Determining rock erodibility parameter « relative block structure » for non-perpendicular hydraulic flow

Lamine Boumaiza[1], Ali Saeidi[1] & Marco Quirion[2]
[1] *Université du Québec à Chicoutimi, Chicoutimi, Québec, Canada*
[2] *Hydro-Québec, Montréal, Québec, Canada*

ABSTRACT
The most commonly used method for assessing the hydraulic erodibility of rock is the Annandale's method. This method is based on a correlation between the force of flowing water and the capacity of rock resistance. This capacity is evaluated using the Kirsten's index which was initially developed to evaluate the excavatability of earth materials. This index is determined according to certain geomechanical factors, such as the compressive strength of intact rock, the rock block size, the discontinuities shear strength and the relative block structure. This last characteristic represents the required effort to erode the rock and, it can be quantified considering the shape and orientation of the blocks. To determine the relative rock structure in the field, the dip and dip direction of closer spaced joint set, as well as the ratio of joints spacing are required. It is found that the Kirsten's concept can be applied only when the direction of flow is perpendicular to the strike of closer spaced joint set. Adjustments are introduced to Kirsten's initial concept concerning the relative block structure parameter. Thus, two equations are proposed which were used to propose a rating for the relative block structure parameter for a case example with non-perpendicular flow.

RÉSUMÉ
La méthode la plus utilisée en pratique pour l'évaluation de l'érodabilité hydraulique du roc est celle d'Annandale. Celle-ci se base sur une corrélation entre la force érosive de l'eau et la capacité de résistance du roc. Cette capacité est évaluée à l'aide de l'indice de Kirsten initialement développé pour évaluer l'excavabilité des matériaux. Cet indice est déterminé en fonction de certaines caractéristiques géomécaniques telles que la résistance matricielle de la roche intacte, la taille des blocs, la résistance au cisaillement des discontinuités et la structure relative des blocs. Cette dernière caractéristique représente l'effort requis pour que le roc soit érodé et, elle peut être quantifiée en considérant la forme et l'orientation des blocs. En pratique, elle se détermine en fonction du pendage et la direction du pendage de la famille des joints les moins espacés, ainsi que le ratio d'espacement des joints. Il est observé que le concept de Kirsten n'est valide que lorsque l'écoulement est perpendiculaire à l'azimut de la famille des joints les moins espacés. Des ajustements sont introduits sur le concept initial de Kirsten concernant la structure relative des blocs. Ainsi, deux équations sont proposées et utilisées pour générer des pondérations de la structure relative des blocs pour un exemple de cas avec un écoulement non-perpendiculaire.

1 INTRODUCTION

Annandale's method (Annandale 1995, 2006) is the most commonly used method for assessing the hydraulic erodibility of earth materials (Mörén and Sjöberg 2007, Hahn and Drain 2010, Pells et al. 2014, Laugier et al. 2015, Rock 2015, Castillo and Carrillo 2016). This method is based on a correlation between the erosive force of flowing water, namely the available hydraulic stream power, and the capacity of rock to resist the flow energy. This capacity is evaluated using the Kirsten's index (Kirsten 1982, 1988), which was initially developed to evaluate the excavatability of earth materials, but has since been adopted to assess the hydraulic erodibility of earth materials. For rock material, Kirsten's index (*N*) is determined according to certain geomechanical factors related to the intact rock and the rock mass, such as the compressive strength of intact rock (Ms), the rock block size (Kb), the discontinuities shear strength (Kd) and the relative block structure (Js). The interest of using Kirsten's index was initially mentioned at a symposium focused on rock mass classification systems (Kirkaldie 1988), where it was argued that the processes of mechanical excavatability and hydraulic erodibility of earth materials could be considered as similar processes (Moore and Kirsten 1988). Since then, many researchers have analyzed the hydraulic erodibility of earth materials by using the excavatability index, where the « direction of excavation » considered in the Kirsten's index has been replaced by the « direction of flow » (Pitsiou 1990, Doog 1993, Annandale & Kirsten 1994, Moore et al. 1994, Van Schalkwyk et al. 1994, Annandale 1995, Kirsten et al. 2000). Hereinafter, the acronyms of « direction of excavation » and « direction of flow » are considered as synonymous term.

Kirsten considers the orientation of a block relative to the direction of flow as an important parameter to be considered in assessing the hydraulic erodibility of rock (Pells 2016). Thus, Kirsten has included the Js parameter in his index, which corresponding to the required effort to move a rock block from the rock mass. It was developed mathematically by considering the shape of the blocks, as well as their orientation relative to the direction of excavation. In practice, as presented by Kirsten (1982), the Js parameter is determined as a function of dip and dip direction of the rock units, as well as the ratio of joint spacing (RJS) which corresponding to ratio of the length and width of block. It should be noted that Kirsten's concept (Kirsten 1982) is perfectly valid only when the rock blocks are oriented in the same direction of flow. However, in practice, the Kirsten's index is applied for all cases by assuming a certain imprecision on erodibility assessment. This paper describes an adjustment introduced on the Kirsten's original concept to consider cases with variable flow direction.

2 LIMITATION OF THE KIRSTEN'S CONCEPT

The Kirsten's concept (Kirsten 1982) considers that the geological formation is mainly fractured by two intersecting joint sets, where an angle of 90° is kept between the planes of the two joint sets (orthogonal fractured system). In practice, the dip angle of the closer spaced joint set and its dip direction relative to the direction of flow are respectively used to determine the Js values (Kirsten 1982, 1988). The dip angle is between 0° and 90°, while the dip direction is determined relative to the direction of flow. In the stereographic example shown in Figure 1, the direction of flow is 270°. If the closer spaced joint set has a dip direction between 180° (270° - 90°) and 360° (270° + 90°), the block is considered to be in the same direction as that of flow. Otherwise, it is against direction of flow. If the closer spaced joint set in Figure 1 is the first joint set, the block will be taken as in the direction of flow. Subsequently, the dip of closer spaced joint and RJS are required also to determine the Js value.

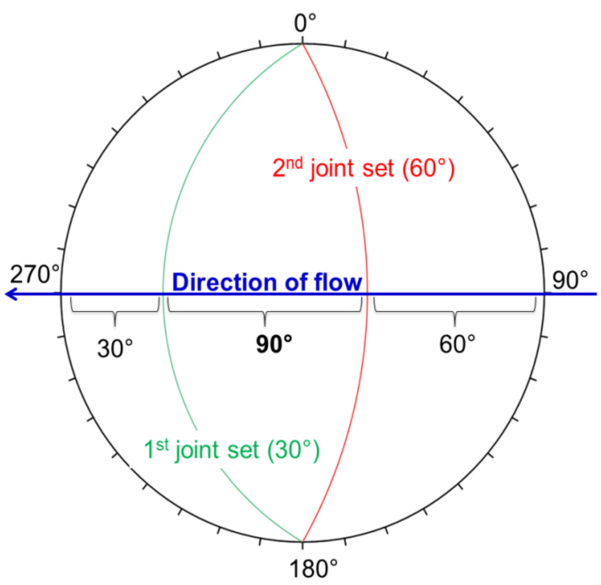


Figure 1. Representation of perpendicular flow

In Figure 2, the two joint sets constitute an orthogonal fractured system. The dip and dip direction of the first joint set are 30° and 270°, respectively; those of the second joint set are 60° and 90°, respectively. The first joint set is considered as the closer spaced joint set, while the direction of flow is 320°. To determine the Js value for such cases, Kirsten suggests taking the apparent dip of the closer spaced joint set, in the vertical plane containing the direction of flow, instead of the true dip of closer spaced joint set. The apparent dip used to determine Js would therefore be 20° (Figure 2). However, it is found that the angle between the two intersecting joint set on the plane containing the direction of flow (this angle is indicated hereinafter as α) is 112° as shown in Figure 1. Remembering that the Js value, when the dip is 20°, was initially proposed by Kirsten with α = 90° (orthogonal fractured system), it is found that Kirsten's concept can only occur when the direction of flow is perpendicular to the strike of the closer spaced joint set (perpendicular flow). Consequently, it is considered valuable to consider the change of α angle along the vertical plane containing the direction of flow to determine the Js value for non-perpendicular flow. Such consideration could improve the global erodibility assessment.

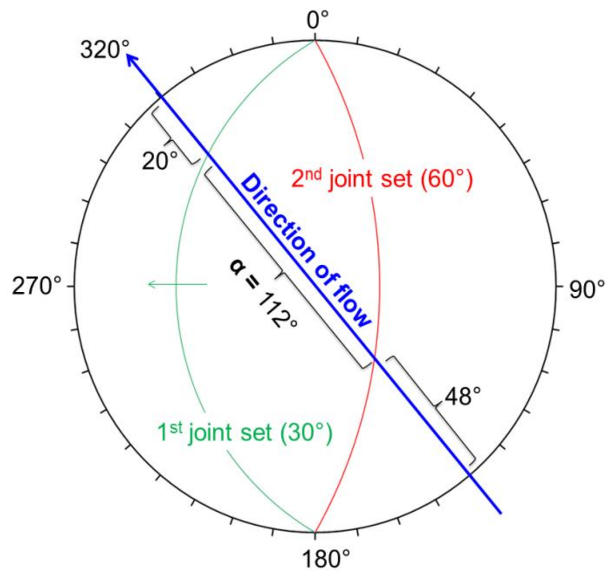


Figure 2. Representation of non-perpendicular flow

3 PROPOSED EQUATIONS FOR DETERMINING Js

According to Kirsten (1982), the relative orientation of blocks and the spacing of joints affect both the possibility of penetrating the ground surface and that of dislodging the individual blocks. Accordingly, Kirsten (1982) determined the effect of orientation and shape of block on the excavatability process by considering the kinematic possibility of penetration (K_p) and kinematic possibility of dislodgment (K_d). To determine the values of Js for non-perpendicular flow, the concepts of K_p and K_d proposed by Kirsten (1982) are adopted. These concepts can be used for α angle deferent to 90°. It should be noted that no change is introduced to the original concept of K_p. However, adjustment is introduced to that of K_d. The followed sub-sections describe the adjusted concept of K_d and give the equations to determine Js.

3.1 Principle of the adjusted concept

Considering that the geological formation is mainly fractured by two intersecting joint sets, the block dislodging act occurs according to the digging process into angle of the first joint set and followed by the riding process on the angle of the second joint set (Kirsten 1982). In the representation of Figure 3, the planes of the two joint sets are plotted in blue and red colors. When the block is oriented in the direction of excavation, Kirsten considered the digging angle θ to be positive angle, while the riding angle Ψ is determined by adding an angle of 90° to θ angle (e.g. θ = 30°, thus Ψ = 30° + 90° = 120°). When the block is oriented against the direction of excavation, Kirsten considered the digging angle θ to be negative, while riding angle Ψ is determined by again adding an angle of 90° to θ angle (e.g. θ = -30°, thus Ψ = -30° + 90° = 60°).

According to Figure 3, when the block is oriented in or against the direction of excavation, the joint spacing S_Ψ is always greater than the joint spacing S_θ. Therefore, the RJS (S_Ψ/S_θ) is of the same order for both blocks, although their orientations differ (Figure 3). This explains why Kirsten (1982) always used the same fixed RJS (1 = 1/1, 2 = 2/1, 4 = 4/1, 8 = 8/1) for both directions of the block (in and against the direction of excavation). However, in the Kirsten's initial concept (Kirsten 1982), where the block is oriented against the direction of excavation, the joint spacing S_Ψ is smaller than the joint spacing S_θ. For this, the corresponding RJS should not be as fixed RJS of 1, 2, 4 or 8. In addition, Kirsten (1982) states that for an RJS > 0.125, Js is determined as if the RJS = 0.125. This value would appear to be derived from a RJS of 1:8, but with a ratio of 1/8 although it must be a ratio of 8/1 = 8 as argued in Figure 3. This confusion has also been noted in Kirsten (1988) and USDA (1997). But, Annandale (1995, 2006) has corrected this confusion by indicating that beyond a RJS of 8 (1:8 = 8/1 = 8), Js could be considered as having a RJS of 8. However, the Kirsten's initial concept (Kirsten 1982) could be adjusted. Indeed, when the block is oriented against the direction of excavation, the digging angle would be Ψ (Figure 3), while Kirsten (1982) represents this angle as θ. Also, when the block is oriented in the direction of excavation, the digging angle would be θ as indicated in Figure 3. Consequently, two equations of K_d will be proposed according to the adjusted digging angles.

3.2 Proposed Kd equation when the block is oriented in direction of flow

The action of block dislodgement can be represented by a unitary horizontal force behind the block, while the latter is free to move in a perpendicular direction to the ground surface (Kirsten 1982). As a result, the kinematic possibility of dislodgement, as shown in Figure 4, can be obtained by the vector product of the principal dislodging force and the principal degree of freedom. These vectors can be decomposed into parallel coaxial components along the sides of the block (Kirsten 1982). The coaxial components are identified as A, B, B' and A' in Figure 4. The coaxial component, identified as A in Figure 4 is in opposite direction to the coaxial component, identified as A' in Figure 4. Accordingly, K_d can be expressed as a function of the other two coaxial components, identified as B and B' in Figure 4. The unknown angle α in Figure 4, as well as the considered coaxial components B and B' can be determined according to the equations numbered 1.

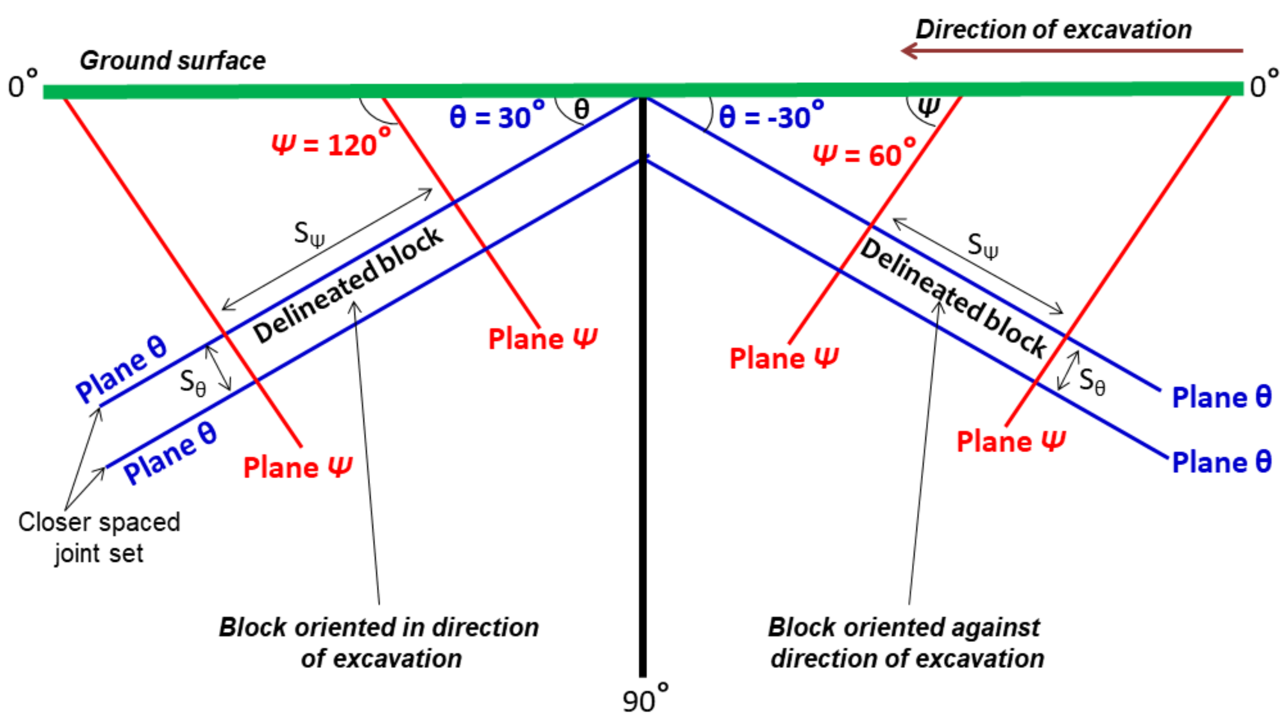


Figure 3. Concept of a delineated block oriented in and against the direction of excavation

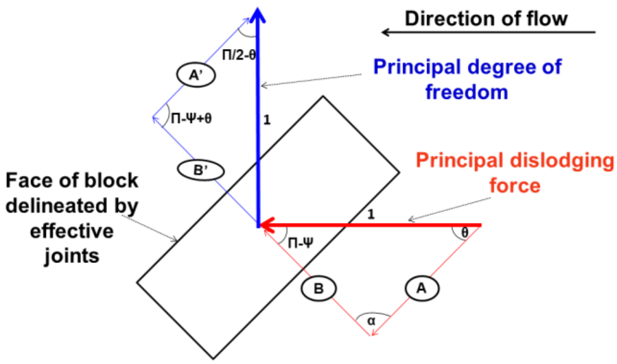


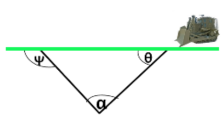
Figure 4. Coaxial components for a block oriented in the direction of flow.

$$\left.\begin{aligned} \alpha &= \psi - \theta \\ B &= \frac{\sin\theta}{\sin\,(\psi-\theta)} \\ B' &= \frac{\sin\,(\frac{\pi}{2}-\theta)}{\sin(\psi-\theta)} \end{aligned}\right\} \qquad [1]$$

The final equation of K_d when the block is oriented in the direction of flow is given by the product of the two components B and B' (Equation 2).

$$K_d = \frac{\sin\theta\ \cos\theta}{\sin^2(\psi-\theta)} \qquad [2]$$

It should be mentioned that Equation 2 can be applied under the following conditions:



$$\rightarrow \quad \left.\begin{aligned} \psi &= \alpha + \theta \\ 0° < \theta &< 90° \\ 90° < \psi &< 180° \end{aligned}\right\} \qquad [3]$$

3.3 Proposed K_d equation when the block is oriented against direction of flow

The concept of a block oriented against the direction of flow is shown in Figure 5.

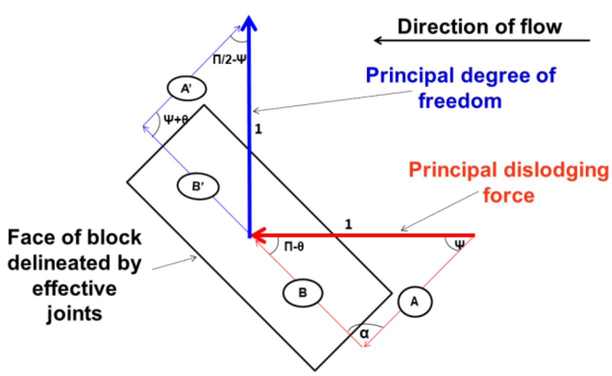


Figure 5. Coaxial components for a block oriented against the direction of flow.

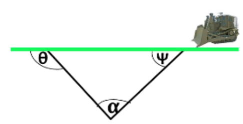
According to the determination of the coaxial components of the principal dislodging force and the principal degree of freedom for a block oriented in the direction of flow (Figure 5), the unknown angle α, as well as the coaxial components B and B' could be determined according to following expressions:

$$\left.\begin{aligned} \alpha &= \theta - \psi \\ B &= \frac{\sin\psi}{\sin\,(\theta-\psi)} \\ B' &= \frac{\sin\,(\frac{\pi}{2}-\psi)}{\sin(\theta-\psi)} \end{aligned}\right\} \qquad [4]$$

The final equation of K_d when the block is oriented against the direction of flow is given by the product of the two components B and B' (Equation 5).

$$K_d = \frac{\sin\psi\ \cos\psi}{\sin^2(\theta-\psi)} \qquad [5]$$

It should be mentioned that Equation 5 can be applied under the following conditions:



$$\rightarrow \quad \left.\begin{aligned} \psi &= \theta - \alpha \\ 90° < \theta &< 180° \\ 0° < \psi &< 90° \end{aligned}\right\} \qquad [6]$$

3.4 Proposed equations for determining Js

Considering that the required effort to move block is equal to 1 minus the kinematic possibility as proposed by Kirsten (1982), Js values can be determined by the proposed Equations 7 and 8. Equation 7 is applied when the blocks are oriented in the direction of flow (Equation 2 of K_d is introduced), while Equation 8 is used when the blocks are oriented against the direction of flow (Equation 5 of K_d is introduced). It should be noted that no change is introduced to the equation of K_p (first term of both equations 7 and 8).

$$J_s = \left[1 - \frac{r\tan\theta + \tan\psi}{a\,(r+1)}\right].\left[1 - \frac{\sin\theta\ \cos\theta}{\sin^2(\psi-\theta)}\right] \qquad [7]$$

$$J_s = \left[1 - \frac{r\tan\theta + \tan\psi}{a\,(r+1)}\right].\left[1 - \frac{\sin\psi\ \cos\psi}{\sin^2(\theta-\psi)}\right] \qquad [8]$$

The prefix *r* in both equations 7 and 8 indicate the RJS (S_Ψ/S_θ). For its part, the prefix *a* is a constant value of 5 that has been proposed by considering the efficiency of the excavatability process relative to direction of excavation (Kirsten, 2016 pers. comm.).

4 DETERMINING J_S VALUES

For non-perpendicular flow, α should be > 90° (from 91° to 180°). To determine the role of the α > 90° on Js, a series of angles can be evaluated (100°, 110°, 120°, 130°, 140°, 150°, 160°, 170° and 180°). However, planes are usually considered, in geomechanics, as being parallel when the angle between the planes is < 20°. Examples of this include the angle between the joint's dip direction and the direction of excavation when determining the orientation factor in the rock mass classification system (RMR) of Bieniawski (1989) and the angle of the joint's dip direction and the direction of slope surface during the analysis of possible planar failure (Wyllie and Mah 2004). Such situation in our study can be occurred when the plane containing the direction of flow is parallel to the strike of the closer spaced joint set. A check-up is performed with DIPS software (Rocsciences 2017) for all non-perpendicular flow with variable orthogonal dips angles. Accordingly, α angle for non-perpendicular flow can be limited to 150°, assuming that for α >150°, the plane containing the direction of flow constitute a parallel plane to the strike of the closer spaced joint set. In this article, just the case of α = 100° is studied. Although the dip of the closer spaced joint set can vary from 0° to 90°, to keep the same considerations as Kirsten (1982), only dip angles used by Kirsten (1982) are taken as being part of the adjustment process. These dips correspond to 5°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 85° and 90°.

The behavior of Js as a function of θ (considered as the dip of the closer spaced joint set) when α = 100° is shown in Figure 6. When the block is oriented in the direction of flow, θ ranges from 0° to 90° whereas this ranges from 90° to 180° when the block is oriented against the direction of flow (Equation 5). However, the latter angles are represented as angles varying from 0° to 90° marked with a negative sign. For example, a θ = 150° corresponds to an angle of 30° (θ = 180° - 150°) in Figure 6. In this Figure 6, Js is not calculated for a θ ≥ 80°. This is explained by a non-favorable geometry applying to the conditions as indicated in Equations 3 and 6. To obtain the Js values, the curves showed in figure 6 are adjusted with the same method followed by Kirsten (Moore and Kirsten 1988). It should be noted that no determination is performed when θ = 0° or 90° as Kirsten considered K_d and K_p to be zero when the joints are sub-horizontal (dip = 0°) or sub-vertical (dip = 90°). For these cases, Kirsten assigned a *Js* value of 1 for the four values of RJS. Indeed, when K_p and K_d = 0, Equations 7 and 8 will have *Js* as the product of 1 × 1 = 1, explaining the *Js* values of 1 when dips are 0° or 90°. For excavatability, Kirsten posits that the ground would not be excavated when *Js* = 1, as the sub-horizontal or sub-vertical joints, relative to the ground surface, would not constitute a situation favorable for excavation. On the other hand, a ground characterized by a *Js* of 1 would have a representative value of its excavatability being determined according to the factors included in Kirsten's index. However, it is practically non-excavatable. Accordingly, the curve adjusting process was undertaken by considering that the curves must be plotted with *Js* of 1 when the dip is 0° or 90°. The final adopted Js values are presented in Table 1.

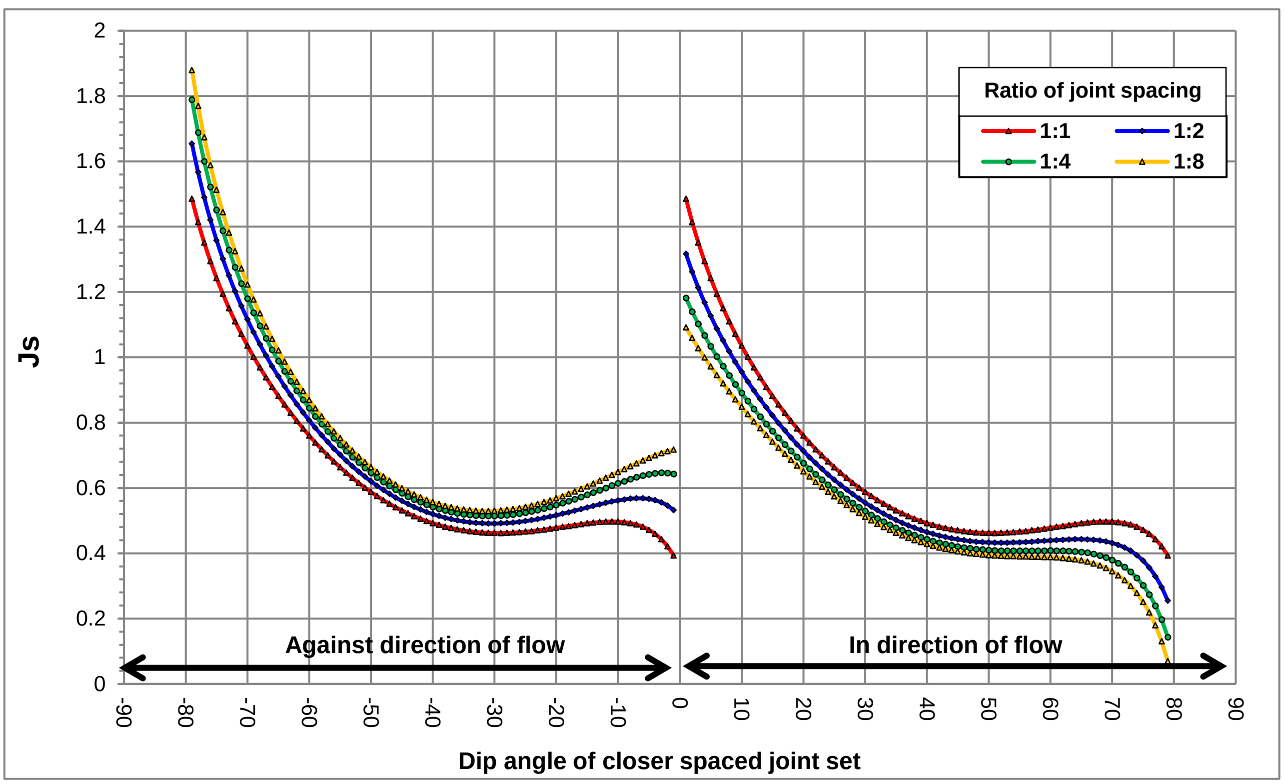


Figure 6. Behavior of Js when α = 100°

Table 1. Js values when α = 100°

	Dip of closer spaced joint set[1]	α Angle 100° Ratio of Joint Spacing 1:1	1:2	1:4	1:8
Dip direction of closer spaced Joint set is in the direction of flow	**70°**	0.50	0.42	0.38	0.34
	60°	0.46	0.41	0.38	0.35
	50°	0.46	0.42	0.40	0.37
	40°	0.49	0.45	0.43	0.41
	30°	0.59	0.55	0.52	0.51
	20°	0.76	0.71	0.67	0.65
	10°	1.04	0.99	0.91	0.85
	5°	1.24	1.13	1.03	0.97
Dip direction of closer spaced joint set is against the direction of flow	**5°**	0.68	0.72	0.75	0.79
	10°	0.50	0.60	0.64	0.68
	20°	0.46	0.52	0.55	0.57
	30°	0.46	0.49	0.51	0.53
	40°	0.49	0.52	0.54	0.56
	50°	0.59	0.62	0.65	0.66
	60°	0.76	0.81	0.84	0.87
	70°	1.04	1.12	1.18	1.22

1: Apparent dip angle of closer spaced joint set in vertical plane containing direction of flow.

5 CASES STUDIES APPLICATION

Three cases examined by Pells (2016), that were originally studied by Van Schalkwyk et al. (1994), are analyzed according to *Js* values proposed in this work for a non-perpendicular flow. The three cases studies are from the spillways of dams located in South Africa: the rock mass section *8E-1* of the Mokolo Dam, the rock mass section *9E-2* of the Hartebeespoort Dam and the rock mass section *13E-3* of the Marico-Bosved Dam. The data for the examined sections as related to Kirsten's index factors, include the compressive strength of intact rock (*Ms*), the rock block size (*Kb*), the discontinuities shear strength (*Kd*) and the relative block structure (*Js*) are presented in (Table 2).

The *Js* values adopted by Van Schalkwyk et al. (1994) assumed a perpendicular flow (α = 90°). From the adopted *Js* value of each examined section, the RJS, dip direction of closer spaced joint set relative to the direction of flow and dip of closer spaced joint set are determined according to Kirsten's rating (Kirsten 1982). This information is then used to calculate the corresponding *Js* when α = 100° by considering the proposed *Js* rating as presented in Table 2. The corresponding *Js* values are presented in Table 2. Subsequently, Kirsten's index (*N*) is calculated according to the corresponding *Js* values (Table 3). The values obtained for Kirsten's index for the three examined cases studies, calculated as a function of α = 100°, are converted into required hydraulic stream power (P_r) using the Annandale's abacus (Annandale 1995, 2006). Note that all examined cases studies of Annandale (1995, 2006) are considered as obtained with perpendicular flow. The determined required hydraulic stream power, for the three examined cases studies, are presented in Table 3.

Table 2. Data of the analyzed case studies

Case study	*Ms*	*Kb*	*Kd*	*Js* (90°)	r[1]-Dir.[2]-Dip[3]	*Js* (100°)
8E-1	140	25.45	0.94	0.81	2-against-5°	0.72
9E-2	70	16.47	1.00	1.20	2-in-5°	1.13
13E-3	140	26.95	1.68	0.69	4-against-10°	0.64

Note :
*1: **r** is the ratio of joint spacing*
*2: **Direction** is dip direction of closer spaced joint set relative to the direction of flow. It is in or against the direction of flow*
*3: **Dip** is the dip angle of closer spaced joint set*

Table 3. Determined required hydraulic stream power

Case study	α = 90° *N*	α = 90° P_r	α = 100° *N*	α = 100° P_r
8E-1	2713	376	2411	344
9E-2	1380	226	1303	217
13E-3	4752	572	4056	508

According to the obtained results, the required hydraulic stream power, using α = 100° for the *13-E3*, *8-E1* and *9-E2* cases studies, is reduced, when compared to the required hydraulic stream power when α = 90° (Table 3). Although the *13E-3* rock mass is characterized by the highest factor values of *Ms*, *Kb* and *Kd* (Table 2), it is marked with more decreasing of the required hydraulic stream power. This is explained by the effect of *Js*. Indeed, the lowest *Js* values, according to α, are noted for rock mass *13E-3* (Table 2). These findings highlight the importance of considering α when determining Kirsten's index to determine the required hydraulic stream power.

6 CONCLUSION

Adjustments are introduced to Kirsten's initial concept concerning the relative block structure parameter. Thus, two equations are proposed to determine the relative block structure parameter when the angle between the planes of the two joint sets, on the vertical plane containing the direction of flow, is defer to the 90° angle considered by Kirsten. The use of the two proposed equations, by varying the angle between the two joint sets, makes it possible to propose a rating for the relative block structure parameter when direction of flow is non perpendicular to the strike of closer spaced joint set. The non-perpendicular flow reflects cases that can be potentially found in the field where rock's vulnerability to erosion will therefore differ if assuming a perpendicular flow. Accordingly, the proposed rating of Js for non-perpendicular flow can provide a more accurate assessment of the hydraulic erodibility of rock.

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