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ORIGINAL ARTICLE

A dynamic separable network model with actor heterogeneity: An application to global weapons transfers*

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Abstract
In this paper, we analyse the network of international major conventional weapons (MCW) transfers from 1950 to 2016, based on data from the Stockholm International Peace Research Institute (SIPRI). The dataset consists of yearly bilateral arms transfers between pairs of countries, which allows us to conceive of the individual relationships as part of an overall trade network. For the analysis, we extend the separable temporal exponential random graph model (STERGM) to account for time-varying effects on both the network level (trade network) and the actor level (country effects). Our investigation enables the identification of potentially differing driving forces that influence the formation of new trade relationships versus the persistence of existing ones. In accordance with political economy models, we expect security- and network-related covariates to be most important for the formation of transfers, whereas repeated transfers should prevalently be determined by the importers' market size and military spending. Our proposed modelling approach corroborates the hypothesis and quantifies the corresponding effects.

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Additionally, we subject the time-varying heterogeneity effects to a functional principal component analysis. This analysis serves as an exploratory tool and allows us to identify countries with exceptional increases or decreases in their tendency to import and export weapons.

KEYWORDS

arms transfers, functional principal component analysis, generalized additive model, security and defence network, varying coefficient model

1 | INTRODUCTION

In this paper, we consider the highly relevant topic of international weapons transfers. The exchange of lethal products is unique from the trade of commercial products and services since it involves questions of political security and defence. We investigate the structure and dynamics of the international arms trade network from a network perspective in order to uncover how the network's evolution depends, on one hand, on the local structure of the network itself and, on the other hand, on economic and political variables. More specifically, we propose to distinguish between the processes of forming new relations in this network and maintaining existing ones. Major weapons transfers are highly risky and require strategic and careful decisions from the perspective of the supplier. If buyers prove to be unreliable, such contracts are often suspended or cancelled all together. Therefore, we follow a modelling strategy that separates the initiation and persistence of trade relationships using a flexible separable network model that shows how the influence of the covariates changes over time and how different countries are situated in the arms trade system.

To situate our work within the field, we first motivate the usage of the network approach to model international trade more generally. We then discuss international arms transfers before we introduce the statistical network model proposed to analyse data on international arms trading provided by the Stockholm International Peace Research Institute (SIPRI).

1.1 | Trade networks

Statistical network analysis provides a good framework for conceptualizing international trade systems. Schweitzer et al. (2009) highlight the enormous interdependencies of economic transactions and propose a network approach for capturing the systemic complexity. Gravity models, which are a standard approach in econometrics for modelling trade data (Head & Mayer, 2014), are usually focussed on relations between country pairs (dyadic relations). Hence, the models exclude important network effects that go beyond dyadic relations. Squartini et al. (2011a, b) showed that gravity models of international trade are, therefore, necessarily incomplete. In particular, they demonstrated that analysing the determinants of link creation is highly important since the binary network carries information that goes beyond the classical gravity model representation. Barigozzi et al. (2010) demonstrated that trade networks are commodity specific, that is, their network structures are quite different across commodities—leading us to conclude that there is also a need to consider arms transfers separately.

This is theoretically challenging since arms transfers constitute a very special trade relationship. The transferred products and services can potentially lead to deadly quarrels between or within states, or they may contribute to stabilization and deterrence. Arms trade is not always a purely economic exchange but may also serve to support aligned countries or groups. In sum, the exchange of weapons is a politically sensitive and security related, but also economically beneficial, relationship. For this reason, we make use of flexible statistical models for network data that allow us to investigate the special incentives in the international arms trade network.

1.2 | Global weapons transfers

At present, there are only a few empirical binary network analyses of the international arms trade. Akerman and Seim (2014) pioneered the analysis of structural features of the binary arms trade network. Their descriptive network analysis is supplemented by an empirical investigation that uses a binarized gravity model without considering network dependencies. In this article, we build on the recently published paper by Thurner et al. (2019). However, our approach extends that work in many aspects. Most importantly, we treat temporal dependencies in a fundamentally different way. In Thurner et al. (2019), the authors found that previous arms trading between two parties was highly predictive of whether subsequent arms transfers would occur, due to inertia. This finding implies that whether trade happened in the preceding year(s) has a considerable impact on the probability of trade in the future. To disentangle the driving network formation forces due to pure inertia, we propose to incorporate this distinction directly in the model. More precisely, the proposed model allows us to investigate whether the mechanisms that result in transfers being formed without an immediate predecessor differ from those that lead to consecutive transfers. This is also of practical importance because governments carefully reflect before deciding whether to authorize arms transfers based on economic and security considerations. Furthermore, they continuously reconsider whether to maintain such trade relations based on potential risks the importer might present to strategic interests or importer violations of once-shared normative standards. (see Garcia-Alonso & Levine, 2007 for the general model and the papers by Blanton, 2005 and Erickson, 2015 for normative considerations).

We expect several necessary conditions to hold for the formation of transfers: the receiving country must be considered at least marginally trustworthy and politically and economically reliable. Hence, passing a threshold of trustworthiness is required for formation, that is, building new trade relationships. The special role of trustworthiness in arms transfers stems from the fact that security concerns play an important role when governments decide whether to license the delivery. We expect network dependencies, regime dissimilarity and formal alliances to play a prominent role in the formation stage to raise a relationship above the minimum threshold level of reservation. Follow-up trades should then be driven by economic considerations like the size of an importer's economy and military expenditures (see Schulze et al., 2017).

While differentiation between trade formation and repetition legitimates the proposed statistical model, we further extend the model to account for time-varying coefficients that are important and, in our view, inevitable because the observational time covers more than 65 years. Hence, the introduction of smooth random effects is needed to build a realistic model. Given the dynamic evolution of the network, the historical developments and the presence of at least one system-wide structural break with the collapse of the Soviet Union, we expect to observe changes in the network's generative mechanisms over time when comparing the included variables from the pre- and post-Cold War period. (See also Akerman & Seim, 2014 and Thurner et al., 2019).

Finally, we argue that not all network activities and trades can be explained by observables and, thus, unobserved heterogeneity remains. We expect primarily actor-specific heterogeneity, which is accentuated by systematic historical accounts (Harkavy, 1975; Krause, 1995). This highlights the self-reinforcing tendencies of highly developed countries' technological advantages, which results in strong heterogeneity in the countries abilities to export (and import). Therefore, the inclusion of actor-specific random effects seems necessary and we expect strong heterogeneity among the countries concerning imports and exports.

1.3 | Statistical network models

Statistical models that are suitable for temporal networks have been developed just in the last 10 to 20 years, and different techniques have been proposed. Robins and Pattison (2001) were the first to extend the static exponential random graph model (ERGM, Holland & Leinhardt, 1981; Lusher et al., 2012) to discrete-time Markov chain models, see also Snijders et al. (2010a). Hanneke et al. (2010) or Leifeld et al. (2018) also consider network dynamics on a discrete time scale. They propose the temporal exponential random graph model (TERGM), which makes use of a Markov structure conditioning on previous network statistics as covariates in the model. A related approach is presented by Almquist and Butts (2014), which discusses assumptions that enable the circumvention of the often computationally intractable fitting process for dynamic network models through the application of logistic regression models. Koskinen et al. (2015) expand the model using Bayesian methods, which allow the parameters in the dynamic network model to change with time. A general perspective on dynamic networks is provided by Holme (2015). It also includes models for continuous time, such as stochastic actor-oriented models (SAOMs, see Snijders et al., 2010b) or dynamic stochastic block models (SBMs, see for instance Xu, 2015). A model that can be seen as a linkage between the ERGM and continuous-time approaches is given by the longitudinal ERGM (LERGM, Koskinen et al., 2015; Snijders & Koskinen, 2013). While the TERGM assumes discrete time steps, the LERGM takes the network dynamics as a continuous-time Markov process with the ERGM as the limiting distribution. Although the model is tie oriented, its mechanics can also be compared well with the actor-oriented SOAM. This is because the LERGM fundamentally builds on micro-steps and in each step, dyadic ties (instead of actors as in the SAOM) are allowed to change the current network, governed by a function that drives stochastic tie changes. Although this approach is an interesting combination between the model approaches of the (T)ERGM and the SAOM, we will choose a discrete-time model for the arms trade network since the data are available only in time-discrete yearly time-steps.

A special variant of the TERGM was proposed by Krivitsky and Handcock (2014). They do not model the state of the network itself but rather focus on network changes which occur either because of the formation of new edges or because of the (non-)persistence of existing ones. Assuming independence between the two processes, conditional on the previous network, leads to the so-called *separable* TERGM (STERGM). Our use of the separable model is motivated by the fact that the two processes under study are highly likely to be driven by different mechanisms and factors.

For many real-world dynamic networks, the generative process changes with time and, therefore, the assumption of stationarity seems to be inappropriate. This is especially the case for network data that span a long period and are potentially subject to structural breaks. Under such conditions, it appears necessary to allow the model parameters to change with time. We take up this idea and extend the STERGM by allowing for time-varying coefficients. More specifically, we propose to rely on so-called generalized additive models (GAMs). This model class was proposed by Hastie and Tibshirani

(1987) and extended fundamentally by Wood (2017) to allow for smooth, semi-parametric modelling of time-varying parameters in a generalized regression framework (see also Ruppert et al., 2009).

Furthermore, the assumption of node homogeneity must be regarded as questionable. We, therefore, allow for heterogeneity in the model (see Thiemichen et al., 2016 for a discussion on node heterogeneity). Accordingly, we follow the p_2 model developed by Duijn et al. (2004) and enrich the STERGM with functional time-varying random effects (Durbán et al., 2005), which leads to smooth node-specific effects. We propose to investigate the fitted functional heterogeneity effects with techniques from functional data analysis (FDA), see, for instance, Ramsay and Silverman (2005). This enables the identification of countries (nodes) that have fundamentally changed their role in the arms-trading network over the observation period.

We proceed as follows. Section 2 presents the data provided by SIPRI. Section 3 introduces the statistical models used to analyse the data. Section 4 provides the results and their interpretation. Section 5 concludes the paper.

2 | DATA DESCRIPTION

Data on the international trade of major conventional weapons (MCW) are provided by the Stockholm International Peace Research Institute (see SIPRI, 2017) in a yearly time resolution. Examples for MCW include aircraft, armoured vehicles and ships. Note that we have excluded all non-state organizations like the Khmer Rouge or the Lebanon Palestinian Rebels from the dataset as well as countries with no reliable covariate information available. See the Supplementary Materials for an overview of the types of arms and the countries included in the analysis.

In the following, we conceptualize arms trade as a network and focus on the binary occurrence of trade, thereby disregarding the exact transfer volumes, and follow Akerman and Seim (2014) and Thurner et al. (2019) in setting the edge value to one if there is a trade flow greater than zero between two countries and edge value of zero otherwise. Additionally, we re-estimated our model with different thresholds and found that the results are quite robust. For details, see the Supplementary Materials. From a network perspective, the international arms trade in a given year can be seen as a collection of nodes (represented by the countries) and directed relations between the countries, called edges, that represent the existence of trade between these two countries in the respective year. Figure 1 at the end of the article shows the binary networks for the years 2015 and 2016 and the Supplementary Materials provides a collection of summary statistics for these networks.

The analysis of the degree distribution is of vital interest in statistical network analysis (Barabási & Albert, 1999) and gives important insights into the basic properties of the network under study. The degree is a node-related property and represents, in this context, the number of arms transfers that are related to a country. Because arms transfers go from an exporter to an importer, the network is a directed one. This allows for differentiation between the outdegree (the number of countries a specific country is exporting to) and the indegree (the number of countries from which a specific country is importing).

We compute the period-average in- and outdegree distributions and provide information on the minimal and maximal value of the realized degree distributions. This is represented in a log–log version in Figure 2 for both the outdegree and the indegree. The plot shows the enormous heterogeneity in the networks. Most of the countries have no exports at all with a period-average share of 78% of countries exhibiting outdegree zero, while the outdegree distribution has a long tail, indicating that there are a few countries which have a very high outdegree. The highest observed outdegree in a year is 66 and is observed for the United States. Other countries with exceptionally high outdegrees for

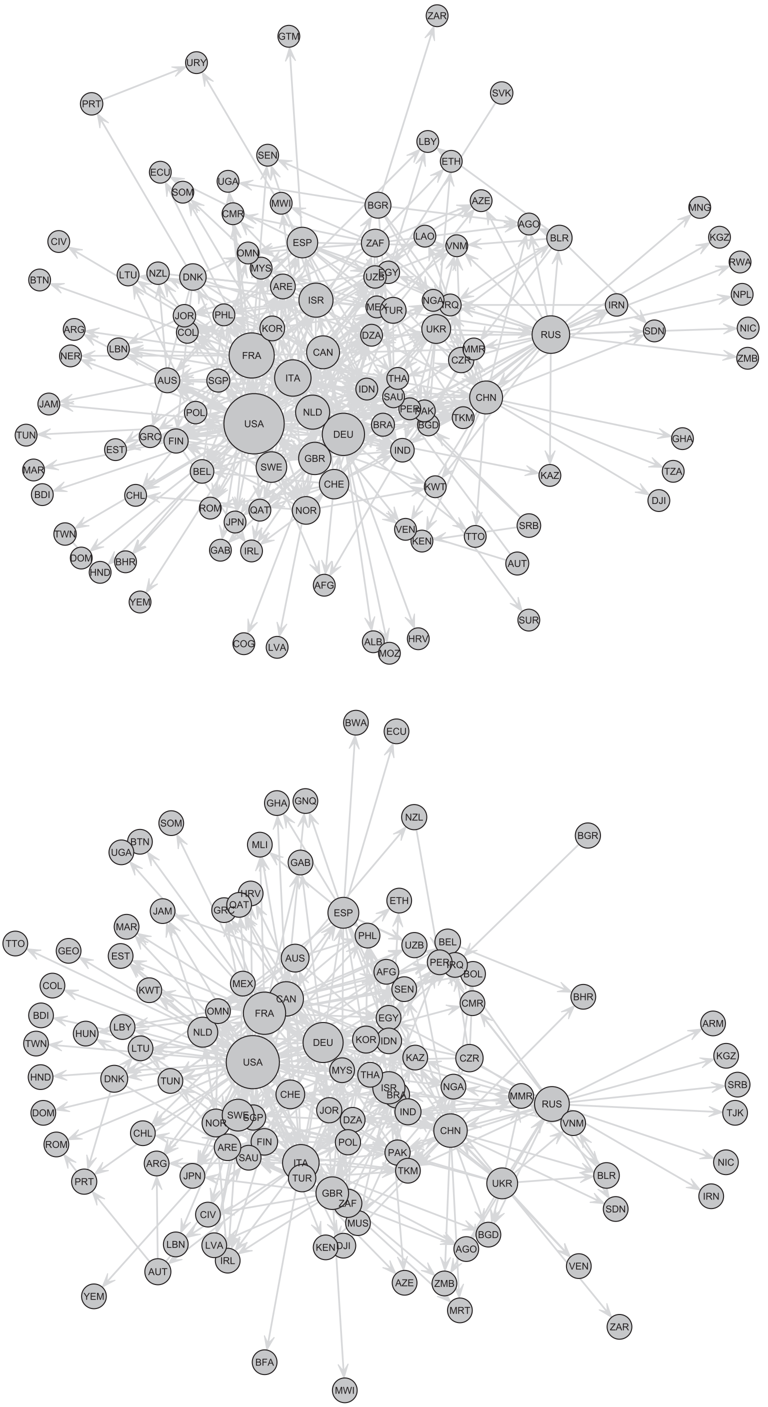

FIGURE 1 The network of international transfers of major conventional weapons (MCW) in 2015 (top) and 2016 (bottom). Countries are represented by vertices. Directed edges represent arms transfers. Vertex sizes are scaled proportionally to the logarithmic outdegree (number of outgoing edges)

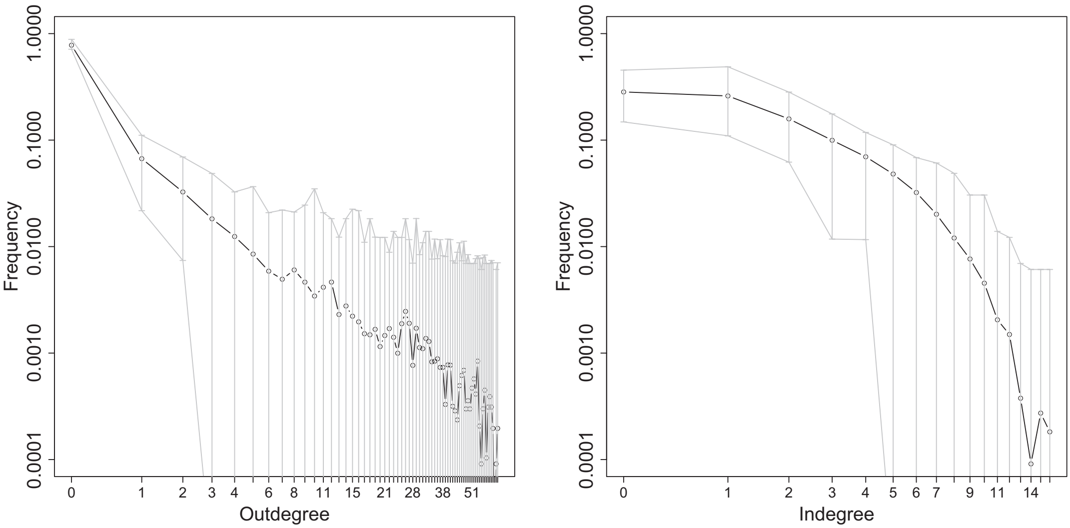

FIGURE 2 Degree distributions of the included countries for the outdegree (number of outgoing edges) on the left and indegree (number of ingoing edges) on the right. Averages over all years are represented by the solid line. The whiskers in grey show the minimum and maximum values realized in all years. Both axes are in logarithmic scale

almost the whole period are Russia (Soviet Union), France, Germany, the United Kingdom, China, Italy and Canada. In the right plot, the indegree distribution can be seen. Here, the pattern is different. The highest value observed in a year is 16 and corresponds to Saudi Arabia. In contrast to the outdegree distribution, the countries with high indegrees changed over time. At the beginning of the observational period, the countries with the highest indegrees were Germany, Indonesia, Italy, Turkey and Australia, but in more recent times the highest indegrees are observed from the United Arab Emirates, Saudi Arabia, Singapore, Thailand and Oman.

In Figure 3 we provide a graphical representation of the stability patterns in the network. On the left-hand side, we present the share of observations (vertical axis) against the number of subsequent transfers (i.e. repeated transfers) on the horizontal axis. Out of roughly 19,000 recorded trading instances only 33% do not have at least one consecutive transfer in the follow-up year of trade. Looking on the right-hand side of Figure 3, we visualize the share of observations (vertical axis) that has at least as many subsequent transfers as indicated by the horizontal axis. It can be seen that roughly the same share of observations (35%) lasts at least 5 years and almost 10% of all dyadic relations last more than 20 consecutive years without any interruption.

As discussed in the introduction, our research questions are centred around the problem of how the formation and persistence of arms trade relationships are differently associated with covariates which include network topologies and exogenous variables. Those are introduced and described in Section 3.3 after the statistical model itself has been outlined.

3 | MODEL

3.1 | Dynamic formation and persistence model

Let Y^t be the network at time point t, which consists of a set of actors (the countries), labelled as A^t and a set of directed edges (the transfers), represented through the index set $E^t = \{(i,j): i, j \in A^t\}$.

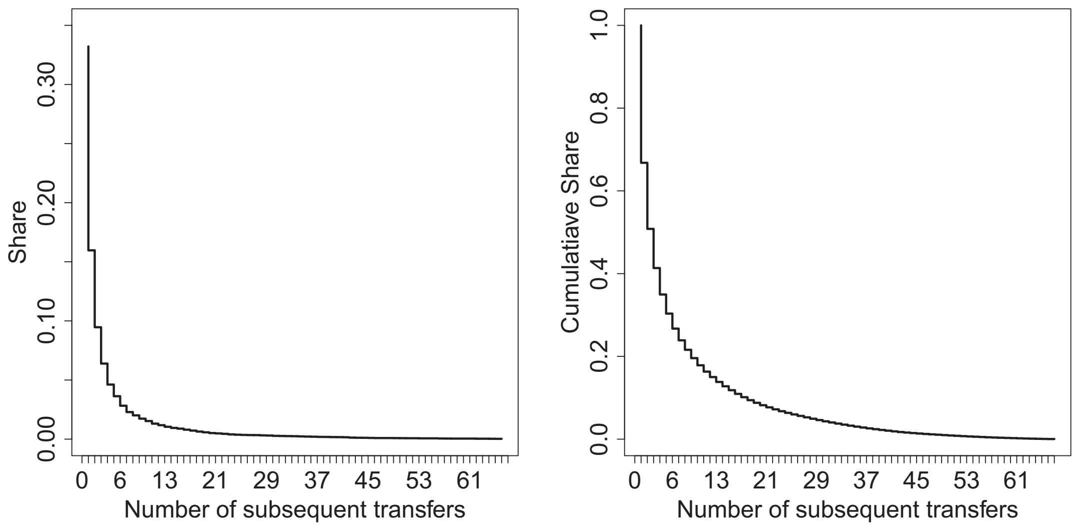

FIGURE 3 Share of subsequent arms transfers (left) and cumulative share of subsequent arms transfers (right). Number of subsequent transfers on the horizontal axis and share of observations on the vertical axis

Note that this is a slight misuse of index notation since Y_{ij}^t does not necessarily refer to the (i, j)-th element if we consider Y^t as an adjacency matrix. This is because the actor set A^t is allowed to change with time so that i and j are not running indices from 1 to n_t, where n_t is the number of elements in A^t. Instead indices i and j represent the i-th and j-th country respectively. We define $Y_{ij}^t = 1$ if country i exports weapons to country j and since self-loops are meaningless, elements Y_{ii}^t are not defined.

We aim to model the network in t based on the network in $t - 1$. To do so, we have to take into account that the actor sets A^{t-1} and A^t may differ. In particular, we have to consider the case of newly formed countries. New countries of interest are those that are present in t but do not provide information about their network embedding in the previous period. For exports this is not a concern as it is seldom the case that a new country starts sending arms immediately after entering the network. Notable exceptions are Russia, the Czech Republic and Slovakia. However, these countries have clearly defined predecessor states (the Soviet Union and Czechoslovakia) which can be used to gain information about the position of these countries in the precedent network. Regarding the imports, there is a share of countries that start receiving arms immediately after entering the network. Notwithstanding, those transactions represent a share of less than 0.3% of the observed trade flows. Therefore, we regard these cases as negligible and include in the model only countries where information on the current and previous period is available. We formalize this approach by defining $Y^{t,t-1}$ as the subgraph of Y^t with actor set $B^{t,t-1} = A^t \cap A^{t-1}$ containing $n_{t,t-1} := |B^{t,t-1}|$ elements. Accordingly, $Y^{t-1,t}$ represents the subgraph of Y^{t-1} with actor set $B^{t,t-1}$. Note that both subgraphs share the same set of actors and $Y^{t-1} = Y^{t-1,t}$ if A^{t-1} and A^t coincide.

From a modelling perspective, we follow Hanneke et al. (2010) and assume that the network in t can be modelled given preceding networks using a first-order Markov structure to describe transition dynamics for those actors included in the set $B^{t,t-1}$. Furthermore, we want to identify the driving forces of a transfer in t if there was a preceding transfer in $t - 1$ in the persistence model, while the formation model considers the process of forming a trade relationship without a preceding transfer. Hence, we call trade 'persistent' if it takes place in two consecutive years and 'formed' if there was trade in a year but no trade in the preceding year. The concepts of formation and persistence can be amended by using broader time windows. We demonstrate the robustness of our results concerning broader time windows in the Supplementary Materials.

Let $Y^{+} = Y^{t,t-1} \cup Y^{t-1,t}$ represent the formation network, which consists of edges that are either present in t *or* in $t-1$. For the persistence network, we define $Y^{-} = Y^{t,t-1} \cap Y^{t-1,t}$, as the network that consists of edges that are present in t *and* in $t-1$. Based on the actor set $B^{t,t-1}$ and given the formation and persistence networks together with the network in $t-1$, the network in t is uniquely defined by

$$Y^{t,t-1} = Y^{+} \backslash (Y^{t-1,t} \backslash Y^{-}) = Y^{-} \cup (Y^{+} \backslash Y^{t-1,t}). \tag{1}$$

Note that both Y^{+} and Y^{-} depend on time t as well, which we omitted in the notation for ease of readability. We assume that for each discrete time step, the processes of formation and persistence are separable. That is, the process that drives the formation of edges does not interact with the process of the persistence of the edges conditional on the previous network. Formally, this is given by the conditional independence of Y^{+} and Y^{-}:

$$P(Y^{t,t-1} = y^{t,t-1} | Y^{t-1,t} = y^{t-1,t}, x^{t-1}; \theta) = P(Y^{+} = y^{+} | Y^{t-1,t} = y^{t-1,t}, x^{t-1}; \theta^{+}) P(Y^{-} = y^{-} | Y^{t-1,t} = y^{t-1,t}, x^{t-1}; \theta^{-}),$$

where the lower case letters y^{+}, y^{-}, $y^{t,t-1}$ denote the realizations of the random networks, x^{t-1} denotes a vector of exogenous covariates and $\theta = (\theta^{+}, \theta^{-})$ represents the parameters of the model.

Note that it is not possible to use the lagged response as a predictor, as by construction it holds that $Y_{ij}^{t-1,t} = 1 \Rightarrow Y_{ij}^{+} = 1$ and $Y_{ij}^{t-1,t} = 0 \Rightarrow Y_{ij}^{-} = 0$. That is, an edge that existed in $t-1$ cannot be newly formed and an edge that was not existent in $t-1$ cannot be dissolved. It follows that the formation model exclusively focuses on the binary variables Y_{ij}^{+} with $(i, j) \in E^{+} = \{(i, j) \colon i, j \in B^{t,t-1}, Y_{ij}^{t-1,t} = 0\}$. This assures that in $t-1$ no edge between actors i and j was present and both actors are observable at both time points. Equivalently, the model for Y^{-} consists of observations Y_{ij}^{-} with $(i, j) \in E^{-} = \{(i, j) \colon i, j \in B^{t,t-1}, Y_{ij}^{t-1,t} = 1\}$, assuring that only edges that could potentially persist enter the model. The time dependence of E^{+} and E^{-} is omitted for ease of readability.

If we use an ERGM for the transition, this would yield the following probability model for the formation

$$P(Y^{+} = y^{+} | Y^{t-1,t} = y^{t-1,t}, x^{t-1}; \theta^{+}, x^{t-1}) = \frac{\exp\{\theta^{+} g(y^{+}, y^{t-1,t}, x^{t-1})\}}{\sum_{\tilde{y}^{+} \in \mathcal{Y}^{+}(y^{t-1,t})} \exp\{\theta^{+} g(\tilde{y}^{+}, y^{t-1,t}, x^{t-1})\}}.$$

The sum in the denominator is over all possible formation networks from the set of potential edges that can form given the network $y^{t-1,t}$. The inner product $\theta^{+} g(y^{+}, y^{t-1}, x^{t-1})$, relates a vector of statistics $g(\cdot)$ to the parameter vector θ^{+}. We will be more precise about the network statistics and the exogenous covariates in Section 3.3. The analogous model is assumed for the persistence of edges and not explicitly given here for the interest of space.

We will subsequently work with a simplified model which is computationally much more tractable. We assume that the formation or persistence of an edge at time point t depends solely on the past state and not on the current state of the network. This is achieved by restricting the statistics such that they decompose to

$$g(y^{+}, y^{t-1,t}, x^{t-1}) = \sum_{(i,j) \in E^{+}} y_{ij}^{+} \% g_{ij}(y^{t-1,t}, x^{t-1})$$

for some statistics $\tilde{g}(\cdot)$. This assumption is extensively discussed by (Hastie & Tibshirani, 1993) and can be well justified by the notion that the lagged network accounts for the major share of the dependency among the edges in the current network. It also allows for intuitive interpretations as can be seen below. Let Y_{-ij}^{+} represent the formation network Y^{+}, excluding the entry Y_{ij}^{+}. Then, for $(i, j) \in E^{+}$ the following logistic regression model holds

$$\log\left\{\frac{P(Y_{ij}^{+}=1|Y_{-ij}^{+}=y_{-ij}^{+},Y^{t-1,t}=y^{t-1,t},x^{t-1};\theta^{+})}{P(Y_{ij}^{+}=0|Y_{-ij}^{+}=y_{-ij}^{+},Y^{t-1,t}=y^{t-1,t},x^{t-1};\theta^{+})}\right\}=\log\left\{\frac{P(Y_{ij}^{+}=1|Y^{t-1,t}=y^{t-1,t},x^{t-1};\theta^{+})}{P(Y_{ij}^{+}=0|Y^{t-1,t}=y^{t-1,t},x^{t-1};\theta^{+})}\right\}=\theta^{+}\tilde{g}_{ij}(y^{t-1,t},x^{t-1}). \tag{2}$$

Note that model (2) describes network dynamics, but does not yet allow the parameters to change with time. Since the process under study is dynamic we replace the parameter θ^{+} with $\theta^{+}(t)$, representing a smooth function in time that enables the inclusion of temporal heterogeneity in the model and leads to a time-varying coefficient model (Hastie & Tibshirani, 1993). The focus of interest is, therefore, not only on the formation and persistence of edges (trade flows) but also on how these effects change over the whole observation period.

3.2 | Country-specific heterogeneity

The proposed network model assumes homogeneity, meaning that all differences between nodes in the network are fully described by the statistics $\tilde{g}(\cdot)$. However, the arms transfer network exhibits a rather small number of countries that are high-intensity exporters and a large number of countries that are restricted to imports. Furthermore, some countries change their relative position in the trade network with time. This indicates a substantial amount of temporal heterogeneity that needs to be taken into account.

This temporal heterogeneity can be accommodated by the inclusion of latent country effects. We follow Durbán et al. (2005) and model country-specific random curves which are fitted with penalized splines. This can be written in a mixed model representation such that the smooth country-specific effects are constructed using a B-spline basis with (a priori) normally distributed spline coefficients. We follow the modelling strategy of Durban and Aguilera-Morillo (2017) and assume that the model includes two time-dependent random coefficients $\phi^{+}_{i,exporter}(t)$ and $\phi^{+}_{j,importer}(t)$ - called exporter and importer effect in the following. The effects are assumed to be a realization of a stochastic process with continuous and integrable functions. For each exporter and importer in both models, the country-specific curves are given by

$$\phi_i(t)=B(t)a_i \tag{3}$$

where $B(t) = (B_1(t), \cdots, B_Q(t))$ is a B-spline basis covering the time range of observations and $a_i = (a_{i1}, \ldots, a_{iQ})$ is the coefficient vector. We impose the prior distribution

$$a_i \sim N(0, \sigma_a^2 D_Q), i.i.d. \text{ for } i=1,\ldots,n$$

where D_Q is the inverse of a difference-based penalty matrix which guarantees smoothness of the fitted curves $\phi_i(t)$ (see e.g. Eilers & Marx, 1996, for details on smoothing with B-splines). Note that for time windows where a country did not exist, the corresponding B-spline takes a value of zero so that no heterogeneity effect is present.

3.3 | Network statistics and explanatory variables

In this section, we specify the covariates x^{t-1} and the network statistics through which the model depends on the network at time $t-1$, that is, the elements of $\tilde{g}(\cdot)$ defined in the previous section.

We start with the network statistics that constitute a very important feature for model-based network analysis. As social network literature has shown, network statistics usually are not just statistical controls but convey substantial meaning (see e.g. Snijders, 2011). In the given context, they can be motivated by political, strategic and economic arguments that refer to real-world processes (see Thurner et al., 2019). Note that we norm all network statistics to range between 0 and 1. This is necessary to make the statistics independent from the varying network size and allows comparability over time. Note, however, that the norming also results in relatively high coefficients since the normed network statistics realize rather small values.

Outdegree: In our application, the outdegree provides information about the number of exporting relationships exhibited by each country. Formally, the outdegree of actor i at time point $t-1$ is defined as

$$outdeg_{t-1,i} = \frac{1}{n_{t,t-1}-1} \sum_{k \in B^{t,t-1}} y_{ik}^{t-1,t}.$$

The arms trade network exhibits a strongly oligopolistic structure with a few high-intensity traders, hence a positive coefficient for the outdegree of the exporter ($exporter.\,outdeg_{t-1,i}$) is plausible. However, it is not clear whether the exporters' outdegree as a global measure is still of relevance once we control for the random country heterogeneity effects.

Only a few advanced countries within NATO export and import at the same time. They have a highly differentiated portfolio, rendering specialization economically reasonable and strategically non-hazardous. To better represent this world-wide asymmetry, we include the outdegree of the importer ($importer.\,outdeg_{t-1,j}$) as a measure of whether countries with many exporting relations also tend to be importers themselves. This should not be captured by the random effects and we expect a clear negative effect, indicating that strong exporters are seldom strong importers.

Reciprocity: This statistic is intended to detect whether there is a general tendency of arms transfers to be mutual. The statistic measures whether the potential importer was an exporter in the dyadic relationship in the previous period:

$$recip_{t-1,ij} = y_{ji}^{t-1,t}.$$

Reciprocation is an essential mechanism in human relations in general, and in trade more specifically. As noted above, very highly developed countries exhibit this feature in the context of arms transfers. Since this group of countries is rather small and specialization-induced transfers between developed countries do not lead to continuous inflows, we expect this mechanism to be rather visible at the formation stage whereas it should not be a dominant feature for permanent repetition.

Transitivity: Within the arms trade network, trade relations often form more complicated structures that go beyond dyadic ones. Often, these relations form collections of triangles. These triadic trade relationships are an effective mechanism for pooling risks in buyer–seller networks (Bramoullé et al., 2019) and for building generalized trust, which is especially important in exchanging security goods. As a measure of these dependencies, we include transitivity, defined as

$$trans_{t-1,ij} = \frac{1}{n_{t,t-1}-2} \sum_{k \in B^{t,t-1}, k \neq i,j} y_{ik}^{t-1,t} y_{kj}^{t-1,t}.$$

The statistic counts how many tuples of transfers from i to j, passing a third country k can be found in $t-1$. Such relations can be interpreted as a direct application of the *Friend of a Friend* logic from social networks to arms trade. This kind of network embeddedness of weapons transfer deals is important for establishing new ones but is also likely to be relevant for the continuation of already existing ones.

Shared Suppliers: We also include a statistic that we call shared suppliers in this context. This statistic counts the shared number of actors that export to a given pair of countries:

$$sup_{t-1,ij} = \frac{1}{n_{t,t-1} - 2} \sum_{k \in B^{t,t-1}, k \neq i,j} y_{ki}^{t-1,t} y_{kj}^{t-1,t}.$$

This statistic allows for the investigation of whether two countries that share multiple suppliers tend to engage in trade with each other. Such a pattern is likely to be induced by a general hierarchy in the network (see Krause, 1995). While the first tier consists of strong exporters, the second tier is populated by countries with the ability to produce and export that are nevertheless mainly supplied by the big exporters. Countries with many shared partners are likely to engage in trade with each other but, on the other hand, they are typically dependent on imports from the first tier. Therefore, relationships among those countries are rather sporadic and unlikely to persist. Consequently, we expect a positive coefficient in the formation model and a negative one in the persistence model.

Naturally, arms trade is not exclusively driven by endogenous network processes but also influenced by variables from the realms of politics and economics. We lag all exogenous covariates by 1 year, first to be consistent with the idea that the determination of the network in t is based on the preceding period and second, to account for the time lag between the ordering and the delivery of MCW. Some of the covariate data are subject to missing values. No time series of covariates for the selected countries is completely missing (those countries are excluded from the analysis) and the major share of them is complete, but there are series with some missing values. This is sometimes the case in the year 1990 and/or 1991 when the former socialist countries split up or had some transition time. In other cases, values at the beginning or the end of the series are missing. We have decided on three general rules to fill the gaps: First, if a value for a certain country is missing in t but there are values available in $t-1$ and $t+1$, the mean of those values is used. If the values are missing at the end of the observational period, the last value observed is taken. In the case of missing values at the beginning, the first value observed is taken. The series on military expenditures are imputed similarly using linear interpolation by employing the `R` package `imputeTS` by Moritz (2016).

Formal Alliance: We regard bilateral formal alliances (including defence agreements and non-aggression pacts) as important security-related criteria that play a central role for the formation of trade relationships during the Cold War period. Therefore, the binary variable $alliance_{ij}$ is included in the model. The variable is equal to 1 if countries i and j had a formal alliance in the previous period and zero otherwise. Given the restriction that the data are available only until 2012 (Correlates of War Project, 2017a), we extrapolate the data, thereby assuming that the formal alliances did not change between 2012 and 2015.

Regime Dissimilarity: Another important security-related variable that potentially acts on the formation of arms trade relationships is given by the differences in political regimes between two potential trading partners. Hence, we include the so-called polity IV score, ranging from −10 (hereditary monarchy) to +10 (consolidated democracy). These data can be downloaded as annual cross-national time-series until 2015, see Center for Systemic Peace (2017) for the data and Marshall (2017) as a basic reference. In our model, we operationalize the distance between political regimes by using the absolute differences between the scores: $poldiff_{ij} = |polity_i - polity_j|$.

GDP: Following the standard gravity model, we include market sizes in our model. The standard measure for market size is the gross domestic product (GDP, in millions). We include the GDP in logarithmic form for the exporter (gdp_i) and the importer (gdp_j). The GDP data are taken from Gleditsch (2013a) and merged from the year 2010 on with recent real GDP data from the World Bank real GDP dataset (World Bank, 2017). Clearly, the market size and economic reliability of the exporter is an important prerequisite for forming and maintaining arms exports.

Distance: For gravity models applied to trade in commercial goods, there exists mounting empirical evidence that distance is a relevant factor for determining trade relations (Disdier & Head, 2008). We do not expect that trade costs and geographical distance impede arms trade because arms transfers establish worldwide alignments of exporters pursuing global strategic interests. Nevertheless, we include the logarithmic distance between capital cities in kilometres (Gleditsch, 2013b) to fulfil the gravity model specification.

Military Expenditures: We propose to include military expenditures of the sending and receiving country. This measure can be used to represent the size of the defence industrial base of the exporter, and the spending power and the intensity of the threat perceptions of the importing country. Accordingly, military expenditure is added separately for the exporter and the importer in a logarithmic form ($milex_i$, $milex_j$). Concerning the distinction between formation and persistence, our expectation is related to the hypothesis that countries with high military expenditures are more likely to import arms repeatedly. We therefore expect a positive and high coefficient for the military expenditures of the importer in the persistence model. The data are available from Correlates of War Project (2017b) in the national material capabilities dataset with Singer et al. (1972) as the basic reference on the data.

3.4 | Complete model and estimation

Putting all the elements together, the specification of the formation model as defined in Equation (2) is given by

$$\begin{aligned}\theta^{+}(t)\%g_{ij}(y^{t-1,t}, x_{ij}^{t-1,t}) = {} & \theta_0^{+} exporter.\, outdeg_{t-1,i}\theta_1^{+}(t) + importer.\, outdeg_{t-1,j}\theta_2^{+}(t) \\ & + recip_{t-1,ij}\theta_3^{+}(t) + trans_{t-1,ij}\theta_4^{+}(t) + sup_{t-1,ij}\theta_5^{+}(t) \\ & + distance_{t-1,ij}\theta_5^{+}(t) + alliance_{t-1,ij}\theta_6^{+}(t) + poldiff_{t-1,ij}\theta_7^{+}(t) \\ & + gdp_{t-1,i}\theta_8^{+}(t) + gdp_{t-1,j}\theta_9^{+}(t) + milex_{t-1,i}\theta_{10}^{+}(t) + milex_{t-1,j}\theta_{11}^{+}(t) \\ & + \phi_{i,exporter}^{+}(t) + \phi_{j,importer}^{+}(t).\end{aligned}$$

Analogously, we define the persistence model. Estimation is carried out with spline smoothing. That is, we replace the coefficients with

$$\theta_k(t) = B(t)u_k,$$

where u_k is penalized through

$$u_k \sim N(0, \sigma^2 D).$$

As in the above, the penalty matrix is appropriately chosen (see e.g. Wood, 2017) and $B(t)$ is a B-spline basis. Hence, smooth functions and smooth random heterogeneity can be estimated in a coherent framework (see Durbán et al., 2005). The identification of the smooth components and the intercept term is ensured by a 'sum-to-zero' constraint. For the smooth time-varying coefficients on the fixed effects, a maximum number of 65 knots is used, combined with a second-order P-spline basis (quadratic splines) and a first-order difference penalty on the coefficients. For the estimation of the time-varying random effects, a first-order penalty with nine knots is employed. The smoothness selection is done by the restricted maximum likelihood criterion (REML). The entire model can be integrated in the flexible GAM framework provided by Wood (2017) (see also Wood, 2006) which is implemented in the `mgcv` package of the statistical programming language `R` (R Development Core Team, 2008) by Wood (2011). Since the dataset is rather big and computationally expensive, we use the `bam()` function of the `mgcv` package that needs less memory and is much faster than other comparable packages (Wood et al., 2015). This function uses discretization of covariate values and iterative updating schemes that allow for the application of parallelization.

Important packages used for visualization of networks and computation of network statistics are the `statnet` suite of network analysis packages (Handcock et al., 2008) and the package `igraph` (Csardi & Nepusz, 2006). For the Tables, the `stargazer` package from Hlavac (2013) was employed. For the model evaluation and visualization, we used the `PRROC` package by Grau et al. (2015).

4 | RESULTS

4.1 | Time-varying fixed effects

The results of the time-varying effects are grouped into network-related covariates (presented in Figure 4) and political and economic covariates (presented in Figure 5). The left and right columns give the coefficients for the formation model and persistence model respectively. In the case of the network statistics, a schematic representation of the corresponding network effects is added on the right-hand side. The values for the coefficients are presented as solid lines with shaded regions,

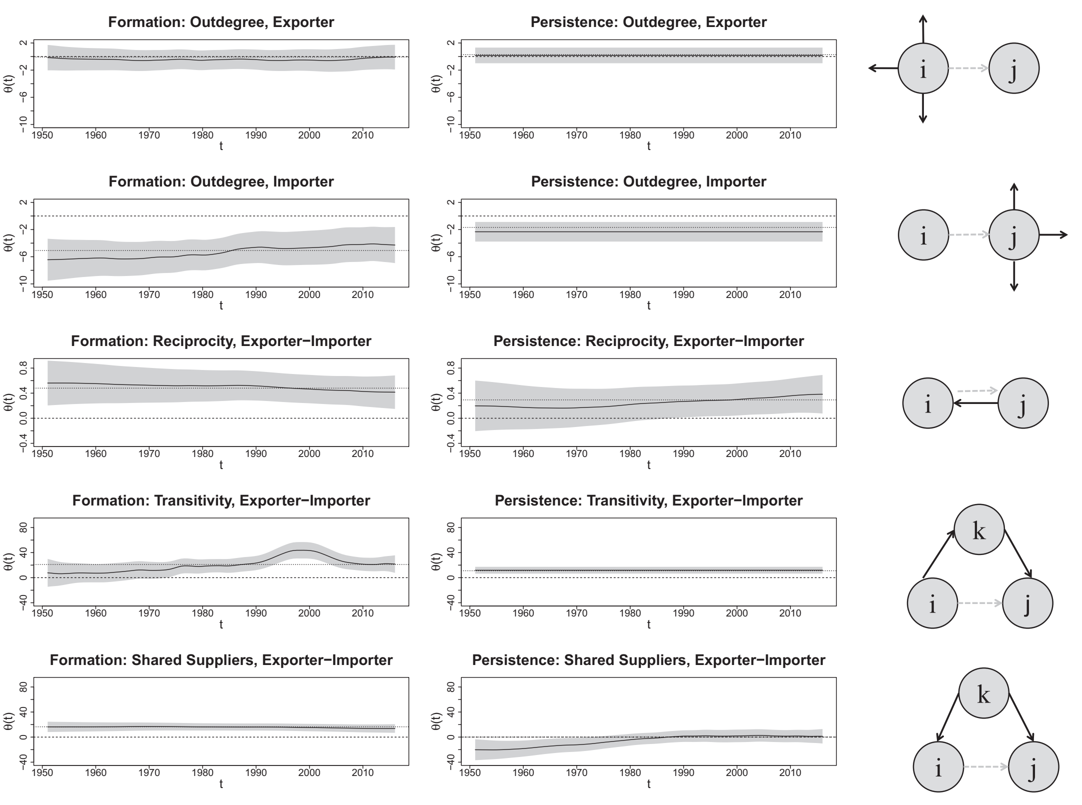

FIGURE 4 Time-varying coefficients of network statistics in solid black. Shaded areas give two standard error bounds. Time-constant effects in dashed grey and zero lines in dotted black. Schematic representation of the network effects on the right-hand side

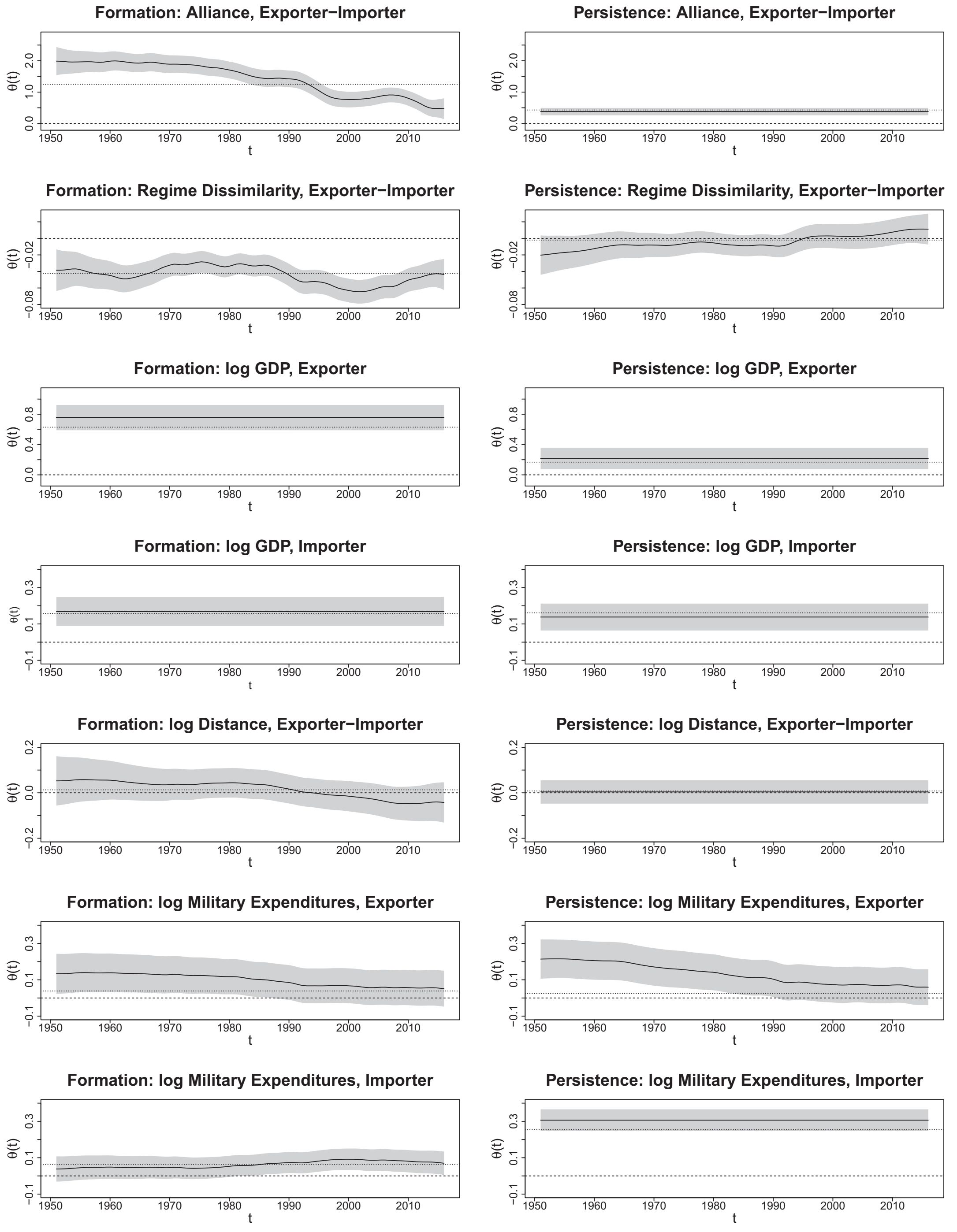

FIGURE 5 Time-varying coefficients of political and economic covariates in solid black. Shaded areas give two standard error bounds. Time-constant effects in dashed grey and zero line in dotted black

indicating two standard error bounds. The zero line is indicated as a dashed line and the estimates for time-constant coefficients are given by the dotted horizontal line. Additionally, for the same coefficient (or coefficients with the same norming) in the formation and persistence models, the effect size can be compared directly. Note that the coefficients at a given time point can be interpreted as in a logistic regression model.

4.1.1 | Network Effects (see Figure 4)

Outdegree: The exporters' outdegree has a coefficient that is almost time constant and close to zero for both models. This stands in contrast to the findings of Thurner et al. (2019), where a strong effect is present. Hence, once we controlled for country-specific heterogeneity (especially the exporter-specific country effect), no population-level outdegree effect for the exporter is present. We show in the Supplementary Materials that the effect is indeed present when country-specific heterogeneity is excluded.

However, the inclusion of country-specific exporter and importer effects does not affect the coefficient on the outdegree of the exporter. On the contrary, the coefficient is consistently negative and increases slightly over time in the formation model. For the persistence model, we find a less pronounced but significant negative effect. We interpret this as clear evidence that countries with high outdegrees are comparatively less likely to import and importers, in turn, tend to export less frequently. According to our experience, this specification captures the trade asymmetries of the oligopolistic market better than just specifying the indegrees of the importer.

Reciprocity: Controlling for the asymmetrical nature of the network, we identify a positive and significant impact of reciprocity in the formation model. Reciprocity in repeated transfers is only a relevant feature after the breakdown of the bipolar block structure. We conclude that the asymmetric structure is more present in persistent trade relations, with importing countries demonstrating large reliance on big exporters.

Transitivity: It can be seen that the variable transitivity has a positive impact on the formation and persistence of arms trade relationships. In the formation model, the effect is insignificant in the first years. This may be influenced by the clear hegemony of the United States and the Soviet Union immediately after World War II, which did not require shared control over the recipient country because the donor was powerful enough to secure the terms of a deal. In the 1980s, middle power countries became technologically more advanced and, especially in the West, they joined the United States in delivering to other countries. The pronounced change between 1990 and 2010 can be explained by the break-up of the two hostile blocs and the interruption of long-standing arm-trading partnerships, which led to a fundamental reorganization until 2010 when the effect came back to the level of 1990. Although these arguments are also valid for the persistence model, we see that transitivity is less relevant for ongoing, repeated transfers (the constant effect in the formation model is twice the size of the one in the persistence model). This finding is also strengthened by the fact that the coefficient is not subject to changes over time.

Shared Suppliers: The coefficients related to the shared suppliers corroborate our expectation that many shared suppliers lead to the formation of new trade relationships (positive and significant coefficient for the whole time period in the formation model). This mirrors the hierarchy of producing countries described above. If importing countries *i* and *j* become acquainted with similar technologies and develop similar levels of production capabilities, this would allow them to exchange arms. Also, the fact that both countries receive from the same supplier means that this country places trust in both importers, which facilities trust between the two importing countries. On the other hand, in the persistence model, the effect of shared suppliers is significantly negative and virtually zero from 1975 on, showing that repetitive trading is not promoted if countries share suppliers.

4.1.2 | Covariate Effects (see Figure 5)

Formal Alliance: The impact of bilateral formal alliances is positive and significant for both the formation and, with a more modest effect, the persistence of new trade relationships. This corroborates our expectation that formal alliances are more relevant for formation, that is, bypassing the required threshold of trustworthiness to initiate new trade relationships seems to decline over time. Hence, while formal alliances play a central role for arms trading after World Ward II, the formation of arms trades become less and less influenced by the existence of a formal alliance between the sending and receiving state. However, given the existence of an alliance, the impact (despite being smaller) continues to be relevant for repeated transfers. This is an important insight as we show for the first time that formalized alliances actually breed a dense web of arms transfers.

Regime Dissimilarity: For the formation model, the coefficient for the absolute difference of the polity scores is negative, significant and shows some time variation. With the decay of the eastern bloc, the resistance to sending new arms to dissimilar regimes increases until 2000. After that, the absolute effect of different polity scores declines again, coming back to the long-term constant effect. Interestingly, we find that regime dissimilarity is irrelevant in the persistence model, showing that if a relationship has started, repetition no longer requires regimes to exhibit shared governance values.

GDP: As expected, the coefficients on the logarithmic GDP for exporter and importer are positive and constant for both models. However, the effect for the exporters' GDP is much stronger in the formation model, model, confirming that it is mostly economically strong countries that can open new markets for arms exports. Together, the coefficients support the 'gravity hypothesis', that is, greater economic power and market sizes of the exporter and the importer increase the probability of forming and maintaining trade relations. However, given that a transfer relationship has started, this effect becomes smaller for repetition.

Distance: The results on the logarithmic distance contradict the standard gravity model. Distance proves to be insignificant in both models.

Military Expenditures: For the military expenditures of the exporter, we find very comparable and declining effects that become insignificant from 1990 on in both models. This indicates that with the end of the Cold War, the dominance of exporting countries with high military budgets decreased. For the importers' military expenditures in the formation model, the effect is positive and turns significant with time. This clearly illustrates that the military expenditures of the importer were not as important in the Cold War period when superpowers often granted military assistance. With the end of the 1980s, we observed a marketization of weapons transfers that resulted in supplier states demanding money for delivery. Given a preceding exchange of arms, we find that importer military expenditures have a very strong effect for the full observational period, indicating that the availability of huge military expenditures is key for understanding the continuous yearly inflow of weapons.

Overall, the results confirm our initial hypothesis. Judging by the size of the coefficients and their significance, we find that the network statistics (reciprocity, transitivity, shared suppliers) and security-related covariates (formal alliance, regime dissimilarity) prove to be highly influential in the formation model. On the other hand, we find weaker (or insignificant) network effects in the persistence model combined with high dominance of GDP and especially the military expenditures of the receiving country. This is not to say that we regard, for example, the positive effect of transitivity or alliances in the persistence model as irrelevant for repeated trading. The special nature of arms trading clearly demands trust for the formation and the persistence of transfers but the effects nevertheless show that the two processes are guided by different mechanisms that attach different priorities to security-related and economic variables.

4.2 | Time-varying smooth random effects

4.2.1 | Functional component analysis

We now turn our attention to the actor-specific heterogeneity. In Figure 6, the country-specific effects for the exporter and the importer countries are visualized for the formation model on the left and the persistence model on the right. Note that in these plots we have truncated the curves for the years where countries are not existent.

At first, interpretation of these plots appears clumsy. We therefore retrieve information by employing a functional principal component analysis (FPCA) to the multivariate time series of

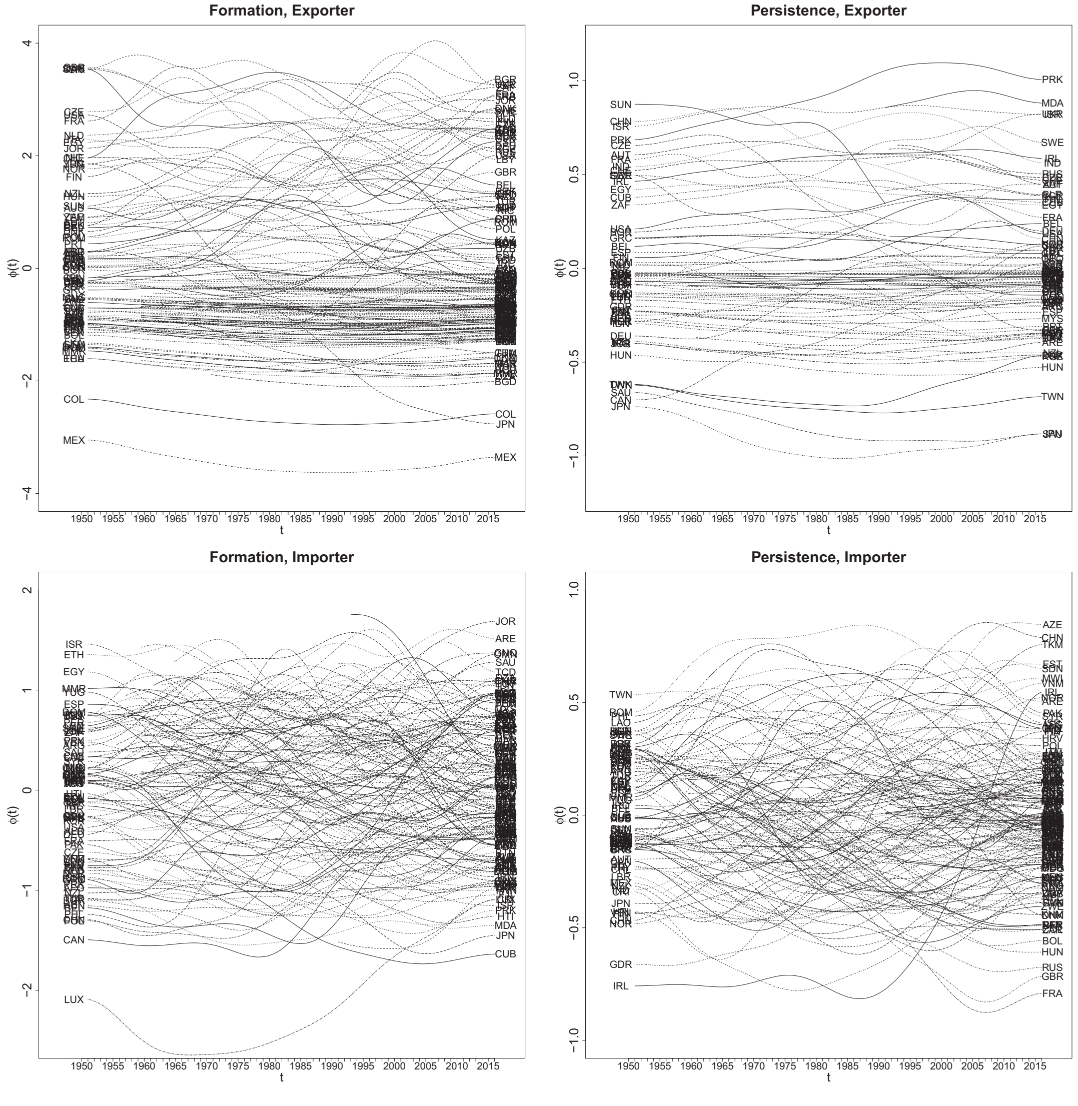

FIGURE 6 Fitted time-varying smooth random effects $\phi(t)$ plotted against time with country codes. The respective models are in the columns (formation on the left and persistence on the right) and the type of random effects in the rows (exporter effect on the top and importer effect on the bottom)

random effects seen in Figure 6 (see also 7 and the Supplementary Materials). The results are shown in Figure 7 for the formation model and in Figure 8 for the persistence model. On the left-hand side, the scores of the first two principal components are plotted. The effect of the two scores, including their contribution to the share of variance explained by the respective component, is visualized on the right-hand side. For example, in Figure 7, the first component explains 95% of the variance, which is shown by the heading PC 1 (95%). The concrete interpretation of the visualization on the right-hand side is given as follows. For the first (top plot) and the second component (bottom plot), adding (the '+' line) or subtracting (the '−' line) the corresponding principal component curve leads to the visualized perturbation from the mean. In all four cases,

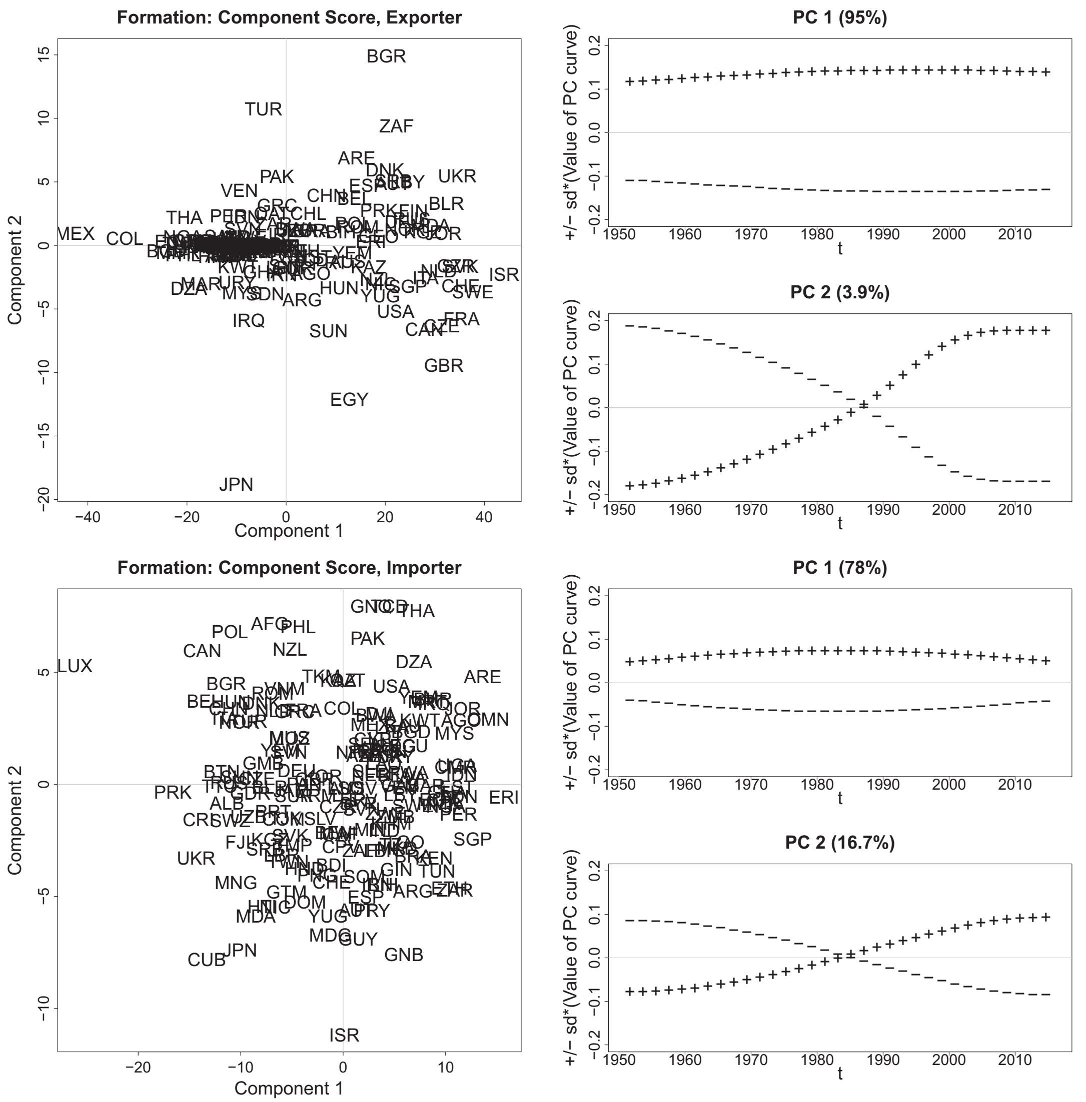

FIGURE 7 Functional principal component analysis of the smooth random effects in the formation model for the exporter (top) and the importer (bottom). Scores of the random effects for the first two principal components are given on the left. Mean principal component curve (zero line) and the effects of adding (+) and subtracting (−) the principal component curve are given on the right

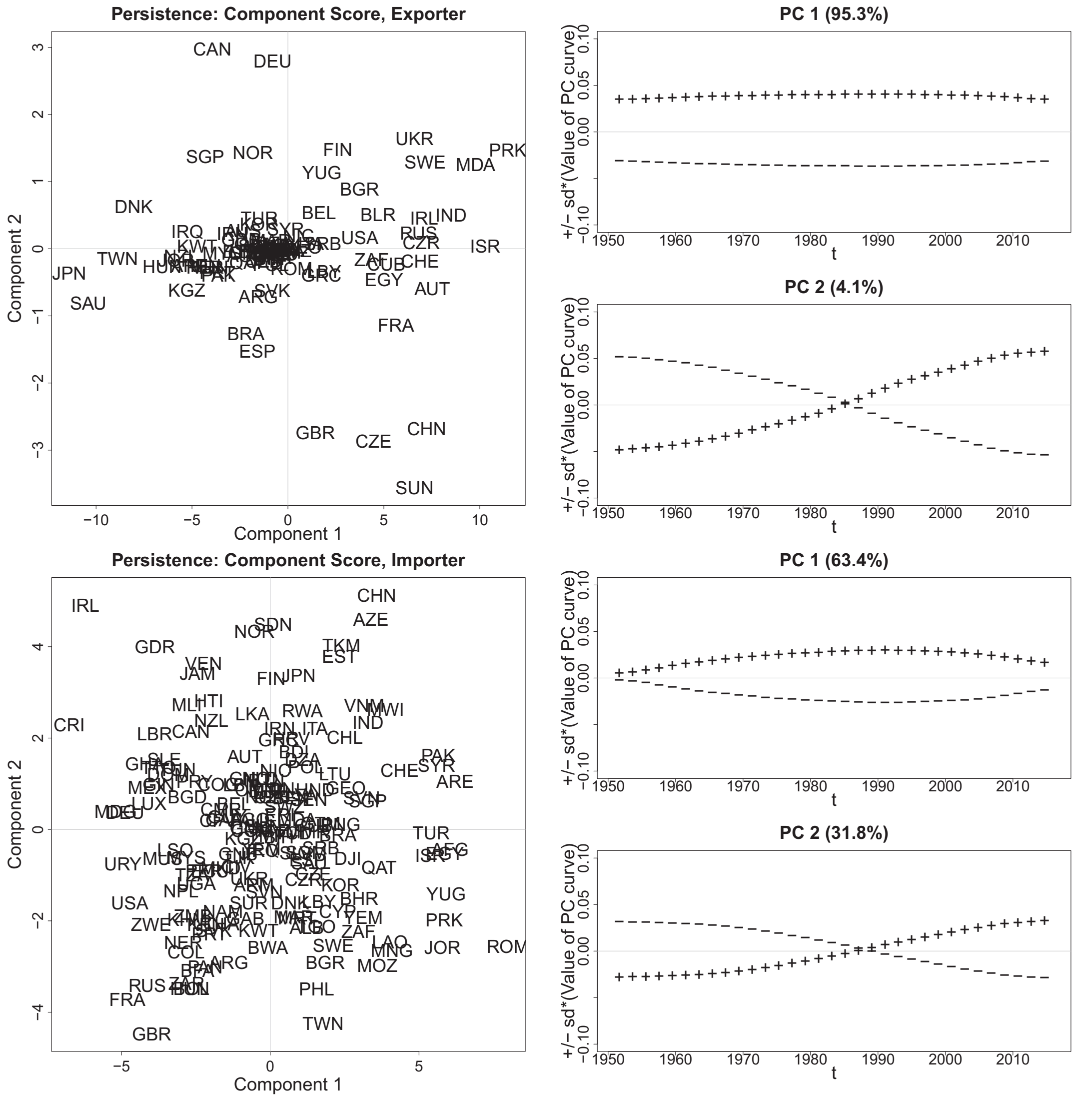

FIGURE 8 Functional principal component analysis of the smooth random effects in the persistence model for the exporter (top) and the importer (bottom). Scores of the random effects for the first two principal components are given on the left. Mean principal component curve (zero line) and the effects of adding (+) and subtracting (−) the principal component curve are given on the right

the first principal component (PC 1) is close to being constant, meaning that large positive scores for the first component indicate countries with a relatively high random effect, and vice versa for large negative scores. The temporal heterogeneity of the random effects is captured by the second principal component, indicating a tendency towards upward movement if positive and downward if negative. Hence, looking on the horizontal axes, we see countries that build up their arms trade links over the years as exporters (importers) on the right-hand side while countries that are reluctant to build up export (import) links are plotted on the left-hand side. Looking on the vertical axes, we see countries that decrease their role as exporter (importer) over time on the bottom, and vice versa countries that increase the number of export (import) links over time on the top. All these effects are conditional on the remaining covariate effects discussed before. Hence, these random effects capture the remaining heterogeneity not included in the remaining model.

4.2.2 | Results of the functional component analysis

Because of the great amount of information condensed in Figures 7 and 8, we restrict our interpretation to a few global patterns and selected countries that take either very special positions in the arms trade network (high or low values for component 1) or exhibit variation over time (high or low values for component 2). Overall, regarding the different levels of the random effects, it can already be seen in Figure 6 that the heterogeneity is much more pronounced in the formation model than in the persistence model. Furthermore, in the formation model, the countries differ more strongly in their ability to export in comparison to their ability to import while this contrast is not present in the persistence model.

A global pattern regarding the temporal heterogeneity of the exporter effect becomes visible since the top left in Figure 7 looks like a 'lying mushroom'. That is, countries that started on a low level (i.e. negative component 1) show, except for Japan (JPN) and Turkey (TUR), minimal upward or downward variability (i.e. low level for component 2). In contrast, countries that have a random effect above zero move more strongly up or down with time. This means that the export dynamics are mainly driven by countries with relatively high exporter effects.

Figures 7 and 8 show clearly that fundamental changes in the system are driven by the end of the Cold War. This can be seen, for example, in the position of the Soviet Union (SUN) and Czechoslovakia (CZE) at the top left in Figures 7 and 8 (both with a high level for component 1 and a low level for component 2). This indicates that these countries left the system shortly after the collapse of the eastern bloc. However, this turning point affected importers as well as exporters, and consequently, the representation of the importer effects of the formation model at the bottom left of Figure 7 is populated with (former) socialist countries such as Cuba (CUB), Ukraine (UKR), North Korea (PRK), Yugoslavia (YUG) and Moldova (MDA). Additionally, we find a prominent position for Romania (ROM), which is a country that has a high level (high value for component 1) but decreased its tendency to be an importer in persistent trade relations (low value for component 2) in Figure 8. However, while some of the countries of the eastern bloc ceased to exist or strongly reduced their exports or imports, we also find a contrary pattern. Countries like Ukraine (UKR) and Bulgaria (BGR) have managed to increase their exporter effect in the formation and in the persistence model over time (high value for component 1 and component 2 in the top left of Figures 7 and 8). This indicates that some leftovers from the collapsed Soviet Union defence industries sold off their stocks and rushed into the global market of military products.

Besides the massive shift initiated by the end of the Cold War, we see that some dominant exporting countries, especially Great Britain (GBR), France (FRA) and Egypt (EGY), lost importance over time. These countries can be found in the fourth quadrant of the top left panels in Figures 7 and 8, meaning their high exporter effects decreased strongly with time. This might seem surprising since France and Great Britain are still among the countries with the highest exported volumes. However, France and Great Britain have lost their dominance over former colonies, leading to a loss of control over many potential importers. The general pattern also carries over to the importer effects. Looking at the scores of Great Britain (GBR) and France (FRA) at the bottom left of Figure 8, we see a strong decrease in their importer effects in the persistence model.

Apart from global patterns, some countries exhibit exceptional scores that can be traced back to country-specific circumstances. We find that Japan (JPN) stands out among the countries with the lowest proclivity to import (see the low scores for components 1 and 2 at the bottom left of Figure 7). Even more pronounced is the very low tendency to export, indicated by Japan's exporter effect in the persistence models (Figure 8, top left) and the strongly declining exporter effect in the formation model (Figure 7, top left). These findings stand in contrast to the fact that Japan is among the wealthiest countries with a highly developed export industry and are clearly due to the highly restrictive arms export principles introduced in 1967 and tightened in 1976. This ban on exports was only lifted in 2014 (see Hughes, 2018 and Ministry of Foreign Affairs of Japan, 2014).

Another very notable case is Israel (ISR), which represents somewhat of an opposite case to that of Japan (JPN). The exporter effects on the top left of Figures 7 and 8 show that Israel (ISR) has an outstanding tendency to establish and maintain arms exports. On the other hand, Israel (ISR) takes a very polar position in the bottom left of Figure 7 as a consequence of a strongly decreased (i.e. low level for component 2) importer effect in the formation model. These results reflect Israel's path of developing highly internationally competitive weapons systems and its rise as one of the most important exporters. This stands in contrast to countries like Mexico (MEX), which is the country with the least tendency to form new trade exports (top left in Figure 7). It appears that this country is not able to be a relevant player in the market despite being among the world's largest economies. We consider these special paths as induced by cumulative advantages and learning over time in the one case (Israel), whereas in the case of Mexico (MEX) we observe the path inertia of a country that has not been able to get its defence products sold externally.

There remain many other interesting cases. For example, the rise of South Africa (ZAF) as an exporter in the formation model (top left in Figure 7) demonstrates the history of the country, which was initially dependent on imports and is now among the major exporters of MCW. We also find that Ireland (IRL) strongly increased its tendency to be a persistent importer after its entry to the European Union (bottom left in Figure 8) while Germany (DEU) and Canada (CAN) strongly increased their roles as persistent exporters (top left in Figure 8).

4.3 | Model evaluation

The evaluation of the out-of-sample predictive power is based on the following steps. We first fit the formation model and the persistence model, based on the information in $t-1$, to the data in t and use the estimated coefficients for the prediction of new formation or persistence of existing ties in $t+1$. As the predictions are probabilistic by nature, we weight the recall (true positive rate, TPR) against the false positive rate (FPR) for varying threshold levels, yielding the receiver operating characteristic (ROC) curve and the area under the curve (AUC) for each year of prediction. Because arms transfers can be regarded as rare events, we also compute the precision-recall (PR) curve and the corresponding AUC (see Powers, 2011 as a basic reference and survey regarding ROC and PR). The results are plotted in Figure 9 with the AUC values that correspond to the PR curves on the left and the one corresponding to the ROC curves on the right. The first row gives the evaluation of the formation model and the second row shows the persistence model. While the AUC values in the formation model are very high when evaluated at the ROC curves they are much lower with the PR curves. This is a consequence of being right quite frequently if a zero is predicted, while it is hard to forecast the actual transfers in the next period in case of the formation model. Interestingly, the opposite holds for the persistence model. In the combined version at the bottom of Figure 9, the AUC values derived from the PR curve show that the model does quite well.

Additionally, we evaluate how well global network structures like the mean outdegree, the share of reciprocity and observed transitivity can be mirrored by the predictions using a simulation-based

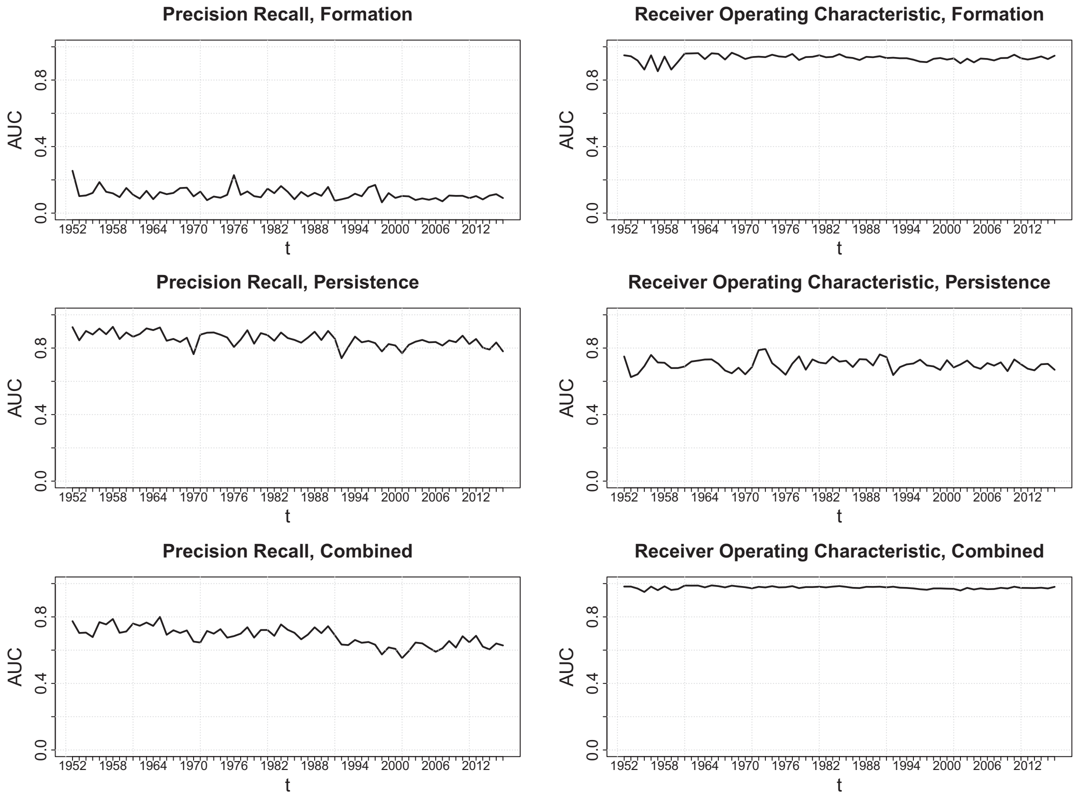

FIGURE 9 Time series of the area under the curve (AUC) values for precision-recall (PR) on the left and AUC values for the receiver operating characteristic (ROC) on the right. Formation model in the first row, the persistence model in the second row and their combination in the last row

approach (see (see Hunter et al., 2008). To do so, we fit the models for the transition between $t-1$ and t and simulate from the formation model and the persistence model 1,000 times based on the information in t. Then, based on Equation (1), the predicted network for $t+1$ is constructed. From this, we evaluate global network characteristics and compare them to the actual characteristics from the true MCW trade network in $t+1$. The corresponding figure is given in the Supplementary Materials. The results are reassuring and the simulated networks acceptably mirror the real network properties.

Clearly, the proposed model is not the only suitable network model. Alternatively, it is possible to analyse the data with a STERGM without random effects and with various variants of the ERGM or the TERGM with and without random effects. Judged by the Akaike Information Criteria (AIC), we find that the values for the formation (49384.11) and persistence model (19278.99) are lower than their counterparts for the formation (56054.33) and persistence models (19867.85) without random effects. We discuss these matters further in the Supplementary Materials and show that the out-of-sample predictive power of our model is superior to other candidate models.

5 | CONCLUSION

In this paper, we employ a separable network model as introduced by Krivitsky and Handcock (2014) and add techniques proposed by Hastie and Tibshirani (1993) and Durbán et al. (2005). This enables us to study the processes of arms trade relationship formation and persistence separately and to include time-varying coefficients and smooth time-varying random effects that are further analysed by methods from functional data analysis as described in Ramsay and Silverman (2005).

Applied to the discretized MCW networks from 1950 to 2016, we find that the mechanisms leading to formation and persistence differ fundamentally. Most importantly, the formation of new trade relationships is driven by network effects and security-related variables, while the persistence of transfers is dominated by the military expenditures of the receiving country. A careful analysis of the random effects indicates a high variation among the countries and along the time dimension. By using functional principal component analysis, we decompose the functional time series of smooth random effects to find countries that have increased or decreased their relative importance in the network. The evaluation of the fit confirms that the chosen model is able to give good out-of-sample predictions.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section.
Supplementary Material

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