Universidade de Aveiro Departamento de Comunicação e Arte
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JULIO EMILIO MARCO FRANCO

CRIAÇÃO DE MÚSICA BASEADA NA PROPORÇÃO ÁUREA: ABORDAGEM TEÓRICA E PRÁTICA À ESCALA DE 34 TONS DE IGUAL TEMPERAMENTO

MUSIC CREATION BASED ON THE GOLDEN RATIO: A THEORETICAL AND PRACTICAL APPROACH TO THE 34-TONE EQUAL-TEMPERED SCALE

IMM Institute for Microelectronics and Microsystems
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Tese apresentada à Universidade de Aveiro para cumprimento dos requisitos necessários à obtenção do grau de Doutor Europeu em Música, realizada sob a orientação científica da Professora Doutora Isabel Maria Machado Abranches de Soveral, Professora auxiliar com agregação do Departamento de Comunicação e Arte da Universidade de Aveiro, e co-orientação do Doutor Diego Luis González-Tisserand, researcher do Istituto IMM-CNR. Bolonha, Itália.

o júri

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Also, my gratitude to the many authors which have lent their artistic images in a copyright-free CC0 format.

palavras-chave

Escala dourada de 34 tons. Escalas de temperamento igual. Tonalidade. Música microtonal. Harmônicos musicais. Intervalos consoantes. Número áurico. Seção áurica. Relação áurica. Proporção áurica. Número dourado. Seção dourada. Proporção dourada. Seqüência Fibonacci. Software de música. Sistemas alternativos de notação de música. Sistemas dinâmicos não lineares. Criação musical contemporânea.

resumo

Os fenómenos sensoriais de perceção musical são considerados substancialmente não lineares. A proporção áurea desempenha um papel fundamental em sistemas dinâmicos não lineares e tem sido reconhecida como um elemento estético em vários contextos ao longo do tempo. Esta investigação desenvolve a escala de 34 notas de temperamento igual (34-TET). Trata-se de um modelo microtonal baseado na proporção áurea, contendo os intervalos harmónicos musicais, e permitindo uma abordagem consistente que abrange os distintos temperamentos ao longo da história, assim como outras culturas musicais. Estas propriedades teóricas estão praticamente expostas em dois portefólios, incluindo exemplos de composição erudita com raízes europeias (desde o Renascimento ao século XX), música popular (bossa nova, tango, swing), maqãm e música indiana. O segundo portefólio contém o trabalho artístico "The Asian Garden," criado no âmbito desta tese, que combina escalas de temperamento igual de 34 e de 12 notas (12-TET), e fornece referências culturais adicionais da China e Japão.

A escala 34-TET oferece uma abordagem global à escala de entonação justa que é mais de duas vezes melhor do que a da escala 12-TET, com todos os intervalos consonantes consideravelmente abaixo do limiar diferencial. Se fosse aceite um valor máximo de impureza não muito diferente do valor acordado quando a escala de 12 tons igualmente temperados foi padronizada (17,65 cents em vez de 15,67 cents), a escala 34-TET tornar-se-ia, adicionalmente, uma ferramenta útil para a aproximação de culturas diferentes.

keywords

34-tone golden scale. Equal tempered scales. Tonality. Microtonal music. Musical harmonics. Consonant intervals. Auric number. Auric section. Auric ratio. Auric proportion. Golden number. Golden Section. Golden Ratio. Golden proportion. Fibonacci sequence. Music software. Alternative music notation systems. Nonlinear dynamic systems. Contemporary music creation

abstract

The sensory phenomena of music perception are considered to be highly non-linear. The golden ratio plays a key role in nonlinear dynamic systems and has been recognized as an aesthetic element in many places over time. This research develops the 34-note equal tempered scale (34-TET). A microtonal model based on the golden ratio, containing the harmonic musical intervals, and permitting a consistent approach that embraces the different temperaments throughout history, as well as other music cultures. These theoretical properties are practically exposed in two portfolios, including compositional samples of art music with European roots (from the Renaissance to the twentieth century), popular music (bossa nova, tango, swing), maqãm, and Indian music. The second portfolio, created within the scope of this thesis, contains the artistic work "The Asian Garden" combining the equal tempered scales of 34 and 12 notes (12-TET), and provides additional cultural references from China and Japan.
The 34-TET scale offers an overall approach to just intonation scale more than twice as good as that of 12-TET, with all consonant intervals well below the differential threshold. If a maximum impurity value was accepted, not appreciably different from that agreed upon when the equal-tempered 12-tone scale was standardized (17.65 cents vs. 15.67 cents), then the 34-TET scale would become, additionally, a useful tool for approaching different cultures.

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VOLUME II	**(Tables, Figures, and appendixes)**	**Page**
	Tables	

Relation of audio & multimedia files

Portfolio I: Common practice, contemporary and popular excerpts

#	*Time*	*File name*	*Genre/Style*	*Scale*
1	2:41	Gloria resurrectionis 34TET	Renaissance	34
2	1:11	Sarabande E major 34TET	Baroque	34
3	1:16	Baroque fantasy 34TET	Baroque	34
4	2:13	Last ball before de battle. Minuet. 34TET	Classical	34
5	2:11	Podolian rhapsody 34TET	Romantic	34
6	1:17	Adrán doors 34TET	Early Contemp./Serial	34
7	2:09	Penelope and the palace intrigues 34TET	Contemporary/Film track	34
8	0:23	March 8. The exam 34TET	Contemporary/Atonal	34
9	0:20	The Milling-machine 34TET	Contemporary/Noise music	34
10	1:57	Que foi do meu violão 34TET	Popular/Bossa nova	34
11	3:06	Tierra mia tango 34TET	Popular/Tango	34
12	1:33	Hello Lara 34TET	Popular/Swing	34
13	0:37	Horse getaway makam 34TET	Maqãm	34
14	1:20	Awaken your soul to meditation 34TET	Hindi	34

Portfolio II: The Asian Garden

15	2:39	The Asian Garden_Prelude	Contemporary-mix	34 & 12
16	4:28	The Asian Garden_Fo's love Aria	Opera - Ballet	12
17	2:06	The Asian Garden_Nuán & Fo duet	Opera	12
18	0:28	The Asian Garden_Restlessness	Contemporary-mix	34 & 12
19	0:38	The Asian Garden_Oboe solo	Contemporary	34
20	3:53	The Asian Garden_Yun's love for Nuán	Opera/ballet	12
21	1:28	The Asian Garden_The storm	Contemporary/ballet	34 & 12
22	4:12	The Asian Garden_The seduction serenade	Opera-Ballet	12
23	1:44	The Asian Garden_The abandonment sadness	Contemporary/mix/ballet	34 & 12
24	2:16	The Asian Garden_Fo's death	Contemporary/mix	34 & 12
25	1:59	The Asian Garden_Despair of Nuán and the fairy's advice	Contemporary/mix	34 & 12
26	1:05	The Asian Garden Final	Contemporary	12

Both WAV and MP4 formats are included for each sample. Multimedia MP4 allows a better following of the musical idea vested in the samples but sound quality is better with WAV.

Pitch Notation Standard

Pitches refer to the IPN (International Pitch Notation) or American Standard Pitch Notation (ASPN, $A_4 = 440$). Pitches for the 34-TET scale, if not stated otherwise, refer to $C_4 = 261.626$ Hz.

Interval Ratios

Note that although in the monochords the octave (with assigned value of one) is divided into fractions ─consequently inferior to one─ in this text, the intervals are very frequently referred as the ratio that increases the root frequency to the corresponding pitch in the ascending octave, and, consequently, they are greater than one. Thus, e.g., a perfect fifth is expressed as 3:2 and not as 2:3. In this way the calculation is facilitated through spreadsheets.

Style information. Disclaimer. Copyright.

See details in last page after scores.

Abbreviations, Conventions and Acronyms Used

BCE	Equivalent to BC. Before Common Era.
ca.	Circa (Near).
CC	© Creative Commons.
CE	Equivalent to AC. Common Era.
comm.	commented (with comments).
Data format	dd/mm/yyyy.
DAW	Digital Audio Workstation.
EACul	East Asia cultures.
(Ed.)	Editor, Editorial.
EDO	Equal Division of the Octave.
(Eds.)	Editors.
e.g.	exempli gratia. (For example).
EP	Electronic piano.
EuCul	Europe and Europeanized cultures.
FM	Frequency modulation.
GR	Golden Ratio.
http:/https	Lowercase, even at the beginning of the paragraph.
i.e.	id est. (that is).
Italic format	Books and web pages titles, and for names in case of journals.
JI	Just Intonation.
JND	Just Noticeable Difference.
MECul	Middle East cultures.
MQD	Mean Quadratic Dispersion (σ).
MS	MuseScore®.
[n.d.]	No date.
[n.p.]	No page.
[sic]	Exactly as written.
p.	Page.
para.	Paragraph.
Phi	Used indistinctly in this work for mathematical τ and – φ.
pp.	Pages.
Rev.	Reviewer.
RMSD	Root Mean Square Deviation (ϑ).
SACul	South Asia cultures.
[*sic*]	Incorrect writing intentionally being left as it was in the original.
SD	Standard Deviation.
TET	Tones Equal Tempered.
vs.	versus.
y/o	years old.
[…]	Ellipsis points to indicate omitted text.
doi®	DOI ® is a trademark of International DOI Foundation (IDF).

> The latter half of the twentieth century saw remarkable advances in our understanding of physical systems governed by nonlinear equations of motion. This development has changed the scientific worldview in profound ways […] (Socolar, 2016, p. 115).

An investigation on the theoretical-practical aspects of the musical scale of 34-tones equally tempered (TET)[1] is presented. The text provides the answer to questions such as: why to propose 34 tones? What are the advantages of using this scheme?

Several authors agree with Adorno in pointing out that, by the turn of the twentieth century, the music entered in a kind of stasis (Slonimsky, 2016; Subotnik, 1976; Tenney, 1984) with compositions resembling each other.[2] Per van der Merwe (2004) "over the period from about 1450 to 1910, composers explored that potential [12-tone] to the limit" (p. 115).

With financial sustainability of the orchestras questioned (Flanagan, 2012; Rosen, 2011), and with a young audience scarcely represented in the concert halls (Flanagan, 2012; Huang, 2011, North, Hargreaves, & O'Neill, 2000; Tarrant, North, & Hargreaves, 2000), the search for elements contributing to the invigoration of the music is a must.

However, the microtonality, seen by some authors as a possible solution (Bohlen, 1978; Carlos, 1987; Castilla-Ávila, 2016; Partch, 1979; Tenney 1984; Wilson, 1975), has had a limited development during the last decades,[3] with their schemes in most cases diverging from the consonant intervals (Marco-Franco, 2018) that seem to play a fundamental role in the musical pleasantness (Cuddy, 1993; Helmholtz, 1954; Jaśkiewicz, Francuz, Zabielska-Mendyk, Zapała, & Augustynowicz, 2016; Terhardt, 1976; Wassum, 1980).

An increasing number of publications are focusing on the nonlinear dynamics,[4] on art in general, and particularly on music (Berndt, 2009; Buttle, Zhang, Song, &

[1] Note that any equal division of an interval is equally tempered, but the case is very different if the scale goes from unison to the octave, or ranges from or to another interval, as will be seen later. EDO would be a more precise acronym than TET, since it clarifies the interval (octave); however, the term is less frequently used.

[2] Slonimsky (2016) refers to the results of a questionnaire published in Le Cas Debussy, Paris, 1910, where the writer Joséphin Péladan declared that as far as 1909 all the masterpieces were similar to each other as they follow the same rules, from Palestrina to Wagner, from Bach to Berlioz and Franck.

[3] No specific statistics have been found, but there is no reference to microtones or microtonality in the headlines of the first 20 pages of a web search for «classic concert events» (November 11, 2018).

[4] According to Antonis Karantonis, «In the physical world with the term dynamical system we define any physical phenomenon evolves [sic] in time. Since a physical system can be described by different physical variables, we can say that a dynamical system is a system in which some (or all) variables evolve in time.

Jiang, 2009; Cartwright, González, Piro, & Stanzial 2002; Cartwright, González, & Piro, 2009; Colucci, Chacón, & Leguizamon, 2016; Fletcher, 1999; Ivanov, 2001; Kramer, 1986; Mouawad, & Dubnov, 2017; Young, Yu, & Reiss, 2005; Yu, & Young, 2000, 2013).

At least since the times of Aristotle and Plato ─although with different perspectives─ art has been linked to the imitation of nature that is considered highly nonlinear (Afraimovich, Tenreiro-Machado, & Zhang, 2016; Fenner, 2003; Kramer, 1986; Ivanov 2001).

As our perceptual systems are mostly nonlinear, it seems reasonable to root the music pleasantness in the nonlinearity (Berndt, 2009; Fletchner, 1999; González, 1987; Kramer, 1986; Ivanov, 2001). But how to include this nonlinearity in music praxis?

Among the factors linked since ancient times to human aesthetics, probably the golden section is second to none, and it also plays "a fundamental rôle in theory of nonlinear dynamical systems" (Cartwright et al., 2002, p. 56).

The golden or auric ratio, proportion, or section (GR) appears frequently as a kind of mythical or magical preferred ratio ubiquitously present. It is found in nature, and in artistic creations throughout the centuries all around the planet, be it in pictures, sculptures, pottery, etc. This *magic* proportion has also been used in the architecture of iconic constructions from Egyptian pyramids to Notre Dame in Paris or the United Nations Secretariat Building; even some typical elements of daily life such as credit cards or well-known trademarks include this proportion (Olsen, 2006).

Many studies have already reviewed the presence of the GR in music metrics (Frey, 2015; King, 1944, Lanza, 2006; Laser, 2012; Madden, 2005; Olsen, 2006; Roberts, 2015).

However, GR-based musical scales have had quite a limited development (Marco-Franco, 2018). Given that the GR is ubiquitously present in nature and art ─both considered to be highly nonlinear─ this exception for musical golden scales seems somehow surprising. One should expect a GR-based musical scale to offer

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Since most of the phenomena we observe in nature are progressing in time it seems clear that the study of dynamical systems is of great interest» (Karantonis, 2000, p.7). The Chaos theory focuses on the behaviour of some of the dynamic systems [those singularly sensitive to the seed value or initial condition]. Nonlinearity could be expressed in a very simple predicate as: if $A + B = C$, then fA + fB ≠ fC (e.g. f a power function).

theoretically excellent possibilities for musical aesthetics; why then this scanty presence?

The dissertation has analysed whether the 34-tone equal-tempered scale (34-TET)[5] could provide the answer to the question. Or, in other words, it developed the hypothesis that the 34-TET scheme, by incorporating nonlinearity through GR, consonance, and equal temperament, can be an effective tool for new musical expression. Once this was established, the following challenges emerged:

- Is 34-TET truly a GR scale?
- Does it incorporate the consonant intervals?
- Why a scale with 34 microtones and no other number?
- Is it possible to combine the two temperaments (12-TET & 34-TET)?
- What could be the future benefits of using the 34-TET scale?

The study has been a marriage of art and scientific-acoustic procedures ─in line with the growing interdisciplinary trends─ searching for a tool that could "express something which has not yet been expressed in music" (Schoenberg, 1950, p. 39).

The dissertation has covered two aspects. On the one hand, it has developed the 34-tone scale, as a further step from the 34-tone non-tempered golden scale (34T-GR) published by Cartwright and co-workers (2002), providing the answers to the theoretical-practical questions: the nomenclature, pitches, symbols for notation, and related methodology for the musical composition as well as its sound reproduction. On the other hand, it raises some possibilities of future uses related to its sonority that will involve an individual positioning.

The basic considerations in this regard are: a) has the 34-TET scale a substantial advantage over the already existing scheme? If yes, in which settings? b) Is pitch proximity the only concern?

If a maximum impurity indiscernible from that accepted when the 12-TET scale was standardized (17.65 cents vs. 15.67 cents, that is less than two cents more) would now be accepted, then the 34-TET scheme could become a tool to be used among different musical cultures, as no possible pitch may be more than half a microtone away from a note in the scale (half of 35.29 cents that is 17.65 cents).

[5] Tone Equal Tempered (TET) and EDO (Equal Division of the Octave) are used interchangeably in this dissertation (see footnote 1).

Interculturality may be a very interesting approach. The sociologist Zygmunt Bauman described the characteristic of our time in an interview by *The Guardian* (Bunting, 2003):

> And we live in a world of communication, so everyone gets information about everyone else. There is universal comparison and you don't just compare yourself with the people next door, you compare yourself to people all over the world […] (Jarosz, 2009, p.144).

Obviously, this has an impact on marketing. Will a global compositional tool be of any help in ameliorating the decline in music revenues?[6]

However, this approach requires facing other questions aside from the pitch, and very importantly keeping in mind the local cultural identity:

> The global economy offers new and exciting ways for companies to extend their influence beyond domestic and regional markets. In order to compete successfully in the global arena, a firm has to develop a strong branding strategy that is consistent with the cultural traditions of the country in which it operates (Krueger, & Nandan, 2008, p.30).

With globalisation of musical culture and genres, and the relatively stable musical style preferences linked to different personality traits since adolescence (Delsing, ter Bogt, Engels, & Meeus, 2008, p. 109; Schwartz & Fouts, 2002, p. 205), there may be more than one creative way to go; ways that the 34-TET scale could offer, as a flexible tool for the development of cultural and personal traits.

That intercultural expansion could even be considered via a common path dictated by the natural phenomenon of nonlinear dynamics.

The 34-TET microtonal richness offers an additional opportunity to create alternative works outside European boundaries. The *maqãm* music is an example. At the beginning of the 20th century, the 24-equal division of the octave with notes rounded to the nearest quartertone was adopted, making the writing of maqãm music now possible using score writing programs (Rechberger, 2018, p. 92). Per Yarman (2008), 34, 41, and 70 divisions of the octave, are better options for Turkish music composition (p. 1).

As mentioned, approaching to other cultural environments is also interesting from the commercial point of view; Asia, for example, seems to be a growing market for art music (Huang, 2011; Paarlberg, 2012).

Could then the 34-TET scale become a future global intercultural element?

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[6] Complete annual reports since 1973 are available at https://www.riaa.com/u-s-sales-database/

The movements towards music standardization coming from Europe have raised some questioning. The prevalence of a colonialist European superiority on this proceeding has been criticized (Rehdling, 2005; Tomlinson, 2007).

> For Riemann it was self-evident that the strange sounds that emanated from the phonograph could be nothing but a consequence of the lack of musical training of the non-Western musicians (Rehdling, 2005, p.132).

Priorities and functions may be different from one culture to another, requiring opportune perspectives, something seemingly missing in the music textbooks as criticized by Catherine Marie Schmidt in her doctoral dissertation. The cultures need to be approached “not with the analytical tools that are compatible only with the music of the Western European heritage” (Schmidt, 1999, p.150). Even the modern maqãm 24-tone division of the octave has some questioning:

> Another mis-representation of Maqam music is the approach to it as a scale system, while the structure of Maqam music will be presented later, it is important to note that one of the most limiting definitions of Maqam music is its view as a scale or modal system, thus subjecting it to the staff logic of European music, whether being notated or practiced orally (Kalash, 2016, p. 16).

The same can be said in relation to using European score writing methods for other cultures, and for India in particular.[7]

Focusing on the composer side, on which this dissertation is framed, the complexity of the new music market requires the authors to be prepared for diverse output demands, needing a very versatile compositional tool, either to produce a romantic consonant song or a more strongly rhythmic contemporary experiment, "a wide range of musical aesthetics throughout the creative production" (Sitsky, 2005, p. 213), while moving outside the timbres of what has considered to be conventional orchestral instruments— another restriction imposed up to now.

Expanded scale compositions may be used in new performances, now growing in interest, including crossover and hybrid (multi-artistic) events making use of the cross-fertilisation for audience increase, particularly for parallel or related fields (Paul, 1999, p. 14). New composition tools could be combined with new performances, and non-conventional market strategies (Levinson, 2011). The multi artistic composition *The Asian Garden*, included in the second portfolio of the dissertation, explores this possibility.

[7] The ethnomusicologist Dr. Vidyadhar Oke has expressed criticism in relation to the use of European schemes for Indian music (personal communication, July 2, 2018).

The expansion of the 34-tone scale may be a way of approaching other musical cultures ─but this requires, as mentioned above, a positioning that goes beyond a mere acceptance of tone impurity─ and, in addition, a step towards the above-mentioned wider and richer use of musical elements.

The answer to whether the 34-TET will be the tool for that evolution is in the artists' hands and their original creative capacities to meet public expectations. The point here is that, again, there are two separate issues: one question is the technical advantages of the scale, and another very different question is its sonorousness.

> The gulf that separates aesthetics and history is recapitulated within the concept of style itself as rift in meaning between style as applied to works or composers and style as applied to ages and nations. The individuality of the composer is an essential element in artistic character, originality being one of the defining criteria of art (Dahlhaus, 1983, p. 18).

Consequently, whether the 34-TET scale is going to be useful in the field of musical composition, and/or if the public in the future will consider its development positively or not, are questions that for the moment cannot be answered. They need to be addressed after several composers have made different creations exploiting the different potentialities of the 34-TET scale together with their individual artistic positioning, and the creations have been analysed with studies of music reception.

As for the text content, Chapter Two comes after this introduction bringing some definitions and constraints, followed by Chapter Three with the purpose, goals and objectives concretization. The state of the art (Chapter Four) is dedicated to tracing the rational bases of the research as a theoretical approach for the development of the 34-TET scheme. Chapter Five on material and methods will describe the notation, and details of the evaluation procedures, and audio reproduction used in this research. Chapter Six offers the results and answers to the questions posed, based on scholar theoretical analyses, and in a series of compositional samples created to evaluate the hypotheses. The main outcomes are discussed in Chapter Seven with mention of problems and criticism; future lines of action are also drawn, including comments for new instrument design, construction, and tuning. A conclusive Chapter Eight summarizes the headlines of this work. After the references, tables and figures, the appendices provide: first, an extended description of the samples included in the two portfolios; second, the libretto of *The Asian Garden;* and third, the scores.

Chapter Two. Definitions and Constraints

Art Music Concept and Periods

Although accepting its many limitations, the concept of art music has been used in this dissertation instead of "Western classical" music.[8]

The Common-practice periods are well known to musicians and little needs to be said in this regard.

For the subsequent period ─following Gagné (2012): starting in 1890 until the present time, and including: "20th-century composers who basically eschewed modernistic devices and wrote accessible works in a tonal idiom, drawing chiefly on classical, romantic, and folk models" (p. 1)─ the generic term of contemporary music has been used.[9]

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[8] First, «Western» comes from west which is a relative concept. One geographical point is west or east, depending on the point taken as reference. Lisbon is west of Rome but is east of New York. If the Greenwich meridian is to be taken as reference, then most of the art music meccas are Eastern: Paris, Berlin, Vienna, Salzburg, Prague, Rome, Milano, Venice, Naples, Budapest, Warsaw, Brussels, Moscow, Saint Petersburg, etc., all have east longitudes, not to say the acclaimed Sidney Opera House, located 151º East (Shanks, & Pottenger, 2005). Even the Royal Opera House in London is east (0° 07'13.17"E) of Greenwich. With the fall of the Berlin Wall, the unification of the two Germanies and the entry into the European Union of countries of the former Soviet Union, it does not seem today that it makes much sense to speak still of East or West in relation to the Iron Curtain that disappeared in the previous century, with many of the former «East» countries now belonging to the «West» European Union.
Second, «Classical» is the term used for the style embodied by music composers such as Mozart, Haydn or Beethoven, that bloomed in a period (broadly) set from the second half of the eighteenth century to the first decades of the nineteenth century. However, using a synecdoche, the baroque or romantic composers have often been included in the term since the nineteenth century (Kallen, 2013; Rushton, 1986). Even more, it is possible to find expressions such as contemporary classical music. The search for «contemporary classical music» on the internet returns over half a million sites. Per Julian Johnson there is no single definition of what constitutes classical music. The author considers that the differentiation from popular music comes from the artistic function as opposite to entertainment: «the label 'classical music' is problematic […] I use 'classical music' simply to refer to music that functions as art, as opposed to entertainment» (Johnson, 2002, pp. 3–4). The Grove Dictionary of Music and Musicians has no specific entrance for «classical music.» The closest term is art music: «one of three categories in a contemporary taxonomy that typically includes folk, art, and popular music, […] although insisting upon too rigorous a separation of types runs the risk of ignoring their intersections and overlaps» (Von Glahn, & Broyles, 2012).
Third, the same wide concept is applied for «popular» music, a concept that the Grove Oxford dictionary of music finds very difficult to define: «A term used widely in everyday discourse, generally to refer to types of music that are considered to be of lower value and complexity than art music, and to be readily accessible to large numbers of musically uneducated listeners rather than to an élite» (Middleton, & Manuel, 2001). However, not to include the term «art» for popular music has clearly important weaknesses as it could easily be derived from it that musicians (e.g.) such as the Beatles, Michael Jackson or Julio Iglesias, with millions of copies sold, would not have the consideration of artists.

[9] The turn of the twentieth century saw the blooming of different musical currents such as Neoromantic, Expressionism, Impressionism, Modernism, Neoclassicism, Serialism, etc. When the twentieth century reached about its second half, the currents (Postmodernism, New Simplicity, Pluralist music, and others) mixed up without a clear predominance, and frequently with compositions not easily classifiable, with composers who do not consider themselves included in the trends in which other authors have included them. It has been considered unnecessary to develop these, fundamentally taxonomic, aspects with little relevancy for the purposes of the dissertation. All music after Romanticism is broadly included as «contemporary» here.

Acronyms for Globe Musical Cultures

Unfortunately, it has been clearly impossible to include samples of all the musical cultures over the planet. Tribal African music, native American Indians and many others, with extraordinary cultural potentials, and varied instrumentation, are not further mentioned in this research (Raine-Reusch, 2008).

As for those mentioned, an agreement about the terms is proposed ─and with the hope that this bona-fide division in cultural areas will not rise, great discrepancies─ the following nomenclature have been chosen:

- The acronym *EuCul* has been used through this text for *Eu*ropean and Europeanized musical *Cul*tures. In this setting EuCul art music will be equivalent to “Western Classical music.”
- The acronym *MECul* stands for *M*iddle *E*ast *Cul*tures.[10]
- *SACul* is the acronym used for *S*outh *A*sia *Cul*tures, defined as those cultures of the Indian subcontinent.
- *EACul* refers to *E*ast *A*sia *Cul*tures, one of the 22 sub-regions of the planet according to United Nations (UN), including China, Chinese special administrative regions of Hong Kong and Macao, both Koreas, Japan and Mongolia.[11]

European Notation System

The music for this project has been written in scores to be played with instruments ─either acoustic or electronic─ consequently, it has not been created with the assistance of synthesizers, samplers or similar computerised technology, although music pieces can be, obviously, reproduced and could be generated in the future, with digital technology assistance.

Although the elements and work methodology are, therefore, frameable in the European art music heritage, the tool developed (34-TET temperament) is born with the hope that it can be used beyond these boundaries.

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[10] It has been preferred to South-West Asia as it includes also Egypt that is not in Asia, and Turkey that is both in Europe and Asia. Although the term has been criticized for denoting a certain British colonialist bias it is preferred to Muslim or Islamic cultures because although Islam is the prevalent religion in the area, there are also non-Muslim minorities: Jews, Christians, followers of the Baha’i faith and numerous other beliefs with a rich musical cultural activity. Also, the term Arab culture would not be appropriate as important cultural areas such as Iran are not Arabic.

[11] The UN countries classification may be retrieved from: https://unstats.un.org/unsd/methodology/m49/

The maintenance of the European musical notation system used in this study, does not imply, by any means, a supremacy positioning. It is enough to review the state of the art (Chapter Four) India or China sections, to realize the author's considerations on the richness and antiquity of those musical cultures.

But that said, another constraint appears: is it possible to reduce such different and rich music cultural features just to a score?

This idea of reducing music only to what is written down in scores ─as cited by Rehdling (2005. p. 155) and shared by other authors such as Johann Nikolaus Forkel (1788, pp. 31–32)─ has aroused some criticism:

> Riemann's worry was that the phonograph […] would allow […] intervals that were unthinkable in the rational system of Western music and had been barred from coming into circulation by the sheer impossibility of writing them down as musical notation (Rehdling, 2005, p.132).

Even using a scale with an extended range such as 34 notes, some losses may be inevitable, and it could not be enough for a perfect reproduction of the enormous musical wealth vested all over the planet (Fletcher, 1994, 2001).

> Western musicians have a tendency to believe that their musical theory and script are sufficient to express everything. When they hear of other intervals, they think of quarter-tones, of commas and other small units […] It is not, however, without amazement that we observe their total inability to put into notations, without completely disfiguring them, the melodies and chords of Oriental music (Daniélou, 1943 p. 21).

The decision on whether to take a perfect ethnomusicological approach versus a single schema valid for different cultures, and the price (loss) to pay for that, is like the decision to move out of perfect harmonic intervals when the twelve note equal tempered scale (12-TET) was adopted, assuming also a notable loss in consonance.

The Just Noticeable Difference

The difference limen, also called differential threshold, or least perceptible difference ─that appears repeatedly in this dissertation─ refers to the just noticeable difference (JND) concept, that although may apply to a wide variety of senses including touch, taste, smell, hearing, sight, or many other aspects, here it is, obviously, referring to hearing. The sound (in our case pitch or frequency) just noticeable difference (JND) "is approximately one thirtieth of the critical bandwidth across the hearing range. Musically this is equivalent to approximately one twelfth of a semitone" (Howard, & Angus, 2017, pp. 125–126), that is about 8.3 cents.

There will no further mention of the complexities for the determination of the JND.[12]

Related to the difference-threshold concept, although progressively less used, it may be convenient to make a comment about the savart. It was established as the differential limen of a well-trained musician with an absolute ear in the best conditions, setting for it a value of about four cents.

This value was rounded by convention to fit exactly four cents matching thus with the 1200 cents for octave (300 savarts per octave). In the few cases that this concept shows up in this text its value is referred to that round value of exactly four cents. Cents is the basic unit used throughout the dissertation.

Golden-Ratio-Based Scales

Golden ratio-based scales have been reviewed in a previous work (Marco-Franco, 2018).[13] The GR is a proportion, and, in this dissertation, a scale is considered golden only if in the construction, the GR keeps the proportional property.

By solving a quadratic equation, the values ─τ and φ, called generically Phi─ of the GR may be determined. Depending on how this Phi value is used, the scheme may be golden or not. E.g., Lange (2013) by combination and manipulation of Phi, reach to the rational simple numbers 1, 2, 3, 4… (para. 8). A scale based on simple rational numbers is considered not to be golden.

The methods contemplated to keep the golden proportion are based on: either the GR itself, the Fibonacci series, or their inverses (convergent) (Marco-Franco, 2018). The inverse values are perfectly suitable, as the absolute values of the solutions of the golden ratio [τ and φ] are inverse to each other.

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[12] The JND values depend on characteristics of the sound wave, including phase, frequency, duration, loudness, and other conditions. It is not constant throughout the frequency spectrum; the maximum is around 0.5-0.6 per thousand (about 8 cents) in the region near 1,000 Hz but it grows significantly for very low or very high frequencies. These questions have been extensively reviewed elsewhere (Zwicker, Flottorp, & Stevens, 1957; Zwicker, and Flast, 1999). A more recent review of JNC in E-book format from Melissa K. Stern and James H. Johnson may be found at *The Corsini Encyclopedia of Psychology* (Wiley Online Library). https://doi.org/10.1002/ 9780470479216

[13] The most popular GR based scales are the schemes of Pierce and Bohlen. They are not based on the octave nor do they contain 1/2 ratios. The equal temperament of Pierce is not equal division of the octave but of the tritave resulting in non-consonant intervals. The Mongoven scales «contain no octaves – or any other pure interval for that matter» (Mongoven, 2010, para. 24). 49-tone square-Phi scale is equal-tempered, reasonably consonant (mean quadratic dispersion (MQD [σ] slightly over five), but it has the inconvenience that the GR is included out of a proportional function.

Musical Controversial Terms. Work Scopes

There is no proof that music composed with a GR scale ─even perfectly matching with the above-mentioned condition─ will improve music pleasantness, however this concept may be considered.

The popularity of the GR scales seems to be very limited, at least in terms of sales and announced performances.[14]

One possible explanation, as advanced in the introduction, is that these scales do not usually contain consonant intervals (Marco-Franco, 2018), a requirement considered by many authors as essential for pleasantness (Cuddy, 1993; Helmholtz, 1954; Jaśkiewicz, et al., 2016; Terhardt, 1976; Wassum, 1980).

But also, alternatively, the supposed scale constraints could be turned aside, or even be lighthearted, in hands of a creative composer: the question of the scale structure versus sonority and creativity emerges once more.

In any case, this limitation of consonance is irrelevant for the 34-TET scale, since, as will be demonstrated, the scale is based on the GR but contains all the consonant intervals.

Some concepts open to debate, such as the definition of what is music, musical pleasantness, etc., have not been discussed in depth, avoiding entering into excessive analyses and debates, no doubt of great musicological interest, but which are not expected to have a substantial impact or originate relevant changes for this project. The reason for avoiding getting into a deeper analysis of these elements not directly related to the project is that it could imply the risk for the thesis to become unfeasible due to its extension, and overall, the risk of diluting the purpose of the scholar work: to present the theorical-practical development of the equal tempered scale of 34 tones, and the advantages of its use.

A generous inclusion of references and/or additional complementary foot notes[15] is testimony of a deeper revision of material that has been excluded for further comments.

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[14] No result was found in an Internet browsing in search of sales or performances with the specific indication of music based on a gold ratio scale, although it is possible to find a few YouTube videos with that or similar label. The finding of music samples using the golden ratio only, as interval, or including the Fibonacci sequence, is less exceptional.

[15] Probably used with greater liberality than recommended following the American Phycological Association (APA) the style manual sixth edition used as reference for this dissertation.

Many concepts included in Chapter Four have only been overviewed, as an in-depth revision will be, again, of little if any benefit in the end: once more, additional references are provided.

Compositional Material: Samples

This is a thesis with compositions, but not a thesis of compositions. Here, rather than the compositions themselves, the essential function of those samples is to support the analyses, possibilities and practical development of the 34-TET scale. The motives are not fully developed, and the compositions portfolios are, thus, illustrative.

In some cases, such as in the SACul sample, the constraints come from the pretension of reproducing through a score the sound of instruments such as the tanpura, which is not played following a score scheme. This may be one of the reasons why the sound library of the score writing software does not contain the timbre for this instrument. Details of these limitations and the solutions adopted may be found in the first appendix with each sample commented on.

Portfolios Styles/Genres

This research does not pretend to exhaust all the possibilities of the scale in its presentation. It remains in the hands of the future composers using the 34-tone scale, its subsequent development, and the inclusion of other styles and peculiarities not represented in the music samples.

Finally, it is pertinent to mention that, in the comparison of the 34-TET versus 12-TET scales analysing which of the two temperaments best approximates historical temperaments, a distinction must be made between the period in which the specific style was born and flourished, and the corresponding techniques of musical composition. These techniques can be used, as done here in this project, out of the time the style flourished.

The references to the supposed date of composition in the samples are simply to situate the musical elements that can be expected in the pieces.

Finally, it must be taken into consideration that the sound quality of the computer generated 34-TET scale notes is limited. The development of sound libraries and 34-TET sound signal improvements are fields open to future research.

Chapter Three. Purpose, Goal, Objectives

Purpose, Goal, Topics Exclusion

The general purpose of this dissertation has been to explore a music composition tool, in the never-ending process of artistic creation and new expressive advancements.

The aim, in this case, was the theoretical and practical development of the 34-tone equal tempered scale ─under the hypothesis of a GR-based musical scheme, and, consequently, linked to incorporating nonlinear systems in musical aesthetics─ and the analysis of its potential advantages, along with the possibility, assuming certain conditions, of becoming a musical composition tool that could bypass borders and be used by other cultures.

Prior to the specific description of the goals and objectives that this dissertation has covered, it is necessary to define the areas that have been excluded from further research, and thus without goals assigned to them.

Although the approach to music through the theory of the nonlinear dynamic system has been one of the primary reasons for using a golden ratio scheme in this research, the studies of nonlinearity in musical perception (Berndt, 2009), and the role of the golden ratio in the theory of nonlinear dynamical systems themselves, have been excluded from additional analysis.

Setting goals related to nonlinearity analyses, music psychology, music perception, and related topics, could have run the risk of blurring the line of research that, as indicated, was designed to remain within the art and the music composition arena.

In addition, issues such as the reasons why the golden ratio has been considered as an aesthetic element throughout the ages, or whether it is needed or not to be aware of the GR for its proper use, have also been considered out of place, with the idea that the human behaviour sciences are more appropriate disciplines to address those questions.

Therefore, the work goals and objectives have been oriented fundamentally towards art and music with the investigation focused on the 34-tone equal tempered musical scale, and what has been considered important was to provide the answers to the questions posed.

The research has included six goals, each one with their corresponding objectives. They followed a rational stepwise progress, beginning with the full description of the tool.

First Goal: 34-TET Scale Theoretical Development

The objectives set for this goal were to:

- Historically review the topic.
- Compute the frequencies, ratios and intervals.
- Analyse the golden component in the 34-TET scale.
- Evaluate consonance.
- Review whether 34 tones is the optimal size for the scale.
- Name the notes.
- Set the notation system in the scores.
- Develop a plugin for the scale.
- Review the audio reproduction possibilities (audio output).
- Check that the notes produced with the chosen audio reproduction procedure really correspond to the theoretical frequencies previously established.
- Determine the corresponding 34-TET note closest to that of the 12-TET scale for each case.

During the implementation it became evident that it was necessary to establish a methodology to analyse the golden component, the consonance, and to compare the 34-TET with the different scales.

Thus, a second goal emerged: the establishment of an objective method for those comparative scale evaluations.

Second Goal: Evidence-Based Methodology

As stated above, an evidence-based approach to be used in the comparative analyses of the different scales has been fixed.

For this second goal of setting up an appropriate investigative procedure, the following established objectives were to:

- Review the state-of-the-art procedures used for scales comparison.
- Select functions and formulas for the study.
- Design data entry spreadsheet methodology.
- Set the appropriate instructions for each function, or equation, in the spreadsheet to process the corresponding input data and place the results in the settled cells.
- Test the procedure to confirm its consistency using known data.
- Set a differential threshold (JND) value.

Third Goal: Comparative Approach to Historical Temperaments by Using the 34-tone or the 12-TET Scales

This objective addresses the question: is there a substantial advantage in using the 34-TET scale instead of the current 12-TET tuned to A 440 Hz for a faithful reproduction of the music created during the Common-practice era?

As the 12-TET scale was virtually not used during the Common-practice periods, since it was standardized in late nineteenth century, in this goal the scales of 34-TET and 12-TET have been studied comparatively, reviewing which of these two temperaments better reproduces the pitches of a series of historical 12-tone temperaments of European culture, used throughout the Common-practice.

The analyses have been done over the author's compositional samples and, also, in pieces of well-known music masters.

The objectives set for this goal were to:

- Select the most representative styles and genres.
- Compose the samples for these styles and genres.
- Select historical pieces of music to complement the analysis.
- Determine the most appropriate 12-tone temperaments and tunings.
- Establish the appropriate number of bars for comparison analyses.
- Analyse comparatively the 34-TET and the 12-TET scales in the author's samples and in well-known compositions of each period.

Fourth Goal: Evaluation of 12-TET Scale Replacement by 34-TET in Contemporary Music & European Popular Music

This goal has evaluated whether the 34-TET scale offers a reasonable approach to 12-TET, for EuCul styles and genres that use that scale.

As the pitch comparison between the 12-TET and the 34-TET was an objective already included in the first goal, what this fourth goal was seeking here is only to evaluate if the music composed with 34-TET gets close to that composed with the standard 12-TET scale.

This goal, not numerically assessed, is for the listener's consideration only.

The objectives set for this goal were to:

- Select styles and genres.
- Compose the samples for these styles and genres.
- Evaluate the results.

Fifth Goal: Evaluation of 34-TET Scale in Non-European Musical Settings

This goal has reviewed whether the 34-TET pitches approach non-EuCul schemes with an acceptable impurity.

Once again, the evaluation depends on the listener's considerations, including the acceptance or not of that impurity, which may be present in certain cases. Its higher value, as previously mentioned, is indistinguishable from that accepted when the 12-TET scale was standardized.

The objectives set for this goal were to:

- Select some non-EuCul musical genres.
- Compose the samples.
- Evaluate the results.

Sixth Goal: Evaluation of the Combined Use of 12-TET and 34-TET scales

The last goal, also non-numerically assessed, has been to review the combined usage of 34-TET and 12-TET scales, while exploring a limited approach to East Asian musical cultures.

A crossover performance project, with multiple artistic activities such as theatre, dance, pictures, digital technology and audience interaction, has been created.

The objectives set for this goal were to:

- Create the libretto.
- Compose the pieces.
- Evaluate the results.

Chapter Four. State of the Art

> I believe that the resemblance between music and mathematics begins at the creative stage; the act of composing music seems to have some affinity with the discovery of mathematical facts. Both arts are essentially abstract, and both can be written down in a notation that is universally accepted (Coxeter, 1962, p. 13).

An Outline on Mathematics, Music Physics, Perception and Harmony

Mathematics, physics and music. As one of the cornerstones of this work is the consideration of nonlinearity in artistic perception, music included, nonlinearity is repeatedly mentioned in the text. Nonetheless, and as advanced in the previous chapter, the studies on dynamic systems and nonlinearity in themselves will not be further detailed in the dissertation.[16]

The following summary of mathematics and physics applied to music has been included for the sole purpose of providing a better contextualisation of the constructions of the scales and the comparisons contained in the research, as it has been considered beyond the scope of this dissertation to go into an in-depth review of concepts previously published in other places and well known by scholars, which are, additionally, tangential to the main topic.

> Music and mathematics having been happily entwined from antiquity, it is not surprising when talent in one accompanies enthusiasm for the other (Putz, 1995, p. 276).

The linkage between numbers and music has aroused interest throughout history, from the earliest times to our days (Assayag, Feichtinger & Rodrigues, 2002; Fauvel, Flood, & Wilson, 2003; Wright, 2009); Leibniz believed, that music was a hidden exercise of arithmetic; and Diderot's ─the Enlightenment contributor to the French Encyclopaedia─ thought: “it is through the numbers and not by the senses that the sublime character of the music is appraised” (Assayag et al., 2002, p. v).

The overview included here has been restricted to a short chronological digest to illustrate that it is not the first time that musicology has relied on numbers for a study, followed by the description of the impurity measurement methods[17] to be found

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[16] This exclusion does not signify, by any means, that those topics are not of scholarly interest. The reason for keeping them out of scope here is that this research moves in the arena of the musical composition, as a doctoral dissertation framed in a department of art and communication, and it is the musical facet, not the number theory, the central core of the investigation.

[17] Although the term has been used with various meanings, impurities here refers to the interval differences between a specific temperament and another (e.g. just intonation) taken as reference: «Temperament in music

later in the text, and ending with a comment about the set theory PC (Pitch Class) that deals with diatonic sets that use discrete mathematical analyses for diatonic, pentatonic and other scales called in this methodology collections.[18] It provides feedback for the later comments on *The Adrán Doors* a serial composition sample included in the study.

Chronological synopsis of mathematics and music. Limiting the comments basically only to EuCul, the scientific production of Nicomachus of Gerasa, the Syrian Greek ─considered by authors such as Porphyry of Tyre or Isidore [the Bishop of Seville] "on par with the Master himself, Pythagoras of Samos" (Levin, 1994, p. 14)[19] with titles such as *Introduction to Arithmetic, Art of Arithmetic, Manual of Harmonics*─ allows us to realize how science joined mathematics and music, as part of a philosophy of universal harmony, in those earlier times.[20]

Music was, in Ancient Greece, a mix of art, science, and philosophy:

> The modern Western concept of 'music' differs from the ancient Greek concept of *mousikē*. For the Greeks, music was both an art and a subject of scientific and philosophical inquiry. It could provide relaxation and entertainment as well as

means the slight adjustment of acoustically pure (or 'just') intervals so that the impurity is spread over an entire scale. It is based on the fact that twelve consecutive fifths (each 3:2) exceed seven consecutive octaves by a small interval» (Sorgne. & Fürbeth, 2010 p. 6). The interval called the Pythagorean comma is not «so small,» since it is about 23.46 cents; These aspects will be commented on later in this chapter.

[18] A collection formed by constant intervals in integer notation is called a generated collection. A single generic interval with moment of symmetry (MOS) has a property called well-formed generated collection. Other properties may be found such as maximal evenness, Myhill's property, deep scale, cardinality events variety and multiplicity. Although in a way linked to Wilson's MOS concept being used frequently in microtonal theory, diatonic set theory details fall out of the scope of this dissertation. More information may be found in the works by Eytan Agmon, Gerald Balzano, Norman Carey, David Clampitt, John Clough, Jay Rahn, Jack Douthett, David Rothenberg, and Erv Wilson, among others. Michiel Schuijer (2008) for Set class analysis and Timothy A. Johnson (2008) for the Diatonic Set theory will expand these points and provide those and other references.
In the set theory PC (Pitch Class) «C» (e.g.) includes all the «C» pitches, and the enharmonic equivalents i.e. B-sharp, and D-double flat. C is a pitch class, but not a pitch unless the octave is indicated, and the exact frequency could be determined. Thus, C is a pitch-class (PC), not a pitch, but C_4 is a pitch.

[19] The reference is not original. It may be found in the translation by M.L. D'Oodge of *Nicomachus of Gerasa* with Studies in Greek Arithmetic by F.E. Robbins and L.C. Karpinski published in New York by Johnson Reprint Corp in 1972 as a reprint of the 1926 edition (p. 78).

[20] However, the link of universal harmony and music is not exclusive to European philosophy. In Asia, the topic is illustrated in the Liji's *Collection of Rituals:* «the original text of which is said to have been compiled by the ancient sage Confucius (551–479 BC)» (The Editors of the Encyclopaedia Britannica, 2013, para. 1); from the *Book of Ritual*: «Music comes from yang, rituals come from yin; when yin and yang harmonize, myriad things are fulfilled (Liji 11:5 [Liji 111]). Music and poetry are the two essential components of ritual acts» (Yeo, 2008, p. 227).
As for Europe, references may be found in Carrozzo, & Cimagalli (2008) and Garland, & Kahn (1995), among others: «From Pythagoras through Plato (*Timaeus*), Augustine (*De musica*), Boethius (*De institutione musica*), William of Conches (*Glosae super Platonem*), Vincent of Beauvais (*Speculum naturale*), to Dufay's colleague Leon Baptista Alberti (*De re aedificatoria libri decem*) came the doctrine that man and his creations need reflect a celestial and universally valid harmony, one that was expressed through numerical ratios» (Wright, 1994 p. 404). «[…] a particular mode of thought embedded in both Ancient and Medieval music theory: a mathematical bent stemming from the idea that Music, based on proportional relationships which embody Number, is an audible symbol of a God-given ontological order» (Ferreira, 2002, p. 1).

> playing a central role in civic and religious life (Mathiesen, Conomos, Leotsakos, Chianis, & Brandl, 2014, Greece, Section: Greece-Ancient, para. 3).

The Greeks laid the theory of the music of the spheres (Godwin, 1993; James, 1995), settling different pitches for each planet,[21] as explained by Nicomachus: "to orbit the earth in their allotted *epochai* calculating from the most distant one ─Kronos─ to the nearest─ the Moon" (Levin, 1994, p. 47).[22]

> It is probable that the names of the notes were derived from the seven stars which traverse the heavens and travel around the earth. For they say all swiftly whirling bodies necessarily produce sounds when something gives way to them and is very easily vibrated (Levin, 1994, p. 45).

Ancient Greek theorists linked numbers, music and the cosmos.

One of Ancient Greece's best-known musical instruments was the four stringed lyra o cithara; a paradigmatic arrangement was the *tetrachordum* (τετράχορδον) or set of four notes or strings, with the two extremes maintaining a relation of perfect fourth.[23]

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[21] Kronos (hypate, E), Zeus (parhypate, F), Ares (lichanos or hypermese, G), Sun (mese, A), Hermes (paramese or trite, B♭), Aphordite (paranete, C) and Moon (nete, D).

[22] The Pythagorean theory of music of the spheres has been reviewed by many other authors (e.g., Plato, Aristotle, Ptolemy, Boethius, Zarlino, Morley, Fludd, Werckmeister, Rameau, Newton, Kircher, Titius, Bode, Haase, Hindemith, etc.), with positions in favour or against. Presumably, the most popular work in this regard is the *Mysterium Cosmographicum* by Johannes Kepler (Tübingen,1596 [Dickreiter, 1973]). He saw in the cosmological model the order, wisdom, and grace of God.
From the publication of *Philosophiae Naturalis Principia Mathematica* in 1687, showing Newton's three laws of motion and its consequent application to the planetary movement (Karam, & Stein, 2009, pp. 49–69), interest was lost in the theory of the music of the spheres. Contributing to this, the development of the acoustics and the finding that sound requires a medium for its transmission, something that does not seem to exist in space (Rossing, 2014).
Although everything seemed to be set, several space probes have detected what seems to be areas with particles of cosmic plasma in which electrons are found to change their movements due to ionization, generating sound waves (Eisenstein, & Bennett, 2008; Peratt, 2015). On this theoretical basis, and after some other space recorded sounds, an active sound space exploration has been maintained and probes have been provided with microphones, from Voyager 1, to the future Mars 2020 project (National Aeronautics and Space Administration, 2018). Thus, the sounds of space seem again to be a trending topic, and we must not forget that Kepler pointed out that it was an inner harmony, not something audible, that «inner harmony governing the human soul» (Cohen, 1984, p. 115).
Recently, in a new twist, the scientists seem to be turning back to cosmic music, linking the theory of strings with the sound (Schomerus, 2017; Susskind, 2006), or in words of the cofounder of the string theory «We are nothing but cosmic music played out on vibrating strings» (Kaku, ca. 2016). It is now possible to calculate solutions for the sophisticated equations that govern the movement of a string, its interaction with the medium of propagation and how «the impedance determines the waves on the string» (Kim, 2010, p. 14).
Ongoing research by Cartwright and co-workers suggests that the same resonances of three frequencies responsible for the pitch have been observed in the movements of planetary bodies (D.L. González, personal communication, September 25, 2018).

[23] There were four tetrachords, two lower ones (with a bass range) separated by a disjunction of a tone, from the two upper ones (treble range). There was also an optional additional tetrachord with a modulating character (Redondo-Reyes, 2006, p. 165). «Tetrachord 1 is the hypaton; 2, the meson; 3, the diezeugmenon; 4 the hyperbolairon; and 5 the synemmenon […]. The tetrachord was regarded by Aristoxenus as the basic musical unit» (Mathiesen, et al, 2014, Section: Aristoxelian tradition, para. 2).

In addition to providing names for the pitches ─as per Nicomachus─ Pythagoras was the first to add an eighth string to the lyre (Levin, 1994, p. 73).

> Pythagoras added an eight string in order to produce the overall range of an octave. He did not add this new string, however at the top, as might have been expected, but rather between the old mese and the paramese (Mathiesen, 1999, p. 214).

This gave the possibility of reaching an octave or even more, with the derived division of the scale into tones and semitones:

> […] the extremes themselves producing with one another the most satisfying consonance, that is the octave in a double ratio, which could not result from the two tetrachords ─intercalated an eight note, which he fitted between mese and paramese and separated it from mese by a whole-tone and from paramese by a semi-tone (Levin, 1994, p. 73).

The Pythagorean innovation was not without risks. The seven-stringed lyre or heptachord was based on a deeply rooted tradition of seven as a sacred number: the seven planets, the sages, the days of the week, the gates of Thebes, or the vowels of the Greek language (Levin, 1994, p. 74).

According to Levin (1975) although tradition assigns to Pythagoras the mathematical determination of the intervals of octave, fifth and fourth, it is only after Nicomachus that this question is recorded (pp. 68–69).[24]

> While this literature is commonly known to modern scholarship as 'ancient Greek music theory', the phrase is a misnomer. […] the earliest surviving independent theoretical works are Aristoxenus's Harmonic Elements and Rhythmic Elements, both of which are fragmentary (Mathiesen et al., 2014, Greece, Section: Music Theory, para. 1).
>
> Ostensibly historical treatments (by such authors as Alexander, Aristoxenus, Glaucus and Heraclides Ponticus) are cited and excerpted or paraphrased in Pseudo-Plutarch's On Music (Peri mousikēs), but the early treatments themselves do not survive (Mathiesen et al., 2014, Greece. Section: Music Live in Ancient Greece, para. 1).

Per Redondo-Reyes (2006) the names of the notes come from the position of the strings in the Greek lyre. It is basically a system of spatial designation, where each name indicates the location of the string in the instrument (pp. 264–265). Thus, (for a diatonic scale): hypate (E) means the string in the highest position; parhypate (F) means the string next to hypate; lichanus [hypermese] G is the forefinger; mese (A) is middle (referring to the finger); trite [paramese] B♭ refers to the third string; and paranete (C), next to the lowest; with nete (D) being the lowest. In order they are: hypate (E)–parahypate (F)–lichanus (G)–mese (A)–trite (paramese, B♭)–paranete (C)–nete (D) (Music of Everyday, 2018, para. 12).
High refers to the string position only, as the frequencies are just the opposite to their names (Sach, 2008, p. 69). Proslambanomenos or «later addition» was the lowest note, whole tone below the hypate-hypaton, added to the tetrachord to complete the octave; it was not considered a part of any tetrachord (Mathiesen, et al, 2014).

[24] Puerto Rican professor Alberto A. Martinez of the University of Texas at Austin goes even further, and after analysing historical sources, concluded that there is no evidence for Pythagoras being the original creator of the mathematical theories attributed to him, although he must acknowledge that his disciples did develop these theories (Martínez, 2012, pp. vii & 17).

Following Pont (2004), “most of the doctrines traditionally ascribed to Pythagoras were really contributions of the older high civilisations, particularly of Mespotamia [*sic*] and Egypt” (p. 17).

The Pythagoreans not only developed the principles of musical consonance, the relation of pitch to the length of the string, and the purpose, expressed in the second chapter of the *Manual of Nicomachus* “to define the essential property of a musical note─ music’s smallest indivisible element” (Levin, 1994, pp. 47 & 173); but also another less notorious but very important finding, the two species of the human voice:

> The genus under which the two species of the human voice are subsumed is motion (*kinesis*), the continuous motion being that of the speaking voice, the interval being that of the singing voice […] the motion in question is a process of change that is directed towards a verbal or, as the case may be, a melodic end. And as any observant Aristotelian would maintain, there is no mention of verbal or melodic ends in mathematics (Levin, 1994, p. 39).

Thus, the germ of concepts such as music melody and music interval seems to be in old Greek music theorists already present.

Also interesting is the early inclusion of rhythm:

> Music in this sense of a performing art was called *melos*. A distinction was made between *melos* in general, which might be no more than an instrumental piece or a simple song, and perfect *melos* […] which consisted not only of the melody and the text (including its inherent elements of rhythm and diction) but also highly stylized dance movement. Melic composition (*melopoiïa*) together with rhythmic composition (*rhuthmopoiïa*) was the process of selecting and applying the various components of *melos* and rhythm to create a complete composition (Mathiesen et al., 2014, Greece. Section: Music Life in Ancient Greece, para. 5).

The different feelings (*ethos*) or moods related to the different modes were known in Ancient Greece:

> […] each scale had a particular feel, a particular mood to it. Aristotle believed that the Mixolydian scale was mournful and solemn but that the Phrygian was too emotional, putting men “into a frenzy of excitement” (Dixon, 2013, Chapter Athens: The Scales of Pythagoras, Secc. 4, para. 6–7).

The relationship of mathematical concepts ─such as size, length, sequence or permutation─ to sound has been recognized since ancient times.

As per William P. Malm (2017), some of these notions were already present in Chinese tradition as far back as the third millennium BCE:

> Chinese writings claim that in 2697 BCE the emperor Huangdi sent a scholar, Ling Lun, to the western mountain area to cut bamboo pipes that could emit sound matching the call of the fenghuang, an immortal bird whose rare appearance signaled harmony in the reign of a new emperor (para. 3).

In addition to calculating the sizes of the strings, flute pipes, or bells, musicians found empirical solutions to mathematical problems that could only be formulated by scholars many years later:

> Change-ringers wish to ring bells in different orders, with no bell moving more than one place in successive rows. The mathematical problem is to devise ways of ringing all possible orders (for example, all 5040 permutations of seven bells) without repetition. English bell-ringers solved this problem more than two hundred years ago; about a hundred years later mathematicians began developing the concepts and terminology to tell the ringers that they had been doing 'group theory' and 'ringing the cosets' all along (Roaf, & White, 203, p. 113).

Moving forward in history to medieval teachings ─leaving without further comment many contributors to mathematical concepts of music and cosmology of the antiquity and post-antiquity periods, such as Plato, Philolaus, Archytas, Aristotle, Aristarchus of Samos, Ptolemy, Aristoxenus of Tarentum, Cleonides, Gaudentius, Boethius, etc.,─ the consideration of music as a mathematical science was still present in the Quadrivium: "a curriculum of the four mathematical sciences: arithmetic, geometry, astronomy and music" (Levin, 1994, p. 17).

A turning point between music and mathematics in the Middle Ages was reached around 800 (CE) when the pneumatic notation was invented for Roman liturgy (Ferreira, 2002, p. 13). About one hundred years later, "probably the first extant reference to polyphony occurs in *De Harmonica Institutione,* a treatise written by the monk, theorist, and composer Hucbald" (Van der Merwe, 2004, p. 57).[25]

During the Renaissance there was renewed interest for linking music to numbers, symbology, cosmology, architecture and universal harmony (Gangwere, 2004).

> The various attempts to align architecture with music and music with cosmology during the Renaissance period indicate that many writers did assume a universal correspondence. Proportion was the thread used to stitch everything together. It was therefore proportion that would provide the bonding for a coherent universe, as well as the link between music and architecture […] But there is one quite effective way to change our understanding of the relation between Renaissance music and architecture; instead of comparing architecture to musical theory, compare it to musical practice (Ebans, 1995, p. 243).

The composition of Dufay *Nuper Rosarum Flores* ─for the consecration in 1436 of Santa Maria del Fiore, Florence's cathedral ─ has been analysed as a work full of numerical symbolic elements, linking the basilica with the temple of Jerusalem, the

[25] His pioneer primitive tablature-like method will be commented on in the scales section. These neumes, although heralding a future musical language, were not indeed «numerical,» since initially they did not include precise information about the pitch or the metrics, as also expanded later in that section.

golden reference for Christian churches (Trachtenberg, 2001; Warren, 1973; Wright, 1994).

The allegory of the seven is continuously present in *Nuper Rosarum Flores*: seven were the pillars in Jerusalem temple and seven results from four plus three, representing the union of the four ─a number traditionally linked to the land (mother, Mary)─ with the three (the Trinity), that is with God (Trachtenberg, 2001, p. 742).

“Dufay's motet is a spiritual vehicle with a symbolic message. Its theme is the divine unity of the Temple and the Virgin Mary” (Wright, 1994, p. 406).

Other Renaissance music compositions have also used numbers in their creations, evidenced e.g., in the piece *Sacrae Cantiones* of Franco-Flemish composer Orlande de Lassus (ca. 1531 – 1594) (Lessoil-Daelman, 1995, p. 47).

Still in the Renaissance period, additional links of numbers with music may be found in the works of professor Bartolomé Ramos de Pareja from Salamanca university, and in the Italian theorist Franchinus Gaffurius (Franchino Gaffurio).

> Ramis [*sic*] [26] compares the tactus to the interval between the diastole and the systole and equates it with the breve […] Gaffurio compares the tactus to the complete pulse, which he equates with the semibreve and conceive it as equally divided (DeFord, 2015, pp. 207–208).

Ramos de Pareja sets measure around 75 beats per minute, which is about the average human heart rate at rest.

The beat sequence of the music has also been postulated as an organic pulse with a relaxation after the effort:

> And if we can breathe life into these irreducibly primitive patterns, could we not do the same for more complex ones? Can chords, modes, or keys behave as if alive? On this question, musical theorists have long been sharply divided (Van der Merwe, 2004, p. 14).

With the music fluctuations compared to breath:

> As for the agogic inflections, it must be emphasized that no music should be played with constant adherence to a rigid tempo. The flow of Baroque music, like that of the music of other eras, is subject to slight fluctuations, reflection the breath of the live organism rather than a machine-like repetition of the metronome’s beat (Kochevitsky, 1996, p. 46).

Or as per Johnson (2002):

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[26] The incorrect spelling of the name of this author seems to be widespread in the English literature: The English version of the only remaining book (in Latin) *Musica Practica* has been published by the American Institute of Musicology in 1993 under the title *Ramis de Pareia, Musica Practica: Commentary and Translation* (Haar, & Corneilson, 1998, p. 87). The authors use the same wrong spelling in their book as did Ruth DeFord above.

> We understand ourselves as particular, physical beings, but we also value the ways we exceed the physical, the ways our capacity for thought, feeling and imagination seem to transcend our bodily existence. Music-as-art performs a similar alchemy: in projecting a content beyond its acoustic materials, it does not deny its physical aspects but redeems it as the vehicle of something that exceeds the physical. (p. 130).

The Renaissance saw a renewed interest in what is probably the main and the best-known mathematical problem of music, the problem of the fifths. An integer number of fifths never matches with any other whole number of octaves (Loy, 2006), or to express it mathematically, $2^x = 3^y$ has no solution for integer numbers.[27] Since this problem is a key element in the development of the scales, it will have its own section later in this chapter.

Following Scimemi:

> The impossibility of obtaining an equal scale via arithmetic was well known in times past. The theoretical difficulties can clearly be seen, for example, in musical-mathematical treatises of the Renaissance, in which approximate arithmetic alternates with geometrical-mechanical approaches. The best known of these books is perhaps Gioselfo Zarlino's *Le Institutioni hamonicae* […] Zarlino was not out to solve equality problems. [...] Theoreticians' dissatisfaction have [*sic*] led them throughout the centuries to suggest other diapason subdivisions, and even very complicated ones (Scimemi, 2002, p. 51).

Of notable interest is the achievement ─before the logarithms and the irrational numbers theory were established using mechanical devices, and geometrical constructions─ of producing "acceptable approximations for the sequence of frequencies for the musical scales. This is exemplified in the Renaissance treatise by Zarlino" (Assayag et al., 2002, p. vii).

The concept of a mathematical foundation of music was actively maintained during the Baroque period. Mathematics and music remained in this epoch closely related to a divine ordination: "[…] composers of that period cited again and again the biblical verse that God had ordered ('disposuisti'), the whole world through measure, number and weight [Wisdom, 11, 20]" (Knoblock, 2002, p. 27).

The order (and symmetry) in Bach compositions has been analysed by several authors (Laser, 2015; Tatlow, 1991; Wacyk, 2014).

Classical period composers also remained greatly interested in numbers.

The "Musikalisches Würfelspiel" [musical dice game] was used by several authors of the time such as Kirnberger, Carl Philipp Emanuel Bach, or Mozart; the

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[27]A solution will require for the number to be odd and even at the same time. This mathematical expression is derived from the ratios of fifth 3/2 and octave 2/1, starting from $(3/2)^x$ and $(2/1)^z \rightarrow 3^x \neq 2^{x+z} \rightarrow 3^x \neq 2^y$.

combinations sequence of precomposed parts of music piece lies on the result of rolling dice (Gerhard, 2009, pp. 36–38). The section dedicated to the GR includes more data about Mozart's (and other authors of the period) fascination for numbers.

Prior to and during the age of Enlightenment additional thoughts were added to the purely mathematical cogitations in music, such as the paradigm regarding the relationship of harmony and melody, with considerations of the harmony as the skeleton and the melody as the filling, something that could be a sort of Kuhnian paradigm, so deeply established that one tends to forget its existence:

> Conventional Western theory has grown so used to interpreting melody in terms of harmony than the very idea of a purely melodic construction has become strange to it. Every mode takes its bearings from harmony; so does every melodic sequence and cadence, so, too, do dissonance, key, and modulation. Yet melody existed for thousands of years without the benefit of harmony, and still manages without it in many parts of the world today. Can it be really relegated to so dependent a status? (Van der Merwe, 2004, p. 20).

As expected in the Age of Reason, the ambiguity was reduced to the minimum and the harmony and melody were ordered and regulated, but still with a cosmic focus:

> Melody, being a part of harmony, was regulated by the same orderly hierarchic system, revolving ultimately round the tonic triad. The resemblance to Newton's clockwork universe was not entirely coincidental (Van der Merwe, 2004, p. 20).

The fascination for numbers did not diminish with subsequent Romantic composers. Per Bauer, & Kerékfy (2018):

> Nearly all the composers we saw here are or were at some time fascinated by numbers, machines, formulae, rational constructions. The inclusion of methods and concepts from mathematics and physics into the structuring of music corresponds to the nineteenth-century Romantic musician's tie to literary, philosophical and descriptive models. In this sense, this whole numeristic view of music can be seen as a continuation of the so-called heteronomy aesthetics of the Romantic period (pp. 35–36).

After Common-practice era, a wide range of mathematical approaches have been applied to music, including Markov chains (Jones, 1981); just to mention as an example, based only on Zipf's laws, over 40 metrics have been tested in music (Manaris et al., 2005). The mathematics have been used to explain the musical process involved in atonality, including the use of a Cartesian coordinate system and the space-p or space of pitches represented by means of integers, positive and negative.[28]

[28] Vazquez talks about «space-a» for *espacio-alturas* in Spanish, which literally means space-heights, but in the context it becomes clear that it refers to **p**itch notation, as explained in the author's paraphrase of his text: now we can approach musical operations as functions. In the space-**p** we will use the traditional numerical notation […]. However, we must bear in mind that everything that is said in relation to traditional notation

This brief chronological review confirms that, although art is not friend of constrictions, and less of numerical constraints, the use of mathematics in music cannot be considered in any way as something alien; the frequencies of the pitches and precise times that musical execution requires confirm the constant presence of the numbers in daily musical practice.

Analysis of consonance. The consonance of intervals may be studied by means of mathematics, measuring their deviations, or impurities. Mathematics also proved useful for tuning (Hall, 1973).[29]

In the analyses of intervals, the statistic function (ϑ) Root Mean Square Deviation (RMSD) between frequencies of the scales under study ─ or in other words, the impurities─ will be frequently used in the dissertation.

The deviations may be multiplied by a weight ─weighted RMSD (ϑ_w)─ related to the interval, or not. The idea of a weighted value is based on the fact that a certain amount of impurity in an already dissonant interval (e.g., a minor seventh) has not the same importance as the same impurity in a perfect consonance (e.g., the octave) (Marco-Franco, 2018).

The RMSD is computed as the square root of the arithmetic mean of the squares of a series of values ($x \ldots x_n$); in this case, a series of impurities:

$$RMSD = \sqrt{1/n\,(x_1^2 + x_2^2 + \cdots x_n^2)}$$

Even in waveform combinations the formula may be adapted for each wave RMSD, resulting:

$$RMSD(Tn) = \sqrt{1/n\,(RMSD_1^2 + RMSD_2^2 + \cdots RMSD_n^2)}$$

And for the case of a 12-tone scale (C–B):

$$RMSD\ (\vartheta\ ) = \sqrt{1/12\left(ImpC^2\ + ImpC\#^2\ + \cdots ImpB^2\ \right)} = \sqrt{\frac{\sum Imp_{C-B}^2}{12}}$$

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will also be valid for the new notation, since both are isomorphic (both notations handle corresponding operations that express the same properties of space-p) (Vázquez 2006, p. 446).

[29] An acoustic beat is a perceived difference between two sounds that are close but with slightly different frequencies. When the frequency difference is about twelve hertz or less, the perception is of a periodic variation in the amplitude,which may be determined mathematically. Interference beats may be used, to check tuning, or for analysing consonant intervals. This is possible as some tuners have the possibility to compute beats.

A second procedure is to compute the impurity in cents; for example, the impurity of a fifth interval (*fi*) in a scale as compared with the just perfect fifth (3/2) may be determined by:

$$\text{cents}=1200 \bullet \log_2([\text{ratio for } fi]/[3/2])$$

The use of cents is familiar to the musicians that have well-known references in the tone and semitone.

The third procedure is to use the Percentage of Beat Pitch (PBP). For the same example, the difference with a just perfect fifth (3/2), may be determined [with zero as optimal value] using the ratio of the interval (*fi*) with the formula:

$$\text{PBP}= 100 \bullet ((2 \bullet [\text{ratio for } fi])-3)$$

As mentioned, when talking about the JNC, the psychoacoustical ability to discriminate between two sounds depends on many factors, not only from the observer's side, but also depending on the place, environmental conditions, frequency, intensity, duration, type of wave, and other circumstances. Zwicker et al. (1957) studied this aspect graphically.

These methodologies may be used for one interval only, or for a full-scale temperament inharmonicity (all notes included) by summing overall the interval difference values.

The consistency of consonant schemas may also be expressed graphically, with arrows representing pure intervals (Broekaert, 2018).[30]

Pitch-class set methodology. The set theory, particularly the branch dealing with diatonic sets of pitches ─(pitch classes set or PC-set) with module 12 analysis─ is a tool used frequently for the study of post-tonal music (Cohn, 1998; Fichet, 2002; Harrison, 1994; Morris, 1991; Schuijer, 2008; Straus, 2005).[31]

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[30] With appreciation to the author for the complete material provided.

[31] Per Richard Cohn (1998), the so called *traditional* (by Cook, 1987) and Schenkerian methods of analysis are suitable for the period of Common-practice while the Neo-Riemannian theory remains particularly useful for the more chromatic music of the late 19th century and the turn of the 20th: «The most prominent transformations of atonal pitch-class theory, transposition and inversion, are fundamental to nineteenth-century theory» (p. 176).

An extended description of the Pitch-class set method of analysis is out of reach here, but it has been considered convenient to include a comment restricted only to setting some definitions that will appear later in the text. This point could also have been included in Chapter Five of material and methods, but since it is not specific to this research, nor are the definitions related to its main topic, and since they are only used in the comments of the musical creation Adrán Doors included in this dissertation as an example of the use of the 34-TET in serial music, it has been found appropriate to include here a brief reference to them as part of the state of the art:

Normal form: the most compact shape with the smallest possible interval on the left.

Finally, it is interesting to see how the relations of cosmos and mathematics established so far back in time are still considered as abstraction levels for current computational mathematics:

Physical Universe→ Mathematical Universe→ Representation Universe→ Implementation Universe (Gomez, & Velho, 2015, p. 2).

To summarize these sections dedicated to mathematics and physics applied to music, this area of knowledge cannot be in any way considered alien to music as it may be traced through the ages until the more recent development of digital instruments for tuning (Blackwood, 1985; Neuwirth, 2002), and provides useful means for scale constructions and comparisons, impurity measurements, and musical analyses.

Music perception. Very interesting, as field for futures studies, is the consideration of whether the perception of music is due to the physically and mathematically related wave characteristics, or else originated in the ear of the listener, and how perception can change with the digital modification of the wave.

The ear hypothesis ─going back to the Aristoxenus rejection of Pythagorean-Platonic tradition, (Ferreira, 2002, p. 10; Risset, 2002, p. 216), with the more recent supporting hypothesis based on the anatomo-physiological properties of the ear's basilar membrane processing the signal (Tymoczko, 2011, pp. 61–62)─ links the psychology of the music with their related psychoacoustic theories (Sethares, 2005), and the expectation (Bissell, 1921).

For physicists and engineers, sound is basically a measurable pressure wave ─with attributes (frequency, intensity)─ propagated through a medium, but there are also the corresponding psycho-acoustic factors (pitch, loudness).

A psychologist will refer to the sound as "the perception that occurs inside the ear, something that is notoriously hard to quantify" (Sethares, 2005, p. 11):

> Similarly, complex sound waves can be decomposed into a family of simple sine waves, each of which is characterized by its frequency, amplitude and phase. [...] This idea of breaking up a complex sound into its sinusoidal elements is important because the ear functions as a kind of "biological" spectrum analyzer (Sethares, 2005, pp. 13–15).

Inverted form (I): difference to 12 (module 12).
Transposed form (Tn): subtracting or adding a value equal to all elements of the PC set.
Main form: selecting from a set and its inversion, both in normal form, the most compact one on the left and transpose it so that its first element is 0.

In that perception, the pitch seems to be the fundamental element, and the pitch of a complex sound is basically a nonlinear response of our ear that does not act as a Fourier analyser simply decomposing a superposition of stimuli.

This is a key aspect, justifying this section, with the sound linked to nonlinearity. In other words, the sum of two Fourier components does not produce the same effect as the sum of the effect of each independently which means that it must be considered as a nonlinear system.[32]

Once the musical perception has been introduced ─ with the main purpose of justifying the inclusion of nonlinearity as a relevant aspect of this project─ moving further away from that point would also require some comments on perception in relation to other issues and considerations ─the understanding of music, music literacy, and allied issues (such as perception and the social group [elite]) (Johnson, 2002, pp. 72 & 83)─ no doubt of great interest in other settings, but beyond the scope of the main purpose of this research, focused on the development of the 34-tone scale.

This work has two starting hypotheses: first, that consonance is associated with pleasantness in art music, and second, the 34-TET scale is capable of including both, the consonant intervals and the golden ratio.

For this reason, the concept of consonance, widely used in the research, deserves some further comments. The next section of the chapter is dedicated to it.

Harmony, consonance & pleasantness. In many English dictionaries (Cambridge, Collins, Merriam-Webster, Oxford…) the entry "harmony" includes concepts such as a simultaneous combination of pleasant musical notes, in contrast to "melody" which involves a succession or progression of music sounds.

However, such a concept of harmony seems far from being clear, since it implies two debatable constituents: music, and pleasantness.

[32] The impressive Fourier analyser (about one-meter length) was developed for simultaneously observing the components of a sound; it includes fourteen adjustable Helmholtz resonators. Helmholtz postulated that the ear could act as a Fourier analyser (Palmieri, 2012, pp. 224–225), that is linearly, but the essentially nonlinear character of the ear is demonstrated by the perception of the pitch of complex sounds (resonances of three frequencies); otoacoustic emissions (self-oscillations); and the compressive response of the cochlea (Hopf bifurcation) (Camalet, Duke, Jülicher, & Prost, 2000; Cartwright et al., 1999, 2001; Robles, & Ruggero, 2001).

The issue becomes worse, not only because of the changes made down through the epochs, or for the differences between distant cultures; it seems that even within Europe, German and English music theorists have differences:

> The German names for the triads on the so-called, primary or tonal degrees of the major or minor scale (I, IV, V) are familiar to English-speaking musicians: "tonic," "subdominant," and "dominant." But the names for the so-called secondary degrees may strike the English reader as foreign in both sound and concept (Gjerdingen, 1990, p. xi).

In any case, the question of pleasantness must be faced interculturally:

> Simultaneously sounded tones that produce beating or roughness typically sound unpleasant to Western listeners, but they evoke positive or neutral reactions in a number of other cultures. For example, Balinese bronze xylophones are deliberately mistuned (e.g., stretched octaves), to produce beating… folk singers in rural Croatia commonly sing duets in parallel seconds […] (Trehub, 2016, p. 390).

Consonance and dissonance have always intrigued theoreticians. Dissonance comes from the Latin *dissonantĭa*, meaning two sounds.

The harmonic sounds have been studied since Ancient Greek times over the ages.

> The understanding of consonance and dissonance as they appear in Western music has been beset by such notorious difficulties and entanglements as have proven the despair of music theorists. Many have abandoned hope of ever explaining to general satisfaction why musical harmony sounds harmonious (Cazden, 1980, p. 123).

Demetrio Santos-Santos (1999) has provided a commented translation into Spanish of *Harmonics*, the classical work by Ptolemy. According to this source, the consonances perceived by the ear are called diatessaron (4/3), diapente (3/2) (whose increment is called tone = 9/8)[33] and the diapason (2/1). Also included are the diapason-diatessaron (8/3), the diapason-diapente (3/1) and the disdiapason (4/1) (Ptolomeo, 1999, p. 27).[34]

> The ancient writers attach great importance to the distinction between 'concordant' and 'discordant' intervals (symphōna, diaphōna). […] The intervals which they regard as concordant are the fourth, the fifth and the octave (or larger intervals compounded from these, octave + fourth, etc.). All other intervals were classed as discordant (West, 1994, p. 160).

[33] It is unclear whether this is an added comment or is found in the original text, but, in any case, it seems wrong, at least for an integer value. The impossibility of such increment comes by the fact that the proportion between 3/2 =1.5, and 9/8= 1.125 is not a whole number but 4/3: (1.5:1.125 = 1.3333…).

[34] These consonance divisions are easily identifiable with current interval names: *diatessaron* (4/3, is now called perfect fourth), *diapente* (3/2, is now called perfect fifth) *diapason* (just octave, 2/1) or *tone* (9/8 is now still called a tone or a major second), etc.

Pythagoreans discovered that the divisions based on a superparticular ratio originated consonant or pleasant sounds (Ferreira, 2002, p. 3; García-Pérez, 2006, p. 16).[35]

> […] the Pythagorean consonances are thus expressed by two types of ratio: multiple xn/n─ from which come two further consonances besides the octave: octave-plus-fifth (3/1, C-g) and double-octave (4/1, C-c_1)─ and superparticular, n+1/n ─ *i.e.* fifth (3/2) and fourth (4/3), the only consonances admitted by the Greeks within the octave. It follows that if an interval is consonant, then it is expressed by a multiple or superparticular ratio (Gozza, 2000, p. 4).

The definition of consonance has not remained unaltered over the ages (Cazden, 1972, 1980; Cohen, 1971; Figaredo, 2010, Grant, 2013): "Sixteenth-century chromaticism is considered a radical alteration of the tonal system, an alteration that marks, or even brought about, the transition from the modal system to major-minor tonality" (Dahlhaus, 1990, p. 163).

> By 1400, theorists generally classified consonances into two types: perfect consonances (unisons, octaves, fifths, and their compounds) and imperfect consonances (thirds, sixths, and their compounds). Seconds, fourths, sevenths, augmented/diminished intervals and their compounds were classified as dissonant (Apel, 2003, p. 209).

As per Dahlhaus (1990) "In the 15th and 16th centuries, dissonance was conceived not in contrast and opposition to consonance, but as a scarcely noticeable interruption in the sequence of consonances" (p. 123).

The idea of a duality, and the "rejection" of one sound by the other, was supported even by Rousseau himself (Masters, & Kelly, 1998, p. 385).

Interval association and affinity are frequently appearing concepts in Dahlhaus' papers analysing the concept of tonality: "According to Riemann, tonality is the embodiment of chordal meanings, and chordal meanings ─subdominant, dominant, subdominant parallel, and dominant parallel─ are based on 'affinities between tones' [Tonverwandstshaften]" (Dahlhaus, 1990, p. 8).

François Joseph Fétis (1784 – 1871), proposes a different theory to that of Riemann to understand the *modern* tonality [from the seventeenth to the nineteenth century]: the contrast between the triad and the seventh chord, "between the consonant

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[35] This ratio between two consecutive integer numbers (n+1/n) is also called epimoric. It has a property: «A superparticular ratio cannot be split exactly in half by a number proportionally interposed» (Ferreira, 2002, p. 8). The author takes the concept from Archytas in the version transmitted by Boethius. Detailed information about the mathematical work of Archytas is found in Huffman, C. (2005). *Archytas of Tarentum: Pythagoeran, Philosopher and Mathematician King*. Published by Cambridge Univ. Press. [The reference about the impossibility to split a superparticular number exactly in half is found on page 419].

harmony called *accord parfait*, which has the quality of rest and conclusion, and the dissonant harmony, which causes tendency, attraction and movement" (Dahlhaus, 1990, p. 9).

Although both theories seem opposed, Dahlhaus proposes a reconciliation:

> Riemann's thesis that only the tonic, dominant and subdominant are "consonances" while the tonic parallel, dominant parallel, and subdominant parallel are "dissonances" seems less strange if following Fétis, one interprets "consonance" as repos and "dissonance as *tendance* (Dahlhaus, 1990, p. 9).

In the end, it seems that tonality requires the presence of a certain interval association ─either for affinity or for contrast─ and a centre ─tonal─ (Dahlhaus, 1990, p. 18). "Even more than harmony itself, the complex interaction of dissonance and discord is the great glory of classical Western music" (van der Merwe, 2004, p. 115). According to this, one tone by itself is just a factor and becomes a musical event only by association:

> A tone taken by itself is an acoustical datum, not a musical phenomenon. It becomes a musical phenomenon only in association with others. A tonal system as a material scale, if it is to pass for musical reality, is thus unthinkable without a tonal system as a complex of tonal relationships (Dahlhaus, 1990, p. 162).

What is understandable is the interest for analysing the intervals and pitches conforming the chords, studying what kind of liaisons they may have; also understandable is the long-lasting debate on natural versus acquired harmony, or, in other words, if the result is a "fact of the nature and not merely the result of a 'convention' [Setzung]" (Dahlhaus, 1990, p. 14).

Per Fètis the tonality is mostly related to the disposition of intervals, not to their individual characteristics:

> The mathematical division of a string and the numerical ratios that determine intervallic proportions are powerless to form a musical scale because, in their numerical operations, intervals occur as isolated facts without requisite connections among themselves, and without anything that determines the order in which they should be lined together (Dahlhaus, 1990, p. 14).

At the beginning of the nineteenth century Jean-Baptiste J. Fourier presented his theorem: the periodic waves are the sum of sines and cosines of frequencies that are integer multiples of a single frequency (the fundamental); these waves are called harmonics (Katz, 2017, p. 543).[36]

[36] According to this fundamental physical characteristic, a free oscillating air column or string oscillates in such a way that the waveform of the oscillation can be decomposed in a sum of pure sinusoidal oscillations, and their frequencies are always integer multiples of the fundamental. If a frequency ratio of two independent oscillators equals 3/2, for example, then the second harmonic of the wave with frequency 3 will have the

Following Helmholtz (1954) two pitches are consonant if their partials (sine waves) are aligned, or else sufficiently far apart so as not to interfere with each other.

> When the two tones produced are too far apart, the vibrations excited by both of them at once in Corti's organs are too weak to admit of their beats being sensibly felt, supposing of course that no upper partial or combinational tones intervene (Helmholtz, 1954, p. 172).

Another drastic change appeared by the end of the Common-practice period. Composers of the early Contemporary period, such as Prokofiev, used dissonances that did not require resolution; per Kholopov (1967), the combination of minor and major chords must be considered in Prokofiev as a different mode, which is neither major nor minor. The tonal function is reinterpreted, including polyharmony, that is, complex chords that seem to be formed by independent parts, and linear harmony, with dissonant chord resolution in another less dissonant, among other part-writing [voice-leading] innovations (pp. 67–68).

Breaking the old rules led to atonality, but that does not mean that the new atonal music had no rules; the next quotation comes from the Spanish text of Catalán & Fernández-Vidal (2012): Schönberg himself, after a compositional silence of almost a decade ─something uncommon in a composer of his category and career, derived his atonality towards the establishment of the twelve-tone technique that is as complex as the tonal system itself. The dodecaphony is a way of treating atonality. Its lack of exclusivity is shown by the fact that, at the same time, another dodecaphonic system was developed, the one proposed by Josef Matthias Hauer; and, if the Schönbergian system prevailed it was simply because of the Darwinian principle that it was better prepared to survive. It was more complete, more practical and more coherent when it was applied (paraphrase of page 12).

Music pleasantness is difficult to define and, consequently, the contributing elements are also matter of debate, facing, additionally, the fact that the same music, in the same person, can produce different feelings over time; music psychology is a complex matter (Hall, Cross, & Thaut, 2016; Huron, 2006, 2015; Krumhansl, &

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same frequency as the third harmonic of frequency 2: indeed 3•2 = 2•3. This is a key fact in music. The lowest integers lead to the lowest (and usually strongest) harmonics, and therefore to the strongest possible beating between those harmonics; this explain why the ratios containing the (low) integers 1, 2, 3, 5 are important, but 7, 11, 13, ... are less important due to their weaker beat (with thanks to Johan Broekaert, for his technical engineering support; personal communication, May 21, 2018).

Agress, 2008), and seems to include a self-organized learning process (Tillmann, Bharucha, & Bigand, 2000).

Plantinga and Trehub (2014) agree with Werner, Fray, & Popper, (2012, p. 235) in asserting that the "origin of Western preference for consonance remains unresolved," but their experiments "go against innate preferences for consonant stimuli" (p. 44). Six-month-old infants exposed to different consonant/dissonant pairs of stimuli selected the first as more familiar stimulus whether consonant or dissonant (Plantinga, & Trehub, 2014, p. 40).

Studies on isolated natives recently done in Tsimane by McDermott, Schultz, Undurraga, & Godoy (2016) go in the same direction:

> The results indicate that consonance preferences can be absent in cultures sufficiently isolated from Western music, and are thus unlikely to reflect innate biases or exposure to harmonic natural sounds. The observed variation in preferences is presumably determined by exposure to musical harmony, suggesting that culture has a dominant role in shaping aesthetic responses to music (p. 547).

Even accepting the "sufficiently separated limit" for consonance, as stated by Helmholtz, this limit seems far from being established universally and fixedly.

Gradual development in harmony and tonality has been attributed to cumulative exposure to tonal music, with progressive incorporation of its rules and principles (Costa-Giomi, 2003; Plantinga, & Trehub, 2014).

> Repeatedly hearing things which one likes is pleasant and need not be ridiculed. There is a subconscious desire to understand better and realize more details of the beauty (Schoenberg, 1950, p. 55).

There is, consequently, a process of memorization in humans, with familiarization of music, and their relative pitches. Infants remember easily music that they have already listened to:

> Infants also exhibit long-term memory for music. After brief periods of daily exposure to instrumental music for one or two weeks, infants distinguish novel music from the music to which they were familiarized […] Relative pitch processing in infancy is demonstrable in a single, brief test session. By contrast, relational pitch processing is very difficult for nonhuman species […] extensive training enables European starlings to recognize transposed conspecific songs but not transposed piano melodies […] More intensive training (thousands of trials distributed over several months) enables rhesus monkey to recognize octave transpositions of Western tonal melodies but not atonal or randomly generated melodies (Arbib, 2013, p. 465).

These, and other theories related to expectation in music, are beyond the scope of this research (Bissell, 1921; Huron, 2006; Pearce, & Wiggins, 2012).

Per van der Merwe (2004): "The New Grove contains seven definitions of 'tonality' ─doubtless an incomplete list, for no musical term has been argued over" (p. 71), and adds:

> It may seem puzzling, in view of the strikingly modern character of some early sixteenth-century dance music, that tonality took so long to develop. But of course the composers of the sixteenth century were not interested in tonality—had, in fact, never heard of tonality (van der Merwe, 2004, p. 86).

This author expands on many enthralling questions such as dissonance and discord (p. 106), evolution of tonality (p. 116), the "harmonic revolution" (p. 53), and the "melodic counter-revolution" (p. 31), again out of the scope of this dissertation, which is not focused on extending musicology debates.[37]

Dahlhaus reference (1990) provide many interesting data related to the origins of harmonic tonality, including almost two hundred references (pp. 371–379).

Thus, and for the purposes of this dissertation, after diverse reviews, even in recent times (García-Pérez, 2006; Grabner 2001; Valentine, 1962),[38] the classification remains basically unchanged: (a) perfect consonances for unisons, octaves, fifths and compounds; (b) imperfect consonances for thirds, sixths and compounds; and (c) dissonances for the remaining intervals, of seconds, sevenths, augmented/diminished intervals and their compounds, with a questioning on where to include the perfect fourth (Apel, 2003, p. 209; Stone, 2018, p. 77).[39]

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[37] Starting for what is to be considered consonance and dissonance, how to determine or measure consonance (Lopresto, 2015), the biological rational for consonance (Gill, & Purves, 2009; Tymoczko, 2011), the classification based on ratios or in other indexes as a reflect of «an eternal and universal law of Nature, pressed today with meticulous attention to indisputable laboratory of frequency ratios» (Cazden, 1972, p. 217), and many other aspects that may be found elsewhere.
Pioneer works about tonality and psychology include, among others, Karl Stumpf's late nineteenth century publication *Tonpsychologie* [tonal psychology], and the works of Meyer (1899), Rèvèsz (1913) and Seashore (1919).
Theodor Lipps was also very active in the study of consonant and dissonant intervals and related them with the concept of empathy in an early (1905) twentieth century German publication (Montag, Gallinat, & Heinz, 2008). Lipps' book was translated by W.E. Thomson as *Consonance and Dissonance in Music* and published in San Marino [CA] by Everett Books in 1995.
Also, James Tenney during his period at Toronto's York University published *A history of 'Consonance' and 'Dissonance'* (1988). In addition to those references, a book recently edited by Albrecht Schneider (2017) about musical acoustics includes chapters for Asian flute and qin peculiarities, and a very interesting chapter by Christiane Neuhaus on methods in neuro-musicology (pp. 341–374).

[38] Extra mention deserves the contributions made by Arlette Zenatti (1967, 1969, 1976, 1980, 1981). She has worked particularly in the field of infant musical perception.

[39] The perfect fourth (P4), despite its name, is considered in counterpoint as dissonant (Stone, 2018, p. 77). This dissonance, however, depends on how the interval is formed. A perfect fourth interval (e.g. C–F), sounds as needed to be solved. It sounds like an inverted fifth remembering a second inversion of a triad, with the bass taking a dominant role, and requiring a resolution with the arrival of a triad whose root is the bass. But if the P4 is supported by lower notes it is consonant. The seventh harmonic (4/7) may be also consonant in certain circumstances (de Klerk, 1979, p. 141). «Although the definitions of the consonances and dissonances

As for pleasantness, and following several authors (Cuddy, 1993; Helmholtz, 1954; Jaśkiewicz et al., 2016; Terhardt, 1976; Wassum, 1980), this dissertation is in favour of the idea that EuCul music pleasantness is solidly based on the consonant intervals, and in both art music and popular music genres, atonality has a minority acceptance.

The importance of the GR in nonlinearity has already been stated: it extends to both chaotic and nonchaotic behaviour models (Cartwright et al, 2002; Chernous'ko, 1990; Majumdar, Mitra, & Nishmura, 2000) and even to space, with research showing that brightness pulses of some dynamic stars have primary and secondary frequencies ratios near the GR (Lindner et al., 2015). However, the next lines will concentrate solely on the GR and music, after a brief initial general overview.

> […] inevitably music and scientism will continue to forget that to generate the sounds one needs to know the proportio aurea studied by giants of thought such as Pythagoras, da Vinci, Bruno and Böhme […] forgetting that *relations between the notes* should follow the same mathematical ratios that govern the reproduction of many species, the phyllotaxis and our solar system (Tuis, 2010; paraphrase of page 13 text in Italian).

The Golden Ratio (GR)

History and calculation of the GR. Although the name of golden ratio is relatively recent (19th century), this quotient (designated with the Greek letter Phi, Φ or φ) stretches back to ancient Europe, Babylonian, Indian, Chinese, Pre-Columbian Maya and other Mesoamerican cultures (Olsen 2006, p. 1) with an early use in art and architecture (MacGucken, 2016).

The larger and the lesser proportion, as it was known before, seems to be a mysterious mystic or magic section that attracted artists and scientists. Pythagoras (ca. 570 – ca. 495 BCE) chose the pentagonal star for his school.[40]

According to Dwivedi and Singh (2013) the metrics in the Vedic and Sanskrit manuscripts show the Fibonacci sequence, as indicated, closely related to the GR.[41]

include the quality of the intervals, those are not typically included in the analyses of counterpoint» (Stone, 2018, p. 77). For the scale analysis in this work, the perfect fourth is included as a consonant interval (see chapter the fifth).

[40] Each line from point to point has three segments with the dimensions: φ–1–φ.

[41] The book cover illustrates the Fibonacci sequence of Veda metrics.

Empirical investigations of the aesthetic properties of the golden proportion date back to the very origins of scientific psychology itself, the first studies being conducted by Fechner in the 1860s (Green 1995, p. 937).

Plato (ca. 424 – ca. 347 BCE) in his *Republic* refers to the uneven division (Olsen, 2006, p. 2), but it was the Greek mathematician Euclid of Alexandria (born ca. 300 BCE), in his work *Elements*, who was the first to register a comment about the ratio and its geometric tracings. Euclid called it the "continuous proportion" (Stakhov, & Olsen, 2009, p. 5). This study has been criticized for not being original, but a collection of data from previous schools, mainly the Pythagorean.

The Roman architect Vitruvio also mentioned the quotient in his *Ten Books on Architecture* (edited in Rome in 1490 by Verulano).

The first complete treatise on the GR had to wait until the Renaissance. In 1509, a Franciscan friar and mathematician, Luca Pacioli, published the first known book (Pacioli, & da Vinci, 1509/2014) about this proportion, with illustrations from Leonardo, and started calling it *de divina proportione* [the divine proportion].

The book, finished in Milano on December 14, 1498, had to wait several years to finally be published in Venice. Pacioli described the ratio as a symbol of God's manifestation: only one number, but three parts, impossible to define in words, unchangeable and omnipresent (p. 20).

Kepler (1571 – 1630) also used the term divina proportione in a letter dated in 1608, but in the *Mysterium Cosmographycum* he called it the "proportional division" (Stakhov, & Olsen, 2009, pp. 3–4); in 1619, he wrote:

> Today both the section, and the proportion it defines are given the title 'Divine', because of the marvellous nature of the section and its multiplicity of interesting properties (Martinez, 2012, p.72).

And added as endnote:

> Two Theorems of infinite usefulness, precious value, but there is a great difference between the two. The former, that the sides of a right-angle bear as much as the hypotenuse, I say that it resembles a mass of gold; the other, of the proportional section, you may call a Gem. It is beautiful in itself, but is worth nothing for the ends of the former, which promotes further knowledge (Martínez, 2012, p. 72).

One very interesting contribution from the author, was the Kepler triangle (Figure 1).[42] He showed interest in the mystical value hidden in Phi, and in its relation to regular solids.

Kepler was a disciple of Michael Maestlin. Interestingly, Maestlin calculated in 1597 at the University of Tübingen, "the first expansion of φ value to a five-place accuracy" [.61803] (Posamentier, & Lehmann, 2012, Chapter 3, para. 2.).

For the Belarussian expert Edward Soroko, the name of Sectio Aurea comes from Claudius Ptolemy referring to the number 0.618… (Stakhov, & Olsen, 2009, p. 5).

In the second edition of his book on mathematics Martin Ohm (1835) mentions the golden section,[43] seemingly the first author using this term (Stakhov, & Olsen, 2009, p. 2).

It was only in 1914 when the American mathematician Cook began using the letter phi (φ) for the golden ratio (Cook, 1914, p. 472); as indicated, phi is the initial of Phidias, but, notoriously, it is also the Greek letter for Fibonacci.

Fascination for the GR has been growing to the point that it is now even taught in elementary schools. Walt Disney himself produced (1959) a short animated educative cartoon film, titled *Donald in Mathemagic Land* (Martínez, 2012, p. 77).

The GR has been studied and analysed from different disciplines and varied points of view (Livio, 2013, p. vii). It has, also, a specific scientific magazine (The Fibonacci Quarterly).

Its ubiquitous presence, and the supposed general aesthetic preferences, have also raised some questioning (Angier, 1903, Green, 1995, Haines, & Davies, 1904).

The GR is closely related to Fibonacci's sequence (0, 1, 1, 2, 3, 5, 8, 13...), a progression of integer numbers whereby each element is the sum of the two preceding ones. The quotient of any element of the series divided by the previous one (F_n/F_{n-1})

[42] As according to APA style indications (6th ed., p. 230) tables and figures must be included in independent pages after the references, it is recommended to open the extra file with figures in an independent screen for better following this text. Although the figures are included in this text, they are additionally provided in an independent file also, for easier access.

[43] In the footnote on page 194 he mentions that the division of any line r into 2 such parts ─or steady proportion─ may probably also be called the golden section [paraphrase from German] (Ohm, 1835, p. 194). This quotation does not show up on first [1834] edition.

tends to the GR, with precision that grows with n. Consequently, any construction including Fibonacci sequence also includes the GR (Dunlap, 1997, p. 92).

But how did the proportion result? The larger and the lesser proportion results from dividing a line into two fragments, one bigger (a) and one lesser (b) in such a way that the ratio of the whole line (a + b) to the bigger segment (a) equals the ratio of the bigger (a) to the lesser (b) or as expressed mathematically:

$$\frac{a+b}{a} = \frac{a}{b}$$

To find the point fulfilling that condition, the equation needs to be solved, something that was not feasible until relatively recently.

Although there are several methods of approach, probably the easiest way is to transform the equation using $a/b = x$:

$$\frac{a+b}{a} = \frac{a}{b} = x = (1+1/x)$$

This results in $x - 1 = 1/x$, then $x^2 - x = 1$, and $x^2 - x - 1 = 0$

The equation $x^2 - x - 1 = 0$, has two solutions formally τ (1.618…) and -φ (0.618...):

$$\tau(\varphi) = \frac{-(-1) \pm \sqrt{5}}{2}$$

It has been stated that the GR was originally discovered as a ratio, not as a number, but the solution offers two possible values fitting in. As both results τ (1.618…) and –φ (0.618...) represent the same ratio, out of the mathematical world, the *absolute values* are frequently named indistinctly as Phi (φ) being customary the case, in the fine arts arena (Dunlap, 1997; Livio, 2013). This makes sense as these absolute values are inverse to each other τ = 1/φ, representing, thus, the same proportional function.

Ubiquity of the GR over time and places. The larger and the lesser proportion may be expressed in several geometrical ways (Figures 1–4). A typical development resulting from the GR is the spiral form (Figure 5). This spiral disposition is found in nature, which offers amazing results about the presence of the GR.

> In concluding geometrical solution (1995) to the spacing of planets, the smallest figure (0.618034) related to Phi (actually the reciprocal of Phi), represents the original distance of the closest planet (Mercury) from the Sun. The original distance of each planet thereafter was calculated with the formula, D= Phi x PO, in which PHI is 1.618034 and PO is the distance (AU) of the previous orbit from the sun (Scarborough, 2000, p. 9).

Hurricanes and galaxies adopt spiral developments (Figures 6–7), and crystallographic structures also adopt rhombicuboctahedron shapes, related to the GR, as studied by Pacioli and da Vinci (1509/2014). Quasicrystals structure has been related to GR showing two-fold symmetry axis as reported by Dunlap (1997, p. 121) in relation to electron diffraction patterns of the icosahedral quasicrystal $Al_{65}Cu_{20}Fe_{15}$ (Figure 8).

Vegetables frequently show a Fibonacci growth (Figures 9–10),[44] and the average temperature for a mammalian is around 38ºC, that is 100 – (100/τ); the DNA has a double-helix disposition (Figure 11) (Fitgerald-Hayes, & Reichsman, 2010, p. 27). The GR is present in snail shells (Figures 12–13) that may be explained in terms of Fibonacci series, as well as in other animals (Figure 14), and in human proportions as that of the da Vinci's Vitruvian man (Figure 15).

The five-point star (Figure 16), has been used by the Pythagorean school, although the form, based on the GR, has been present since much earlier times. As indicated, the three segments point to point have the dimensions: φ–1–φ.

Constructions considered to be related to the GR and the five-point star may be traced back to Stonehenge,[45] over twenty centuries BCE (Figures 17–18):

> The Fibonacci progressive arithmetic series numbers are the fundamental design basis for Stonehenge. The combination of sarsen columns counts and selected linear dimensions fully conform to the first nine items of the Fibonacci arithmetic progression series; 1, 2, 3, 5, 8, 13, 21, 34, 55. First consider the principal element of the entire Stonehenge design, a Trilithon =1, and it comprises two columns 2, and a third lintel stone 3. here are five trilithons 5. Five trilithons are arranged around an oval whose short axis measured 8 faethms, and the long axis 13 faethms. The radius of the site measured from the true centre to its outer edge, the ring of Aubrey Holes = 21 faethms. There are thirty-four sarsen columns = 34. The distance along the midsummer dawn axis from the Heel stones pair to the south-west = 55 faethms (Thomas, 2013, pp. 15–16).

The trilithons adopt a five-point star that has also been considered to represent a Dodman (Thomas, 2013, p. 10).[46] Other early constructions based on the GR are the Egypt pyramids (Meisner, 2012) (Figure 19).

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[44] Due to copyright restrictions, published figures analysing the GR in nature, buildings, ceramics and art have not been included. Instead, new drawings have been done over CC0 photos. Original references and sources are tangential information for the purposes of this dissertation, but they are available in a previous publication (Marco-Franco, 2018, pp. 89–92). My warmest gratitude again to all the authors that have lent CC0 images not requiring recognition.

[45] Faethm, ancient measure also called a megalithic rod equivalent to slightly over two meters [2.073 metres] (Thomas, 2013, p. 6).

[46] Thomas describes the figure as «The outline figure of a man holding two vertical staves portrays I believe the ancient surveyor of prehistoric millennia, the Druid man, the Dodman» (p. 4).

The enumeration of other GR-based architecture includes the Mesoamerican pyramids (Figure 20); the Parthenon (Figures 21–22); the great mosque of Kairouan (Figure 23); Notre Dame in Paris (Figure 24); the Vatican spiral based scale (Figure 25); the Taj Mahal (Figure 26); the Eiffel tower (Figure 27); the Toronto tower (Figure 28); the Louvre museum pyramid (Figure 29); the UN General Secretariat Building (Figure 30); the USA defence building pentagon (Figure 31), and many others.

But not only nature and human constructions are based on the GR; in one ancient cave in Spain (La Pasiega, Maltravieso, Ardales) a striking Neanderthal creation[47] has been found with a ladder-like drawing of unknowable precise meaning showing golden proportions (Harris, 2018; Pike, & Standish, 2018) (Figure 32).

The bigger and the lesser ratio is also present in Gorham cave, at least 39,000 y/o., (Nuwer, 2014) (Figure 33), and in Altamira bison (Figure 34), ca. 13,450 y/o.

Famous paintings of all times from *La Gioconda* (Figure 35), to *The Feast of Bacchus* (Figure 36), or Mondrian rectangle works (Figure 37) give evidence of the artistic use of the golden ratio.

As for sculpture, per Dr. Elliot McGucken the *Honle Fels Venus*[48] (Figure 38) follows the GR that may be traced back also to *Nefertiti* bust proportions (Figure 39), and to Ancient Greece and Rome sculptures (Figure 40).

According to Amabilis-Dominguez (1956), Mesoamerican cultures have also used the GR (Figures 41–42), and the same may be found in China (Figure 43), India (Figure 44), or Africa (Figure 45).

Pottery from different continents has GR proportions (Figures 46–47).

The presence of the GR has continued until the present time where it may be found in commercial logos and daily objects (Figures 48–49).

This overview confirms the widespread presence of the GR.

The GR and music. Per Casey Mongoven (2010) the first reference to music and GR comes from the German philosopher Adolf Zeising a few years after Ohm's first use of the term golden section. Zeising (1856) considered this ratio as natural law

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[47] Presumably the oldest known artwork, at least, 65,000 y/o, according to uranium-thorium dating of calcium carbonate that had formed on top of the petroglyph.

[48] Made of mammoth ivory, it is estimated to date from the very beginning of the Upper Palaeolithic time, between 35,000 and 40,000 y/o.

(p. 127), a position taken with scepticism by other scholars as was the case of Gustav Theodor Fechner (1871):

> As a result, he [Zeising] would clearly be forced instead of the pure sixth, to declare the impure sixth which corresponds exactly to the golden ratio, to be the musically most pleasant proportion (Fechner, 1871, as translated from German by Mongoven, 2010, p. 127).[49]

Mongoven also gathers Schönberg's doubts about the Golden Ratio, as expressed in his (1922) *Harmonielehre* [Harmony] that a single element could be responsible for musical pleasantness (Mongoven, 2017, p. 128).

Schönberg linked music aesthetics to an internal process generated from its sound, in a theory close to that of expectation. The composer stated that the sensual perception triggers associations and the need for the three elements ─sound, ear, and sensation─ to cooperate, but just as a chemical compound shows different properties to the elements that compose it, the artistic effect has other characteristics than those that arise from each of its components (Schönberg, 1922, p. 15).[50]

The GR in music with non-golden scales. There are basically two ways to incorporate the GR into music. The first ─and its oldest use─ is to include it in the structure, basically in metric, progressions, symmetry and similar elements, using a non-golden 12-note scale (Mongoven, 2010, pp. 127–138).

Bartók's *Music for Strings, Percussion and Celesta* (movement III) is an example of a Fibonacci composition: "the work, naturally, is 13 bars long and is structured in phrases of increasing length: 1, 1, 2, 3 and 5 bars."

Moreover, Bartók used a Fibonacci-based scale with the notes 1 (unison), 2 (second major), 3 (minor third), 5 (perfect fourth), 8 (minor sixth) and 13 (augmented unison) (Van Gend, 2014, p. 76).

It is recognised that Fibonacci and Phi were used for the layout of instruments (Kertsopoulos, 2011; Meisner 2012; van Gend, 2014) (Figure 50).

Kertsopoulos also designed a guitar based on GR (Figure 51).

The existence of GR in art music ─and the corresponding analysis from Sabaneev─ is summarised by Olsen as follows:

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[49] The author alludes to using the golden ratio (1.618034…) as a musical interval. It results in about 833.09 cents, or 423.32 Hz, for the C_4 octave. This interval falls between A♭ and A.

[50] Schönberg's empirical postulate keeps a notorious concordance with the current concept of nonlinearity in music perception.

> The golden section has been used by composers from Dufay […] to Bach, Bartok, and Sibelius, as a way of structuring a work of music. Russian musicologist Sabaneev discovered in 1925 that the golden section particularly appears in compositions by Beethoven (97% of works), Haydn (97%), Arensky (95%), Chopin (92%, including almost all of his Etudes), Schubert (91%), Mozart (91%), and Scriabin (90%) (Olsen, 2006 p. 8).

As for the Baroque period of EuCul music, and in relation to Bach, Jeff Laser indicates:

> The Golden Ratio appears in two of Bach's preludes from Well Tempered Clavier, No. 1 in C major and No. 3 in C Sharp major, and serves to dictate the form of the pieces on a macro-level. Specifically, in the latter prelude, Bach creates a textural and rhythmic climax on the measure corresponding to the Golden Ratio. This is achieved through harmonic tension with a dominant chord as well as a break in the constant sixteenth note motion of the composition (Laser, 2012, p. 306).

On the internet there are some other examples, such as the video by Margaret Wacyk (2014) analysing the Fibonacci sequence in another Bach composition, his *Prelude in C*.

Mozart was very fond of mathematics (May 1996), going as far as calculating his odds of winning a lottery in the margins of this *Fantasia and Fugue* in C major (King, 1944, p. 178), and used the GR in composition:

> The first movement of the first sonata, K. 279, is 100 measures in length and is divided so that the Development and Recapitulation section has length 62. Note that 1009, rounded to the nearest natural number, is 62. (These lengths are necessarily natural numbers because they are measure counts.) This is a perfect division according to the golden section in the following sense: A 100-measure movement could not be divided any closer (in natural numbers) to the golden section than 38 and 62. This is true of the second movement of this sonata as well. That is, a 74-measure movement cannot be divided any closer to the golden section than 28 and 46 (Putz, 1995, p. 277).

Putz (1995) also evaluated the degree of consistency in these proportions, by using a scatter plot (pp. 277 & 279), concluding: "This is impressive evidence that Mozart did, with considerable consistency, partition sonata movements near the golden section" (p. 277).

The GR in Beethoven fifth symphony has been analysed by Angela Frey:

> In 1978, Derek Haylock argued about the presence of the golden section in the first movement of Beethoven's fifth. Claiming that the opening motto occurs exactly at the golden mean point of 0.618, namely in bar 372 of 601. What's more, the coda is 129 bars long, and, if you divide it using the golden section, you get 49:80. After the first 49 bars of the coda, Beethoven actually introduces a completely new tune that has not appeared in the movement so far, a real first in the history of classical music composition (Frey, 2015, para. 7).

Even authors showing considerable criticism about the GR in music had to recognize its presence, at least in some cases: Madden (2005) recognizes, in his conclusions about the Romantic period, that "the golden proportion was found in several movements of Mendelssohn" as in symphonies No. 3 and No. 4, although not in relation to Chopin’s prelude No. 4 (p. 247).

Following van Gend (2014), Debussy also used the GR. In *Reflets dans l’Eau* [reflections in the water] the author has detected a pattern for right hand: C (1); C (1); D (2); E (3); G (5); A (8) and F (13), then left hand B (21). After a silence reverse descending pattern; in other bars a similar process is repeated (p. 75).

Erik Satie and Béla Bartók were two relevant contemporary musicians that used the GR:

> Erik Satie composed Trois sonneries de la Rose+Croix while he was composer and chapel master of the Ordre de la Rose-Croix Catholique, du Temple et du Graal. Even though the three sections are written with no bar line, which suggests a free metric structure, the whole composition displays an interplay of two juxtaposed themes, with occasional departure from the initial exposition. In a 1988 study, Alan Gillmor of Carleton University, Ottawa, observed that all three movements have ratios of beat counts of the complementary sections that near the Golden Ratio (Frey, 2015, para.7).

The presence of GR has been analysed in latter contemporary composers such as György Kurtág (Mongoven, 2010):

> Kurtág’s laconic musical language requires a more specific micro-level examination. The Fibonacci Series may be applied to numerous miniatures in Jatekok through intervallic content, rhythm, and structure. For example, in many of the microludes examined, Kurtág chooses the intervals of minor second, major second, minor third, and perfect fourth. These intervals consist of one, two, three, and five half steps - all Fibonacci numbers (Laser, 2012, p. 306).

The GR is found in popular music, as in the case of the rock group Tool and the 2001 composition *Lateralus*.[51]

In addition, it is possible to find recorded music with note succession following the Fibonacci digit sequence (Korotitsch, 2012).

Another interesting point is the question of Mendelssohn’s awareness of using the GR, as indicated in Madden’s reviewed work by Alcides Lanza (2006, p. 83).

But, is there a need to be aware of the GR to use it?

Were prehistoric painters (Figures 32–34), or the sculptor of *Hohle Fels* Venus (Figure 38) aware of using the GR?

The previous are just a few samples intended as non-exhaustive reference to the many composers who have used the GR in music metrics.

[51] Even the syllabic progression of the lyrics has been analysed in that way.

However, this is not the only way to use the golden ratio in music since there are also golden-ratio-based scales that can be used either in combination or not with the metric.

GR-based musical scales. This topic has been reviewed in a previous work, and only a brief résumé is included here (Marco-Franco, 2018).

Different golden scales were analysed and rated in relation to: equal temperament, rate for GR, Mean Quadratic Dispersion (MQD), adjustment to Pythagorean and Just Intonation (JI) intervals, and compatibility with the 12-TET scale (Table 1). Rating for golden ratio in a golden-ratio-based scale may seem odd. But, as mentioned, the first and very essential point regarding golden scales is how the GR is included in the scale design. Or in other words how “golden” the scale in question is.

In Chapter Two it has been advanced that with successive transformations of Phi it is possible to reach any number, and mentioned how Lange (2013a) indicates a procedure to reach the series of integer numbers using the Phi-powers:

> Phi-1+Phi-2=Phi0=0,61803399+0,38196601=1,00000000
> Phi1+Phi-2=1,61803399+0,38196601=2,00000000
> Phi2+Phi-2=2,61803399+0,38196601=3,00000000
> Phi2+Phi-2+Phi0=2,61803399+0,38196601+1,00000000=4,00000000
> (Lange, 2013b, para. 8).[52]

But is the series of whole numbers a golden series? Obviously not.

A second example, this time in relation to Erv Wilson (1992) schemes. His scale constructions (p. 2), have been recently reviewed by Terumi Narushima (2018): “The generator size of this scale is calculated by applying the number from the branches to the formula (a+c* Phi)/(b+d* Phi) = (1+1* Phi)/(6+7* Phi) = 0.151102276…” [n.p].

How much of the “power” of the golden proportion is kept with this formula? What elements of equality or proportionality share this value 0.151102276 …with Phi (1.618033988...)?

[52] Incidentally Lange and co-workers have published a very interesting paper on the golden musical system and the mathematical connections between prime numbers and the «landscape» in the theory of strings (Lange, Nardelli, & Bini, 2013b). According to string theory, the physical laws that operate in our world depend on the way in which the extra dimensions of space are rolled up into a tiny «bundle.» A map of all the possible configurations of the extra dimensions produces a «landscape» within which each valley corresponds to a stable set of laws. The entire visible universe exists in a region of space associated with a valley of the landscape that casually gives rise to physical laws compatible with the evolution of life forms (Bousso & Polchinski, 2004, para. 1).

One more case is Thorvald Kornerup (1935) meantone scale formula: (2{([b * (8- φ)] /11) + a}]), again, far from including the GR as a “proportion.”

Per Monzo (2005a, para. 4) the generator is the fifth meantone: v=2[(3Φ+1)/(5Φ+2)] = ~1.49503444952678 = ~696.2145 cents:

> The golden meantone ‘5^{th}’ or generator is thus near the middle of the meantone spectrum, with an interval size only a tiny bit larger than the ‘optimal’ 7/26-comma meantone (advocated by Woolhouse and Erlich) and a little smaller than 1/4-comma (see the graph near the bottom of the meantone entry) […] Expressed as a fraction-of-a-comma meantone, the golden meantone generator is nearly indistinguishable from that of 4/15-comma meantone […] Golden meantone is also audibly indistinguishable from the 7/26-comma ‘optimal meantone' discovered independently by Woolhouse and Erlich […] Any meantone interval can be designated in the form 2a *vb, where v is the generator (2[(8-Φ)/11] in this case), b is the number of generators needed to produce the octave-invariant form of that interval, and a is the amount of octave that must be added or subtracted to fit it into the reference octave (Monzo, 2005a, para. 14).

Other constructions, using quite important modifications of Phi, are the square root of Phi (Sqrt Phi) scales, based on the exponents of Phi ─an approximation to Lucas numbers. It is needless to say that as φ is an irrational number, any exponent or root is just approximate. The 416.54515 interval has been used in this case (Vajda, 2008).

As mentioned above, utilizing the value of 1.618033988... as the ratio for an interval from Middle C ($C_{4\,=}$ 261.63 Hz) results in a frequency close to a pitch between A and A♭. It is well known that this is not the most consonant interval, consequently the matching of consonance with the GR must be achieved through an alternative way.

But there is a serious risk of losing the “magic” of the proportion if the GR is included through complicated formulas or equations that could cause the loss of that mysterious constituent of the GR.

The conditions used in the dissertation for considering a scale to be golden-ratio-based have been described in the corresponding section in Chapter Two.

What could be the best technique to construct such a scale? It seems that the continued fraction (Hardy, & Wright, 1975, pp. 154–177), as will be expanded upon later, is the best option, but if another methodology is used, the proportional factor must be maintained.

The presence of consonant intervals in GR-based scales is another important point to take into account in connection with previously expressed evidence of EuCul music pleasantness solidly based on the consonant intervals (Cuddy, 1993; Helmholtz, 1954; Jaśkiewicz, et al, 2016; Terhardt, 1976; Wassum, 1980).

> Others remarked that atonal music could not express the wide range of human emotions in an appropriate way. One could translate Shakespeare's plays into hundreds of different languages, but one could not translate a Beethoven symphony into an atonal equivalent. The language of music was not as arbitrary as the normal languages. Atonality was even described as "not music" or "incomprehensible" (Sfetcu, 2014, Atonality, para. 10).

This by no means implies that composers using golden scales are always compelled to create music with consonant intervals, in the same way as composers with the 12-TET scale may also compose atonal music.

The advantage of a versatile scale ─such as the 34-TET─ is that it offers wider possibilities to be used either to compose tonal, or atonal, music.

It has been mentioned that GR-based scale builders have found difficulties in integrating both elements (golden ratio and consonance).

One hypothesis, already advanced, for this mismatch is the fact that consonant intervals result from quotients of simple whole numbers: octave 1/2, perfect fifth 2/3, perfect fourth 3/4, etc, while the golden ratio values result from solving a quadratic equation that includes an irrational value ($\sqrt{5}$); therefore, the approximation does not seem easy.

What seems clear is that many constructors of golden scales (Bohlen, 2012; Enrich, 2002; Finnamore, 2001; Freivald, 2017; Kornerup, 1935; Lange, 2013b; Mongoven, 2010; Pareyon, 20111; Sevish, 2017; Vajda, 2008; Walker, 200), have put aside the consonance, although this is not the case for Kornerup (1935), Vajada (2008) or Wilson's (1975) constructions.

By analysing the well-temperament, Tennenbaum (1992) concludes that it is based on the GR:

> The Classical well-tempered system is itself based on the Golden Section. This is very clearly illustrated with the following two series of tones, whose musical significance should be evident to any musician: C–E♭–G–C, and C–E–F#–G. In the first series, the differences of the frequencies between the successive tones form a self-similar series in the proportion of the Golden Section. The frequency differences of the second series decrease according to the Golden Section ratio (p. 49).

In his figure (three) accompanying the text, the author points out that the frequency values are ordered according to the GR: Setting G–C' as 1, then C–E♭ is $1/\varphi^2$, and E♭ – G is $1/\varphi$. Setting C – E as 1, E – F# is $1/\varphi$, and F# – G is $1/\varphi^2$.

Barberá-Saiz (2012) used the golden ratio to create a scheme based on the fuzzy logic with formulas derived from Weber-Fechner, with three variants: double tempering (bilinear), parabolic, and exponential. However, the difficulties in using the GR as a root value induced the author to simplify 8.331 to 8 (p. 19). The scales have differences over 33 cents with the equal temperament for F, and for D# in two out of the three variants the impurity is close to 40 cents. As the 12-TET gamut has a maximal impurity of 15.67 cents, in relation to just intonation, that means that the impurity of this scheme is even higher.

In addition to the inconveniences of using a function non-proportional such as the exponential, the scale is not equally tempered.

Meantone and Wilson's scale GR farness have already been commented on, and in square Phi scales the GR is expressed in its exponential form.

The conclusion, after reviewing all these GR-based developments, is that it seems that either the schemata get close to the consonant intervals or to the GR, but not to both (Table 1).

However, Cartwright, and co-workers (2002) have found a way to join both consonance and GR in a 34-tone scale.

As their work constitutes a cornerstone of this dissertation, the scheme will be expanded on in a compared review with the 34-TET scale as part of the results (Chapter Six).

GR and computerised music. Sound generation using computer technologies is a growing area. The work of Assayag et al., (2002) provides several chapters dedicated to computing musical sound, including synthesis models (p. 221), imitation of instruments (p. 223), composition of textures (p. 224), communication chain (p. 243), symbolic representation (p. 246) and computational models for music sound sources (pp. 257–283), among other aspects.

One relevant case of joint usage of the computers and music is the architectural work of Iannis Xenakis (1990).

There are also works about Fibonacci computers (Stakhov, & Olsen, 2009, p. 416).

Stria was a famous work by John Chowning. He began the project as far back as 1972 but without the possibility of further experiments he returned to Stanford (Chowning, 2007).

As mentioned in the introduction, this project is not framed in computerized music but in score-written music.

As a final consideration in relation to the magical or mystical properties of the golden ratio: although this thesis presents a musical scheme based on the GR, it is beyond its scope to analyse the reasons for these mystical or magical effects, neither in art in general, nor in the case of music.

Also, whether the artists using the golden proportion were aware or not of its use was not considered to be a topic for further study in the research.

Musical Scales

This work links a number ─or better say a ratio (golden)─ to a musical scale and, consequently, the question of musical scales and intervals[53] deserves some comments, again necessarily restricted, since the *Scale Omnibus* contains 1,018 names corresponding to 392 different scales (Balena, 2014), and extensive additional information about the historical temperaments may be found in available sources such as Dolmetsch online database (Blood, 2018).

In Asia, there are scales for days, seasons and even for mood states (Balena, 2014). The EuCul do not have such a wide range of scales, but it has specific melodies for birthdays or festivities such as Christmas, or funeral songs, and moods have been linked to certain tonalities, just to mention some examples.

“The oldest records of organized and systematized music are Sumerian and Egyptian. Sumerian texts written in the third millennium BC frequently speak of ecclesiastical music” (Sachs, 2008, p. 58).[54] This is very probably the origin of the later knowledge that has arrived at EuCul societies through Ancient Greece.

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[53] As explained at the beginning of the text, interval ratios are usually expressed in values greater than one. A perfect fifth, is usually referred as 3:2 and not as 2:3. This (3/2) is the ratio to multiply the frequency of the root note to obtain the frequency of the note of the corresponding interval; in consequence the ratios (in the ascending scale as usually referred in EuCul current music theory) are always greater than one. Ratios may occasionally appear in the text, lesser than one, corresponding to the monochord, whose octave ranges from 1/2 to 1. In this case, all ratios are comprised between 0.5 and 1, which means that all are less than 1. A glance to see if the value is greater or lesser than one makes the scheme clear. For that reason, this specification has not been included case per case.

[54] This author based this comment on the work *Babylonian Liturgies* from Stephen Langdon (1913), published in Paris by Librairie Paul Geuthner [in English]; but the own Langdom recognized in his introduction «The

> No direct source betrays the nature of Hebrew melodies; nor do we know how the temple singers of Egypt and Babylonia regulated their chants. But one thing is certain: wherever a higher class of musicians was distinguished from a lower class, wherever the official standard of an educational center was respected, there must have been law and logic, measure and reckoning. Heman, Asaph, Jeduthun, and their brethren in Mesopotamia and Egypt had the concepts 'correct' and 'faulty' music, they had a system (Sach, 2008, p. 64).

Early schemes. Traceable oldest melodies seem to be based on two tones only, with frequent presence of the fourth as interval:

> The earliest melodies traceable have two tones. The two-tone style, in its narrowest form, comprises melodies pendulating between two notes of a medium level, the distance of which is a second or less. And the melodic span is narrow: the themes, or rather motifs, are extremely short and often consist merely in a single step up (Sachs, 2008, p. 32).[55]

Such a simple schema may still be found in Vedda pygmoids in Ceylon, and the Botocudos in East Brazil, while the pygmoids in Central New Guinea repeat the two notes one fourth apart (Francès, 1988, p. 159; Sachs, 2008, pp. 32–33).

> Though rudimentary, these melodies are not order less. As they are indefinitely repeated, they follow the same principle of co-ordination that children use when they annoy their parents with endless reiterations of a tiny scarp of melody; […] Most of these patterns are vehicles for words, not autonomous pieces; they are expected to be heard, not to be listened to (Sachs, 2008, p. 33).

"The fourth is, again, the structural interval in North American Indians, while the octave is traced back in Australian tribes" (Sachs, 2008, pp. 42–43).

Ancient Greece. As mentioned, "All scales (according to Greek theory) are built up from 'tetrachords', that is, from a system of four notes spanning the interval of a four" (West, 1994, p. 160).[56]

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volume which is here presented to the public contains for the most part fragments of Sumerian liturgies copied from the library of Asurbanipal, none of whose originals in their final form antedate the Cassite period» [ca. two millennia BCE] (p. v). Thus, the existence of a «systematized music corpus» dating circa three millennia BCE ─although probable, considering the source─ does not seem to be convincedly proved with this only reference. The sections about musical scales of India and China will provide additional background on the already-existing musical organization in these cultural settings.

[55] There are sometimes called ditonic scales, but the term may be confusing as it is also employed for other scales having a ditone (two tones) as the biggest interval.

[56] Ancient Greek scales start on the uppermost note and move downwards. «The tetrachord […] is built on assumption of the following rules: The smallest concord is the fourth with the upper note its tonic. This space cannot be divided by more than two intermediate notes. No interval smaller than a quarter tone can be produced or discriminated. In the division of a fourth, when the upper note is tonic, the lowest interval must be equal to or less than the middle, and less than the highest» (Music of Yesterday, 2018, para. 5). It must be taken into consideration that the Greek tetrachord was a unit, and considered as such by the Greeks, but there were diverse ways to split it ─in fact they tuned the strings differently to play the *harmoniai* o *tonoi* later called *modes* in Latin ─ but with first and fourth strings fixed.

The constituting elements or units of "indivisible sizes or steps" are called genus (*genos*) (Sach, 2008, p.65), a word also used to name the division or schema resulting from them.

> Conventional Western 12-tone scale genus is the semitone […] Western diatonic genus, for example, has existed both in numberless forms of unequal temperament and in the equal temperament of modern keyboards (Sach, 2008, p. 66).
>
> In Aristoxenus any variant of the standard genus of scale is called a *chroa* ('shade', 'colour'), and the so-called chromatic genus admitted more such *chroai* than did any other. […] The main forms of the 'chromatic genus' are unlikely to have been systematically analysed before Aristoxenus (Barker, 1984, p. 225).

The tetrachords may be linked by a share note, or by an extra note called disjunction. There are three different options for this later case: tetrachord-disjunction-tetrachord, tetrachord-tetrachord-disjunction and disjunction-tetrachord-tetrachord (West, 1994, p. 161).

There are, also, the possibilities for the division of the tetrachord, although their denominations do not match exactly with current homonymous concepts: enharmonic, chromatic and diatonic.

> Aristoxenus recognized three basic genera of tetrachords: the enharmonic (also known as *harmonia*), the chromatic (also known as *chrōma*, i.e. 'colour'), and the diatonic, the last two of which exhibited various shades (*chroai*) (Mathiesen, el al., 2014, Greece. Section: Aristoxenian tradition. Genera, para. 1).

The enharmonic division includes two tones and two quarter tones. The chromatic division comprises two semitones and a tone and half. The diatonic two consecutive tones and a semitone.

The nomenclature of these intervals does not seem to be consistent among authors.[57]

A basic Ancient Greek musical structure was the Dorian harmony. Four *descending* notes, two whole tones and one semitone apart (for the case of the diatonic

[57] Sometimes these quarter tone intervals are called *diesis*, as did Aritoxenus, but Philolaus, and Marchetto of Padua use this name for a different interval (Monzo, 2005b), so it may be better to maintain the interval name descriptive. «The Pythagoreans were also concerned with the measurement of intervals smaller than the 4th, which they identified through mathematical processes. The tone, for instance, was shown to be the difference (9:8) between the 5th and the 4th, and various sizes of 'semitone' were identified, such as 256:243 (the 'limma'), 2187:2048 (the 'apotomē'), and 'semitones' that could be created by proportioning the ratio 9:8 to create any number of small subdivisions (e.g. 18:17:16 or 36:35:34:33:32 etc.). The size of the semitone and the addition of tones and semitones to create 4ths, 5ths and octaves became a subject of heated controversy between the Pythagoreans, with their fundamentally arithmetic approach, and the Aristoxenians, who adopted a geometric approach to the measurement of musical space» (Mathiesen, et al, 2014, Greece. Section: Music Theory. Pytagoreans, para. 4).

genus) with the Dorian as the basic tetrachord (beginning in hypate meson E[58]–D–C–B), including two of these tetrachords (T–T–S), each other separated by a **whole tone**), to form a larger system (white notes of the piano down from E to e': E–D– C– B–**A**–G–F–E).

With two additional tetrachords linked by shared notes, the range comes up to two octaves minus one whole tone. For that reason, the Proslamban-omenos [later addition] note (**A**) was added, to reach the two-octaves, or *disdiapason*, called the Greater Perfect System: ([A–G–F–[E] – D–C–[B] –**A** –G–F–[E]–D–C–B]─**A**) (Magil, 1998, p. 134).

As per the Roman theoretician Boethius in his *De Institutiona Musica* (ca. 500 CE), the Aristoxenian schemas also included a Lesser Perfect System with three conjunct tetrachords plus an added note (as A to d') and the Immutable System combining both Lesser Perfect and Greater Perfect (Magil, 1998; Wiering, 2001).

The initial Ancient Greece tonoi[59] were seven: Dorian, Hypodorian (or Locrian), Phrygian, Hypophrygian, Lydian, Hypolydian, and Mixolydian.

> The later Aristoxenians expanded this discussion to include consideration of the ways in which the tetrachords are combined to produce the Greater and Lesser Perfect systems, […] the octave species are the most important because of their apparent relationship to the tonoi; they are commonly described and named as follows: hypate hypaton–paramese (b–b′), Mixolydian; parhypate hypaton–trite diezeugmenon (c′–c″), Lydian; lichanos hypaton–paranete diezeugmenon (d′–d″), Phrygian; hypate meson–nete diezeugmenon (e′–e″), Dorian; parhypate meson–trite hyperbolaion (f′–f″), Hypolydian; lichanos meson–paranete hyperbolaion (g′–g″), Hypophrygian; and mese–nete hyperbolaion (a′–a″), called, Locrian and Hypodorian (Mathiesen, et al, 2014, Greece: Section scales, para. 2).

The combination of tetrachords could be *authentic* when built downwards with two identical tetrachords separated by a whole tone (Dorian, Phrygian, Lydian and Mixolydian) or *plagal*, obtained by moving those tonoi one fourth down. Thus, they keep the same name but with the "hipo" [under of] prefix (Hypodorian, Hypophrygian, Hypolydian, Hypomixolydian), giving thus a total of eight tonoi.

Of some other concepts such as: *chroma malakon, hemiolion, toninion, diatonon malakon, diatonon malikon, syntonon toninon, syntonon, ditoniaion or homalon* there is is lack of complete information or it is not coincident between key

[58] The modern Dorian mode however corresponds to the intervals of the white keys of the piano from D to D' (see later).

[59] As mentioned, the term mode was not used by Greeks but by later Latin authors. Although tonoi are also called harmoniai it is preferred to use tonoi. According to Mathiesen harmoniai could signify up to three different things.

authors such as Aristoxenus and Ptolemy (Ambros, Nottebohm, von Sokolowsky, Becker. & Reimann, 1887). “Unfortunately the passage in which Aristoxenus enumerated the tonoi has not survived [...]. They are not, therefore, modes, as has so often been thought, but transposition scales rising from semitone to semitone” (Bélis, 2001, Section Aristoxenus, para. 9).

The Tables 2 & 3 depict the details of the seven primitive Greek tonoi, (Dorian, Hypodorian, Phrygian, Hypophrygian, Lydian, Hypolydian, and Mixolydian), plus the later addition of Hypomixolydian.[60]

The tonoi took the Latin name of modes and went through successive changes, summarized by the Swiss music theorist Heinrich Glarean in his publication *Dodecachordon* (1547).

He proposed increasing the modes to twelve, including authentic and plagal Aeolian (9–10) and Ionian (11–12). The Table 4 reproduces an abridgment from the first page of the *Dodecachordon*.

A summary of the different correspondences between the Ancient Greek tonoi and the modern diatonic modes systematized by Glarean is presented in Table 5.

Pythagorean tuning. Due to its importance over the centuries, a comment on the Pythagorean scheme is required. It is based on two elements, the octave and the circle of fifths:

> The Pythagorean system is based upon the octave and the fifth, the first two intervals of the harmonic series. Using the ratios of 2:1 for the octave and 3:2 for the fifth, it is possible to tune all the notes of the diatonic scale in a succession of fifths and octaves, or, for that matter, all the notes of the chromatic scale. (Barbour, 2004, Chapter I, para. 2).

Although the development was mainly theoretical, “this scheme is also consistent with practical tuning of an eight stringed instrument in the ancient Greek Phrygian mode” (Frazer, 2015, Pythagorean tune, para. 3).

The diatonic major scale comes thus, as the joining of two tetrachords, with the schema T–T–S–(T), “consistent with an eight stringed instrument in the ancient Greek Phrygian mode” (Frazer, 2015).[61]

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[60] Note that the «reciting tone» is not the final note. For example, for Dorian reciting tone is A, for Phrygian is C, for Mixolydian, D, etc. It is «generally [not always] the fifth scale-degree of the authentic octave, a third lower in the plagal mode» (Alwes, 2015, p. 6).

[61] This reference includes ─in addition to the analysis of all possible intervals with this tuning scale─ the possibility to hear the notes as melodic or harmonic interval by left or right clicking into the corresponding cell in the table. Retrieved May 15, 2018 from http://www.peterfrazer.co.uk/music/tunings/greek.html#2.7

> This mode has symmetrical tone, semitone, tone intervals within each tetrachord. The manner of tuning may be illustrated with modern note names. First tune an octave between the two outer strings, D D. The other end of each tetrachord is a fourth from one of these or a fifth from the other, D A G D (descending). Two further notes lie a fourth from these middle notes, D C A G E D. The final two notes lie a fifth from these D CB A G FE D. Unspaced letters indicate semitone steps. (Frazer, 2015, Pythagorean Tuning, para. 3).

The scheme based on cyclic fifth intervals may be calculated by going on descending and ascending cycle of fifths, and then adjusting the values obtained to the same octave.

Thus, the procedure starts with unison, moving on 3/2 steps: (3/2), $(3/2)^2$, $(3/2)^3$…, and the same on inverse motion (2/3), $(2/3)^2$, $(2/3)^3$… (Barbour, 2004; Sethares, 2005).

For pitches not being in the same octave it is needed to increase (•2) or reduce (•1/2) their frequency values to come up or down into the same octave, that is to multiply or divide by 2, as many times as needed (Table 6). Once ordered, and using current standard C_4 frequency as reference, they have been depicted in Table 7.[62]

The Pythagorean tuning remained as the basic reference for EuCul music for centuries, although the number of slightly different tunings grew over time.[63]

It must be taken into consideration that the chromatic intervals were developed during the Middle Ages, and preferences for sharp or flats may vary from one author to another. E.g., Hugo von Reutlingen (ca. 1285 – ca. 1360) used a schema with two flats and three sharps. A typical Pythagorean tuning including G# instead of A♭, is presented in Table 8 (Barbour, 2004, Chapter V, para. 2).

As per the English theorist Walter Odington, in the early fourteenth century the singers tended towards using the harmonic intervals 5/4 (1.25 instead of 1.2657), and 6/5 (1.2 instead of 1.185) (Barbour, 2004, Frazer, 2004).

There are not many data on instrumental music and tunings before 1450, but in the following three decades, several works of music theorists confirm a growing interest in the construction of instruments and music.

[62] The scale results to have a tone of 9/8 (203.91 cents) and two different semitones: the major, chromatic or apotome of $3^7/2^{11}$ (113.685006057712 cents), and the diatonic, minor or limma of $2^8/3^5$ (90.22499567 cents). These semitones are enharmonic equivalents but with a difference of about 23.46 cents, a Pytagorean comma.

[63] Liu Lian-Sheng (2017) has fully reviewed ─with impressive calculations support─ the standard format of heptachords in 792 kinds of Pythagorean tuning different structures.

One of those pioneers was Henri Arnaut de Zwolle (ca. 1400 – 1466), physician, astronomer, astrologer, and organist to Philip Duke of Burgundy.

Zwolle published detailed schemas with illustrations for keyboard musical instruments, both of plucked (harpsichord) and percussed strings (clavichord) and for organs, contributing to the fifth problem by suggesting a small fifth (B–F#) (Atlas, 2002, p. 256; Le Cerf, & Labande, 1972).[64]

These scheme results in eight Pythagorean major thirds, with the remaining ─slightly smaller─ four major thirds very close to purity (cents: 90–204–294–408–498–588–702–792–906–996–1110) (Kelletat, 1982, p. 149).

With the time, the old problem become more evident. For the singers, the major thirds were too sharp, while the minor thirds too flat. In fact, the major and minor thirds and sixths differ from the true harmonic values in a small amount (about the syntonic comma [21.45 cents]).

It seems that the first European theoretician to break away from the Pythagorean system was professor Ramos de Pareja from the university of Salamanca, already mentioned in this text. In his new monochord some notes are joined by perfect fifths as in former Pythagorean tuning, but others have about one comma difference.

A similar tuning, ─with the same idea of looking for a better consonance on the thirds─ was found in an anonymous German pre-sixteen century document in the Erlangen University Library.

Table 9 shows the comparisons of the monochord of Ramos de Pareja, the one from the Erlangen library, and the Pythagorean. It must be considered that the sharps and their enharmonic flat notes do not match exactly. There is a schisma (2 cents) difference:

> Ramis must have been a good practical musician. Although his system would not now be called a temperament, we might do well to take him at his own evaluation and hail him as the first of modern tuning performers (Barbour, 2004, Chap. V. para.10).[65]

The germ of the meantone temperaments had arisen.

[64] The works of Henri Arnaut de Zwolle (in Latin), including drawings of instruments, are archived in the National Library of Paris (file 7295); information about intervals is included in folios 111, 128, 129 & 131. Partial reproduction, available on the internet, was retrieved on May 17, 2018 from http://organ-au-logis.pagesperso-orange.fr/Pages/ArnautI.htm. The National Library has published several editions of the treaties of Henri Arnault improving that of 1932. Particularly interesting is the edition (Georges Le Cerf and Edmond René Labande) of 1972 with facsimile reproductions of the original drawings.

[65] It has been mentioned that the wrong spelling *Ramis* instead of *Ramos* is frequently found on English texts.

Thus, music theory was ready to leave Pythagorean fifth cycles as the only generating schema and move back to Ptolemy's idea of using natural harmonic thirds:

> The influential theorist Franchino Gaffurio (1451 – 1522) uncovered the tuning system devised by the ancient Greek Ptolemy which used natural harmonic thirds. Gaffurio himself was a conservative and opposed to the introduction of a new tuning system. The new scheme was championed primarily by Lodovico Fogliano in Musica Theorica of 1529 and Gioseffo Zarlino (1517 – 1590) in Institutioni Armoniche published in 1558 (Frazer, 2004, Just Intonation, para. 3).

The Pythagorean gamut was defended even as late as the seventeenth century. William Holder (ca. 1616 – 1698), in his *Treatise on the Natural Grounds and Principles of Harmony* published in 1731,[66] proposed a division of the octave based on 53 parts (1/53 or Holder comma).

Before leaving the Pythagorean monochord it must be said, however, that according to the definition of just intonation as a tuning system based on small natural numbers, the Pythagorean scheme must also be included into the just intonation category. The syntonic tuning continuum is depicted in Figure 52.

Hexachords. Guido d'Arezzo: Guidonian hand. Birth of solfeggio. In the hexachord, the semitone occupies a central position:

> In simplest terms, a hexachord is a set of six notes arranged to form intervals of two whole-tones, a central semitone, and two more whole-tones. We may represent this arrangement as T–T–S–T–T, with "T" standing for a whole-tone (Latin tonus), S for a semitone (semitonium) (Schulter, 2000, para.1).

Hucbald of Saint-Amand (ca. 845 – ca. 930) describes in his treatise *De Harmonica Institutione* (ca. 880) a six-stringed instrument arranged as T–T–S–T–T, establishing a notation system of six lines representing the six strings separated for spaces ─labelled as T or S─ with C as the lower note of the instrument.[67]

> […] the syllables are connected with diagonal lines, providing a kind of visual graph of the melodic contour. Melismas, passages in which a single syllable is set to many notes, are shown in Hucbald's notation by repeating the syllable (Schulter, 2000, para. 4).

Whether inspired by the Carolingian musical theorists ─as the aforementioned Hucbald of Saint-Amand─ or not, the merit for the paternity of current solfeggio is however attributed to the monk Guido of Arezzo (ca. 991 – ca. 1033).

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[66] Holder, W. (1731). *A Treatise on the Natural Grounds and Principles of Harmony.* London: W. Pearson. Also retrievable from https://archive.org/details/treatiseofnatura00hold/page/n5

[67] A facsimile of this notation may be found in The New Oxford History of Music (Page, 1990, p. 458). Per Christopher Page this should be considered as a sort of tablature (p. 465).

Arezzo used the words of the hymn in honour of Saint John the Baptist *Ut queant laxis*[68] associating the series of six notes from C to A, in *protus* or Dorian mode ─that is with D as the final or point of repose─ with the initial easily remembered six syllables of the first stanza half-lines of the hymn:

> ***Ut** queant laxis*
> ***re**sonare fibris*
> ***Mi**ra gestorum*
> ***fa**muli tuorum,*
> ***Sol**ve polluti*
> ***la**bii reatum,*
> ***S**ancte Iohannes.*[69]

These syllables were called "aretinian" and could be applied to the first six notes of the diatonic scale, starting at any pitch desired. In this case, the group of six notes from C to A, is called the "natural" hexachord.

Guido of Arezzo used this mnemonic technique of linking syllables to sound (solmization) for teaching, creating the bases of solfeggio (Alwes, 2015, p. 7; Harman, 1969, p. 3).

He added a lower G with the name of the Greek letter Γ (*gamma)*:

> Guido d'Arezzo […] devised a system of three overlapping hexachords to identify each individual pitch. The entire range of possible notes was arranged as a scale of overlapping hexachords [..] because it began on G (written as the Greek letter Γ or *gamma*, which was the first and lowest possible *ut*, hence *gamma ut*, which became the word "gamut," signifying the whole range) (Alwes, 2015, p.7).

Later, *ut* was renamed as do,[70] and a final note was added to the scale as *ti* (si).

> Since Guido and successors conceived musical theory in terms of overlapping hexachords rather than diatonic scale the syllable ut could represent any of the three pitches capable of sustaining the overlapping hexachords that made up the system; these were C, F, and G. While ut might vary here was only one gamma-ut (The Editors of the Encyclopaedia Britannica, 1998, para. 1).

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[68] The hymn is nonetheless attributed to Paul Wilfridus the Deacon (*Paulus Diaconus*), and the musical motive is found in Horace's «Ode to Phyllis» (Odes 4.11) (Lyons, 2007).

[69] There are several translations to this first stanza, but frequently in them, the idea expressed in the Latin text is virtually lost in favour of the maintenance of the musical notes; the stanza asks to unchain the sinful lips to sing the miracles and could be paraphrased as: **Do** let our voices/ **Re**sonate, singing freely/ **Mi**racles that marvel/ **Fa**r away/, **So l**eave your servants/ **La**vishing them, unleashing our stained lips/ **S**aint John.

[70] It was attributed to the Italian musicologist Giovanni Battista Doni (ca. 1593 – 1647) although there have been speculations about an Arab origin of this change.

Gamut eventually became a generalized word designating the complete range or extent of an entire series of musical notes or as defined in Cambridge dictionary “the whole range of things that can be included in something.”[71]

At the time of Hucbald of Saint-Amand music still did not use a stave, but the music texts started to include some signs called *pneumas*.[72]

The nine century musical texts were stave-less (*campo aperto pneumas* or *cheironomic*). The information provided by the signs was only partial and complementary, as songs were transmitted, basically, orally.

Later, some copyists started to indicate the main note ─tonic─ and in some codices they included a line marked in red or with a graphite drawing, as a primitive indication of pitch.

This elemental line drawing was progressing to a four-line stave. The innovation has been attributed to Guido of Arezzo:

> Parallel lines have already been proved of higher antiquity than the time of Guido; but the regular staff of four lines was not generally used in the church till the thirteenth century. […] The description, however, which Guido has given of different coloured lines to ascertain the sound of C and F has encouraged an opinion of his having first suggested the idea but even that contrivance is not indisputably his property: for in the Magliabecchi library at Florence I found a MS. missal, said to be of the tenth century, in the old ecclesiastical notation, with two lines, the one red, and the other yellow. Sometimes indeed there was but one line, which was red. (Burney, 1782, pp. 468–469).[73]

By the early eleventh century, the *pneumas* started to be written at varying distances from the text to indicate the melody shape; those neumes are called “diastematic,” or “heightened,” although still without references to ruled lines (Knighton, & Fallows, 1992, p. 387).

The neumes evolved to square notation, but the Gothic German variant continued until the Renaissance.

By the thirteenth century, the Gregorian chant included square notation, four lines stave and clef, although still without a uniform systematization:

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[71] Retrieved April 18, 2018, from https://dictionary.cambridge.org/dictionary/english/gamut. See also in the refences «Gamut» from the editors of the Encyclopaedia Britannica (1998).

[72] From the homonymous Greek word, to enter later into the English language as neume. The Greek word pneuma means breath (also spirit). In their early use, neumes indicate voice inflections only.

[73] This text corresponds to the first volume with critical and historical notes by Frank Mercer in modern English. It may also be found in page 87 on the original text in old English, second volume, first edition printed by the author in London, and folded by J. Robson, New Bond-Street, and G Robinson, Paternoster-Row.

> In the Romish Missals, Breviaries, Antiphonaries, and Graduals, only four lines are used in the notation of the chants; with two clefs, the base tenor, or those of F and C, which are removable; and two kinds of notes, the square and the lozenge; the first for long syllables, and the second for short. In some modern French Missals a third species of note is used, generally at a close; this is square with a tail added to it, and is of longer duration than either of the other two. However, the Italians seldom use any other than square notes in their Canto Fermo, nor did the French, in their more ancient books […]. The only accident allowable in Canto Fermo is a flat to B, which is removed by a ♮ (Burney, 1782, p. 419).

Both Hucbald six-stringed instrument, and Guido single hexachord had limited musical possibilities, and a larger range was required.

Guido found the solution in having more than one hexachord available. The singer then changed or mutated from one hexachord to another (Schulter, 2000, para. 9).

The hexachord system used by Guido of Arezzo spans from gamma-ut to E–la. As indicated before, gamma-*ut* (Γ) represents the lower (G_2); E–la is E_5, being the only note on the back of the hand (Figure 53).

Music using the seven diatonic tones plus B♭ (corresponding to the frequently used mode G Dorian: G–A–B♭–C–D–E–F) was called regular, proper or *recta musica*.

Other accidentals, apart from B♭, were defined as *extra-manum* [out of the hand] and they were not part of Guido's basic system (Schulter, 2000, para. 34), although after the fourteenth century they were regularly included in music, and in that sense, all music then became *ficta or extra-manum.*

> The term 'musica ficta' has two meanings […] In medieval and Renaissance music theory, the term applied to transgressions of what was considered the 'proper' tonal vocabulary […] In current musicological use, musica ficta applies to the performers' and editors application of accidentals not specified in practical sources. The identification of situations that demand such application is still an issue of debate […] (Wegman, 1992, p. 265).

The Guidonian hand is just a mnemotechnic tool to help singers that may be found in some references even before Guido's time, but this author was the one that popularized it in his teachings (Figure 53).

Chord development. Between the fourteenth and sixteenth centuries, chords were also in development. The common chord seems to have been established when the three usual musical voices (middle, melody and bass) were arranged with the middle part a perfect fourth below the melody and the bass a third over, but in the octave below (e.g. $E_{3\text{-bass}}$, $G_{3\text{-middle}}$, $C_{4\text{-melody}}$).

Transposing the top note down an octave resulted in the common triadic chord (Curtis, 1997; Frazer, 2001), which could be major or minor.

The sestina was a chord recognized later including the intervals of unison–octave–fifth–fourth–major third, and minor third, with pitches related in the series 1: 2:3:4:5:6 (Frazer, 2001).

The concept of music in the late medieval and early common practice periods was, in many ways, different from the common view of music as an element for enjoyment. Much of the music of the time was religious and not intended to produce pleasantness.

> Even the most basic contact with Mass music of the late medieval and early Renaissance periods suggest that it cannot be listened to in the same way as, say, a Classical symphony. This is not merely a function of their different purposes, that Mass music is strictly and adjunct to liturgy, whereas a symphony is to be listened to as music for its own sake. It is also because sonata form, like most other forms of the seventeenth, eighteenth and nineteenth centuries, is dynamic: it has within it a sense of progress, and it is the logic of this progress which gives aural coherence. By contrast, the music with which we are concerned here is essentially decorative (Curtis, 1997, p. 154).

Temperaments and the tonal system. Art music finally got the canonical heptatonic scale (with seven intervals) and two modes: the major mode based on the intervals T–T–S–T–T–T–S, called by Glarean the Ionian mode, and the natural minor scales based on T–S–T–T–S–T–T, the Aeolian mode.

> Late 14th- and early 15th-century keyboard instruments that were fully chromatic, that is, with 12 notes in the octave, were normally tuned with 11 pure 5ths among the naturals and hence automatically with one sour 5th, a comma smaller than pure, which was, by most accounts between B and F♯ (the evidence for this comes mainly from Italy and southern France: Dijon, Liège, Padua, Mantua, Parma, Ferrara, Milan, Florence, Urbino). Such a tuning happens to yield virtually pure 3rds between the naturals and sharps (D–F♯–A–C♯–E–G♯–B–D♯) and much less nearly pure 3rds, a comma larger or smaller than pure, among the naturals (D–F–A–C–E–G–B–D) and between the naturals and flats (D♭–F–A♭–C–E♭–G–B♭–D) (Lindley, 2009, Section: Quasi-Pythagorean temperament, para. 1).

But, although with the development of the tonal system these two schemas or modes become predominant, to arrive at the 12-tone equal tempered scale (12-TET) with 12 equal division of the octave (12-EDO) there was still quite a way to go.

> The seeds of just intonation had been sown early in the Christian era, when Didymus and Ptolemy presented monochords that contained pure fifths and major thirds […]. But they remained dormant during Middle Ages. Even after the seeds had sprouted near the beginning of the modern era, the plants were to bear fruit only occasionally and haphazardly (Barbour, 2004, Chap. V, para. 1).

The name of octave comes from the Latin "octo" (eight). As indicated above, the inclusion of an eight string in the Ancient Greek lyre above is attributed to

Pythagoras. The key element is that frequency of the octave is double that of the unison tone.

> […] The octave is the one unequivocal universal in all the world's music. It is, of course, simply the doubling of frequency, a ratio of 2:1, and it`s so perfect that if you play two notes exactly an octave apart, they will still really only sound like one note ─just richer and fatter.
> Interesting enough, an adult male voice and an adult female voice are pretty much exactly on octave apart (Dixon, 2013, Chapter Athens: The Scales of Pythagoras, Secc. 4, para. 2).

But assuming an early incorporation into music of the octave due to the speculative reason of that difference in voice range between women and men, a very different question is how this octave is divided.

> The reason there are so many different ways to divide the octave and display such a range of scales can be found in the fact that there are [*sic*] no formula that can fit the octave perfectly. The different ratios expressed in numbers are prime inter-related, so a common divisor is not possible in an octave - unless some notes or keys are sound disharmonious. […] Different musical traditions embrace this schism depending on what they consider best fit for their musical expression. The culture in which the musical scale has emerged is a profound reflection of that particular culture. […] The Eastern music tradition considers the fine-tuned intervals of much more importance than the Western, which prefers first harmonious chords in any key. Consequently there are intervals which are perceived as consonant in the West, but considered dissonant in the East (Hightower, 2009, para. 11–13).

The possibilities for dividing the octave are many: fixed steps, proportions, progressions, generators (such as the fifth), mixed systems as the meantone, etc. (Hightower, 2009, para. 10).

Per Gleen Dixon there are solid mathematical reasons to divide the octave in 12 steps:

> If you start with an octave and divide the two frequencies exactly in half, you get a perfect fifth. […] But then if you take that same perfect fifth, go up a full octave from there (say the perfect fifth of a C scale ─ G and go up to the high G above that) then divide those frequencies in half again, you get second of the original scale (in other words you get D ─ the second note in the C major scale). So, if you just keep going up dividing the octaves in a perfectly mathematical way… you will eventually go through twelve steps before you come back to the original note. This gives you the 12 notes of the chromatic scale […] this is what the Greeks worked out two and a half thousand years ago. (Dixon, 2013, Chapter Athens: The Scales of Pythagoras, Secc. 4, para. 3).

Just intonation: Zarlino. *Le Istitutioni Harmoniche* (Zarlino, 1558)[74] is "a landmark in the history of music theory, he achieved an integration of speculative and

[74] [Fundamentals of harmony]. Public domain first edition (1558), second (1562) and third (1573) are downloadable from https://imslp.org/wiki/Le_Istitutioni_Harmoniche_(Zarlino%2C_Gioseffo. The treatise is based on five blocks of «rationings,» it includes orderly definitions, proposals and demonstrations with the constant help of tables and graphs. Particularly enlighten are: the scheme of harmonic and in-harmonic

practical theory and established Willaert's methods as models for contrapuntal writing" (Palisca, 2001, para. 1).

Zarlino is a turning point for harmonic and melodic systems based on tonality, leaving behind the old Greek modal system.

This author reviewed the basics of music from various disciplines such as philosophy, theology, history, literature, cosmology and mathematics; very importantly, after analysing the Greek tonal system, the contributions of Ramos de Pareja, Gaffurius, Spataro, Fogliano, and other authors, he found a way to organize the intervals of third and sixth, ─which according to the Greek Pythagorean relations had imperfect consonances─ in a system that would allow these six intervals (*numero senario*) to have a pleasant sound (Table 10).

> The intervals that are most consonant for harmonic sounds are made from small integer ratios such as the octave (2:1), The fifth (3:2), and the fourth (4:3). These small integer ratio intervals are called just intervals, and they collectively from scales known as just intonation scales (Shetares, 1998, p. 6).

Thus, the modern series of eight (six, plus unison and octave) consonant intervals were incorporated to art music composition (Palisca, 2001, para. 5).[75]

> The Pythagoreans had limited the class of intervals they called consonant to those produced by the first four divisions of a string: the octave, 2:1; 5th, 3:2; 4th, 4:3; octave plus 5th, 3:1; and double octave, 4:1. Zarlino extended the upper limit to the divisions of the string into two, three, four, five and six equal segments. Thus the number six, 'numero senario', epitomized the formal cause – the 'sonorous number' ('numero sonoro') – that generated consonances out of the 'sounding body' ('corpo sonoro'). The elevation of the determinant of consonance from four to six permitted the admission of several more intervals: the major 3rd, 5:4; the minor 3rd, 6:5; and the major 6th, 5:3. The minor 6th, 8:5, which remained outside this sanctuary, had to be rationalized as the joining of a perfect 4th and a minor 3rd (Palisca, 2001, para. 5).

The idea behind these changes ─already present in the Greek theorist Claudius Ptolemy (85–165, CE) (Hardegree, 2001,[76] p. 10)─ was to include notes whose

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proportions (p. 114); the descriptive figure of the tone, comma, diathesis and apotome (p. 167); the development of the regular diatonic monochord in the fourth reasoning including the numerical values (tables in pp. 233, 254, & 250); the division of the diatonic regular monochord (p. 284) and the graphic descriptions of the main or authentic mode (p. 301), plagal (p. 303) and the natural order of all modes (p. 306).

[75] Corresponding in the monochord to 1/1, 1/2, 2/3, 3/4, 3/5, 5/8, 4/5 y 5/6. The ratios that Zarlino used in the Le institutioni harmonicae are: ut (1) – re (9/8) – mi (5/4) – fa (4/3) – sol (3/2) – la (5/3) – si (15/8) – ut (2) (Kelletat, 1982, p. 147; Scimemi, 2002, p. 51). It is notorious that Zarlino (1558) depicts a keyboard instrument with 19 intervals in the octave, a number that the modern methodology of impurity analysis shows as excellent in terms of consonance (p. 141).

[76] The publication date (not included in the text) has been provided by personal communication of the author, April 22, 2018, with thanks.

frequencies result from ratios of small whole-numbers, considered to be harmonically pure, perfect or in just intonation (JI).[77]

Thus, any monochord following this rule must be considered a JI scheme:

> Technically, Just Intonation is any system of tuning in which all of the intervals can be represented by whole-number frequency ratios, with a strongly implied preference for the simplest ratios compatible with a given musical purpose (Boty, 2002, p. 1).

Consequently, there is not one single schema or unique JI scale, but a family of tunings based on those small integer ratios, leading "to obtain the power of just intonation, (in other words of playing in tune)" (Thompson, 1860, p. 9)[78] (Table 11).

Through the centuries other authors —such as Johannes Kepler (1571 – 1630), Salomon de Casus (1576 – 1626), Marin Mersenne (1588 – 1648), André de Barrigue de Montvallon (1678 – 1779), Alexander Malcolm (1685 – 1763), Leonhard Euler (1707 – 1783), Jean-Jacques Rousseau (1712 – 1778), Friedrich Wilhelm Marpurg (1718 – 1795), Jean Baptiste Romieu (1723 – 1766), and many others— proposed a variety of different tunings to fulfil the goal of a pure harmony (Barbour, 2004).[79]

As result of the "sonorous numbers" the modes are reorganized, and after Zarlino's work *Dimostrationi Harmoniche* (1571), the reference octave started to be that of C rather than A as in Boethius theories.

Since 12-TET was not the gamut used during Common-practice, and its ratio for A = 1.6818, is 15.67 cents impure in relation to the JI ratio of 1.6667, in the great majority of the historical temperaments A = 440 Hz and C = 261.63 Hz do not match.

It is well known that current standardisation of tuning was not without controversy: "various countries had adopted their own standard of pitch so that Middle C could vary from a frequency of 256, preferred by scientists, to 262 and above"

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[77] The terms pure, simple or perfect harmony, or just intonation, will be used in this dissertation frequently; they refer to this concept from Zarlino. The consonant intervals used have been already commented on.

[78] One of the scales that may be constructed with simple integer ratios is the scale of harmonics, occasionally named natural harmonic scale, whose values start in *f*, the fundamental frequency, progressing by whole numbers as harmonic series 2*f*, 3*f*, 4*f*…. etc. The scheme follows a secondary heptatonic scale starting as Lydian and ending as Mixolydian (Lydian-mixolydian T–T–T–S–T–S–T) called acoustic scale. Although frequently used interchangeably, it is preferred to use the term harmonic series rather than overtone series when the values change by a whole-number value (Bain, 2003). The use of harmonic series in music has been reviewed by Paul Hindemith in the book *The Craft of Musical Composition* (1942). This or close schemata are used in other cultures, and in jazz music (Bain, 2003; Hulen 2006). Hulen reference provides the details for the construction of true and non-true overtone scales.

[79] Tuning for these and other monochords may be found in Gary Hardegree (2001).

(Osborne, 2009, p. 56). Since the tuning for A = 440 Hz was adopted[80] using the 12-TET scale whose ratio for A is 1.6818 “all other notes must be adjusted to accordingly. By calculation Middle C […] ends up as 261.65 cycles per second […]” (Osborne, 2009, p. 56).

To sum up the point, for revisiting historical temperaments, it is necessary to choose to tune either to A or to C.

Using this second option, the JI scale published by Urquhart (1945) for C = 260.71 Hz was recalculated for 261.626 Hz and presented in Table 12.[81]

Per Fonville (1991) this scheme includes nearly all “the basic just scale degrees 1/1, 16/15, 9/8, 6/5, 5/4, 4/3, 3/2 8/5, 5/3, 16/9 and 15/8” (p. 111).[82]

As reviewed by this author, Ben Johnston explored the results of sound purity and tuning in an extended form beyond the first six partials:

> Just intonation is simply the easiest way to tune musical intervals by ear. It results in greatly heightened purity and clarity of sound for two reasons: it eliminates acoustic beats to the maximum possible, and second, it exploits resonance by utilizing harmonically simple combination of pitches. The term extended refers to the use of higher overtones than the first six partials (Fonville, 1991 p. 107).

For fair comparison with 12-T scales it may prove convenient to use a semi-extended JI 16-T scale (Table 13), as the one based on Burns (1999), and Solà-Soler (2012), already used in a previous work (Marco-Franco, 2018, p. 61).

Fonville (1991) also includes different scales and contributions from Ben Johnston. A 21-tone just enharmonic scale, unequal and with adjacent intervals of 25/24, is depicted in Table 14 (p. 112). However, Fonville considers the harmonic potential of this scale poor:

> […] this scale is not very useful for harmonic usage. A more useful just scale collection in regard to harmonic potential […] provides major and minor triads on, and dominant triads on, all the basic just scale degrees” (p. 111).

The 22-tone scale proves useful, as it includes enharmonic[83] sharps and flats differentiated (Table 15). The schema is virtually the same as in Kelletat (1982, p. 148), with differences only in F♭, B♭ and C♭; it has been labelled as standard for comparisons. The extended approach for Fonville (1991) includes 25 notes (Table 16).

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[80] This question will be discussed further in this chapter.

[81] The advantages of tuning for C instead of A will be discussed in Chapter Six.

[82] 16/9 is not included but 9/5.

[83] The term may be misleading as it is correct for 12-TET scales (e.g. C#/D♭) but not totally correct for the scales with different values for sharps (C#) and for flats (D♭), but it has been kept as descriptive.

As mentioned, the JI intervals are also present in contemporary alternative scales:

> Much of the current xenharmonic music is written in just intonations and other scales that are closely related to harmonic timbres. Many of the most popular equal temperaments (7, 11, 19, 21 and 31, for example) contain intervals that closely approximate the intervals of scales related to harmonic timbres (Shetares, 1998, p. 254).

However, the big practical problem with the JI scales does not come from their varied constructing possibilities, but by the fact that an instrument sounds tuned only for a specific key, and out of tune for the rest (Heman, 1964),[84] and that it is not possible to modulate with a JI scale to a different key in the same piece of music.

> Zarlino acknowledged that the numerical criteria that he established in parts i and ii for the tuning of the consonances did not apply to instrumental music, which employed artificial tunings made necessary by the imperfection of instruments. But in the natural medium of the voice it was possible, he maintained, to realize all the inherent perfection of harmony (Palisca, 2001, para. 8).

For that reasons several solutions were adopted, one of the most popular being the meantone temperaments.

Meantone scales: wolf intervals. The design of musical scales must face three main difficulties:

> Difficulties arise with the notion that each note entertains a variety of relationships with the others: B is not only the seventh degree of C major scale, but also the fifth of the fifth on A, the triple fifth on D, etc. It is this latter relationship, that of superimposed fifths, which generally determines the tuning of instruments. […] The result was that such not-quite identical notes ended up sharing a key which sounded perfectly in tune with a smaller number of its relatives but out of tune in several other of its "natural contexts". This compromise is the reason why most works written before 1720s were confined to keys close to C or G major (Bruhn, 1951, p. 10).

The first one is the fifth problem. As outlined, this problem of the unsatisfactory division of the octave may be expressed mathematically as: $2^x = 3^y$ has no solution for integer numbers: a series of fifths never attains exactly the expected frequency. This requires mentioning the concept of the comma.

The comma has been described as "a small pitch interval of fundamental importance to temperament and tuning" (Greated, 2001, para. 1).

The Pythagorean comma is defined as "the difference between twelve fifths and seven octaves" (Greated, 2001, para. 1), with a value is 23.46 cents[85] (Figure 54).

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[84] Christine Heman is the correct name of the author, although the German composer appears occasionally as Herman (https://www.hebu-music.com/en/musician/christine-heman.53375/).

[85] Let us assume that the middle C frequency (C_4) is the current standard taken with an accuracy of seven decimal places: 261.6255653 Hz, ─although for this example it may be used any other value ─ and let us

> In a Pythagorean scale, all intervals form perfect just fifths with the scale tone seven steps above except for one called the *wolf* […] When extending the scale to a complete tuning system (continuing to multiply successive terms by perfect 3/2 fifths), it is impossible to ever return to the unison. (Sethares, 2005, p. 54).

Another useful value is the syntonic comma, also called comma of Didymus, Ptolemaic comma, Chromatic dieses, or diatonic comma, detailed as:

> […] the difference between a just major 3rd and four just perfect 5ths less two octaves which is 21.51 cents[86] […] For practical tuning purposes, the difference between the two types of comma is often ignored and the comma is taken to equal 24 cents (Greated, 2001, para. 1).
>
> A series of seven 5ths ascending from F yields a diatonic scale comprising the naturals on the keyboard; the 3rds and 6ths in this scale, however, differ from their justly intoned equivalents by a syntonic comma, and therefore do not meet medieval and Renaissance criteria of consonance implied by such terms as ‘perfection’ and ‘unity’ (Lindley, 2001a).

The difference between the commas (Pythagorean and syntonic) called schisma is very close to 2 cents.[87]

The second problem for a scale design is how to improve the thirds and sixths, one of the main reasons why JI schemas originated.

The solutions proposed for this problem brought a third complication: the impossibility to modulate and the requirement of different tunes for each tonality.

The idea to find a solution for these three issues has generated many proposals along the 16th and 17th centuries. The fifth-tempering changes based on thirds, called meantone temperaments became very popular at the time.

The Grove Oxford Music Dictionary define the meantone as:

> In its most restricted sense the term refers, like its German equivalent mitteltönige Temperatur, to a tuning with pure major 3rds (frequency ratio 5:4) divided into two equal whole tones (whereas in Just intonation there are two sizes of whole tone corresponding to the ratios 9:8 and 10:9); to achieve this the tuner must temper the 5ths and 4ths, making the 5ths smaller and the 4ths larger than pure by

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increase the frequency exactly in twelve 3/2 (1.5) steps. As seen in the figure (clockwise movement) it will result in →392.44 Hz (G_4) →588.66 Hz (D_5)→882.99 Hz (A_5) →1,324.48 Hz (E_6) → 1,986.72 Hz (B_6) → 2,980.08 Hz (F_7#) →4,470.12 Hz (C_8#) →6,705.18 Hz (G_8#) → 10,057.77 Hz (D_9#) →15,086.65 Hz (A_9#) →22,629.98 Hz (E_{10}#), and finally →**33,944.96** Hz (B_{10}#). This note should be equivalent to C_{11}, however, the real value of C_{11} resulting from the successive doubling of the initial frequency of C_4 (261.6255653 • $2^{(11–4)}$) is **33,488.07** Hz. The difference is about 456.89 Hz. Computed in cents comes to be about 23.46 cents (the Pythagorean comma). Another way to arrive at the Pythagorean comma is by the difference between the Pythagorean semitones: 113.69 and 90.23 cents.

[86] The value is more precisely 21.5062896 cents.

[87] This is quite close the difference between Pythagoric limmas: A major limma could be expressed as the difference between two major tones and a minor third, 135/128 close to 92.1787 cents. The minor limma has already been commented as one of the semitones in the Pythagorean scale (90.23 cents). Thus 92.1787 – 90.225= 1.95 cents). Details over *schisma*, *Phytagorean comma*, *major limma*, *Phytagorean limma* and *diaschisma*, can be found elsewhere (Benson, 2007, p. 171).

> a quarter of the syntonic comma, hence the label '¼-comma mean-tone', a more specific name for the same kind of tuning (Lindley, 2001b, para. 1).

It may be interesting to add the reason why "mean" is included in the definition:

> The 'mean' in the expression of 'meantone' refers to the fact that all the meantone systems divide the major third (whether it is pure, larger than pure, or smaller than pure) into two whole tones of equal size. In standard one-quarter comma meantone, for example, the whole tone results from a division of the pure 5:4 major third into two equal tones by geometrical mean proportional ($\sqrt{5/2}$ or $\sqrt{5/4}$) (Panetta, 1987, p. 16).

Thus, in summary the temperament is regular,[88] with smaller fifths and major thirds equally divided in two equal intervals. Meantone temperament, again, is not a single tuning but a family of musical scales that share some characteristics:

> In all mean-tone temperaments the diatonic semitone is larger than the chromatic semitone, so that E♭ is higher than D♯, A♭ higher than G♯ and so forth; and a diminished 7th (e.g. G♯–F) is larger than a major 6th (A♭–F), a diminished 4th (G♯–C) larger than a major 3rd (A♭–C), etc (Lindley, 2001, para. 3).

The concept of meantone may be used with three different meanings: in the case of a set temperament with pure major thirds divided into two equal whole tones; to designate a family of temperaments; or for a mathematical mean pitch (tone) resulting from certain mathematical operations with intervals (Monzo, 2005a).

As stated, in the basic meantone temperament design there are only two ─rank two─ generating elements: the octave (2:1) and the perfect major third (5:4 = 386 cents), all other are derived or deducted from these two, for that reason it has been called a tuning by major thirds (Hardegree, 2001, p. 12).

To construct this scale, the first step is to set all major thirds (386 cents): F–A, C–E, G–B. Then, each major third is divided into two equal intervals (each one $\sqrt{5/4}$) that is approximately 193 cents). The only remaining values missing are now the half step intervals of B–C' and E–F, but as C–C' is 2:1 = 1,200 cents, then these intervals must equal 117 cents.

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[88] «Regular» is considered to be a temperament in which all frequency ratios can be obtained as a product of powers of finite numbers, called generating frequency ratios, or simply generators. When only two generators are needed and one of them is ½ (octave), the regular temperament is called linear. The meantone is the best-known example. A disquisition over musical linear temperaments ─relating them to the abstract algebra concept of abelian or commutative group, the Prüfer or torsion rank, and related questions─ is out of scope. Here simply, a rank will be equivalent to the number of independent generators. Detailed explanations of construction of musical scales using generators and their associated concepts such as rational continuum, layout-mapping, dynamic tuning and others may be found elsewhere (Milne, Sethares. & Plamondon 2007). A review of temporary linearity in music may be found in Kramer (1986).

The problem with this scheme is that it overcorrects the fifths. Every four fifths have a reduction of one syntonic comma, and in a full cycle of 12, the reduction is three times that value, $(80/81)^3$ (about 0.96342, instead of the expected value of 1) with a large wolf needed to close the fifth circle (wolf 5th).

To reduce the wolf, several variations have been developed. Probably the most popular tuning was the one-quarter-comma meantone, although there were other trendy members of the meantone family with alternative variants such as one-third-comma, or the two-sevenths-comma (Panneta, 1987, p. 14–20).

An early description of one-quarter-comma meantone is found in the treatise of the Florentine Piero Aron (ca. 1480 – after 1545) *Toscanello de la Musica* (1523).[89] In this method, the perfect fifth is flattened by one quarter of a syntonic comma, with respect to just intonation, and the major third is thus justly intonated to 5/4.

Several theoreticians including Zarlino himself in his *Dimostrationi Harmoniche* (1571) and Francisco Salinas in *De Musica Libri Septem* (1577) have made mathematical contributions to explain the benefits of the one-quarter-comma meantone tuning (Panneta, 1987, p.19).

That century (16th) and the following, were very active in the search for a solution to the problem of the circle of fifths, with important contributions from Giambattista Benedetti (1563); Guillaume Costeley spreading the meantone tuning to France with his *Musique* (1570); Pietro Cerone with his impressive 22 volumes of *El Melopeo y Maestro* (1613); Michael Praetorius with his *Syntagma Musicum,* a work in three volumes published in 1620; and many others such as Jehan Titelouze; Elias Nikolaus Ammerbach (Orgel oder Instrument Tabulaturbuch [Treatise of organ and other instruments tablature]); and Giovanni Paolo Cima (Lindley, 2009, section Regular mean-tone temperaments to 1600, para. 4).

The one-quarter comma meantone temperament is presented in Table 17; values in bold, correspond to those of Kelletat (1982, p. 150).

This gamma ─except for minor second, where ratio 1.06992 has been preferred─ is one of the temperaments used in this dissertation for comparative analyses (Hardegree, 2001; Monzo, 2005).

[89] Although even in this link, as in other references, the title is given as Thoscanello, and the author's name as Pietro Aaron, in the original text, as appreciated in the facsimile itself, the spelling is as stated above.

The simple meantone temperament has been reported as a being a suitable tuning for Bach compositions:

> Even the Bach example, originally chosen because I knew it included both G# and A♭, went very well in meantone because the G#'s (as well as some D♭'s) appeared only very ephemerally, never joining intervals to serve strong harmonic functions as did the A♭'s (and C#'s) (Hall, 1973, pp. 286–287).

It has been stated that during the active historical period of scale building, some particularly dissonant intervals were found, being called wolf intervals.

Wolf intervals are characteristic features present in most tunings of the meantone family. “While some intervals are harmonious, others, such as the wolf fifth, set our teeth on edge” (Loy, 2006, p. 55).

In the one-quarter-comma meantone temperament, as commented on, one typical wolf interval is the wolf fifth ─classified as diminished sixth─ spanning seven semitones.

This interval is extremely dissonant and sounds like a howl of a wolf, from which it takes its name. By extension, all intervals that depart from just intonation are considered wolf intervals.

In the one-quarter-comma meantone, significant differences with JI intervals are found in minor 7th, diminished 5th, augmented 4th, and major 2nd, as appreciated in Table 18.

Well-temperaments. This expression has two different meanings. In the way used by Bach in his *Das Wohltemperierte Klavier* [The well-tempered clavier] and musicians in the seventeenth century it means a family (once more, not a specific scheme) of tone systems, developed about 1680 – 1885.[90]

The second meaning for “well-temperament” is the one used in twentieth century musical theory. Details and comparative historical information may be found in the impressive publishing activity on the question of Herbert Kelletat from 1960 to 1984.[91]

As per professor Jan Charles Haluška (2004) a well-temperament, in the broadest sense, is any temperament that permits all relevant intervals in a given musical style playable in any transposition (a temperament “avoiding wolves”) (p.

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[90] A relation of 268 different well temperaments may be found at http://www.hpschd.nu/tech/tun/tmp-well-tempered.html (retrieved May 14, 2018). The site also offers software for the different tunings.

[91] Publication and edition activity may be found at http://www.worldcat.org/identities/lccn-n87-832543/

183) or in other words a tuning allowing to play all major and minor keys without perceiving any note out of tune: “Historical well temperaments generalize the subclass of meantones having a sequence of 11 equally tempered fifth and one single wolf fifth” (p. 175).

The author (Haluška) provides a complete mathematical definition of well-temperament scales used in Bach and Mozart periods,[92] including many examples of Werckmeister tunings, the tonal system devised by Charles Fisk and Harald Vogel, and the irregular temperaments of Kirnberger and Vallotti-Tartini.

Johan M. Broekaert, has reviewed numerical aspects related to tuning and musical scales, and based on Kelletat (1981, 1982), offers another definition for well-temperament:

> Well temperaments are temperaments wherefore the diatonic scale of C-major has an impurity lying between an assignable absolute mathematical minimum, and a maximum corresponding to the value for the Equal Temperament, whereby no major thirds larger than Pythagorean thirds are allowed, and fifths must have a ratio between 1.491 and 1.509 (Broekaert, 2018).

The basic idea behind all these developments is to solve the three problems of the scales commented on above.

They fulfil the criteria for using any tonality and chord, have enharmonic flats and sharps, an absence of wolf intervals, and the possibility of modulation, although with better results for three or fewer key signature accidentals.

Irregular well-tempered tunings arose almost at the same time as meantones but developed slowly, being introduced customarily only by the end of seventeenth century (Padilla, 2016).

Andreas Werckmeister (1645 – 1706) proposed several such tuning systems, the most popular probably being the [Werckmeister] III (1691), that Bach seemed to use in some of his compositions.

As per Kelletat (1981) well-temperament means the establishment of an acoustic-mathematic, practical musical tonal system within a scale of 12 tones allowing a perfect use of all keys based on the natural harmonic system with diatonic intervals respected as much as possible. There occurs as proportional-bound slight

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[92] Werckmeister (1681) in *Orgelprobe* [Organ examples] was probably the first to use the term well-temperament, for a pleasant harmony, as indicated in the title’s page (paraphrased from German): Lessons on how to temper and tune a piano through the instructions and assistance of monochord, so that according to today's style one hears all *modos fictos* in a bearable and pleasant harmony (Norrback, 2002, p. 18).

widening or stretching of the meantone systems, with uneven half-tone but with equal temperament (paraphrased text from German, p. 9).

The underlying idea was to combine Pythagorean and tempered intervals, having as basis the Pythagorean comma (23.46 cents, or its corresponding ratio 1.01364326). The Pythagorean comma impurity is distributed among four fifths (C–G, G–D, D–A, & B–F#) to preserve the basic consonant intervals:

> In the earliest surviving edition of his Musicalische Temperatur, […] Werckmeister gives precise details of a number of temperaments. After just intonation and meantone, he writes down a temperament usually called Werckmeister III in which the fifths C–G, G–D, D–A, and B–F# are each narrowed by one-quarter Pythagorean comma while all the other fifths are pure […] The sum of the errors of all the fifths is always a Pythagorean comma, […] or approximately 23.5 cents. By approximating further and calling the Pythagorean comma 24 cents, the numbers are greatly simplified without introducing errors of practical importance (Barnes, 1979, p. 239).

Per Barnes, narrowing of fifths may be none or about six cents, and widening of major thirds range from 4 to 22 cents.

This scheme has the obvious consequence that not all the tunings have the same characteristics, and consequently the chords may rank from the best combination (F) with perfect fifth and mayor third only four cents too wide, until the major B triad, results in a fifth six cents too narrow while the third is sixteen cents too wide. Pure fifths are found in F#, C#, and G# but the thirds are 22 cents too wide (Barnes, 1979, p. 239).[93]

The Figure 55 shows the differences resulting in cents of the different tunings of the Werckmeister III scale.[94]

The well-tuned piano of the minimalist composter La Monte Young (1935 –), which “may well be the most important American piano work since Charles Ives's Concord Sonata-in size, in influence, and in revolutionary innovation,” was an initially secret tuning that Young started using in 1964, based on E♭ 1/1. The cents series are: 0 (E♭)–177–204– 240–471–444–675–702–738–969–942–1173 (Gann, 1993, p. 134). This scheme has no relation with Thomas Young (1773 – 1829) tunings.

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[93] This reference also includes extended information about purity and error figures, based on his own work and on those of Kelletat and Kellner.

[94] Thanks to Peter A. Frazer for his authorization to reproduce the data. By left or right clicking at each corresponding cell of the original table, (whose link appears at the foot of the figure), it is possible to hear the notes as melodic or harmonic intervals.

Per Chuckrow (2012) The tuning (Thomas Young II) is summarised as follows: C0 – G (-1/6) – D (-1/6) – A (-1/6) – E (-1/6) – B (-1/6) – G♭0 – D♭0 –A♭0 – E♭0 – B♭0 – F0 – C0. Table 19 depicts the original scheme from F and the transposition to C (Chuckrow, 2012). The table also includes the values for A = 440 Hz; some figures have been recalculated as in the reference text there are some printing errors.

Kirnberger III was another very popular well-tempered scheme: in this version, the syntonic comma impurity is distributed (1/4 comma each) among four fifths. C–E remains as a pure interval.

Following Padilla (2016b), the characteristics of this temperament are:

- All keys and chords are usable.
- The flats and sharps are enharmonic.
- The tonalities and chords with fewer alterations have the most rested and pure sonority.
- The tonalities and chords with the greatest number of alterations have harsher and more characteristic sonorities.
- The tonalities present "colours" that distinguish them.
- Modulation to any tonal region in all tones is possible.
- There are no "wolf" intervals.
- It divides the Pythagorean coma irregularly between four fifths: C–G, G–D, D–A, A–E.
- Major third C–E is a pure interval.
- The colours of the tonalities with few alterations, either sharp or flat, are very close.

The Kirnberger III scheme is presented in Table 20 where G, D, A, & E fifths have been reduced by (1/4 comma each), the remaining fifths are tuned in Pythagorean steps down from C and up from E.

The system has about two cents error, which is well under the JND, and with enharmonic notes. Monzo’s reference includes a table with the matrix in cents calculated for all the chromatic scales of this temperament (2005c).

The series (in cents) for 1/2-comma version are: 0–90–204–294–386–498–590–702–792–895–996–1088–1200 (Barbour, 2004, p. 133).

Francesco Antonio Vallotti (1697 – 1780) was a Franciscan priest, composer, theorist and organist that became the maestro in St. Antonio in Padua. There, he met another theorist, violinist and composer Giuseppe Tartini, and they joined to produce the so-called Vallotti-Tartini temperament (Padilla, 2016a).

The Pythagorean comma impurity is symmetrically distributed here into six fifths, resulting in six fifths pure and six tempered in 1/6 of comma (about 3.91 cents): F (-1/6) – C (-1/6) – G (-1/6) – D (-1/6) – A (-1/6) – E (-1/6) – B – F# – C# – G# – E♭ – B♭ – F (Dolata, 2016, p. 117; Jorgensen, 1991, p. 72; Schneider, 2017, p. 424).

The schema (Table 21) is very simple to put into practice, resulting in a comfortable temperament with sharps and flats enharmonic, without wolf fifth and able to modulate to any tonality, having remarkable success.

Vallotti's description of the 1/6 comma temperament appears in his second book of the treatise: *Della Scienza Teorica e Pratica della Moderna Musica* [On the theoretical and practical science of modern music] (Padua, 1779).

Although it seems that he worked on the details for about fifty years, the text was published just the year prior to his death; even so, it seems that the diffusion of the scheme had to wait until the mid-twentieth century.

> Today's most popular 1/6-comma-based irregular keyboard temperament was designed by Francesco Antonio Vallotti (1697 ─ 1780). Although Vallotti claimed to have devised it in 1728, and it was referred to in 1754 by the violinist Giuseppe Tartini (1692─1770), who worked with Vallotti in Padua, it was not actually published until 1950 (Dolata, 2016, p.117).

The schema is frequently equated to Young II temperament:

> Vallotti is often uttered in the same breath as Young, as in Vallotti/Young, as if they are the same temperament. They're not. One of the temperaments of Thomas Young (1773─1829) devised in 1800, his No.2 is similar to Vallotti's with the difference that it tempers a series of fifths by 1/6 Pythagorean comma starting on C rather than F with rest of the fifths pure. Although some consider Young to be a transposition of Vallotti, Young seems to have been totally unaware of Vallotti's temperament, which calls into question whether Vallotti's temperament ever achieved any widespread use beyond the confines of Padua (Dolata, 2016, pp. 117–118).

As indicated, Kelletat series *Zur Musikalischen Temperature* [On musical temperament] volume one (1981) & two (1982)[95] provide more data on these and other well-tempered tunings such as Silbermann, Mattheson, Euler, Anonymous, Malcolm,

[95] There is a third volume (1994) of the series basically dedicated to Romanticism/Schubert.

Kirnberger, I, II, Werckmeisnter I, II, III, IV, Neidhardt I, II, III, including tables of comparation; classical period temperaments are covered/presented in volume two.

Johan Norrback's book (2002) offers additional information for temperaments in organs, and a half hundred audio-files of Bach's compositions for organ (BWV 540, 542, 544, 552, 588 & 656) and Jorgensen (1991) provides further insights into eighteenth and nineteenth centuries tunings. The historical overview of temperament tuning is also covered by Barbour (2004) and in a later section in this chapter.

Equally tempered scales. Towards the end of the nineteenth century, the use of the 12-TET scale began to generalize, although it represented only a small step over the quasi-equal-tempered gamut used in the previous decades of Romanticism.

The construction of pianos with the 12-TET scale contributed to setting the scheme. A preeminent supporter of this tuning was the Bostonian instrument builder Edward Quincy Norton, and his publication "Construction, Tuning and Care of the Piano-forte" (Norton, 1887).

> Because of the dominance of the piano in musical culture, even its tuning affected musical life generally. Edward Quincy Norton produced an equal-tempered piano tuning by 1887, partly in response to the growing difficulty of hearing the upper harmonics of the strings and therefore of achieving any mean-tempered system. This new system of tuning pianos made a difference to the tuning of other instruments, especially when they were used with the piano, and it created an ideal medium for the creation of atonal and twelve-tone music (Parakilas, 2001, p.158).

The early approaches to this temperament, however, must be traced many centuries back. As per Rechberger the origin of the "diatonic-chromatic" scheme goes back to Aristoxenus in Ancient Greece, a school that, as per the author, took many ideas from the Babylonians (2018, p. 21).

The equal division of the octave, the spiral of fifths, and the commas, seems to have been known in China since very early times:

> Next in line comes Ching Fang, a diviner, mathematician, astronomer and acoustician of the Former Han Dynasty (202 B.C. – 9 A.D.) who flourished around 45 B.C. He extended the traditional up-and-down principle from 12 to 60 steps of perfect fifths and fourths, creating a spiral of five arcs defining 60 microtonic intervals. Selecting the 12 pitches among the 60 which came closest to the quantities of an equal temperament then only dimly surmised, he achieved a creditable approximation to the theoretically correct values (Kuttner, 1975, p. 172).

Later, The Chinese increased the number of divisions to have one tune per each day of the year "Fang, calculated a precise subdivision of the octave into 60 micro-intervals, a feat that was raised to 360 subdivisions by another theorist around 400 A.D" (Kuttner, 1964, p. 124)

> The work of Ho Ch'eng-t'ien [seventh century CE] proves that the 360-division tunings were developed within the context of a concern with temperament and with perfectly cyclic tunings─and awareness that pure fifths 3:2 and pure fourths 4:3 produce only spirals (McClain, & Shui-Hung, 1979, p. 219).

As per Kuttner (1975), "Chronologically, there is no doubt that Prince Chu was the first to offer, in 1584, a nine-digit monochord of the 12 pitches of equal temperament" (p. 172). The author refers to Chinese non-royal prince of the Ming dynasty Zhu Zaiyu, or Chu Tsai-Yü (朱載堉) (1536 – 1611).

Helmholtz[96] recognized the precise description of the musical scale of 12 tones equal-tempered done by prince Zhu Zaiyu ─an erudite in music, choreography, mathematics and physics─ in the sixteenth century. His detailed calculations may be found on the third exhibit room of the Zhu-Zaiyu Memorial Hall.[97]

In Europe, and about the same epoch, the values of the scale were calculated by the Dutch scientist Simon Stevin (1548-1620). However, his unfinished manuscript *Van de Spiegheling der Singconst* [Reflections on the art of singing] "was drafted in the 1580s, subsequently rewritten during the 1610s, but was eventually left unpublished at the author's death in 1620" (Rusch, 2006, p. 205). The publication came out in the late nineteenth century.

According to this author (Rusch) it contains several errors: "Stevin's equal-tempered monochord, however ingeniously calculated for its time, is of rather poor quality. Many figures are one or two units off what they should be" (2006, p. 205).[98]

More comments about Zhu Zaiyu (or Chu Tsai-Yü) contributions to equal temperament theory may be found in Huynh (2012), and Kuttner (1975), including outer and inner diameters of the Lü pipes (Kuttner, 1975, pp. 176, 177, & 191)

> Anyway, his five maximum deviations from "ideal" equally tempered pitches are between 1.77 and 1.07 cents, while all other errors are of the order of 0.5 and 0.75 cents. This is so close to maximum tuning accuracy by the finest craftsmen that, from a practical point of view, no improvement is possible or necessary (Kuttner, 1975, p. 173).

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[96] In *Die Lehre von den Tonempfindungen als Physiologische Grundlage für die Theorie der Musik* [The theory of sound sensations as a physiological basis for the theory of music], 3rd edition (1895), p. 258.

[97] This hall was built over the remains of the court palace of «Marquis» Yi of Zeng (Suizhou, Hubei) (Huynh, 2012, p. 49).

[98] Stevin reluctancy to write in Latin probably played a role in the limited diffusion of his work.

The precision of China's developments in relation to equal temperament may be well appreciated from a comparison of the data of three Chinese cyclic tunings based on 12-tone schemas reported by McClain & Shui-Hung (1979): (i) the Huai NanTzu twelve division (ca. 122 BCE); (ii) the Ching Frang (78 – 37 BCE) based on the 60 division of the octave Hou Han Shu; and (iii) the Ch'ien Lo-chih (ca. 450 CE), a reconstruction based on the 360 division. The biggest deviations reported are 19.55, 9.79 and 1.46 cents respectively. Ch'ien Lo-chih scheme total deviation was 8.47 cents with median deviation of 0.77 cents (p. 221).

Renaissance European authors, following the Aristoxenus wake, also showed interest in equal temperament schemes. Vincenzo Galilei suggested a semitone of 99 cents for the lute (Galilei, 2003).[99]

> […] it appears to have been used on fretted instruments such as the lute and viol at least since the early 16th century […]. The ratio 18:17, familiar to theorists from well before the Renaissance and recommended by Vincenzo Galilei in 1581 for equal temperament on the lute, corresponds mathematically to a semitone of 99 cents, virtually indistinguishable from the 100-cents semitone of equal temperament (Lindley, 2009, Equal temperament to 1735, para. 1).

Although no proposal for a keyboard instrument tuned to 12-TET has been found from that period, its use in fretted instruments such as lute and viol, may be traced back at least to early sixteenth century; "some 16th-century composers appreciated the enharmonic advantages of equal temperament" (Lindley, 2009, sec. Equal temperament to 1735, para. 1).

Following Lindley, musicians of the epoch interested in equal temperament included: Willaert (1490 – 1562), Orso (1522 – 1281), Zarlino himself (1517 – 1590), and others of subsequent decades as Frescobaldi (1583 – 1643), or Froberger (1616 – 1667).[100]

The author (Lindley) adds:

> The practical history of equal temperament, then, is largely a matter of its refinement in various respects and its gradual acceptance by keyboard musicians from the late 1630s, when Frescobaldi endorsed it, to the 1870s, by which time even the conservative English cathedrals were won over (Equal temperament to 1735, para. 1).

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[99] This lutenist, and theorist, former pupil of Zarlino, was the father of Galileo Galilei. It is not unlikely that his *Dialogue on Ancient and Modern Music* [translated to English by late professor of Yale Claude V. Palisca] could have influenced other music theorists of his time, including Simon Stevin.

[100] Dates are included only with the intention of better following the interest for equal temperament over the years; accuracy has not been contrasted with individual biographical studies.

By the turn of the seventeenth century, several German theorists such as Werckmeister (1645 – 1706), Meckenheuser (1666 – ca. 1726), Neidhardt (ca. 1685 – 1739) or Mattheson (1681 – 1764), also became interested in the equal temperament (Lindely, 2009).

During the age of Enlightenment, a renovated interest was seen in the "rational" or "reasonable" equal tempered scale:

> In 1749, the register of the Academie Royale des Sciences in Paris reported that Jean-Philipppe Rameau's support of equal temperament was cause for serious consideration. During a lifetime of debate, he managed to convince many others of its possibilities as well. Indeed, his revolutionary approach to music theory and its artistic implications edged the next generations toward equal temperament's inevitable adoption (Isacoff, 2003, p. 224).

The idea was also considered by Mozart (1756 – 1791), Vogler (1749 – 814), Türk (1750 – 1813), Hummel (1778 – 1837), and others.

> The musicologist Thomas McGeary surveyed twenty-two tuning manuals and keyboard tutors published in the eighteenth and nineteenth centuries ─discarding works that merely theoretical in favour of practical, "how-to" methods─ and came to the conclusion that the equal temperament was the prevailing choice (Isacoff, 2003, p. 242).[101]

Finally, the 12-tone equal tempered scale with twelve equal steps of 100 cents was set, after so many and such long-lasting trials on interval adjustments "following various mathematical schemes to compensate for the slight intervallic impurities produced by strict adherence to Pythagorean ratios" (Johnson, 2008, pp. 2–3).

Before leaving the section of equal temperament, some words are required to comment on other equal tempered scales and the "quasi-equal" temperaments.

Jorgensen (1991) labels as quasi-equal temperaments those schemas made with the clear intention of becoming equal tempered but that nowadays "resulted in something below today's standards for that tuning" (Isacoff, 2003, p. 241).

The approach to equal tempered scales, either with a 12-tone, or with another division, requires setting some additional mathematical concepts.

A fixed interval scheme should not be confused with equal interval division. The combination of fixed intervals, for example in steps and half-steps, if they are not multiples of each other, does not constitute an equal division.

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[101] See McGeary (1989) original reference for details.

E.g., the 747 Golden Scale is divided systematically in fixed steps of 108.204 Hz and half-steps of 66.873 Hz, but as the ratio of those values is not an integer number, it is not of EDO (Marco-Franco, 2018, p. 35).

An EDO scale must have equal intervals, no matter what considerations or names are later given to two or more joined intervals. The scale of 12-TET is formed by tones and semitones, but a tone is exactly equal to two semitones, and it can be considered as a division of the octave in 12 equal half tones. If that was not the case, it would be a case of *systematic division*, but not *equal division* (EDO) of the octave.

When using a procedure for equally-tempered scales comparison, it must be considered whether the method ─for example the root-mean-square deviation─ is applicable to the interval range or not. E.g., Pierce's scale of 14 steps is equally divided in terms of one fixed step of 143.31 Hz, but uses a double octave, thus the scale is not based on the octave, and it is not an equally divided *octave* (EDO) but equally divided *tritave* (Marco-Franco, 2008, p. 43).

Methods designed for EDO analyses may not be appropriate for this case, or for other equal division schemes not based on the octave. All EDO schemes follow a logarithmic progression of frequencies with a ratio between each note computed with the formula $r_n= \sqrt[n]{2}$ (the nth root of 2), with n equalling the number of tones in the scale. This formula is not applicable (e.g.,) to Pierce's tritave design.

The ratio (r) of one semitone in 12-TET (one step) is computed with the formula: $r_{12}= \sqrt[12]{2} = 1.0594630943593$. The frequency of the next note is computed by multiplying the previous frequency by this ratio.

Alternatively, each frequency may also be determined by multiplying its own ratio, obtained with the formula $r_n= \sqrt[n/12]{2}$, by the root frequency.

The semitone may be further divided into 100 cents, each with a ratio: C = $\sqrt[100]{1.0595} = \sqrt[1200]{2}$ = 1.00057778950655 (Howard, & Angus, 2017, p. 169).

> Xenharmonic is a term coined by Ivor Darreg from the Greek words "xenos," meaning strange, foreign, or inhospitable, and "harmonikos," (harmonic, etc). Darreg used it to refer to any non-12-EDO (microtonal) scales, which presented strange and wonderful new intervals and sonic worlds to explore (Rechberger, 2018, p. 218 [footnote]).

There are reports of equally-tempered scales up to 100 steps or more.[102]

[102] It must be considered that as the JND results in above 8 cents, there is a limit to the number of notes in the scale (around 150) that may be differentiated by the ear in normal conditions.

It is interesting to mention the 6-TET, which is also called the whole-tone scale with all intervals equal to 200 cents, while others such as the 10-TET have no intervals near the familiar fifths or thirds (Rechberger, 2018, p. 218). From 12-EDO onwards, the scales are microtonal.

The antiquity of references to the 24-TET scale is remarkable ($r_{24} = 1.029302$). “The earliest confirmed written reference to an interval of this size range was by Philolaus (ca. 480 – 385 BC), who called it ‘diachisma’” (Rechberger, 2018, p. 240).

As will be commented on later, this scheme is being used in MECul for modern (20th century) maqãm music.

The roll of EuCul contemporary musicians that also have used the 24-tone gamma include: Charles Ives, Richard Stein, Ivan Wyschnegradsky, Alois Hába ─the latter designed several instruments built for it─ Arnold Schoenberg, Anton Webern, Alban Berg, and others.

31-EDO was used in the Renaissance; in 1555 “Nicola Vicentino produced a 31-step keyboard instrument, the archicembalo” (Rechberger, 2018, p. 244).

The 34-EDO (34-TET) will be detailed later, since this is the scale on which the research is focused.

40-EDO has been found in Indonesian gamelan tunings (Rechberger, 2018, p. 253).

43-EDO is close to 1/5 comma meantone temperament:

> Its thirds and fifths have an equal and opposite error of slightly over four cents, thus making it somewhat inferior to the 34-division, although the equality of the error may have some weight in ranking the two systems. Since 43 is a number occurring in a useful series for multiple division ─12,19,31,43,55…─ this division was treated by Romieu, Opelt, Drobisch and Bosanquet (Barbour, 2004, p.22).

The 72-EDO has been used by many microtonalists (Ezra Sims, Joe Maneri, etc); it fits very well with just intonation intervals (Rechberger, 2018, p. 259).

A list of many authors interested in the different equal tempered scales may be found in Monzo et al. (2013); however, this information has some inconsistencies: e.g., it is mentioned that Dirk de Klerk used the 34-TET scale, but according to the author’s own information he did not.[103]

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[103] Personal communication, August 20, 2018, with thanks.

Contemporary trends. In the nineteenth and twentieth centuries, additional types of scales were explored: the chromatic scale (twelve notes); the whole-tone scale (six notes or hexatonic) already mentioned; the pentatonic scale (five notes); the octatonic or diminished scales (eight notes); the Phrygian dominant scales (actually, a mode of the harmonic minor scale); the Oriental or Pseudo-Oriental scales; and many others, such as those used in Folk music traditions, e.g. Hungarian minor scale, Gypsy, scales, Greek scales, etc. (Rechberger, 2018, p. 26).[104]

A musical gamut may also be represented using geometrical figures, with circular or spherical models and vectors.

Zarlino (1571)[105] himself used graphics to assist his work. Computers and algorithms have also been used for complex analyses particularly of post-tonal music (Cope, 2009; McLean, & Dean, 2018).

Summarising this long race, the Pythagorean temperaments favour the perfect fifths, but resulted in problems with the thirds; the just intonation is harmonious, but it did not allow modulation, requiring different tunings for each key; the meantone temperaments improved the thirds, but gave rise to the wolf intervals; the well-temperament allows playing in any key, but they are not equal tempered; and the equal-tempered solution moves significantly away from consonance in benefit of modulation, making it possible to play in any key with a single tuning.

Each scheme pays a price, but the 12-TET was so far the best compromise, although it paid the price of an impurity up to 15.67 cents in a consonant interval, as detailed elsewhere in this dissertation.

More about microtonal scales will be commented on later.

A comprehensive account of musical scales in other cultures would be out of place here. However, some references in this regard are needed, particularly when this research holds that the 34-tone scale has good possibilities for approaching other musical cultures out of the European art music heritage, and samples corresponding to Middle East and Asian musical cultures are included in this study.

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[104] An extended analysis on tuning and temperament all over the planet may be found in the works by Michael Hewitt (2013), J. Murray Barbour (2004), and Herman Rechberger (2018).

[105] E.g. pp. 68, 112, 161–163, 166–168…etc.

The following lines concentrate only on those cultural settings that have samples in this research.

Temperaments in other cultures, and less frequently used scales. Moving out of the above-mentioned schemata, many diapasons may be found:

Pentatonic scales. Any gamut with five tones should be considered a pentatonic scale, but the name is usually applied to a scheme following fifths, without semitones (*anhemitonic*). That is, e.g., starting in C, then its fifth (G), then the fifth of G (D), and so on. After ordering, the schema comes to: C–D–E–G–A: “the strict sense of the term pentatonic is justified, given the rarity of hemitonic pentatonic scales (e.g. the Japanese *In* or the Korean *Kyemyŏnjo*)” (Day-O`Donell, 2001, para. 1–2).[106]

> Once described variously as the ‘Chinese scale’ or the ‘Scotch scale’, the pentatonic scale has impressed commentators since at least the mid-19th century for its astonishing ubiquity. A significant feature of such diverse musical traditions as those of the British Isles, West Africa and Amerindian America (among countless others), pentatonicism may well be a musical universal […] (Day-O`Donell, 2001, para. 3).

Pentatonic scales have been used in a variety of styles and genres, both in religious (spiritual), and in popular expressions, with a wide usage in art music, rock or jazz. They are found in African slavery songs, and in Asian cultures. “A common scale in Chinese, Japanese, Korean and South-East Asian (Vietnam partly excluded) music follows the principles of the non-tempered pentatonic scale” (Rechberger, 2018, p. 92).

However, there are innumerable variations in how pentatonicism is approached by the diverse cultures, and through the ages. According to Henry (2010), it may be traced back in Africa, Asia, the Native Americans, and Europe (including melodies from Scotland and England).

As per Rechberger (2018), the Bacovian pentatonic scale may be found in Romanian and Moldavian folk music (p. 32); in the *Chad Gadyo* (jiddish) song “constructed on the five notes of a natural minor scale” (p. 33); in the Chinese scale F, G, A, C, D (p. 193);[107] in Celtic folk; in Northwest Greece; in Southern Albania; in “Polish fold music from the Tatra Mountains”; in Ethiopian string instrument *Krar;*

[106] *In* scale is also called Sakura after the popular spring song Sakura-Sakura [cherry blossoms].

[107] Chinese ancient baseline tonality was set in a half of the octave (that is F# if counted from C) as the baseline standard for art music (von Falkenhausen, 1993, p. 255).

and in the (non-tempered) Indonesian gamelan. “The melodies of Japanese, Chinese and Korean Classical music also use the pentatonic scale material” (p. 31).

Another remarkable use of the pentatonic scale is in jazz music:

> Pentatonic scales, as used in jazz, are five note scales made up of major seconds and minor thirds. Within a scale there are two minor thirds leaps in an octave, thus producing a gap […]. In addition there is no leading tone [..] nor, for that matter, any half step within the scale. For these reasons, the scales act as chords, and are invertible […] each pentatonic has five possible inversions […] With five possible modes and twelve half steps in an octave, there exist sixty different pentatonics. To have every pentatonic at full command the student should be able to pay [*sic*] five different pentatonics from each note in the chromatic scale. A tall order for anyone! (Ricker, 1976, p. 2).

Per Day-O’Connell (2001):

> […] the Chinese system – for which the very propriety of the term ‘pentatonic’ has been questioned […] – is based on a universe of 12 perfect 5ths, featuring a pentatonic ‘core’ plus two ‘exchange tones’ (bianyin), embellishments that in both theory and practice fill in the minor 3rds […]. The sléndro tuning of Javanese gamelans is pentatonic, though in this case the intervals are more nearly equidistant (para. 3).

For this authorized voice ─Chairman of Music Department at the Arts Skidmore College [Saratoga Springs, N. York] selected for his long trajectory in the research of pentatonism by the Grove Music Online editors for the “pentatonic” term─ the universal hypothesis seems convincing as well as the historical importance of the pentatonic scale:

> Still, such modal and tuning issues notwithstanding, the universalist hypothesis seems compelling, and although the question of primordial scales is far from resolved, a general consensus does exist concerning at least the importance of the pentatonic in the history of music (Day-O`Connell, 2001, para. 4).

The scale ─used in learning methods as the Orff system─ is considered very appropriate for improvisation and children’s tuition. “Orff instruments make often use of xylophones and metallophones with removable bars, leaving only those corresponding to the pentatonic scale” (Rechberger, 2018, p. 33).

Musical scales of Middle East cultures (MECul). Some of the traditions in the area, and particularly Arabic, Turkish, and Persian music traditions span over two thousand years.

They include instrumental and vocal, secular and sacred, containing important features of artistic inspiration, and improvisation (Touma, 2003), with pitch notation (*Abjad*) traced back to the ninth century (Yarman, 2007, p. 52).

The following lines will focus on *maqãm (makam*) as the most representative of those traditions. A maqãm (plural *maqãmãt* but frequently used *maqãms* instead) is a traditional MECul set of relationships between the notes keeping a pattern.[108]

> Makam in Turkey, and homonymously elsewhere across the Middle East from Morocco to Uyghur autonomous region of China, designates a musical mode, or a family of kindred modes, consisting of a set of "more or less" fluid pitches (called *perdeler*), with distinctly embedded intonation (*baskı*) and inflexion (*kaydırma, oynaklık*) attributes, the entirety of which remains dependent on the classical rules of thematic flow (called *seyir*) (Yarman, & Rakaosmanoğlu, 2014, p. 175).

Maqãm (makam in Turkey) is best defined and understood in the context of the mentioned rich MECul music repertoire. The nearest equivalent term in EuCul would be a modal system.

Maqãm modes are organized in classes or families. The world maqãm means place, or position. It is not restricted to one single country or culture:

> I adopt the "Maqam Music" nomenclature for this reason. The saying "Turkish Maqam Music" ought not imply different instrumental ensembles, performance styles and manners, but – just as with Classical-Contemporary Western Music – the repertory based on maqams which are more or less the equivalent of keys/modes. Thereby, when the "Turkish" prefix is omitted, we may consider the works also based on maqams/dastgahs by neighbouring nations under the umbrella of "Maqam Music" – just as is the case with Brahms, Shostakovich, and Copland making Classical-Contemporary Western Music despite the fact that they belonged to different nations. Hence, Abd al-wahhab and Umm Kulthum become part of our multicultural identity. Moreover, it therefore becomes possible to evaluate our Armenian, Greek, and Jewish musicians outside of the scope of a Turkish nationalism that, from time to time, borders chauvanism [*sic*].
>
> As an additional reason […] no racial/ethnic distinction can be made between westernized modernist Turks and conservative Turks who uphold Eastern values, and because the East-West conflict still dominates a huge part of our lives, only saying "Turkish Music" does unfortunately nothing but confuse minds.
> (Yarman, 2008, p. 2).

The (maqãm) modal structure is basically a melodic organization; there are over 200 maqãmãt, with 74 main rows or scales, each one made of a series of consecutive notes. These series are called *jins* (plural *ajnās*) coming from the Greek word genus, meaning gender or kind, with the same meaning as in early EuCul modal music (as plainchant for example).

The majority of ajnās are tetrachords, but there are also trichords and pentachords (Rechberger, 2018, p. 93):

> In current Middle Eastern ─and particularly Arab─ music theory, modes are conceived in terms of a combination of elements: scale, which consists of a

[108] My warmest and very special gratitude goes again to Prof. Dr. Ozan Yarman from Istanbul University for his great assistance and material provision for this section. Translations of Turkish references come from him.

> juxtaposition of tetrachords (or pentachords) ─known in Arabic as ajnās (sing. *jins*)─determined by inervallic succession, register, and final note, and in some cases characteristic melodic features (such as cadential formulas) (Jarjour,2018, p. 84).

Each scale has a tradition defining the melodic development, the mood, the note sequences, modulation, etc.

As indicated in its name, maqãm only describes the relations (spatial) between the notes, creating a melody or mood, but there is no rhythmic organization. Most is left to improvisation. Pitches may be microtonal, and their frequencies are not necessarily equally related.

The maqãm includes the starting, ending, dominant and emphasized or prominent notes (leading note) among other characteristics.

The learning of maqãm is basically auditory:

> […] what defines maqam in the reciter's mind is not the intervallic structure of the maqam or even the stepwise catalogue of notes, but rather the characteristic phrases and variations, activated only when put into play by the human voice (Rasmussen, 2009, p. 74).

Although the different theoreticians do not reach a consensus on the classification of the ajnās, there is much of agreement about the eight basic schemas; using the EuCul stave system there are: D-based (*Hijāz, Kurd and Bayyati families*); C-based (*Nahawand*, *Ras*t, and *Nawa Athar* families); *Ajam* family based on B♭; and *Sikah* family based on E.

Alternatively, *Saba* and *Nakriz (Nikriz)* have also been considered apart from their respective families of *Bayyati* and *Nawa Athar*.

There are also moods traditionally associated with each maqãm (Powers, 2005, pp. 9–28). However, the same note does not always have the same pitch but depending on which maqãm is included there may be slightly different pitches for it: "The E𝄳 in MAQÃM RÃST is commonly understood to be higher than E𝄳 in MAQÃM BAYÃTI" (Rechberger, 2018, p. 92).

As per Dr. Gilbert Yammine (2016) these are the most frequent scales:

Ajam – عجم

Shawk Afza - شوق أفزا

Nahawand – نهاون

Nawa Athar - نوا أثر

Nakriz – نكريز

Kurd – كرد

Hijaz – حجاز

Zanjaran – زنجران

Hijazkar – حجازكار

Rast – راس

Suznak – سوزنك

Bayyati – بياتي

Karjighar – قارجغار

Saba – صبا

Siga – سيكا

Huzam - هزام

The complexities of the system make it very difficult to find a theoretical model to correctly assign the measured relative pitches to the right scale (Bozkurt, Yarman, Karaosmanoğlu, & Akkoç 2009, p. 67).

Maqãm computer-assisted music composition is thus very complex; the *Centre de Recherche en Informatique et Création Musicale* from the University of Paris, has modelized and created a library using the computing programming language *CSound*, but that requires compiling data for each instrument.[109]

The musical system is not equal in all the regional cultures: "Persian art music is organized in twelve systems of dastgãh […] nevertheless it should not be considered a scale system or as a modal system such as 'rãga' or 'maqãm'" (Rechberger, 2018, p. 113). Interval division has also wide regional variations.

Arabic nomenclature and cent values do not reflect the situation in Turkey or Iran and establishing a table of cent value equivalents for each pitch and case is required.

Although it is possible to transpose a maqãm, this is seldom used, as in addition to changing the mood, the tonics will not fall on open strings in the oud, the most used instrument, and the sound is much better on open strings.

[109] As explained by Belhassen (2014) in this paraphrase [original in French]: the difficulties at this level, and at first glance, will be to consider the creation of UDO [User-Defined-Opcodes database] specific to each instrument, which are to be studied according to a logic of musical execution appropriate to each instrument. We will consider the implementation of hybrid sounds to exploit the potential of sound synthesis. This phase will allow the culmination of other forms of musical interpretation such as heterophony for example (p. 111).

These subtle nuances later disappeared with regular division of 24 equal tones; "the starting note may be different, but the intervals remain the same" (Mukhtar, 2018, pp. 9, 51, etc.). Now, the transposing allows mixing different maqãmãt:

> D and C–based Maqamat are seldom mixed together in the same song or improvisation. […] But if we transpose either group to G, then the whole world of modulation possibilities from the family opens up. (Powers, 2005, p. 43).

One additional element to the mode or melody is the so called *tarab*. The word tarab refers both to the state of ecstasy evoked by music and to the music itself.

According to Racy, in a broader sense, the term also refers to the social environment of tarab makers and consumers in cities such as Cairo, Beirut, and Damascus, including the *sammi'ah* music followers (those who hear "properly"), who have the right level of musical initiation and an innate disposition to listen with attention and dedication (Racy, 2004, p. 40).

Maqãm music does not normally include harmony or chords. This is because microtones are not easy to harmonise as it will require non-existing notes to complete the chord, to keep a pleasant sound interval.[110]

As a rule, MECul music is monophonic. When a small ensemble (*takht*, or the Turkish *Fasil*) ─or a large orchestra─ is playing, all instruments play in unison or in octaves (Muhssin, 2007, para.1).

One illustrative example of the complexity of MECul music is that even the most typical instrument, the oud, is not always tuned in the same way. The most frequent Arabic tuning (from high to low) is C, G, D, A, F, C, in sets equally tuned. The most frequent Turkish tuning (again from high to low) is D, A, E, B, F#, C# (Toshich, 2008, pp. 7–8).

Another difference is that since the beginning of the nineteenth century (Sultan Mahmud II), Turkish notes have a fourth difference between written note and concert pitch.

Castelo-Branco has recorded a typical small ensemble instrumentation:

> The instrumental section of the *takhat* consisted of a *qānUn* (a trapezoidal plucked zither with twenty-six courses of three strings), an '*ūd* (a fretless short-necked lute with five or six pairs of strings), a nāy (an end-blown cane flute with seven holes) a violin designated *kamaān*, or *kamaānja*, and a *riq* (a round frame drum, approximately 25 centimeters in diameter, with jingles). The solo vocalist (*mutrib*,

[110] This is not the case for microtonal equal division of the octave as in 24-TET or 34-TET diapasons.

> masculine; or *mutriba*, feminine) was the central figure in a takht performance; there was much solo vocal improvisation (Castelo-Branco, 2002, p. 561).[111]

53 notes per octave mirror the *Arel-Ezgi-Uzdilek* system with an error of about one cent (Yarman, 2007, p. 51). However, as stressed by the author, the instruments have limited frets in number, and it is convenient to reduce tuning to less than 53 tones.

Moreover, the Arel-Ezgi-Uzdilek system[112] is missing some intervals required for certain maqãmãt:

> For one thing, characteristic middle second intervals peculiar to certain maqams do not exist in the "Arel-Ezgi-Uzdilek" system. In order to compensate for these, additional frets are fastened to tanburs and mandals affixed to qanuns. On the other hand, there will inevitably be mathematical relationships between pitches. For instance, we shall be able to say that the interval between rast-neva, dügah-hüseyni, and segah-evc will always be a fifth, be it pure or tempered (Yarman 2008, p. 4).

Yarman further suggests using a 34 or 41-tone division with the additional advantage for 34-tone of being more compatible with traditional 17 (*Safi al-Din al-Urmawi*'s) schema "due to being divisible by this number. All of these are favourable for transpositions and polyphony" Moving to a theoretical ground of high resolution a 79-tone scale would be recommended (Yarman 2008, p. 6).

As indicated, at the beginning of the twentieth century, following Mikhail Meshaqah's system of ratios, a 24 equal quarter tone division of the octave with notes rounded to the nearest quarter tone was adopted, making possible to write the maqãm music using score writing programs. "The distance between each successive note is a quarter tone (50¢). In practice the octave is tuned to a different tuning system using the so-called Arabian comma, which is roughly 22.6415¢" (Rechberger, 2018, p. 92).

This diapason, now equally tempered, is also used for modern Arabic music (Rechberger, 2018, p. 240).

Turkish music has nine intervals (called *komma*), but only five of them are recognized in the 24-tone system. Not all the experts agree with this simplification:

> It has been established on the basis of frequency measurements and analyses, that the 24-tone Pythagorean tuning is not a model suitable to express all the intervals of Turkish Maqam Music and does not suffice to compensate practice contrary to what is claimed (Karaosmanoğlu & Akkoç, 2003). It has been made manifest that characteristic middle second intervals representable by such superparticular ratios (based on the formula n+1/n) and roughly corresponding to 2/3, 3/4 and 4/5 whole-

[111] Per Prof. Dr. Yarman, the word *mutrib* means the person that plucks (uses a plectrum), and refers to the instrumentalist, not the vocalist, in the Turkish sphere. *Mutriban* is another word for the ensemble of players.
[112] with 24 keys although four of them (bûselik, dik geveşt, dik acem) are in disuse in Turkish art music, and also unavailable in the Safi al-Din 17 tone-scale (Can, 2002, p. 175).

> tone like 13:12, 12:11, 11:10 are particularly used in maqams such as Uşşak, Saba, Hüzzam, Karcığar, and that these are not haphazard deviations (Yarman, 2008, p. 3).

In any case, when selecting a system, the threshold (JND) should be considered in order to assess whether the increases in the number of tones in fretted instruments with the resulting design difficulties are worthwhile:

> But what about tanburs? No matter how long their necks are, the frets which can be fastened are limited in number. We ought to think of the bağlamas as well. Under these circumstances, it might be necessary to focus on tunings "less voluminous" than 53-tET. These are, from small to large, 34-tET, 41-tET and 46-tET (Yarman, 2008, p. 6).

A worthy goal is to find a compromise in the number of equal divisions of the octave: "When all is said and done, this 'magic number 17' when taken as equal, gives a semblance of maqãm; but with 34 equal, you have more options and subtlety to get that semblance with the same intonational quality and rigor" (Yarman, personal communication, May 11, 2018).

The improvement with the use of a 53 division of the octave vs. 34, as will be commented on, is under JNC (about 6 cents).

Musical scales of Indian subcontinent (SACul). Also, a very ancient and wide range of musical traditions may be found in the different countries (Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan and Sri Lanka) usually included under the geographic denomination of the Indian subcontinent (Alison, 2000).

The following text is limited only to the art music or erudite music of India, which split into two major musical traditions around the sixteenth century: the Hindustani (North India) and the Carnatic [Karnatic] (South India), although keeping many common elements (Sorell, & Narayan, 1980).

These traditions include many complex concepts, elements and structures alien to EuCul,[113] which are impossible to detail in a few pages, so what follows is only a very limited overview.

[113] A glossary of Indian musical terms may be found at https://www.culturalindia.net/indian-music/music-glossary.html

As advanced, some ethnomusicologists, both from the Middle East and India, consider that there is no EuCul scheme that can faithfully reproduce the maqãm or the raga.[114]

In origin, Indian music seems to be rooted in Brahmanism (Vedism or ancient Hinduism), and the songs for fire sacrifices (Sama-Veda) of the ancient Indo-Aryan culture, with a mythology that included male heavenly beings, called *Gāndharva*, with excellent musical skills married to the female spirits of the clouds and waters *Apsaras*, outstanding dancers, accompanied by players of musical instruments the *Kinnaras*, present in many Hindu cultures, particularly in the south (Beck, 2012, Gandharva Sangita, para. 3). These musicians and dancers are typically represented performing in the court (heaven) of Lord Indra, the king of gods.

> The earliest Hindu musical expression was the singing of Sama-Veda Hymns (Sama-Gana), rendered during Soma sacrifices, and Gandharva Sangita, performed during Puja (services to the Hindu gods) associated with early religious dramas (Beck, 2012, Ancient India, Yajna and Sama-Gana, para. 1).
>
> As a gift from the gods, Gandharva Sangita was considered similar in kind to the music performed and enjoyed in Lord Indra's court in Svarga, or heaven. Viewed as a replica of heavenly archetypes, this ancient religious music was primary vocal but included instruments such as the Vina (harp or Zither), flutes, drums and cymbals (Beck, 2012, Gandharva Sangita, para. 2).

Although dance (and consequently music) is mentioned by Rājaśekhara (ca. 920 – ca. 880 BCE) in his *Karpūramañjarī*,[115] the detailed musical elements ─including performance specifications─ and music theory are described in the *Nāṭyaśāstra*,[116] a treatise attributed to Bharata-Muni (ca. 200 BCE).

According to this reference (Ghosh, 1961, [vol. 2]):

> […] the world Gāndharva in the sense of music may well be pre-Buddhistic. Ant it is certainly not later than 200 BC […] In earlier times *gīta* (song) and *vādya* (instrumental music) were seperately [*sic*] mentioned, or the compound word *gītavāditra* (Pali, *gītavāditta*), represented music in its tonality. But dance and

[114] My appreciation goes here to Dr. Vidyadhar Oke for his suggestions and comments. He is one of the supporters of the thesis that there is no way to reproduce Indian pitches in perfect tune with any other methodology but the use of the 264 shrutis (personal communication, July 2, 2018).

[115] Available in English since 1972 publication edited and translated by Manomohan Ghosh: *Rājaśekhara's Karpūramañjarī: a Prakrit play.* Calcutta: World Press.

[116] Although the original Nāṭyaśāstra text was thought to be lost, after a laborious gathering of several sources it was reconstructed and translated from Sanskrit by Dr. M. Ghosh as: *The Nāṭyaśāstra*: *A Treatise on Hindu Dramaturgy and Histrionics*. The first volume was edited in Calcutta by The Royal Asiatic Society of Bengal (1950), (Retrievable from https://jambudveep.files.wordpress.com/2012/03/natyashastra.pdf). This volume does not include much information about music, which is basically found in the second volume, (Chapters XXVIII─XXXIII) published by the same editors in 1961 (Retrievable form https://ia800709 .us.archive.org/22/items/NatyaShastraOfBharataMuniVolume2/NatyaShastraOfBharataMuniVolume2.pdf). Mind that each chapter starts page numbering again. According to this source the text is from 200 BCE (Introduction, p. 8).

> drama […] were very closely associated with music vocal and instrumental, from ancient times, possibly long before the time of Buddha (The Ancient Indian Theory and Practice of Music: The Indian Conception of Music, p. 5).

Also, there is a mention of the music melodic centre and the intervals between notes in terms of consonance, assonance, dissonance and sonant: "[…] an interval of [more or less] Śrutis, […] are of four classes, such as Sonant (*vādin*), Consonant (*saṃvādin*), Assonant (*anuvādin*, and Dissonant (*vivādin*)" (1961, vol. 2, Chapter twentyeight [*sic*], On the instrumental music, p. 5).

The closest Indian equivalence to a musical scale is a *Grāma.* There are three Grāmas: *Ṣaḍja*, *Madhyama* and *Gāndhāra* (the latter long obsolete).

Each Grāma is the source of seven *Mūrchanās*, which have been considered to be (more or less) like the Ancient Greece modes (Kuiper, 2011, p. 249).

The notes are called *svara* [*swara*], the musical instruments are *ātodya*, and the songs *gāna*.

Instrumental and vocal music are closely related to dance; they "were so intermixed with spoken word of the play text that each became inseparable from the other at the moment of performance" (Mehta, 1995, p. 248).

The central element of both Hindustani and Carnatic music traditions is the *raga (rāga),* a word meaning colouring or dyeing, without a direct equivalent in EuCul music; it is not a tune nor a scale.

Different rāgas may share the same scale. It may be explained as the intention to produce a feeling in the audience, "to colour the mind."

Rāgas use certain combinations of notes, (normally at least five), and motives, with some progressions allowed and frequently repeated, while others are not used.

Improvisation, as in maqãm, is an essential part of the music, particularly in Hindustani tradition while Carnatic is more subject to the composition.

Rāgas are frequently related to season, time and certain moods (Ravikumar, & Bhavan, 2002).

> More than two dozen classical Ragas (male melodies) and Raginis (females melodies) are mentioned with regard to their presiding deities […] Legions of technical terms are also present, such as *svara* (note), *sruti* (microtones), *jati* (patent scale), *murcchana* (scale variation), *vadi* (principal note), *samvadi* (secondary note), *anuvadi* (penultimate note), *tana* (rapid improvisatory passages), and so on (Beck, 2012, Sri Krishna, para. 12).

Some combinations of notes are extremely characteristic of a given rāga, while others can sound out of place, even using the notes in the rāga; the combination is mostly intuitive, with no fixed rules. There are currently two hundred rāgas (Avtar, 2006).

After Pandit Vishnu Narayan Bhatkhande (1860 – 1936) a modern simplified classification of the Hindustani ragas was adopted, grouping them into ten families called *thaat* or *thāt*, which, by evolution of the twelve initial notes, all have seven notes (Nayar, 1989, p.144).

Like the movement in MECul against the use 24 microtones for maqãm, Bhatkhande classification has been criticised as an unscientific oversimplification that forces some rāgas into an inappropriate thāt.

The number of groups in Carnatic music is much higher and, instead of thāt, the rāgas are classified in 72 *Mēḷakartas*, including the parent rāgas and others derived from them (Bhagyalekshmy, 1990).

Typically, a Mēḷakarta rāga has also seven notes ascending (*ārōhanam*) and descending (*avarōhanam*) (Bhagyalekshmy, 1990).[117]

> Ragas in the Carnatic music fall into two categories, the base or melakarta ragas and the derived or janya ragas. The 16 swaras form the basis for the melakarta scheme. Melakarta ragas have a formal structure and follow a fairly rigid scheme of scientific organization whereas the janya ragas are rooted in usage and are liable to evolve with the music. In fact many janya ragas change their character over time (Kumar, 2003, pp.4–5).

The *pakad* is the heart of the rāga, usually a phrase or two with just a few notes, which has been defined as a brief snapshot or key sequence, while the *chalan* is the set of characteristics that allows the connoisseur to identify the raga by the sequences of notes (with one or more pakads), and the rhythm:

> Certain characteristic phrases *have* to be used to give the *raga* its character. These are like the features of a person. Indeed, the word in Hindustani is *'sakal'* meaning 'the face'; one has to render a *raga* to bring out its *'sakal'* distinctly. This can be done only if melodic designs of some definition are employed. These characteristic phrases are called variously as *prayoga* (use), *pakad* (to hold or grasp), *chalan* or *sanchara* (movement) (Deva, 1995, p. 12)

Hindu music is characteristically referenced to daily periods. Although there are different names for rāgas according to the hours of the day: "All the ragas are divided into two broad groups—Poor Ragas and Uttar Ragas. The Poor Ragas are sung

[117] The reference compiles over 450 popular Carnatic ragās. The mēlakartas must follow certain rules and they take different names according to the sequence of notes. Sampūrṇa rāgas must contain all seven svaras both ascending and descending. In a Krama sampūrṇa raga the svaras must follow a strict ascending and descending sequence, and so on.

between 12 noon and 12 midnight. The Uttar Ragas are sung between 12 midnight and 12 Noon" (Kumar, 2003, p. 4).

For music time measurement the function tāla is used:

> Time was considered to be eternal by itself and therefore unlimited. Therefore the fundamental question was whether time could measured and if so how. […] The concept of *tāla* is based on the understanding of time-space. Time itself is not measurable: what is measurable is the action or event that takes place in it (Kumar, 2003, p. 107).

The time regulation is syllabic-based:

> The temporal aspects of the chant were regulated, as one would expect, by the syllabic quantities of the text—as measured in *mātrās* (a unit equal to one short syllable) and their multiples. Three basic durations were recognized: short, long and protracted (*pluta*), in an invariant ratio of 1:2:3 *mātrās*. The same durations form the basis for the system of *tāla* (Rowell, 1992, p. 67).

Understanding the metrics of Indian music requires a different approach than that used for EuCul music. As frequently found in ethnomusicology, it is one of the many systems on the planet not based on metric units and bars (Clayton, 2000, p.28).

The thaalam is not only a tool for musical tempo, with *mātrās* as standard time-units, but also reflects the mood. Although a syllabic-based system, it must be taken into consideration that the *akṣharas* (The Hindi syllable-like units) are not always equivalent to drum-syllabes (*ṭhekā*):

> In the Hindustani scene, *mātrā* is the conceptual standard time-unit with an arbitrary time-value depending on the *laya* (tempo); it is concretized in terms of numbers 1, 2, 3, 4 etc. Why the concretization in terms of *akṣaras* has been dropped is a pertinent question. It could be conjectured that the strong identification of the *tāla*-cycle with the specific set of drum-syllables (*ṭhekā*) could be responsible for this phenomenon. In a *ṭhekā* each *mātrā of the tāla* is not necessarily represented by one drum-syllable; in *cautāla,* for example, 5 out of the 12 *mātrās* are represented by double syllables, whereas in *dhamār* four of the 14 *mātrās* are represented by only two syllables. The understanding of *akṣara* in terms of drum-syllable could not, therefore, be directly linked with *mātrā* and hence *mātrā* has become the sole term for the smallest time-unit in *tāla*. (Katz, 1992, p. 152).

Frequently, the rhythmic pattern is guided by percussion instruments:

> […] it is important to understand that melody cannot exist without reference to time; yet, a distinction needs to be made between free time and organised time. Melody, ie., *rāg*, in Indian classical music can exist in a free rhythmic form as *ālāp*, in which time durations are determined by its own internal logic as interpreted by the expression of the performer, […] Indian melody can also be presented in its metric form under the constraint of organised time as dictated by a particular time measure, *tāl* (Jairazbhoy, 1995, p.5).

Rasa (*Blavas*) is another element, not easy to define. In a way it may be considered as the emotion, sentiment or feeling that the rāga is intended to transmit:

> Rasa is difficult to translate. In common parlance it can be 'taste'. It can also be 'essence'. But in aesthetics, it means more. It is not mere experiencing or being in a state of emotion. It is a condition of 'observation' of one's involvement (Deva, 1995, p. 50).

It is generally accepted that there are nine rasas (Table 22). In addition to those nine rasas found in old Sanskrit texts, some modern rhetoricians are also including two additional rasas related to religious feelings: *Vatsalya Ras* (वात्सल्य रस), linked to the supreme personality of godhead in his childhood feature. In this avatar, the feeling is for a child god to be protected by the devotee, and *Bhakti Ras* (भक्ति रस) as a feeling of devotion and love for god (Brooks, 1989, pp. 188–193).

In summary, a rāga may be considered as a well-timed melodic combination of tones freely interpreted by the musicians but subject to certain rules, linked frequently to a time or season, created to promote an emotional state in the audience. This definition is close to that included in *Brihaddeshi* Sanskrit text,[118] well over one thousand years ago.

The treatise provides early information about musical notation and rāgas, including their classification as *marga* or *desi*. Their difference comes from the Sanskrit literature, where marga represents the rigid serious texts linked to the initiated, and to the religion ─the marga rāgas would be used in formal religious music─ while desi represents the popular, and in the case of music, the desi rāgas are those for folk, songs and popular profane music.

The history of Indian music itself is divided in two periods, sacred and profane:

> The history of Indian music can be divided into two main periods, *vaidika* (Vedic) and *laukika* ─ sacred and profane. The *sámagána*, together with its various forms, constitutes the fabric of the *vaidika* music, while the *gándharva* and formalised *desi* music form that of the *laukika* music (Prajnanananda, 1963, p.11).

As stated above, another interesting feature found in *Brihaddeshi's* treatise ─attributed to Matanga Muni─ is the musical notation system. The octave may have 7 notes or 12 notes:

> The *murcchanás* evolved from the *grámas* as their base […]. Each *murcchaná* possessed a special unit of aesthetic sentiment. Though Nárada has roughly said about twenty-one and Bharata about fourteen […] yet by different arrangement of

[118] The text was edited (1992) by prof. Prem Lata Sharma under the title *Mataṅga and his Work Hṙhaddeśī* [Brhaddesi], with a currently available edition (2001) from Delhi: Sangeet Natak Akademi. Prof. Dwaram Bhavanarayana Rao, transliterated this book into modern Telugu in 2002 (Retrievable from https://ia800709.us.archive.org/35/items/Brihaddesi/Bruhaddesi.pdf).

> seven tones […], 84 (7x12=84) variations of *murcchanā* might have evolved (Prajnanananda, 1963, pp. 25-26).

Following Nayar (1989), Matanga Muni defined the shruti as follows: "Everybody recognises the *Pancham* of *Shadja* grama and the *Madhyam* grama. The gap obtained by the difference of the two *Panchams* of these two above-mentioned *gramas* is the sound measurement of a *shruti*" (p. 117).[119]

Brihaddeshi analyses and gathers those microtonal intervals existing for centuries (shrutis) that may be traced back many centuries before his text (at least back to 500 BCE):

> The microtones (*shrutis*) are the minute perceptible ("shravanayogya") tones or musical sound units that constitute the structures of seven tones like *shadja, rishabha, gándhára, madhyama, panchama, dhaivata and nisháda* (corresponding Vedic tones, *chaturtha, mandra, atisvárya, krusta, prathama, dvitiya, tritiya* (Prajnanananda, 1963, p. 15).

The old initial five classes of the 22 shrutis: *diptá, áyatá, karuná, mridu* and *madhyá* were systematized and arranged by Nárada of the Shiksá during the first century (CE), creating on that basis 22 *játis* or shrutis scales (Prajnanananda, 1963, p.16–17).[120]

This division was initially Hindustani, but it was later accepted by the Carnatic tradition.

> [Bhatkhande] first step in this regard was to establish the exact position of the present day *swaras* on the gamut. […] Like the ancient and mediaeval authors he accepted the the s*hurtis* as microtones, each one placed at a very small but cognisable interval from the other. Twelve out of these -- namely, *shruits* with nos. 1,3,5,6,8,10,12,14,16,18,19,21-- are known as *swaras*. They were selected for forming the *ragas* (Nayar, 1989, p. 130).

Table 23 presents the differences in frequency and in cents between the shrutis of the scale of C (Shaphet 1) as compared with the 34-TET scheme. The dissimilarities

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[119] This confirms, once more, that the same note (svara) in this case *pancham*, is not tuned equal in every case. But again, taking *shadja* as 1, it may be alternatively considered that *pancham* ratio is ─always and in both traditions─ 1.5, as the «other *pancham*» ratio of 1.48148148 is only used when *Madhyá* is taken as the new *shadja* (making *Madhyám grama*), operating as a lower *rishabha* (1.111). These technical ethnomusicological disquisitions are out of the scope of this dissertation.

[120] A complete reference of the 264 frequencies of the 22 shrutis for the 12 scales may be found in Dr. Oke site (http://www.22shruti.com/research_topic_44.asp). Several cases have less than 4 cents difference. For example, 8-*saphet*-1 (327,03 Hz) and 2-*kali*-2 (327.77 Hz) have only 3.9 cents difference. 22-*kali*-1 (526.21) and 3-*saphet*-7 (526.81 Hz), have only 1.9 cents difference.
Although much more respectful with the rich musical tradition of ancient India, the system of shrutis has obvious practical difficulties in the case of being included in a wider musical system. As already commented on, the JNC sets the limit of different microtones that can be differentiated by ear to around 150 notes; tuning a difference of 1.9 cents by the ear seems to the author an extremely difficult task if not impossible.

are, in the worst of the cases, 15.647 cents, with only seven shrutis over 10 cents of difference.

The musical notes (svaras/swaras), with solfa-syllables (*sargam*), *sa*, *ri* [Carnatic, or *re* Hindustani], *ga, ma, pa, dha, ni* emerged in the post-Vedic period and, according to professor Sámbamoorthy, were "the earliest landmark in the history of music" (Prajnanananda, 1963, p. 20).

Such enthusiasm is understandable, when compared to Guido d'Arezzo's solfeggio introduction in the 10th century (CE):

> Naranda (an historical person, not the Vedic Rishi), explained how these seven notes were directly determined from the original three Vedic accents: Udatta into *ni* and *ga*, Anudatta into *ri* and *dha*, and Svarita into *sa*, *ma* and *pa*. The seven notes are also named here, metaphorically, after the sounds of different birds an animals *sa*/peacock, *ri*/bull, *ga*/ram, *ma*/crane, *pa*/cuckoo, *dha*/horse, *ni*/elephant. The F (*ma*) may be raised, and the notes D E A B (*rig ga dha ni*) may be lowered (like the Western sharp and flat notes, respectively) to create multiple variations of scale formulas, while the tonic C (*sa*) and the dominant G (*pa*) generally remain fixed (Beck, 2012, Gandharva Sangita, para. 12).

In both traditions, the *vadi* is the tonic svara, the most important and repeated note, and sets the reference hour of the day for the music to be played.

The notes may be natural (*shuddha*) or with accidentals or *vikrit* tones (flat for *komal*, and sharp for *teevra*):[121]

> Out of the 12 notes of the gamut, seven were recognized as *shuddha* and five *vikrit*. The *swaras* were respectively *Shadja, Komal Rishabh, Shuddha Rishabah, Komal Gandhar, Shuddha Gandhar, Shuddha Madhyam, Teevra Madhyam, Pancham, Komal Dhaivat, Shuddha Dhaivat, Komal Nishad* and *Shudha Nishad*. So *Shadja* and *Pancham* had no changed form while the rest of the 5 notes had one *vikrit* each. These *swaras* were written and pronounced in practice in the abbreviated forms, namely, *Sā, Re, Ga, Ma, Pa, Dha, Ni* respectively while their *vikrit* forms mentioned with the names of *Komal* and *Teevra* added to them (Nayar, 1989, p. 113).[122]

[121] The vikrit tones are usually represented simply by their lowercase letter. What really changes is the tonic not the positions of the 22 shrutis in terms of percentage of length of the string. As the tonic changes to maintain the harmony (*samvad*) the frequencies need to change accordingly. Teevra means in Sanskrit higher volume. The correct name for higher frequency is *tara*. Again, these disquisitions are beyond the scope of this work.

[122] Per Oke, this comment is inaccurate. 12 shrutis are selected from the 22 for the 12 notes (svaras) to be used in a rāga. This selection of shrutis depends on each rāga structure and tends to provide a maximum number of natural ratios (mostly 1.25, 1.33, 1.5, and 1.66). Although the svaras are written in Hindi in the same way in both traditions, the author is using the North Indian transliteration. In Carnatic tradition there are pronounced (and transliterated) slightly different: *Ṣaḍja, Ṛṣabha, Gāndhāra, Madhyama, Pañcama, Dhaivata, Niṣāda.*

The melody is the most crucial element in Indian music: “The music of India offers a most complete example of melody untouched by harmony” (Banerjee, 1983, p. 155). The author defines Indian harmony as an “innocent system.”

Per Kumar (2003): “In India, where harmony never proceeded beyond the drone bass, melody was unfettered and could evolve its own laws” (p. 39).

Beyond the scope of this dissertation would be a detailed description of the musical instruments of India, which are basically classified as:

Percussion (drum, tabla, mridangam, pakhawaj, dholak, nagada, dholki, kanjira, tavil, pung, damaru, chenda, edakka … etc,).

Wind (bansuri, shenhai, pungi, harmonium, shankh, nadaswaram, kombu…etc).

Stringed plucked (sitar, rebab, sarod, saraswati veena, ektar, tambura [tanpura], santoor, gottuvadyam...etc).

Bowed (sarangi, violin, dilruba, pena…etc), adopting different names and characteristics according to the different traditions (Deva, 1995, Dutta, 2008; Lata, 2013).[123]

Music notation and scores are not a part of the Indian master musicians’ practice, although they are frequently used for learning.

There are several studies about pitches, both traditional and adapted to match with current A = 440 Hz standard.

However, the shrutis are not only pitches but also "what is heard" [the Sanskrit translation of the term], involving a concept of authorless revelation, linked to the earlier Hinduism concepts, with shrutis even considered divine by some Hindu schools.

Therefore, abandoning, changing or adapting the shrutis may have implications that extend beyond the musical field.

Table 24 summarises two well-known Indian tunings.

[123] A comprehensive and beautifully illustrated reference for both Hindustani and Carnatic instruments evolution and history may be found in the successive editions of Prof. Dr. Suneera Kasliwal’s Classical Musical Iinstruments (Kasliwal, 2006), also incorporating chapters for «western,» and electronic instruments. In addition to her outstanding scholarly work she is also an active and highly reputed sitar player. The book provides additional interesting information about the influences of Muslim stringed instruments in India.

Musical scales of East Asia cultures (EACul).[124] As per an ancient legend, the study of music in China started in the times of the legendary sovereign Huangdi[125] (mythically dated 2,698 – 2,598 BCE).

The emperor ordered Ling Lung (吕氏春秋), his wise man, to study the musical-cosmic order. After a period outside the court, in the mountains, Ling Lung (ca. 2,697 BCE) returned with a flute (panpipe) made of five bamboo pipes cut in diverse sizes, tuned to the sounds of different birds.

The pentatonic scale, thus created, had five basic tones: *宫 gōng (C), 商 shāng (D), 角 jiǎo (E), 徵 zhǐ (G) and 羽 yǔ (A)* (Shi, 2016, p. 9).

Using the flute pitches, the emperor is said to have ordered the casting of bells, in tune with these pitches.

However, the discovering of musical remains going back as far as the Neolithic Age proves that the legend fell short in dating the musical presence in China.

> Chinese music has a long history. From the "twelve Lüs," with a history of more than 4,000 years, to the bone flute found in Henan dating to the Neolithic Age more than 8,000 years ago, a large variety of musical relics have been discovered, reflecting the skill of the ancient Chinese people (Jin, 2010, p.1).

Since those earlier times, musical notes are not that far from current pitches. The Neolithic flute had seven holes. In the ancient (at least two millennium BCE) fretless-seven-stringed zither qin, first and sixth strings share the same tone as second with seventh (Table 25).

F# tone appears here instead of G, and B instead of C but the scheme is still pentatonic: A1–B1–D2–E2–F3#–A2–B2 (Waltham, Coaldrake, Koster, & Lan, 2017, p. 59).[126]

A very well-developed musical praxis seems to have been present during the Zhōu dynasty (ca. 1,045 BCE – 256 BCE).

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[124] Although, as indicated in Chapter Two, according to the United Nations, the region includes China, the Chinese special administrative regions of Hong Kong and Macao, both Koreas, Japan and Mongolia, the focus of this section will be concentrated basically on China with a brief comment on Japan.

[125] Per Shiji [Chinese records of the grand historian] ancestral name was Gongsung and given name Xuanyuan. Huangdi (皇帝) literally means emperor, in this case the first emperor, or the emperor par excellence. Shiji Chinese-English text is retrievable from https://ctext.org/shiji/wu-di-ben-ji.

[126] The instrument is also called quin, ch'in, guqin or qixianqin. Following J. Kenneth Moore (2003) from the Metropolitan Museum of art «Ideographs on oracle bones depict a qin during the Shang dynasty (ca. 1600 – 1046 B.C.), while Zhou-dynasty (1046 – 256 B.C.) documents refer to it frequently as an ensemble instrument and record its use with another larger zither called the se» (para. 1). The instrument was not tuned to absolute pitch. Detailed information about the qin tuning may be found in Yeung, 2010. The tuning out of absolute pitch is a characteristic feature of ancient Chinese musical culture.

The impressive discoveries in a tomb belonging to the Marquis[127] Yi date back to the fifth century BCE (ca. 433), evidencing an outstanding knowledge of physics, acoustics, metallurgy, design and musical theory.

Findings in the tomb include 64 *bianzhong* bells, each of them producing two different tones (depending on whether rang in the centre or in the side), covering a range of five octaves.[128]

In the grave there were also tubular bells (chimes), several stringed instruments, (*se, qin, zhu*), *paixiao* and wind instruments such as flutes, and *shengs*, totalising 124 instruments of eight types among over 10,000 burial goods (Chang et al., 2005, p. 241). The findings also provide proof of an advanced tuning system.

There were many other interesting musical aspects in the Zhōu dynasty such as the belief in a relation between music harmony, society, and cosmos ─and the yin-yang or female-male duality part of the Taoist philosophy─ with each system representing a whole-tone scale, and when combined symbolizing the perfect binary opposed balance of the society (Rechberger. 2018, 190):

> Ancient Chinese philosophers believed that disturbing the five tones of the basic pentatonic scale named gong (C), shang (D), jiao (E), zhi (G) and yu (A), would be damaging to society (Hartong, & Bor, 2006, p. 10).

or the standardisation of a pitch for all the empire, under the responsibility of a public official:

> To the end, the first *yuefu*, or government office of music, was established under the Zhou dynasty (c. 1045 – 256 B.C.E.) and was made responsible for setting an absolute pitch standard for the empire as well as training musicians and supervising performances. Each succeeding dynasty followed the example of the Zhou, and each new office of music began by returning the standard pitches to correct the disharmony that had evidently contributed to the fall of the previous dynasty (Provine, Tokumaru, & Witzleben, 2002, Officers of music, para. 1).

Ceremonial dance and music were closely related, and there were different genres and styles of music categorized after Confucius in *yayue* (literally elegant

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[127] The title of Marquis for this noble governor is confusing, although very frequently used in relation to this tomb, found in 1977 and excavated one year later in Suizhou, the former Zeng state (province of Hubei). This title (marquis) was created during the Carolingian empire for the rules of the rural borders or frontier (buffer) areas or *marcas,* governed by a Marquis acting in name of the emperor and thus with power over all other nobles in the *marca*. Occasionally Yi is mentioned as duke. Probably a very broad equivalent European title would be Lord (Seigneur) as it is a very spread out name for a ─only relatively independent─ noble ruler (in this case subordinated to the kingdom of Chǔ) (McKitterick, 1995, p.176).

[128] A detailed text about the Chime-Bells may be found in von Falkenhauser (1993). Footnote [20] in page seven of his introduction includes ample information on reports and scholarly articles on MarquisY's tomb.

music, called cultivated music), *suyue* for popular or uncultivated music, and *huyue* (barbarian or foreign music) (Ho, 2007, p.85).

The Spring and Autumn period (770 – 476 BCE) has in Confucius an outstanding reference:

> Confucius, […] (born 551, Qufu, state of Lu [now in Shandong province, China]—died 479 bce, Lu), China's most famous teacher, philosopher, and political theorist, whose ideas have influenced the civilization of East Asia. Confucius's life, in contrast to his tremendous importance, seems starkly undramatic, or, as a Chinese expression has it, it seems "plain and real." (Ames, 2018).

In addition to those well-known areas where Confucius has been recognized for centuries, he has also made important contributions to music.

First, it must be considered that the goal of music education for Confucius was not only to acquire pure knowledge. Music was seen as a way to reach harmony, with enjoyment closely related:

> The fact that "music" and "enjoyment" are represented by one and the same graph would appear to be far from accidental. It is an indication of an association between the quality of achieved harmony and the consequent possibilities for enjoyment. To describe the relationship between music and political administration, Mencius plays with this intersection of "music" and enjoyment" as alternative terms for expressing harmonious order (Hall, & Ames, 1987, p. 281).

Consequently, community, harmony, and music are, in Confucius, linked:

> Music for Confucius is an expressive medium for the king of aesthetic order that can be achieved by a person in his community, a harmony consequent on a lifetime of cultivation, the full expression of his own personhood, and his virtuosic attunement to his world (Hall, & Ames, 1987, p. 280).

The ritual is a basic cultural element linked to the music; Confucius asserts (13/3): "[…] when the use of ritual action and music does not prevail, the application of laws and punishments will not be on the mark […]" (Hall, & Ames, 1987, pp. 269–270).

> Confucius said, "if one should desire to know whether a kingdom is well governed, if its morals are good or bad, the quality of its music will furnish the answer" (Tame, 1986, p. 345).
>
> Confucians incorporated Chinese "refined" music (Yayue) into moral education. Music, together with ritual, was thought to provide a route to the achievement of an ideal life and an ideal state of mind (DeWoskin, 1982, Wang, 2004). Music and rites were viewed as pathways to human perfection, bringing human beings together with cosmic harmonies (Ho, 2007, p. 85).

In addition to the importance of teaching music as a discipline integrated in society and the cosmos, the musical education content was also very noticeable, and what Confucius considered to be good music.

The philosopher considered the music of *Wu* beautiful, and a good expression of martial courage, but lacking the expression of goodness. S*hao* music was for Confucius a landmark of excellence (7/14):[129]

> When the Master was staying in Ch'I he heard the *shao* music. For three months he did not even notice the taste of meat. He said, "I had no idea that the performance of music could reach these heights." (Hall, & Ames, 1987, p. 279).

His method of teaching music, tailored for each student as a visual art in a steady and patient progression free from pressure, is an outstanding modern schooling methodology:

> […] visual art as part of the holistic scholarly education in the Far East has been important since the time of Confucius (see Tu, 1985). In the West, integrating the arts into other school subjects has become an area of great interest in recent years. Whereas the aesthetic education movement emphasized *perceiving the visual* within art disciplines, attention has gradually moved to broad themes than reach across disciplines. In doing so, a counter balance has taken shape than can be characterized as *visualizing engagement* (Irwin, & Chalmers, 2007, p. 182).

Qin dynasty (221 BCE – 206 CE) also deserves a comment for the improvements in the organization of the Musical Bureau.

By this time, the empire continued to recognise the importance of the establishment of a musician office for carrying out diverse tasks related to music and poetry, and the old musician office gave way to the Imperial Musical Bureau, greatly expanded under emperor Han Wu Di (140 – 87 BCE).

> […] the Music Bureau (Yüeh-fu 樂府), which reputedly was responsible "for the collection of songs (in accordance with old tradition), the maintenance of orchestras and choirs, and for providing musical performances at certain occasions of state such as court audiences, banquets, and religious services; the office also provided martial music, in accordance with the old instructions of war." […] the Music Bureau was already in use in the early Han and probably existed as early as the Ch´in […] even remote kingdoms such as Nan-yüeh 南越 had their own *yüeh-fu* music academies (Knechtges, 2014, p. 61).

The period of the Tang dynasty (618 – 907 CE)[130] represented a great developmental time for music in general, and for some instruments, such as the qin (Picken, 1985).[131]

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[129] «*Shao* and *Wu* were the most important types of music played in the imperial courts of all dynasties; the former, as the ritual music of the supreme hierarchal level, was played until the end of the Qing Dynasty (1644 – 1911)» (Jin, 2010, p. 13).

[130] Further data about Tang Court music expansion to China and Japan, with many previously unpublished texts in English, Japanese and Chinese gathered by several collaborators, may be found at the Cambridge publication edited by Laurence Picken (1985).

[131] The publication of Cambridge University by Jie Jin (2010) will provide additional information about music during the different dynasties, the meaning of ancient music with the natural way (*Dao*), and the inwardness

Yayue (known as *gagaku* in Japan, *aak* in Korea and *nha nhac* in Vietnam) music ─well-developed during Tang dynasty (618 – 907 CE)─ was born as a ritual with a set of conventions, to be performed at the China imperial court, together with dance in certain special and very rare circumstances.

Yayue was an expression of noble sophistication, power and cosmic religious harmony with shamanistic elements.

Typical instruments included the seven-stringed old zither (qin), the transverse bamboo flute dizi, the stone tablets bianqing and the free reed mouth organ sheng.

With the growth of commercial and political relationships, China exchanged influences with other musical traditions, markedly with India, even adopting (sixth century) the heptatonic Indian scale for a time.

Later, the culture also received influences from Europe, particularly with the establishment of the Jesuits in Asia.

For the predominant Han ethnic group tradition, Chinese music sound (*shēng*) deals with four elements: temperament (*lǚ*), the scale or mode (*diào*), the melody or tune (*qiāng*) and the rhythm (*pāi*) (Rechberger, 2018, p. 190).

Considering the importance given in ancient China to equanimity, centrality and equilibrium in music, and the early knowledge of the 1/2 octave proportions, the circle of fifths and related questions, it is not strange that the starting note in Chinese music is reported to be the central pitch of the octave (F#).[132]

This ancient Chinese vocal tuning (F#) is the note in the seventh place in case of equal division of the octave, in the median position (600 cents), splitting the octave into two equal parts with six notes each (its frequency match in both 12-TET and 34-TET scales).

> The *lü*, carefully fixed in ancient times by the Music Office of the Ministry of Norms, were the basis of the musical system of the Chinese. They were determined by a cycle of either ascending fifths or descending fourths which started from the central note of *huang chung*: *F* sharp, *C* sharp, *G* sharp, *D* Sharp, *A* sharp, *E* sharp or *F, C, G, D, A, E* and *B.* As the Chinese conceive the cosmos as the harmony of *yang* and *yin* – the male and female force– and as they understand music as a substitute and representative of the universe (Sachs, 2006, p. 176).

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of Zen, Chinese instruments, folk, singing and dancing, the *Quyi* artistic combination developed with the New Culture Movement, and the other issues of current music in China.

[132] As evidenced by remaining instruments (Engel, 1874, p. 53).

Not surprising, either, is the connection of the notes in the Chinese pentatonic scale with aspects such as nature, organs, flavours, odours, and colours: *宫 gōng (C)* is related to palaces, the heart, the center, the earth, the sweet, the fragrant and the yellow; *商 shāng (D)* is related to business, the liver, the west, the metals, spicy flavor, fishy odor and white color; *角 jiǎo [jué] (E)* relates to angle, the spleen, the east, the wood, the bitter flavor, the mutton odor and the blue/green color; *徵 zhǐ (G)* relates to drafting, the lungs, the south, the fire, the sour flavor, the burned odor and the red color; *and 羽 yǔ (A)* is related to the feather, the kidneys, the north, the water, the salty flavor, the rotting odor and the black color (Rechberger, 2018, p. 190).

The absence of a tradition of social dances in China is notable. Remarkably, also, the harmony does not bear the melody:

> China historically has had various dance types, notably dance associated with state rituals in which regularity of musical phrase was the norm, individualized state dance associated with traditional opera, and other individualized types such as 'flower drum' dance-songs, but no social dance tradition that is, one in which males and females dance together, requiring predictable melodic lengths. Similarly, Chinese melody is not supported by harmonic structure. Harmony has a regularizing effect on melody and, when absent, melody is not restricted by harmonic rhythm (Tharsher, 2008, p. 80).

As mentioned above, there were two combinable genres: the instrumental music representing the Yang, ─the masculine, the sky, the sun that comes over the land─ and the vocal music representing the Yin, that land, dark and fertile that receives and is penetrated by the Yang.

This union attains the universal equilibrium, a fruitful paradox of simultaneous unity and duality in perpetual movement. In this case, each system is represented by a whole-note scale (Table 26) combined to reach the musical-cosmical equilibrium (Rechberger, 2018, p. 190).

The bells found in the "Marquis" Yi's tomb confirm, once more, that the basic instrumental pitch was close to modern F, then called *gōng*.

The schema was later replaced with gōng becoming used for C (Rechberger, 2018, p. 190).

Due to the different modes and modulations required for the rich Chinese musical traditions, extra notes were added to the pentatonic scale.

These new notes were created as a change (alteration) of previously existing ones. These "changes" are based on major thirds (four half-tones ascending or descending from the basic o reference note).

The denomination of the accidental or "changed" note (called *biàn shēng*) derives from joining the term *jué* or *zēng* following the name of the note to be changed. If the reference note is a third below, then (jué) [meaning cut short][133] is used, indicating, thus, that the jué note is four half-tones higher than its reference. Analogously, if (zēng) [meaning to add or increase] is added, then the reference note lies a major third above and the altered note is a major third lower than its reference (Table 27).

The twelve names of the notes were later modified following the Chinese astronomical calendar, also based on the number twelve:

> This system however was later replaced by the more common 12 names of the *shí èr lǘ* (十二率), the chromatic scale from where Chinese modes emerge. In the ancient Chinese calendar, a year was divided into 12 lunar months and the day into 12 double hours. There were also 12 earthly branches (*shíèrzhī* 十二支) of the astrological animals engraved in the *Marquis Yi* bells. From these engravings we learn also about the association of the 12 notes with the 12 earthly branches (十二支) of the astrological animal figures (Rechberger, 2018, p. 191).

The 12 earthly branches: 子 (*zǐ*), 丑 (*chǒu*), 寅 (*yín*), 卯 (*mǎo*), 辰 (*chén*), 巳 (*sì*), 午 (*wǔ*), 未 (*wèi*), 申 (*shēn*), 酉 (*yǒu*), 戌 (*xū*) and 亥 (*hài*) were used cyclically in the calendar and as ordinal sequence.

Each one corresponds to a Zodiac animal and to a musical note of the 12-tone chromatic scale starting in C or *huángzhōng* corresponding to *zǐ* (子) and successively with the twelve cited earthly branches (Rechberger, 2018, p. 191).

The new names for the notes according to this system were: C, *huángzhōng;* C#/D♭, *dà lǘ;* D, *tài cù*; D#E♭, *jiā zhōng*; E, *gū xiǎn*; E#/F, *zhōng lǘ*; F#/G♭, *ruí bǐn*; G, *lín zhōng*; G#/A♭, *yí zè*; A, *nán lǘ*; A#/B♭, wú yì; and B, yīng, *zhōng* (Rechberger, 2018, p. 191).

However, the number five keeps its paramount importance and the fifth interval continues to be the basis for tone genesis.

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133 *Jué* has several meanings. It is also employed to name the third note above (E) in the pentatonic scale, but here it is used to denote short or down.

The fifth as resulting from 3:2 harmonize heaven and earth. This number is constantly present in Chinese cultural life with five elements, five relationships, five virtues, etc. (Rechberger, 2018, p. 192; Tharsher, 2008, p. 84):

> In the theory of Chinese music the interval of the fifth is the basis of tone generation. […] One of the first European writers on Chinese music has said that the ancient Chinese cut a second bamboo tube to two-thirds the length of the first one, thereby producing the interval of a perfect fifth, 702 cents […] a far more reasonable explanation of the theory of tone generation can, it seems to me, be found in the manner in which string players in China and Japan today tune the perfect fifth. Two methods are used. Either the string is stopped at two thirds of its length, which produces a perfect fifth, as on the Chinese ch'in; or in the case of the Japanese koto, two strings separated by the interval of approximately a fifth are tuned until the harmonic partial structures of both pitches are perfectly synchronized (Barfias, 1975, p. 57).

As mentioned, the diatonic scale, the principle of fifths, and the 12-tone equal temperament were known at a much earlier age in China than in EuCul:

> How the Chinese derived their scales dates back to 3000 B.C., when people in the Western hemisphere were still beating on logs and stones. The prevalent opinion in the West about the superiority of our musical culture should hereby be moderated. The diatonic scale was the foundation for Ancient Chinese music. The musical scale was developed by the circle of P5s, up to 60 degrees, called the 60 Lü, though the Chinese usually used only the first 5 Fifths in their pentatonic music system. […]. The seven note scale with B and F# has been used already as far as the Zhōu period (Rechberger, 2018, p. 191-192).

There is documented evidence of a modal system in China as far back as the seventh century BCE with two predominant pitches in each mode, one fifth apart (Rechberger, 2018; Tharsher, 2008).[134]

Although derived from Chinese theories, the creation of pitch ratios by means of successive division of a bamboo pipe arrived in Japan around the eighth century BCE. The Sanbun son'eki system of deriving pitches ─a similar procedure as the Pythagorean fifths─ calculates the initial pentatonic notes as G= C–1/3; G= C+1/3; D= G+1/3; A= D–1/3; E= A+1/3; B= E–1/3 and F#= B+1/3 (Rechberger 2018, p. 194).

The elegant style (*Gagaku*) has been used at the Japanese imperial court since ancient times. Scalar structure and modal practice are complex in Japan with variations among the different genres.

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[134] «The five pitches of each mode are primarily of structural importance in that they tend to be melodically stressed and dominate cadential points» (Thrasher, 2008, p. 81). The modes are called diàshì (調式) and have basically five different forms with pentatonic structure (gōng, shāng, jué, zhǐ, and yǔ). The zè mode (仄) is equivalent to the EuCul major scale. «Each mode may actually be transposed to any of the seven notes in the traditional scale» (Rechberger, 2018, p. 192).

As per Tokita & Huges (2017), based on the works of Koizimi and Rokushiro, “nearly all Japanese modes can be usefully understood in terms of three-note ´tetrachords’ a pair of pitches a perfect fourth apart, framing one intermediate, variable infixed pitch serving as auxiliary” (Section scales and modes, para. 4, 2017).

Except for Gagaku, absolute pitch also has little significance in Japanese music (Tokita & Hugues, 2017, scales and modes, para. 5).

> Melodies are brought to life through dynamics, timbral variety and ornamentation, as opposed to rapid movement through pitches, and therefore may seem quite slow in much traditional music […] The emphasis on the interval of fourth has already been noted […] The Western construction of harmony – that is, chords organized in hierarchical relationships with each other […] does not exist in Japanese traditional music; […] Although the simultaneous sounding of pitches on the *shamisen* in “Kasuga Sambasō) may sound harmonious,” these combination of pitches should not be understood as harmony (Matsue, 2016, pp.88–89).

It falls outside the scope of this dissertation to go into a deeper analysis of other rich musical traditions of East Asia Cultures (EACul).[135]

Microtonal scales. The use of less than a semitone (100 cents) intervals has been verified historically for centuries and in different cultures and continents (Geer, 2016, Keislar, 1987, 1988).

There were some antecedents in EuCul, but it was, however, during the twentieth century that the interest in microtonality and microtonal instruments grew:

> In the years from the turn of the century to World War II, many musicians were interested in 24-tone equal temperament, which provides microtones while retaining the familiar 12-tone temperament as a subset. In 1892 A. Behrens-Senegalden built his “achromatisches Klavier;” this quarter-tone instrument was followed over the next four decades by a good number of other quarter-tone pianos and harmoniums (Keislar, 1988, p. 7).

Any microtonal gamut falls, basically, into one out of three categories: first) a systematic microtonal equal division of the octave, already commented on; second) an octave-non-equal division procedure following another principle, and interestingly for this dissertation, following a GR schema; and third) a division based on an interval

[135] Garfias’ publication (1975) will provide interesting information about the expansion in the seventh and eighteen centuries (CE) of *Tōgaku* to Japan from the Court of *T’ang* in China (p. 4), with many useful Chinese – Japanese equivalences, detailed mode analyses, etc. Jiazi Shi doctoral dissertation *meets* Chinese music with EuCul methodology (Shi, 2016). Alan Thrasher (2008) book offers additional information with special focus on South China, about *Sizhu,* including *Yuelin* Confucian foundation (pp. 25–44), *Quigshang* music and historical legacy (pp. 53–65), *Yuelü* music theory and practice (pp. 75–95), repertoires, performance practices and much more. Rechberger source (2018), expands the information with dozens of additional scales and details for China minorities, Japan, and Korea and Stevens (2008) about popular Japanese music.

other than the octave (not based on the octave), either using the GR as reference such as Mongoven's (2010), or Pierce and Bohlen's schemas, or not.

This latter procedure may follow an equal division (ED) scheme but, obviously, it will not be an equal division of the octave (Not EDO).

There are also three basic specific problems that the microtonality must face, starting with the construction of instruments. As indicated, the instruments have frets and keys limited in number, and the microtone increase has a limit (Yarman, 2007, p. 51).

Besides, the number of octaves may be reduced considerably with a microtonal scale design. For example, a standard keyboard of 88 keys has 7.3 octaves with a 12-TET tuning, but only 2.5 with 34-TET, which is 2 octaves plus 20 more notes, and less than the range of a conventional classical guitar.

With the advent of modern digital technology this problem becomes less important:

> The flexibility of pitch afforded by computer technology suggests the use of new input devices optimized for playing in arbitrary tuning systems. In particular, keyboards are well-suited for polyphonic playing, and there is a legacy of historical microtonal keyboards that can serve as models for controller design (Keislar 1988, p. 1).

The second problem for using microtones is the compatibility of some digital file formats. Musical Instrument Digital (MIDI) format does not file audio (sound) notes directly. It records them indirectly as a sequence of fixed distances or intervals, originally, not designed for microtones.

In the case of using a MIDI format it must be tested whether it supports microtonal intervals, and that no displacements occur during the process:

> As for this writing, MIDI does not support nonstandard tunings [footnote omitted]. However at least one manufacturer has marketed an inexpensive synthesis unit (the Yamaha FBO1) that can generate microtones by responding to system-exclusive (i.e., specific to Yamaha) "fractional note numbers." (Keislar 1988, p. 14).[136]

The third question, to which the following lines will be devoted, is how to notate, ─particularly with a score writing software─ the microtonal music.

Although in 1974 an international conference was hosted by the State University of Ghent [Netherland] aimed at setting standard for microtonal notation, it

[136] Many improvements have taken place since 1988, but this question must be checked when using files with non-audio formats.

seems that there have been not many more reviews on this question, and there is not a universally accepted notation system for microtones (Sabbe, & Stone, 1975).[137]

The developments for the 34-tone scale notation will be commented on in results (Chapter Six).

For quarter tones ─particularly after the new MECul musical schema of 24 notes for both old maqãm and modern music─ the agreement is higher (Figure 56), but for the remaining intervals there are still a variety of options used by the different compositors.

The Figure 57 depicts a sample of the symbols used for microtonal notation. The Helmholtz-Ellis system designed in 2004 by Marc Sabat and Wolfgang Schweinitz may be found elsewhere (Sabat 2005).

Additional information for microtonal notation is available in Rechberger (2018).[138]

Tuning

The tuning pitch for EuCul music has fluctuated considerably over time (Haynes, & Cooke, 2001; Joutsenvirta, 2005).

During the Common-practice era, once an organ was constructed, it was usually not replaced until it became old; even though tuning trends changed, the instrument remained with the original tuning pitch set at the construction time: "Over the last 400 years in Europe, the point that has been considered optimal for pitch standards has varied by about six semitones, depending on time and place" (Haynes, & Cooke, 2001, para. 2).

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[137] The book published by Gardner Read (1990), considered by some as the best review of twentieth century microtonal notation, has been subject to criticism «[…] readers who already have a thorough understanding of microtonal composition may find this book interesting for the great variety of notational systems it gathers together in one place, but are likely to be as annoyed by the errors, inconsistencies, and contradictions in the text» (Doty, 1992, p. 1309).

Stone reference for twentieth century notation (1980) provides nearly no information about microtones (a few lines in pp. 67–71, 70–71 & 76, in a text of 357 pages).

The MIDI microtonal design by Christian Texier was very interesting but the link has not been working for the last months (September 2018): http://pros.orange.fr/christian.texier/mididesi/

Besides, another site, which until recently, was the source of approximately 1,000 musical fonts (https://elbsound.studio/music-fonts.php), has turned into ─according to them by imperative of the German laws─ a commercial website.

[138] Good quality printed symbols for 19-, 21-, 24-, 36-, and 72-note scales may be also found on Ted Mook's site (http://mindeartheart.org/micro.html).

Another significant constraint was that the relatively high pitch, used in organs and other non-stringed instruments, was too high for the possibilities of the strings in those days, which were made of gut.

Current violin steel strings can sustain tensions over 110 Newtons (N);[139] nylon, over 90 N; and silk and gut over 80 N (Lin, 2002), but in Baroque times the technical construction of strings was not so advanced, and those tensions made strings break, so they tended to tune at a lower pitch.

> It is rarely possible to generalize about pitch standards. Even when the exact period and location are known, different kinds of music often had their own standards (reflected in names such as 'opera pitch', 'chamber pitch' and 'choir pitch'). Although the levels shifted with time, the breaks were rarely clean, so older standards overlapped with newer ones (Haynes, & Cooke, 2001, para. 7).

The music theorist and member of the French Academy Joseph Sauveur (1653 – 1716) pioneered the measuring of pipe organs and vibrating strings, establishing the frequency of middle C in 256 oscillations per second (Sigerson, LaRouche, & Wolfe, 1992).

As per Tennenbaum (1992) "no musical tuning is acceptable which is not based on a pitch value for middle C of 256 Hz" (p. 47)[140]

> The first explicit reference to the tuning of middle C at 256 oscillations per second was probably made by a contemporary of J.S. Bach. It was at that time that precise technical methods developed making it possible to determine the exact pitch of a given note in cycles per second. The first person said to have accomplished this was Joseph Sauveur (1653–1716), called the father of musical acoustics. He measured the pitches of organ pipes and vibrating strings, and defined the "ut" (nowadays known as "do") of the musical scale at 256 cycles per second.
> J.S. Bach, as is well known, was an expert in organ construction and master of acoustics, and was in constant contact with instrument builders, scientists, and musicians all over Europe. So we can safely assume that he was familiar with Sauveur's work (Tennenbaum, 1992, p. 54).

Thus, 256 cycles per second for C pitch, corresponding to A about 427 – 430 Hz (depending on the ratio assigned in the scale for the interval C–A) was a frequently found reference for tuning but with quite a wide range of other possibilities.

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[139] One lbs = 0.454 kg (or kp) = 4.448 newtons (N). Fracture limit for typical string material in International System (Pascals) may be found at http://www.speech.kth.se/music/acviguit4/part4.pdf (For conversion Pascals, 1Pa = 1 $kg \cdot m^{-1} \cdot s^{-2}$). A typical tension for violin E string found in the market is around 17 lbs (7.7 Kg = 75.6 N).

[140] Tennenbaum not only links the C = 256 Hz proposal to Leonardo, Kepler and Pacioli scientific theories but to a «very specific internal geometry, whose most direct visible manifestation is the morphological proportion of the Golden Section» more over «everything in music must be coherent with the Golden Section» (p. 49). His interpretation of the well temperament as golden-ratio-based has already been commented on.

The Italian sixteenth century *tutto punto* A was around 440 cycles per second and the *mezzo punto* was around 464 cycles per second.

In France, tunings for opera (*Ton d'Opéra*) and chamber (*Ton de Chapelle*), initially similar, were later (1683) during the seventeenth and early eighteenth centuries different. Values for A were deducted from the tone used in the king's [Louis XIV] chamber (*Ton de la Chambre du Roi*): "If 'Ton de l'opera' was actually 393 (as attested by surviving woodwinds), then the pitch of Versailles chapel would have been 406, very close to the level found on the higher group of contemporary woodwinds and organs" (Haynes, 2002, p. 119).

According to this author (Haynes) frequency for A in "organs constructed in France in the period 1670-1700" average 408 Hz (2002, p. 118):

> […] the frequency of Ton de la Chambre du Roy was probably about 404-409 Hz, only about 60 cents higher than Ton d'Opéra, enough to be considered a semitone but not a full 100 cents above it. There was of course no reason for the two pitches to have been in transposing relationship of an exact semitone (in fact, even had they been a semitone apart, transposition would have been impractical in the general tuning schemes of the period based on meantone) (Haynes, 2002, p. 119).

These values seem to be in partial discrepancy with a previous work by the same author, or else the tunings substantially changed around 1680:

> The known pitches of most large organs built in France before 1680 range from a′ = 388 to a′ = 396 (A–2); this was the principal level associated with organs in France right into the 19th century. According to Mersenne's dimensions and illustrations of the 1630s, French wind instruments (which in this period never played in church) were at a level similar to the Italian mezzo punto (A+1). In France it was called Ton d'écurie, and woodwinds continued to be made to it until the 18th century (hence the C-hautboys that appear to be 'in D' and the F-recorders 'in G'). But most woodwinds were played at two other pitch standards: Ton de l'Opéra and Ton de la chamber (Haynes, & Cooke, 2001, France, para. 1).

For cornetts, however, tuning pitch seems to be higher, even over current 440 Hz standard.

> Cornetts were commonly used as a reference for pitch frequency in Italy, Germany and the Habsburg lands. Cornetts made in Germany in the 16th and 17th centuries range in pitch from a′ = 450 to a′ = 480, but most are close to a′= 465 (Haynes, & Cooke, 2001, Germany, para. 10).

As a consequence of the Civil War (1642 – 51), many organs were destroyed in England and tuning rules evolved:

> At the beginning of the seventeenth century the primary English church standard was Quire Pitch, similar to Praetorious's Kammerton. After the nearly complete destruction of organs during the Civil War and Commonwealth, remaining organs and new instrument were initially tuned to their old pitch (up to a' = 473; modern b♭). Gradually the influence of the newer French woodwinds (pitched at Ton de la

> Chambre) brought about the lowering of English organs to Consort Pitch (a' = 433), […] later a' 452 […] (Bush, & Kassel, 2006, p. 414).
>
> […], a′ ≈ 423 or Q-2 […] was to become the dominant organ pitch in England in the 18th century and into the 19th, identified at least once as Chappell-pitch; […] when it was later adopted by orchestral instruments in about the 1730s, it was called 'new Consort-pitch' (Haynes, & Cooke, 2002, p. 91).

There are references about Händel's Messiah, and Mozart's piano to be tuned to A = 422 Hz.

> A famous early specimen pitched at a' = 422.5 Hz is reported to have been used by Handel for a 1751 performance of Messiah; the piano manufacturer J.A. Stein (whose clients included Mozart) is said to have possessed a fork at a' = 421.6 Hz (Lawson, & Stowell, 1999, p. 86).

As seen, the situation was utterly confusing, even more so after Russian Czar Alexander requested a brighter sound at the Congress of Vienna (1815). "By 1850 chaos reigned, with major European theatres at pitches varying from A = 420 to A = 460, or even higher at Venice" (Bose, 2013, p. 28).

A frequently reported pitch for the later period, under Viennese influence, was A = 430 – 440 Hz, although considering the instruments that have been preserved, the uppermost values of the rank really may be a little too high (Bose, 2013, p. 28):

> Charles Delezenne (1854) reported pitches at various theatres in Paris in 1823 as a′ = 424, 428, 432 and, in 1834, 440. The Italian influence on pitch throughout Europe was reinforced by the dynamism of the so-called 'Wiener Klassik'. By the second third of the century performances in Vienna were generally at a′ ≈ 430–435. Pitch remained at this level in Viennese instrumental, dramatic and much church music until the end of the century (Haynes, & Cooke, 2001, Classical pitches, 1765–1830, para. 4–5).

During the second half of that century there were some efforts to standardize the pitch, both in France and in Italy, but at the new Vienna Congress (1885) the British opposed setting A = 432 Hz.

> The tendency for a pitch to climb continued in the nineteenth century, the Paris Opéra pitch being measured successively at a' = 423 Hz (1810), a' = 432 Hz (1822) and a' = 449 Hz (1855). Similar situations obtained elsewhere, for example in Dresden (Lawson, & Stowell, 1999, p. 86).

In May 1939, under German Minister Goebbels' influence, an international meeting was held at the Broadcasting House (BBC) in London. Britons and Germans agreed to establish A = 440 Hz as standard. French musicians were not consulted, and they opposed; they established their own so-called *diapason normal* A = 435 Hz.

In a second meeting held again in London (1953), and once again without the presence of a French delegation, the value for A = 440 was reaffirmed. This provoked a referendum promoted by Professor Dussaut, in which more than 23,000 French musicians voted in favour of A = 432 Hz (Lawson, & Stowell, 1999, p.86).

Even after recommendations from the European Union, the values are far from being uniform, with British organs remaining down (A = 430 Hz) while in other places such as Vienna over A = 440 Hz:

> In fact, A = 440 has never been the international standard pitch, and the first international conference to impose A =440, which failed, was organized by the Nazi Propaganda Minister Joseph Goebbels in 1939. Throughout the seventeenth, eighteenth, and nineteenth centuries, and in fact into the 1940s, all standard U.S. and European textbooks on physics, sound and music, took as given the physical pitch or scientific pitch of C = 256, including Helmholtz's own texts […] all early music scholars agree that Mozart tuned at precisely C= 256 […] Christopher Hogwood, Roger Norrington, and dozens of other directors of orginal-instrument [*sic*] orchestras established the practice […] of recording all Mozart's works at precisely A= 430, as well as most of Beethoven's symphonies and piano concertos. Hogwood, Norrington, and others have stated, in dozens of interviews and record jackets, the pragmatic reason: German instruments of the period 1780-1827, and even replicas of those instruments, can only be tuned at A = 430 (Bose, 2013, pp. 27–28).

Nonetheless, the value of 440 Hz for A is now included as standard in the Grove Encyclopaedia of Music, and the International Organization for Standardization set it in 1955 and reaffirmed in 1975. Pitch for A = 440 Hz is ISO 16:1975 (https://www.iso.org/standard/3601.html).

Music and Mood

The power of music to induce emotions has been well known since antiquity. This circumstance has been exploited in military marches, lullabies, religious music, love songs, national anthems, film soundtracks and in many other cases. Music is now used in elevators, in shopping centres ─where it is specifically selected to induce purchase─ hymns are created to represent countries, religions, civil groups, sport organizations, and specific compositions are used for celebrations including anniversaries, weddings, births, or funerals.

Aerophones such as bugles, whistles, horns, and drums are used for transmission of commands in parades; to convey feelings of football team fans in a match; to look for notoriety and group cohesion in union or political demonstrations; as a wake-up call by town criers, post or transit officers, sport referees; etc. Post offices in many countries still have a bugle in their emblems.

Art music has strategically been used to prevent young people from loitering in public stations "I've noticed that bus and train stations now pipe canned classical music, day-in, day-out, through their speakers as a way of stopping young people hanging around" (Service, 2009, para. 1), and with a similar reasoning, unpleasant music sounds have been used to torture prisoners or as a demoralizing element for enemy countries.

Many people declare that when they get home, they turn on the television or play music. "they surround themselves constantly with noise. At work, they have talk radio on. In the car, they play music. When they get home, they turn on the television and become distracted with their eyes as well as their ears" (Sproul, 2013). Repetitive sound patterns have been used throughout history to induce situations of ecstasy, or to encourage dancing.

Music is used as a therapy in a discipline called musicotherapy, now extending even to neonatology units where musicians interpret live music adapted and modified according to vital signs and other data recorded on the monitors of new-borns (Loewy, Stewart, Dassler, Telsey, & Homel, 2013).

In the early 1990s the social psychology of music blossomed with contributions from Hargreaves, Ivaldi, Juslin, North, O'Neill, Sloboda, and others (Deliège, & Davidson, 201, p. 118).[141]

Aspects such as harmony or consonance are not universal, and what seems to be an unpleasant and dissonant piece of music for EuCul can be perfectly pleasant in other cultural environments, or even in certain collectives within the same cultural frame, e.g., the case of using art music in the train stations mentioned above.

Focusing on art music in EuCul, on which this dissertation is mainly based, musical modes first, and tonalities later, have been associated with certain moods since the early times of Greek music theorists.[142] However, it seems that there are no comprehensive scientific studies about tonal music and moods.

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[141] The emotional power of music has been approached from multiple perspectives (Cochrane, Fantini, & Scherer, 2013; Rakowski, 2016). Obviously, such a broad scholarly activity is covered by many disciplines (biology, medicine, sociology, marketing, psychology, sports, religion ...), and beyond the scope of this investigation. Based on extensive empirical work, it seems that assessing those emotional experiences requires a domain-specific approach, and a validated scale has been developed (Geneva Emotional Music Scale). The method evaluates nine items: wonder, transcendence, tenderness, nostalgia, peacefulness, power, joyful activation, tension and sadness (Zentner, Grandjean, & Scherer, 2008, p. 499). It has been confirmed that it can be used in various musical genres (art music, jazz, rock, pop) in different contexts (Cochrane et al., 2013).

[142] Plato, as wrote in *The Republic*, wanted to banish most of the scales, thinking that they would lead to laziness and lack of purpose. He thought they should keep the Phrygian and Dorian scales only, because one was good to use in battle and the other was good for reassurance in times of misfortune. All the rest were

Most references go back to Christian Schubart's book *Ideen zu einer Aesthetik der Tonkunst* [Ideas for an aesthetics of the art of sound] (1806), and, particularly, to the English translation by Dr. Rita Katherine Steblin.[143]

According to the PhD dissertation of Ted Alan DuBois (University of Southern California, 1983) under the title "Christian Friedrich Daniel Schubart's *Ideen zu einer Aesthetik der Tonkunst*: an annotated translation," Schubart's text included a limited number of references and some inaccuracies later taken for good:[144]

> There are several instances where Schubart mistakingly attributes a passage to the wrong person or in some way makes a statement which contains factual inaccuracies, leading one to believe that indeed he was without reference materials. The problem here is that many of these false statements are perpetuated in later writings (DuBois, 1983, p. 4).

In fact, the original text *Charakteristikstück der Tone* [Characterization of the tonalities] is simply reduced to a few pages in the second volume of the book, which does not offer any source or base for the information yielded.

It starts straightforwardly with the following paraphrased text from German: each tonality is either dyed or not dyed, innocence and simplicity are expressed with uncoloured key-signatures, gentle, melancholic feelings, with flat key-signatures; wild and strong passions with sharp key-signatures (Schubart, 1806, p. 377).

According to Holzer (1905), Schubart's list had been based on previous publications in the *Vaterländische-Chronik* [Patriotic chronicle] (1787), and in the *Vaterlands-Chronik* [Fatherland Chronicle] (1789) (pp. 133–134). "His characterization of the keys was the subject of discussion for both Beethoven and Schumann" (DuBois, 1983, p. 433).

Once these considerations were expressed, the Table 28 summarised the information from Schubart's original source.

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good for nothing but pathos, just useless emotions (Dixon, 2013, Chapter Athens: The Scales of Pythagoras, Section 4, para. 6). Per Zentner and colleges, Aristotle Politics book VIII, includes a «relatively detailed descriptions of the emotional effects of different musical modes. Whereas the Mixolydian mode tend to make people sad, the Phrygian mode inspired enthusiasm» (Zentner, et al., 2008, p. 494).

[143] This Vienna-based well-renowned music scholar included Schubart's source on her PhD thesis (University of Illinois at Urbana-Champaign, 1981) and later, on her book: *A History of Key Characteristics in the 18th and Early 19th Centuries*, published first (1983) in Ann Harbour [MI] by UMI Research Press, and reissued by the University of Rochester [NY] Press in 1996.

[144] Even Schubart's autobiography must be taken with caution, as it was dictated during his period in the prison of Hohenasperg to a fellow prisoner and edited much later. He used to dictate his works including that in question (DuBois, 1983, p. 9).

Summing up this chapter, music and mathematics have been linked since antiquity.

Thanks to the use of methodology taken from mathematics and acoustic physics, it is possible to build and evaluate comparatively different musical temperaments.

With the JND as reference, it is possible to determine if the variations between the frequencies of the notes of the scales under comparison are significative or not and try to find the ideal gamut.

Musical scale designs must consider that music pleasantness in EuCul is linked to consonance (Cuddy, 1993; Helmholtz, 1954; Jaśkiewicz et al., 2016; Terhardt, 1976; Wassum, 1980).

In addition, there is a need to solve the problem of the fifths.

The ideal would be a scale with a single tuning for the different tonalities, and modulation capacity; a reasonable practical number of notes in the octave; and if possible, combinable with historical instruments such as organs built for ancient tunings; it would be ideal of it could approach to tonalities of the different cultures.

The task is of enormous complexity. It must be admitted that some aspects are irreconcilable. The harmonic series of the just intonation and the 12-TET scheme are far apart (unweighted impurity comes to about 15.67 cents, that is about two times the JNC); therefore, it is hardly possible (at least for a reasonably sized scheme) to be close to both at the same time; a certain commitment becomes necessary.

The GR is present in nature, human constructions, and other developments notably artistic, as a kind of mysterious or magic ratio offering interesting possibilities to be used in music. It plays an important role in the nonlinearity, a vanguard area of research.

However, as opposed to metrics where it has a tradition of being used for centuries, truly GR-based scales have a limited development and seem difficult to match with consonance. Cartwright, and co-workers (2002) offer a way to overpass this handicap, but their 34-tone scheme is not equally tempered.

The next chapters review if the 34-tone equal tempered scale gets close to the above mentioned ideal.

Chapter Five. Material & Methods

The 34-tone equal tempered scale has been constructed following the general procedure for equally tempered schemes explained in Chapter Four. In this case, it is based on the ratios resulting from: $r_n = \sqrt[n/34]{2}$, with *n* ─ranging from one to thirty-four─ divided by the total number of notes in the octave (34). They were calculated with the function r_n = POWER(2;(*n*/34)).[145]

For the 34-tone golden scale (34T-GR), the data from Cartwright et al. (2002) have been recalculated and the notes transposed to the fourth octave (middle C); the unison was ranked as one, as usual in music, instead of zero (Table 29).

The formula used computation of cents (*c*) between two frequencies Hz_a and Hz_b, was $c = 1200 \cdot \log_2(Hz_b/Hz_a)$ [function LOG(([*cellforHz*$_b$]/[*cellforHz*$_a$];2)·1200].

In the case of knowing one frequency (Hz_a), and the cents interval (*c*), the second frequency was determined by the formula $Hz_b = Hz_a \cdot 2^{(c/1200)}$ using the power function: Hz_b = [*cellforHz*$_a$]·POWER(2;([*cellforcents*]/1200)).

Savarts (*s*) have not been included in the tables of this research. The formula: $s = 1000 \cdot \log_{10}(Hz_b/Hz_a)$ and the function: = 1000·(LOG10([*cellforHz*$_b$]/[*cellforHz*$_a$])) provide the results in savarts for the interval $Hz_b - Hz_a$. Note that with this formula, the octave has 301.03 savarts. In this research, and as indicated in Chapter Two, the differences between two frequencies have been expressed in cents.

Spreadsheets Microsoft© Excel have been employed for computations.

Direct recordings of samples by means of a 34-tone tuned instrument, an electronic piano Yamaha YDP-143, have been made using the CSE software developed by Aaron Andrew Hunt[146] adjusting the frequencies for each key. The central A position was kept as spatial reference.

Some compositions using the 12-TET scale have been created directly from the piano and transferred by USB port via cable in MIDI format, to the score writing software, using the transferring software provided by Yamaha (USB-MIDI Driver V3.1.4) for Win 10/8.1/8/7 (64-bit).

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[145] All pitches are referred to the IPN (International Pitch Notation) or American Standard Pitch Notation (ASPN). Unless otherwise indicated, all tunings of this dissertation refer to C_4 = 261.63 Hz.

[146] http://hpi.zentral.zone/cse (Retrieved November 15, 2016).

The Cakewalk Sonar Platinum,[147] and the Logic Pro 9-Scala[148] were tested for the evaluation of different computer creation possibilities.

MuseScore (MS) has been the score writing program used. The study began with version 2.1.0, and finished with version 2.3.1, (https://musescore.org/). MS was selected as an open source free music notation software.

MS, as most other similar programs ─in its preconfigured form─ does not change the frequency of the notes after the incorporation of a microtonal alteration in the stave; it is only in the case of including a standard (100 cents) sharp or a flat alteration that the frequency is automatically changed. To get the desired frequency for a microtonal alteration it is needed to enter the corresponding value in cents.

It is important to note that, even if the software had the possibility of assigning an adjustment in cents to a microtonal accidental, this function would be of no help in the case of the 34-TET scale, since the adjustments vary from note to note.[149]

Manual changes of pitch frequencies were easily made by means of the F8 ─inspector─ function. The value, in cents, to enter [$c = 1200 \cdot \log_2(Hz_b/Hz_a)$] was that corresponding to the interval between the baseline (root, or reference note) frequency (Hz_a), and the frequency to be obtained with the microtonal alteration (Hz_b).

A plugin was developed later for automatic frequency adjustment. More information is presented in chapter the sixth.

Pitch values, once obtained, have been tested with a digital audio frequency meter.[150]

No standard sharp or flat symbols have been used except in the physical notation method of double stave. The changes in frequency of the notes start from the basal pitch instead of an already sharped or flatted note.

The physical notation with double stave was the first system developed, as initially the use of existing microtonal symbols was not considered, since the intervals of the 34-TET scale do not match any of those usually associated with microtonal symbols, and it was thought that it could be cause for confusion.

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[147] https://www.cakewalk.com/Products/SONAR/Versions (Retrieved January 12, 2017).
[148] http://www.huygens-fokker.org/scala/ (Retrieved April 20, 2017).
[149] This question will be expanded in results.
[150] With thanks to the Acoustic Laboratory of the Department of Physics. University of the Balearic Islands. Spain.

However, as there is no standardization of the symbols associated with microtonal intervals, as commented on in chapter the fourth ─and the use of already existing microtonal accidentals substantially simplified the composition allowing the use of a plugin that changes automatically the pitches, minimizing errors─ a new notation (musical) system was later developed, with two variants.

The frequencies of the notes, in each the three notation options, are obviously the same, although their names may differ. E.g., the sixth note starting from C may be D in the physical nomenclature, D♭II in the musical option of using both sharps and flats, or C#III for the option using only sharps, but in all three cases its ratio is 1.107310 and its frequency 289.7 Hz.

To avoid confusion with the musical concept of grade in a scale, this term has not been used to indicate the position of the notes relative to the unison. The position of a note in the scale is called here the rank, and its number appears with the symbol # (not to be confused with a musical sharp) in the tables. As explained above, a note of a given rank has (or may have) different names according to the notation system used.

The note names were set following the basic rule that those having a near equivalent in the 12-TET scale kept the name. It must be said, however, that despite the coincidence in names, the frequencies do not match between the two scales 12-TET and 34-TET, except in two cases (C and F#).

This had two consequences: first, even the notes without accidentals needed some pitch adjustments; changes are required for the notes A, B, D, E, F, G. If the adjustments are done manually (F8 function) these notes do not need to include a natural.

When the plugin is used, all the notes need to include accidentals, as the naturals provide the information for changing the corresponding pitch to each natural note. The plugin has been designed to identify the notes without accidentals as errors. For this reason, even C needs a natural, which has no consequences in the pitch (no frequency change) but helps to prevent an error from appearing.

The second consequence is that the pitch closest to a note on one scale may not be the pitch of the note bearing the same name on the other scale. This point will be widely commented on in chapter the sixth.

The *extra* notes have been identified by adding a corresponding Roman numeral (I, II, III), immediately after the reference note, but, alternatively, it could be indicated as 1,2,3. The Roman numeral is, however, preferred to avoid confusion with the reference to the octave. Thus, for a 34-note scale, the note just after C (second rank) that lies between C and C# ─ still in the same ledger line─ is CI, and so on.

For notes after the sharp, again it would be the possible to use C#I or CI#, but the former is preferred since C#I is not the sharp of CI (that is, it is not a 100-cents higher pitch) as it could be deducted with the indication (CI#). The same procedure has been followed later with the sharp and flat notation. The octave may be indicated in text ─if required─ as a sub-index (e.g.): C_4#I.

For notation of those extra notes the tremolo symbol has been selected, and this procedure has been patented with the Ministry of Culture in Spain (Marco-Franco, 2017) (Figure 58). The number of strokes through the stems of the notes match with the number of bars (I, II, III) in the note's name (one for CI, etc.). It was chosen because, nowadays, it is a non-essential symbol, but still present in all score writing programs, and it may be easily replaced by the equivalent value notes, or by writing over the note Trem-I, Trem-II, Trem-III. This nomenclature was considered appropriate for both tempered and non-tempered 34-tone scales.

A number to the left of the stave, before the clef, indicates (if needed) the corresponding octave following the International Pitch Notation (IPN) or American Standard Pitch Notation (ASPN), that, as stated at the beginning of the text, is the standard used in this dissertation. In any case, there are no restrictions in the use of clefs with the 34-TET scale.

The words GR or TET, also to the left of the stave, will indicate which 34-tone scale is used, if required (Figure 59), but only the 34-TET scale was used in the samples, and this indication is absent in the music sheets.

As for score writing software, as commented on, the physical nomenclature system with double stave was used initially (Figure 60).

With this first method (double stave) there is one stave for the score and another for sound reproduction; the first stave (a) (visible) is the one seen in the music sheet containing the notes with the corresponding strokes through the stems as required. In MS (view – mixer) its volume must be set to 0 and the mute cell marked. A parallel second stave (b) (hidden: edit→ instruments→ deselect visible) with conveniently modified microtonal notes to get the appropriate pitches exactly matched in metric with the visible music sheet.

Consequently, for each voice there will be two staves, although only one is finally visible. As the sound-stave is hidden, any symbols before the notes with the appropriate time values may be used. The only conditions are to be consistent with the selection and that time value must be the same in both staves.

To summarize, the check points for this procedure are:

- Assure that each note is paired.
- Change frequencies in stave (b) using the inspector (F8) function.
- Match time values between staves.
- Assure that the sound in stave (a) is turned to 0/ and the cell "silent" selected.
- Set the sound stave to not visible.

A pattern with 34-tone pairs was constructed. This facilitated the use of copying and pasting,[151] minimizing the errors of continuous changes of frequencies with the F8 function. MS transportation to a different octave (or clef) maintains the adjustment in cents, consequently it was not needed to have more than one clef pattern-sample.

During the development of the project, a second notation based on a single stave with microtonal symbols was incorporated (Figure 61). This musical notation has two possibilities, either to use only sharps, or to combine sharps and flats.

For compositions using any of the two variants of this second (musical) system, the same note seen in the score is the one producing the sound. The upper part in the Figure 61 shows the sharp only notation modality, while the sharp and flat is depicted in the lower part. The cents in values needed for pitch adjustments are also advanced in this figure but the rational for these changes will be expanded in the Chapter Six of results.

The concept of consonance has been reviewed in Chapter Four. For this dissertation consonant intervals included: 1/1 unison; 1/2 octave; 2/3 perfect fifth; 3/4

[151] Note: MuseScore does not allow copying and pasting over irregular groups.

perfect fourth; 3/5 major sixth; 5/8 minor sixth; 4/5 major third; and 5/6 minor third. The intervals, second, seventh and tritone are considered not to be consonant.

For the theoretical analysis of consonance several procedures have been followed, simple difference in cents, disharmony coefficients (weighted or not), and root-mean-square deviation (RMSD) for the case of a 12-tone scale (C–B):

$$RMSD\ (\vartheta) = \sqrt{{}^{1}/_{12}\left(ImpC^2 + ImpC\#^2 + \cdots ImpB^2\right)} = \sqrt{\frac{\sum Imp_{C-B}^2}{12}}$$

weighted and unweighted.[152]

Also, the information about MQD (Mean Quadratic Dispersion) available in the publication of Cartwright et al. (2002) was used but with the data simplified into Likert scale steps. MQD or RMSD methodologies may be used for tempered and for non-tempered scales, but in this second case it will require the computation of the values for all the different tunings and the average of the results, a cumbersome procedure that was not employed in this research.

It is important to take into consideration that the condition required for using MQD or RMSD is an integer (equal) division of the octave (EDO) such as in 12-TET (12 x 100 cents intervals). As already indicated, other designs, e.g., Pierce's gamut that is an equal division but based on the tritave (ratio 3/1 instead of 2/1), are not considered appropriate, requiring an analysis for each key as if the scale was not equally tempered.

The second procedure follows that of Cartwright et al. (2002) giving a weight coefficient to each interval "the square of the difference between the note of the equally-tempered scale that best approximates each harmonic interval, multiplied by the relative weight of each interval and summed over all the intervals" (p. 56).

The rationale is that an impurity has a different impact in a consonant interval (e.g., a perfect fifth) than in an already dissonant interval (as e.g., a minor seventh).

The authors (Cartwright et al.) include a figure (p. 56) to determine the MQD value (σ) by simply entering the number of notes of the (equal-tempered) scale. This computation has been simplified by modifying the figure to get the σ in ranks from +3 to 0 (Marco-Franco, 2018, p. 17) and depicted in Figure 62.

[152] See Chapter Four for details.

Here, the method is similar but using the author's own calculated weighted coefficients (Table 30),[153] applied in a simplified way to each of the differences in cents between the notes of the scales to compare, and then the twelve corrected values are added up in the so-called adjusted coefficient of disharmony in cents of the scale (Marco-Franco, 2018, p. 34).

Compositional samples are collected in two portfolios. The first one contains different pieces in assorted genres and styles for the Common-practice periods: Renaissance, Baroque, Classical and Romanticism; serial, noise, soundtrack, and atonal music representing the twentieth century; non-European pieces; and popular music including bossa nova, tango and swing. The second portfolio contains the multi-artistic project *The Asian Garden* and combines both scales (34-TET and 12-TET).

A part of the dissertation is devoted to studying the advantages of using the 34-TET scale, starting with the analysis in order to review whether the 34-TET scale offers advantages in relation to 12-TET for reproducing the historical temperaments set at a *probable tuning* of the period.[154]

Among the different possibilities reviewed in Chapter Four, two tunings have been selected for the analyses, under the consideration of probability: A = 422 Hz, Händel's Messiah, Mozart's piano, and eighteenth-century English organs; and C = 256 Hz, that results in about A = 430 Hz, depending on the interval C–A used (Bose, 2013; Haynes, & Cooke, 2001; Lawson, & Stowell, 1999).

Also, based on the estimation of popularity, the temperament of Arnault de Zwolle was selected for the Renaissance samples, Werckmeister III, and one-quarter-tone meantone for Baroque, and Tartini-Vallotti for Classicism.

Following the mentioned methodology, the differences in cents for the notes in each of the samples composed in Renaissance, Baroque and Classicism styles have

[153] This procedure follows the idea of professor Michael C. Lopresto (2015) from the University of Michigan to rate musical intervals using dissonance metric (pp. 226–228). A metanalysis pooling information from several sources (Cariani, 2009, para. 11; van de Geer, Levelt, & Plomp, 1962, p. 308; Plomp, & Levert, 1965, pp. 556–557; Sethares, 2005, p. 47) was performed, and the dissonance variables of these methods (pitch instability salience, sensory dissonance, class dissonance, etc.), have been combined, setting a coefficient factor (a) for each of the 12-TET intervals; The values were then simplified to items of a Likert scale (b) (Marco-Franco, 2018).

[154] It must be accepted that due to the wide variations in tuning pitches summarized in chapter the fourth, and the lack of statistics of the historical uses of temperaments and tunings, the decision remains in the field of a supposition or probability, and other temperaments or tunings could have been selected with the same or even better merits. However, as seen in the results, the uniformity of the differences favouring the 34-TET scale supports the conclusion that the results would probably be consistent with other temperaments and tunings.

been determined, and the same procedure was followed in well-known masterpieces of music of those periods.

A brief review of the musical context has been included prior to each of the pieces, followed ─as indicated in the case of composition in Renaissance, Baroque, Classical and Romanticism samples─ by the selection of the musical scale that most likely would have been used if the pieces had really been composed at the time.

In all cases the evaluation has included each of the two selected tunings (A = 422 Hz, and C = 256 Hz) for each of the historical schemes with pitches calculated for the corresponding tonalities, comparing which of the two temperaments (34 or 12 TET) provided a better approach to each historical gamut.

As for the methodologic details, the procedure started by choosing at random (https://www.random.org/integers/) the bars, representing, at least, ten percent of the total in each piece evaluated. The notes in those bars were identified and the number of times they repeated were counted.

Once set how many times (*n*) each note was present, this number (# *n*) was multiplied by the corresponding differences between the frequencies expressed in cents, (a – b) [historical – 12-TET], and (a – c) [historical – 34-TET], for each of the notes and summed over all, to see which of the two scales (12-TET or 34-TET) added fewer cents (or in other words had less impurity or got closer to the historical gamut). Weighted and unweighted RMSD methodology was used in all the cases. The procedure, as indicated, was repeated for all samples and in well-known music pieces, with the results depicted in tables.

For the Romantic style sample (*Podolian Rhapsody)*, it has been assumed that the scale that could have been used would be the quasi-equally tempered scale of 12 notes. As in previous cases the probable tuning during Romanticism was determined (A = 432 Hz) and the computation was, in this case, to evaluate how much the 34-TET scale approached the 12-TET scale tuned for A = 432 Hz, or in other words the capacity of the 34-TET schema for a reasonable reproduction of a romantic piece.

As for the remaining pieces in portfolio I, including those of popular music, no numerical methodology was included, allowing the listener to decide whether the 34-TET scale compositions are appropriate for these cases.

The set theory Pitch-Class [PC] module 12 analysis (Morris, 1991; Straus, 2005) (commented on in Chapter Four) has been used for the serial piece *Adrán Series.*[155]

For the composition of Indian subcontinent culture (SACul) the traditional Indian tuning of Bhatkhande has been used (Nayar, 1989, p. 137); the ratios correspond quite well with those obtained at the Mahaaraja Sayajirao University of Baroda using an oscilloscopic frequency meter (Vaishnav 2007, pp. 48–62).

Table 31 depicts the notes, values and corresponding notes from Bhatkhande schema, and the 34-TET closest pitch used.

Due to the microtonal structure of traditional Indian music, with 22 shrutis, the 34-TET scale offers a better approach than the 12-TET schema for these pitches, as any shruti may be only 17.65 cents away ─in the worst and most unfavourable case─ from the note of the equal tempered scale of 34 tones, as compared with 50 cents for the 12-TET most unfavourable case (Nayar, 1989; Prajnanananda, 1963).

To preserve the bass notes that are so characteristic of Indian music in the sample, since the tanpura does not follow a score scheme and, in addition, it is a fretless instrument, with four metal strings which do not even play in a melodic rhythm but in a continuous loop of a pedal bourdon (drone) harmony, a “false” melodic bass line has been assigned to tanpura (including more than 4 pitches) which would not be possible to be reproduced as written in a real performance. In a real performance the written bass melody would need to be reassigned to a fretted bass instrument (e.g. the surbahar), with the tanpura, then, in its real bourdon role.

Moreover, the sound library of MS has restrictions for these non-EuCul instruments: neither the tanpura, nor the surbahar or the bansuri flute are reproductible with it. The final solution adopted has been to use the sound effects of sitar, shehnai, and a bass clef (for prim) with synthetic bass (4) reproduction of the bass melody. This and odder details will be expanded in the comments included in appendix the first with each sample.

[155] Extended PC-Class analysis and Neo-Riemannian procedures such as clock-face diagrams or requiring vector transformations and complex tonal spaces graphics [tonnetz] have not been included in this work, but may be found elsewhere (Cohn, 1998; Harrison, 1994; Morris, 1991). Common set theory standard notation C = 0, C# = 1, etc., was used.

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Samples totalize 26 exhibits. Portfolio I contains 14 compositions, from Renaissance style to noise-music created with the 34-TET scale, and incorporates pieces approaching other non-European cultures.

Portfolio II ─the multi-artistic performance *The Asian Garden* with 12 musical pieces─ shows the possibilities of combining both scales 12-TET and 34-TET offering additional artistic material for the project.

It also represents an appreciation of East Asian cultures. The libretto of *The Asian Garden* may be found in the second appendix.

Once the compositions were finished, the function export file allowed the exporting (PDF) and/or printing of the score with the microtonal notes. All the music sheets (portfolios I & II) have been joined in a third appendix.

Exporting the file as audio (WAV) provided the sound, either of the hidden stave ─containing the altered notes corresponding exactly to those printed in the music sheet for the case of physical notation─ or in the case of using the musical notation and the plugin, the single stave having both functions.

The list of audio files has already been included in the index and the audio samples themselves can be found in the corresponding folder labelled as *audios.*

A limited audio-visual (AV) edition of each piece made with Windows® 10 editor (MP4 format) has also been included.

Copyright-free images (mostly CC0) were incorporated mainly from Pixabay® and similar sites. Each image, other than those with CC0 license, includes the source and the copyright details.

The AV creations have been incorporated with the idea of adding artistic value to the dissertation, helping to evoke the atmosphere and the narrative ideas underlying each sample; they may be found in the folder labelled as *videos*.

They may also be found in the free sharing web side YouTube®; the URL addresses are included in the footnotes of each sample commented on in the first appendix.

Audacity 2.1.0 has been used, very sporadically, for some final sound adjustments in basses and trebles. However, no audio mix or digital manipulations of the sound have been used.

Chapter Six. Results

Historical Antecedents of Thirty-Four-Tone Scales

The following lines present the results of an overview research on prior information and data existing about the 34-tone temperaments.

A survey about microtonality has already been included in Chapter Four. Geoff Geer (2016), in his work for Anglia Ruskin University, offers extended historical information on this regard, containing a table of intonation systems, and the ratios based on Harry Partch & Ptolemy, for tempered scales from 5 to 72 notes (including also 34EDO) (p. 25).

The subsequent review focuses only on the scales with 34 tones.

Non-tempered scales Several 34-tone non-equal temperament schemes have been found, none of which were more than theoretical projects:

- Cartwright et al. (2002) 34-tone non-equal division of the octave.
- Erv Wilson's 34-tone harmonic tuning (Wilson, 1994) shared to Neil Haverstick. Wilson developed several microtonal schemes, but his 34 tone non-tempered design has had no practical development.[156]
- Wendy Carlos "gamma" design (Carlos, 2008, para. 6; Wilkinson, 1988, p. 81), with sharp octaves close to Indonesian Gamelan gamut.[157]

Although the GR scales have already been studied in a previous work (Marco-Franco, 2018), some comments in relation to Cartwright et al. (2002) 34-tone golden ratio temperament (34T-GR) need to be added, as this project is essentially an evolution from the GR gamut described in that paper, although set up on the equal division of the octave.

Based on the continued fraction (Hardy, & Wright, 1975, pp. 154–177), Cartwright and co-workers (2002) pointed out that "the construction of a musical scale is then a problem involving approximating irrational numbers by rationals […] the best

[156] Neil Haverstick personal communication, August 22, 2018. With thanks to the composer, guitarist and friendly colleague for the information provided, including material from his forthcoming book, and the authorization to use his material.

[157] The values that she described are not exactly a 34-tone division: «alpha» = 78.0 cents/step = 15.385 steps/octave, «beta» = 63.8 cents/step = 18.809 steps/octave, and «gamma» = 35.1 cents/step = 34.188 steps/octave. More designs by Wendy Carlos, may be found in the book by Wilkinson (1988), which also includes a foreword by her. As per Haverstick she never recorded with gamma scale (personal communication August 22, 2018).

such approximation […] consists of writing the irrational number a continued fraction" (pp. 53–54).[158]

Hardy, & Wright (1975) description of the continued fractions follows:

> We shall describe a function
>
> $$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}} \quad + \frac{1}{a_N}$$
>
> of the $N + 1$ Variables $a_0, a_1, \ldots a_n, \ldots a_N,$ as finite continued fraction, or, when there is no risk of ambiguity, simply as a continued fraction. Continued fractions are important in many branches of mathematics, and particularly in the theory of approximation to real numbers by rationals (p.129).

As for Phi, the value (φ) may be expressed as a continued fraction where the $a_0 \ldots a_N$ values are one:

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$$

The golden component in the construction of Cartwright et al. (2002) is included through the convergence series of Fibonacci (1/1, 1/2, 2/3, 3/5, 5/8, 8/13…). Following the first convergent after the octave (1/2), the interval is divided by the next (2/3); to compensate, another mirror note is set. The division principle is that the intervals cannot be greater than the quotient of $c_{n-2}:c_{n+1}$ or less than that that of $c_n:c_{n-1}$ (Figure 63).

The procedure is repeated until it reaches a central space that does not have a rational value for solution, so it is filled with the tritone, which is the only irrational that, due to its central location, satisfies the mathematical conditions and symmetry.

Once the 12 notes (convergent 5/8) are reached, the process continues (8/13) until reaching 34 notes.

[158] The practical demonstration of this approach, and the explanation why the 12-TET scale has precisely that (twelve) number of notes may be found in chapter eight of the book by Daniel Adam Steck (2002, pp. 105–112) with an easy musical approach including exercises.

The Table 29 shows the 34-scale (34T-GR) with cents and frequencies for C_4 and for C_8, as it was the eighth octave the one used by Cartwright and co-workers (2002) in their publication (p. 56).[159]

Equally-tempered scale. The first EuCul reference related to the 34-TET gamut seems to come from Thuringia in a treatise entitled *Nova & Exquisite Monochordi Dimensio* by Cyriac Schneegass (1546–97) a disciple of Johannes Lindemann.[160]

Per Barbour, the 34-tone temperament appears in chapter III:

> The 34-division is a positive system, like the 22-division. That is, its fifth of 706 cents is larger than the perfect fifth, being the same size as for the 17 division. Its thirds is about 2 cents sharp. This it provides slightly greater consonance than the 31-division. But, like the 22-division, it has remained one of the stepchildren of multiple division, largely because it is in a series for which ordinary notation cannot be used. There is a surprising mention of the 34-division by Cyriac Schneegass in 1591 (see Chapter III), but his own monochord came closer to the 2/9 comma division. Bosanquet had indicated the relation between the 22-and 34-division and has praised the 56- and 87- divisions also as similar systems. Opelt, too, has included it in his fairly short list (Barbour, 2004 p. 121).

The "surprising mention of the 34-tone division" could not be found in the text, neither in chapter III (better said in the two chapters headed as chapter III) nor in any other place in the text.

The monochord is described by Schennegass through a scalene triangle figure. Additionally, the essential values of the monochord are clearly indicated in the "second" chapter III:[161] diapasson 2/1, disdiapasson 4/1, diatessaron 107/80, diapente

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[159] The only difference is that the unison here is ranked as one as usual in music, while in the original work it is zero. The terminology rank for the notes has already been specified in Chapter Five. The indication # in the tables represents the rank number of the note with the unison ranked as one.

[160] Both studied in Gotha and had familial links (Schneegass was married to Dorothy, daughter or cousin of Lindemann); he was a pastor in Friedrichroda (18 km from Gotha).
Through Thomas Bouillon from the Erfurt library (with gratitude), a scanned facsimile of Schneegass' book ─ in the text in Latin the author appears as Snegass─ has been obtained; another copy was facilitated by Birgit Busse (also with thanks), from the State Library in Berlin: http://resolver.staatsbibliothek-berlin.de/SBB00020A4500000000. Both facsimiles correspond to Nicolaj Rostby (1592) ex libris from Isagoges Musicae (1591); Per Buelow (1972) there is also a Deutsche musica (1592) edition (p. 48). According to Thomas Bouillon information, the original text is available in the libraries of: Berlin, State Library of Prussian Cultural Heritage; Koblenz, Library of the Stiftung Staatl. Görres-Gymnasium; Vienna, Austrian National Library; and Wolfenbüttel, Herzog August Library (personal communication August 27, 2018). Both scanners are identical and although the text pages are not numbered, the publisher followed the procedure that the first word at the beginning of each page appears at the lower right margin of the previous page, so both texts seem to be complete. The place, and date for the conclusion of the manuscript appear at the end of the preface: Friedrichroda, July 6, 1590 [Fridrichrodæ ad Syluam semanam, pridie nonæ Iulij, anno Epoches Christianæ, 1590]. The treaty was dedicated to Lindemann [Doctrina ac virtute ornatissimo viro, Domino Iohanni Lindemanno, Musico Egregio, Eclesia Gothanæe Cantori, affini suo charißimo]. Per Rose (2016) this devoted dedication indicates a clear close and friendly relation between the two (p. 236).

[161] Which corresponds really to chapter IV since chapter V follows.

160/107, and tone 12800/11449. Therefore, the scheme has an octave (*r* = 2), a fourth of 107/80 (*r* = 1.3375), a fifth of 160/107 (*r* = 1.4953) with a total of twelve notes ─confirmed by the information in the tabula dimensiones monochordi [last page]─ the tone has a ratio of 1.118, that is 193.1 cents. In conclusion, Schneegass' monochord is not of 34 notes.

However, looking at the figure of the triangle (drawn in chapter III [the first]), under the inferior line, there is a ladder-like drawing starting in Γ (Gamma), and ending in G, (thus an octave), with several divisions (each one with two equal subdivisions). The first eleven of those divisions are numbered, while the remaining have no number, with a total of 34 intervals in the octave. The possibility for a 34-tone division seems to be deduced from the figure, not from the text.

Barbour analysed the monochord as follows:[162]

> Schneegass gave an interesting geometrical construction for what was much like the common meantone temperament, but more like the 2/9 comma temperament. His contention was that the diatonic semitone contains 3 ¼ "commas" and the chromatic semitone 2 ¼. (These commas of 35.3 cents have nothing in common with either the diatonic [23.5] or the syntonic [21.5] comma). Thus the tone contains 5 ½ commas, and the octave 5 x 5 ½ + 2 x 3 ¼ = 34 commas. As shown in Chapter VI, the 34-division has fifths that are almost 4 cents too large and thirds that are 2 cents too large (Barbour, 2004 p. 37-38).

Aside from the historical mentions of microtonal scale designs in China, and the above-mentioned sixteenth century treaty of Schneegass *Nova & Exquisite Monochordi Dimensio,* it seems that no additional treatises, instruments or practice with a gamut of 34 divisions of the octave were recorded prior to 1993,[163] when Larry A. Hanson published a notation system called Selene:

> "#" means the pitch is raised by two (2) scale steps from the natural tone; "b", pitch lowered by two (2) scale steps from the natural tone; "/", pitch raised by one (1) scale step from the nominal tone; "\", pitch lowered by one (1) scale step from the nominal tone (Hanson, 1993, p. 39).

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[162] It is true that the difference of the thirds expressed in cents (5/4) = 386.31 cents to (6/5) = 315.64 cents equals 70.67 cents, which is about a double tone of the 34-TET scale (70.59 Cents), but this is an incidental finding and no theory generating the scale based on this value has been found, to the best of the author's knowledge, based on the publications so far reviewed.

[163] Pioneer Mexican microtonal composer Augusto Novaro constructed different guitars ca. 1927, but per Çoğulu (2018) none with 34 equal divisions of the octave. Particularly interesting for this dissertation is his opening sentence from his work A Nature System of Music Based on the Approximation to Nature (1927/2008): «Music is a combination of art and science in which both complement each other in a marvellous fashion»

Unfortunately, no more information about this method has been found, and some questions, e.g., how he notated the third flat of D, or his enharmonic third sharp of C, remain unsolved.

The 34-TET scale has between C and D or between A and B five notes, so an extended notation is required.

The interest in guitars with microtonal division of the octave may be traced back at least to the nineteenth century, with an enharmonic design by Thomas Perronet Thompson, built by luthier Louis Panormo.[164]

Erv Wilson sketched on July 28, 1994, a 34-tone scale by Larry Hanson (July 26th), with alternating tones of 21.51 cents and of 41.06 cents (Wilson, 1994).

However, if this is the same scale that Hanson mentions in his paper of 1993, this scheme, as already commented on, is a *systematic* division of the octave, but not an *equal* division of the octave (not EDO).

The difference is better appreciated with a practical case: when tuning for C_4 = 261.63 Hz, Hanson's temperament has pitches matching perfectly well with the intervals of consonance (as in JI); but e.g., if E_4 = 327 Hz ─a major third based on C_4 = 261.63 Hz, with a ratio of 1.251388─ is taken as tonic, its major third comes to be 408.75 Hz, but with the tuning for C (261.63 Hz), the closest pitch is 407.0 Hz (twenty-third note of the scale) that is over seven cents out of tune.

Larry Hanson also included the image and details of an electric guitar for 34-tone:

> The standard 12-tone per octave fretted neck of a standard Fender© Telecaster ® was removed; and a new neck, fretted for 34-equally tempered tones per octave, was attached. This work was done by Carruthers' Guitars in the Los Angeles area (Hanson, 1993, para.2).

As per Hanson (1993) this was the first time that a 34-tone instrument was constructed: "Although having received passing mentioned for centuries, no-one (to my knowledge) has previously advocated or promoted the use of any instrument employing the 34-equal tuning" (para. 4).

[164] A detailed history of this topic may be found in Schneider (1985) with additional information and images gathered in a recent video by Tolgahan Çoğulu (2018) ─a young Turkish guitarist and scholar with particular interest in microtonal guitar (http://www.tolgahancogulu.com/en/)─ but Çoğulu has never used a 34-TET instrument and he is not aware of any academic musician using one (Personal communication, August 25, 2018, with thanks).

Neil Haverstick is a pioneer in the use of the 34-tone guitar and temperament. According to his forthcoming book, his first contact with the 34-TET was about 1995, when Larry Hanson shipped a 34-tone guitar to him to try.[165]

Although Hanson considered 53 tone to be a temperament closest to the harmonic series, he thought that 53 were too many frets for a guitar and decided on 34 instead. The work was done by Carruthers Guitars in Los Angeles (Hanson, 1993). More details of this pioneering 34-tone acoustic guitar and the bass built as well, and generously offered by Hanson to Haverstick, and how Hanson heard Haverstick play it for the first time in Los Angeles, will hopefully be found soon in Haverstick's book.[166]

Aside from guitars, no other reference for custom-made instruments for 34-TET have been noticed.

Recordings. An incidental reference to 34-TET recording has been noticed: "19-tone, 24-tone, 34-tone, 36-tone, and 72-tone musics of Ivan Wyschnegradsky, Julian Carrillo, Alois Haba, Ezra Sims, Neil Haverstick, and others," (Gann, 2016, para. 11) but the citation does not provide concrete and specific data for 34-TET.

Consequently, the first consistent EuCul data about performances with the 34-TET scale seem to be those of the guitarist Neil Haverstick available on CDs (*Acoustic Stick* [CD 1997, recorded live at microfest in El Paso]; *Other Worlds [CD 1999];* and *If the Earth Was A Woman [CD 2002]*. His website (htttp://microstick.net/music/) also provides some free samples of them.

As per August 23, 2018, the Naxos database contained only nine microtonal recordings, none of them labelled as 34-tone. A limited frequency analysis survey[167] resulted in no match with 34-TET.

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[165] Larry Hanson knew about Neil Haverstick playing a 19-tone guitar during the microfest in New York in 1994 for the American Festival of Microtonal Music (AFMM). Lary Hanson was not a musician but engineer by profession, although very interested in music tuning and related concepts. The idea of this scheme was to get closer to the intervals of the harmonic series. (Neil Haverstick personal communication, August 22, 2018).

[166] With warmest gratitude to Neil Haverstick, and his active information interchange (personal communications August 20–26, 2018). Images of 34-tone fretted guitars may be found on his website (http://microstick.net/guitars/).
The site https://www.last.fm/tag/microtonal+guitar/artists offers additional data of top microtonal musicians. The blog of Nacho Belido (http://theguitar-blog.com/?p=83) offers instructions on how to convert a standard guitar into a microtonal one including the formulas for calculation the frets places.

[167] The frequency analyses commented on in the following lines have been done out of laboratory over a very limited number of pitches from an audio file from the internet and must be taken with great caution. Appropriate lab methodology will be required for precise evaluation.

At a concert in Vernon in 2010, Ascension on a 34-TET guitar, Chris Gianitti on bass and Jack McChesney on drums, recorded an improvised piece for popular music, out of program (https://archive.org/details/Ascension_105). Again, a limited frequency analysis suggests 34-TET scale tuning for A = 440 Hz.

Claudi Menehim (2012) offered a MIDI organ version of his *Padanian Canon* (second version) in just intonation, 34, 38, 46, and 50 EDO.

Two years later (2014) the author proposed an organ version of Scott Joplin's *Maple Leaf Rag* tuned into 34 EDO, played at midi organ. Menehim explains the procedure for tuning as follows:

> The original 12-tone system of the piece has been turned into 34edo by establishing an appropriate sequence of local tonal centres, then mapping notes which are ±4*N (N≥0) fifths apart from each tonal centre to notes which are ±N 34edo natural major thirds (388.235 cents) apart from the tonal centre itself. Residual stacks of fifths (including the case N=0, i.e. ±0, ±1, ±2 or ±3 fifths) have been mapped to stacks of 34edo superpythagorean fifths, as large as 705.882 cents each.

Information for tuning software was also provided: it was done with Scala, implemented with Timidity ++. Sound font: organPipe.sf2.

A review of the frequencies of this piece seems to match with 34-TET in a high percentage of the pitches.

Discrepancies (minor) may be due either to MIDI format or to the technique of measuring the pitches from an audio file on the internet.

On November 24, 2014. Robin Perry uploaded two pieces of popular music labelled 34EDO: *Black Salt-white Pepper* (http://soonlabel.com/xenharmonic/archives/2593) and *Uncomfortable in Clouds* (http://soonlabel.com/xenharmonic/archives/2590).

On June 30, 2016, Perry also uploaded another piece of popular music with Brazilian brushstrokes on 34-TET temperament (again a limited review of the frequencies suggests tuning for A = 440 Hz) (https://www.youtube.com/watch?v=FXTM0HeuExk).

David Stützel (2016a, 2016b) plays on YouTube what he describes as Georgian tetrachords 34 EDO. He seems to use a sort of Zillion-Keyed keyboard.[168]

[168] http://cdm.link/2007/01/zillion-keyed-keyboards-new-musical-layouts-and-microtonal-gadgets/

The limited review of frequencies may loosely coincide with 34-TET tuned for A = 440.

David Victor Feldman [n.d.] reports on Soundcloud© *Five Rags* that seem to have been done also with 34-EDO tuned for A = 440.

Tolgahan Çoğulu (2014) shows on YouTube two different possibilities for microtonal guitar frets, including [at the end of the video] a small sample of 31-TET makam, following the theory of Professor Dr. Yarman (2007, 2014) about the good possibilities of the microtonal scales for makam.

As indicated, 34-TET has not yet been used by this guitarist.

Thus, this historical overview confirms the information advanced by Çoğulu that he was not aware of any art music guitarist using the 34-TET temperament.

Additionally, this research has not found any other instrumental soloist, ensemble or art music orchestra using this scheme nor a score writing report. To the best of the author's knowledge this is the first report of a score technique and 34-TET-scale art music creations.

Thirty-Four-Tone Equally-Tempered Scale: From Theory to Practice

Tuning, frequencies, intervals. Physical and musical nomenclatures. As stated, one of the particularities of the 34-TET scale is that, starting in C = 261.63 Hz, there is no 440 Hz note and if tuning for A=440 Hz, then there is not a 261.63 Hz note.

Consequently, a decision must be taken whether tuning to middle C (261.63 Hz) or tuning to middle A (440.00 Hz).

The information could be eventually added to the scale if needed, (e.g. 34-TET_C for C tuning or 34-TET_A for A tuning) and indicated at the beginning of the stave (Figure 59).

As C and F# pitches match in the three scales 12-TET, 34-TET and 34T-GR, it is proposed to use C = 261.63 Hz as reference, instead of A = 440 Hz.

If an A based tuning would be required, then A = 435.5 Hz, or 444.5 Hz (pitches usually within adjustable A tuning range) would be the options; else, C would diverge from 261.63 Hz in approximately 17.5 cents.

Of the two options, the tuning to C = 261.63 Hz offers a slightly better approach to 12-TET, particularly with better matching for unison, perfect fourth and fifth; this becomes clearer with the weighted value (14.45 vs. 14.65); C tuning was the option used in this dissertation (Table 32).

As advanced in Chapter Five three options for notation have been developed, as shown in Table 33. The first one, labelled as physical (Ph), follows Cartwright et al. (2002) scheme; the other two are called musical options, with the possibility either to use only sharps, or flats and sharps; they are closer to music theory taxonomy.

As already stated, the pitches, logically, remain unchanged independently of the option used, as they are only denominative. It has been iterated that comparisons must be based on frequencies or ratios and not in note names.

The first schema (physical notation) follows the names of the notes as in the work of Cartwright et al. (2002), allowing easy comparisons between the 34-tone golden scale (34T-GR), and the 34-tone equal tempered scale (34-TET).

The only change here, already commented on, has been to start the unison in one (in this case the tonic C ranks one) as usual in musicology, instead of zero, and the frequency values have been transposed to the fourth octave (C_4).

It is important to note that due to the construction system used in 34T-GR, based on the Fibonacci convergent series, the number of notes included in a given interval of the 12-TET scale may contain different numbers of notes in the 34-tone scale, depending on the starting note, as also mentioned.

The same holds for 34-TET; e.g., while from C to D (a second major in the 12-TET scale) there are five notes on the 34-TET scale, from G to A (also a second major in the 12-TET scale) there are only four. Similar differences may be found in other cases.

Consequently, one interval may easily not fall into the expected note according to the standard musical theory learned at the conservatories for 12-TET, as repeatedly indicated.

This is particularly confusing when using the physical notation for interval determination, and this procedure is not recommended, but to use the frequencies, the ratios, or the number of cents instead. E.g. taking the homonymous D (sixth note) for the best matching note of D (12-TET) in the 34-TET scale is worse than taking the seventh note (DI), instead of the sixth; the difference of 23.5 cents improves to only 11.8 cents.

Thus, to get a frame closer to musicology, a second schema called musical notation was developed, with two variants, one using only sharps signs and the second using both flats and sharps.

It must be noted that with the 34-tone scales a flat and the corresponding sharp enharmonic note, according to 12-TET scheme, are different notes. This scale has its own enharmonic equivalences (Table 33).

Using the 22-notes of JI temperament (Table 15) as reference, the names of the notes in the 34-TET scale were assigned to the nearest pitch with the idea of keeping the terminology as close to the standard 12-TET notes as possible.

However, the terms *enlarged* and *reduced*— for microtonal intervals of 35.3 cents up or down— have been used here to avoid confusion with the augmented and diminished/minor concepts, which are used in music theory for 100 cents stepping.

The coefficient between two successive chromatic note ratios is 1.059463094 for 12-TET and 1.02059591 for 34-TET.

Tones and chords development. The scheme in Table 34 helps to construct the frequently used major and minor mode diatonic musical scales with the 34-TET scale and their corresponding chords. The 34-tone scheme opens the possibility of new sets of chords.

The chromatic scale for F is presented in Table 35. It is brought to visualize the point that using only the names to set an interval may be misleading. With the physical nomenclature —the fifteenth rank note, a perfect four in the table— is not the expected A# but AIII. The musical nomenclature in this case is consistent with A#.

Score writing. Neither a notation based on commas nor even one based on the small schismas (1.98 cents) are appropriate for 34-TET scales, since the microtone of about 35.3 cents does not fit with any multiple of commas or schismas.

A scholar work for microtonal notation can be found in Duckworth (2012) although it does not include information about 34-TET.

Rechberger proposes combinations of symbols for a great range of scales, including 34-TET (2018, p. 251).

This proposal presents three inconveniences: first, it is not standardised, so it is just a suggestion like any other; second, the symbols are rather small and confusing;

and third, they cannot be written with conventional score writing programs unless an adaptation or plugin would be developed for this purpose.

Hasegawa (2007) has used open-headed arrows for adjustments of 33 cents, but the 34-TET intervals are 35.29 cents and, again, arrows are not usually standard in score writing programs.

It seems that Larry Hanson's schema, commented on above, has never been put into practice.[169] Moreover, the information is incomplete and again the symbols cannot be written with computer programs, and the same may be said for some other options suggested for microtonal composition in Chapter Four (Figure 57).

The ideal solution will be to use symbols not associated with already defined intervals, and available in current score writing software.

The initial proposal (see Chapter Five) was to use the standard notation but adding strokes (as in tremolo) through the stems of the notes matching with the Roman numerals (I, II, III) corresponding to the extra notes (Figure 58) (Marco-Franco, 2017).

The system was chosen because nowadays the tremolo stroke, although not that essential a symbol, is still present in all score writing programs.

If needed, the tremolo could be easily replaced by the equivalent value notes, or by simply writing over the note Trem-I, Trem-II, Trem-III.

As commented on the fifth chapter, with the double stave method there is one note in the stave for score and another paired note, with same time value, in a hidden stave for sound reproduction (Figure 60). In each case there is one note seen in the score but not heard, and another paired hidden note that is the one that sounds.

Very importantly, a microtonal alteration only applies to the note just after it. Other subsequent notes in the bar, either with the same pitch or octave/s apart, are not affected by the accidental; in other words, microtonal accidentals do not alter all the notes in the bar, but only the concrete one following it.

As mentioned, the accidentals are mandatory for all notes; unaltered notes require a natural, if the plugin is used. They are also recommended, although not strictly needed, when manual adjustment is used. It must be remembered that other

[169] According to Neil Haverstick, Hanson first heard him play the 34-TET guitar in Los Angeles, when his (Hanson's) health was very poor, and Haverstick never used music sheets for this temperament (personal communication, August 22, 2018).

than C, and F# all the notes in the scale 34-tone have pitches different from those of the 12-TET scale.

The physical nomenclature uses standard flats symbols (#). It was designed in that way as the sharp (#) is easily found in any keyboard, but not the flat (♭), a special symbol not always available. This has no influence on composition in any tonality either with sharps or flats. This variant with only sharps has the sole objective of facilitating the typing of the notes with a keyboard.

Key signatures with so many notes may be confusing and result in additional unwanted changes (see later); they are not recommended, although tonality may be indicated, if needed, at the beginning of the score as e.g., C major [CM] in Figure 59.

In case of using the physical notation, if notes are beamed together it is good practice to print the beam descending (downwards) to reduce confusion, particularly if a semiquaver, demisemiquaver or other short-lasting notes are included. This helps to differentiate the strokes from the beams, as strokes normally go upwards.

Methodological details for score writing and music reproduction with the physical notation have already been presented in Chapter Five: double stave, one (b) for sound (hidden) and one for score (a) (soundless) (Figure 60).

The two musical notation systems use only one stave with microtonal alterations that may be found in most of the score writing programs (Figure 61). This has made possible the creation of a plugin for automatic adjustment of pitches, with sound generation from them.

This procedure was not considered initially as it could create some confusion, particularly with quarter tone microtonal alterations that are more extended, but finally the decision to use existing microtonal symbols was chosen for the benefit of the plugin automatization, and the lack of standardisation.

The Table 36 summarises the result of the calculations for changes in ± cents required to get the 34-tone frequencies (also included in Figure 61 for the musical notation). The frequency of each reference note (basal, or root note) on which the changes are applied is also indicated.

It is important to note that changes made, either by entering the cents in MuseScore with F8 (inspector→ note→ tuning) option or with the plugin, require the MS software tuned to A = 440. Thus, it is mandatory to check that the tuning is initially set to A = 440 Hz before starting with the changes (Create→ Synthesizer→ master tuning).

As commented on in Chapter Five, no standard sharps or flats are included in the musical notation. The reason is that, depending on how the note is entered, errors may be caused. For example, entering GI  after G# can cause error; the # symbol automatically changes all subsequent notes in the bar, including the following GI ─that is taken as a natural G for the program, and consequently automatically modified─ with the result that, in addition to its own adjustment done with the F8 function (inspector), or with the plugin, the prior sharp/flat alteration ─a 100 cents change─ is also (wrongly) affixed.

Notice that it is not possible to add a natural before GI to cancel the previous sharp or flat alteration, as this note has its own microtonal alteration, and MS does not allow entering double accidentals together in the same note (except double sharp or flat included as single symbols)

Contrary to sharp and flat accidentals, the microtonal alterations included in MS do not change the pitch by themselves, thus taking the adjust from the basal note, as indicated, provides extra security of reaching the desired frequency. This particularity may change from one score-writing software to another and must be checked with a different software.

If the physical notation of double stave with manual changes is used, it may prove helpful to have a standard pattern table of notes, as indicated, with the adjustments already done in another MS file, taking the notes from it using the copy and paste function.

As for transposition, MS automatically transposes the adjusted pitch up or down the octaves, and between clefs. So, it is not needed to have a sample for each clef.

Other MS automatic transposition based on 12-TET intervals other than the octave, as e.g., the one used in transposing instruments, may produce wrong results. 34-TET scale score writing must be done with the cell concert pitch (Notes→ Concert Pitch) set on.

If transposing is required, it will be needed to do it manually, or through a plugin; it has been mentioned that the intervals do not match those of 12-TET.

The next step after setting the scale details was to perform a comparative review of the scheme (Tables 37–45) based on three main aspects: golden properties, consonance, and optimal size, each with its corresponding section following in this chapter.

Golden properties of the 34-tone equal tempered scale. The possibility of expanding the research to the 34-tone equal temperament variant was already present in the original publication from Cartwright et al. (2002, p. 57).

Moving the core of this dissertation to the 34-TET scale was a decision made during the research period in the Institute for Microelectronics and Microsystems of the National Research Council of Italy in Bologna, under the supervision of Dr. Diego Luís González-Tisserand, one of the co-workers of Cartwright's publication:

> […] the thirty-four-note equal-tempered scale can be used as a very good approximation to the golden one, with the benefit that in the equal-tempered scale a musical composition can be transposed without difficulty. Moreover, we can change tonality within microtonal intervals, by going to a non-twelve interval available in the thirty-four-note scale (Cartwright et al., 2002, pp 56–57).

Due to its construction the 34T-GR is a perfect golden-based scale, but how well does the 34-TET approach to the GR, or in other words, can it still be considered a golden-ratio-based scheme? This was the essential and first aspect to verify.

Comparison to the 12 standard notes for both scales 34-TET and 34-GR (what the authors called the twelve-note golden scale) was already done by Cartwright and co-workers (2002, p. 57).

However, it must be emphasized, once more, that the closest note to a certain value may be different from that expected by the interval's name.

This is clearly seen with a new example (Table 37). Cartwright and co-workers took D for the second major (34T-GR) (five in their paper as they count from zero, but six in the table), resulting in a difference with the 34-TET scale of about six cents (last column on the left of the table). But if the seventh note (the sixth on the paper) were selected, the difference would drop to 0.70 Hz, which is about 4 cents.

The same holds for A#; by selecting another note (29) with closer frequency the difference of about six cents will come to -1.01 Hz, which is less than four cents. This made it possible to reduce the difference in about two cents in both cases.

It is important to remember this point: using only the interval names did not prove to be the best option for comparisons.

Technical questions aside, what is important from the data in Table 37 is that the differences for the 12 notes (golden scale) between the 34T-GR and the 34-TET are under the JND, as indicated by the authors and confirmed here. The detailed comparative analysis of the 34-T GR and 34-TET is presented in Table 38. The differences in cents are, in most of the cases, under or close to the JND (no more than 8.5 cents) or in other words the 34-TET scale must be considered as GR-based.

Consonance in 34-tone scales. As stated, most of the golden ratio-based scales seem far from consonance (Marco-Franco, 2018). One possible hypothesis is that the consonant intervals are related to frequency ratios that follow simple natural number quotients (1/2, 2/3, etc.,) while the GR is an irrational value.

This difficulty in joining natural and irrational numbers, together with the public favoring natural (consonance), may be the speculative reason for the limited development of the golden scales. But the novelty of the scale constructed by Cartwright et al. (2002) is that, although based on the GR, it also includes all the consonant intervals as a just intonation scale (Table 39). For the equal tempered variant (34-TET) the differences of consonant values are well under the JND (Table 40).

It may also prove interesting to analyse how the 34-TET gamut compares with the extended JI scales. The reference used has been the 22-tone JI scheme (Table 15), and the results (unweighted) are included in Table 41. It has been ascertained that, even dissonant intervals of the 34-TET diapason diverge less than twice the JND from any JI scheme, including those of the 25-tone scale (Table 16) not illustrated in a table.

Prior to entering into detail in the next tables it is necessary to return to the methodology used for evaluation of the intervals, although the formulas have already been mentioned in Chapter Five, as well as the concepts of weighted and unweighted differences.

The index of consonance used by Cartwright et al. (2002) was the weighted mean quadratic dispersion (MQD). The procedure is based on “the square of the difference between the note of the equally-tempered scale that best approximates each harmonic interval, multiplied by the relative weight of each interval and summed over all the intervals” (Cartwright et al., 2002, p. 56). According to one of the co-workers (O. Piro, personal communication November 4, 2017), the use of a relative weight for each harmonic interval did not substantially change the results. This has been confirmed in the analyses carried out in this dissertation with the RMSD methodology and its own weight coefficients (Marco-Franco, 2018).

Based on the MQD (Figure 62) the equal tempered scale with 53 notes has the best σ value. For scales with 50 notes or less, the best alternatives are 46, 34 and 41 tones, with very close σ values. The 12-TET ranks in a medium zone. It is interesting to note that the 11 and 13-tone schemas have the worst σ. As indicated above, these results have been confirmed using the RMSD (Table 42). The comparative analysis included in the table is expanded on the next section.

Notice that the use of MQD (or RMSD) for non-tempered scales (34T-GR included) will require computation for each of the 34 keys and an average of the results. This long ─and out of the artistic arena─ procedure has not been used in the dissertation, and the RMSD values are calculated only for one key, being thus indicative. The JI is not equal tempered, and it would be needed to compute the data for the JI scale in every key and average the results.

Taking the 12-TET scale as reference (Table 43) it is evident that JI, 34T-GR, and 34-TET scales are similarly far from it (ϑ_w =13.3, 13.6, and 14.4 respectively). Values are very close, meaning that the three schemata are near each other.

Another analysis, this time with the whole (12) JI scale as reference, confirmed that the 34T-GR scheme is very near, (Table 44, ϑ_w= 2.2), with perfect matching of consonant intervals, and with the remaining weighted impurities under JND. The 34-TET comes next (ϑ_w= 4.1), with dissonant impurities no more than the JND, but on the limit, and if they would step away more they will fall out of range. The 12-TET scale has a value for ϑ_w= 13.3 with weighted differences as important as 31, 27, and 16 cents. I.e., these two tables show that any further attempt for the 34-TET gamut to get closer to the 12-TET will represent a loss of the golden properties and consonance.

Finally, a simple comparison between the 12- and 34- TET schemes (Table 45) may prove convenient, (e.g., in the case of combinations of instruments with a historic organ that cannot be tuned to the 34-tone temperament, or for some of *The Asian Garden* pieces included in portfolio II) always keeping in mind that it was the 12-TET scale that moved out of consonance for the well-known practical reasons, although much has already been presented on this comparison.

As seen in the Table (45) there are two options for D# and A, but with the same difference in cents. Although the impurity is the same, it may result more convenient to choose one or the other depending on the musical context and tonality.

The comparative analyses show that the 34-tone scale has significant differences to 12-TET for minor 2nd, minor 3rd, major 3rd, minor 6th, major 6th, and major 7th (names referred to 12-TET), with lesser differences for perfect 4^{th}, and 5th, but these differences are also found in the 12-TET scale in relation to JI.

To summarize, 34-TET and JI extended scales are very close in consonance, and the greatest difference for a non-consonant interval is less than twice the JND.

Scale sizes & comparisons. How to determine if the 34-tone division is the optimum size scale? In other words, how to verify if 34-TET is the scheme with the least possible number of notes incorporating the three properties: to be based on the golden-ratio, with best consonance, and equal temperament?

This evaluation required returning to the construction procedure used by Cartwright et al. (2002) based on the Fibonacci convergents: their model starts with five intervals: the pentatonic scale. C (1/2), D# (3/5), F (2/3), G (3/4), A (5/6) and C (1/1) (p. 54). This scheme is clearly insufficient as it does not include the major third (4/5) a basic interval in music, nor the minor sixth (5/8).

Following the procedure to the next convergent, the scale comes to 12 notes (the so called golden 12-tone scale).

As the scale is non-tempered, the only option is to take the 12-TET as the closest approximation; but 12-TET has seven out of twelve notes over JNC, in comparison with the golden scheme as seen in Table 44, column (a/b); in other words, the 12-tone equal tempered scale, as is well known, cannot be considered consonant.

Moving to the following convergent, the scale comes to 34 tones, which fulfils all the requirements of being GR-based, and consonant. With the next convergent the size of the scale would increase considerably. This point of moving over 34 notes will be discussed later; what is important now is whether 34 is the optimal *minimum* size for a golden scheme to get close to the harmonic intervals, and with this method, the 34-tone proved to be that size.

A different (opposite) approximation may be made by selecting the best equally-tempered scales in terms of those harmonic intervals and then analysing wether the impurities in relation to the golden ratio intervals still allow them to be considered golden.

The scales that best fulfil that condition of consonance are 34, 31, 46, and 41 EDO (Table 42). The 31-tone scheme will be commented on later in comparison with the 34-tone, but it must be advanced now that it has three consonant intervals with impurities over the JNC. The computation has been done with weighted as well as unweighted coefficients, and again the *minimum* condition (with consonance as requisite) favours the 34 notes division.

Thus, this alternative procedure corroborates that the 34-TET scale is the best option among those EDO schemata with 50 or less steps, in terms of both GR properties and harmonic intervals, with the minimum possible number of notes.

53-TET is a better option (Cartwright et al., 2002; Holder, 1731; Yarman, 2007) (Figure 62). In the 34-TET gamut the worst possibility for any pitch is to be (1200/34/2) = 17.647 cents off. For 53-TET (1200/53/2) this value comes to 11.32 cents. The difference is less than 6.5 cents in the worst case, or in other words, the improvement is under JND, with all the implications resulting from the increase of notes.

Summing up the data (Tables 37–45), the 34-TET scale is the optimal approach for less than fifty equal divisions of the octave in terms of combining perfect consonance, a golden-ratio based scheme (with the associated nonlinearity), and the best possible approach to 12-TET temperament without losing the other properties mentioned.

Recording and reproduction methodologies. Three possible methodological approaches for musical reproduction of the 34-TET scale have been analyzed: (a) recording directly from a conveniently tuned instrument; (b) use of a digital audio workstation (DAW); or (c) use of a music notation software, with or without the assistance of a plugin. Each of these options have some problems to be commented on.

Recording and playing directly from a tuned instrument. As indicated in Chapter Five, a Yamaha YDP-143 key-board was tuned using the CSE software developed by Aaron Andrew Hunt, adapting manually the frequencies for each key. The central A position was kept as spatial reference.

The first complaint arising from this system was that the application showed constant instability and a tendency to deconfiguration, switching to the restricted demo mode in three different computers, although the rather uncooperative designer insisted that these problems were not related to his software.

A second difficulty stemmed from the fact that the Yamaha software recording system uses only MIDI format output, and it was found that some pitches were modified to fit within the 12-TET scale.

This may not necessarily be due to the Yamaha export system as it may also be originated during the computer process required until the notes appear in the score. In the meantime, recording and further direct cable transmission of the MIDI file to the score writing software is not recommended unless it can be assured that the transmitting procedure does not alter the pitches.

The third constraint is the reduction of playing range. In the 88 keys of the usual keyboards only 2.5 octaves (88/34) of the new scale can be included. The usual wide piano range is sharply reduced (coming close to that of a classical guitar).

A fourth problem is the physical distance between the notes. Due to this, it is virtually impossible to play some of the frequently used diatonic chords.

Microtonal composer Dolores Catherino[170] suggested the use of alternative keyboards in future developments, such as the Hex keyboard Microzone U-648 with 288 keys, or a musical performance controller, such as the Linn Instrument.[171]

External microphoning would be needed for the recording of audio files in high quality formats such as DSF, FLAC, ALAC, WAV, AIFF, CAF, etc., but transferring to score writing programs, again, is usually done through a MIDI format, with the serious additional technical handicap that audio and MIDI formats are not directly interchangeable.

Reproduction using a digital audio workstation (DAW). As indicated, the Cakewalk Sonar Platinum and the Logic Pro 9 – Scala have been tested.

Preliminary evaluation has shown that the Sonar software appears as a rather complex option with a steep learning curve.

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[170] Personal communication (November 14, 2016) with thanks. https://dolorescatherino.bandcamp.com/

[171] More information may be found at https://www.starrlabs.com/product/microzoneu648/ (Last access November 15, 2016) and http://www.rogerlinndesign.com/linnstrument.html (Last access November 15, 2016)

Pro-9, seemed to have a limited offset (around +/– 100 cents) while some changes as high as 176.5 (C#III) are required; it uses MIDI format files, and it is Apple software. Although there are versions for Windows, none was found for Windows 10.

As this dissertation is framed in the artistic arena and has no intention of covering areas of computer science ─undoubtedly very interesting but out of scope─ that require a highly specific background, the line of investigation of computerized musical development with the 34-TET scale remains open to future investigations.

Reproduction from a score writing software. This option, as the standard way for art musicians to compose, has had priority for evaluation and development.

It has allowed fast creation progressing, immediate changes of instrument timbres, and simultaneous generation of the score and the audio file.

The main problem with this option, when using the physical notation, has been that the software reproduces the tremolo of written notes. Thomas Bonte, MS developer, suggested as a solution to present the problem in the open forum.[172]

This was not done since the double stave system was developed.

Later, the musical notation with only one stave using a plugin was adopted.

At first a plugin option was also considered,[173] but due to the structure of the 34-TET the approach was not easy (see next section).

Due to the difficulties, the initial solution adopted was to adjust manually the cents for the 32 notes (as C, and F# do not need changes) with the option 'Inspector' (F8), using the double stave method.

Once the array with the 34 pitches was created, this file was used for composition, with the option of copy and paste, adjusting the time values, once copied, in the new stave.

As already commented on, the MS software allows automatic transposition to the octave and/or to different clefs.

The changes in cents required for each case and the reference notes where to apply this adjustment are shown in Table 36 and in Figure 61.

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[172] Personal communication (June 2, 2017) with thanks.

[173] With thanks to Dr. Gilbert Yammine (www.gilbertyammine.com/ Last access June 2, 2017), and to senior software engineer from Zurich, Mr. Christian Stettler for previous trials.

The outputs of frequencies resulting after these changes have been tested with results consistent with an error of less than 1 Hz.

Plugin Development

Information about theoretical and programming aspects of microtonal computer-aided composition may be found at Bancquart, Agon, & Andreatta (2008).

The following lines refer only to the plugin *Retune* created specifically for the 34-TET scale, to be used with the MuseScore software.[174]

MS is an open source desktop software that allows musical composition in a conventional way by writing notes on the stave.

As a free software it has a wide diffusion and has been used even in elementary schools (Musilli, 2013).

MS has a WYSIWYG editor (Crowder, 2010, pp. 47–58) with full support to reproduce scores, and import or export MusicXML (Steyn, 2013, pp. 187–191), and standard MIDI files.

It also includes the possibility of importing notes, in addition to entering them directly onto the score, printing or exporting the music sheet as PDF, and exporting the sound in different audio formats.[175]

[174] Developed with the altruistic collaboration of Ramón Arnau-Gómez, CEO of ARTECO Consulting (http://www.arteco-consulting.com), with my warmest appreciation.

[175] MS is programmed in C ++ and uses the graphics environment multiplatform and open source Qt (Rischpater, 2013). The version 2.3.2 used in the development of the plugin allows to expand or change the standard functionalities through the execution of third-party plugins.

The third-party plugins are text files in QML (Qt Modelling Language) syntax, a variant of JavaScript created by the programmers of the Qt graphics libraries that allows binding between the properties of the objects of the script and the graphic environment. That is, when modifying a property of an object it will be automatically reflected in the execution. And vice versa, when executing the program, the script will be able to obtain the different values of the properties as the execution moves on.

Therefore, it is possible to create a script using the QML syntax and the JavaScript expressions (values) generated from the matrix (a table containing the required adjustments of notes frequencies expressed in cents), generating a sequence of statements that can interact with the configuration objects of the score and change how MS will interpret the sounds, resulting in a modified pitch based on the expression values coming from the matrix.

It works in a very similar way to a web browser, which via the ECMAScript standard of the W3C consortium (World Wide Web Consortium) offers a JavaScript object called Window to interact with Web pages.

The structure of the plugin is determined by QML and the entry point is always the MS object. This object has a function that MS will always execute at the moment the plugin is activated: onRun.

With onRun it is possible to make calls to other functions that are defined within the MS object, as well as access to global variables.

There is the possibility of configuring dialogs from the user interface allowing the composer to indicate input parameters to the plugins. With QML it is feasible to designate windows, add text fields, labels and other visual components to improve the user experience and at the same time configure the plugin at the time of execution.

One problem in programming changes of the pitch linked to a certain symbol with the 34-TET scale is that the alterations are not uniform throughout the scale. While with the 12-TET scale, every alteration (e.g., a sharp) keeps a constant relation to the basal note (in the case of a sharp the pitch moves invariably 100 cents up), and consequently, it is possible to establish an algorithmic instruction (if # → then 100), linking thus the alteration (#) to a pitch increase of 100 cents, in the 34-TET that is simply not possible.

The 34-TET scale variations associated to accidentals are notable. E.g., a sharp, changes the frequency only in 58.8 cents in the case of E#, but 70.6 cents for C#, 82.4 cents for D#, 88.2 cents for A#, and so on.

The same holds true for intermediate notes: AI is only 17.6 cents, but DI is 47.1 cents, over the reference pitch.

The objective of the plugin *Retune* was to modify the pitch of every note in the stave, when they include a mark [an accidental]. That is the reason why all notes must have the accidental (including naturals).

The pseudo code that describes the algorithm to be implemented is the following:

- For each note in the score, get the accidental.
- If there is no accidental move to next note.
- If there is an accidental get the accidental index (i) and note index (j)
- Go to matrix and get the adjustment needed for (i, j)
- If not, continue.

The complex part of the plugin is to route over the notes of the score.[176]

[176] To perform this route, it is needed to use the object Cursor accessible as a property named curScore of the MS object:

```
onRun: {
    var cursor = curScore.newCursor();
    cursor.rewind(0);
}
```

The rewind method accepts an integer with the following possible values: 0, beginning of the score; 1 beginning of selection; and 2 final of the selection.

A score can be composed of one or several voices or instruments organized into staves. This structure requires that the course of notes must be made for all the voices or instruments and for all the staves. The number of staves can be obtained by consulting curScore.nstaves

Once each note is obtained, the last step is to apply the modification of the tuning to the note according to the corresponding accidental. It is very important that all notes including unaltered ones have the corresponding accidental (a natural in this case). As has been mentioned (Table 36 & Figure 61) except for C and F# all notes need some cents adjustment.

For easy review of the results after the execution of the plugin, the accidentals appear in orange in the case of an incorrect combination of indexes based on the refence matrix. Notes without accidentals turned red indicating error.

The *Retune* plugin was designed to be used with both musical notations, either using only sharps or the option with flats and sharps.

It must be remembered that the musical notation systems do not use standard sharps or flats, consequently there are no key signatures, and in any case, they will be incompatible with the use of this plugin that requires an accidental for each previously unaltered individual note. It has already been said that, if required, the tonality could be indicated to the left of the stave (Figure 59).

The «note» object of QML in MS represents the note that is being revised at that moment according to the previous loop (property tpc: tonal pitch class). The tuning property is found inside, and the objective of the plugin is to modify it. Also, in «note» there is the property that indicates whether it has an accidental, and if so, which one it is.

Therefore, the accidental and the note become the two elements necessary to change the tone; once converted into indices (i, j) the matrix is accessed to find the change in cents, to be applied to the note. Keep in mind that unlike the 12-TET accidentals here the same accidental does not change all the notes in the same number of cents, consequently a value is needed for each accidental and each possible note.

For the conversion of the note into an index, the following relationship has been used:

```
// función tpcToIdx
switch(note.tpc){
 case 14: return 0; //c
 case 16: return 1; //d
 case 18: return 2; //e
 case 13: return 3; //f
 case 15: return 4; //g
 case 17: return 5; //a
 case 19: return 6; //b
 default:
   return null;
}
```

While for the conversion of the accidentals in the other index, this conversion has been used:

```
// function accToIdx
if (note.accidental != null){
 switch(note.accidental.accType){
 case Accidental.FLAT_SLASH2: return 1;
 case Accidental.FLAT_SLASH: return 2;
 case Accidental.FLAT_ARROW_DOWN: return 3;
 case Accidental.NATURAL: return 4;
 case Accidental.SHARP_SLASH: return 5;
 case Accidental.SHARP_ARROW_DOWN: return 6;
 case Accidental.SHARP_ARROW_UP: return 7;
 case Accidental.SHARP_SLASH2: return 8;
 case Accidental.SHARP_SLASH3: return 9;
 case Accidental.SHARP_SLASH4: return 10;
 default: return -1;
 }
```

The code of the plugin in Github is available at: https://github.com/ramonchu/musescore-plugin/blob/master/tuning.qml.

In summary, the 34-TET scale score composition and its sound reproduction are now feasible, either using a manual procedure, or using the designed plugin *Retune.*

Creative Applications. Samples

The second part of the dissertation deals with sonority, and musical creation. This implies value judgments and positioning of a more subjective nature, which may vary from one listener to another.

Twenty-six samples, showing the practical use of the scale, have been included in two portfolios. The portfolio I contains a range of compositions from Renaissance to Popular music, and additionally, approaches to non EuCul music traditions. The portfolio II contains the pieces of the multi-artistic composition *The Asian Garden.*

Extended comments and details for each exhibit are available in the first appendix with the corresponding scores in the third appendix. The libretto of *The Asian Garden* may be found in the second appendix.

The first goal of this second part of the thesis (third goal in Chapter Three) has been the review of the ability of the 34-TET as compared to the 12-TET scale in approaching a variety of historical temperaments, selected on the basis of their popularity in each period.[177] Five samples (audio files 1–5) were created for the analyses, and the methodology of comparative impurities (RMSD) assessment was used (Tables 46–61).[178] In addition to the author's own compositions, well-known pieces have also been analysed.

The results, favourable to the 34-TET temperament in every case may be summarized in:

- Renaissance style: author's *Gloria Resurrectionis*, (audio file 1, Table 46–48), the *Kyrie* of William Byrd Kyrie from the *Mass for Four Voices* (Table 49), and the four voices madrigal No 22, of Orlando di Lasso*, Canzon se l'Esser Meco* (Table 50), in Zwolle temperament.

[177] As there is no exact information for every Common-practice composition and the corresponding scale, the selection has been estimative, based on remaining instruments and other sources. The two most probable tunings have been also selected (see Chapter Five for details).

[178] In some analyses of well-known compositions, the alternative tunings are not presented in new tables but as a footnote comment in order not to excessively increase the tabulations. The advantage for 34-TET has been confirmed in all the cases, either shown in tables or not.

- Baroque style: author's *Sarabande*, (audio file 2, Tables 51–53), and Händel's *Sarabande* of the Suite No 4, in D minor (HWV 437) (Table 54), for one-quarter coma meantone temperament; author's *Baroque Fantasy* (audio file 3, Tables 55–56), and Bach's *Well-tempered Clavier* (Table 57) in Werckmeinster III temperament.
- Classical style: author's minuet da capo (*Last Ball Before Battle*) (Audio file 4, Tables 58–59), and Haydn's *Sonata No. 48* Hob. XVI:35 in C major (Table 60), in Vallotti-Tartini temperament.
- For Romantic style, considering the quasi equal temperament of the time, the evaluation went in the opposite direction, that is, on how well the 12-TET scale could be replaced by the 34-TET scale. Author's *Podolian Rhapsody*, (audio file 5) was analysed tuning to A = 432 Hz (Table 61). As expected from the theoretical aspects already commented on, the 34-TET gamut came near the 12-TET scale, with RMSD (ϑ_w =178.18; ϑ = 94.37). Notice that the average values for the 12-TET scale in author's samples plus the compositions of well-known artists were much higher (ϑ_w = 724.75; ϑ = 394.35). Consequently the 34-TET gets much closer to 12-TET temperament than the 12-TET did to the historical schemes of common practice reviewed.

It must be concluded that the 34-TET scale offers significant advantages over the 12-TET for those musicologists wanting to reproduce the music notes near to the pitches in which they were conceived, with the additional advantage of the equal temperament.

Following the idea (fourth goal) to evaluate the 34-TET scale for its use in contemporary and popular compositions replacing the 12-TET scheme, seven additional pieces have been composed (audio files 6–12). These snippets have not been individually analysed numerically, as the differences between the 12-TET and the 34-TET have been already reviewed. Four of these samples (6–9) are dedicated to Contemporary art music: (a) *Adrán Doors*, (audio file 6, Tables 62–65) is a development of the hexaphonic series inspired by Messiaen but including the additional potential of the 34-TET scale, with more possibilities for whole-tone scales

─a partial set theory analysis was included for this case;[179] (b) *Penelope and the Palace Intrigues* (audio file 7) was composed in recognition of the soundtracks of the films; (c) *8th of March: the Exam* (audio file 8) is an example of wide atonality possibilities of the 34-TET scale, and (d) *The Milling Machine*, (audio file 9) exploring the possibilities of creating Noise music with the 34-TET scale.

Three more samples (10–12) have been devoted to Popular (Eucul) music: (a) The bossa nova *What Happened to my Guitar*? (audio file 10); (b) The tango *Oh, my Homeland, How Much I Miss You*! (audio file 11); and (c) the swing *Hello Lara* (audio file 12).

Here for all these pieces that should be typically composed with the 12-TET scale it is for the listener to decide if the 34-TET replaces that gamut reasonably well.

With the highest respect for other cultures and musical developments, the fifth line of action (fifth goal) has sought an approach to the musical cultures out of EuCul.

The rãst maqãm composition *Horse Gateway* (audio file 13) is an approach to MECul. Data of the schema used may be found on Tables 66–70.

The sample *Awaken Your Soul to Meditation* (ध्यान के लिए अपनी आत्मा जागृत करें।) (audio file 14) has sook getting near SACul with the 34-TET scale (Table 71).

The combined used of 34-TET and 12-TET scales is shown in the second portfolio (*The Asian Garden*, audio files 15–26) This creation also pays tribute to EACul music.

The appendices provide complementary data: expanded information, musicological details for each of the 26 pieces, together with many other details of *The Asian Garden* (Tables 72–74, and Figures 64–69) are included in the first appendix. The libretto may be found in the second appendix. Music sheets are available in the third appendix.

The folder audio includes all the samples in WAV format. The folder video contains the same creations with images (MP4 format), also retrievable from YouTube.

[179] Let us take an example, the 12- tone whole-tone scale, called by Oliver Messiaen (1956) the first mode of limited transposition which is, according to the author, «transportable twice,» that is there may be only three transpositions (C-flat, C and C-sharp), and any other will be contained in one of these three (Taylor, 1991, 246-247), but with the 34-TET scale these serial possibilities increase to 17 (sample 6) (Table 64).

Chapter Seven. Discussion. Problems and Future Developments

It may be convenient to start the discussion with the initial reasoning that originated this investigation.

The study of the 34-tone equal tempered scale did not begin as a process of searching among the EDO scales to find the best one in terms of consonance, although it is true that according to the root-men-square deviation (RMSD) it is an excellent option, particularly among the scales with fewer than fifty tones per octave, with next options based on the RMSD/MQD for 46, 31 and 41 notes (Table 42 & Figure 62).

The first lights for this work emerged from the nonlinear dynamical systems, an area of growing interest that allows a newer and wider way to understand different phenomena in varied scientific areas (Buttle et. al., 2009; Cartwright et al., 2002; Colucci et al., 2016; Fletcher, 1999; Mouawad, & Dubnov, 2017; Young et al., 2005; Yu & Young, 2000, 2013).

Nonlinearity also offers new ways to explain artistic perceptions (Ivanov, 2001, p. 66; Wilson, 2002, p. 235).

Particularly for art related to sound: “In the case of audio signals, complex perceptions are appraised in a listener’s mind, that trigger affective responses that may be relevant for well-being and survival” (Mouawad & Dubnov, 2017, p. 1727). Acoustic analyses have shown that the voice ─perhaps the oldest primitive musical instrument─ is “a highly nonlinear, dynamic system” (Sataloff, 2017, p. 336).

The revision of the theories of the nonlinear systems themselves, and the cogitation of the role of the golden ratio on them, have been intentionally kept out of this research, since such academic work would move this thesis away from the art arena.[180]

[180] The field of dynamic systems is extending even to complex motions and chaos theories. In the case of nonlinear signals, data may be seen as a multifrequency spectrum (Afraimovich et al., 2016, p. 3): «[…] the multifrequency presentation is *valid* for a description of different types of *harmonic beatings* that naturally appear in any nonlinear signal to be analysed […] It is interesting also to note the case where the famous so-called golden ratio between relative frequency distances arises. If we insert into expression […] then we obtain a dispersion law […] where the calculated roots coincide with the values of the famous golden ratio» (Afraimovich et al, 2016, pp. 31–32).

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This work is presented as a vanguard foray into a new terrain, framed by the scholarly curiosity to explore, despite the ignorance of its internal mechanism, new horizons of musical art and compositional tools.

History offers numerous cases in which a breakthrough was made before knowing its intimate rationale, or even as a consequence of a conceptual error. Mendeleev formulated his Periodic Law and the Periodic Table in 1869, even foreseeing new elements without knowing that this ordination derives from the number of protons, discovered decades later. Columbus discovered America thinking that he had arrived in Asia.

Prehistoric caves with painted lines following the GR have already been commented on (Figures 32-34). However, to calculate the GR it is necessary to solve a quadratic equation, either with a complex geometric approximation, or with the formula developed thousands of years later.

This dissertation started with the hypothesis that if nonlinearity is a growing area of interest, and music perception is linked to nonlinearity, this bond could offer new elements to invigorate the art music, which has been reported to be in stasis (Subotnik, 1976, Tenney, 1984), and with the 12-TET scale already “explored that potential [12-tone] to the limit” van der Merwe (2004, p. 115).

In other words, the catalyst was the consideration that art music, being related to nature and to nonlinear systems, could benefit from a nonlinear approach.

The second forethought for this work came from the fact that it seemed surprising ─when analysing the presence of the golden ratio all over the planet and throughout time─ to see how scarce is the presence of GR in music as compared to its permanent presence in other artistic manifestations.

This shortage is additionally surprising considering the known important role of GR in nonlinearity (Afraimovich, et al, 2016; Cartwright, et al., 2002).

Questions then arose: why? Is there a way to face the hitherto important obstacle for incorporating the GR ─and consequently nonlinearity─ into music?

As per Booth and co-workers, every research project must deal with a clear problem or question (Booth, Colom, & Williams, 1995, p. 57), and per Umberto Eco (2001, p. 26), a doctoral thesis must face a significant problem.

The research area seemed optimal as it opened one possibility of solving the problem of musical exhaustion, through cutting-edge research, linked, in addition, to nature, so often related from remote antiquity to music through the golden ratio.

Those thoughts had led nonlinearity, golden ratio, scales, and art aesthetics to condense later in the problem and question: is there any reasonable chance of joining the golden ratio-nonlinearity, aesthetics, and consonance in a musical gamut?

Does such a scheme ─if it exists─ represent an advantage over current existing developments?

It was during the process of searching for material for the study that the original article by Cartwright et al. (2002) came forth.

As a nice surprise ─although not directly included in the title but hidden within the word aesthetics─ the 34-tone scale developed by these authors provided to be golden and consonant, although not of equal temperament, with the well-known problems of this feature (Heman, 1964; Zarlino, 1571).

In the paper (2002), Cartwright and co-workers opened the possibility of moving towards an equal tempered variant, and performed an initial study showing that the differences between the 12 notes for both scales ─the 34-tone golden and the 34-tone equal tempered─ are under the threshold limen (JND), as confirmed in this study for the 12 notes, and also for the entire gamut of 34 notes (Tables 37–38).

This diapason, equally tempered, golden-ratio-based, and consonant, was selected as the starting point for the dissertation to grow from there, with a developmental plan based on the six main research lines (goals) as stated in Chapter Three.

The first goal has been to set the theoretical framework of the scale.

After an overview of the scale antecedents, this goal has included computation of the 34 ratios and frequencies; transposing the original data of Cartwright et al. to the more comfortable central octave (C_4) instead of the C_8 used in the original paper; and new labelling beginning with one for unison (root) ─ as usual in music─ instead of zero.

Ratios, intervals and frequencies, have been calculated using the general formula for equal tempered scales $r_n = \sqrt[34/n]{2}$ presented in chapters the fourth, and the fifth, with (n) ranking from one to thirty-four.

During the process, the question of tuning appeared: a middle C (261.63 Hz) tuned 34-TET scale does not include standard A tuning (440 Hz). If tuning to A = 440 Hz then the C comes to be 264.31 Hz, not 261.63 Hz.

The decision for tuning C to 261.63 Hz, after a comparative evaluation with 12-TET (Table 32), was taken. The main reason for including C = 261.63 Hz was to get closer to the 12-TET scale, something that prove to be of interest for the case of combined use of the two scales; consequently, the option closest to 12-TET was chosen although the difference has been found to be really very small.

Additionally, with C = 261.63 Hz, the 34-TET gamut has two options near 440 Hz ─435.54 Hz (A) and 444.51 (AI)─ that usually fall within the tuning range for A in current instruments. In addition, the option (C = 261.63 Hz) makes C and F# (central point of symmetry in the scale) frequencies identical to the 12-TET scheme.

Due to the construction of the scheme with intervals of about 35.29 cents, it was found that the number of notes included within a given interval of the 12-TET scale may contain different numbers of notes in the 34-tone variant, depending on the note taken as root. As iterated, this point is important because the use of note's names to determine the intervals can be misleading (Table 37). E.g., in the original article by Cartwright et al. (2002), a frequency of 4,651.11 Hz (182.4 cents) is assigned to D_8 (sixth note in Table 29). The note has a difference of 17.65 cents with the value of 200 cents that should have a second major, but if DI (4,741.47 Hz/ 215.7 cents) were taken for comparison, there would be only 15.7 cents instead of 17.65 cents difference over the theoretical desired value of 200 cents.

For the 34-TET scheme the case is the same: taking the sixth note following the physical scheme (Table 36), the note (D_4) with a frequency of 289.7 Hz (176.5 cents) falls 23.5 cents off the theoretical value (200 cents), but the seventh note (DI) with 295.7 Hz (211.8 cents) is only 11.8 cents off, which is an improvement of 11.7 cents.

All these provide additional reasons for developing a new system, so-called musical notation, looking for an approach closer to musicology.

In some cases of comparison, there may be two options: e.g., for the 34-tone note closer to D# (311.1 Hz) in the 12-TET scale there are two values equally distant, both with a difference of about 17.65 cents: 307.97 Hz (D#) and 314.31 Hz (D#I/Eb), one in positive and the other in negative. Same holds for A with two possibilities to choose A (435.54 Hz) or AI (444.51 Hz), again with ±17.65 cents of difference (Table 45).

Notation for the scale was then developed. Initially, with the schema based on Cartwright et al. (2002) labelled as physical nomenclature (Figures 58). This procedure required the explained double stave method (Figure 60) per each instrument, one for sound (hidden in the score) and one for the notes with sound in mute.

The system uses tremolo strokes for the additional notes as explained in the preceding chapters.

During the research process, the second (musical) notation using one stave only was developed with two variants, the first one with sharps only, and the second one with sharps and flats (Figure 61).

Needless to say, the frequencies in the three options are the same, but not the cents to enter for each case, as the root note where that change (in cents) is to be applied is not the same in all the cases (Table 36).

The musical notation variant with two options offers an approach closer to musicology. It uses a single stave per instrument, with frequencies arranged conveniently either manually or with the plugin that has been developed. The use of microtonal alterations and the adjustment of frequencies (Table 36, & Figure 61), either manually with the F8 function (inspector) or with the plugin, lets to compose with only one stave for instrument, as described.

The plugin is valid for the two musical notations indistinctly, allowing easy tuning of the 34-TET scale. The plugin automatically changes the pitches, and the action may be repeated as many times as needed.

A natural in all the notes without accidentals is required, as those notes also need some slight frequency changes in the 34-TET scale, and the plugin uses the naturals to individualize their adjustments. For another score writing software specific designs will be needed and this plugin information may prove useful for them.

The pitches thus obtained have been tested with a frequency-meter at the department of acoustics of the University of the Balearic Islands, with an error of less than 1 Hz.

The possibility of audio-reproduction of the 34-TET compositions has been analysed in three ways: obviously, the first one was to use specifically designed instruments, or instruments that may be tuned for the scale of 34 notes.

Most of the digital keyboards may adjust the tuning of A to frequencies other than 440 Hz, but do not change each key frequency independently.

However, with the assistance of specifically designed software, a Yamaha EP was tuned to the different temperaments including the 34-TET, with manual change of each key frequency to the desired value. With this procedure there was a tendency to a constant instability and deconfiguration; moreover, the export procedure uses a MIDI format that proved to be unreliable, with an occasional tendency to change the pitch to the nearest 12-TET frequency.

More studies will be needed with other keyboards and file exportation formats for further evaluation with MIDI versions accurately exporting microtones.

In any case the main problem for including 34 notes in the standard 88-tone keyboard is that the wide range in octaves of the piano is reduced to (88/34), that is, only about 2.5 octaves; in addition, the distance among the keys makes it difficult, or simply impossible, to play diatonic chords.

Specially designed keyboards, with increased number and/or design of keys, or other solutions such as the possibility of changing the octave by a foot pedal or similar procedures, would be interesting options to consider in the future.

A preliminary evaluation of music reproduction using a sampler proved a laborious process for a conservatory-trained score-writing musician. This line is open to future studies by experts in this area.

The final decision taken for audio reproduction of the scores has been based on the score writing program itself, changing the frequencies, first manually, and later with the assistance of the developed plugin.

Once the musical notation and reproduction systems were established, and the frequencies obtained contrasted with a frequency meter with errors less than 1 Hz, the next step was to compare scales.

Prior to the review it was necessary to establish the comparison methodology. As the development of this analytic procedure has no direct link with the scale of 34-tone nor with any other scale, it became a goal by itself (second goal).

The development of this comparison methodology required a review of mathematic and physic concepts related to music, the summary of which has been incorporated in the state of the art, including the different options for determining the differences between two notes.

These differences or “losses” needed to be systematized although that brought the inconvenience of introducing numerical methodology into the artistic project.

But even following that evidence-based scientific methodology, there is a final decision on how much loss is acceptable, and for this point, the values obtained though analyses are just a mere aid.

Comparison in cents has been chosen, as cents are well-known to musicians; the values may be compared taking the JND as reference. Cents have been calculated with spreadsheets by using a logarithmic formula.

Comparative analyses have been undertaken with the review of the differences between the two 34-tone scales, to confirm whether the 34-TET schema must be considered as a golden-ratio-based scale, and the results presented in Table 38.

The outcomes of the extended comparation confirmed what Cartwright et al. (2002) had advanced for the 12 notes: the differences were no more than the JNC.

Next step was the evaluation of consonance. This was done with three different methods: simple difference in cents, difference weighted or disharmony coefficient, and the root-mean-square deviation (RMSD).

A simple approach to consonant intervals only ─using the difference in cents methodology (Table 40) ─ confirmed that the 34-TET design is almost as consonant as the 34T-GR (Table 39) already published by Cartwright et al. (2002).

In the second method (adjusted coefficient of disharmony) all the notes of the scale were included ─not only the harmonic ones─ weighting the cents differences with the dissonance factors, obtained from a previous work, into a summed overall adjusted coefficient of disharmony (Marco-Franco, 2018, pp. 34–35).

This procedure was first done taking the 12-TET scale as reference (Table 43). As appreciated, the three values for JI, 34T-GR, and 34-TET schemata are very close to each other (104.7/ $\vartheta_{w=}$ 13.3; 111.2/ $\vartheta_{w=}$ 13.6; and 121.0/ ϑ_{w}= 14.4 respectively).

The procedure was then repeated, but this time taking JI scale (II) as reference (Table 44). Here it must be considered that it is the 12-TET the scale that moved away from consonance (coefficient of 104.7/ ϑ_w 13.3), with 34-TET in middle position (40.0/ ϑ_w 4.1) and 34T-GR very close to JI (13.1/ ϑ_w 2.2), with most of the values of dissonance in zero. The consequence is that any attempt to move the 34-TET scale closer to 12-TET will represent moving it away from just intonation harmony, in addition to the loss of equal temperament and the risk of losing the golden proprieties.

The final step on this theoretical approach was to analyse whether the 34-equal division was the best smallest option possible, or if there could be better alternatives.

Moving back to the original method used by Cartwright et al. (2002), using the 5/8 convergent, their scale reaches to 12 notes non-equal tempered; its closest tempered approximation is 12-TET which has seven out of twelve notes over JND compared to just intonation (Tables 44, column a/b), and eight out of twelve if compared to the golden scale (Table 43, column a/d). Therefore, the next convergent must be included, with the process already explained, reaching to the 34 tones scale.

To go to the next convergent ─stepping up to 53 notes in the octave, with all the difficulties in keyboards and frets designs─ will only suppose an improvement of about six cents for the most extreme case (1200/34/2 versus 1200/53/2, that is 17.65 cents versus 11.32 cents). A maximal improvement so small ─of about 6 cents, which is under the JNC─ compared to the considerable increase in the number of notes and the associated complications, seems pointless.[181]

A second approach to analysing the optimal equal-division size was done, with the MQD (σ) methodology, using the values already published by Cartwright et al. (2002, p. 56), with author's addition of a Likert scale with four steps to classify the σ value as a function of the number of notes (Figure 62).

This method has been used in a previous work to compare different golden scales (Marco-Franco, 2018, p. 35), and the summary is depicted in Table 1. This

[181] Although the merit of finding a solution to combine the Pythagorean gamma (harmonic 3), with the Ptolemaic (harmonic 5) in a scale of 53 tones, and the creation in 1977 (constructed by luthier Germano Banchetti) of an instrument for it called dinarra is attributed to the Uruguayan pharmacist Sabat (1994), receiving several distinctions for that (Prince of Asturias, Inventors of Seoul, UNESCO), according to Çoğulu (2018), as early as 1927 the Mexican composer Augusto Novarro had made designs of guitars with 15, 29, 31, 53 and 72 notes per octave. The proposal of a division of the octave in 53 steps may traced back, at least to the seventeenth century with William Holder (1694. *A Treatise on the Natural Grounds and Principles of Harmony*). It must be recognised that the 53-tone design gets closer to the intervals of just intonation or harmonic series than the 34 gamut, but Harverstick in reference to Hanson wrote «Larry thought 53 eq was too many frets for a guitar, so he decided on 34 instead» (Harverstick, personal communication, August 22, 2018).

figure (62) confirms the excellent position of the 34-TET scale in terms of MQD value (Likert rank 3+).

All scales ranked 3+ using a Likert method were additionally taken for a comparative analysis (Table 42), using the MRSD numeric methodology both with and without weighted coefficients. Values have been consistent in indicating that the 34-equal division of the octave is the best option in terms of proximity to the just intervals and to the golden values.

As per Yarman (2007) the 34-TET scale is the optimal size, considering that the frets of the instruments have a limit (p. 51).

To conclude, under 50 notes in an octave, the 34 TET scale is the best option in joining consonance, golden ratio, equal temperament, and lesser size.

Summarising, the first and second goals have established the theorical development of the scale, and the development of evidence-based comparative scale methodology, with data supporting the 34-TET scale as the optimal solution, creating the basis for selecting the 34-TET scheme for the following steps.

The second part of the dissertation deals with comparative sonorousness and musical creation using this gamut. It implies value judgments and positioning of a more subjective nature that may vary from one listener to another.

The third line of research (third goal) has evaluated the potential advantages of using the 34-TET scale for historical European art music styles and genres prior to twentieth century, analysing how well the scale approaches to the temperaments and tunings of the common practice periods as compared with 12-TET.

Compositional samples (audio files 1–5, portfolio one) have been created with a brief introduction of their corresponding environments included as first appendix.

Several historical temperaments, reported as popular during the common-practice periods, have been selected. In every case, the evaluation included tuning to A = 422 Hz, and for C = 256 Hz.[182]

Frequency values have been computed for each tonality used, and the cents difference in the notes of 10% of the bars, taken at random, analysed. Each historical gamut has been compared to 34-TET and to 12-TET scales. The analyses included not only author’s samples, but also well-known compositions for each style/genre.

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[182] Additionally, in some cases also for A = 440 Hz

The results (Tables 46–60) based on RMSD (weighted and unweighted) have been consistent with better results for the 34-TET as compared with the 12-TET scale, in all the author's samples and also in all the samples from other composers, for all the historical temperaments chosen, and for each of the tunings (A = 422 Hz, or C = 256 Hz).[183]

Besides, the 34-TET scale has frequencies close to other (former) pitches of schemes not included in this study, e.g., seventeenth century Venice A_4-tuning to 465 Hz, could be approached using AIII (463.0 Hz); eighteenth century French tuning A 392 Hz, using G (392 Hz), and the so called "lower baroque tuning" (A = 415 Hz), using G# (418 Hz).

The possibilities of the 34-TET scale in contemporary music were evaluated in four samples (audio files 6–9), including an example based on a whole-note scheme (Audio file 6, and Tables 62–65).

As seen in Table 64, the possibilities with the scale of 34 notes are greater than with the scale of 12 notes for serial music. A limited analysis for this snippet based on the set theory is presented in the first appendix.

The remaining compositions have not been evaluated numerically as the differences between the 34-TET and the 12-TET have been widely exposed earlier in this text.

Audio files 10–12 explore more possibilities of the scale, this time in popular music.

In the opposite analysis, i.e., about how well the 34-TET approaches 12-TET, some losses are inevitable. The issue here is not how to avoid those losses, an impossible task, but whether the schema may be assumable overall, a positioning that may change, therefore, from one musician to another. Choosing a convenient tonality for scale combination or for the best approach to 12-TET may be of help.

Simply to exemplify the point, being the scale of 34-TET consonant, and therefore with frequencies close to those of the JI gamut, it cannot be, at the same time, close to the 12-TET scale that departed from it in seven out of the 12 notes, as said.

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[183] It has been commented on that not all the results are fully presented in tables, but all the values are consistent with the advantage for the 34-TET scheme. A summary of some data resulting for an alternative tuning are indicated as a footnote in some tables.

The 34-tone and the 12-tone scales have significant differences for minor 2nd, minor 3rd, major 3rd, minor 6th, major 6th and major 7th (names referred to 12-TET) with lesser differences for perfect 4th and 5th.

The weighted 12-TET scale step out of consonant intervals proves to be as significant as 27 cents for a major third or 31 cents for a major sixth (Table 44); it is therefore impossible to construct a scale close to 12-TET and simultaneously close to perfect harmonic intervals.

Throughout history, the decision on whether to maintain a perfect consonance versus the capacity for modulating, and single tuning for the different tonalities (the so-called called dilemma of consonance versus equal temperament) has always been resolved in favour of the second option.

The fifth goal has analysed the closeness of the 34-TET scale to two non-EuCul diapasons, represented in the audio samples by maqãm (sample 13) and Hindi music (sample 14).

This goal is by no means a pretentiously or arrogantly demonstration of the superiority of EuCul, but a humbly recognition and manifestation of a genuine interest for the long historical tradition and the immense musical values found in non-European cultures, such as Indian or Chinese, which have been analysed so many times in the past under the prism of colonial rationalism. It may be convenient, in this regard, to bring again the quotation:

> Riemann's worry was that the phonograph […] would allow […] intervals that were unthinkable in the rational system of Western music and had been barred from coming into circulation by the sheer impossibility of writing them down as musical notation (Rehdling, 2005, p.132).

The aim for this fifth goal has been just to offer an additional EuCul methodology, simply as one more option, to the many possible approaches already present in those rich cultures, but with no idea of depreciating or belittling the enormous musical values vested on them.

The key point ─for the goals in general and in particular for this fifth line of research evaluating the 34-TET scale closeness to other culture musical schemes─ is to set how much disharmony (impurity) is acceptable in paying for the advantage of using a single gamut all over the planet, under the limitations, as stated, that a perfect unique ethnomusicological approach to fit anytime and anywhere will hardly be possible. This price is again rather subjective. Diverse opinions can be argued validly.

Per Dr. V.G. Oke[184] the (22 • 12) shrutis, or each of the resulting 264 microtones, are indispensable for the right performance of proper Indian art music. Even the Indian classic Bahtkhande schema is considered by this author inappropriate and out of tune.

However, an overview of Oke's table (http://www.22shruti.com/research_topic_44.asp) allows one to see that shruti 19 in D (saphet 2) has a frequency of 522.07 Hz, and shruti 10 in G (saphet 5) is 522.66 Hz. This difference of less than one Hz proved to be about two cents.

The small difference of about two cents may be found between the frequencies of shruti 1 in C# (kali 1) of 554.37 Hz and shruti 14 in F# (kali 3) with 554.99 Hz.

The above-mentioned comment does not pretend to devaluate the opinion of a Hindi ethnomusicology expert such as Dr. Oke, but only to emphasize that depending on where the requirements are set it will be possible to use the 34-TET scale more or less extensively.

There is has an antecedent for a similar question. It was raised when the 12-TET was standardised. By then, it was assumed that an unweighted difference of about 15.61 cents for a minor third or 15.67 for a major sixth in relation to the harmonic intervals was assumable (Table 43, column a/b). Once weighted, this value comes to about 31 cents, as indicated.

If a similar reasoning would be followed here, the maximum impurity that a note of the 34-TET scale can have with any pitch around the globe would be slightly higher than the maximal 12-TET impurity that musicologists have been accepting for more than a century; specifically, 17.647 cents, that is, just under 2 cents more.

This value results from the fact that the intervals in the 34-TET scale are about 35.294 cents, and the maximal difference will be for a pitch falling just in between two notes, with the impurity then of 35.294/2 = 17.647 cents. The value is about twice the JND, and less than one fifth of a semitone.

Thus, the question: is that maximal difference still acceptable? If that impurity in relation to the exact pitch was accepted ─as indicated less than 2 cents more─ then, the 34-TET could become a unique scheme valid for all musical cultures throughout the planet.

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[184] Personal communication, July 1, 2018 with appreciation.

But at this point another query arises: is tuning to perfect pitch the only requirement for reproducing the vast richness in nuances and music characteristics of the diverse cultures of the planet?

It has been mentioned that the Indian shrutis have a religious connection that goes further than a simple difference of frequencies.

The last line of investigation (sixth goal) has reviewed the possibility of combining the two scales 12-TET and 34-TET, something particularly interesting in the case of some instruments (such as pipe organs e.g.,) that have a specific temperament design difficult to change.

For this goal, a multi-artistic performance has been created (*The Asian Garden*) including: music ─instrumental and vocal─ dance, theatre, painting and engraving, digital technology ─with audio-visual media, multiple screen display, and interaction with the public─ odours, and physical perceptions.

The Asian Garden plot synopsis, artistic cast, requirements both human and technical, prop, budget, and details for each composition may be found in the first appendix.

The libretto (with dialogues in Spanish) may be found in the second appendix and the scores in the third appendix (portfolio II).

The portfolio (II) contains the 12 pieces of this work (15–26), combining both (12 & 34) TET schemas, in many cases even in the same piece (15, 18, 21, 23–25). For operatic arias the 12-TET has been chosen for the singer's sake.

Additionally, this composition renders homage to East Asian musical cultures (EACul), including to Chinese and Japanese rich instrumentation.

In summary, the intention with these samples is to show, in practice, the possibilities of joining in a single scale (34-TET) the nonlinear theories of music perception (here represented by the golden ratio), the harmonic series, and a design with a reasonable number of notes, by means of compositions in different genres and styles.

The research opens new possibilities for approaching historical temperaments, other cultures, including microtonal music ─an area which according to several authors may have a good future (Fox, 2003a; Gilmore, 1998; Loy, 2006; McHard, 2006; Partch, 1979)─ and composing music for films with new previously unheard material (Powrie, & Stilwell, 2006).

As for limitations of the study, the first one has already been mentioned: to considerer whether the size of 34 tones is reasonable and the consequences derived from this decision. Data confirming the best minimal size option to reach harmonic intervals has also been commented on (Table 42). At a reasonable number of divisions, no other scheme is better in consonance than the 34 tones.

But even accepting the 34-note option as the best choice, the distances between the keys make it impossible to play conventional diatonic 3/5/7 chords. The richness in octaves of the keyboard piano will disappear, getting a reduction to 88/34), that is two octaves and twenty additional notes. This could be solved with microtonal custom-made keyboard designs.

The second limitation has also been commented on extensively and depends on each individual criterion: whether a maximal impurity of 17.647 cents is acceptable or not. Per Shethares (2005), consonance was sacrificed in favour of multiple key modulations (and transposition) when the 12-TET was standardized (p. 177). The decision was hard because humans prefer consonance since early childhood, according to the studies of different authors confirmed experimentally (Plantinga & Trehub, 2014, pp. 40–49; Valentine, 1962, pp.196–227; Zenatti, 1993, pp. 177–196).

A third limitation comes from the fact that standardisation of the 12-TET and the tuning to A = 440 Hz made standard a C–A interval (r = 1.6817928) far from consonance (r = 5/3 = 1.6666667). Any (reasonably sized) consonant scale if tuned to A = 440 Hz gets C (261.626 Hz) out of tune, and if tuned to C = 261.626 Hz, then the pitch 440 Hz is not in the scale. With A = 435.5 Hz the 34-TET scale coincides with C = 261.63 Hz. The tuning to C = 261.63 Hz gets closer pitches to 12-TET particularly for unison, perfect fourth and perfect fifth. The RMSD (ϑ) is slightly favourable to C tuning and, additionally, the fifth is twice as good (Table 32).

A fourth caution must be considered: exporting a file (usually in a MIDI format) when using an electronic instrument, it is necessary to confirm that the pitches remain unaltered. Some developments with *Scala*, a software tool, have been made to facilitate the transfer of files with microtones (Op de Coul, 2017).

The fifth consideration is that the combined use of 12-TET and 34-TET need to be planned to minimize the differences that may reach, as mentioned, in some cases (D# and A) up to 17.647 cents.

Scale combination is important for instruments, such as pipe organs, or even some kettledrums, that prove impossible or very difficult to change their tunings. Some kettle-drum designs use a ratchet clutch system with fixed positions and although a slight additional change in tuning may still be possible, it may not be enough. Timpani tuning has proved to be one of the most complex in whole orchestra (Friese & Lepak, 1985, pp. 35–69).

In the combined use of scales it is important to emphasize, once more, that interval determination based on the names of the notes may provide confusing or incorrect information and its use is not recommended; better to use the frequencies, the ratio or the number of cents instead; while from C to D (a second major on the 12-TET scale) there are five notes on the 34-TET scale, from G to A (also a second major on the 12-TET scale), there are four. Similar differences can be found in other cases.

Alex Zorach [n.d.] has posted some criticism about the 34-TET scale showing his preference for 31-tone division of the octave. His first questioning is about the whole-tone intervals (para. 3). He seems to consider two possibilities for 34-TET gamut whole-tones, a major and a minor variant.

The interest in decimalisation grew importantly during the nineteenth century, but what is important in music, as known since Pythagoras, are the interval ratios, which as well known to musicians, do not follow a decimal sequence.

As commented on above, it was only when the 12-TET was standardized that consonance was lost in favour of modulation and of playing different tonalities with one tune; in the case of whole-tone intervals [considering a whole-tone as 200 cents] the closest values in 34-TET temperament are: 211.8–388.2–600.0–811.8–988.2–1200 cents. Two of them have no deviation in the decimal value and the remaining four have an impurity of 11.8 cents. It makes no sense to use what the author seems to call minor whole-tone: 176.5–352.9–600–776.5–1023.5 cents. Why call these pitches, some with a difference of 47.2 cents (almost a quarter tone), whole-tones?

As for the second criticism (para. 4): "While 34-ET matches the perfect fourth, major third, and minor thirds very closely, it provides a poor match to most intervals involving the 7th and 11th overtones."

At this point it may be important to start with the concept of overtone. According to the Grove dictionary of music (Campbell, 2001) "the overtone is one of the frequency components of a sound other than that of lowest frequency [fundamental]. Usually overtones are numbered consecutively in ascending order of frequency; *they need not be harmonic*" (para 1). Each of the simple waves is a partial [including the fundamental], and Fourier found that the harmonic partials are those whose frequencies are numerical integer multiples of the fundamental (Katz, 2017, p. 543) So, it is convenient not to mix the concepts. An overtone does not necessarily need to be harmonic, and a seventh interval is not a consonant one.

As additional questioning here is "match," but to what? To the 12-TET scale that it is known to have moved away from harmony? The 12-TET is 11.76 cents off harmony for a seventh major interval, while the 34-TET is 5.88 cents (Table 44).

For the eleventh interval (compound fourth interval), starting from C_4 = 261.63 Hz and the corresponding ratio (eleventh or fourth compound r = 2.660624116), results in a pitch of 696.09 Hz (F_5). The JI scale (as not tempered) does not include this value, being the closer 697.67 Hz. The nearest values for 31-t and 34-t are, respectively, 699.76 and 696,00 Hz, which is 5.2 cents out of pitch for 31-t while the 34-t is only 4.1 cents off. The nearest value for 12-TET is 698.5 Hz, about six cents out of JI scale. So again, the 34-TET pitch is the one that gets closer to the eleventh or fourth compound interval among 12, 31 and 34 equally divided octave temperaments.

Zorach's third questioning (para. 5) is: "34-ET has more intervals (3, or 6 counting inversions) that are not harmonically useful in the context of most tonal music, whereas 31-ET only has one (or two, counting its inversion)"

This is just an opinion not supported by any concrete data. Those intervals may be useful or not, depending on the style of music and/or other cultural settings. Based on the mean quadratic dispersion (σ) as calculated by Cartwright et al (2002, p. 56), and confirmed using RMSD both weighted and unweighted (Table 42), the values for 31-TET (ϑ_w = 2.5; ϑ = 1.4) are poorer than those of 34-TET (ϑ_w = 1.6; ϑ = 0.9).

Fourth criticism (para. 6): “For the large number of intervals it contains, 34-ET does not distinguish between the greater (10:7) and lesser (7:5) septimal tritones, matching both of these with an equal division of the octave in half.”

The tritone of 12-TET, 34T-GR and 34-TET are just coincidental (370 Hz, 600 cents) with zero error, dividing the octave in two halves; they are slightly under 10 cents off the just intonation scale (Table 44). Nearest ratios for 31-T (1.3998 & 1.4301) result in 19.35 cents [almost double] off. Even following the reasoning, 10:7 comes to 1.42857 and 7:5 to 1.4. Those ratios are not present in the 31-TET scale either, although it is true that close values are found: 1.43011289 and 1.398490998, both about 2 cents off. But it is well known that the tritones are not considered consonant intervals. It is difficult to imagine ─particularly in an author who advocates for tonal music─ the relevance of giving preference to those two septimal tritones non-consonant intervals that are not musicologically consolidated within EuCul, over the central tritone note.

Fifth Zorach’s criticism (para. 7):

> Besides the perfect fourth, fifth, and major and minor thirds being more in-tune, the other intervals of 34-ET are so out-of-tune to their natural harmonics that they would be difficult (probably impossible for most people, even trained musicians) to accurately sing or play on any instruments except those with a fixed pitch. There are only 14 intervals (7 and their inversions) of the 34-step scale in 34-ET that are in-tune and usable in the sense that they are in-tune and can be put in harmonic context.

As presented in Table 40 the 34-TET scale consonance is excellent with the greatest differences to the intervals of the JI scale of less than 4 cents, and thus well under JND. Even the most dissonant interval of 34-TET, the minor seventh is less than 12 cents off the JI scale (Table 41). The perfect fourth, perfect fifth and major sixth intervals are better in 34-TET than in 31-TET, with better RMSD values (Table 42) advantage already anticipated by the mean quadratic dispersion (σ) (Figure 62).

Zorach [n.d.] also comments the interest for the 41-TET scale:

> 41-ET seems to combine the strengths of both 31-ET and 34-ET. Unlike 34-ET, 41-ET provides a good match for the intervals involving the 7th and 11th harmonics. It also addresses one shortcoming of 31-ET: it distinguishes between the undecimal major third (14:11) and the septimal major third (9:7) (para. 12).

In fact, according to the values of MQD (σ) and RMSD (ϑ) the 41-tone schema provides closeness to 34-TET but not better (Table 42), and as the same Zorach indicates, “I'm obviously an advocate of 31-ET, due to its simplicity” (para. 11).

Zorach [n.d.] also questions the intervals of the 34-tone scale and the need to properly line-up the notes for the “few useful intervals.” But as demonstrated in the samples it is possible to play a variety of harmonic chords with the 34-TET diapason.

However, the diffusion of the 34-TET scale will not be free from difficulties. New software will be required for the different score writing programs, although the plugin developed in this research may be of great help in this regard, and instruments should need to be designed to play the 34 notes of the scale.

Chords and harmonies will probably need to be revisited. Unless special keyboards be specifically designed, the keys provide too great a distance to play diatonic chords in an 88-key standard piano retuned for 34-TET. New chords that make use of the advantage of the extended scale may be developed.

Also, this work opens new opportunities for computerized music research, and a great opportunity to innovate by filtering and enhancing the overtones. By modifying the sound files with computerized algorithms or other means, there is the possibility of enhancing the sound by generating new musical timbres, which will allow the creation of richer instrument colors, as well as improve the quality of the sound output.

Another interesting field for future research may be to analyze the audience reception, taking different compositions with the 34-TET scale from several authors, in a comparative survey with other schemes.

The shadow of the *dissonant* 12-TET ─particularly in musicians and musical experts who have developed their long learning process based on the 12-TET scale and have those pitches internalized─ will not facilitate progress.

To paraphrase Sir Winston Churchill’s vision of what is to come: “Now this is not the end. It is not even the beginning of the end, but it is, perhaps, the end of the beginning” (The Lord Mayor’s Luncheon, Mansion house, November 10, 1942).

The 34-TET scale, may offer new opportunities to invigorate the music, particularly for those who, instead of a pessimistic vision for the forthcoming (Breer, 2008, Freeman, 2014, Vanhoenacker, 2014; Wheatcroft, 1998), believe in a more promising future for art music (Beltin, 1984, 1994; Getz, 2015; Robin, 2014), relying on the never-ending process of advances and improvements taking place in the academic world.

Chapter Eight. Conclusions

The problem and the questions faced in this dissertation may be summarised as: is there any reasonable chance of joining the aesthetics linked to golden ratio-nonlinearity with consonance in an equally-divided octave scale?

Does such a scheme ─if it exists─ represent an advantage over current existing developments? Is a likelihood that it could contribute to art music invigoration through new musical expression and a wide future expansion?

The answer provided in this dissertation is the 34-tone equal tempered scale. It includes a series of unique features: it proves to be a golden model, and thus linked to the current nonlinear approach to art. It is consonant, very close to the harmonic series, something sought by musical theorists for centuries; and, additionally, the octave is of equal temperament, facilitating tuning, modulation and music transposition.

Making use of the 34-TET scale it has been possible to create examples in different tunings of genres and styles of the periods of the common practice era (Renaissance, Baroque and Classicism) with evidence-based results consistently superior to those of the 12-TET gamut in its ability to approach the pitches of those historical temperaments and former tunings, both in the author's samples and in well-known selected masterpieces of art music.

A diapason with 34 notes also allows wider compositional creations in contemporary styles and genres originally developed with the 12-TET temperament, including atonal, serial, noise, film soundtracks and popular music, and it has been proved possible to combine both temperaments in compositions.

The 34-TET scale approximates other music cultural contexts outside of Europe, with 17.65 cents as the greatest possible difference with any pitch, a value of maximal impurity virtually identical (under two cents more) to that accepted when the 12-TET scale was standardized. Authors such as professor Dr. Ozan Yarman from Istanbul University considered that the 34-TET offers additional advantages for Turkish makam music, even over the accepted 24-note quarter tone scale, including better compatibility with the Safi al-Din al-Urmawi's scheme.

The Indian sample *Awaken Your Soul for Meditation* and *The Asian Garden* imitating Indian, Chinese and Japanese music ─with the impurity limitation indicated─ provide additional samples of creations with Asian brush strokes.

It must be recognized that this applicability to Non-European tradition may have some limitations. The 34-TET temperament is not the panacea that solves all the problems. Some musical cultures do not use scores, and there may be features that could eventually prove impossible to be included in the stave; but the difficulties could be lessened by imaginative use of additional text indications, in the same way as art music scores include indications such as rubato, a tempo, lusingando, pizzicato, and many others; the advantages of using the same scale all over the planet become huge. In any case, the approach must be based on respect for local values and the absence of any colonialist attitude.

The microtonal construction of the scale opens possibilities of new chords, colours and expression possibilities beyond the 12-ptich classes and diatonic intervals. The size of the scale with 34 notes seems still manageable. Even for standard keyboards, there are enough keys for over two octaves, and with digital technology it is nowadays possible to have more than one row of keys or change the octave if needed. Multiple keyboard technology may offer a solution to the current difficulties in playing diatonic chords. However, instrument designs may have problems and limitations, e.g., the distance between frets in a guitar, which will require new construction schemes.

The standardization of a global temperament can offer tremendous opportunities for commercial expansion in regions such as Asia, which is currently showing a growing interest in novel art music developments.

Crossover events with 34 different possible tones ─notably if also left behind the old-fashioned dresses, the dark and silent concert halls liturgy, the digital interaction absence, and the European colonialist superiority─ could attract young population (now underrepresented) from different continents to art music events.

Another field open to future developments is the enhancement of sound signals, and the generating of new musical timbres, with computer assistance, taking advantage of the expanded 34-tone diapason possibilities.

Further studies, including ones on audience reception of the music, will be needed in order to confirm the advantages foreseen in this pioneering development of a hitherto unexplored musical scheme.

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De acordo com as normas de apresentaçao de teses em vigor na Universidade de Aveiro, os anexos que neste caso também incluem tabelas, figuras e partituras (pp. 207–534) não são impressos

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The thesis style and formatting follow the rules for the submission of doctoral theses of the University of Aveiro, including the model-130-cover, and the two-side tabulations of long (over 40 words) text citations. The publication *Manual of the American Psychological Association* (APA), Sixth Edition (2010) has been used as complementary reference.

Running head has been omitted in the cover pages due to University of Aveiro's own format cover (section 2.01, p. 229). Page format follows the recommendations of Aveiro University: 3–3–2.5 (after headings)–2.5 cm. Extra 0.5 cm has been included on the left for binding.

Headings follow the five-level APA format with the introduction starting without heading (Manual sections 3.02–3.03, pp. 62–63). Headings have not been labelled with numbers or letters in accordance with p. 63 of APA Manual.

Quotations follow the instructions of section 4.08, p. 92, and sections 6.03–6.10, pp. 170–174. Digital Object Identifier [DOI] ® references have been included with the new (effective March 2017) Crossref format when indicated in the papers, or found, using the Crossref-finding software (https://www.crossref.org/guestquery/) (Manual pp. 188–192). Foreign language quotations are included as paraphrases, but in the reference list the source appears in the original language with translation in square brackets.

According to the sixth edition of the Manual, tables and figures have been included after the references on separate pages (p. 230).

Five spaces ident has been used at the beginning of each paragraph except for the abstract (p. 229), and complementary information; line spacing follows instructions of page 180. Capitalization of only the first letter in the references and letters format follow sections 4.15, 4.21 (italics) and 4.07 (quotation marks). Capitalization after the colon follows section 4.05 (p. 90), and sections 4.14–4.15 (p. 101) of the Manual. Italics have not been used for foreign languages and abbreviations, and in the case of key terms they were used only the first time (p.104–105).

Web retrieval date is not included, conforming to Manual (p. 192), unless the document is regularly updated, or the date is missing.

Punctuation marks after DOIs or URLs (full stops) are omitted in reference list entries, to keep the live links. Note that URL references may be broken (according to Manual, p. 192) in the case of paragraph justification. Please delete any empty space in web addresses if found, before linking. If publications have been issued in more than one place, only the first location is included. In the case of small towns, or in cases such as Vienna or Cambridge in the USA, with homonymous cities in Europe, additional information, such as the state initials, is included in square brackets.

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