Optimal capital structure and the debtholder-manager conflicts of interests: a management decision model

Title – Optimal Capital Structure and the Debtholder-Manager Conflicts of Interests: a management decision model

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Abstract

Purpose - The issuance of debt as a corporate governance mechanism introduces a different agency problem, the asset substitution problem noted as the agency cost of debt. Thus, there is a recognised need for models that can resolve the agency problem between the debtholder and the manager who acts on behalf of the shareholder, leading to efficient financial management systems and enhanced firm value. The purpose of this paper is to model the debtholder-manager agency problem as a dynamic game and resolve the conflicts of interests.

Design/methodology/approach - This paper uses the differential game framework to analyse the incongruity of interests between the debtholder and the manager as a non-cooperative dynamic game and determine the optimal capital structure which minimizes the marginal agency cost of debt and further resolves the conflicts of interests as a cooperative game via a Pareto-efficient outcome. The model is applied to a case study company.

Findings - The optimal capital structure required to minimize the marginal cost of the agency problem is a higher use of debt, lower cost of equity and withheld capital distributions. The debtholder is also able to enforce cooperation from the manager by providing a lower and stable cost of debt and greater debt facility in the overtime framework.

Originality/value - The study develops a model based on the integrated issues of capital structure, corporate governance, agency problems and differential game theory and applies the differential game approach to minimize the agency problem between the debtholder and the manager.

Keywords: Modelling, Simulation

1 Introduction

The optimal capital structure is a classic issue in corporate finance and management. There has been a lot of contributions to observe the effect of the asset substitution moral hazard problem on the firm's capital structure. However, there is a need for more research and development of models to mitigate this problem in a dynamic framework. Implementing good corporate governance mechanisms and contracts in determining optimal capital structure will result in efficient financial management by minimizing the effects of agency problems.

The issuance of debt as a corporate governance mechanism (Jensen & Meckling 1976) introduces the agency cost of debt known as the asset substitution problem (Green, Richard C. & Talmor 1986). Due to the limited liability of shareholders, debt finance provides shareholders with an impetus to select riskier projects to maximize his value against the preferences of debtholders (Jensen & Meckling 1976). The agency cost of asset substitution results in the reduction of a company's total firm value due to consequences of risk-shifting (Vanden 2009). This implies that the company's 'first-best' exercise policy of maximizing the firm value is replaced with the 'second-best' exercise policy of maximizing equity value (Wang, H, Xu & Yang 2018). The problem is also significant because a company's payout policy is influenced by the extent of the agency conflicts between its shareholders and debtholder (Lepetit et al. 2018). Although agency cost does not consistently increase with the use of debt (Mao 2003), higher tax rates exacerbate the risk-shifting incentives and debt-overhang problem (Wang, H, Xu & Yang 2018). The agency conflicts of interests are also worsened by director interlocks (Ramaswamy 2019).

Debt contract can be perfectly represented as an incentive contract. In the incentive contract, the principal induces the agent by taking penal actions when the agent commits a sub-optimal effort. Similarly, in the debt contract, the debtholder legally obliges the firm to enforce its interest payments (cost of debt) irrespective of the firm's financial position. The debtholder can impose a penalty on the firm and its manager when cheating arises (Shah & Abdul-Majid 2019).

A manager encounters the following significant issues: What is the optimal level of coupon or cost of debt and cost of equity that minimizes the marginal effect of the agency cost of debt on the firm? What is the optimal capital structure required to minimize the impact of the debtholder's penal actions on the firm in the case of non-cooperation? What are the incentives provided by the debtholder to discourage risk-shifting? In contrast, the debtholder faces how to maximise his investments in the company and the design of sufficient and sustainable incentives to induce the manager to protect his interest in the firm.

In this paper, we provide an analysis to observe the impact of moral hazard on the firm's capital structure as a non-cooperative game and further obtain a Pareto-efficient outcome to minimize the agency conflicts of interests between the debtholder and the manager in a dynamic framework. Following Beladi and Quijano (2013), in this study, the manager is assumed to act on behalf of the shareholder due to the firm's indebtedness.

The goals of the study are to a) analyse the debtholder-manager dynamic agency relationship as a non-cooperative game using the Nash open-loop and feedback equilibrium outcomes, b) obtain the Pareto-efficient outcomes for the debtholder and the manager as well as the optimal capital structure for the firm via differential game model, and c) design contracts and strategies to minimize the agency conflicts of interests between the debtholder and the manager for specifying the optimal capital structure of a company in a dynamic framework.

The paper is organized as follows, Section 1 and 2 introduces and provides background for the study. Section 3 discusses the material and methods. This includes the Nash equilibrium game analysis and Pareto analysis of the model. Section 4 discusses the implication of the study for optimal capital structure, corporate governance and dynamic agency theory. Finally, Section 5 summarizes and concludes the study.

2 Background

This study is linked to several clusters of literature in management and finance. The first cluster are the studies of optimal capital structure such as Fischer, Heinkel and Zechner (1989), Leland (1994, 1998), Elton and Gruber (1974), Goldstein, Ju and Leland (2001), Titman and Tsyplakov (2007), Tian (2016), Schorr and Lips (2019), to name few. These studies examine the optimal static/dynamic capital structure for a firm and were limited to results for a shareholder. The major difference in this new study is that it examines the determinant of capital structure arising from the agency costs of debts due to the debtholder-manager conflicts of interests, while the previous studies do not. Their studies do not consider the impact of conflicts of interests (due to the use of debts) on optimal capital structure.

The second cluster of literature fundamental to this study introduces the moral hazard problem called the asset substitution or risk-shifting problem such as Jensen and Meckling (1976), Heinrich (2002), and Wang, H, Xu and Yang (2018). Jensen and Meckling (1976) establish that the use of external financing in the form of debt can modify the optimal operating strategy of a firm by giving shareholders an impetus to select riskier projects against the preferences of debtholders. Moreover, the payoff of a shareholder is convex in the profit stream of an indebted firm whereas the payoff of a debtholder is concave (Heinrich 2002). The debtholder anticipates more risk-shifting due to the manager's increased equity assets in the company, and hence imposes a higher cost on the firm which alters its required optimal capital structure (Beladi & Quijano 2013). This creates a problem for the firm and the manager.

The third cluster are studies that have evaluated the significance of the asset substitution problem on firm value. This includes via theoretical frameworks Leland (1998) and Ericsson (2000), simulation methods Parrino and Weisbach (1999), managerial surveys Graham and Harvey (2001), empirical research Eisdorfer (2008), optimization Moreno-Bromberg and Rochet (2018), Lepetit et al. (2018), etc. The difference between these previous studies and our new study is that the former seeks to establish the impact of the risk-shifting problem on the firm value but does not resolve the problem.

The last cluster relevant to this study examines the elimination of cost or the impact of asset substitution in a static or dynamic framework. The first group under this cluster are literature focused on minimizing the negative impact of asset substitution on the debtholder while maximizing the opportunistic benefits to the company such as Childs, Mauer and Ott (2005) and Vanden (2009). Vanden (2009) and Childs, Mauer and Ott (2005) suggest the use of structured financing, identified as a company having the adequate financial flexibility to continuously manage its degree of short-term debts. These studies seek an internal solution to minimize the resultant loss on total firm value and consequences for the debtholder but retain the opportunistic benefits for the firm. The model by Vanden is also limited because it eliminates the tax effect in the model design.

The second group are literature focused on seeking agency-based approach to minimize the asset substitution problem. These strategies seek to defend the interests of the debtholder. These studies include Myers (1977), Green, Richard C (1984), Hennessy and Tserlukevich (2008), Burkart and Ellingsen (2004), (Chod 2015). Myers (1977) recommends that the productive life of a company's asset should be evened with the debt maturity offered by the debtholder. However, this does not disincentives a manager from risk-shifting. Smith Jr and Warner (1979) and Wang, J (2017) recommend debt covenants. The drawback is that debt covenants may limit the firm's level of investment as a covenant cannot fully distinguish between a non-rewarding and a rewarding investment. Thus covenant may unduly impede a good investment (Edmans & Liu 2011). Brander and Poitevin (1992) and Edmans and Liu (2011) examine managerial compensation contracts; however, with a significant assumption that the manager does not take actions on behalf of the shareholder. In this new study, we assume that the manager takes actions on behalf of the shareholder due to the firm's indebtedness. Further, Green, Richard C (1984) suggests replacing straight debt financing with the use of convertible debt financing, while the convex and concave domains of the debt contract are stabilized to present the security locally in the form of equity. Hennessy and Tserlukevich (2008) prove this to be an unrealistic solution in a dynamic context because equity remains risk-loving as a firm tends to bankruptcy. Burkart and Ellingsen (2004) and (Chod 2015) propose trade credit as an agency-based measure to mitigate asset substitution. However, their result is limited because it only favours the possibility of lending goods rather than lending cash, which is not always a realistic alternative for all companies. Short-term debt has been recommended as one panacea to the moral hazard problem of asset substitution because they are less reactive to the change in the company's asset (Barnea, Haugen & Senbet 1980). Moreover, it bridges the information gap between the debtholder and manager, since it spurs a frequent reporting by the manager on the company's performance and operating risk (Jun & Jen 2003). Contrarily, Lopez-Gracia and Mestre-Barberá (2015) find evidence that some Spanish Small-Medium Enterprises (SMEs) defer to long-term debt to moderate the conflict of interests between the manager and the debtholder. This current study improves this literature by developing a model that is flexible for analysis in both a long-term and short-term (debt maturity) period. The model is developed to enhance a long-term debt contract if the manager does not renege on the terms of the contract. Sudheer, Wang and Zou (2019) propose dual ownership can minimize the extent of covenants a company is bound by in its debt contract. If a debtholder simultaneously holds both equity and debt in company, this can minimize the incentive conflict by increasing the debtholder's monitoring scope and internalizing the conflict. The limitation of this proposition includes that; debtholders will not always seek an equity interest in a company, not all debt providers have the legal rights to buy equity interests, and not all firms will be willing to sell equity interests to its debt provider in order to avoid excess monitoring.

Finally, our paper is related to Liu et al. (2017), Antill and Grenadier (2019), Tran (2019), Sterman (2010). Liu et al. (2017) examine the impact of incomplete information on the optimal capital structure under a significant assumption of unobservable firm's growth rate. Our study is different because it considers the moral hazard problem of asset substitution. Antill and Grenadier (2019) analyse the debtholder-manager relationship; however, with a focus on a manager who deliberately selects a preferred time to default. In our study, the manager finds the contract and relationship of benefit to the firm. Tran (2019) furthers the literature on the use of debt covenant in addition to reputation-building as mechanisms to minimize the agency problem. Sterman (2010) examines system dynamics and decision-making between various agents in organisational design. The study noted that decision rules should align with managerial practices.

In this study, using a dynamic optimization approach, the debtholder selects optimal or equilibrium strategies as well as trigger strategies which induce the manager from risk-shifting once the debt contract is active. Similarly, the manager selects the optimal capital structure that minimizes the effect of the debtholder's penal actions on the firm.

Hence, the study employs corporate governance mechanisms to minimize the conflicts of interests between the debtholder and the manager and simultaneously optimizes the capital structure of the firm. This modelling work is helpful for managers in making optimal financing decisions as well as maximizing the debtholder relationship. Differential game theory is considered because of its suitability in analysing non-cooperative games as specified above and its use of mathematical optimization approach. Another advantage of differential game theory founded in system dynamics is that it has both rigorous mathematical foundations and it is also valuable for policy makers in solving crucial organisational problems (Sterman 2010). In a differential game, the objective of one decision maker, here as (debtholder and manager) impacts the objective of the other and hence the problem from the strategic interaction becomes a game (de Zeeuw 2014).

3 A Dynamic Principal-Agent Game Model for an Optimal Capital Structure

3.1 The model setup

In this section, we first specify a dynamic principal-agent model between the debtholder and the manager with the moral hazard problem for determining an optimal capital structure. The model incorporates the firm's capital structure in a continuous-time framework. The exogenous contract implies that the manager takes actions that are not in the best interest of the debtholder. We present underlying assumptions for tax environment, debt contract structure and the dynamic game problem. It is assumed that the company only issues limited-liability securities (loans), such as bilateral loans, etc.

The model development process is stated as follows:

I. Company's liquid reserve

The company's liquid reserve is significant because it covers the company's ongoing operating expenses such as its cost of debt or current finance cost. The liquid reserve $M(t)$ otherwise tagged as current asset evolves by adding the operating income $\beta S(t)$, the financial income $rM(t)$ (liquid reserve is assumed to be renumerated at rate r) minus cost of debt $c(t)$ and the cost of equity $l(t)$. $S(t)$ is the firm's productive asset and β is the asset payout rate. This is consistent with Moreno-Bromberg and Rochet (2018) and Vanden (2009). The evolution of $M(t)$ can be referred to as the company's net earnings stated as:

$$\dot{M}(t) = \beta S(t) + rM(t) - c(t) - l(t) \tag{1}$$

II. Tax and debt financing

A simple tax setting is considered. The firm's income is taxed at the effective tax rate θ, when $\theta > 0$, the use of debt shields some of the firm's income from tax charges. $c(t)$ denotes the cost of debt associated with the use of debt $D(t)$ at any time t. We assume that the company's value of debt changes throughout the lifecycle of the firm depending on its need for new financing in the next period. Thus, the capital structure is dynamic, a distinction from most capital structure models. However, based on the agency relationship between the debtholder and the manager, the debtholder promises to provide more or less debt facility to the firm depending on the manager's discretion to act opportunistically or not in a previous period. Hence, more debt facility may serve as an incentive. The company's value of debt is defined as its cost of debt plus its need for new debt, where α represents the ratio of the new value of debt to the existing value of debt.

$$\dot{D}(t) = \alpha D(t) + c(t) \quad (2)$$

III. Productive assets

The company's productive asset impacts the value of the company in any period. The debtholder may specify that the firm keeps a minimum value of productive assets throughout the contract. The value of the company's productive assets $S(t)$ is assumed to grow or decline exponentially depending on the difference between the riskless rate (r) and the payout rate (β):

$$S(t) = S_0 e^{(r-\beta)t}, \quad (3)$$

IV. Value of equity

In a company's statement of financial position, total equity $E(t)$ is defined as:

$$E(t) = M(t) + S(t) - D(t) \quad (4)$$

The equilibrium/optimal strategies selected by the manager and the debtholder impact the optimal outcomes of Equations (1 - 4) known as the **state variables**.

The exogenous debt contract

In the finite horizon differential game, the debtholder makes the first move by offering a debt contract to the manager. The manager initially accepts the terms and conditions of the contract but has incentives to renege, by taking unobservable actions (risk-shifting) that can cause it to default on his debt by maximizing $rM(t)$. This is called the moral hazard problem. The output process, $M(t), D(t), S(t)$ are observable by both the debtholder and the manager. Thus, the game is said to be one with **perfect information** but **incomplete information** because the preference of the manager is unknown to the debtholder. Since the debtholder does not provide the management fee, his incentive options to induce the manager are limited.

Differential game problem and utility functions

For simplicity, it is assumed that the firm's flow of earnings is discounted at a constant risk-free rate $\rho \geq 0$. The agency conflict of interests is formulated as a nonzero-sum game problem between two players. Next, we show the differential game problem for the manager and the debtholder, respectively.

3.1.1 *The formulation of the manager's (agent) problem:* The manager's objective is to minimize the company's cost of finance and maximize the value from its asset substitution. The weighted average cost of capital (WACC) is a compelling and extensively applied financial theory by both investors and company management. It is referred to as the cost of financing a company's activities, otherwise known as the cost of capital. This is the minimum return a company must realize on its capital asset base as anticipated by its providers of capital (Reilly & Wecker 1973). In addition, a lower cost of capital reduces the company's development and production costs (Sterman 2010). Therefore, the primary financial goal of a company is to find the optimal capital structure which yields the lowest weighted average cost of capital and maximizes the value of the company (Zelgalve & Bērzkalne 2011).

The WACC is, therefore set, as the cost function the risk-loving manager seeks to minimize, while maximizing the financial income of the company $rM(t)$, the rate of return on the company's liquid reserve from asset substitution. To achieve an optimal capital structure, it is assumed that the manager prefers the responsibility of cost of equity (or dividend) $l(t)$ to the responsibility of the cost of debt (interest payment) $c(t)$. Cost of debt increases the performance pressure on managers and requires more measurable efforts (Harris & Raviv 1988). In addition, the manager prefers to dilute the company's shares when he fears overreliance on debt. Therefore, the manager's problem is to select the optimal cost of equity $l(t)$, his **control variable/strategy** that minimizes cost of capital and maximizes income.

The objective functional of the manager over a finite time horizon is:

$$J_1 = \min \int_0^T e^{-\rho t}\left(\omega_1 \frac{E(t)}{V(t)} l(t)^2 + \omega_2 \frac{D(t)}{V(t)} c(t)^2 (1-\theta) - \omega_3 r M(t)\right) dt \qquad (5)$$

The ratio of the company's capital $V(t) = E(t) + D(t)$, financed by equity $E(t)$ can be represented as $\frac{E(t)}{V(t)} = \mu(t)$, such that the remaining ratio financed by debt $D(t)$ is $\frac{D(t)}{V(t)} = 1 - \mu(t)$. The first two elements of equation (1) are specified as the WACC, and the last element represents the maximization of the company's financial income.

The objective functional of the manager over a finite time horizon is therefore restated as:

$$J_1 = \min \int_0^T e^{-\rho t}(\omega_1 \mu(t) l(t)^2 + \omega_2(1-\mu(t))c(t)^2(1-\theta) - \omega_3 r M(t))dt \qquad (6)$$

Where $\omega_1, \omega_2, \omega_3 > 0$ are balancing cost factors. The debtholder's objective functional is introduced next.

3.1.2 The formulation of the debtholder's (principal) problem: The risk-averse debtholder provides the company a debt finance based on the company's market value, credit rating and existing relationship. These parameters are used by the debtholder to categorise the borrower as a safe borrower, hence relying on the theory of reputation. The debtholder who is assumed to be a secured and senior debtholder ultimately seeks to maximize the principal value of debt $D(t)$ issued to the company at $t = 0$ which comes an opportunity cost γ_2 while minimizing the monitoring costs γ_1 of obtaining his interest payments $c(t)$. The debtholder's problem is to consistently select the optimal cost of debt $c(t)$ in each period as his strategy that achieves this. The principal value of debt and the cost of debt accrued are the fixed claim available to the debtholder (Sudheer, Wang & Zou 2019).

The debtholder's payoff functional is specified as:

$$J_2 = \max \int_0^T e^{-\rho t}(\gamma_2 D(t) - \gamma_1 c(t)^2)dt \qquad (7)$$

Where $\gamma_1, \gamma_2 > 0$ are balancing cost factors. The debtholder has no power of decision-making in the firm but is only keen on the firm's debt valuation and ability to recover his investments. Equations (6) and (7) represents the different objectives of the debtholder (principal) and the manager (agent) and the conflicts of interests between them after debt issuance.

Parameters used in the model are in Table 1:

Table 1 Parameters used in the model

Parameters	**Definition**
r	Rate of return on Liquid reserve $r > 0$, assumed to be constant
β	payout rate of company's productive assets, assumed to be constant
$c(t)$	cost of debt (interest payment)
$l(t)$	cost of equity (dividend)
$E(t)$	Market value of company equity

$M(t)$	Liquid Reserve
$S(t)$	Value of company's productive assets
$D(t)$	Market value of company debt
$V(t)$	Company's total capital; $V(t) = E(t) + D(t)$
θ	Effective tax rate
α	Ratio of new debt to existing market value of company debt
ρ	Discount rate

3.1.3 Balancing cost factors

In specifying the objective functionals, it is presumed that there are certain costs associated with optimising elements of the objective functionals, known as the balancing cost factors. $\omega_1, \omega_2, \omega_3$ are specified as inherent transaction and operational costs incurred by the manager in order to meet its finance costs and maximize its financial income. Similarly, the debtholder incurs an opportunity cost γ_1 on the principal debt value $D(t)$ and monitoring cost γ_2 to recover the cost of debt $c(t)$. It is to be noted that the values of the weight assigned to the balancing cost factors as specified in Table 3 are merely theoretical for illustrative purposes.

Varying the balancing cost factors

To obtain interesting and useful results for the model, the weight or value assigned to the balancing cost factors can be varied to understand the impact of certain cost of optimizing the players' objectives. The varied balancing cost factors are denoted as Encounter 1 (E1), Encounter 2 (E2), Encounter 3 (E3).

Encounter 1 – $[\omega_1, \omega_2, \omega_3] = [2\ 2\ 5]$ and $[\gamma_1, \gamma_2] = [5, 2]$. This implies that the cost of maximizing the company's financial income is higher than the cost of minimizing its finance cost. In the same encounter, it is hypothetically stated that the debtholder incurs a higher cost to optimize its debt face value than the cost of debt.

Encounter 2 – $[\omega_1, \omega_2, \omega_3] = [2\ 2\ 5]$ and $[\gamma_1, \gamma_2] = [5, 10]$. The costs associated in encounter 2 are similar to those of encounter 1, however, with a significant increase in the cost of recovering the cost of debt than the debt face value.

Encounter 3 – $[\omega_1, \omega_2, \omega_3] = [50\ 2\ 2]$ and $[\gamma_1, \gamma_2] = [5, 2]$. In encounter 3, there is a significant weight on the operational cost of minimizing the company's cost of equity than other variables.

This provides different outcomes for the optimal states of the game and the optimal capital structure of the firm that minimizes the agency problem and thus provides useful insights.

Summarily, the model is therefore set out as:

<u>Manager-Debtholder game</u>

$$Manager: J_1 = \min \int_0^T e^{-\rho t}(\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1-\theta)(1-\mu(t)) - \omega_3 r M(t)) dt$$

$$\text{Debtholder}: J_2 = \max \int_0^T e^{-\rho t}(\gamma_2 D(t) - \gamma_1 c(t)^2) dt$$

Subject to:

$$\dot{M}(t) = \beta S(t) + rM(t) - c(t) - l(t),\ M(0) = M_0$$

$$\dot{D}(t) = \alpha D(t) + c(t), D(0) = D_0$$

$$S(t) = S_0 e^{(r-\beta)t}$$

$$E(t) = M(t) + S(t) - D(t),$$

where $\mu(t)$ can also be represented as $\mu(t) = 1 - \frac{D(t)}{M(t)+S(t)}$.

The differential game problem is analysed and solved via adequate equilibrium concepts, first as a non-cooperative game using the Nash open-loop and Nash feedback solution concepts. Second, as a cooperative game using the Pareto solution concept to obtain the optimal results for the capital structure.

3.2 Model solutions

In this section, we solve the agency problem via differential game theory. The general case with moral hazard is specified as a non-cooperative game. We derive the open and closed-form solutions by solving the ordinary differential equations (ODEs) with the associated initial and terminal (boundary) conditions. To minimize the conflicts of interests, we assume that the manager and debtholder may be able to agree and cooperate if the debtholder provides enough incentive for the manager, thus providing a pareto-efficient outcome. The results are obtained using approximate analytical methods and by further applying the model to financial data from a company.

3.2.1 Non-cooperative Game Analysis - Nash Equilibrium

3.2.1.1 Open-loop Nash Equilibrium (OLNE) Solution Concept

The agency conflicts of interests between the debtholder and the manager stipulate the problem as a non-cooperative game. The manager does not comply with the no-risk-shifting terms of the debtholder. Given the debtholder selects an optimal strategy, the manager must select his optimal strategy to optimize the firm's capital structure in a way that minimizes the impact of the debtholder's penal actions. As a non-cooperative game, we solve the model using the open-loop Nash equilibrium solution concept where the only available information for action at time t is that of the initial states $M(0)$ and $D(0)$. The information scheme does not give the players knowledge about the changes in state variables, known as pre-commitment (Bressan 2011). This implies that the debtholder and the manager do not revise their actions nor reconsider the debt contract throughout the debt maturity.

Open-loop System of the Game

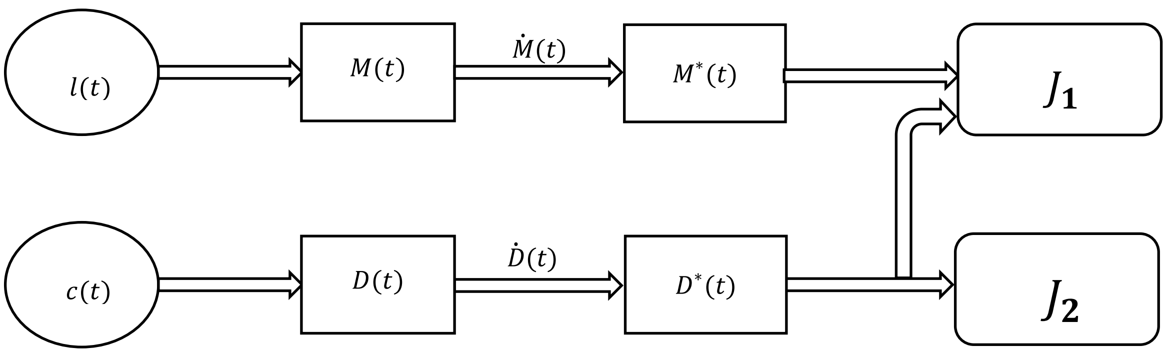

Figure 1 Open-loop System of the Game

Figure 1 describes the open-loop system of the game. Both players' strategies $l(t)$ and $c(t)$, cost of equity and cost of debt influence the states of the game $M(t), D(t)$. The systems of the game, $\dot{M}(t)$, $\dot{D}(t)$ react to the information from the strategies and states of the game and produce equilibrium state trajectories at the Nash pair of strategies for which a player cannot improve his outcome (J_1, J_2) if he moves from this strategy while the other player sticks to his.

The necessary conditions developed by Pontryagin and his co-workers (Boltyanskii, Gamkrelidze & Pontryagin 1956) are derived by generating the Hamiltonian. This is obtained by adjoining the state equations to objective functional for each player with adjoint or co-state functions, $\lambda_j, j = 1, 2$ for player 1 (the manager) and $\phi_j, j = 1, 2$ for player 2 (the debtholder). Hence the Hamiltonian for the manager-debtholder game is defined as

$$H_1 = \frac{\omega_1 l^2(M+S-D)}{M+S} + \frac{\omega_2 c^2 D(1-\theta)}{M+S} - r\omega_3 M + \lambda_1(\beta S + rM - c - l) + \lambda_2(\alpha D + c) \quad (8)$$

$$H_2 = -\gamma_2 D + \gamma_1 c^2 + \phi_1(\beta S + rM - c - l) + \phi_2(\alpha D + c) \quad (9)$$

Where J_2 is multiplied by minus to change the maximization problem to a minimization problem. The set of necessary conditions makes it possible to identify the equilibrium time path for the variables and proffers implications for the ideal financial management policies. The **first** part of the principle states that each control variable/strategy selected at any moment in time must have an effect that maximises or minimises the Hamiltonian. This imply:

$$\frac{\partial H_i(t)}{\partial l(t) \text{ or } \partial c(t)} = 0 \qquad \text{for all } i = 1, \dots, I \text{ and all t}$$

The equilibrium conditions are:

$$\frac{\partial H_1}{\partial l} = \frac{2\omega_1 l(M+S-D)}{(M+S)} - \lambda_1 = 0$$

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1(M+S-D)} \quad (10)$$

This calculation means that the optimal cost of equity for the firm is the ratio of the value of an added dollar of debt or earnings multiplied by the firm's total assets, to the firm's equity multiplied by two times the balancing cost factor of the use of equity at any time t. This implies that with an increase in the weight on the cost of implementing equity, the ratio of the company's total assets to its equity is reduced. A lower asset to equity ratio may mean that the company has more of its assets financed by equity providers.

$$\frac{\partial H_2}{\partial c} = 2\gamma_1 c - \phi_1 + \phi_2 = 0$$

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2) \quad (11)$$

From equation (11), the optimal cost of debt for the firm is the ratio of the value of an added dollar of debt to the Debtholder's cost of monitoring times two. In contrast to the first result of Modigliani and Miller, in this study, the required optimal cost of equity was found to be lesser than the required optimal cost of debt when the conflicts of interests is introduced into the optimal capital structure model. This result is also contrary to those of Elton and Gruber (1974), where the cost of equity funds equals the cost of debt funds without the moral hazard problem.

Equations (10) and (11) above are the characterisations of the Nash strategies.

The second necessary conditions necessitate the rate of change with respect to time of each co-state variable to be equivalent to the negative of the partial derivative of the Hamiltonian with respect to the correlated state variable.

The starting or ending conditions for the adjoint variables can be logically deduced from the structure of the problem. For example, the present value of a dollar earned in the infinite future is zero (Elton & Gruber 1974).

The third condition requires that the state equations are achieved.

The optimality system which generates the equilibrium outcomes is a forward-backward system of differential equations stated as follows

$$\dot{M} = \beta S + rM - c - l, \quad M(0) = M_0 \tag{12}$$

$$\dot{D} = \alpha D + c, \quad D(0) = D_0 \tag{13}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial M} = (\rho - r)\lambda_1 + \omega_3 r - \frac{D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} \tag{14}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} = (\rho - \alpha)\lambda_2 + \frac{\omega_1 l^2 - \omega_2 c^2(1-\theta)}{(M+S)} \tag{15}$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial M} = \phi_1(\rho - r) \tag{16}$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} = \phi_2(\rho - \alpha) + \gamma_2 \tag{17}$$

$$\lambda_1(T) = 0 \qquad \lambda_2(T) = 0 \qquad \phi_1(t) = 0 \qquad \phi_2(T) = 0$$

with Nash equilibrium strategies:

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1(M+S-D)} \tag{18}$$

$$c^* = \frac{1}{2\gamma_1}(\phi_1 - \phi_2) \tag{19}$$

Next, some of the optimal state and adjoint variables are obtained analytically.

The adjoint equations (16) and (17) are independent of other unknown variables and hence can be solved analytically. First, equation (16);

$$\dot{\phi}_1 = (\rho - r)\phi_1$$

And the solution is

$$\phi_1 = k_1 e^{(\rho - r)t}$$

Where k_1 is the constant of integration, and solving for the constant of integration using the terminal (transversality) condition this gives:

$$\phi_1 = 0 \qquad (20)$$

For equation (17):

$$\dot{\phi}_2 = \frac{d\phi_2}{dt} = (\rho - \alpha)\phi_2 + \gamma_2$$

Using the integrating factor method of integration for $\rho \neq \alpha$, where $e^{-(\rho-\alpha)t}$ is the integrating factor, we obtain:

$$\phi_2 = \frac{-\gamma_2}{(\rho - \alpha)} + k_2 e^{(\rho - \alpha)t}$$

From the transversality condition $\phi_2(T) = 0$, the constant of integration k_2 is determined. Hence,

$$k_2 = \frac{-\gamma_2}{(\rho - \alpha)} e^{-10(\rho - \alpha)}$$

Therefore:

$$\phi_2 = \begin{cases} \gamma_2(t - T), & \rho = \alpha \\ \frac{\gamma_2\left(e^{(\rho-\alpha)(t-T)} - 1\right)}{(\rho - \alpha)} & \rho \neq \alpha \end{cases} \qquad (21)$$

From equation (24), the Nash strategy $c(t)$ associated with debtholder is:

$$c(t) = \begin{cases} \frac{\gamma_2(T-t)}{2\gamma_1}, & \rho = \alpha \\ \frac{\gamma_2\left(1 - e^{(\rho-\alpha)(t-T)}\right)}{2\gamma_1(\rho - \alpha)} & \rho \neq \alpha \end{cases} \qquad (22)$$

Hence the solution for the optimal state for $D(t)$

$$D(t) = k_3 e^{\alpha t} - \frac{\gamma_2}{2\gamma_1(\rho-\alpha)}\left[\frac{1}{\alpha} + \frac{e^{e(\rho-\alpha)(t-T)}}{(\rho-2\alpha)}\right] \quad (23)$$

With $D(0) = 0.27$ in Table 2, we have

$$D^*(t) = 0.27e^{\alpha t} + \frac{a}{\alpha}(e^{\alpha t} - 1) + \frac{b}{(\rho-2\alpha)}(e^{\alpha t} - e^{(\rho-\alpha)t}) \quad (25)$$

3.2.2 *Feedback Nash Equilibrium (FNE) Solution Concept*

The alternative to the open-loop Nash case which only relied on the initial state information, the feedback Nash equilibrium uses information about the current state of the game in addition to the initial state or remain memoryless, this eliminates the problem of information non-uniqueness from the equilibria (Yeung & Petrosjan 2006). This lends to the co-learning theory in which all players attempt to learn their optimal strategies concurrently (Sheppard 1998). This can also be described as the learning process of a decision-making system where the sensors receive a signal (Roberts & SenGupta 2020). Figure 2 describes the feedback system of the game.

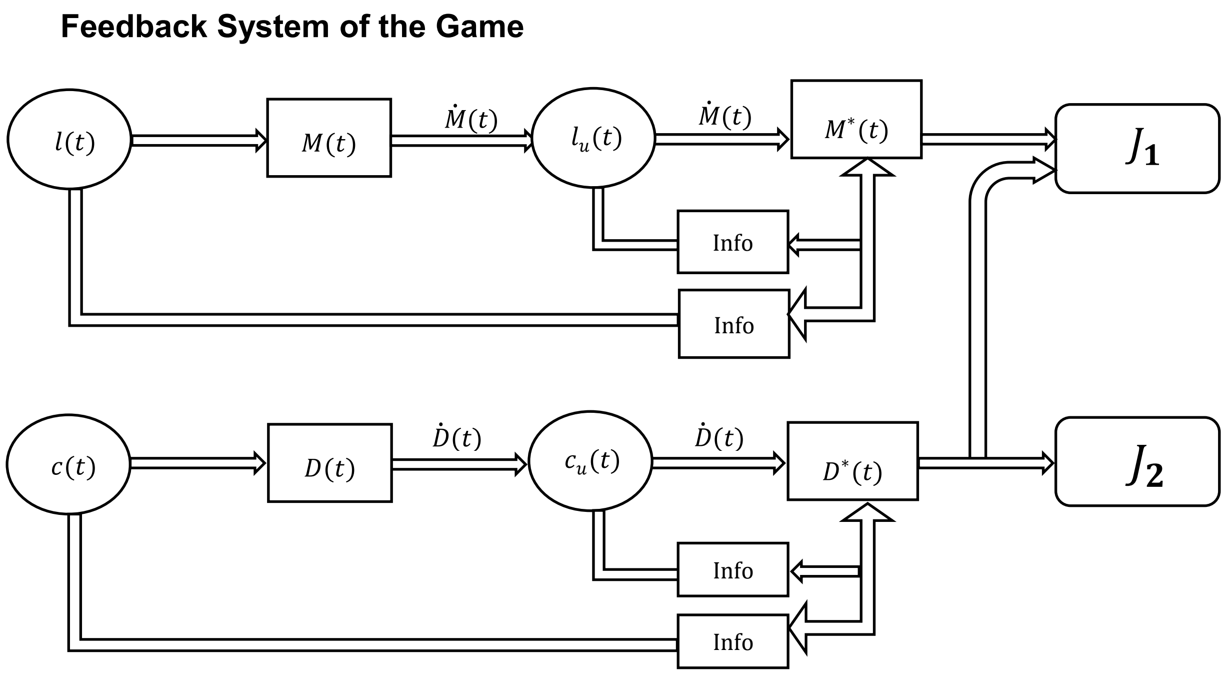

Figure 2 Feedback System of the Game

In the feedback system of the game, the debtholder and manager choose to consider the current states of the debt contract at any time t via the reported outcomes of the company and update their strategies with this information. Hence, the equilibrium state trajectories and final utility of the players are functions of information (Info) from the initial strategies $l(t)$, $c(t)$ and updated strategies $l_u(t)$, $c_u(t)$. This feedback information system is reflected in the solution method as a cross-derivative of the Nash strategy of one player in the Hamiltonian of the other.

The set of necessary conditions to be satisfied in the FNE case are similar to those of the open-loop Nash Equilibrium case. Although the definition of the optimal strategies of the manager and debtholder are the same as the open-loop case, there exists a significant difference in the adjoint equations. The adjoint equations for each player in the feedback case incorporate the response of the other player to changes in the state variables thereby impacting the decision making of that player as seen in equations (26) and (27). This is expressed as a cross-derivative and updates the Nash pair of strategies of both players as necessary, specifying how each player feeds existing information in the game back into their decision-making process.

From the Hamiltonian function (7) and (8), the adjoint equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H_1}{\partial M} - \frac{\boldsymbol{\partial H_1}}{\boldsymbol{\partial c}}\frac{\boldsymbol{\partial c^*}}{\boldsymbol{\partial M}}$$

$$\dot{\lambda}_1 = (\rho - r)\lambda_1 + \omega_3 r - \frac{D\left(\omega_1 l^2 - \omega_2 c^2(1-\theta)\right)}{(M+S)^2} + \left[\frac{2\omega_2 c(1-\theta)D}{(M+S)^2}\right]\left[\frac{2\omega_2 c(1-\theta)D}{(M+S)} - \lambda_1 + \lambda_2\right] \qquad (26)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H_1}{\partial D} - \frac{\boldsymbol{\partial H_1}}{\boldsymbol{\partial c}}\frac{\boldsymbol{\partial c^*}}{\boldsymbol{\partial D}}$$

$$\dot{\lambda}_2 = (\rho - \alpha)\lambda_2 + \frac{\omega_1 l^2 - \omega_2 c^2(1-\theta)}{(M+S)} - \left[\frac{2\omega_2 c(1-\theta)}{(M+S)}\right]\left[\frac{2\omega_2 c(1-\theta)D}{(M+S)} - \lambda_1 + \lambda_2\right] \qquad (27)$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_2}{\partial M} - \frac{\boldsymbol{\partial H_2}}{\boldsymbol{\partial l}}\frac{\boldsymbol{\partial l^*}}{\boldsymbol{\partial M}}$$

$$\dot{\phi}_1 = \phi_1(\rho - r)$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial D} - \frac{\boldsymbol{\partial H_2}}{\boldsymbol{\partial l}}\frac{\boldsymbol{\partial l^*}}{\boldsymbol{\partial D}}$$

$$\dot{\phi}_2 = \phi_2(\rho - \alpha) + \gamma_2$$

$\dot{\phi}_1$ and $\dot{\phi}_2$ shows that the debtholder does not modify his strategies with the updated information available about the firm's change in the cost of equity $l(t)$ or liquid reserve information, since the cross-derivative information of the debtholder's response to changes in $M(t)$, $D(t)$ yields zero. Therefore, the debtholder does not incorporate any new information in his selection of an equilibrium strategy. The manager, on the other hand, updates his optimal strategies due to new information available in the game, as seen in equations (26) and (27).

$$\dot{M} = \beta S + rM - c - l$$

$$\dot{D} = \alpha D + c$$

Also, these co-states functions satisfy the terminal conditions:

$$\lambda_1(T) = 0 \qquad \lambda_2(T) = 0 \qquad \phi_1(t) = 0 \qquad \phi_2(T) = 0$$

The third condition remains that the state equations are achieved.

3.3 Cooperative Game Analysis - Pareto Outcome

The non-cooperative game analysis discussed above elucidates the incongruity of interests between the players, and thus does not fully resolve the agency problem but provides optimal strategies to minimize the marginal agency cost of debt. To elicit corporate governance in the selection of an optimal capital structure and optimizing the interests of the manager and debtholder, cooperation may be sought between the players. The Pareto solution concept, also known as the cooperative form of the game, jointly optimizes all players utility functions over the time interval. It is therefore presumed that the equilibrium of a cooperative game will be Pareto optimal. This implies that it is impossible to allocate resources in a way that make a player better off without leaving the other player at least worse off (Yeung & Petrosjan 2006). Although the plausibility of cooperation in a typical non-cooperative game may be argued, due to the difficulty of ensuring congruity, it may be otherwise argued by the so-called Coase Theorem, this states in part that when one player is affected by the externality from the other player's actions, both players (if rational) will transact to reach a Pareto optimal solution (Coase 1960). That is, if a rational debtholder observes the acute effect of the manager's actions on the company's default tendencies, he will readily negotiate on a Pareto optimal outcome.

Pareto System of the Game

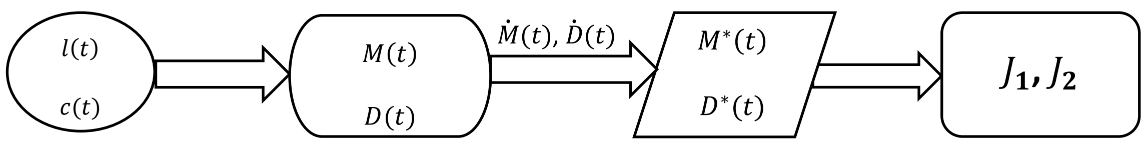

Figure 3 Pareto System of the Game

Figure 3 describes the Pareto system of the game. The game becomes a seemingly optimal control system, here, both players agree to jointly optimize their objectives with respect to a weight assignment as a corporate governance mechanism. The results from the optimal states and strategies are then imputed in each players utility function to derive a Pareto Frontier to compare the outcomes for both players.

In the solution concept, the interests of both players are prioritized with respect to the assigned constant φ such that

$$\varphi J_1 + (1-\varphi)J_2$$

However, a controversial question in most multi-objective literature is the basis for weight assignment; one way out of this dilemma is to create a Pareto front consisting of possible weight assignments. Thus, the joint objective functional of the game now becomes

$$min \int_0^T \left(\varphi e^{-\rho t}\left(\omega_1 \mu(t) l(t)^2 + \omega_2 c(t)^2 (1-\theta)\big(1-\mu(t)\big) - \omega_3 r M(t)\right)\right) + e^{-\rho t}(1-\varphi)\big(\gamma_1 c(t)^2 - \gamma_2 D(t)\big) dt \qquad (28)$$

The Hamiltonian for the game is specified as

$$H = \varphi\left(\frac{\omega_1 l^2 (M+S-D)}{M+S} + \frac{\omega_2 c^2 D(1-\theta)}{M+S} - r\omega_3 M\right) + (1-\varphi)(\gamma_1 c^2 - \gamma_2 D) + \lambda_1(\beta S + rM - c - l) + \lambda_2(\alpha D + c) \qquad (29)$$

The optimal conditions are:

$$\frac{\partial H}{\partial l} = \frac{2\omega_1 \varphi l(M+S-D)}{(M+S)} - \lambda_1 = 0$$

$$l^* = \frac{\lambda_1(M+S)}{2\omega_1\varphi(M+S-D)} \qquad (30)$$

$$\frac{\partial H}{\partial c} = \frac{2c\omega_2\varphi(1-\theta)D}{(M+S)} + 2\gamma_1 c(1-\varphi) - \lambda_1 + \lambda_2 = 0$$

$$c^* = \frac{(\lambda_1-\lambda_2)(M+S)}{2\omega_2 D\varphi(1-\theta) + 2\gamma_1(M+S)(1-\varphi)} \qquad (31)$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial M} = (\rho - r)\lambda_1 + \varphi\omega_3 r - \frac{\varphi D(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)^2} \qquad (32)$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial D} = (\rho - \alpha)\lambda_2 + \gamma_2(1-\varphi) + \frac{\varphi(\omega_1 l^2 - \omega_2 c^2(1-\theta))}{(M+S)} \qquad (33)$$

The optimal cost of equity for the firm from equation (30) is the ratio of the value of an added dollar of debt or earnings multiplied by the firm's total assets, to the firm's equity multiplied by two times the balancing cost factor of the use of equity times the assigned φ at any time t. The higher the weight on the cost of implementing equity, the lower the ratio of the company's total assets to its equity. This implies that the cost of implementing equity can lower the company's asset-to-equity ratio. Similarly, the greater the weight φ assigned to the manager's objective function, the lower the optimal cost of equity required to attain optimality.

The optimal cost of debt is impacted by the ratio of the total assets to the tax-deductible value of the use of debt finance and the debtholder's assigned weight. From equation (31), the higher the weight φ assigned to the manager's objective function, the higher the optimal cost of debt required by the debtholder. The contraposition is that the higher the weight assigned to the debtholder $(1-\varphi)$, the lower the optimal cost of debt. Thus, it is more optimal to assign a lower weight or priority to the manager's utility function.

4 Results

Nikooeinejad, Delavarkhalafi and Heydari (2016) thoroughly discuss the difficulty in solving two-points boundary value problems analytically and the need for numerical solutions for dynamic games. Due to the non-linearity of the developed model, the remaining solutions are obtained via a numerical algorithm. The model is applied to financial data from a company. The numerical code was simulated in the Matlab2018b (64-bits) programming environment. The numerical algorithm was devised to generate an approximation for a pair of Nash equilibrium piecewise continuous strategies that yield the optimal state values and optimal capital structure for the non-cooperative game analysis. Similarly, they produce optimal results for the Pareto case. The fourth order Runge-Kutta (RK4) numerical method is used to solve the boundary value problem using the forward-backward sweep approach. The procedure for the RK4 forward-backward sweep approach is as follows: initial guesses are provided for the control or strategy variables $l(t)$, $c(t)$ specified as zero, using the initial values of the state variables $M(t)$, $D(t)$ collected from the financial statements, the states are solved forward in time following the differential equations in the optimality system, using the transversality condition $\lambda(T) = \phi(T) = 0$, and the values for ($l(t)$, $c(t)$, $M(t)$, $D(t)$), $\lambda(t)$ and $\phi(t)$ are solved backward in time, $l(t)$, $c(t)$ are updated using the values of $M(t)$, $D(t)$, $\lambda(t)$, $\phi(t)$ in the characterization of the optimal strategies, finally, convergence is confirmed if the values of the variables in a current iteration is close to the last iteration such that $\delta\left||l(t)|\right| - ||l(t) - oldl(t)|| \geq 0$ and $\delta\left||c(t)|\right| - ||c(t) - oldc(t)|| \geq 0$, else the process is restarted until convergence is attained.

The results obtained are computed graphically, discussed and compared to provide implications of the model. Financial variables obtained from the company's 2018 financial statements to obtain numerical results for the model application are presented below.

Table 2 Financial data from a company

Parameters	Definition and Code	Data
r	Rate of return on Liquid reserve $r > 0$, assumed to be constant - Current (As of May 2019) Government bonds yield for 10-year residual maturity	0.02

β	Payout rate of productive assets, assumed to be constant - Assumed	0.30
$M(0)$	Initial value of liquid reserve - financial data (AUD $b)	2.40
$S(0)$	Initial value of company's productive assets - financial data (AUD $b)	1.30
$D(0)$	Initial market value of company debt - financial data (AUD $b), calculated as the interest-bearing current liabilities plus total non-current liabilities	0.27
θ	Applicable effective tax rate, ranging between 0 and 1	0.28
α	An average of the rate of change in use of debt over a 6-year financial period (2013 - 2018)	0.01
ρ	Discount rate - Assumed	0.001

The simulated results for the open-loop case are given as follows in table 4. The Nash Equilibrium strategies over time are presented in figures 4. Where unspecified on the figure, the parameters do not have units on the y-axis.

Table 3 Open-loop Nash Equilibrium Outcomes for Encounters (1 - 3)

Encounter	ρ	$M(0)$	$D(0)$	$M(T)$	$D(T)$	$\omega_1, \omega_2, \omega_3$	γ_1, γ_2
1	0.001	2.400	0.270	2.494	0.374	[2 2 5]	[5 2]
2	0.001	2.400	0.270	2.092	0.778	[2 2 5]	[5 10]
3	0.001	2.400	0.270	2.480	0.374	[50 2 2]	[5 2]

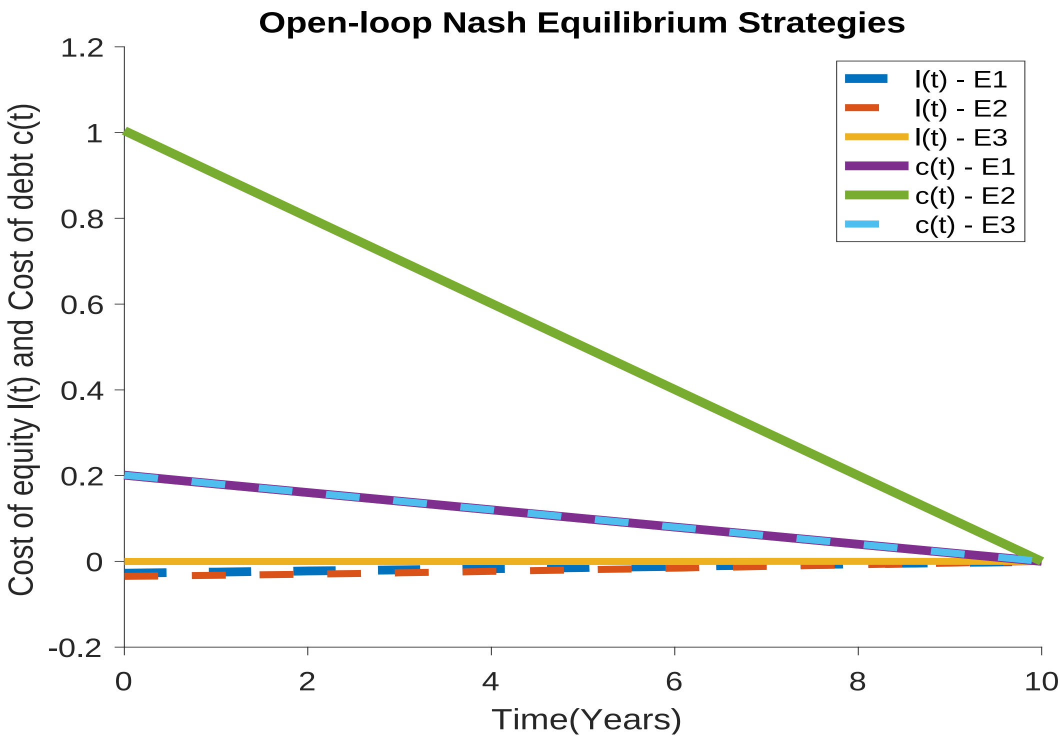

Figure 4 Open-loop Nash equilibrium pair of Strategies

For the open-loop case, the optimal capital structure from the study suggests a higher cost of debt than a higher cost of equity. In addition, payouts (cost of equity) should be returned into the firm's fund pool rather than as a cash outflow. This can be done by repurchasing shares rather than paying out dividends. Moreover, share repurchases may be encouraged by low capital gain rates (Allen & Morris 2014). Although payout policy conveys information to the capital market about the health and ability of a company to produce cash flows (signalling motives), a firm is limited by the availability of its free cashflows (Copeland, Weston & Shastri 2014).

The feedback Nash outcomes are presented in table 4 and figure 5.

Table 4 *Feedback Nash Equilibrium Outcomes for Encounters (1 - 3)*

Encounter	ρ	$M(0)$	$D(0)$	$M(T)$	$D(T)$	$\omega_1, \omega_2, \omega_3$	γ_1, γ_2
1	0.001	2.400	0.270	2.494	0.374	[2 2 5]	[5 2]
2	0.001	2.400	0.270	2.099	0.778	[2 2 5]	[5 10]

3	0.001	2.400	0.270	2.480	0.374	[50 2 2]	[5 2]

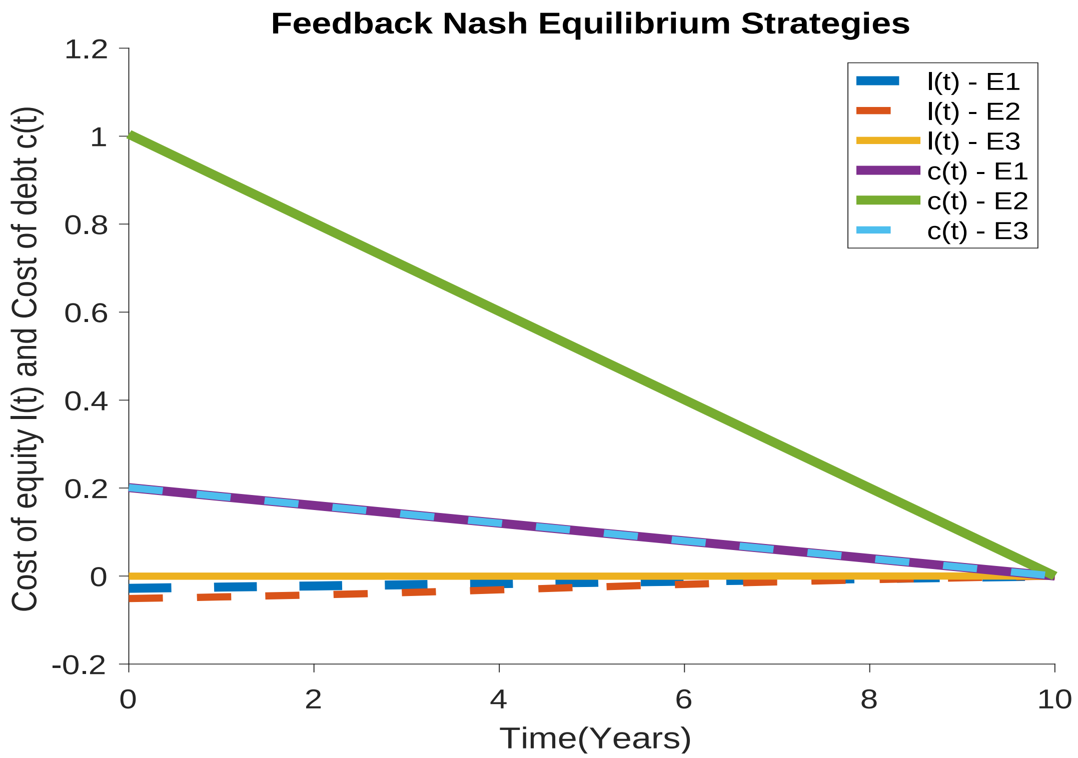

Figure 5 Feedback Nash equilibrium pair of Strategies

With a minimal reaction from the debtholder, from figure 5, the result for the FNE case is similar to those of the open-loop case, except that the manager adjusts his equilibrium strategy to a higher cost of equity, and thus show only a marginal difference in the outcomes. In contrast to Liu et al. (2017), in which the optimal cost of debt is found to be increasing, in this study the optimal cost of debt required declines over the time period due to the long-term relationship.

Achieving cooperation between the players present a mechanism that resolves the agency problem. An optimal solution found for each weight φ, $0 < \varphi < 1$ yields a point on the Pareto frontier. To obtain the Pareto frontiers, the optimal strategies and states were obtained for each $\varphi = [0.1, \ldots 0.9]$. Weights 0 and 1 have been excluded because a player will not remain in a game if his interest is set to 0. The returned equilibrium values at each weight share are then imputed into the individual payoff functions (objective functionals) of each player independently, thereby producing the manager and debtholder's payoff for each weight share. The values for the manager's payoff are then plotted against those of the debtholder, to observe the outcome for both players in the Pareto frontiers as seen in Figure 6. From the Pareto frontiers, the weight assignment with the optimal payoff is at (0.1, 0.9).

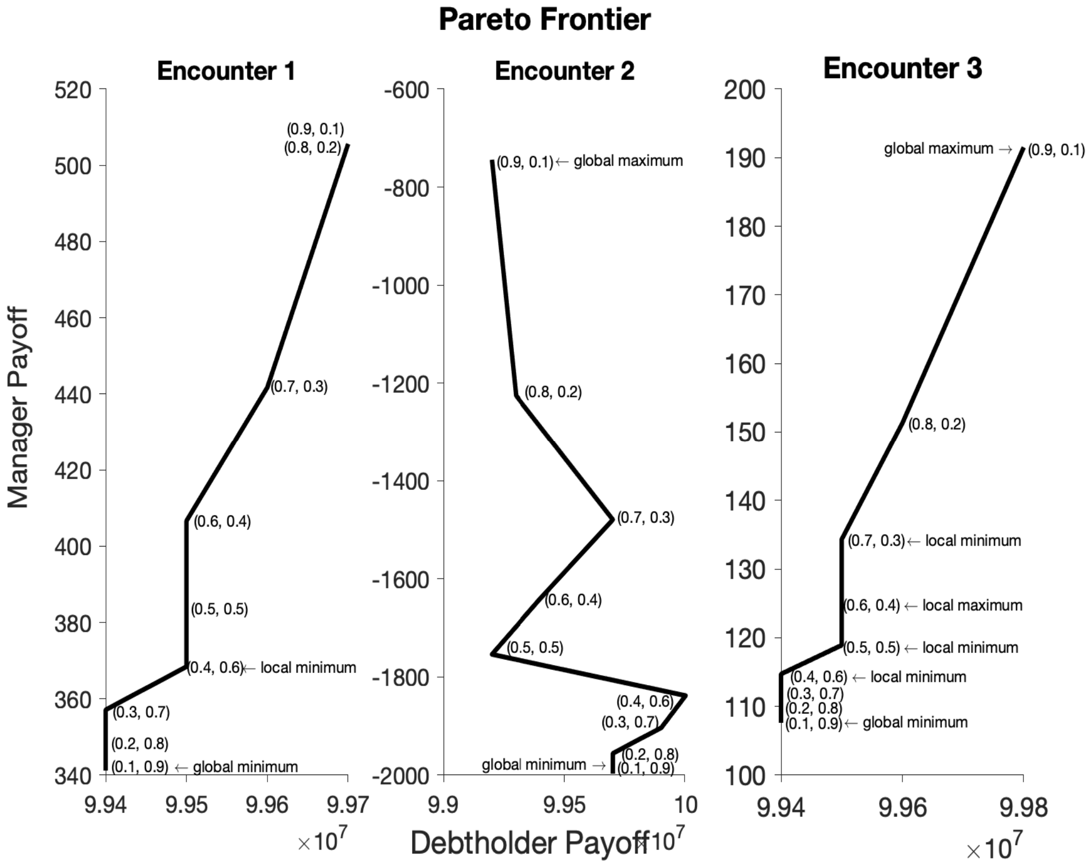

Figure 6 Pareto Frontier

4.1 Comparison of the Open-loop, Feedback and Pareto Solution for the weight share [0.1, 0.9] and [0.7, 0.3]

It offers insights to compare the over time outcomes for the value of liquid reserve $M(t)$ and debt $D(t)$ of the three solution concepts. The sub-optimal [0.7, 0.3] and optimal Pareto outcomes [0.1, 0.9] are compared with the open-loop and feedback Nash outcomes for all encounters in figure 7 and figure 8. This is done for liquid reserve and value of debt over time. This is also done to identify the trigger strategies presented by the debtholder when the manager shifts from the optimal strategy and reneges on the terms of cooperation.

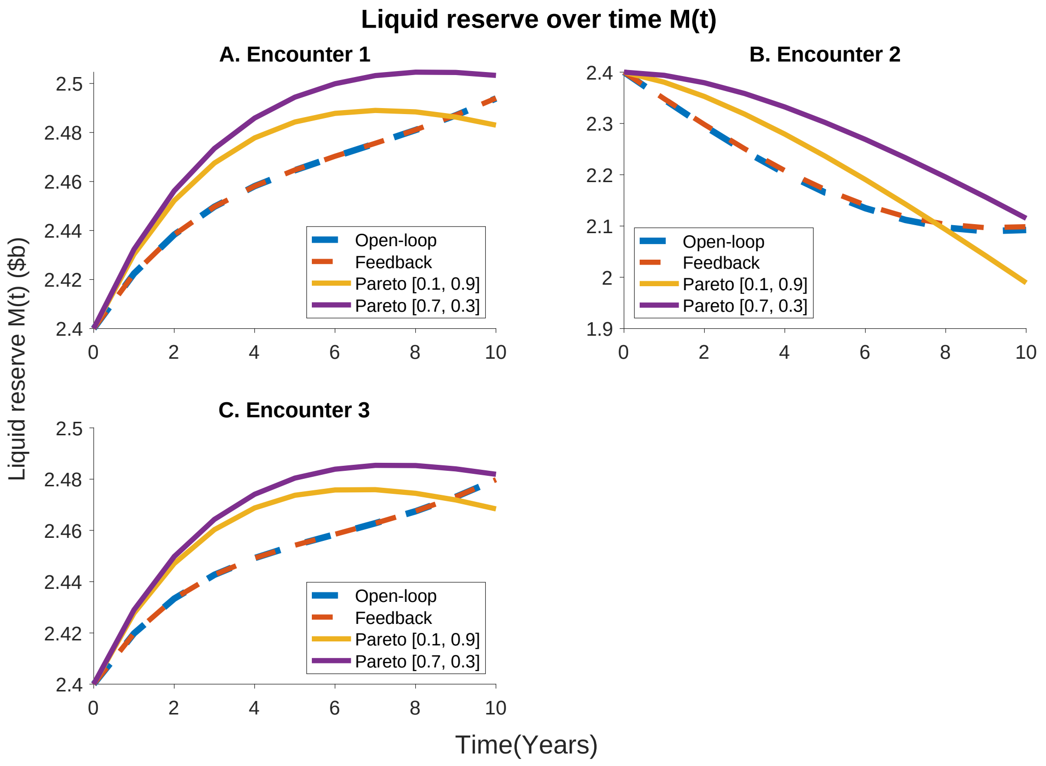

Figure 7 Firm liquid reserve over time

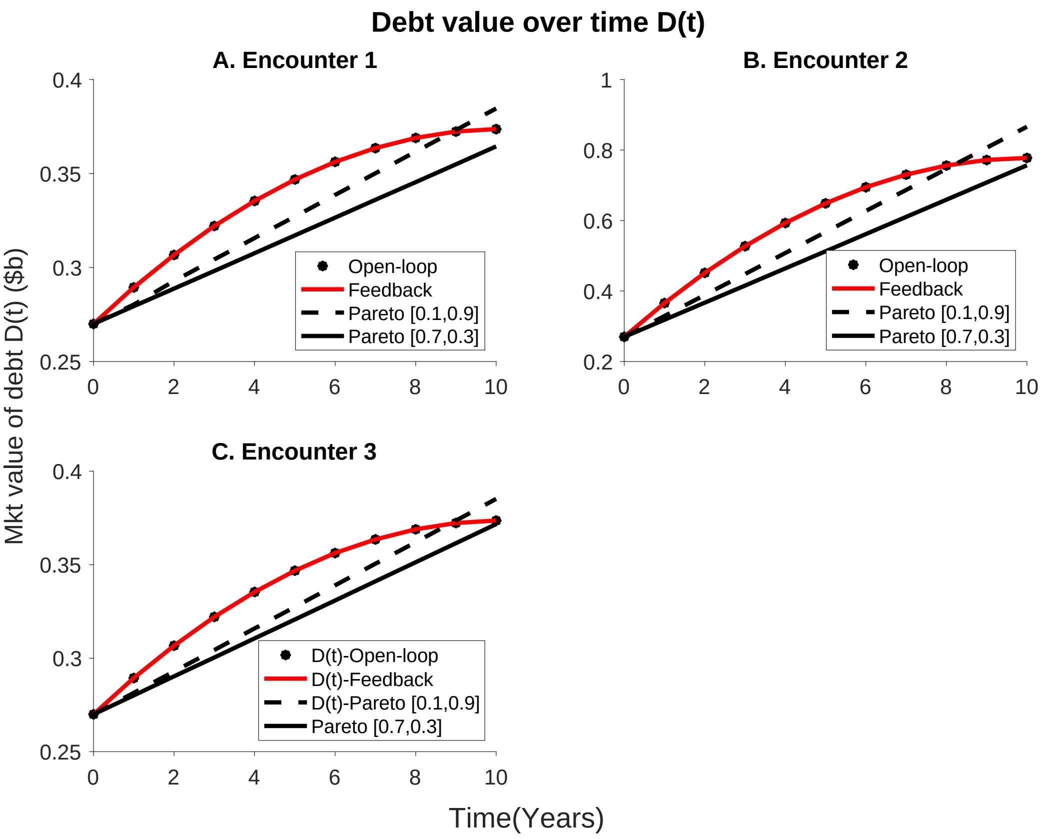

Figure 8 Market value of debt over time

Figure 9 below presents the optimal capital structure over a ten-year period. This is compared for the open-loop and feedback case, as well as the pareto case at the optimal and sub-optimal weight assignment.

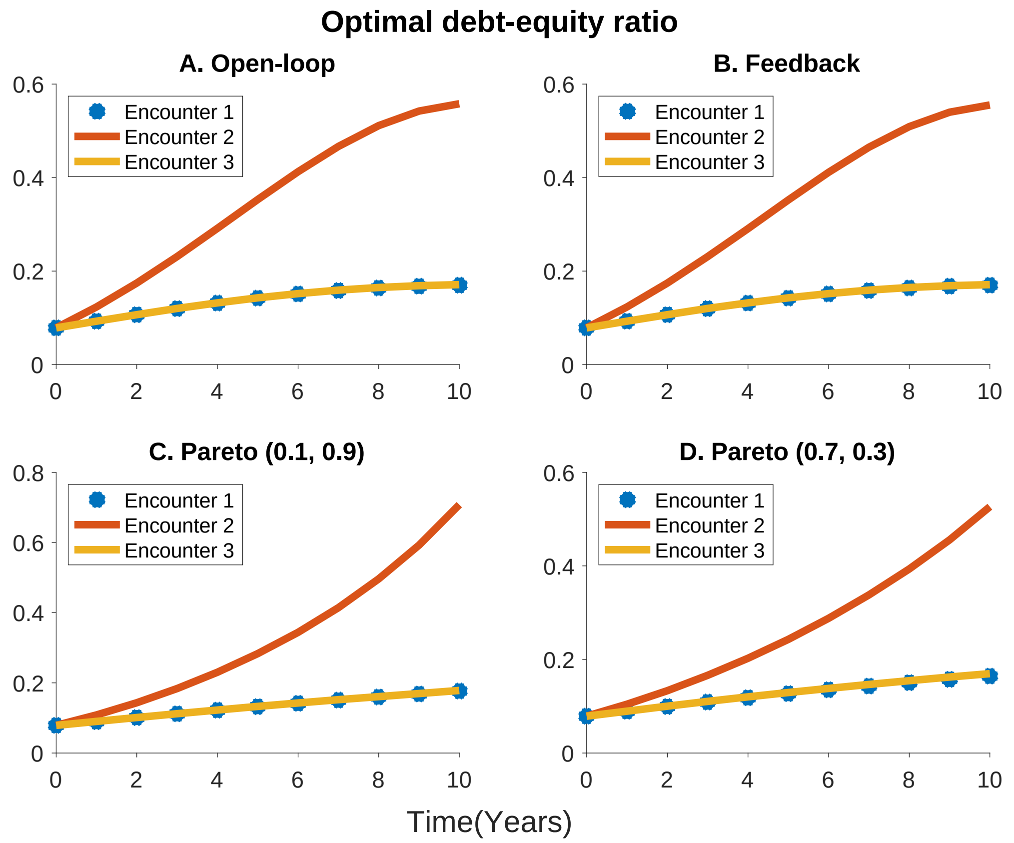

Figure 9 Optimal Debt-Equity Ratio (Open-loop, Feedback and Pareto cases)

The optimal capital structure obtained enhances the firm's ability to finance potential investments as the firm's financial income and operating income increases. This empowers the manager to make appropriate investment decisions in the time-period considered. The graph of the optimal capital structure in the cooperative game is slightly convex or concave up while the graph of the optimal capital structure for the non-cooperative game is concave down. This implies that a marginally lower debt-equity ratio is required when the players can reach cooperation in contrast to the non-cooperative case. Since the Pareto case presents more gains than the non-cooperative case, any of the encounters are wealth maximizing for the players, however with a preference for encounter 3 where the debtholder enjoys the optimal payoff, see figure 6. Further implications are discussed in section 4.2.

4.2 Implications for Corporate Governance, Optimal Capital Structure and Dynamic Game Theory

The implications of the results of the differential game theory-based financial management model for corporate governance, optimal capital structure and dynamic game theory are discussed below.

1.) The Nash strategies proposed by the optimal cost of debt and equity as shown in figures 4 and 5 causes an improvement in the firm's liquid reserve over time in encounter 1 and 3 for the feedback and open-loop non-cooperative cases. The result for the liquid reserve levelled out towards a maximum at the near end of the contract when the debtholder does not incur an excessive monitoring cost on the firm. This shows the decreasing marginal effect of the use of debt towards the end of the contract. This provides salient recommendations for a manager in estimating the weighted average cost of capital required to optimize the capital structure while cooperation is yet to be attained.

2.) Similarly, in the absence of cooperation, during the debt maturity period, it is recommended that the company's payouts, that is cost of equity should be returned into the company's fund pool rather than as a cash outflow, e.g., via share repurchase. Thus, a capital distribution may be avoided. This result agrees with Lepetit et al. (2018) which established that a company's payout policy is significantly dependent on the degree of agency conflicts between shareholders and debtholders. Further, the optimal payout policy (cost of equity) can serve as a complementary mechanism for the firm in cushioning the effects of the debtholder's stringent actions on the firm, particularly when cooperation is yet to be attained.

3.) Additionally, the cost of debt which is the debtholder's response to the agency issue declines overtime because long-term relationship can minimize information asymmetries between the debtholder and the firm and can thus reduce agency problems, consequently the agency cost of debt (Fukuda & Hirota 1996).

4.) When compared with interest bearing borrowings and other long-term debt, the initial values of the financial data reveal a low debt to equity ratio was maintained at the start of the dynamic game. The optimal capital structure obtained in this study permits for a greater use of debt to equity than is currently being used, up to the maximum recommended by the optimal debt-equity ratio obtained in figure 9. This result is consistent with He (2011), Mu, Wang and Yang (2017) and Qu et al. (2018) which suggest a need for higher leverage when moral hazard is present even between the shareholder and manager in contrast to Leland (1994), a no moral hazard problem. However, when the two players can reach cooperation, a lower use of debt is required for an optimal capital structure. This implies that cooperation reduces the weight of the optimal leverage required by the company. The optimal capital structure and optimal cost of financing obtained are provided as corporate governance mechanisms that minimizes the marginal agency cost of debt associated with the issuance of debt.

5.) Cooperation as a mechanism via the Pareto case minimizes the conflicts of interests between the two players by disincentivising the manager from substituting the company's asset, which jeopardises the debtholder's value maximization. From the results of the study, the incentives proposed by the debtholder includes the provision of a lower and more consistent cost of debt as well as more debt facility for the company. These are described as a fair distribution of the gains from cooperation (Trost & Heim 2018). The relationship between the company's cost of debt, new debt and total debt was linear in the Pareto case but non-linear in the non-cooperative case. Thus, suggests a more reliable relationship between the players over time. The Pareto optimal solution in the cooperative analysis is to assign a lower weight φ to the manager's objective functional, and a higher weight to the debtholder's objective functional. This is logical because it enhances the interest of the debtholder in the debt contract or strategic game relationship.

6.) During the cooperation, a selfish manager has an incentive, albeit minimal to shift from the optimal pair of weight [0.1, 0.9] to an opportunistic weight assignment [0.7, 0.3], as this provides the firm a minimally higher liquid reserve as seen in figure 7. This proves the theory of Pareto optimality, which states in part that it is impossible to allocate resources in a way that makes one player better off without making the other player worse-off. If a selfish manager reneges from the Pareto optimal strategy, the debtholder responds by increasing the firm's cost of debt and reducing its available debt facility. This is observed by the lower value of debt finance available to the firm as seen in

Figure 9 when the optimal pair of weight [0.1, 0.9] are compared to the suboptimal weight assignment [0.7, 0.3]. These are trigger strategies that enforce cooperation and ensure renegotiation-proofness.

7.) Over time, in the dynamic game relationship, the private information held by the manager may be revealed through the company's regulatory reporting such as annual reports, annual corporate governance statements and other forms of external reporting demanded by the debtholder. Additionally, in a dynamic game, the constrained efficiency of the contractual outcome should be affected by the repeated interactions (Bolton & Dewatripont 2005). From the results, due to the repeated interactions in the optimal contract observable from the pareto-efficient outcome, it is observed that the company enjoys a stable and an efficient cost of debt, a greater provision of debt facility, and a higher liquid reserve overtime.

5 Summary and Conclusion

One main drawback of debt as a key corporate governance mechanism as established by Jensen and Meckling (1976) is that it introduces the asset substitution moral hazard problem in the debtholder-manager agency relationship. Most studies have focused on observing the impact of the moral hazard problem on a firm's capital structure. However, there has been a number of studies designed to minimize the problem. We have offered a more tractable framework using differential game theory to design and observe the contract dynamically. We obtain a Pareto-efficient outcome that minimizes the agency problem and compare these outcomes with non-cooperative scenarios to highlight the benefits of the joint optimisation approach. These provide recommendations for a manager about the optimal financing strategies and the optimal capital structure required for the firm when there are significant effects of agency cost of debts.

For an optimal capital structure in the non-cooperative game, the manager adopts a higher cost of debt and lower cost of equity for the company and avoids capital distribution until the debt matures. The pareto-efficient outcome provides incentives and trigger strategies which serves as corporate governance mechanisms to align the interests of the two parties. Generally, the gains of cooperation were higher than the open-loop and feedback non-cooperative cases for the manager and thus induces him to select the pareto-efficient outcome. The gains include provision of more debt facility with lower and more consistent cost of debt and improved earnings.

The study has modelled the strategic interactions between the debtholder and manager as a dynamic game, and designed mechanisms to minimize the inherent conflicts of interests for specifying an optimal capital structure. Optimal mechanisms are important for company's growth. However, managers may make financing policies at the expense of an effective debt-management policy. The modelling in this paper laid a template for efficient and effective interactions between manager and debtholders. When such optimal strategies are followed, it provides a framework for successful organizational management.

Future research in line of this study will include the signalling use of the state variables and the use of other complementary corporate governance mechanisms in minimizing the highlighted agency cost of debt.

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