2, 3, 5].

It turns out that after choosing any admissible control u and any nonzero non-negative initial density  $p_0$  the solution of model equation (1) uniformly converges by  $t \to +\infty$  to stationary solution  $p = p_{u,\infty}(x)$  [3]. Thus, when an admissible control is selected, the value of functional (2) becomes in fact a function of the selected control. We show that there exist an admissible control and the respective steady state of the population distribution that provide the maximum of functional (2), that is the maximum income from the exploitation in the long run.

The research is carried out by support of the Russian Science Foundation grant 19-11-00223.

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# Elementary geometry is dead. Long live (experimental) elementary geometry!

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By "elementary", I do not mean Euclidean, axiomatic, high school geometry, nor do I mean that these results are expected or easy to obtain: I use this term to distinguish this subject from differential geometry. I have a collection of recent results that fall into this category, and I shall present a sampler; in most cases, these results were discovered in computer experiments and were motivated by the theory of completely integrable systems. The topics will include the circumcenter of mass of polygons, a new take on Steiner's porism, new projective configuration theorems, and lesser known geometrical properties of Poncelet polygons.

**Keywords:** Elementary geometry, theory of completely integrable systems, computer experiments

# Estimation in the behavioral approach

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The estimation of an unmeasurable system variable from another which can be measured is a standard problem in linear state-space systems theory. A simple case is the estimation of the state from the output, but other situations can be of interest, such as the estimation of a linear function of the state, under the presence of unknown disturbances, based on the sole knowledge of the output (in case the input is taken to be zero).

In this talk we study the more general problem of estimation within the context of the behavioral approach, in particular, multidimensional behavioral systems. We consider the case where there are no disturbances as well as the case where the system dynamics is perturbed, and provide necessary and sufficient conditions for the solvability of the corresponding estimation problems together with the construction of a solution, if it exists.

Finally, we also show how this theory, in the 1D case, can be applied to the particular case of state-space systems.

**Keywords:** Multidimensional behavioral systems, observer, disturbance, state-Space systems.

This work was partially supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

# Estimation of moments and density of first passage times for optimal sustainable harvesting policies in random environments

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In random varying environments, we can describe the evolution of a fished population size using stochastic differential equations. Previously, we have compared the profit performance of two harvesting policies, one with variable harvesting effort, called optimal policy, and the other with constant harvesting effort, called optimal sustainable policy. The former is characterized by fast and abrupt variations of the harvesting effort associated with the frequent variations in population size due to the random environmental fluctuations. This type of policy is inapplicable due, for instance, to the logistics of the fisheries being incompatible with abrupt and frequent changes in the harvesting effort. It also poses social problems during the periods of no or low harvesting effort. Furthermore, this type of policy requires the knowledge of the population size at each instant and estimating population size is an inaccurate, lengthy and expensive task. The optimal sustainable policy considers the constant application of the same harvesting effort and leads to population sustainability, as well as to the existence of a stationary probability density for the population size. This policy has the advantage of being easily applicable and there is no need to estimate the population size at every instant. The performance of the two policies was compared in terms of the profit over a finite time horizon. Using data based on a real fished population, we showed that there was only a slight reduction in profit by using the optimal sustainable policy (based on constant effort) instead of the inapplicable optimal policy (based on variable effort).

For the class of models with constant fishing effort, mathematical extinction (population size X(t) converging to zero) occurs with zero probability. However, since we work with ergodic processes, all states in the interior of the state space are attainable with probability one in finite time. In particular, we can consider a threshold y > 0 and study how long it takes for the process X(t) to reach y for the first time. This threshold can be a low biological reference point y < X(0), i.e., a minimum biomass value below which the population self-renewable capacity is endangered. It can also be some high biomass level y > X(0) that is important for the fishery, such as a warning level of danger

to the survival of another species or to possible deviations from the optimal fishing effort.

Based on general expressions for the mean and standard deviation of first passage times by lower and upper thresholds, we compute such values for the particular cases of the logistic and the logistic-like models with weak Allee effects and for several lower and upper threshold values y. For a fixed threshold value, we also present a way to estimate, by numerical inversion of its Laplace transform, the probability density function of the time to reach the threshold. To check the quality of this estimate, we compare the mean and standard deviation of the first passage time obtained by using this estimated probability density function with the mean and standard deviation obtained directly.

**Keywords:** First passage times, fisheries management, Laplace transform, profit optimization, stochastic differential equations.

### G-adjoint equation for stochastic optimal control

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In my proposed presentation, I will work on reaching the adjoint process and the adjoint equation of the stochastic maximum principle in optimal control problem for stochastic differential equations driven by G-Brownian motion (G-SDEs in short), in which the set of admissible controls is convex by using the classical way of the convex perturbation method.

**Keywords:** Optimal control, stochastic maximum principle, G-Brownian motion, G-Stochastic differential equation.

# Generalized convolutions, differential operators, and Lévy-like processes

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In some cases, it is possible to construct Lévy-like processes where actions by random elements of a given semigroup play the role of increments. I discuss some cases where such construction is possible and cases where it can be shown to be impossible. The different outcomes are related to spectral properties of differential operators.

**Keywords:** Differential operator, semigroup, Lévy-like processes

#### Hesitant fuzzy sets are observers

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In this talk we have an overview on the notion of observer and the systemic results of it. By introducing the papers on observers, which have been published before presenting the notion of hesitant fuzzy sets, we see that hesitant fuzzy sets are observers. Multidimensional observers as a tool in dynamic control are considered.

**Keywords:** Observer, relative dynamical systems, entropy

# Individual growth modelling with stochastic differential equations

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To describe individual growth dynamics with environmental random variations, regression methods are inappropriate. So, we use stochastic differential equation (SDE) models that, in the most general form, can be written as dY(t) = b(a - Y(t))dt + sdW(t), where Y(t) = h(X(t)) with X(t) being the weight of an animal at age t and h an appropriate strictly increasing C1 function. The parameters are a = h(A), where A is the maturity weight of the animal, b the rate of approach to maturity and s the intensity parameter of the random fluctuations. W(t) is a standard Wiener process.

In previous works, we have studied estimation, prediction and optimization issues using cattle weight data from females of Mertolengo cattle breed. In this new project, we have adjusted and extended the methodologies and applied them to the weight data of males of Mertolengo cattle breed and Alentejana cattle breed. Since model parameters may vary from animal to animal and variability can be partially explained by their genetic differences, we introduce the extension of the study to SDE mixed models in order to incorporate the effect of environmental variability and genetic effects. For this, we allow the parameters of the model to depend on the genetic values of the animal.

The estimation of parameters for this type of models may present some difficulties, in particular, for the maximum likelihood method there are sometimes convergence and bias problems. To overcome these shortcomings, we have studied some alternative methods like weighted maximum likelihood. This method allows to overcome data limitations of the database, where frequent weighting

of the animals was lacking and the animals (with the exception of the animals selected for reproduction) were sold to the meat market at a quite young age, thus lacking weight observations at later ages. Using a numerical approach, we also evaluate the optimal choice of the weight function.

The SDE methodology is much more appropriate, and therefore more precise. The farmer's expected profit as a function of the animal's selling age can now be determined on an individual animal basis by considering the animal's genetic values, thus improving profit optimization.

**Keywords:** Application to cattle data, maximum likelihood estimation, stochastic differential equations, weighted maximum likelihood estimation.

# Long-time dynamics of a tumor growth model

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This talk is devoted to the long-time behavior of a tumor growth model, which models a tumor tissue as a mixture of cancerous and healthy cells. The long-time dynamics of this model have been studied so far by ignoring the chemotaxis and active transport, which effects the tumor growth significantly. In this research, we consider this model with chemotaxis and active transport. We obtain that the weak solutions of the problem is asymptotically compact in the whole phase-space. We prove the existence of the global attractor in a phase space defined by means of mass conservation. Furthermore, we establish that the global attractor is finite dimensional.

**Keywords:** Tumor growth model, Cahn-Hilliard equation, global attractor.

# Mathematical analysis, forecasting, and optimal control of the reaction diffusion epidemic SICA model

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The key contribution in this paper is to extend the work due to Silva and Torres (2017) to spatio-temporal dynamics, intending to take into account not only the local reaction of the occurrence of new infected individuals, but also the global diffusion in order to incorporate an additional amount of arguments into the system. We model the spatial behavior by adding a diffusion term with the Laplacian operator for which we devote one section to justify and interpret, mathematically and physically, its use in this context. The paper is organized as follows. Firstly, we begin by introducing our model. Secondly, we prove existence and uniqueness of solution to the system, adequately using semigroup theory. Thirdly, we characterize the latter solution in order to gather the maximum of information about it. Fourthly, we pose an optimal control problem, whose solution is naturally established by a constructive method. Fifthly, appropriate numerical simulations are implemented to illustrate different results of our model system.

**Keywords:** Epidemic SICA Model, spatio-temporal dynamics, optimal control problem.

This work was partially supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

# Necessary conditions and numerical methods for optimal control involving sweeping processes

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This talk focuses on joint work with Maria Margarida A. Ferreira and Gueorgui Smirnov on optimal control problems involving sweeping processes. Here the sweeping system is defined as the 0 level set of twice continuously differentiable function. A remarkable feature of such problems is that the set valued function defining the dynamics fails to be Lipschitz. We derive results asserting the existence of solution as well as new necessary conditions using an ingenious

sequence of approximating problems. We cover both problems with and without end state constraints. Exploring the nature of the approximating problems, we also produce numerical methods for these optimal control problems.

**Keywords:** Optimal control problems, sweeping process, necessary conditions, approximating problems

## New explicit formulae for sub-Finsler geodesics in Heizenberg groups $H_n$ in terms of convex trigonometry

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Left-invariant sub-Finsler problems on  $H_n$  are given by arbitrary 2n-dimensional compact convex set  $\Omega$  containing 0 in its interior. Convex trigonometry allows to write down explicit formulae for geodesics in these problem if  $\Omega$  admits some natural generalization of spherical coordinates. For example, in the  $L_p$  case, geodesics can be obtained in terms of Shelupsky's trigonometric functions (or equivalently by incomplete Euler B function).

### Nondegenerate abnormality, controllability, and gap phenomena in optimal control with state constraints

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In optimal control theory, infimum gap means that there is a gap between the infimum values of a given minimum problem and an extended problem, obtained by enlarging the set of original solutions and controls. The gap phenomenon is somewhat "dual" to the problem of the controllability of the original control system to an extended solution. We present sufficient conditions for the absence of an infimum gap and for controllability for a wide class of optimal control problems subject to endpoint and state constraints. These conditions are based on a nondegenerate version of the nonsmooth constrained maximum principle, expressed in terms of subdifferentials. In particular, under some new constraint qualification conditions, we prove that: (i) if an extended minimizer is a nondegenerate normal extremal, then no gap shows up; (ii) given an extended solution verifying the constraints, either it is a nondegenerate abnormal extremal, or the original system is controllable to it. An application to the impulsive extension of a free end-time, non-convex optimization problem with control-polynomial dynamics illustrates the results.

**Keywords:** Optimal control problems, Maximum Principle, state constraints, gap phenomena, controllability, nondegeneracy.

# Non-linear optimization with perturbed objective function

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Perturbations in non-linear optimization problems will be modelled asymptotically in the setting of nonstandard analysis, e.g. a small perturbation will have a concrete representation by the set of infinitesimals. Noting that the sum of two infinitesimals is infinitesimal, such a set is a convex group, a so-called neutrix. An external number is the sum of a real (non-standard) number and a neutrix. Being stable under some shifts, and giving rise to an ordered structure in which many algebraic laws are valid [1], this setting enables an effective form of robust set-valued analysis. Concepts of continuity and derivative can be defined, satisfying classical laws like a chain-rule. Both an optimal solution (a form of Dedekind completeness holds) and a near-optimal solution can be properly defined.

While [2] dealt with linear programming with near optimal solutions, we give here conditions such that some approximate forms of classical theorems on non-linear optimization are valid, like the near-maximum of a continuous function on a compact interval and Fermat's Lemma.

Adapting the notion of partial derivative, we also study multivariate functions. We obtain general conditions on the existence of near-optimums, and also forms of the inverse and implicit function theorems. For constrained optimization problems with perturbed objective functions the results are strong enough to define a Lagrange multiplier to locate the near-optimums.

**Keywords:** Nonlinear optimization, perturbations, near-optimum Lagrange multiplier, external number.

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#### Obstacles to stabilization

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It is well known that a continuous feedback stabilization on an attractor is often impossible because of the mismatch of the topologies of the attractor and the configuration space of the system. I will discuss the implications on the topology of the locus of discontinuity of the feedback, and some examples motivated by control of robotic swarms.

**Keywords:** Feedback stabilization, attractor, control of robotic swarm

#### On applications of Cayleys graphs in networks under Denial-of-Service attacks

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In the paper discrete multi-agent systems under Denial-of-Service (DoS) attacks are considered. Since in the presence of DoS attacks the stability of the whole system may be disturbed, sufficient stability conditions for the multi-agent system under DoS attack are delivered. Some practical solutions for companies of the deployment of links in a network of cooperative devices (servers, computers, robots and etc.), according to adjacency matrices of Cayley graphs are given. Theoretical consideration is illustrated by numerical examples.

**Keywords:** Multi-agent systems, Denial-of-Service attacks, stability Cayley graphs.

# On decomposition approach to stability and stabilizability of the singularly perturbed system with delay on time scales

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The problem of stability and stabilizability of linear time-invariant singular perturbed systems with constant delays on homogenous time scale T is studied. To this aim, based on a non-degenerate change of variables, this system is separated into slow and fast subsystems with a different velocity dynamics. The decoupling transformation is presented in the form of asymptotic series on time-scale.

It is shown that stability of the slow and fast subsystems on the general model of time yields the robust (on a parameter) stability of the original system. Parameter-free spectrum-type sufficient conditions of exponential stability of the singular perturbed system with delay on time scales are obtained. The conditions are valid for all sufficiently small values of the parameter of singular perturbation, i.e., they are robust with respect to this parameter.

A stabilization problem for a given class of control systems is solved by a design of a linear well-conditioned parameter-free composite state-feedback control which is a sum of a stabilizing slow and fast control.

**Keywords:** Singularly perturbed system, delay decomposition, time scales, stability, stabilizability.

# On decoupling transformation for three time-scale linear time-invariant singularity perturbed control system with state delay

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For a linear time-invariant singularly perturbed control system with multiple commensurate delays in slow state variables and with two perturbation parameters of different orders of magnitude at the highest derivatives of the system variables, a decoupling transformation is constructed that generalizes the known Chang-type transformation and reduces the original three timescale system to an equivalent system of three independent subsystems smaller dimension with different rate of change of variables. The solvability of the matrix Riccati and Silvester equations with respect to the functional matrices determining the constructed non-degenerate transformation is proved. It is shown that these matrices can be found in the form of asymptotic series in powers of small parameters. It is proved that although the resulting decoupled subsystems are systems with infinite delay, they are approximated by systems with finite delay.

An approximation of the splitting transformation with any order of accuracy can be implemented in the form of programs for computer algebra systems, the results obtained can be used to solve problems of analysis and synthesis of three time-scale linear time-invariant singularly perturbed system with multiple delays in slow state variables.

**Keywords:** Singularly perturbed control system, delay, three time-scale, decoupling transformation.

#### On the fractional Cucker–Smale optimal control problem

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The Cucker-Smale flocking model is an elementary mathematical model describing a general tendency of multiple interacting agents to coordinate their behavior based on the dynamics of their neighbors. More precisely, it is focused on the alignment of agents' velocities (in the general context called consensus parameters), when taking into account positions and momenta of each individual. Despite the fact that self-organization is rather ubiquitous in natural bio-groups, in social or engineering multi-agent models it is legitimate to ask whether - in case of lost cohesion - additional forces acting on the agents of the system may restore stability and achieve consensus. One of the ways, that are proposed in the literature, to deal with this problem is by introducing an external controller. We develop a sparse flocking control for the fractional Cucker-Smale multi-agent model. The Caputo fractional derivative, in the equations describing the dynamics of a consensus parameter, makes it possible to take into account in the self-organization of group its history and memory dependency.

**Keywords:** Fractional differential systems, flocking model, multi-agent systems, consensus optimal control.

This work was partially supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

### On the growth and oscillation of fixed points of solutions of complex linear differential equations

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The determination of the order and the type of complex functions plays an important role in the Nevanlinna's value distribution theory. Very recently, some researchers have used more general scales called the  $\varphi$ -order and the  $\varphi$ -type to investigate the fast growth of entire solutions of the linear differential equations

 $f^{(k)} + A_{k-1}(z)f^{(k-1)} + \dots + A_0(z)f = 0$  (1)

where  $k \geq 2$  and the coefficients  $A_0, \ldots, A_{k-1}$  are entire functions.

In this presentation, we discuss the growth and the oscillation of fixed points of solutions of equation (1) when the coefficients are meromorphic functions on the whole complex plane. We describe the relationship in terms of growth and exponent of convergence between the meromorphic solutions of equation (1) and the domination coefficient. Some previous results considering the concept of iterated order will be improved and extended.

**Keywords:** Meromorphic function, order type, exponent of convergence, linear differential equation

#### Optimal bacterial resource allocation

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The problem of synthesizing a metabolite of interest in continuous bioreactors through resource allocation control is addressed. The approach is based on a self-replicator dynamical model that accounts for microbial culture growth inside the bioreactor, and incorporates a synthetic growth switch that allows to externally modify the RNA polymerase concentration of the bacterial population. The optimal control problem exhibits two ubiquitous phenomena: Fuller and turnpike. Joint work with J.-L. Gouzé and A. Yabo (Sophia)

**Keywords:** Resource allocation control, self-replicator dynamic model, optimal control

#### Optimal control applied to a COVID-19 epidemic model

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In this talk, we propose a compartmental SAIRP mathematical model, for the transmission dynamics of SARS-CoV-2, given by a system of ordinary differential equations, which is fitted to the number of active infected individuals with COVID-19 in Portugal. We apply optimal control theory to the SAIRP model, to maximize the number of people returning to "normal life" and minimizing the number of active infected individuals with minimal economical costs while warranting a low level of hospitalizations.

After the SAIRP model is generalized, considering piecewise constant parameters, and we construct a complex network of dynamical systems, in order to take into account the mobilities of individuals, which are known to play a decisive role in the dynamics of the epidemic. We prove the existence of pseudoperiodic solutions of the epidemic model and analyze its relation with multiple epidemic waves in COVID-19 pandemic. The model with piecewise constant parameters is calibrated in order to fit with the real data of the COVID-19 active infected individuals in six regions of Portugal mainland. Through numerical simulations, we explore the effect of the topology of the network on the dynamics of the epidemics (disposal of connections and coupling strength) and identify which type of topology minimizes the level of infection of the epidemic.

This work is based on references [1, 2] developed under the Project Nr. 147 "Optimal Control and Mathematical Modeling of the Covid-19 Pandemic: contributions to a systemic strategy for community health intervention", in the scope of the "RESEARCH 4 COVID-19" call, financed by the Portuguese Foundation for Science and Technology (FCT).

**Keywords:** Optimal control, COVID-19, piecewise constant parameters, pseudoperiodic solutions, complex network, epidemic waves.

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This work was supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

#### Optimal control with distorted probability distributions

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We study a robust optimal control of discrete time Markov chains with finite terminal T and bounded costs using probability distortion. The time inconsistency of these distortion operators and hence its lack of dynamic programming are discussed. Due to that, dynamic versions of these operators are introduced, and its availability for dynamic programming is demonstrated. Based on dynamic programming algorithm, existence of the optimal policy is justified and an application of the theory to portfolio optimization along with a numerical study is also presented.

**Keywords:** Probability distortion, Markov decision processes, Dynamic Programming, risk management, mathematical finance.

#### Optimal error estimate of a finite element method for a non coercive system of elliptic qvis related to the management of energy production

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This work deals with the finite element approximation of a noncoercive system of elliptic quasi-variational inequalities related to the management of energy production problem. More specifically, a optimal  $L^{\infty}$  error estimate is established, using the finite element spatial approximation and the concept of subsolutions.

**Keywords:** The management of energy production problems; Quasi-Variational; Inequalities; Subsolutions concept; Finite Elements;  $L^{\infty}$  Error Estimate

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### Optimal sizing of sand replenishment in intermittent and uncertain environment

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Sand replenishment is a major countermeasure against sediment depletion in water environment such as dam-downstream rivers and eroded coastal areas. Designing a cost-efficient storage size of sand replenishment is a critical issue. Key points in applications are that only temporally discrete replenishment is possible and that sediment transport is intermittent and uncertain. We approach this new designing problem from a viewpoint of a stochastic differential game between an environmental manager and nature. Sediment transport is represented by a stochastic differential equation driven by a (tempered) stable Levy subordinator with physical constraints of sediment storage. Sand replenishment is considered as an impulse control where observation/replenishment chances arrive at jumps of a Poisson process. If sand is replenished only when the stored sediment is depleted, then we show that the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation as the optimality equation is exactly solvable under uncertain transport rate and/or observation rate and that the corresponding optimal storage capacity is obtained. Furthermore, we argue that the derived solution is a unique continuous viscosity solution to the HJBI equation. We also show a computational example using real data of Hii River in Japan. Issues concerning a related backward stochastic differential equation and probability density function under the optimized sediment storage are discussed as well.

**Keywords:** Optimal sizing, Levy-driven storage model, Hamilton-Jacobi-Bellman-Isaacs equation, analytical solution, finite difference scheme.

### Pontryagin maximum principle for distributed fractional order sytems

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Distributed-order fractional non-local operators have been introduced and studied by Caputo at the end of the 20th century. They generalize fractional order derivatives/integrals in the sense that such operators are defined by a weighted integral of different orders of differentiation over a certain range. The subject of distributed-order non-local derivatives is currently under strong development due to its applications in modeling some complex real world phenomena. Fractional optimal control theory deals with the optimization of a performance index functional subject to a fractional control system. One of the most important results in classical and fractional optimal control is the Pontryagin Maximum Principle, which gives a necessary optimality condition that every solution to the optimization problem must verify. In our work, we extend the fractional optimal control theory by considering dynamical systems constraints depending on distributed-order fractional derivatives. Precisely, we prove a version of Pontryagin's maximum principle.

**Keywords:** Fractional calculus, distributed-order operators, Pontryagin's maximum principle.

This work was supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

### Quantum confinement for the curvature Laplacian $-\Delta + cK$ on 2D-almost-Riemannian manifolds

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Two-dimension almost-Riemannian structures of step 2 are natural generalizations of the Grushin plane. They are generalized Riemannian structures for which the vectors of a local orthonormal frame can become parallel. Under the 2-step assumption the singular set Z, where the structure is not Riemannian, is a 1D embedded submanifold. While approaching the singular set, all Riemannian quantities diverge. A remarkable property of these structures is that the geodesics can cross the singular set without singularities, but the heat and the solution of the Schrödinger equation (with the Laplace-Beltrami operator Delta) cannot. This is due to the fact that (under a natural compactness hypothesis), the Laplace-Beltrami operator is essentially self-adjoint on a connected component of the manifold without the singular set. In the literature

such phenomenon is called quantum confinement. In this talk we study the self-adjointness of the curvature Laplacian, namely  $-\Delta + cK$ , for c > 0 (here K is the Gaussian curvature), which originates in coordinate free quantization procedures (as for instance in path-integral or covariant Weyl quantization). We prove that there is no quantum confinement for these types of operators.

Keywords: Quantum confinement, 2D-almost-Riemannian manifolds

### Reachability and stabilizability for positive nonlinear systems

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Positive reachability of a positive system means that system trajectories starting from the origin can reach all the points of the nonnegative cone. Positive stabilization of such a system means feedback stabilization that preserves positivity of the system. It is shown that if a positive (linear or nonlinear) continuous-time system is positively reachable from the origin, then it is positively feedback stabilizable. On the other hand, such implication does not hold in general for discrete-time systems, even linear ones, though it holds for one-dimensional systems. These differences between continuous-time and discrete-time systems follow from different characterizations of positive reachability for these two classes of systems. This work was supported by the National Science Centre under the grant No. 2017/25/B/ST7/01471.

**Keywords:** Positive system, positive reachability, positive stabilizability, time scale.

#### Refinements of Kusuoka Representations on $L^{\infty}$

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We study Kusuoka representations of law invariant coherent risk measures on the space of bounded random variables, which says that any law invariant coherent risk measure is the supremum of integrals of Average Value-at-Risk at  $\alpha \in [0,1]$ , denoted by  $\text{AVaR}_{\alpha}$ . We refine this representation by showing that the supremum in Kusuoka representation is attained for some probability measure  $\mu^*$  on [0,1]. Namely, we prove that any law invariant coherent risk measure on the space of bounded random variables can be written as an integral of the Average Value-at-Risk measures on [0,1] with respect to some probability measure  $\mu^*$ . This representation gives a numerically constructive way to bound any law invariant coherent risk measure on  $L^{\infty}$  from above and below. The results are illustrated on specific law invariant coherent risk measures along with numerical simulations.

**Keywords:** Coherent risk measures, law invariance, average value-at-risk, comonotonic risk measures, robust performance measure, disutility minimization.

## Regularization of linear copositive problems: methods and algorithms based on the immobile indices

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We study new ways of regularization of copositive programming problems, which consists of transforming a problem to an equivalent form, where the Slater condition is satisfied and, therefore, the strong duality holds. Our approach is based on the concept of immobile indices which plays an important role in the feasible cone representation. Given a linear copositive problem, we obtain new representations of the minimal faces containing its feasible set and describe a regularization algorithm based on the obtained representations. This algorithm is compared to some regularization procedures developed for a more general case of convex conic problems and based on a facial reduction approach.

**Keywords:** Copositive programming, facial reduction, minimal face, Slater Constraint Qualification, regularization algorithm.

This work was partially supported by the state research program "Convergence" (Republic Belarus), Task 1.3.01, by Portuguese funds through CIDMA - Center for

Research and Development in Mathematics and Applications, and FCT - Portuguese Foundation for Science and Technology, within the project UIDB/04106/2020.

## Sensitivity analysis of a parametric optimal control problem applied to daily irrigation

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With the purpose to study the water usage for the daily irrigation of a given farmland, we propose the study of a parametric optimal control problem associated with the one analysed in [1], considering some changes with respect it.

First of all, it is important to note that in the proposed parametric optimal control problem, we guarantee that the field crop is kept in a good state of preservation, by considering that  $x(t) \geq x_{\min}$  for all instant of time t under study, where x(t) represents the water quantity in the soil along time and  $x_{\min}$  is the hydrological need of the crop. In second place, we have to consider a quadratic control in the objective function, instead linear as in [1], in order to apply the sensitivity analysis theory described in [2]. The motivation of this work is sustained by the fact that irrigation processes may be subject to perturbations that are modelled by the percentage of water losses due to the run-off and deep infiltration. Such percentage is represented by parameter  $\beta \in [0, \beta_{\max}]$ . Then, the main goal of the current research is to know how the solution of optimal control problem depends on  $\beta$ . So, the idea of this work consists in:

i. solving a nominal optimal control problem for a nominal value of  $\beta$  given by  $\beta_0 \in [0, \beta_{\text{max}}]$ ; ii. obtaining an approximation of the numerical solution, using the nominal solution associated with the previous point, which depends on time and on  $\beta$  in a neighborhood of  $\beta_0$ ; iii. comparing the numerical solutions provided by the sensitivity analysis, associated with the previous point, and the ones obtaining by solving numerically the respective non-parametric optimal control problem.

Actually, we can repeat the previous steps for several nominal values  $\beta_0$  and, then, we can compile all these results. It is important to emphasize that we are interested to do this, since we intend to cover an interval of admissible values with respect to  $\beta$ .

**Keywords:** Sensitivity analysis, parametric optimal control, daily irrigation, water losses percentage.

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This work was partially supported by The Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

### Singularities of minimum time affine control systems in dimension 4

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This talk will focus on recent developments on optimal time control affine control systems, with in mind application to the controlled Kepler and circular restricted three body-problems (CRTBP). We are interested in minimizing the final time for affine control systems,

$$\begin{cases} \dot{x}(t) = F_0(x(t)) + u_1(t)F_1(x(t)) + u_2(t)F_2(x(t)), & t \in [0, t_f], \quad u \in B, \\ x(0) = x_0, \quad x(t_f) = x_f, \\ t_f \to \min, \end{cases}$$

(3)

where the control u is contained in the euclidean ball B, the  $F_i$  are smooth vector fields, and the phase space M is a four dimensional manifold. (Most results are valid of 2m dimensional manifold with m controls.) Pontrjagin Maximum Principle provides the following necessary condition: Optimal trajectories are projections on M of so-called extremals, that is of solutions of the Hamiltonian system defined on  $T^*M$  by

$$H^*(z) = H_0(z) + \sqrt{H_1^2(z) + H_2^2(z)}, \ z = (x, p) \in T^*M$$

with  $H_i(x,p) = \langle p, F_i(x) \rangle$ , i = 0, 1, 2. The associated control is

$$u = \frac{1}{\sqrt{H_1^2 + H_2^2}} (H_1, H_2).$$

The set  $\Sigma = \{z \in T^*M, H_1(z) = H_2(z) = 0\}$  defines a singular locus, and we are interested in the behaviour of the Hamiltonian flow in the neighborhood of  $\Sigma$ . When this set is crossed, the control admits a discontinuity, called a switch. We partition  $\Sigma$  into three subsets  $\Sigma_-$ ,  $\Sigma_+$ , and  $\Sigma_0$ , on which we study the flow. We will reformulate this problem as as a system with paramater that admits a bifurcation at  $\Sigma_0$ . We make the following generic assumption:

$$\det(F_1(x), F_2(x), F_{01}(x), F_{02}(x)) \neq 0$$
, for almost all  $x \in M$ , (A)

This assumption is in particular valid for every second order controlled mechanical system of the form (V denotes a potential)

$$\ddot{q} + \nabla V(q) = u. \tag{4}$$

We use a blow up and give a normal form for the extremal system, which allows us to build a stratification of the phase space on which the flow is smooth. Furthermore, when crossing the strata, the flow admits log-type singularities (and thus belongs to the log-exp category).

In a tubular neighborhood of  $\Sigma_+$ , the flow is smooth and no extremal crosses  $\Sigma_+$ . Actually, we will show that whole situation can be seen as a bifurcation around a point  $\Sigma_0$ . For those points, that are nilpotent equilibria for the regularized flow, this last case is much more delicate to study. We prove that for generic systems (3), either there exists a unique trajectory passing through  $\bar{z}$ , or there exist unique trajectory going out of  $\Sigma_0$  at  $\bar{z}$ . In the first case, this trajectory is connected to the singular flow in  $\Sigma_0$ . Furthermore, the local flow is continuous.

**Keywords:** Optimal control, geometric control, Hamiltonian systems, singularities, dynamical systems, blow up, three body problem

#### Some new aspects on the Krein C-determinantal range

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Given a control system on a smooth manifold M, any admissible control function generates a flow, i.e. a one-parametric family of diffeomorphisms of M. We give a sufficient condition for the system that guarantees the existence of an arbitrary good uniform approximation of any isotopic to the identity diffeomorphism by an admissible diffeomorphism and provide simple examples of control systems on  $\mathbb{R}^n$ ,  $\mathbb{T}^n$  and  $\mathbb{S}^2$  that satisfy this condition. This work is motivated by the deep learning of artificial neural networks treated as a kind of interpolation techniques.

**Keywords:** Control system, diffeomorphisms, artificial neural network

### Stochastic approximation for high-frequency observations in data assimilation

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With the increasing penetration of high-frequency sensors across a number of biological and physical systems, the abundance of the resulting observations offers opportunities for higher statistical accuracy of down-stream estimates, but their frequency results in a plethora of computational problems in data assimilation tasks. The high-frequency of these observations has been traditionally dealt with by using data modification strategies such as accumulation, averaging, and sampling. However, these data modification strategies will reduce the quality of the estimates, which may be untenable for many systems. Therefore, to ensure high-quality estimates, we adapt stochastic approximation methods to address the unique challenges of high-frequency observations in data assimilation. As a result, we are able to produce estimates that leverage all of the observations in a manner that avoids the aforementioned computational problems and preserves the statistical accuracy of the estimates.

**Keywords:** Data assimilation, high-frequency observations, data modifications, stochastic approximations, stochastic gradient descent, Kalman-based stochastic gradient descent.

### Stochastic differential games within the framework of regime-switches

Gerhard Wilhelm Weber<sup>1</sup> (gerhard.weber@put.poznan.pl), Emel Savku<sup>2</sup> (esavku@gmail.com)

We study on zero-sum and nonzero-sum stochastic game approaches in a continuous-time Markov regime-switching environment. We use the dynamic programming principle for applications of two optimal investment problems. The first application is a zero-sum game between an investor and the market, and the second one formulates a nonzero-sum stochastic differential portfolio game as the sensitivity of two investors' terminal gains. We derive regime-switching Hamilton-Jacobi- Bellman-Isaacs equations and obtain explicit optimal portfolio strategies with Feynman-Kac representations of value functions. We illustrate our results in a two-state special case and observe the impact of regime switches by comparative results.

**Keywords:** Stochastic differential games, Dynamic Programming, Hamilton-Jacobi-Bellman-Isaacs equations

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#### Sub-Riemannian structures on homogeneous manifolds

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This talk, based on joint work with Velimir Jurdjevic and Irina Markina, deals with sub-Riemannian structures associated with homogeneous spaces M=G/K induced by a transitive left action of a semi-simple Lie group G on a smooth manifold M, where K is the isotropy subgroup relative to a fixed point in M. A large class of sub-Riemannian systems on Lie groups that admit explicit solutions with certain important properties will be uncovered. The relevance of these results to the action of Lie groups on Stiefel and Grassmann manifolds will be emphasized.

**Keywords:** Lie groups, sub-Riemannian systems, Stiefel and Grassmann manifolds

#### Sufficient optimality conditions in Optimal Control

Laura Poggiolini (laura.poggiolini@unifi.it) University of Florence, Italy

This talk focuses on sufficient second order conditions for strong local optimality of Pontryagin extremals in optimal control problems with a control affine dynamics and bounded controls. If the cost is smooth, then the typical structure of extremal trajectories is the concatenation of bang and singular arcs. We review some results for these kind of concatenations, obtained in collaboration with Gianna Stefani, and then consider the case of a Bolza problem, studied in collaboration with Francesca Carlotta Chittaro, where the integral cost has a  $L^1$ -growth with respect to the control. Indeed, with such a cost, extremals trajectories present also a new kind of arcs, which in the literature are known as "zero arcs" or "inactivated arcs".

**Keywords:** Optimal control problem, Pontryagin extremals, strong local optimality, Bolza problem, zero arc

### Sustainable-robust aggregate production planning considering workforce satisfaction

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Aggregate production planning (APP) is known as a medium-range production and organizational planning that cope with the main issues of manufacturing industries, such as in-house production and outsourcing quantities, employment and unemployment rates and workforce satisfaction level. By the help of APP, decision-makers can make successful decisions to meet demand at a minimal cost in terms of employees, overtime development, backorders, subcontracting or inventory levels. Furthermore, the sustainable development of APP is regarded as a striking issue in line with global concerns, specifically in environmental aspects, as well as economic ones, and nowadays, human-related factors in terms of workforce satisfaction level as social aspects. Accordingly, in this research, the objectives are decided to simultaneously minimize total cost of the production system, minimize total environmental pollution and maximize workforce satisfaction level. The research proposes a novel multiobjective mixed-integer linear programming (MOMILP) model to formulate the sustainable-robust APP problem under uncertainty. Thereby, two stages are performed so as to design an effective research methodology for the proposed aims. In the first stage, one of MCDM techniques like Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is employed to evaluate and to decide most important criteria in order to create new parameters. Herewith, data collection via matrix questionnaires (two ones) is considered the first compulsory step in advancing processes in the first phase. The questionnaires has been fulfilled on the manufacturer's side and by interviews with managers/experts/foremen in production/quality assurance (QA) and managers in human resources (HR). Throughout questionnaires, new data will be added including Human Factors such as needed Technical, Common and General Skills along with social and infrastructural variables, alongside archived data in the company. At the further steps, robust techniques are applied to formulate the sustainable APP problem robustly with uncertain demand. Some MODM techniques will be applied at the second phase to refine the main objectives of the issues subject to real-life constraints. Moreover, a numerical example is then examined to evaluate the performance of the proposed methodology. Finally, the sustainability and robustness of the problem is investigated against the fluctuations of key parameters based on unstable real-world situation and managerial insights and decision aids are given. To cope with the multi-objectiveness of the suggested mathematical model, a lexicographic approach is implemented.

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**Keywords:** Aggregate production planning, human factors, decision making, robust Optimization, sustainability.

## Time-symmetric optimal stochastic control problems in space-time domains

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We present a pair of adjoint optimal control problems characterizing a class of time-symmetric stochastic processes dened on random time intervals. The associated PDEs are of free-boundary type. The particularity of our approach is that it involves two adjoint optimal stopping times adapted to a pair of filtrations, the traditional increasing one and another, decreasing. They are the keys of the time symmetry of the construction, which can be regarded as a generalization of "Schrödinger's problem" (1931-32) to space-time domains. The relation with the notion of "Hidden diffusions" is also described.

**Keywords:** Bernstein processes, stochastic optimal control, hidden diffusions, free boundary PDEs

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#### Dynamic Control and Optimization International Conference – DCO 2021

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