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# Bubble: Experimenting with Feynman's sum over paths approach in the secondary school

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**Summary.** — We discuss a teaching experiment on the introduction of elements of quantum physics already at the level of the fourth year of high school (17–18 years old students) using Feynman's sum over paths approach. More precisely, the educational sequence is constructed on the juxtaposition of the wave and quantum theories of light, and it constitutes an attempt to anticipate the current status of the past, secular debate on the nature of light to young students, while providing them with a unifying perspective on different approaches, models and theories which are encountered in high school. The experimentation is part of the research on quantum physics education in secondary school conducted by the Physics Education group in Pavia, and it was developed by the author as a thesis work in the context of the IDIFO-6 Master coordinated by the University of Udine. The focus of the work is on the gradual introduction of innovative elements in the traditional high school didactics. Analysis of the experimentation data shows very rich and complex patterns, allowing to identify activities which may be more productive for students and to uncover weak points and student's difficulties.

## 1. – Introduction

Starting from the last two decades of the twentieth century, many studies in physics education have investigated the possibility of introducing elements of quantum physics already at the level of secondary school. In many countries, including Italy, political decisions have followed on the path opened by educational research, and reforms increasing the modern physics content of physics curricula have been introduced. Quantum physics constitutes today a body of knowledge of such cultural relevance and central importance for the technological development of society, that it should be made accessible to all young students, especially to those who are trained to be future scientists of any kind.

As more than a hundred years have elapsed from the initial formulations of the idea of a light quantum, many researchers and teachers have felt the need of going beyond the *quasi-historical* [1] approach offered by a majority of textbooks, and of proposing students a deeper conceptual reconstruction of quantum physics, emphasizing its structure as "a rational theory, with its own rules, its own language and first of all, its own facts" [2]. Thus, the value of the theory as a breaking point with the classical concepts should not (or not only) be simply stated within a meta-scientific, historical-like narrative, but become apparent from the rigour and internal coherence of the non-classical model developed to explain physical facts.

The research group in Physics Education at the University of Pavia has worked for several years on the use of Feynman's sum over paths approach in high school education. As we have argued elsewhere [3] it might not even be necessary to wait for the last year of secondary school to introduce the rules of Feynman's approach: students could, instead, first encounter them at the time they are introduced to the wave theory of light, as a possible way to reconcile seemingly illogical facts of nature. Furthermore, such an approach could help overcoming significant educational problems which are connected with the traditional way of teaching the different theories and models for light. In fact, in the Italian curriculum and textbooks, ray optics and wave optics are presented in very different parts of the course, corresponding to very different student ages: geometrical optics in the second year and wave optics in the fourth one. Then, electromagnetic waves and a very basic (mainly phenomenological) introduction to the photon concept are taught in the fifth year. Typically, textbooks devote very little or no space to discussing the compatibility, or respective domains of validity, of all the aforementioned theories. Such an approach could possibly exacerbate student difficulties, and increase confusion and hybridization between mental models corresponding to wave and ray representations of light, which have been observed even at University level [4].

In part in response to this situation, our group has proposed to already introduce the photon concept, using Feynman's sum over paths approach [5], at the fourth year of secondary school, presenting it as the currently accepted synthesis of the secular debate on the nature of light. Feynman's approach allows to interpret all the traditional wave phenomena (interference, diffraction, etc.) in terms of photons, and also provides a clear picture of how Fermat's principle, which is the foundation of ray optics, emerges from the sum over paths rules, in the limit of small wavelengths with respect to relevant length scales of the problem. Following this route, students can be led to juxtapose and compare theoretical models, and critically examine the relationship between them.

The experimentation presented in this article constitutes a first attempt at transposing such idea in the educational practice. The teaching-learning sequence, realized and tested in the context of the IDIFO-6 Master coordinated by the University of Udine, is intentionally designed as a moderate departure from the traditional syllabus. Its main focus is on the juxtaposition of the wave and photon theories of light, and the need for a statistical interpretation of the results of wave optics, explained in terms of Feynman's approach. In broad terms, the sequence comprises two main blocks of activities: the initial part is tightly connected to experimental evidence, as the phenomena of interference, diffraction and iridescence (thin film interference) are observed and analyzed in the laboratory, and explained in terms of the wave theory of light; in the second part, closely following the track of Feynman's book *QED* [6], students are presented with novel, inescapable evidence demonstrating the existence of photons as fundamental constituents of light, and the sum over paths approach is introduced as a way of making sense and re-interpret the phenomena previously observed in view of the existence of photons.

## 2. – The Bubble sequence

The educational sequence was nicknamed *Bubble* following the name of an art installation in which German artists created, using rather sophisticated techniques, a giant soap bubble which could remain intact for several days [7]. The reason for the nickname is that a point of departure, motivating experiment, and common thread in the sequence are everyday interference phenomena, like soap bubble iridescence [8] and similar examples of thin film interference (*e.g.*, from oil stains).

**2**˙1. *Structure of the sequence.* – We present in advance a schematic description of the sequence structure in order to render more comprehensible the detailed description of content, educational choices and steps, which will be presented in the following subsections. Table I reports the order of lessons, experimental and theoretical content, and the type of activities in which students are involved. As anticipated, the first three lessons are mainly experimental; the experiments are interpreted through a wave theory which is directly derived from an analogy with sound and mechanical waves, as is usual in the high school treatment. In the fourth lesson, which is the heaviest one in theory content, we pose the problem of evidence for light quantization, and of the inconsistency of such evidence with both classical wave and particle interpretations. Feynman's approach of the sum over all possible paths is introduced in its simplest form [6,9,10] and, in the final three lessons, the approach allows to explain and re-interpret the experimental evidence of light interference phenomena. In the last lesson, which circularly closes the sequence, students work individually on a structured problem which returns on thin film interference in view both of the photon concept and of the Feynman rules to compute detection probabilities.

**2**˙2. *Key ideas.* – During the phase of design we searched for a small number of key ideas which could constitute a constant reference point, and to which the teacher could return often during classroom and lab discussions. The central ideas we identified are a) significance of a *physical model*; b) juxtaposition of wave and photon explanations for the behaviour of light; c) need for an unambiguous language to express new concepts; d) probabilistic and statistical interpretation of phenomena; e) visualization of the mathematical model of quantum theory.

The centrality of the concept of the model is mentioned in the Italian national indications for the physics curriculum and often stressed in the research literature [11]. The main effort in our work is to make students conscious and aware of the process of modelling, by placing them, through guided inquiry laboratory activities, in similar conditions to those of the scientist who needs to interpret a phenomenon from scratch. Given a system which behaves in a certain way, one first constructs a mental model, a qualitative image of what is happening; then he chooses a language to describe and communicate the contents of his model and, at the same time, attempts to formalize it using the rules of mathematics. The dialectic relationship between these three moments allows to improve one's own understanding both of the phenomena studied and of the model itself, which is tentatively being built. Such activities have also the objective of stimulating metacognitive reflection on the students' own reasoning processes, and meta-theoretical reflection on the nature of physics.

After students have familiarized with the basics of wave theory, the evidence of the existence of photons breaks into the certainties students have acquired as a classical moment of cognitive conflict. From here on, the comparison of the wave and photon concepts

TABLE I. – *Structure of the Bubble teaching-learning sequence.*

| Lesson (Location and time) | Experiments | Theory | Methods |
|---|---|---|---|
| 1. Iridescence (Physics lab, 1 h) | Observation of iridescence in soap bubbles | Wave theory explanation of iridescence | Group worksheet on the explanation of iridescence |
| 2. Interference (Physics lab, 1.5 h) | Experiment on two slit interference | Wave theory explanation of two slit interference | Laboratory group work |
| 3. Diffraction (Physics lab, 1.5 h) | Measurement of the width of a hair | Wave theory explanation of light diffraction | Laboratory group work |
| 4. Feynman's approach (Classroom, 1.5 h) | | Evidence for the existence of photons and introduction to Feynman's approach | Frontal lesson |
| 5. Applications (Classroom, 2 h) | | Explanation of interference, diffraction and refraction using Feynman's approach | Work in pairs on interference from multiple slits.<br>Simulations on diffraction and refraction |
| 6. Visualization (Computer lab, 1 h) | | Explanation of thin film interference using Feynman's approach | Group task on the construction of a GeoGebra simulation on thin film interference |
| 7. Working with the new theory (Classroom, 2 h) | | | Individual worksheet on thin film interference using Feynman's approach |

as explanations for the behaviour of light serves a dual role: on the one hand, it shows that the new theory must be more *fruitful* than the previous one [12], that is, it must be able to explain all evidence which was previously explained, and also predict or explain new facts. On the other hand, the comparison poses a problem of *language*: although the mathematical formalization is for many problems identical in the two descriptions, from a physical point of view they describe very different events; thus, a new language must be devised, which resolves, as far as possible, all ambiguities due to objects of the newer theory which have a mathematical analogue in objects belonging to the older one. For example, although photons are ordinarily described as elementary particles in modern physics, the term "particle" is too loaded with meaning from classical physics, carrying with itself a semantic area comprising the concepts of "position", "velocity", "trajectory" and so on. Thus, as many authors have remarked [10, 13], it is preferable to avoid the terms "particle" or "corpuscle" in an educational setting, replacing them with the term "quantum object". The centrality of language issues in the teaching of quantum physics was highlighted by several authors [13, 14].

Because of the limited time available for the experimentation and the lack of mathematical tools by students at this age, there is essentially only one, central idea about quantum physics which our sequence aims to communicate: the radical difference with classical physics in how probabilities are computed. We compactly recall this concept, which constitutes the starting point of Feynman's 1948 paper [15] and a recurring theme in the book *QED* [6]. Suppose there is an event $E$ (for example, the measurement of the presence of a particle at some point of space) which can happen due to two different, mutually exclusive processes or chains of events $A$ and $B$ (for example, a particle passing through either one of two slits in a screen). Suppose we know the process $A$ alone happens with probability $P_A$, and the process $B$ alone with probability $P_B$. Then, for *any* classical probabilistic theory (which may not be deterministic due, for example, to either the inclusion of noise or ignorance of initial conditions) the probability that the event $E$ happens will be $P_E = P_A + P_B$. On the contrary, quantum physics will, according to certain rules, associate to the realization of processes $A$ and $B$ two amplitudes, $\psi_A$ and $\psi_B$, which can be represented as vectors with a modulus and a phase, in such a way that $|\psi_A|^2 = P_A$ and $|\psi_B|^2 = P_B$; the probability of the event $E$ will be given by $P_E = |\psi_A + \psi_B|^2$. Since $\psi_A$ and $\psi_B$ are summed as vectors, the result will in general be different from the classical one, and give rise to interference phenomena.

When comparing with students the wave and quantum theories, we often return on the idea that although the two approaches are conceptually very different, they do not make different predictions for the behaviour of light at values of intensity which are similar to those encountered in everyday life. Feynman's rules provide a probability for the detection of a photon in any point of space(1), and such probabilities, for many photons, translate into statistics of photon counting which give rise to light intensities agreeing with those predicted by wave theory. The quantum theory for the photon emerges as arguably the only possible model for individual discrete objects which can be compatible in a statistical sense with the predictions of wave theory.

Finally, a central theme in our sequence is the possibility of visualizing the mathematical model underlying quantum theory, a possibility provided to students mainly through *GeoGebra* simulations [16, 17]. The issue of providing a visual representation of quantum objects has long been debated in the research literature, with several authors arguing that such representation may favour the persistence of classical hybrid quantum-classical conceptions. By design, the simulations we use, or guide students to produce, do not contain any direct visual representation of quantum objects, but only represent elements of the experimental setup (source, detectors and physical constraint), and elements of the model, such as paths and their associated tiny arrows or phasors. Even so, great attention to students' choice of words during their work is necessary to prevent overly concrete interpretations of elements of the model (*e.g.*, the possible paths as alternative trajectories, or phasors representing amplitudes as vectors connected to some physical quantity) which have sometimes been reported for the educational use of Feynman's approach [18]. It is also worth mentioning that in this experimentation we tried to have

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(1) And, if necessary, time, although time of detection remains undefined in an elementary, energy-fixed treatment, see ref. [9] for a discussion.

students take a more active role, involving them not only in the manipulation of existing simulations, but also in the creation, guided by the teacher, of a new one representing the phenomenon, discussed several times within the sequence of thin film interference.

**2**˙3. *Methods and detailed sequence account.* – With the aim of obtaining a high level of student participation and motivation in the activities, it was decided to avoid almost entirely traditional frontal lessons; frontal teaching has been necessary only within the fourth lesson, for the introduction of the "rules of the game" of Feynman's approach. The privileged educational strategy was group or pair work, both in the physics laboratory, in the computer lab and in the classroom, using work sheets for guided inquiry experiments, guided problem solving, and activities involving the use or the construction of simulations. The materials used were in part original and in part were adapted from those previously used by the Pavia group in the experimentations discussed in ref. [18] which, however, were performed with one-year-older students. The realization of guided worksheets allowed to collect a large amount of data with important indications on the critical points in the development of conceptual understanding.

*Lessons 1, 2 and 3* were devoted to the study of the physical phenomena of iridescence, two slit interference and diffraction. The study of luminous phenomena observed on soap bubbles opens the sequence (also with the aim of a motivating activity), closes it and returns periodically as a common thread. In the first lesson, after observing soap bubbles, students performed group work using a work sheet guiding them to use known concepts (wave theory) to attempt an interpretation of a phenomenon not previously studied. Data from the activity were also used as a reference for the starting levels of the different classes involved in the experimentation. In the second lesson, students performed an experimental activity on two slit interference in the physics laboratory. Students collected data on the relationship between the spacing of interference maxima and the slits spacing, or the slit-screen distance. Using guided laboratory sheets they interpreted such data in view of the wave theory of light. In the final part of the activity, students also measured the distribution of intensity on the screen using sensors and, again, attempted to interpret the result using a wave model. In the third lesson, reversing the logical order of the previous activities, students first predicted the expression for the position of diffraction minima as a function of the width of a thin obstacle and used such expression to determine the width of a hair belonging to one of the students. At the end of the three lessons, wave theory has for students a strong empirical validation, stemming from its ability to interpret and predict experimental results.

*In the fourth lesson* we discussed the breakup of both classical wave and corpuscle light theories, and introduce the sum over paths approach. Students were proposed the same reasoning used by Feynman in *QED*: light is made of discrete objects called photons, whose existence is experimentally demonstrated by photomultiplier clicks and low intensity interference experiments. This experimental evidence is clearly not compatible with wave theory, but interference phenomena cannot be explained using any classical corpuscle theory either. In order to demonstrate the last point, we used Feynman's example of light reflection and transmission from thin glass sheets of slightly different widths, demonstrating interference effects (percentage of reflected light intensity varying cyclically with the sheet width, from a minimum of 0% to a maximum of 16%). The same example was used as a stage to introduce the basic rules of Feynman's approach.

*The fifth lesson* was entirely devoted to applications of the sum over paths approach. The first step was asking students whether the model they learned can reproduce the formulas for interference maxima and minima in the two slits interference, and discussing the issue with them. It is stressed that the results of the quantum model must be interpreted in a probabilistic and statistical sense: for example, maxima are those points in which the *probability* of detecting each individual photon is highest, and thus in which, for many photons directed towards the screen, the count of photons is higher in a statistical sense. In the following part of the lesson students worked in pairs on sheets guiding them step by step to solve problems concerning light passing through multiple slits, either on the same screen (three and four slit interference) or on successive screens (see ref. [18]). Student's solutions were then collectively discussed in class. The worksheets used in this lesson are an important means of monitoring and improving student's understanding of the new model and language that had just been introduced. In the last part of the lesson we treated light diffraction and refraction in the sum over paths approach, using the GeoGebra simulations of refs. [16, 17], in order to allow an immediate visualization. In the case of refraction, the simulation allowed to reach the limit of very small wavelengths, obtaining the results of geometrical optics as a limit case, thus establishing a connection and relationship with a different theory of light, which students had studied two years earlier.

*In the sixth lesson* students worked in the computer lab. They were given a sheet containing instructions on how to construct a simulation of reflection from a thin film [16] using GeoGebra. Besides providing instructions for the construction of the simulation, the protocol given to students helped going through the logical passages needed for solving the problem with an adequate language and terms, and helped students interpret correctly the objects that were visualized.

*The last lesson* aimed at closing the circle, returning on the theoretical explanation of iridescence and thin film interference, this time using the sum over paths approach. Students were given a problem with numerical data (fig. 1), and an individual worksheet guiding them to solve the problem through a series of questions. The problem was similar, in structure, to the simulation students coded in the previous lesson, so that the work in the computer lab could contribute to their solution of

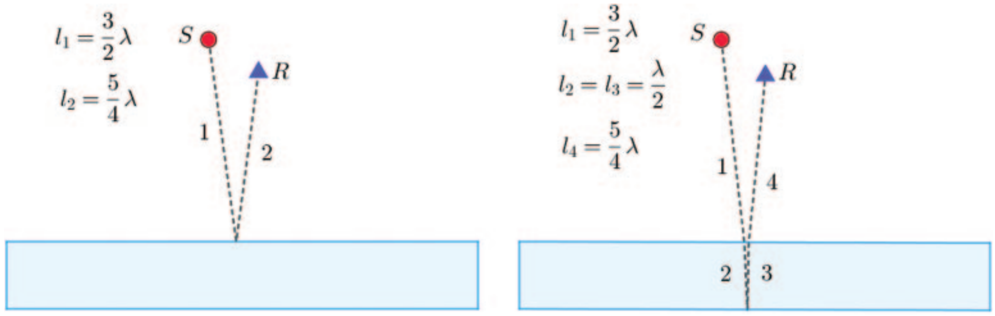

Fig. 1. – Part of the worksheet on thin film reflection. Students are required to solve the problem of finding the reflection probability for two different wavelengths of light. The incidence of light is considered as approximately normal to the surface, and multiple internal reflections are ignored (but the issue is discussed with students).

the physics problem given. At the end of the lesson, the individual sheets filled by students were given to the teacher and constituted a main data source for evaluating individual learning outcomes.

## 3. – Context

The experimentation involved four classes (around 80 students) of the fourth year of science-intensive high school ("Liceo Scientifico") Taramelli in Pavia. The duration was about 10–11 hours in each class, in the months of November and December 2017. The experimentation was conducted by the author of this article, which was not the regular physics teacher in the classes. At the end of the experimentation, each physics teacher of the classes involved decided whether and how to further evaluate students' results on the subject of the experimentation. On a qualitative level, students' participation and interest during the experimentation was generally high, and they participated to the proposed activities with interest and positive attitude.

## 4. – Results

Data collected during the experimentation includes the laboratory sheets and reports, worksheets used by students for exercises and problems, and the notes taken during the activities on students' comments, questions and difficulties. Here we sintethically report the main indications we gathered from the analysis of such data:

- The sequence was appropriate for the age of the students, both in its formal and conceptual content. The analysis of the exercise and problem worksheets in most cases shows a majority of correct and complete answers to each of the questions

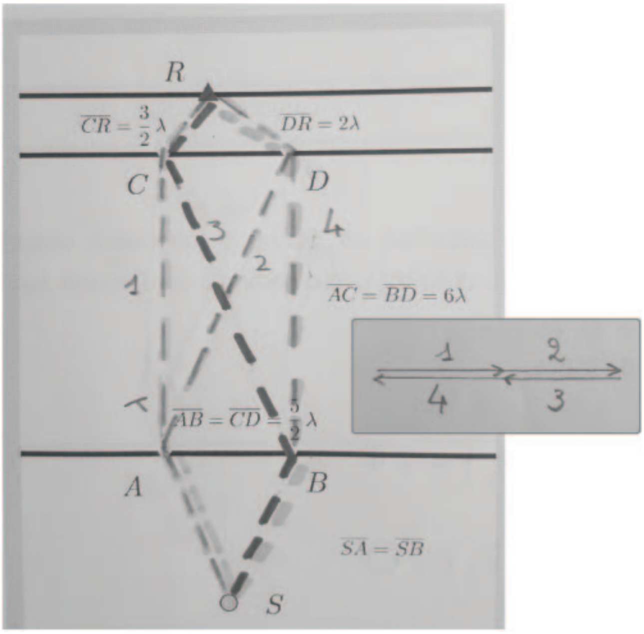

Fig. 2. – Pictures produced by a student pair of the possible paths and (in the inset) phasors for computing the detection probability for a problem of photon interference from successive slits.

proposed; this is especially true in the case when students are required to work in pairs (problem worksheets used in lesson 5, see fig. 2). When working individually, students may have difficulties with the details of Feynman rules (*e.g.*, the phase shifts due to reflection) but appear to master reasonably well the general conceptual meaning of the approach. For the final worksheet on thin film reflection, completed by students individually in lesson 7, the percentage of answers displaying a general understanding of the model was 73%, while answers completely correct in all details were 49% (sample $N = 76$ students).

- Students were in general well disposed towards a probabilistic/statistic reading of natural phenomena and the new language introduced by Feynman's approach. However, sporadic cases of strong epistemological resistance were observed. For example, a student objected that she did not consider it sufficient to learn how to compute probabilities, but she desired instead a deeper understanding of why Nature behaves in such a way. Since the epistemological aspects of quantum theory were not the focus of the present experimentations, students were not encouraged to entertain in philosophical debates for long times. Cases of tenacious opposition to the tenets of quantum physics by secondary school students were observed and discussed recently in the physics education community [18, 19].

- The objective of providing a comprehensive, integrated perspective on the theories of light appears to have been partially met. Students understand and correctly describe the probabilistic-statistic relationship between the wave and photon theories. However, ray optics appears to be too far in the past in their school curriculum (they studied it two years earlier) to recall or apply correctly its rules, and to grasp the connection with Feynman's approach. This is due, in part, to the fact that in the Italian tradition geometrical optics is very rarely taught as stemming from Fermat's principle but more often as a phenomenological theory. Future experimentation will explore strategies for further reinforcing the student's understanding of the relationship between the sum over paths approach and ray optics.

## 5. – Conclusions

The proposed teaching-learning sequence proved to be adequate for students of the fourth year of secondary school, both on a formal and on a conceptual level. The compared analysis of the same phenomena using either the wave or photon concepts was productive for students, as it helped them consider critically the relationship between model and reality, and build a more integrated and consistent mental model of light. Helping students to integrate also ray optics in such perspective appears to be more problematic, especially since most Italian students in advanced secondary school recall geometrical optics as a collection of empirical rules concerning lenses and mirrors, rather than as a simple and elegant theory based on a minimum principle.

In informal discussions, students showed appreciation of two main aspects of the experimentation: first, the general structure of the activities, which always required them to take an active role; second, the proposed exercises and problems, which helped them understand a subject which would probably have been too abstract for them otherwise. A particularly successful strategy appears to have been to require students to first approach exercises and problems in groups or pairs; in this way students were allowed to discuss their difficulties among peers, and were encouraged to develop the first rudiments of a quantum language.

In conclusion, data from our experimentation indicates that the strategy of introducing an elementary quantum theory of the photon, using Feynman's approach, already at the level of the fourth year of secondary school, in parallel with the wave theory of light, may be viable and productive. Of course, it would be desirable to operate a follow up study on the classes involved, with the objective of determining how such an early introduction would impact their understanding of quantum theory, which, according to the official curriculum, is to be delivered the following year. Unfortunately, due to external constraints, such follow up study will not be possible.

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