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Auctioning Annuities

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Keywords

Annuity Contract, Annuitization Costs, Auctions, Mortality

Disciplines

Economics

Auctioning Annuities

Gaurab Aryal, Eduardo Fajnzylber, Maria F. Gabrielli, and Manuel Willington

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In this paper, we empirically analyze a market for annuity contracts. To this end, we use multi-stage and multi-attribute auctions, where life insurance companies that have private information about their annuitization costs compete for retirees' savings by offering pensions and bequests, and retirees choose "winners" that maximize their *expected present discounted utilities*. We establish the identification of our model parameters and estimate them using rich administrative data from Chile. Our estimates suggest that (i) retirees with lowest savings quintiles value firms' risk ratings (e.g., AAA, AA) the most, while the high savers do not; (ii) almost half the retirees who choose an annuity do not value bequest, and (iii) firms are more likely to have low annuitization cost for retirees in the top-two savings quintiles. Counterfactuals show that private information about costs harms only these high savers, and simplifying the current mechanism by using English auctions *and* "shutting-down" risk ratings lead to higher pensions in equilibrium, but only for the high savers.

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JEL: D14, D44, D91, C57, J26, L13.

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1 Introduction

Most countries have social security programs to help provide retirees with financial security. However, these programs are experiencing enormous pressure to remain solvent and viable. For example, the OECD (2019) notes that "...pressure persists to maintain adequate and financially sustainable levels of pensions as population aging is accelerating in most OECD countries." At the same time, too many people do not have enough retirement savings. Policymakers have proposed several *fiscal measures* to improve these programs, but there is a growing belief that it is fruitful to also use a competitive, market-based system, to provide better retirement products, e.g., annuities, (Feldstein, 2005; Mitchell and Shea, 2016).

Despite this, there is little empirical research about how a market for annuities works, or how the demand and strategic supply interact to determine equilibrium pensions and retirees' welfare. In this paper, we answer these questions in the context of the annuities market in Chile where firms have private information about their annuitization costs and they compete in multi-stage multi-attribute auctions for savings of risk-averse retirees with different mortality risks and preferences. An annuity is an ideal retirement product because it insures against longevity risk (Yaari, 1965; Brown et al., 2001; Davidoff, Brown, and Diamond, 2005), so a better understanding of an annuity market can help many retirees.

For instance, retirement income in Chile is considered low: The median replacement rate (ratio of pension to the last wage) is 44%, whereas the International Labor Organization recommends 70%. According to the antitrust authority (Quiroz et al., 2018) pensions are low because Chile uses complex multi-stage bidding mechanism, and retirees have poor understanding of the role of firms' risk ratings (AA+ vs. AA), which soften competition. There is a proposal in Chile to use English auctions and prohibit the use of risk ratings. Using our model estimates and counterfactuals we evaluate the effect of this proposal on pensions and retirees' welfare. We find that these changes lead to an insignificant increase in pensions *except* for those with high savings. The estimates of the cost distributions suggest that the current system is competitive when it comes to retirees with savings that is less than 60% across all retirees, and for the rest 40%, firms are more likely to have lower costs. So, using English auctions improves pensions only for the high savers.

These and our other estimates of bequest preferences and welfare can be useful for countries that have adopted the "Chilean model" or are considering private a market for annuities. For example, the *SECURE Act of 2019* incentivizes businesses and communities in the United States to band together to offer annuities but is silent about "designing" such a market.

Chile provides an ideal setting to study and evaluate a market for annuity contracts. It

is one of the first countries in the world to adopt a market-based system for annuities. In 1981, Chile replaced its public pay-as-you-go pension system with a new system of privately managed individual accounts. Moreover, since 2004, all retirees use a centralized exchange (known as SCOMP) to choose between an annuity, from among those offered by insurance companies, or a programmed withdrawal option, which is a default "self-insurance" product.

SCOMP provides access to high-quality administrative data that span more than a decade. We observe everything about retirees that firms observe before they make their participation decisions. In particular, for each retiree, we observe her demographic information, savings, names of the participating firms and their offers for different types of annuities (e.g., immediate annuity, an annuity with ten years of guaranteed payments), her final choice, and her date of death, whenever applicable. All annuities are fixed and standardized.

We propose a flexible but tractable model of demand and imperfectly competitive supply for annuities to capture the key market-features.[1] There are at least four main components of a retiree's demand for annuity: Her savings, mortality risk, preferences for bequest and firms' risk ratings, and the monthly pensions.[2] So, we model each retiree as an "auctioneer" who chooses a firm and an annuity that gives her the highest *expected present discounted utility*.[3] To determine each pension's expected present discounted utility, we follow the extant literature and assume that the preferences are homothetic with constant relative risk aversion utility, and retirees' mortality follows a Gompertz distribution.

In Chile, there is uncertainty about the role of firms' savings quintile in retirees' decisions. To capture this uncertainty, we assume that retirees are *rationally inattentive* decision makers, and do not know their preferences for risk ratings. However, they learn about their preferences in the first stage by processing some costly information. We use the discrete choice framework in Matějka and McKay (2015) to model the first stage's decision process. In the second stage, we assume that retirees know their preferences.

On the supply side, we assume that life insurance companies observe everything about the retirees. They know their annuitization costs before participating in a retiree auction. The per-dollar annuitization cost is also known as the *Unitary Necessary Capital* (UNC), and it captures the cost of making a survival-contingent stream of payments. In particular, UNC is the expected amount of dollars required to finance a stream of payments of one dollar until the retiree's death and any proportional obligations to her surviving relatives if any.

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[1]Berstein (2010); Alcalde and Vial (2016, 2017); Morales and Larraín (2017); Fajnzylber and Willington (2019) and Illanes and Padi (2019) also use data from Chile. However, they focus on demand (retirees) and we consider both sides of the market.

[2]Bequest preference affects demand for annuities (Kopczuk and Lupton, 2007; Lockwood, 2018; Illanes and Padi, 2019; Einav, Finkelstein, and Schrimpf, 2010), but we let its distribution to have a "mass-at-zero."

[3]Similar considerations arise when U.S. states bid for firms (Slattery, 2019), and in Internet service markets (Krasnokutskaya, Song, and Tang, 2020), where the "winner" is not necessarily the highest bidder.

For example, if the UNC of a firm is $200, it means that the firm's expected cost to provide a pension of $100 is $20,000. Participating firms bid simultaneously on all of the annuity products that the retiree is interested in. If the retiree chooses from the first round, then the game ends, or else she bargains with the participating firms, where she has imperfect information about firms' annuitization costs and what they can offer.

We establish the identification of our crucial model parameters by relying on the exogenous variation in retirees' demographics, savings, and market interest rates. Intuitively, as demographics and interest rates vary, they affect firms' costs directly (via returns on investment) and indirectly (via mortalities), affecting firms' participation decisions and their offers. For instance, all else equal, a retiree with stronger bequest preferences is more likely to choose annuities with a larger present expected value of bequests, and vice versa.[4] To identify preferences for firms' risk ratings and the conditional distributions of annuitization costs, we use only the second-stage data. We can express the chosen pension as a sum of the difference in utility from the two most competitive firms' risk ratings and the losing firm's annuitization costs. These two competitive firms' identities vary with retirees and with them the differences in risk ratings and the cost for the losing firm, which allow us to apply the identification strategies from random coefficient models and English auctions to our setting.[5]

Our estimates suggest that those who have higher savings have lower information processing costs. This result is consistent because those with more considerable savings tend to be more educated and possibly have better financial literacy. Interestingly, we find that those who use sales agents or directly contact insurance companies behave as if they care a lot more about risk rating than others. One interpretation of this result is that while everyone starts with a prior that puts much weight on the risk ratings, those with lower information processing cost revise their weights downwards.

Approximately half of all retirees who choose annuity show no preference for a bequest. There is, however, considerable heterogeneity among those who value bequests: Those in the lowest and highest savings quintiles, on average, care 1.92 and 2.82 times more about their spouse than themselves, respectively.

Using our demographic information, we also estimate the survival probability for each retiree. Comparing the expected mortality with the model-implied annuitization costs, we find that retirees who may live longer have higher annuitization costs. We find significant heterogeneity in these costs across retirees' and across retirees' savings. However, the average annuitization costs do not increase with savings, despite the fact that in our estimation those

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[4]Existing literature (e.g., Lockwood, 2018) identifies the bequest preference only indirectly from savings.

[5]Our identification strategy does not rely on optimal bidding in the first stage, which involves submitting bids for several types of annuities. Without the first-stage model, however, we cannot determine the ex-ante expected profit, so we cannot identify the entry costs.

with higher savings live longer, which in turn is consistent with other studies (e.g., Attanasio and Emmerson, 2003) that document a negative correlation between wealth and mortality. The average costs do not increase with savings quintiles because for the top two quintiles with 0.14 probability, firms' annuitization costs are less than actuarially fair self-annuitization costs. This probability drops to 0.06 for the rest.

To quantify the effect of asymmetric information on pensions and retirees' ex-post expected utilities, we simulate the equilibrium pension under the assumption that the firms observe each other's annuitization costs while shutting down the risk ratings. We find that the gap between the observed pensions and the complete-information pensions is the largest for retirees who belong to the top two savings quintiles.

This result is even though low savers value risk ratings the most, and they stand to benefit the most. However, our estimates suggest that because firms' risk ratings are not too different, at least for the two most competitive firms, differences in costs are more important in determining pensions than risk ratings' preferences.

Next, we evaluate the effect of replacing the current pricing mechanism with simpler English auctions while shutting down the role of risk ratings. Similar to the complete information counterfactual, we find that using English auctions increases pensions for everyone. However, the gain is minimal for the retirees whose savings belong to the lowest three quintiles. However, we find that these changes do not translate into large gains in ex-post expected present discounted utilities because either the increase is minimal (for those with lower savings) or they increase (for those with high savings). These retirees have higher pensions than other retirees, and because of diminishing marginal utilities, the utility gains from English auctions are minimal.

In the remainder of the paper we proceed as follows. In Section 2, we introduce the institutional detail; and in Section 3, we describe our data. Section 4, presents our model and Section 5 discusses its identification. Sections 6 and 7 present estimation and counterfactual results, respectively and Section 8 concludes. The appendix includes additional details.

2   Institutional Background

The Chilean pension system went through a major reform in the early 1980s, when it transitioned from a *pay-as-you-go* system to a system of fully funded capitalization in individual accounts run by private pension funds (henceforth, AFPs). Under this system, workers must contribute 10% of their monthly earnings, up to a pre-predetermined maximum (which in

2018 was U.S. \$2,319), into accounts that are managed by the AFPs.[6]

Upon reaching the minimum retirement age –60 years for woman and 65 years for men– individuals can request an old-age pension, transforming their savings into a stream of pension payments. In this paper, we focus only on those retirees who have savings in their retirement accounts, that are above a certain threshold, who can, and must, participate in the electronic annuity market.[7]

Regulation

The Chilean government regulates and supervises AFPs, who manage retirement savings during the accumulation phase, and life insurance companies, who provide annuities during the decumulation phase. In addition, at the time of retirement, the government provides subsidies to workers who fail to save enough during their working years (Fajnzylber, 2018).

Moreover, the life insurance industry is heavily regulated. The current regulatory framework for life insurance companies providing annuities recognizes that the main risks associated with annuities are the risk of longevity and reinvestment. Longevity risk is taken care of through the creation of technical reserves by insurers that sell annuities, which consider self-adjusting mortality tables. The government also regularly assesses the risk of reinvestment via the Asset Sufficiency Test established in 2007. Under this regulation, every insurance company is required to establish additional technical reserves, if and when there are "insufficient" asset flows, following the international norm of good regulatory practices in insurance industries. Bankruptcy among life insurance companies is rare in Chile, but the government guarantees every retiree pensions up to 100% of the basic solidarity pension, and 75% of the excess pension over this amount, up to a ceiling of 45 UFs (see footnote 6). Thus, there are enough safety nets for retirees to feel protected in case of a bankruptcy.

2.1 Pension Products

Retirees participating in the electronic market have three main choices: programmed withdrawal (PW), immediate annuity (IA), and deferred annuity (DA).[8] Under PW, savings remain under AFP management and are paid back to the retiree following an actuarially

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[6]This maximum, and annuities in general, are expressed in *Unidades de Fomento* (UF), which is a unit of account used in Chile. UF follows the evolution of the Consumer Price Index and is widely used in long-term contracts. In 2018, the UF was approximately equivalent to U.S. \$39.60.

[7]The threshold is currently established as the amount required to finance a *Basic Solidarity Pension*, which is the minimum pension guaranteed by the state. Retirees with insufficient funds will receive them from the AFP based on a programmed withdrawal schedule.

[8]There is a fourth, rarely chosen, pension product which is a combination between a PW and an IA.

fair benefit schedule. In the event of death, remaining funds are used to finance survivorship pensions or, in absence of eligible beneficiaries, become part of the retiree's inheritance. PW benefits are exposed to financial volatility and provide no longevity insurance so that, barring extraordinarily high returns, the pension steadily decreases over time.

Under both IA and DA, the retiree's savings are transferred to an insurance company of her choice that will provide an inflation-indexed monthly pension to her and her surviving beneficiaries. In deferred annuities, pensions are contracted for a future date (usually between one and three years), and in the meantime the retiree is allowed to receive a temporary benefit that can be as high as twice the pension amount.

Thus, the main trade-off between an annuity and a PW is that an annuity provides insurance against longevity risk and financial risk whereas under a PW a retiree can bequeath all remaining funds in case of an early death. Moreover, while annuitization is an irreversible decision, a retiree who chooses a PW can switch and choose an annuity at a later date.

Annuities may also include a special coverage clause called a guaranteed period (GP).[9] If an annuity includes, for instance, a 10-year guaranteed period, the full pension will be paid during this period to the retiree, eligible beneficiaries or other individuals. Once the guaranteed period is reached, the contracts reverts to the standard conditions (implying a certain percentage of the original pension and only for eligible beneficiaries).[10]

For illustration of how benefits change with the annuity products and marital status, consider a male retiree who is 65 years old, has a savings of U.S. $200,000 and is retiring in 2020. Suppose he is unmarried and chooses an annuity with GP=0 and DP=0, then he gets a constant pension until death (blue '◇' in Figure 1-(a)), but after that his beneficiaries gets nothing (blue '◇' in Figure 1-(b)). But if he chooses an annuity with GP=20, then while alive he gets lower pension (compare red '+' and blue '◇' in Figure 1-(a)), but if he dies within 20 years of retirement, his beneficiaries get a strictly positive amount (purple 'x' in Figure 1-(b)) for 20 years, and after that they get nothing. If he was married, then even with GP=0 and DP=0 (blue '◇' in Figure 1-(c)), the beneficiaries will get a positive amount (blue '◇' in Figure 1-(d)) after the retiree dies.

2.2 Retirement Process

The process of buying an annuity begins when a worker communicates her decision of considering retirement to her designated AFP. We assume that she is then exogenously matched with one of four intermediaries or "channels" who can help her choose a product and firm.

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[9]Another rarely chosen clause is the spouse's percentage increase, which maintains the full payment to the surviving spouse, instead of the mandated 50% or 60% for regular contracts.

[10]In our sample, 99.9% of the chosen annuities correspond to contracts with 0, 10, 15, or 20 years of GP.

Figure 1: **Benefit Schedules, by Annuity Type**
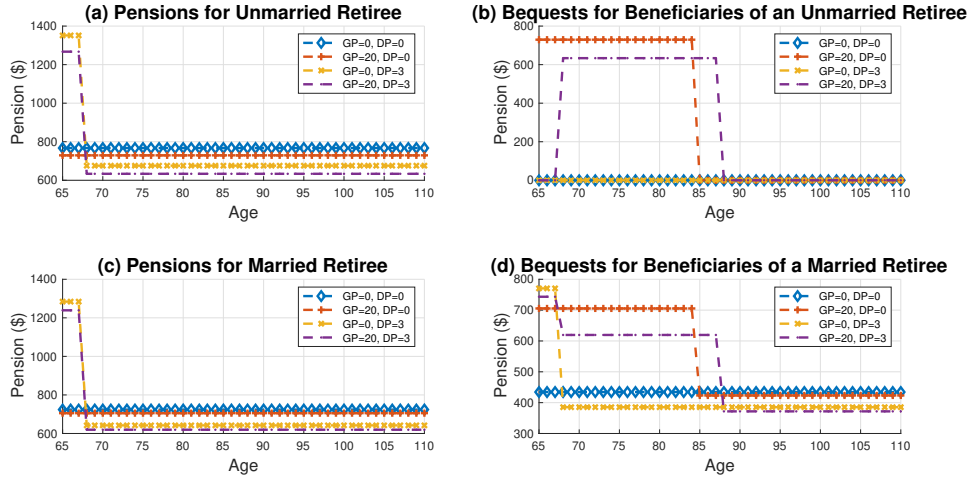
Note: The figure shows the survival-contingent benefit schedules for retirees and their beneficiaries for a representative retiree in our data, who is a 65-year-old male with savings of U.S. $200,000. Subfigures (a) and (b) shows the pension and bequest schedules, respectively, for four types of annuities and if he is unmarried. Similarly, subfigures (c) and (d), show the pension and benefit schedules when he is married. All calculations are performed by the authors using the official 2020 *mortality table*. GP stands for guaranteed period (in years) and DP stands for deferred period (in years).

Out of these four channels, two (AFP and insurance company) are free and the other two (sales agent and independent advisor) charge fees. Retirees must also disclose information on all eligible beneficiaries.[11] The AFP then generates a *Balance Certificate* that contains information about the total saving account balance (henceforth, just savings) and her demographic characteristics. Then the decision process can be described in the following steps:

1. The retiree requests offers for different types of pension products (described above).[12] Upon request, insurance companies in the system have eight business days to make an offer (for every requested annuity products).

2. These offers (i.e., bids) are collected and collated by the SCOMP system and presented to the retiree as a *Certificate of Quotes.* The certificate is in the form of a table, one for each type of annuity, sorted from the highest to the lowest pensions along with the company's name and risk-rating.[13]

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[11]The main beneficiaries are the retiree's spouse and their children under age 24.

[12]Retirees can request quotes up to 13 different variations, including PW and annuities with different combinations of contractual arrangements.

[13]In the case of guaranteed periods, the certificate also includes a discount rate that would be applicable in the event of death within the GP. In absence of legal beneficiaries, other relatives can receive the unpaid benefits in a lump sum, calculated with the offered discount rate. For an example see Figure 1.

3. The retiree can choose from the following five options: (i) postpone retirement; (ii) fill a new request for quotes (presumably for different types of annuities); (iii) choose PW; (iv) accept one of the first round offers for a particular type of annuity; or (v) negotiate with companies by requesting second round offers for one type of annuity. In the latter case, firms cannot offer lower than their initial round offers, and the individual can always fall back to any first round offer.[14]

3 Data

We have data on the annuity market in Chile from January 2007 to December 2017. We observe everyone who used SCOMP to buy an annuity or choose PW during this period. As mentioned before, we observe everything about a retiree that participating life insurance companies observe about them before they make their entry decisions and their first round offers. We observe all the offers they received, their final choice, and whether they chose it in the first round or the second round. Our working assumption is that retirees use the first round offers to decide between different types of annuities in most cases. Furthermore, conditional on choosing the annuity type, they bargain with companies for better pensions in the second round.

3.1 Retirees

Table 1: **Share of Pension Products**

Product	Obs.	%
PW	78,161	32.7
Immediate annuity	87,115	36.4
Deferred annuity	73,272	30.6
Annuity with PW	343	0.9
Full Sample	238,891	100

Note: The table shows the distribution of retirees across different annuity products. We restrict ourselves to annuities with either 0, 10, 15 or 20 years of guaranteed periods or at most 3 years of deferment.

We focus on individuals without eligible children considering retirement within ten years of the "normal retirement age" (NRA), which is 60 years for a woman and 65 years for a man. The result is a data set with 238,891 retirees, with an almost even split between PW,

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[14]A firm that does not offer in the first round cannot participate in the second round.

immediate annuities, and deferred annuities: see Table 1. Less than 1% of retirees choose annuity *with* PW, and so we exclude them, leaving a total of 238,548 retirees.[15]

Table 2: **Age Distribution, by Gender and Marital Status**

Retiring Age	S-F	M-F	S-M	M-M	Total
Before NRA	1,871	1,771	4,714	22,142	30,498
At NRA	20,789	22,475	17,114	72,572	132,950
Within 3 years after NRA	14,470	16,797	4,447	19,086	54,800
At least 4 years after NRA	6,900	6,715	1,251	5,434	20,300
Full Sample	44,030	47,758	27,526	119,234	238,548

Note: The table displays the distribution of retirees, by their marital status, gender and their retirement ages. Thus the first two columns 'S-F' and 'M-F' refer, respectively, to single female and married female, and so on. NRA is the 'normal retirement age,' which is 60 years for a female and 65 years for a male.

In Table 2, we present the sample distribution by retirees' marital status, gender, and age at the time of their retirement. Around 56% retire at their NRA, and 79% retiree at or at most within three years after NRA (rows 2 and 3), and married men are half of all retirees. Retirees also vary in terms of their savings; see Table 3. The mean savings in our sample is $112,471, while the median savings is $74,515 with an inter-quartile range of $85,907. Savings are higher for men and for those who retire before NRA.

Table 3: **Savings, by Retirement Age and Gender**

	Mean	Median	P25	P75	N
Retiring Age					
Before NRA	185,660	129,637	73,104	245,857	30,498
At NRA	89,907	60,023	41,521	103,680	132,950
Within 3 years after NRA	115,666	87,126	54,353	135,562	54,800
At least 4 years after NRA	141,673	101,594	58,815	168,202	20,300
Full Sample	112,471	74,515	46,449	132,356	238,548
Gender					
Female	97,308	81,180	51,817	121,633	91,788
Male	121,955	69,372	43,818	147,184	146,760
Full Sample	112,471	74,515	46,449	132,356	238,548

Note: Summary statistics of savings, in U.S. dollars, by retiree's age at retirement, and by retiree's gender.

3.1.1 First Round Offers

A retiree receives approximately 10.6 offers for several types of annuity, and the number of offers increases with savings. For example, retirees with savings at the 75th percentile of our

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[15]The fourth option allows retirees to split their savings into PW and annuity.

sample get an average of 12.4 offers, and those at the 25th percentile get an average of 7.8 offers. It is reasonable to assume that retirees with higher savings are more lucrative for the firms, and therefore more companies are willing to annuitize their savings. If those with higher savings, however, also live longer than those with lower savings, then it means that annuitizing higher savings is costlier for the firms. To determine which of these two opposing forces dominate, we estimate the annuitization costs and mortality by savings.

Moreover, there is also substantial variation in the pensions offered across life insurance companies and retirees; see Table 4. For an immediate annuity, retirees get an average offer of $570, and for deferred annuities, the average offer is $446. On average, women get an offer of $479 for immediate annuities and $412 for deferred annuities, while for men, they are $631 and $473 respectively. These features are consistent with men having larger savings *and* shorter life expectancy than women (see Table 7).

Table 4: **Monthly Pension Offers, by Annuity type and Gender**

Annuity Type	Gender	Mean	Median	Savings Q1	Savings Q2	Savings Q3	Savings Q4	Savings Q5
Immediate	Female	479	414	202	288	385	510	857
	Male	631	435	200	269	372	585	1329
	Full Sample	570	423	201	278	378	556	1152
Deferred	Female	412	374	190	258	349	463	714
	Male	473	356	187	241	331	529	1019
	Full Sample	446	365	189	248	339	500	882

Note: Summary of average monthly pensions (in U.S. dollars) offers received in the first round.

In our empirical model, we rationalize this variation in pension offers by allowing firms to have heterogeneous costs (UNCs) of annuitization. We assume that only the firm knows its annuitization cost, which can depend on retirees' savings. An important exogenous factor affecting UNCs is the market interest rate, which affects the opportunity cost of offering a pension at retirement. Our sample spans a decade, so we observe substantial variation in interest rates, which causes exogenous variation in annuitization costs.

3.1.2 Chosen Annuities

Once the participating companies make first round offers, one for each type of annuity the retiree requests quotes for, she can either choose from one of those offers or buy a PW, or initiate the second round bargaining phase. Table 5 displays the distribution across these stages. Most retirees who choose PW choose that in the first round (98.1%), and most

retirees (86.9%) who choose annuity choose in the second round. As we can see, 2,979 retirees opt for the second round but choose an annuity quote from the first round.

Table 5: **Number of Retirees who choose in First or Second Round**

Round/Choice	PW	1st round	2nd round	Total
1st round	76,690	18,001	0	94,691
2nd round	1,471	2,979	139,407	143,857
Total	78,161	20,980	139,407	238,548

Note: Round refers to whether retirees chose in the first or second round.

In Table 6, we present information about the chosen annuities: (i) the total number of accepted offers by the type of annuity; (ii) the average number of first round and second round offers received for the chosen annuity; (iii) the number of accepted second round offers; (iv) the average percentage increase in pension offers from first round to second round (only for the accepted choice); (v) the percentage of retirees who requested at least one second round offer; (vi) the percentage of retirees who chose the highest-paying alternative; and (vii) the percentage of retirees who chose a dominated option, in terms of either pension (with the same risk-rating) or risk ratings (with the same pension) or both.

Table 6: **Summary of Accepted Annuities**

GP Months	# Accepted	Average # of 1st Round Offers	# Accepted in 2nd Round	Increase	Average % Requested 2nd Round	Best	Dominated
			Immediate				
0	21,292	11.3	16,357	1.5	80	59	22
120	26,907	11.1	23,463	1.3	89	51	28
180	24,452	11.6	22,070	1.4	92	49	29
240	14,464	11.8	13,020	1.5	92	51	29
Total	87,115	11.4	74,910	1.4	88	53	27
			Deferred				
0	11,703	10.9	8,919	1.5	79	53	23
120	26,119	11.0	23,390	1.4	91	46	31
180	26,775	11.4	24,324	1.4	92	42	34
240	8,675	11.0	7,864	1.3	92	42	34
Total	73,272	11.1	64,497	1.4	90	45	31

Note: The table shows the number of chosen annuities by type of product, the average number of first round offers received for the chosen annuity, the number of accepted offers that resulted from second round offers, the average percentage increase between the first round and second round offers (for the accepted choice), the percentage of individuals who requested at least one second round offer, the percentage of retirees who chose the highest-paying alternative option and the percentage of individuals who chose an offer that was dominated by another alternative with same (or better) credit rating.

From Table 6, we see that some retirees *do not* choose the annuity with the highest pension. One way to rationalize this behavior is to recognize that besides pensions, retirees also care about firms' risk ratings. After all, risk-rating is a proxy of financial health, and

it is also widely advertised as such. A retiree can prefer lower pensions from healthier firms to a higher pension from a less healthy firm.

This rationalization, however, begs the follow-up questions: Is there an objective (i.e., correct) trade-off between pension and risk-rating, and should it be homogeneous or vary across retirees? If it is heterogeneous, should it increase or decrease with savings? On the one hand, because of the regulation, those with lower savings are less exposed to the risk of firms defaulting than those with higher savings; those with higher savings should care more about the risk ratings than those with lower savings. On the other hand, savings positively correlated with education, so those with higher savings will process publicly available information about past defaults. Thus, in the case of Chile this suggests that retirees should not care much about the risk-rating. Finally, how does this trade-off vary with preferences for bequests? To determine which of these countervailing forces dominate and how pensions and utilities would change under alternative market rules, later we estimate a structural model.

3.1.3 Mortality

A determinant of annuity demand and supply is the retiree's expected mortality. We observe every retiree when they entered our sample, i.e., their retirement age and their age at death if they die by the end of our sample period. Using this information, we estimate a mixed proportional hazard model (defined shortly below) and use the estimated survival function to predict the expected life conditional on being alive at retirement.

Let the hazard rate for retiree i with socioeconomic characteristics X_i at time $t \in \mathbb{R}_+$, that includes includes i's age, gender, marital status, savings and the year of birth, be $h_{it} = \lim_{dt \to 0} \frac{d \Pr(m_i \in [t, dt) | X_i, m_i \geq t)}{dt} = h(X_i) \times \psi(t)$, where m_i is i's realized mortality date, $\psi(t)$ is the baseline hazard rate. Furthermore, let the hazard function $\psi(t)$ be given by Gompertz distribution, such that the probability of i's death by time t is $F_m(t; \lambda_i, \mathfrak{g}) = 1 - \exp(-\frac{\lambda_i}{\mathfrak{g}} (\exp(\mathfrak{g}t) - 1))$, and let $\lambda_i = \exp(X_i^\top \tau)$.

The identification of such a model is well established in the literature (Van Den Berg, 2001). The maximum likelihood estimated coefficients of the hazard functions suggest a smaller hazard risk is associated with younger cohorts, individuals who retire later, with females, those who are married, and those with higher savings.[16] Using these estimates, we report the median expected lives, by gender and savings quintile, and their standard errors in Table 7. Overall, 50% of males expect to live until 86 years, and 50% of females expect to live until they are 94.9 years old. As we can see, those who have larger savings also tend

---

[16] For robustness, we estimated the Gompertz model using data from before the introduction of SCOMP. The estimates are qualitatively the same. For instance, the predicted median expected life at death is 85 and 96 for males and females, respectively. Both of these results are available upon request.

to live longer than those with lower savings.

Table 7: **Median Expected Life, by Savings Quintile**

| Savings	Male	Female	Overall
Q1	85.15	93.80	86.89
	(5.79)	(6.03)	(5.82)
Q2	85.86	94.24	87.64
	(5.81)	(6.06)	(5.84)
Q3	86.45	94.83	88.23
	(5.83)	(6.09)	(5.88)
Q4	87.62	95.48	89.40
	(5.88)	(6.12)	(5.95)
Q5	90.87	97.25	93.52
	(6.01)	(6.21)	(6.11)
Total	86.75	94.91	89.57
	(5.82)	(6.09)	(5.94)

Note: The table shows the predicted median expected life at the time of retirement implied by our estimates of the Gompertz mortality distribution. Standard errors are reported in the parentheses.

3.2 Intermediary Channels

We observe retirees with one of the four intermediary channels (AFP, insurance company, sales agent, or independent advisor) to assist them with their annuitization process. If and when such an intermediary's incentives do not align with those of a retiree, then retirees do not always choose the "best" option for them. The misalignment of incentives may be particularly relevant for sales agents, who receive their intermediation fee only if the retiree chooses the sales agent's firm. In other words, it is possible and very likely that those with a sales agent would appear to value the non-pecuniary benefits of a company more than the pecuniary benefits. We allow preferences for risk ratings and information processing costs to depend on the channel to capture this effect.

To account for observed differences among retirees, we consider the money's worth ratio (henceforth, mwr), which is the expected present value of pension per annuitized dollar. If mwr $= 1$, then it means the retiree expects to get \$1 pension (in present value) for every annuitized dollar. In Figure 2, we display the distributions of the mwr offered in the first round (left panel) and mwr accepted by the retirees (right panel). The mean and the median mwr of the offers, by channels (AFP, insurance company, sales agent, advisor), are $(0.989, 0.988, 0.984, 0.987)$ and $(0.990, 0.989, 0.986, 0.988)$, respectively, but the means and medians for accepted offers are $(1.010, 1.010, 0.990, 1.007)$ and $(1.010, 1.009, 0.991, 1.007)$, respectively. Thus, the final accepted offers are on average better than the first round offers, and those with sales agents have lower mwr.

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Figure 2: **CDFs of Offered and Accepted MWR, by Channel**
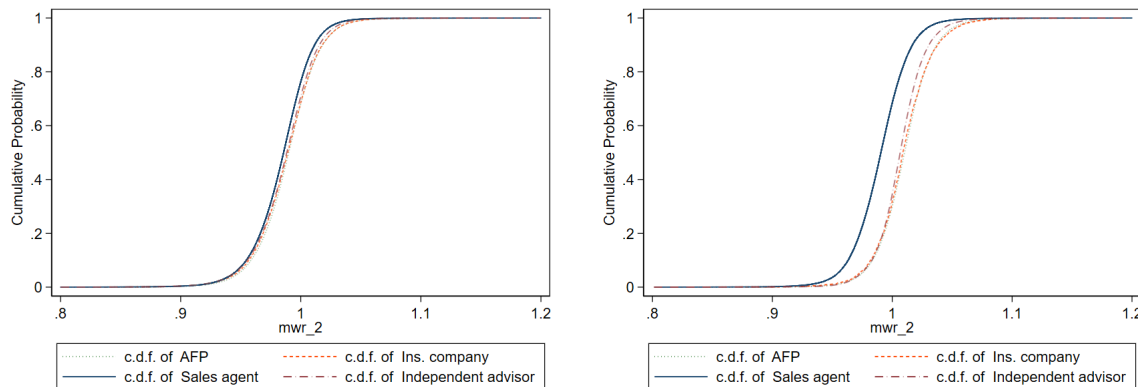
Note: Distributions of the offered and chosen `mwr` (left panel vs. right panel), by channel.

We use a multinomial logit model to consider if observed differences among retirees can explain the differences in their channels see; Table 8. In particular, we estimated the log-odds ratio of having one of the three intermediary channels relative to the AFP and found that some characteristics and the channel are correlated. For instance, those who have lower savings, retire early, are male or unmarried are more likely to use sales agents than AFP.

Table 8: **Intermediary Channel - Estimates from Multinomial Logit**

Regressors \ Channels	Insurance Company	Sales-Agent	Advisor
Savings ($million)	0.629***	-0.857***	-0.130***
	(0.128)	(0.0436)	(0.0447)
Age	0.0131	-0.0408***	-0.0816***
	(0.00857)	(0.00189)	(0.00218)
Female	0.437***	-0.0588***	-0.124***
	(0.0546)	(0.0120)	(0.0140)
Married	0.0245	0.0620***	0.0874***
	(0.0491)	(0.0107)	(0.0127)
Constant	-5.029***	2.333***	4.326***
	(0.560)	(0.123)	(0.142)
N	238,548	238,548	238,548

Note. Estimates of multinomial logit regression for channels, where the baseline choice is AFP. Standard errors are in parentheses, and ***,**,* denote p-values less than $0.01, 0.05$ and 0.1, respectively.

We treat the channel as exogenous for model tractability. There are two reasons why we believe this is not as strong an assumption in our context as it might appear. First, several anecdotal evidence from Chile suggests that most people rely on word-of-mouth when it comes to a channel. Second, and as mentioned previously, we observe everything the firm observes about a retiree when making the first round offers. When we estimate

the preference parameters, we estimate them separately for several groups that we define based on age, gender, savings, and channels. Estimating preference parameters separately for each group allows us to control the effects of potential selection based on unobservable characteristics.

For instance, from Table 9, we see that channels affect the outcomes. Out of 109,786 retirees who choose AFP, only 25.1% choose the second round, whereas the shares are 85.2%, 92.0%, and 87.8% for insurance company, sales agents, or advisors, respectively. Most of those who choose PW have AFP, and those with sales agents are least likely to choose PW.

Table 9: **Retiree choices, by Intermediary Channel**

	N	**Requests 2nd Round**	**Chooses PW**	**Chooses in 2nd Round**
AFP	109,786	0.251	0.661	0.235
Company	2,169	0.852	0.066	0.817
Sales-agent	79,120	0.920	0.030	0.907
Advisor	47,473	0.878	0.066	0.846
Full Sample	238,548	0.603	0.328	0.584

Note: Proportion of retirees separated by their choices and their channel.

Our empirical framework can capture the effect of channels on outcomes. In particular, we posit that channels affect the cost of acquiring information about the importance of risk-rating. For instance, we allow those retirees who use sales agents to act "as if" they have a higher cost of acquiring information about the trade-off between risk-rating and pensions. We assume that in the first stage, retirees are rationally inattentive with respect to their preference for risk ratings, but they know their preferences in the second stage.

3.3 Firms

In our sample, we observe 20 unique life insurance companies, and they differ in terms of their annuitization costs, which are unobserved, and in terms of their risk ratings. Table 10 shows the distribution of risk ratings. The ratings mostly remain the same over time, and most companies have high (at least AA) risk ratings. For our empirical analysis, we treat these ratings as exogenous, and group them into three categories: 3 for the highest risk rating of AA+, 2 for all the risk ratings from AA to A, and 1 for the rest.

Although there are 20 unique firms, not all of them are active at all times, and not all participate in every auction. On average, 11 companies participate in a retiree auction, which suggests that the market is competitive. We define potential entrants (for each retiree auction) as the set of active firms that participated in at least one other retiree auction in the same month. In our sample, retirees have either 13, 14, or 15 potential entrants.

Table 10: **Risk ratings**

Rating	Frequency	%	Cumulative %
AA+	155	24.64	24.64
AA	245	38.95	63.59
AA-	171	27.19	90.78
A+	2	0.32	91.1
A	15	2.38	93.48
BBB+	1	0.16	93.64
BBB	6	0.95	94.59
BBB-	15	2.38	96.98
BB+	19	3.02	100
Total	629	100	

Note: The table shows the distribution of quarterly credit ratings from 2007-2018.

The participation rate, which is the ratio of the number of actual bidders to the number of potential bidders, varies across our sample from as low as 0.08 to as high as 1, with mean and median rates of 0.73 and 0.78, respectively, and a standard deviation of 0.18.[17] Thus, it is likely that a firm's decision to participate depends on its financial position when a retiree requests quotes and this opportunity cost of participating can vary across retirees.[18] To capture this selection, in our empirical application, we follow Samuelson (1985) to model firms' entry decisions, which posits that firms observe their retiree-specific annuitization cost before entry. This entry method is a reasonable assumption in our setting because firms have sophisticated models to predict retirees' mortality and the expected returns from the savings.

We treat firms as symmetric bidders with annuitization costs independently and identically distributed with some (unknown) distribution function. We do not observe firms' annuitization costs, and so, we cannot directly test this assumption. However, we can perform a diagnostic test and check if the firm-specific pension (bid) distributions are different. If they are not different from one another, then our symmetry assumption is a reasonable first step.

However, to perform this test, we have to "control" for all relevant factors that can affect the pension. For instance, retirees with high savings can be lucrative because the total gain from annuitizing their savings will be large. However, as we have seen above, these retirees are expected to live longer. To compare the bids across firms, we have to estimate the

---

[17]Using a Poisson regression of the number of participating firms on the retiree characteristics, we find that one standard deviation increase in savings, which is approximately $87,000, is associated with roughly one more entrant. Moreover, women have 0.61 more participating companies than men, while sales agents and advisors are associated with approximately 0.19 fewer participants than the other two channels.

[18]We tested this selection by estimating a Heckman selection model with the number of potential bidders as the excluded variable and found strong evidence of negative-selection among firms.

expected discounted life for each retiree, which we refer to as UNC_i where the subscript i refers to retiree i. This UNC_i is different from UNC_j, where the latter refers to a firm j's cost. We formally define UNC_i when we present our model's supply side, and in Appendix A.1 we detail how we use the estimates from the mortality distribution to calculate UNC_i. But for now, it is sufficient to know that UNC_i depends on i's estimated mortality and the discount factor. A retiree who expects to live longer will have a larger UNC_i and will be costlier for firms to annuitize, but these costs are unobserved.

For each of the 20 firms, in Figure 3 we present the histograms and scatter plots of monthly pension per annuitized dollar (which is known as the monthly pension rate) and the UNC_is of all the retirees that the firms make offers to in the first stage. Using pension rates instead of pensions allows us to compare across different retirees. As we see, indeed UNC_i and pension rates are negatively correlated, and there are no differences across firms. Now, using these UNC_is, we can compare pensions across firms. We normalize the offered pension rates (ratio of monthly pension to annuitized savings) across firms, and compare the distributions across firms. We say that firms are asymmetric if the distributions are different and symmetric otherwise. For each firm we estimate

$$
\begin{aligned}
\texttt{Pension-Rate}_{i,j} \;=\; & \texttt{constant} + \beta_1 \times UNC_i + \beta_2 \times \texttt{Age}_i + \beta_3 \times \texttt{Gender}_i \\
& + \beta_4 \times \texttt{Marital Status}_i + \beta_5 \times \texttt{Spouse's Age}_i \\
& + \beta_6 \times \texttt{Guaranteed Months}_i + \beta_7 \times \texttt{Potential Bidders}_i + \varepsilon_{i,j}, \quad (1)
\end{aligned}
$$

using ordinary least squares method, and predict the residual $\hat{\varepsilon}_{i,j}$ for retiree i and firm j. In Figure 4, we show the Kernel density estimate of the firm-specific distribution of $\hat{\varepsilon}_{i,j}$. We can see that these 20 distributions are very similar, so it is reasonable to say that firms have symmetric cost distribution.

4   Model

In this section, we introduce our model. To model the demand, we consider the decision problem facing a retiree who uses SCOMP to choose a company to annuitize her savings. To model the utility from an annuity, we closely follow the extant literature on annuities, particularly, Einav, Finkelstein, and Schrimpf (2010), with a modification that accounts for heterogeneous preferences for firm characteristics.

As we have shown before, retirees do not always choose the best offer. To rationalize this, we posit that besides the pecuniary aspect of an annuity, retirees also care about a company's risk ratings, which is a proxy for the likelihood of default. That said, we assume

Figure 3: **Pension Rates and UNC_i for each Firm**
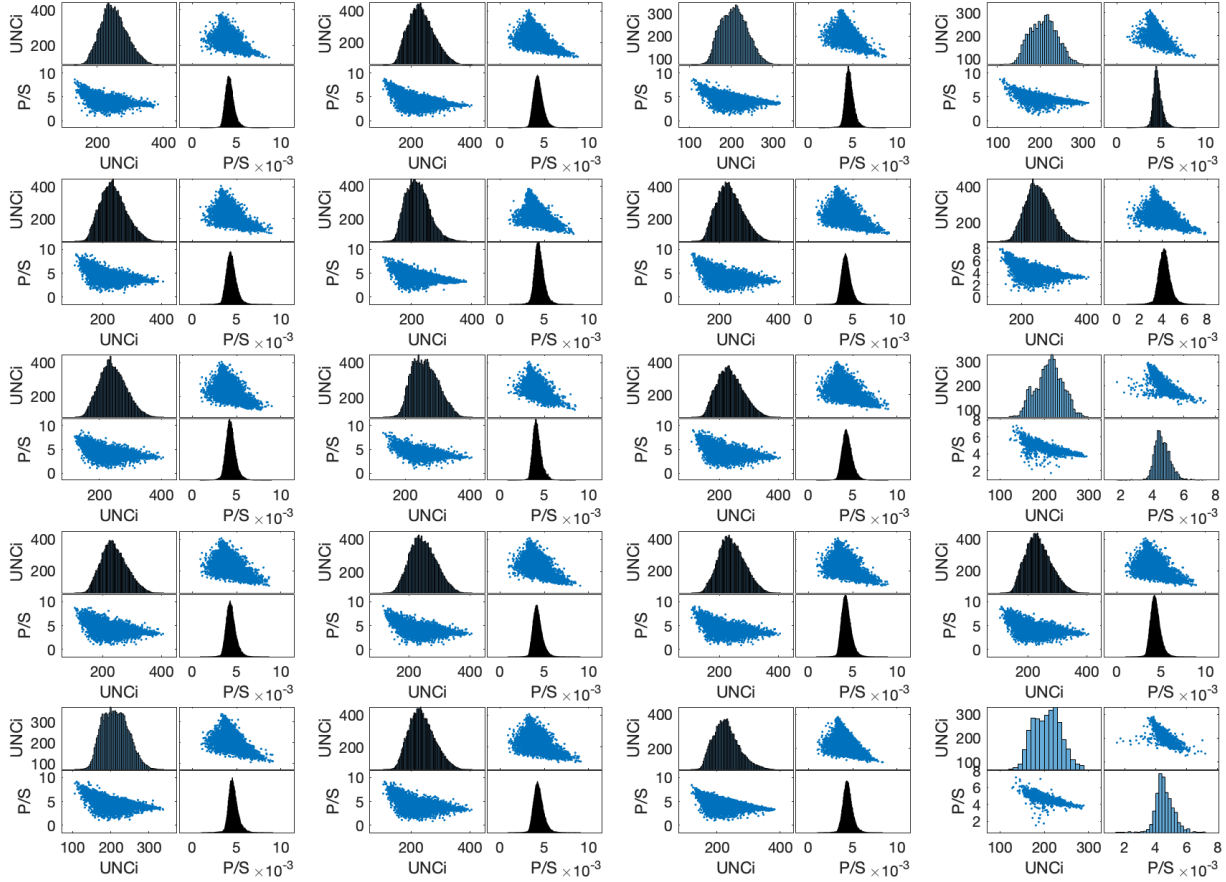
Note: These are histograms and scatter plots of monthly pension rate, i.e., the ratio of monthly pension to annuitized savings, and the UNC_i of the retirees the firms make an offer. There are 20 firms, so there are 20 sets of four subfigures each. Clockwise, the first subfigure is the histogram of UNC_i, and the second subfigure is the scatter plot of the pension rates (on the x-axis) and UNC_i (on the y-axis). The third subfigure is the histogram of the pension rates, and the last subfigure is the scatter plot of UNC_i and the pension rates.

that all retirees have a prior that puts much emphasis on risk-rating, and only those who spend some resources learning about the likelihood of default will update their prior and choose accordingly. To capture the trade-off between pension, risk ratings, and information gathering, we follow Matĕjka and McKay (2015) and model the retiree as a rationally inattentive decision maker. If a retiree chooses to go to the second round bargaining, we assume that she knows her risk ratings preferences.

On the supply side, we model the imperfect competition using an extensive form game where the first stage is a first-price auction with independent private value and endogenous

Figure 4: **Distributions of Homogenized Pension Rates, by Firms**
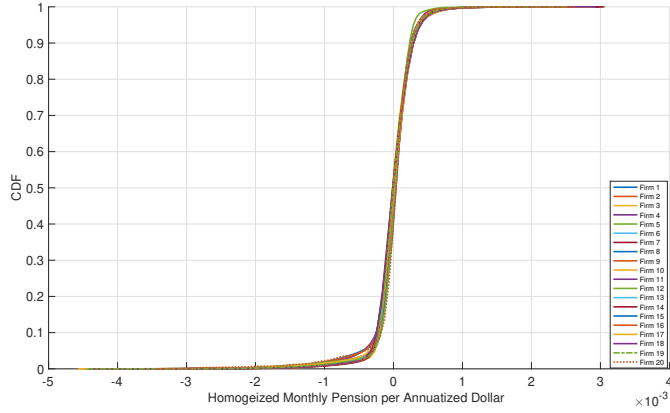
Note: Kernel estimates of the distribution residuals $\hat{\varepsilon}_{ij}$ from Equation (1), one for each firm.

entry (Samuelson, 1985). If there is a second stage, then it is multilateral bargaining with one-sided asymmetric information. The winner of the game is not always the firm that offers the highest pension because the probability of winning depends on the bids and the preferences for risk-rating and bequest, which can vary across retirees.

4.1 Demand

Here, we consider the problem faced by an annuitant i who has already decided which annuity product to choose (e.g., an immediate annuity with 0 guaranteed period) and is considering between J_i firms who have decided to participate in the auction for i's savings S_i. The retiree will choose the firm that provides her the highest indirect utility.

We assume that the utility from an annuity consists of three parts: the *expected present discounted utility* from the monthly pension that the retiree enjoys while alive, utility she gets from leaving bequest (if any) to her kin, and her preference for firm's risk rating. Retirees may value the risk ratings because they may dislike firms with lower risk ratings. However, they may not know the "correct" weight to put on these risk ratings. To capture this uncertainty, we model retirees as rationally inattentive decision makers. We explain this aspect shortly below, but for ease of exposition, we begin without rational inattention.

Let (θ_i, β_i) denote i's preferences for bequest and risk-rating, respectively, and given savings S are distributed independently and identically across retirees as $\tilde{F}_\theta(\cdot|S) \times F_\beta(\cdot|S)$ on $[0, \bar{\theta}] \times [\underline{\beta}, \overline{\beta}]$. To capture the fact that retirees might not be able to afford bequest, and therefore will act as someone who does not care about bequest we allow $\tilde{F}_\theta$ to have a mass

point at $\theta = 0$. Letting $\zeta \in (0,1)$ be the probability that the retiree has $\theta_i = 0$, and let $F_\theta(\cdot) = \zeta \times H(0) + (1-\zeta) \times \tilde{F}_\theta(\cdot)$ where, $H(0)$ is a Heaviside function and $\tilde{F}_\theta$ is the continuous distribution on $(0, \bar{\theta}], \bar{\theta} < \infty$.

Let P_{ij} denote the pension offered by firm j to retiree i. Given the type of annuity and the pension P_{ij}, i's expected mortality and the mortality of her beneficiaries determine the bequest, which we denote by $B_{ij}(P_{ij})$. Whenever it is clear from the context, we suppress the dependence of B_{ij} on P_{ij}. Let i's indirect utility at retirement from choosing an annuity with pension and bequest (P_{ij}, B_{ij}) from firm j with risk rating $Z_{i,j} \in \{1,2,3\}$ be

$$U_{ij} = \underbrace{\mathfrak{U}(P_{ij}, B_{ij}; \theta_i)}_{i\text{'s discounted utility}} + \underbrace{\beta_i \times Z_j,}_{i\text{'s preference for } j\text{'s risk-rating}} - \underbrace{\mathfrak{U}_{0i}(S_i),}_{\text{outside utility}} \tag{2}$$

where the utility $\mathfrak{U}_{0i}(S_i)$ is the utility associated with the outside option.

Next, we explain the expected present discounted utility, $\mathfrak{U}(P_{ij}, B_{ij}; \theta_i)$. For simplicity, consider only the first month after retirement, and let q_i be the probability of being alive one month after retirement. Then, the expected present discounted utility will be

$$\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i) = u(P_{ij}) \times q_i + \theta_i \times v(B_{ij}(P_{ij})) \times (1 - q_i),$$

where $u(P_{ij})$ is the utility from P_{ij}, and $v(B_{ij})$ is the utility from leaving a bequest B_{ij}. Thus, the marginal utility from leaving a bequest B_{ij} upon death is $\theta_i \times (1 - q_{it}) \times v'(B_{ij})$. Now, let us consider two periods after retirement. We have to adjust the probability that the retiree survives two periods given that she is alive at retirement and take into account that the bequest left upon death will also change, which in turn depends on whether the annuity product under consideration includes a guaranteed period.

In practice, we do not know for how long i expects to live. So, to determine expected longevity at retirement, we estimate a continuous-time Gompertz survival function for i and her spouse (if she is married) as a function of her demographic and socioeconomic characteristics. Once we have the survival probabilities, the expected discounted utilities become the product of $u(P_{ij})$ and the discounted number of months i expects to live, where the discount factor is the market interest rate.

Even with a bequest, $\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i)$ has an intuitive structure. It is a sum of two terms, one of which is the product of $u(P_{ij})$ and the discounted number of months i expects to live, and the other term is the product of $v(B_{ij})$ times the discounted number of months i's beneficiaries expect to receive B_{ij}. Legally, i's spouse is entitled to 60% of i's pension, and 100% during the guaranteed periods; the amount B_{ij} may change over time.

Thus, we can write $\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i)$ as

$$
\begin{aligned}
\mathfrak{U}(P_{ij}, B_{ij}(P_{ij}); \theta_i) &:= u(P_{ij}) \times D_i^R + \theta_i \left(v(0.6 \times P_{ij}) \times D_i^S + v(P_{ij}) \times D_i^{S,GP} \right) \\
&\equiv \rho_i(P_{ij}) + \theta_i \times b_i(P_{ij}),
\end{aligned}
\tag{3}
$$

where D_i^R is the discounted expected longevity of the retiree (in months, from the moment the annuity payments start) and $D_i^{S,GP}$ is the discounted number of months that the spouse (or other beneficiaries) will receive the full pension because of the guaranteed period. Furthermore, D_i^S is the discounted number of months that the spouse will receive 60% of the retiree's pension.[19] If the annuity has a deferred period, then the retiree gets twice her pension until the annuity payment begins. So $\rho_i(P_{ij}) = u(P_{ij}) \times D_i^R + u(2P_{ij}) \times D_i^{R,DP}$ where $D_i^{R,DP}$ is the expected life during the deferred period.[20]

However, a retiree can have additional wealth, besides S_i, that she can use for consumption or bequest, especially those who are wealthy. However, we do not observe her consumption (after retirement) or her wealth, so following the literature (Mitchell et al., 1999; Einav, Finkelstein, and Schrimpf, 2010; Illanes and Padi, 2019), we assume that retirees have homothetic preferences. In particular, we assume that all retirees have CRRA utility $u(c) = v(c) = \frac{c^{(1-\gamma)}}{1-\gamma}$ with $\gamma = 3$. Homothetic preferences imply that the retiree's annuity choice does not depend on the unobserved wealth. In Appendix A.1, we detail the steps to estimate $\rho_i(P_{ij})$ and $b_i(P_{ij})$.

Substituting (3) in (2) we can express i's indirect utility from annuity P_{ij} from firm j as

$$
U_{ij} = \rho_i(P_{ij}) + \theta_i \times b_i(P_{ij}) + \beta_i \times Z_{ij} - \mathfrak{U}_{0i}(S_i).
\tag{4}
$$

Thus, (4) shows that there is a trade-off between higher pensions and lower risk ratings, but as mentioned above, we assume that i does not know her β_i, but only its distribution.

We follow Matëjka and McKay (2015) and assume that before the retirement process begins, i has a belief that $\beta_i \overset{i.i.d}{\sim} F_\beta(\cdot)$ with support $[\underline{\beta}, \overline{\beta}]$, and if i wants to learn her preference, she has to incur information processing cost, valued at $\alpha > 0$ per unit of in-

---

[19]These "discounted life expectancies" also have interpretation in terms of the annuitization costs. Assuming firms use the same mortality process as us and invest retirees' savings at an interest rate equal to the discount rate, then D_i^R is the necessary capital to provide a one-dollar pension to the retiree until she dies. Similarly, $D_i^{S,GP}$ is the necessary capital to finance a dollar of pension for the beneficiaries once the retiree is dead and until the guaranteed period expires. Finally, D_i^S is the necessary capital to finance a dollar of pension for the beneficiaries between the retiree's death or the guaranteed period is over (whichever occurs later) and until the spouse dies. The gains from trade between retirees and insurance companies come from the differences in risk-attitude between retirees and life insurance companies and potential differences between the discount rate of retirees and firms' investment opportunities.

[20]For simplicity, we are disregarding survival benefits during the deferment period. Deferred periods in our sample are at most three years. Thus, death probability is quite low.

formation. So, i has first to decide how much to spend learning about β_i, and after that make the decision. Let $\sigma : [\underline{\beta}, \overline{\beta}] \times \mathcal{P} \to \Gamma := \Delta([0,1]^{J+1})$ denote the strategy of a retiree with preference parameter β, with offered pensions $\boldsymbol{P}_i := (P_{i1}, \ldots, P_{iJ}) \in \mathcal{P}$. The strategy is a vector $\sigma(\beta, \boldsymbol{P}_i) \equiv (\sigma_1(\beta, \boldsymbol{P}_i), \ldots, \sigma_J(\beta, \boldsymbol{P}_i), \sigma_{J+1}(\beta, \boldsymbol{P}_i))$ of probabilities, where $\sigma_j(\beta, \boldsymbol{P}_i) = \Pr(i \; \texttt{chooses} \; j | \beta, \boldsymbol{P}_i) \in [0,1]$. For notational simplicity, we suppress the dependence of choice probabilities on the offers ($\boldsymbol{P}_i$). Then, by adapting Matějka and McKay (2015)'s choice formula to two periods, the probability that i chooses j is given by

$$\sigma_{ij}(\boldsymbol{P}_i) = \sigma_j(\beta_i, \boldsymbol{P}_i) = \begin{cases} \dfrac{\exp\left(\log \sigma_j^0 + \frac{U_{ij}}{\alpha}\right)}{\sum_{k=1}^J \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}, & j = 1, \ldots, J \\[3ex] \dfrac{\exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}{\sum_{k=1}^J \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)}, & j = J+1. \end{cases} \tag{5}$$

4.2 Supply

Next, we present the supply side, where J insurance companies participate in an auction run by "auctioneer" i with characteristics $X_i \equiv (S_i, \tilde{X}_i)$. For simplicity, we suppress the dependence on X_i and treat J as fixed, but account for selection in our empirical application.

Companies differ in terms of their UNCs. Thus, if j can annuitize i cheaper than j', then j has an advantage over j' because all else equal, j can offer a higher pension. Let UNC_j^R be j's unitary necessary capital to finance a dollar pension for the retiree. Similarly, we must consider the costs related to the bequest, which may come from two sources: a guaranteed period, during which after the death of the retiree the beneficiaries receive the full amount of the pension, and the compulsory survival benefit, according to which the spouse of the retiree receives after the retiree died and after the guaranteed period is over, 60% of the pension until death, see Equation (A.1). We denote by $UNC_j^{S,GP}$ and UNC_j^S the present value of the cost of providing these two benefits. Then, j's expected cost of offering P_{ij} is

$$C(P_{ij}) := P_{ij} \times (UNC_j^R + 2 \times UNC_j^{R,DP} + 0.6 \times UNC_j^S + UNC_j^{S,GP}) \equiv P_{ij} \times UNC_j. \tag{6}$$

Here, the 2 in (6) follows from our assumption that the life insurance company made the pension payments during the deferred period. Let UNC_i be the unitary cost of a pension calculated with the retirees' discount rate and the mortality process we estimate. For the same retiree i, firms' UNCs may differ from UNC_i due to the differences in their (i) mortality estimates, (ii) investment opportunities, and (iii) expectations about future interest rates. For these reasons, it is more likely that only firm j knows its UNC_j. Moreover, the ratio of UNC_j to UNC_i captures j's relative efficiency selling an annuity to i. Henceforth, we

call this ratio $r_{ij} \equiv \frac{UNC_j}{UNC_i}$, j's relative cost of annuitizing a dollar. Working with r, we can compare retirees who otherwise will have different UNC_i's.

We assume the cost r_{ij} is private and is distributed independently and identically across companies as $W_r(\cdot|S)$, with density $w_r(\cdot|S)$ that is strictly positive everywhere in its support $[\underline{r}, \overline{r}]$. Thus, we assume that firms are symmetric, and this is consistent with what we observe in the data; Figure 4. Allowing the cost distribution to depend on S captures the fact that those who have higher savings tend to live longer and, therefore, are costlier to annuitize.

Ignoring for now the second round, and the multi-product nature of the first round, j's net present expected profit from offering P_{ij}, to a retiree i with S_i is

$$
\begin{aligned}
\mathbb{E}\Pi_{ij}^I(P_{ij}) &= (S_i - P_{ij} \times UNC_j)) \times \Pr(\texttt{j is chosen by offering } P_{ij}|\mathbf{P}_{i-j}) \\
&= S_i \times (1 - r_{ij} \times \rho_i^*(P_{ij})) \times \sigma_{ij}(\mathbf{P}_i), \quad\quad\quad (7)
\end{aligned}
$$

where $\rho_i^*(P_{ij}) \equiv P_{ij} \times UNC_i/S_i$ is the money worth ratio (`mwr`) computed using the retirees' discount rate, and $\sigma_{ij}(\mathbf{P}_i)$ is the probability that i chooses j given the vector of offers $\mathbf{P}_i$. Considering the second round, and denoting by $\tilde{P}_{ij}$ the second round offer of firm j

$$
S_i \times (1 - r_{ij} \times \rho_i^*(P_{ij})) \times \sigma_{ij}(\mathbf{P}_i) + \sigma_{iJ+1}(\mathbf{P}_i) \times \mathbb{E}\Pi_j^{II}(\rho_i^*(\tilde{P}_{ij})|r_{ij}, \mathbf{P}_i), \quad\quad (8)
$$

is its ex-ante expected profit, where $\sigma_{iJ+1}(\mathbf{P}_i)$ from (5) is the probability that i takes the bargaining option in the second round with expected profit given by $\mathbb{E}\Pi_j^{II}$.

The two rounds are connected. First, more generous offers on the first round may lower the retiree's probability of going to the second round. Second, and more importantly, each firm's first round offer is binding for the second round: A firm cannot make any second round offer below its first round one. Our focus in the empirical analysis will be on the second round. For the first period, it suffices for our purposes to argue that firms will never make first round offers that, if accepted by the retiree, would render expected non-positive profits.

Now, when we include the fact that i might request offers from A_i types of annuities, insurance companies have to solve a multi-product bidding problem. As mentioned in the timing assumptions, once i receives all the offers $\{P_{ij}^a : a \in A_i, j \in J\}$, she chooses $a^* \in A_i$ and then chooses the firm. Thus, with a slight abuse of notations, we can express the expected profit of a firm $j \in J$ from an auction where i requests offers for A_i types of annuities as $\mathbb{E}\Pi_{ij} := \sum_{a \in A_i} \mathbb{E}\Pi_{ij}(a) \times \Pr(\texttt{i chooses } a|\{\mathbf{P}_i^b\}_{b \in A_i}; \theta_i)$.

Thus, in the first round, when choosing P_{ij}^a, firm j has to consider the competition from other firms for product a and all other types of annuities in $A_i \backslash \{a\}$. It also has to consider competition from its offers $P_{ij}^b, b \in A_i, b \neq a$, which is the self-cannibalization consideration facing multi-product firms. Determining the equilibrium bidding strategies

for the first round auction, although conceptually straightforward, will require us first to determine the equilibrium in the bargaining phase. However, irrespective of the first round offers, to estimate F_β and W_r it is sufficient to only consider the equilibrium outcome in the second round. Under the assumption that by the second round, the retiree would already know her β_i and has already decided which $a \in A_i$ to choose, the choice problem facing the retiree is relatively straightforward: choose the offer that maximizes the utility (3). Henceforth, we focus only on the second round bargaining, which is relatively simpler to model and to use for estimation.

This multi-product feature means that to characterize the equilibrium first round offers require solving a multi-dimensional bidding problem. The problem becomes more complex when we consider the fact that at the time of making the first-round offers, it is unlikely that firms know (β_i, θ_i).

In our empirical application, we only use the chosen offers from the second round to infer the annuitization costs' distributions. Moreover, in the second round, it is more reasonable to think that firms can learn retirees (β, θ) from the retirees. First, there are many interactions between firms and retirees, so it is reasonable to assume that the firms will be able to (at least) update their priors belief about β_i. Second, given our assumption that retirees choose the type of annuity in the first round, it is reasonably likely that in the second round, firms will be able to know more about θ_i than they did in the first round.

We recognize that this is a strong assumption, but it allows us to keep the second round bargaining game tractable. If retirees' preferences were their private information, it would lead to a bargaining game with two-sided asymmetric information. Even then, we would have to make assumptions about firms' updated beliefs about (β_i, θ_i), and if and how the updating varies across retirees. So from here, we assume that firms know (β_i, θ_i) for those who opt for the second round.

We model the second round as an alternating offer bargaining process. The game's timing is as follows: In an arbitrary order, firms sequentially choose whether to improve their previous offer by a fixed amount ε or to "stay." The process ends after the round with all firms consecutively choosing to stay. Finally, the retiree then chooses any of the offers. In Lemma 1, we formalize the analysis, with the proof in the Appendix A.3.

Before we proceed, we introduce some new notation. Let $P_{ij}^{\max}$ be the maximum firm j can offer to i without losing money, i.e., $P_{ij}^{\max}$ solves $C(P_{ij}^{\max}) = P_{ij}^{\max} \times UNC_j = S_i$, or equivalently $1 = r_{ij} \times \rho_i^*(P_{ij}^{\max})$ and let j_i^* denote the firm that can offer the highest utility without losing money, i.e.,

$$j_i^* := \arg \max_{j \in J} \rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij}.$$

Lemma 1. In the bargaining game, firm j_i^* wins the annuity contract and, as ε goes to zero, ends up paying a pension $\tilde{P}_{ij_i^*}$ such that

$$\beta_i \times Z_{ij_i^*} + \theta_i b_i(\tilde{P}_{ij_i^*}) + \rho_i(\tilde{P}_{ij_i^*}) = \max_{k \neq j_i^*} \left\{ \beta_i \times Z_{ik} + \theta_i b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \right\}. \tag{9}$$

The symmetric behavioral strategies that sustain this perfect Bayesian equilibrium are:

1. For the retiree, choose whichever firm made the best offer (including non-pecuniary attributes), i.e., retiree i chooses firm j_i^* if

$$j_i^* = \arg\max_{j \in J} \rho_i(\tilde{P}_{ij}) + \theta_i \times b_i(\tilde{P}_{ij}) + \beta_i \times Z_{ij},$$

   where $\tilde{P}_{ij}$ refers to the last offer of firm j (or to its first-stage offer if it did not raise it during the bargaining game).

2. For a firm j, play I iff $\tilde{\tilde{P}}_{ij} + \varepsilon < P_{ij}^{\max}$ and

$$\beta_i \times Z_{ij_i^*} + \theta_i b_i(\tilde{\tilde{P}}_{ij_i^*}) + \rho_i(\tilde{\tilde{P}}_{ij_i^*}) < \max_{k \neq j_i^*} \left\{ \beta_i \times Z_{ik} + \theta_i b_i(\tilde{\tilde{P}}_{ik}) + \rho_i(\tilde{\tilde{P}}_{ik}) \right\},$$

   where $\tilde{\tilde{P}}_{ik}$ refers to the standing offer of firm k (or to its first-stage offer when we are in the initial round of the bargaining game).

5 Identification and Estimation

In this section, we study the identification of the model parameters, which include the conditional distribution of bequest preferences $F_\theta(\cdot|S)$, the distribution of preferences for risk ratings $F_\beta(\cdot)$, the distribution of costs $W_r(\cdot|S)$, and the channel- and savings-specific information processing cost α. We observe outcomes of the annuity process described above for N retirees who choose one of the several annuity products, where N is large.

For each retiree $i \in N$, we observe her socioeconomic characteristics $X_i = (\tilde{X}_i, S_i)$, her consideration set $\mathbf{A}_i$, which is the list of annuity products that she solicits offers for, the set of firms $\tilde{J}_i$ who could participate, the set of participating firms $J_i \geq 2$, their risk ratings $\{Z_j \in \mathbb{R} : j = 1, \ldots, J_i\}$ and their pension offers each product and the implied discounted expected utilities $\boldsymbol{\rho}_{ia} := (\rho_{1a}, \ldots, \rho_{J_i a})$ for all $a \in \mathbf{A}_i$. For each offer, we can determine the corresponding bequest, if any. So, for each $a \in \mathbf{A}_i$, we also observe the implied discounted expected utilities from bequest $\boldsymbol{b}_{ia} := (b_{1a}, \ldots, b_{J_i a})$.

Let $D_i^1 \in \{1, \ldots, J+1\}$ denote i's choice in the first stage, such that $D_i^1 = j$ means i

chose firm j, and $D_i^1 = (J + 1)$ means i chose to go to the second round. Conditional on $D_i^1 = (J + 1)$, we also observe j's final choice and the chosen company's identity.

5.1 Distribution of Bequest Preference

Here we study the identification of the distribution of the preference for bequest $F_\theta(\cdot|S)$ with support $[0, \bar\theta]$. To this end, we rely on the fact that we observe her final choice for each retiree, which means we know her chosen bequest. Comparing the chosen bequest and the foregone bequests, we can identify her bequest preference. In this exercise, we use only the winning firms' offers to "control" for the effect of risk-rating on choices.

For intuition, let's consider the case where the consideration sets have only two annuity products, where product 1 offers a smaller bequest–and larger pensions–than product 2. Let $a \in \{1, 2\}$ denote the two products. Using (4), we can write the utility from product a as

$$U_{ij_i^* a} = \beta_i^\top Z_{ij_i^*} + \rho_{ij_i^* a} + \theta_i \times b_{ij_i^* a} - \mathfrak{U}_{0i}(S_i),$$

where $j_i^* \in J_i$ is the firm chosen by retiree i. Let $\chi_i \in \{1, 2\}$ denote i's choice $a = 1$ or $a = 2$. Suppressing the index for retiree and winning firm, $\chi = 1$ if and only if $U_1 \geq U_2$, or equivalently $\theta \leq \frac{\rho_1 - \rho_2}{b_2 - b_1}$. Then the probability that a retiree with characteristics X chooses the annuity with the smallest bequest is

$$\Pr(\chi = 1|X) = F_{\theta|S}\left(\frac{\rho_1 - \rho_2}{b_2 - b_1}\Big|S\right) = F_{\theta|S}\left(-\frac{\Delta\rho_{12}}{\Delta b_{12}}\Big|S\right).$$

The left-hand side probability $\Pr(\chi = 1|X)$ can be estimated, and we also observe the "indifference ratio" $\left\{\frac{\Delta\rho_{12}}{\Delta b_{21}}\right\}$. So if there is sufficient variation in $\tilde X$ across retirees, and sufficient variation in the indifference ratios across retirees and firms, we can "trace" $F_\theta(\cdot|S)$ everywhere over $[0, \bar\theta]$. Formally, if for $t \in [0, \bar\theta]$ there is a pair $\{\Delta\rho_{12}, \Delta b_{21}\}$ in the data such that $t = -\Delta\rho_{12}/\Delta b_{12}$, then the distribution is nonparametrically identified.

If there are more than two products in the consideration set, i.e., $\mathbf{A} \geq 2$, then we can order them from that with the lowest bequest to the highest bequest, the probability that a retiree with X chooses the annuity with lowest bequest is given by

$$\Pr(\chi = 1|\tilde X, S) = \int_\Theta \mathbb{1}\{U_1 \geq U_a, a \in \mathbf{A}|\theta\} \, dF_{\theta|S}(\theta|S) = F_{\theta|S}\left(\min_{1 < a \leq A}\left\{-\frac{\Delta\rho_{1a}}{\Delta b_{1a}}\right\}\Big|S\right).$$

Similarly, the probability of *not choosing* the annuity with the *largest* bequest is given by

$$\Pr(\chi \neq |\mathbf{A}|\big|\tilde X, S) = F_{\theta|S}\left(\max_{1 \leq a < A}\left\{-\frac{\Delta\rho_{Aa}}{\Delta b_{Aa}}\right\}\Big|S\right).$$

So, we can use these two equations to identify $F_\theta(\cdot|S)$, where, as mentioned above, the identifying source of variation are, $\tilde{X}$, annuitization costs across firms, number of participating firms, which in turn lead to variations will induce variation in pensions and bequests.

5.2   Information Processing Cost

Here, we verify that we can identify the channel- and savings-specific information processing cost. Let $\mathcal{J}$ denote the unique values of J_i across all $i \in N$. Consider the subset of retirees with $|J_i| = J$. Then, we can identify the conditional choice probability for $j \in (J+1)$, including the option, being chosen, given $X = x$, $Z = z$ and $(\boldsymbol{\rho}, \mathbf{b})$, by

$$\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J) = \sum_i \frac{\mathbb{1}[D_i^1 = j, X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}{\sum_i \mathbb{1}[X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}; \quad \hat{\sigma}_{J+1}(\tilde{x}, z, \boldsymbol{\rho}, \mathbf{b}|J) = 1 - \sum_{j=1}^{J} \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}). \tag{10}$$

Applying (10) to the relevant subsample, we can identify $\{\sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J)\}_{j \in J}$ for all $J \in \mathcal{J}$. We can also identify the probability that there are J participating firms as $p(J) = \#\{\texttt{retirees with } J_i = J\}/N$, and together we identify $\sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}) = \sum_{J \in \mathcal{J}} \sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J) \times p(J)$. Integrating (5) with respect to F_β and using the definition of $\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b})$ gives

$$\hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}) = \int \frac{\exp\left(\log \sigma_j^0 + \frac{U_{ij}}{\alpha}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ij}}{\alpha}\right) + \exp\left(\frac{\mathbb{E}U_i}{\alpha}\right)} dF_\beta(\beta). \tag{11}$$

Taking the derivative of (11) with respect to ρ_j identifies the cost $\alpha = \frac{\sigma_j(x, z, \boldsymbol{\rho}, \mathbf{b})(1 - \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}))}{\frac{\partial \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b})}{\partial \rho_j}}$.[21]

Thus, the information processing cost depends on the choice probability elasticity with respect to ρ. Consider an extreme case when the choice for j is insensitive to changes in premium, i.e., $\frac{\partial \hat{\sigma}_j(x, z, \boldsymbol{\rho})}{\partial \rho_j} \approx 0$, then it implies that $\alpha \approx +\infty$ because the only way to rationalize the fact that retirees do not respond to changes in pension is that their information processing cost is extremely large. If the demand is elastic with respect to the pensions, then the cost of processing information is low, and vice versa. To identify the cost as a function of the channel and savings, we can use the appropriate subsample and follow the above steps.

5.3   Risk-Rating Preferences and Annuitization Costs

To identify the preference distribution F_β and the cost distribution W_r, it is sufficient to consider only those who buy annuities in the second round, where the chosen pension and

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[21]To estimate α, we use a logit specification to model the LHS of (11) so the derivatives are well defined.

Figure 5: **Histogram of Winner and Runner Up**
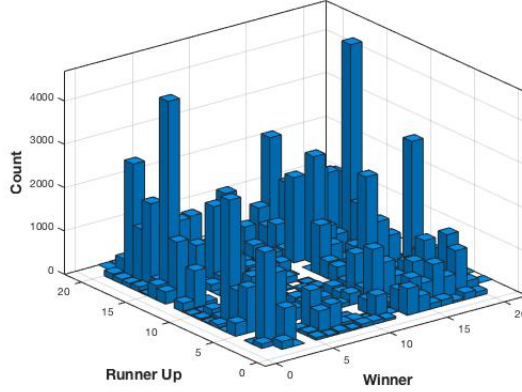
Note: The histogram shows the identity of the winning firms (on the x-axis) and the runner-up firms (on the y-axis). The runner-up firm for a retiree is the firm that has the largest probability of being chosen by the retiree after excluding the chosen firm, where the probabilities are estimated using logistic regression as explained in Appendix A.2.

bequests are given by (9). Let $\tilde{P}_{ij_i^*}$ be the chosen offer. Then from (9) $\tilde{P}_{ij_i^*}$ satisfies

$$\rho_i(\tilde{P}_{ij_i^*}) + \theta_i b_i(\tilde{P}_{ij_i^*}) = \max_{k \neq j_i^*} \left\{ \beta_i \times Z_{ik} + \theta_i b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \right\} - \beta_i \times Z_{ij_i^*}. \tag{12}$$

Let k_i^* denote the runner-up company in i's auction. Then we can re-write (12) as

$$
\begin{aligned}
\rho_i(\tilde{P}_{ij_i^*}) + \theta_i b_i(\tilde{P}_{ij_i^*}) &= \mathfrak{U}(\tilde{P}_{ij_i^*}, B_{ij}(\tilde{P}_{ij_i^*}); \theta_i) = \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \theta_i b_i(P_{ik_i^*}^{\max}) + \rho_i(P_{ik_i^*}^{\max}) \\
&= \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \mathfrak{U}(P_{ik_i^*}^{\max}, B_{ik_i^*}(P_{ik_i^*}^{\max}); \theta_i) \\
&\equiv \beta_i \times (Z_{ik_i^*} - Z_{ij_i^*}) + \varpi_i,
\end{aligned}
\tag{13}
$$

where the first equality follows from (3), and $\varpi \sim F_\varpi$ is the highest gross utility that the runner up firm k_i^* can offer to retiree i. Notice that we can use the chosen annuity to determine the left-hand side terms, and if we view ϖ as an error, then (13) is the random coefficient model. From the literature on random coefficient (Hoderlein, Klemelä, and Mammen, 2010), we know that the distributions F_β and F_ϖ are nonparametrically identified under our maintained assumption that $(Z_{ik_i^*} - Z_{ij_i^*})$ and β_i and ϖ_i are uncorrelated and there is sufficient variation in $(Z_{ik_i^*} - Z_{ij_i^*})$. The runner-up and the winner firm pairs vary across retirees, which ensures the difference $(Z_{ik_i^*} - Z_{ij_i^*})$ also varies as can be seen in Figure 5.

The next step is to show that we can determine $W_r(\cdot)$ from $\{F_\beta, F_\theta\}$. The argument is based on the following steps. First, note that for each draw $\theta_i \sim F_\theta$, the distribution of the LHS in (13) is also the distribution of the second largest value of the RHS in (13).

Second, from this distribution of the order statistics, we can identify the parent distribution of the RHS in (13). Third, this parent distribution is a convolution of the distribution of $\beta_i \times (Z_{ik_i^*} - Z_{ij_i^*})$ and the distribution of ϖ_i, which in turn identifies the distribution of ϖ_i. Fourth, we know that there is a one-to-one mapping from ϖ_i to $P_{ik_i^*}^{\max}$ –the maximum pension runner-up firm k_i^* can offer to retiree i, see Equation (A.7), which together with the definition that $C(P_{ik_i^*}^{\max}) = S_i$ identifies the distribution of $r = \frac{UNC_{k_i^*}}{UNC_i} = \frac{S_i}{P_{ik_i^*}^{\max} \times UNC_i}$. We formalize these steps in the following result, and provide the proof in Appendix A.3.

Lemma 2. $W_r(\cdot|S)$ can be nonparametrically identified from $\{F_\beta, F_\theta\}$.

Selective Entry. Let $\tilde{J}$ be the set of companies that are interested in selling annuities to i with characteristics X_i. When i requests an offer for a product, company $j \in \tilde{J}$ observes its cost r_j, and all firms simultaneously decide whether or not to participate, and it costs (the same) $\kappa \geq 0$ for each company to participate. This cost captures the opportunity cost to participate, and it can vary across retirees. Let $J \subset \tilde{J}$ denote the set of participating companies. All the firms that participate simultaneously make their offers.

Under the symmetric Perfect Bayesian-Nash equilibrium, the entry decision is characterized by a unique threshold $r^* \in [\underline{r}, \overline{r}]$ such that firms participate only if their costs are less than r^*. Then the cost distribution among the participating firms is $W_r^*(r; \tilde{J}) := W_r(r|r \leq r^*; \tilde{J}) = W_r(r)/W_r(r^*; \tilde{J})$. Let $r_{\tilde{J}}^*$ be the threshold with $\tilde{J}$ potential bidders, and suppose $\tilde{J} \in \mathcal{J} := \{\underline{J}, \ldots, \overline{J}\}$, where $\overline{J}$ is the maximum number of potential bidders and $\underline{J}$ is the smallest number of potential bidders. All else equal, $r_{\tilde{J}}^*$ decreases with $\tilde{J}$, so $W_r(r)$ is identified on the support $[\underline{r}, r_{\underline{J}}^*]$.

5.4 Estimation Steps for Risk-Rating Preference and Annuitization Cost Distribution

Here, we present the steps that we take to estimate the conditional distributions of β and r. Although we can nonparametrically identify $F_\beta(\cdot|X)$, we impose parametric assumption about the density for estimation. In particular, we divide retirees into separate groups based on gender, three age groups and savings quintiles, and three channels, which gives us a total of $G = 90$ groups, and further assume that β_i in (13) is normally distributed, $\beta_i \sim \mathcal{N}(\beta_{g(i)}, \sigma_{g(i)})$ where $g(i) \in G$ is i's group. Thus, we allow each group to have a group-specific mean and variance of β. Similarly, we assume that savings affect r through the savings quintiles S_q, i.e. $r \sim W_r(\cdot|S_q)$, where S_q is the $q \in \{1, \ldots, 5\}$ quintile of savings.

Let $N_{q,J}$ denote the subset of retirees in the qth–quintile and have $J \in \{13, 14, 15\}$ potential bidders. Then, we can re-write our estimation equation (13) with group-specific

coefficients for each $(q, J) \in \{1, \ldots, 5\} \times \{13, 14, 15\}$ pair as

$$\rho_{j_i^*} + \theta_i \times b_{j_i^*} = \beta_{g(i)} \times (Z_{k_i^*} - Z_{j_i^*}) + \varpi_{k_i^*}; \quad i = 1, 2, \ldots, N_{q,J}, \tag{14}$$

and $\beta_g = \beta_0 + \upsilon_g$ where $\mathbb{E}(\upsilon_g) = 0$ and $\mathbb{E}(\upsilon_g^2) < \infty$. Applying GLS to equation (14), we estimate group-specific β_g and $\varpi_{k_i^*}$ for all $i \in N_{q,J}$.

Next, using the estimated $\hat{F}_\theta(\cdot|S_q)$, we can "integrate-out" θ from the estimation equation. For each (q, J) and each $i \in N_g$, we generate i.i.d. samples $\{\theta_{i,\ell}\}_{i=1}^{|N_g(i)|} \sim \hat{F}_\theta(\cdot|S_q)$, and estimate $\{\hat{\beta}_g^\ell : g = 1, \ldots, G\}$ applying generalized least squared method to

$$\rho_{j_i^*} + \theta_i^\ell \times b_{j_i^*} = \beta_{g(i)} \times (Z_{k_i^*} - Z_{j_i^*}) + \varpi_{k_i^*}^\ell. \tag{15}$$

We repeat this exercise for $L = 10,000$ sample draws of θ, which, for each group $g \in G$ gives us 10,000 estimates $\{\hat{\beta}_g^\ell\}_{\ell=1}^L$, and averaging across those samples give $\hat{\beta}_g = L^{-1} \sum_{\ell=1}^L \hat{\beta}_g^\ell$.

To estimate $W_r(\cdot|S_q)$, we focus on the sub-sample of retirees that have the top two firms with the same risk ratings. In our sample, close to 60,000 retirees are in this group and have $(Z_{k_i^*} - Z_{j_i^*}) = 0$. Substituting this in (15) for $J \in \{13, 14, 15\}$ gives

$$\rho_{j_i^*} + \theta_i^\ell b_{j_i^*} = \varpi_{k_i^*}^\ell, \tag{16}$$

where the left hand is the known winning utility and the right-hand side is the unobserved maximum utility the runner-up firm can offer without incurring loss. Thus, the estimation problem in (16) becomes similar to the estimation problem in a standard English auction where only the winning bid is observed. The key difference here is that everything is expressed in terms of winning utility and not the bid. From the estimated distribution of $(\rho_{j_i^*} + \theta_i^\ell b_{j_i^*})$, we can estimate the parent distribution of $\hat{r}_{ij}$, i.e., $W_r^*(\cdot|S_q, J)$ using a Kernel Density Estimator.

6 Estimation Results

Preferences for Bequests. In Figure 6, we display the estimates of the conditional distributions of preferences for bequests, given savings quintiles $\{\hat{F}_\theta(\cdot|S_q), q = 1, \ldots, 5\}$. Our estimates suggest that approximately 40% of retirees do not value leaving bequests, so there is a positive mass at $\theta = 0$. In Table 11, we present some summary statistics associated with these conditional distributions. As we can see, the median θ is either 0 or very close to 0, but the average θ is around 2. We also find considerable heterogeneity, both within and across different savings quintiles, as shown by the last two columns in Table 11.[22]

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[22]Kopczuk and Lupton (2007) provides an excellent discussion about variation in bequest preference.

Figure 6: **Estimated Distributions of Bequest Preferences**
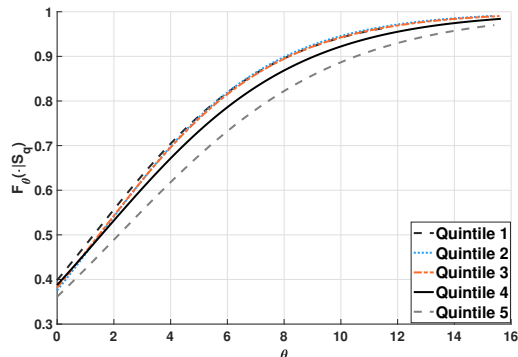
Note: This figure displays estimated conditional distribution of preference for bequests $F_\theta(\cdot|S_q)$ given savings quintile $S_q, q = 1, \ldots, 5$, as we move from the left to the right.

The fact that the conditional distributions in Figure 6 "shift right" with saving quintile suggests that the average bequest preference increases with savings. We can also see this pattern by comparing $\mathbb{E}(\theta|S_q)$ across savings quintiles in Table 11. This result is consistent with the hypothesis that with decreasing marginal utility from a pension, the marginal utility of bequest for an altruist retiree increases with savings. From Table 11, we also see that although the mean of θ suggests that, at the margin, retirees value bequest twice as much as they value self-consumption, the median is almost zero.

Table 11: **Summary Statistics of Preference for Bequests**

Savings	Mean	Median	Std. Dev.	IQR
Q1	1.92	0	2.82	3.34
Q2	2.22	0.1	3.22	3.77
Q3	2.25	0	3.27	3.85
Q4	2.41	0	3.5	4.13
Q5	2.82	0.35	3.82	5.01

Note: Mean, median, standard deviation and inter-quartile range of preference for bequests, by saving quintiles. These statistics are calculated using simulated θ from $\{\hat{F}_\theta(\cdot|S_q)\}_{q=1}^5$ as shown in Figure 6.

Risk ratings and Information Processing Costs. Next, we present the estimates of the preference for risk-rating. Figure 7 displays the group-specific means of β, with their corresponding 95% confidence intervals. The estimates of β suggest that those in the lowest two savings quintiles are the only ones who value firms' risk ratings. Moreover, even among these retirees, males have a stronger preference for risk ratings than females.

Figure 7: **Group-Specific Mean of Preferences for Risk Ratings**
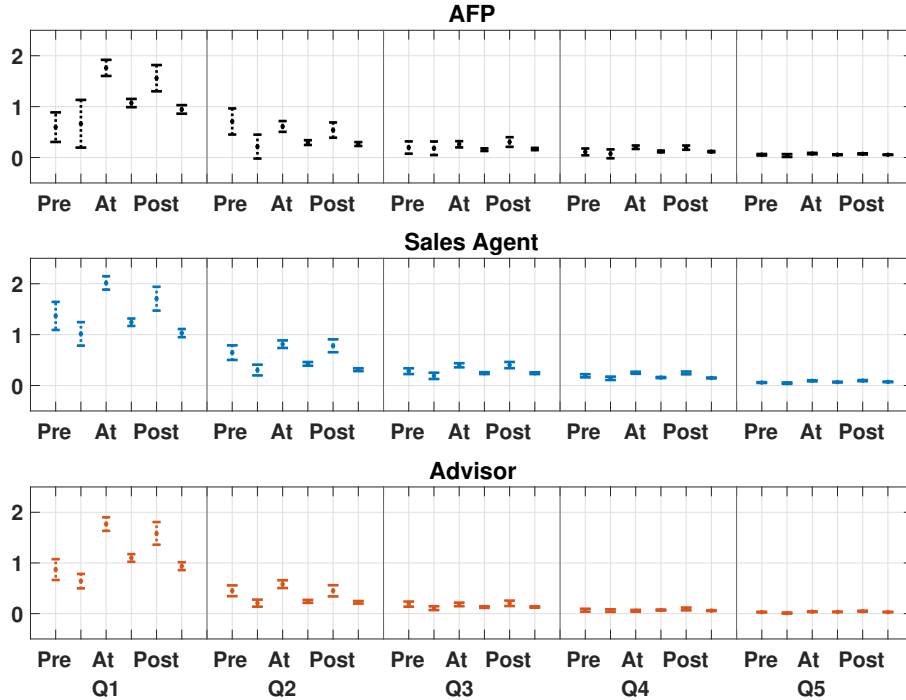
Note: These figures display the estimates for group-specific mean of $\mathbb{E}(\beta_g)$, from (13). Each panel (row) corresponds to a channel, and each channel is divided into five quintiles. And within each channel-quintile box, parameters are ordered by retirement age (before, after or at NRA), and for each age group, the two estimates correspond to male and a female respectively. The two bars represent 95% confidence intervals.

On the face of it, the fact that preference for risk ratings decreases with savings is counter-intuitive. There are at last two reasons why it is so. First, if the risk-rating is a proxy for financial health, then there should be no heterogeneity across retirees. Second, because of the government's guarantee, those with higher savings are more exposed to the "bankruptcy risk" than those with lower savings, so the high savers should care more about the risk ratings than the low savers. One way to explain this finding is to consider the information processing cost. If these groups have different information processing costs, then the rational-inattention model suggests that the group with lower-cost processes more information and learns more.

In Table 12, we present the estimated group-specific information processing costs ($\hat{\alpha}_g$). We find that the cost decreases with savings, and the absolute decrease is largest among the retirees with the lowest quintile and who have sales agents because those with higher savings tend to be more educated. So, even if the prior suggests that the risk ratings are essential,

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those with lower information processing cost revise their preferences downwards to reflect that bankruptcy is a rare event in Chile and that most of the firms have good ratings.

Table 12: **Information Processing Cost**

Savings	AFP	Sales Agent	Advisor	Full Sample
Q1	0.009	0.027	0.006	0.021
Q2	0.006	0.019	0.004	0.016
Q3	0.005	0.013	0.003	0.013
Q4	0.005	0.012	0.003	0.005
Q5	0.005	0.012	0.003	0.006
Overall	0.005	0.013	0.003	0.009

Note: Estimates of the median of information processing cost, by savings quintiles and intermediary channel.

Annuitization Costs. Figure 8i presents the estimates of the conditional distributions of costs given the savings quintile. Recall that $r_{ij} := UNC_j/UNC_i$ is the ratio of firm j's UNC to retiree i's UNC and is thus unit free.[23] So, $r_{ij} > 1$ means that firm j's cost of annuitizing i's savings is larger than an actuarially fair cost, and vice-versa.

Figure 8: **Conditional Distributions of Annuitization Costs**

(i) Over the Full Support

(ii) Focusing on $r < 1$.
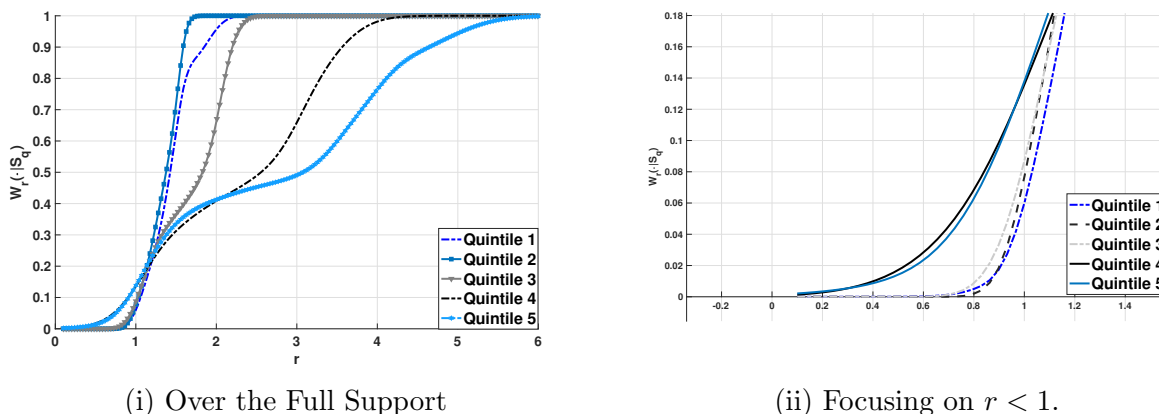
Note: The first sub-figure shows the estimated conditional distribution of relative annuitization costs, by savings quintiles. The second figure shows the same distributions but focuses only on the support $r < 1$.

As we can see from Figure 8i, because the distributions "shift right" with savings, it suggests that the annuitization cost also increases with savings. In Table 13, we present the summary statistics of r by savings quintiles, and we can see that the mean annuitization costs do not seem to increase with savings quintiles. This finding is at odds with the prior

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[23]Recall that we work with r_{ij} instead of UNC_j because each retiree is unique and has different mortality and normalizing UNC_j by UNC_i homogenizes cost across retirees. Then we can "pool" the data from different retiree auctions together and make inter-retiree comparisons.

research finding that wealth rankings are essential determinants of mortality, conditional on the initial health status.

The shapes of the distributions when $r < 1$ (see Figure 8ii) can explain this pattern. Firms are twice as likely (14% versus 6%) to have $r < 1$ when the retiree belongs to the top two savings quintiles than when they do not. Thus, this "crossing" of the conditional distribution functions lowers the overall average costs for high savers.

Table 13: **Summary Statistics of r**

Savings	Mean	Median	Std. Dev.	IQR
Q1	2.74	3.1	1.47	2.7
Q2	2.75	3.11	1.47	2.7
Q3	2.73	3.07	1.46	2.69
Q4	2.77	3.12	1.47	2.69
Q5	2.76	3.12	1.48	2.72

Note: The table displays mean, median, standard deviation and inter-quartile range of the annuitization costs r. These statistics are calculated using simulated r from $\{\hat{W}_r(\cdot|S_q)\}_{q=1}^5$ as shown in Figure 8i.

In equilibrium, pensions are determined by the lowest two order-statistics of the cost, which in turn depends on the left tail of the distributions (Figure 8ii). So, the fact that firms are twice as likely to have $r < 1$ for the highest two quintiles than the other three quintiles translates into a larger gap between what firms offer and their break-even offer. The best way to illustrate this is to use the estimated cost distributions and determine the maximum pension that firms can offer without making a loss.

To this end, we run the following simulation exercise: (i) for each savings quintile, we identify the retiree with the median income (among this subsample); (ii) simulate $\{r^{(\ell)} : \ell = 1, \ldots, 1000\}$'s from the relevant distribution $\hat{W}_r(\cdot|\cdot)$; (ii) using the savings and the estimated UNC_i of the retiree identified in step (i), for each draw $r^{(\ell)}$ determine UNC_j and from that the maximum pension is is given by the zero-profit condition, i.e., $P_{ij} = S_i/UNC_j$. Figure 9 shows the resulting distributions of these pensions. Furthermore, we can see, those firms would offer more for those with higher savings than with lower savings per dollar.

7 Counterfactual Results

Based on these results, we consider ways to improve the market, some of which are also under debate in the Chilean parliament: (1) simplify the current system by replacing it with the standard English auction; (2) remove risk ratings from the supply side to increase

Figure 9: **Distributions of Maximum Pension** $P^{\max}$
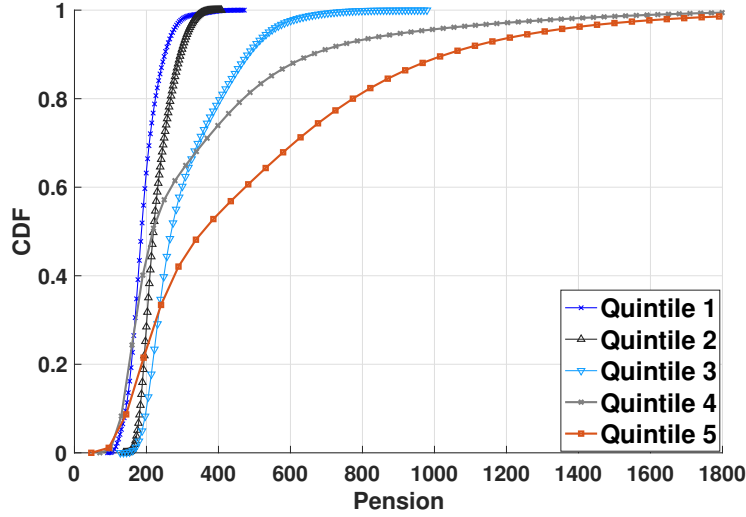
Note: Conditional distributions of maximum pensions for retiree with median savings within each quintile. For each savings quintile $1 \leq q \leq 5$, we simulate several rs from $\hat{W}_r(\cdot|S_q)$ that is displayed in Figure 8i, and we determine the median savings among this group. Using these rs and the median saving, we determine the maximum pensions $P^{\max}$ that firms can offer without making loss and estimate the distribution of $P^{\max}$.

competition by selecting the firm that pays the highest pension, and (3) automate the system, so retirees do not use risk ratings to choose a firm.

We present pensions and retirees' gross utility under the current system, full information, and English auction. However, first, we determine the pensions and utilities under first-best full information outcomes, i.e., when firms' annuitization costs are publicly known, and the winning firm offers the break-even pensions. First-best outcomes allow us to compare results across different mechanisms, they are interesting per se because we can use them to quantify the effect of asymmetric information on pensions and retirees' utilities.

7.1 Complete Information

We begin by considering the effect of asymmetric information on the pension and the margin and how that varies across different savings quintiles and competition. To determine the pensions under full information, we divide retirees into 15 groups based on their savings quintiles and their corresponding potential number of bidders. Then, for each retiree in a group, we simulate as many r from the appropriate $\hat{W}_r(\cdot|S_q)$ as the potential number of bidders present and determine the lowest cost among those draws. The winner will be the bidder with the lowest cost. Then, we determine the zero-profit pension the winning firm can offer. For every retiree, we repeat this step 10,000 times and calculate the average pension.

Figure 10: **Estimated Distributions of Pensions (in 1000s of US$)**
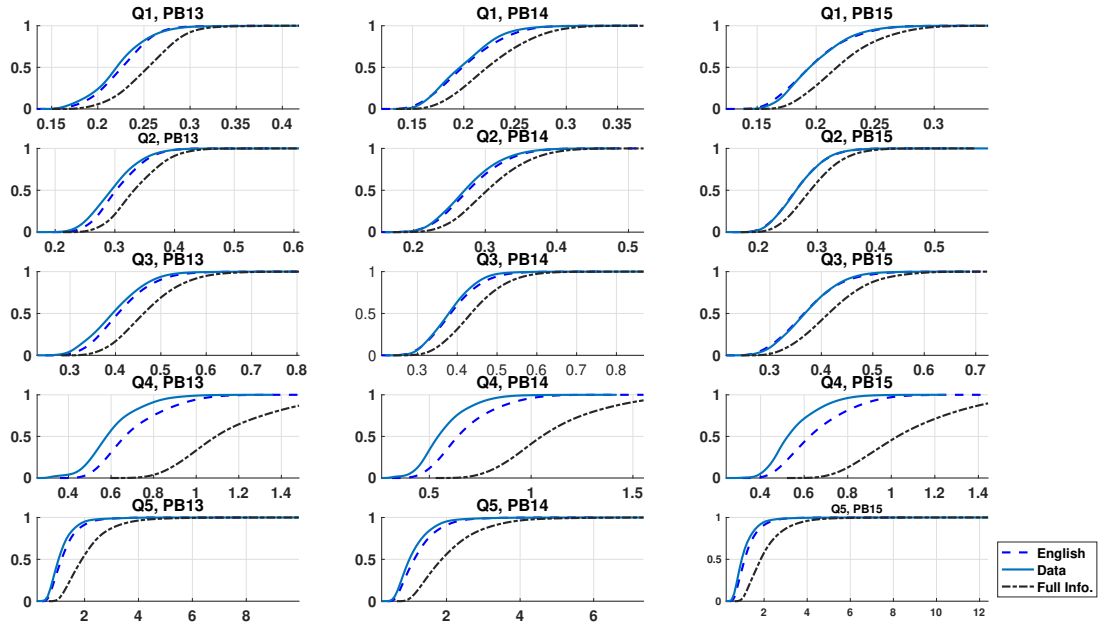
Note: Distributions of pensions (in thousands of U.S. dollars) under the current system (solid blue), under English auction (dashed blue) and under full information (dotted red), by savings quintiles (rows) and the number of potential bidders (columns). The sample includes only those who choose in the second round.

In Figure 10, we present the distributions of chosen pensions (in solid blue line) and the pension if there was no private information about r (in the dotted red line). As expected, the pension distribution under full information, first-order stochastically dominates the distribution of the observed pensions. Interestingly, the gap between the two distributions is substantial for those with higher savings, suggesting that firms have a more considerable margin from this group.

In Table 14, we present the mean and median of current pensions as a percentage of the full information pensions for each group. We find that for the lowest three savings quintiles, the numbers are at least 85%, whereas for the top-two saving quintiles, they are significantly lower. These results are consistent with the shape of the cost distributions, as shown in Figure 10.

Next, we also consider the money's worth ratio for each group. Money's worth ratio measures the generosity of an annuity contract, see Mitchell et al. (1999). As we have explained earlier in Section 3, the money's worth ratio is the return a retiree can expect to earn per annuitized dollar. Suppose this ratio is more than (respectively, less than) one. In

Table 14: **Pensions under Current and English auctions, relative to Full Info.**

Savings \Potential Bidders	13	14	15
Q1	(87%, 87%)	(91%, 91%)	(93%, 93%)
	(88%, 88%)	(89%, 89%)	(89%, 89%)
Q2	(89%, 90%)	(94%, 93%)	(97%, 97%)
	(90%, 90%)	(91%, 91%)	(91%, 91%)
Q3	(88%, 90%)	(92%, 93%)	(95%, 96%)
	(87%, 87%)	(88%, 88%)	(88%, 88%)
Q4	(54%, 55%)	(55%, 56%)	(56%, 56%)
	(60%, 60%)	(60%, 60%)	(60%, 60%)
Q5	(55%, 56%)	(56,% 57%)	(57%, 57%)
	(60%, 60%)	(60%, 59%)	(59%, 59%)

Note: Mean and median of pensions under the current system and under English auction, expressed as a percentage of the pension under full information, separated by savings quintile (rows) and the number of potential bidders (columns). Each entry has two rows: The first row corresponds to the current system and the second row corresponds to the English auction.

that case, a retiree expects to earn more than (respectively, less than) what she annuitizes.[24]

Instead of presenting money's worth ratios for each retiree, in Table 15, we present the group-specific money's worth ratios, which are equal to the ratio $(\sum_i P_i \times UNC_i)/\sum_i S_i$, where the sum is over all retirees in the respective group. As we can see from the first column, those with AFP (the first row within each quintile) get better money's worth ratio than the other two channels under the current system. We also see that those with higher savings get a slightly better offer than those with lower savings. If we compare the first and the last columns in Table 15, we see that as before, the gap between the current system and that under the full information is the largest for those with higher savings.

7.2   English Auction

One way to increase pensions is to make the system more competitive. To this end, we can replace the current system with the standard English auction, and also "shut down" the risk ratings in the supply side by picking the winner to be the firm that offers the highest pension. Simplifying the process should improve outcomes for those who choose in the first round. Similarly, shutting down risk-rating should force firms to bid more aggressively; the benefits should be larger for lower savers than higher savers. The former have stronger preferences for risk ratings, which means without risk ratings, the firms should be more aggressive if the retiree is of lower savings. However, because the gap between the chosen pension and the full information pension is the largest for those with higher savings, they may benefit the

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[24]Formally, the money's worth ratio for i, under the current system, is equal to i's chosen pension times her UNC_i divided by her savings S_i.

Table 15: **Money's Worth Ratio, by Savings Quintile and Channel**

Savings Quintile	Channel	Current	English	Full Info.
Q1	AFP	0.99018	0.93229	1.04419
	Sales Agent	0.95663	0.93128	1.04327
	Advisor	0.95969	0.93019	1.04237
Q2	AFP	1.02480	0.95833	1.04920
	Sales Agent	0.99589	0.95728	1.04841
	Advisor	0.99624	0.95608	1.04748
Q3	AFP	1.04418	0.96340	1.08998
	Sales Agent	1.02315	0.96216	1.08906
	Advisor	1.01623	0.96067	1.08796
Q4	AFP	1.06109	1.13492	1.86677
	Sales Agent	1.04144	1.13166	1.86129
	Advisor	1.03278	1.12759	1.85429
Q5	AFP	1.09793	1.12368	1.87748
	Sales Agent	1.07350	1.12027	1.87109
	Advisor	1.06609	1.11688	1.86514

Note: Each row denotes a different group, and each entry is money's worth ratio for that group, which is equal to $(\sum_i P_i \times UNC_i)/\sum_i S_i$, where the sum is taken over all retirees in the group. There are 15 groups based on 5 savings quintiles and 3 channels. Each column corresponds to a different pricing mechanism, where English is the English auction.

most from the new mechanism.

Next, we implement the standard English auction by treating the potential bidders as the actual bidders. Our results are an upper bound on the effect of English auction on pensions and retirees' ex-post expected present discounted gross utilities. We follow the same steps as in the full information counterfactual, except under the English auction, the winning pension is the maximum pension a firm with the second-lowest-cost (r) can offer, at zero profit.

We present the Kernel density estimates of the distributions of winning pensions under English auction in Figure 10. Although English auction leads to higher pensions, most of the benefits accrue to those in the top two savings quintiles. We can also see this in the second row of Table 14 for each quintile, where we present the mean and the median pension under English auction expressed as a percentage of the pension under full information. Similar results hold if we consider the money's worth ratio; see the first two columns in Table 15.

We are also interested in determining the effect of using the English auction on retirees' ex-post utilities. We do not know the utility from the outside option, but we can determine the ex-post gross expected discounted present utility which is equal to $\beta_i \times Z_j + \rho_{ij} + \theta_i b_{ij}$.

For each retiree and each mechanism using the "winning" pensions, we first determine the bequest (if any) and then calculate the ex-post expected present discounted utilities. To shed light on the effect of shutting down risk ratings on the retirees' utilities, for each

mechanism, we calculate two utilities: one with $\beta_i \times Z_j$ and one without Z_j by setting $\beta_i = 0$. To calculate the utility from the risk-rating, we use simulated data under the assumption that β_i is normal with estimated group-specific mean and variance.

We present the average utilities across different groups in Tables 16 and 17. In Table 16, we group retirees by their savings quintiles and the potential number of bidders and in Table 17 we group retirees by their savings quintiles and their channels. In each table, and for each mechanism, we have two columns, one with and one without (asterisk) β, respectively.

Note that for each quintile in Table 16, by comparing the rows, we can see that the utilities increase with the number of bidders because the pensions increase when there are more firms. However, despite the large gap between the pensions under the current system or the pensions under English auction and the pension under the full information (Figure 10), our estimates show that the gap in utilities is almost negligible.

From Table 17 we can see that similar results hold even if we group retirees by their savings quintile and their channel. Nonetheless, what is new and here is that those who have sale agents (second row in each savings quintile) have higher utilities than other channels, and this difference decreases with savings.[25]

8   Conclusion

In this paper, we develop an empirical framework to study an imperfectly competitive market for annuities. We used a rich administrative data set from the Chilean annuity market to estimate our model. In the market, risk-averse retirees use *first-price-auction-followed-by-bargaining* to select from different types of annuity contracts and a firm. Life insurance companies have private information about their annuitization costs, and for each retiree auction, they decide whether to participate and compete by making pension offers. The Chilean data gives us a unique opportunity to examine the role of private information about cost, retiree's preferences, and market structure on the outcomes of a market for annuities.

Our main contribution is to study the current market system by estimating both the demand and supply of annuities and evaluating a simpler mechanism that may improve the system. We find that while there is a gap between the observed pensions under the current system and pensions under the full information regime, the gap is significantly larger for those with higher savings. We also determine the effect of replacing the current system with a simpler one-shot English auction, where the winning firm offers the highest pension on pensions and ex-post expected present discounted utilities. We find that while the new

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[25]Adding an optimal reserve price has an insignificant effect on the outcomes. Results are available from the authors upon request.

Table 16: **Average Gross Utility, by Savings Quintile and Potential Bidders**

Bidders	Current	English	Full Info.	Current*	English*	Full Info.*
13	8.8176	8.8180	8.8191	-0.0054	-0.0049	-0.0039
14	6.9852	6.9851	6.9866	-0.0073	-0.0073	-0.0058
15	11.9204	11.9200	11.9215	-0.0073	-0.0077	-0.0061
13	3.5616	3.5618	3.5622	-0.0027	-0.0026	-0.0021
14	3.5055	3.5054	3.5061	-0.0038	-0.0040	-0.0033
15	4.2757	4.2753	4.2760	-0.0042	-0.0046	-0.0039
13	2.5903	2.5903	2.5907	-0.0015	-0.0014	-0.0011
14	2.6788	2.6787	2.6791	-0.0018	-0.0020	-0.0015
15	2.8087	2.8084	2.8089	-0.0021	-0.0024	-0.0018
13	2.4089	2.4091	2.4095	-0.0007	-0.0005	-0.0002
14	2.4462	2.4464	2.4468	-0.0009	-0.0007	-0.0003
15	2.4724	2.4726	2.4731	-0.0010	-0.0008	-0.0003
13	2.3357	2.3358	2.3359	-0.0003	-0.0002	-0.0001
14	2.2684	2.2684	2.2686	-0.0004	-0.0003	-0.0001
15	2.3018	2.3019	2.3021	-0.0004	-0.0004	-0.0001

Note: The table displays the gross utility, see Equation (4), under 4 (current, English auction, and full information) pricing mechanisms, averaged over subgroups defined by savings quintile and potential bidders. Each quintile is separated by a horizontal line, and within each line, the rows reflect the number of potential bidders $\{13, 14, 15\}$. The first four columns use the estimated β (c.f. Figure 7) in calculating the utility and the last four columns (with asterisk) set $\beta = 0$ in (4).

mechanism increases pensions for almost every retiree, pensions increase the most for those in the top two savings quintiles, albeit the increase in utility is minimal.

There are several possible avenues for future research on related topics. First, we can also include the choice between PW and annuities and consider an imperfectly competitive market with two-sided asymmetric information. On the demand side, retirees will have private information about their mortality forces and their bequest preferences. On the supply side, as in our case, firms have private information about their annuitization costs. Another interesting extension of our model is to allow for the possibility that bargaining in the second round is costly for some retirees. Introducing such a cost would require us to embed search friction into the second stage, but it might provide us with a complete picture of the market.

Table 17: **Average Gross Utility, by Savings Quintile and Channel**

Channel	Current	English	Full Info.	Current*	English*	Full Info.*
AFP	9.2078	9.2073	9.2087	-0.0066	-0.0071	-0.0057
Sales Agent	11.7779	11.7778	11.7794	-0.0075	-0.0075	-0.006
Advisor	9.239	9.2388	9.2402	-0.0068	-0.0069	-0.0055
AFP	3.7995	3.799	3.7998	-0.0038	-0.0043	-0.0036
Sales Agent	4.4095	4.4092	4.4099	-0.0041	-0.0043	-0.0036
Advisor	3.585	3.5848	3.5854	-0.0038	-0.004	-0.0033
AFP	2.6741	2.6738	2.6743	-0.0019	-0.0022	-0.0017
Sales Agent	2.9609	2.9607	2.9611	-0.002	-0.0022	-0.0017
Advisor	2.5351	2.535	2.5354	-0.0019	-0.0021	-0.0016
AFP	2.4637	2.4639	2.4644	-0.0009	-0.0008	-0.0003
Sales Agent	2.5845	2.5847	2.5852	-0.001	-0.0008	-0.0003
Advisor	2.2824	2.2826	2.283	-0.0009	-0.0007	-0.0003
AFP	2.3075	2.3076	2.3078	-0.0004	-0.0003	-0.0001
Sales Agent	2.3537	2.3537	2.354	-0.0004	-0.0004	-0.0001
Advisor	2.2215	2.2216	2.2218	-0.0004	-0.0003	-0.0001

Note: The table displays the gross utility, see equation (4), under four (current, English auction, and full information) pricing mechanisms, averaged over subgroups defined by savings quintile and the three channels: AFP, sales agents and advisors. The first four columns use the estimated β (Figure 7) to calculate the utility and the last four columns (with asterisk) set $\beta = 0$ in (4).

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Appendix

A.1 Expected Discounted Present Utilities

Here we explain how we determine the discounted expected utility given by Equation (3). To provide intuition, while keeping the notations manageable we only explain a simple case where the mortalities are known and common across all individuals. Once we understand this simpler case, it is straightforward to allow for individual specific longevity prospects but notationally messy, and for brevity, we do not describe that case here.

The major difficulty in determining Equation 3 is the fact that unlike a pension which is fixed, bequest (the wealth left for her estate) varies over time and across retirees. In particular, it depends on having legal beneficiaries, the type of annuity (in particular, whether it has a guaranteed period), and the time of death (before or after the guaranteed period). Chilean law states that certain individuals are eligible to receive survivorship benefits upon the death of a retiree. As mentioned in Section 3, we focus on retirees without eligible children (but with or without spouses), which is the most common case in our sample. The spouse is eligible for a survivorship annuity equivalent to 60% of the retiree's original pension.

When the annuity includes a guaranteed period (of G months), and the annuitant dies before G, say in $G' < G$ months, her spouse will continue to get the same pension for the next $(G - G')$ months and after that he gets 60% of the original pension. If at the time of death there is no surviving spouse (either because the retiree was single when contracting the annuity or because the spouse died before the retiree), the 100% is paid to the designated beneficiaries in the annuity contract. We assume that the retiree values her spouse or other beneficiaries in the same way, with utility $v(B_{it})$. Using these rules we can write Equation (3) as

$$
\begin{aligned}
w(P, B; \theta_i) &= u(P) \times D_i^R + \theta_i \times \left(\sum_{t=0}^{G} \frac{(1 - q_{it})}{(1 + \delta_t)^t} \times v(P) + \sum_{t=G+1}^{T} \frac{(1 - q_{it}) q_{it}^*}{(1 + \delta_t)^t} \times v(0.6 \times P) \right) \\
&= u(P) \times D_i^R + \theta_i \times \left(v(P) \sum_{t=0}^{G} \frac{(1 - q_{it})}{(1 + \delta_t)^t} + v(0.6 \times P) \sum_{t=G+1}^{T} \frac{(1 - q_{it}) \times q_{it}^*}{(1 + \delta_t)^t} \right) \\
&= u(P) \times D_i^R + \theta_i \times \left(v(P) \times D_i^S + v(0.6 \times P) \times D_i^{S,GP} \right), \quad\quad\quad \text{(A.1)}
\end{aligned}
$$

where q_{it}^* is the probability that the spouse will be alive in t.

Next, we explain how to calculate the net present expected value (NPEV) of an annuity (ρ_{ij}, b_{ij}) from pension offer P_{ij} in Equation (3). For this, we model the force of mortality as a continuous random variable distributed as Gompertz distribution. Let t_0 denote the age

at retirement, expressed in months and let $\delta \in (0, 1)$ denote the discount factor. An annuity pays a constant benefit P from t_0 until retiree's death, so NPEV is calculated at t_0. We start by considering immediate annuity with no spouse. Such annuity does not pay anything to the beneficiaries upon death, therefore, $b_{ij} = 0$.

Let $F_m(t|X)$ be the conditional distribution function for the time of death of retiree with characteristics X, and let $f_m(t|X)$ be the corresponding conditional density. For notational simplicity, we suppress the dependence on X. The probability of being alive at time t, i.e., that death occurs after t, is given by the survivor function $\overline{F}_m(t) := 1 - F_m(t)$. Since the analysis is from the perspective of a retiree who is alive at t_0, henceforth, all relevant functions are conditional on being alive at t_0. Then the NPEV is

$$\rho_{ij} = \int_{t_0}^{\infty} u(P)\overline{F}_m(t|t > t_0)e^{-\delta(t-t_0)}dt. \tag{A.2}$$

As introduced in Section 3.1.3, we assume that F_m is a Gompertz distribution, so the conditional survival functions as $\overline{F}_m(t \mid t > t_0; \lambda, \mathfrak{g}) = e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t}-e^{\mathfrak{g}t_0})}$. Substituting this in (A.2) gives

$$\rho = u(P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t_0}} \int_{t_0}^{\infty} e^{-\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t}} e^{-\delta t}dt \right\} = u(p) \times D^R. \tag{A.3}$$

To allow demographic characteristics X to affect the mortality, we let $\lambda = \exp\left(X^{\top}\tau\right)$, and estimate the parameters $(\mathfrak{g}, \tau)$ using maximum likelihood method. Finally, we set the discount factor $\delta = \ln(1 + \tilde{r}_{t_0})$, where $\tilde{r}_{t_0}$ is the annual market rate of return at t_0.

Deferred Annuity. If the annuity contracts include a deferred period clause for d months, then the pensions start from t_0+d. In the meantime, the retiree receives a "temporal payment," which is almost always twice the pension. The annuity component of the NPEV expression in (A.3) remains the same, except the lower limit is $t_0 + d$ and an additional term reflecting the temporal payment to be received during the transitory period:

$$\rho = u(2P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t_0}} \int_{t_0}^{t_0+d} e^{-\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t}} e^{-\delta t}dt \right\} + u(P) \times \left\{ e^{\delta t_0} e^{\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t_0}} \int_{t_0+d}^{\infty} e^{-\frac{\lambda}{\mathfrak{g}}e^{\mathfrak{g}t}} e^{-\delta t}dt \right\}. \tag{A.4}$$

Annuity with Guaranteed Periods. In addition to deferment, annuity contracts can also have a guaranteed period clause, which implies that if the retiree dies within a certain period (denoted as g months) from the start of the payment (either t_0 or $t_1 = t_0 + d$), the total pension amount (P) will be paid to the retiree's spouse or other beneficiaries specified in the contract until the end of the guaranteed period. The NPEV of benefits to be received by the retiree is the same as (A.3) if $d = 0$ and (A.4) if $d > 0$. As the retiree's beneficiaries are now eligible for benefits in the event of death within the guaranteed period, we let b as

the NPEV of benefits to be received by these beneficiaries, i.e., bequests. Recall that the instantaneous utility associated with beneficiaries receiving a pension P is given by $\theta \times v(\cdot)$.

The bequest b, assuming a deferment period until $t_0 + d$ and a guaranteed period of g is similar to (A.2), except that the upper integration limit is given by the guaranteed period and the instantaneous probability function corresponds to $F_m(t \mid t > t_0; \lambda, \mathfrak{g})$:

$$
\begin{aligned}
b &= \theta \times v(P) \times \left\{ \int_{t_1}^{t_1+g} F_m(t|t > t_0; \lambda, \mathfrak{g})e^{-\delta(t-t_0)}dt \right\} \\
&= \theta \times v(P) \times \left\{ \int_{t_0+d}^{t_0+d+g} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t}-e^{\mathfrak{g}t_0})})e^{-\delta(t-t_0)}dt \right\}.
\end{aligned} \tag{A.5}
$$

Allowing for Eligible Spouse. When a participant is married at the time of retirement, the spouse is eligible for a survivorship benefit in the case he or she outlives the retiree. This benefit is until death and, in the absence of eligible children, equivalent to 60% of the original pension benefit. Once again, the formula for the NPEV associated with benefits to be received by the retiree (ρ) is not affected by the presence of spouse (except for the fact that the offered pension will be lower, to account for the additional contractual entitlements).

The formula for the NPEV of bequest must then include an additional term, to account for the additional benefits to be paid in the case the spouse outlives the retiree, after the guaranteed period has elapsed. We assume that the two mortality processes are independent and follow the same Gompertz distribution (same $\mathfrak{g}$ parameter, but different λ_{sp} parameter for the spouse). In this case, the expression for NPEV of bequest is given by:

$$
\begin{aligned}
b &= \int_{t_0+d}^{t_0+d+g} \theta \times v(P) \times F_m(t|t > t_0; \lambda, \mathfrak{g}) \times e^{-\delta(t-t_0)}dt \\
&\quad + \int_{t_0+d+g}^{\infty} \theta \times v(0.6 \times P) \times F_m(t|t > t_0; \lambda, \mathfrak{g}) \times \overline{F}_m(t - \Delta|t - \Delta > t_0 - \Delta; \lambda_{sp}, \mathfrak{g}) \times e^{-\delta(t-t_0)}dt \\
&= \theta v(P) \int_{t_0+d}^{t_0+d+g} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t}-e^{\mathfrak{g}t_0})}) \times e^{-\delta(t-t_0)}dt \\
&\quad + \theta \times v(0.6 \times P) \times \int_{t_0+d+g}^{\infty} (1 - e^{-\frac{\lambda}{\mathfrak{g}}(e^{\mathfrak{g}t}-e^{\mathfrak{g}t_0})}) \times (e^{-\frac{\lambda_S}{\mathfrak{g}}(e^{\mathfrak{g}(t-\Delta)}-e^{\mathfrak{g}(t_0-\Delta)})}) \times e^{-\delta(t-t_0)}dt, \tag{A.6}
\end{aligned}
$$

where Δ is the age difference between the retiree and the retiree's spouse.

A.1.1 Recovering Pension from Expected Present Value

In this section, we consider the reverse problem of determining pension P from ρ and b for a retiree with bequest preference θ. This exercise is important because, if we can uniquely

determine pension from the expected present value, then it will allow us to go back and forth between the monetary value of an annuity (for the supply side) to utility for the retiree (for the demand side). From (3) we know that $w(P, B; \theta) = \rho(P) + \theta b(P)$, and letting $\varpi = w(P, B; \theta)$ we get

$$
\begin{aligned}
\varpi &= u(P) \times D^R + u(2 \times P) \times D^{R,DP} + \theta \left(v(P) \times D^S + v(0.6 \times P) \times D^{S,GP} \right) \\
&= \frac{P^{-2}}{-2} \left(D^R + \frac{D^{R,DP}}{4} + \theta \left(D^S + \frac{D^{S,GP}}{0.36} \right) \right),
\end{aligned}
$$

where the second equality follows from $u(c) = v(c) = \frac{c^{-2}}{-2}$. Then we can solve for the pension as

$$
P = \sqrt{\frac{\left(D^R + \frac{D^{R,DP}}{4} \right) + \theta \left(D^S + \frac{D^{S,GP}}{0.36} \right)}{-2 \times \varpi}}.
\tag{A.7}
$$

A.2   Determining the Runner-Up Firm

We define the runner-up firm in round one as the firm with the highest prob of being chosen in the first round once we exclude the chosen firm. And under the assumption that the runner-up in round one is one of the two most competitive firms in the second round then we can identify the runner-up firm for the second round as well.

To construct a measure of the probability of being selected in the first round, we estimate a series of *alternative-specific conditional logit model*. To allow for the most general estimation, we divided the sample into 90 different groups, based on the age at retirement (below, at, and above the NRA), gender, channel (recall that we combine insurance companies and sales agents into one so there are three channels) and balance quintiles. For each group, we estimate the model where the choice of an individual depends on firms' characteristics such as the ratio of reserves to assets, the fraction of sellers employed by each firm, the ratio between the fraction of complaints and premium of each firm, and the risk rating and also the `mwr`. The random utility associated with j's offer to i is given by the following expression

$$
\eta_{ijt} = \gamma_j^0 + \gamma^1 \times Z_{jt} + \gamma^2 \times \mathtt{mwr}_{ij} + \varepsilon_{ijt},
\tag{A.8}
$$

where γ_j^0 is a company-specific constant, and γ^1 is a coefficient vector for firm-specific variables. Then the probability of observing a particular choice is then given by $\Pr(D_i^1 = j) = \frac{\exp(\hat{\eta}_{ijt})}{\sum_{j=1}^{J} \exp(\hat{\eta}_{ij})}$. Using these estimated probabilities for a retiree i, we say that a company

j is the runner-up if j provides the highest utility to individual i among the set of companies ultimately not chosen by i.

A.3 Proofs

Proof of Lemma 1.

Proof. Note first that, given the proposed strategies, as ε goes to zero, the winner is the firm with the maximum $\rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij}$. We introduce some notation and then check that the proposed strategies are optimal for any $\varepsilon > 0$:

- Given a history $\mathfrak{H}$, let $\tilde{\tilde{\mathbf{P}}}_{\mathbf{i}}$ be the vector of standing offers.

- Given a history $\mathfrak{H}$ at which j plays, let $\mathcal{E}_1$ be the event that $j = \arg\max_{j \in J} \Big\{ \rho_i(P_{ij}^{\max}) + \theta_i \times b_i(P_{ij}^{\max}) + \beta_i \times Z_{ij} \Big\}$; and let $\mu_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_1)$.

- Given a history $\mathfrak{H}$ at which j plays and player k is winning (it could be the case that $j = k$), let $\mathcal{E}_2$ be the event that $\tilde{\tilde{P}}_{il} + \varepsilon > P_{il}^{\max}$ for all $l \neq j$ and $l \neq k$. Let $\tilde{\mu}_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_2)$ and $\tilde{\tilde{\mu}}_j(\mathfrak{H}) \equiv \Pr(\mathcal{E}_1 \wedge \mathcal{E}_2)$.

- Given $\mathfrak{H}$ and conditional on $\mathcal{E}_1$, define P_{ji}^* as the expected value of P such that

$$\beta_i \times Z_{ij_i^*} + \theta_i \times b_i(P) + \rho_i(P) = \max_{k \neq j} \Big\{ \beta_i \times Z_{ik} + \theta_i \times b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \Big\}.$$

  Note that $P_{ji}^* \leq P_{ij}^{\max}$.

- Given $\mathfrak{H}$ and conditional on $\mathcal{E}_1 \wedge \neg\mathcal{E}_2$, define $\tilde{P}_{ji}^*$ as the expected value of P such that

$$\beta_i \times Z_{ij_i^*} + \theta_i \times b_i(P) + \rho_i(P) = \max_{k \neq j} \Big\{ \beta_i \times Z_{ik} + \theta_i \times b_i(P_{ik}^{\max}) + \rho_i(P_{ik}^{\max}) \Big\}.$$

Assume first $\mathfrak{H}$ is such that j is not the current winner, then j's expected payment from choosing I is greater than the one from choosing S:

$$\mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*) \geq (1 - \tilde{\mu}_j(\mathfrak{H})) \times \mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*).$$

Assume $(\mathfrak{H})$ is such that j is the current winner. Then j's expected payoff of choosing S_i is

$$\mu_j(\mathfrak{H}) \times (S_i - UNC_j \times P_{ji}^*) = (\mu_j(\mathfrak{H}) - \tilde{\tilde{\mu}}_j(\mathfrak{H})) \times (S_i - UNC_j \times \tilde{P}_{ji}^*) + \tilde{\tilde{\mu}}_j(\mathfrak{H}) \times (S_i - UNC_j \times \tilde{\tilde{P}}_{ji}^*),$$

which is greater than or equal the expected payment of choosing I, so that

$$(\mu_j(\mathfrak{H}) - \tilde{\tilde{\mu}}_j(\mathfrak{H})) \times (S_i - UNC_j \times \tilde{P}^*_{ji}) + \tilde{\tilde{\mu}}_j(\mathfrak{H}) \times (S_i - UNC_j \times (\tilde{\tilde{P}}^*_{ji} + \varepsilon)).$$

$\square$

Proof of Lemma 2.

Proof. For notational simplicity, we denote the LHS of Equation (13) as $\mathfrak{U}$ and the RHS as a sum $\tilde{\beta} + \varpi$. Consider auctions with J firms. From the observed chosen pensions and F_θ, we can identify the distribution of $\mathfrak{U}$, which is, by definition, also the distribution of the second-highest value of the sum $\tilde{\beta} + \varpi$. We denote the latter distribution by $F^{(J-1:J)}_{\tilde{\beta}+\varpi}(\cdot)$. However, there is a one-to-one mapping between the distribution of order-statistics and the "parent" distribution. In particular, the parent distribution of the sum $F_{\tilde{\beta}+\varpi}(\cdot)$ is pinned down by $F_{\mathfrak{U}}(t) = F^{(J-1:J)}_{\tilde{\beta}+\varpi}(t) = J(J-1)\int_0^{F_{\tilde{\beta}+\varpi}(t)} (\xi^{J-2} \times \xi) d\xi$.

And since $F_{\tilde{\beta}+\varpi} = F_{\tilde{\beta}} * F_\varpi$, is a convolution, where $*$ is the convolution operator, we can identify the distribution of ϖ via deconvolution. Lastly, we observe that there is a one-to-one mapping from ϖ to $P^{\max}$ –the maximum pension runner-up firm can offer to retiree (see Equation A.7), which we denote by a function $P^{\max} = m(\varpi) = S/UNC_k$. Then we get

$$
\begin{aligned}
W_r(\xi) &= \Pr(r \le \xi) = \Pr\left(\frac{UNC_k}{UNC_i} \le \xi\right) = \Pr\left(\frac{S}{P^{\max}} \le \xi \times UNC_i\right) \\
&= \Pr\left(P^{\max} \ge \frac{S}{\xi \times UNC_i}\right) = 1 - \Pr\left(P^{\max} \le \frac{S}{\xi \times UNC_i}\right) \\
&= 1 - \Pr\left(m^{-1}(P^{\max}) \le m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right) = 1 - \Pr\left(\varpi \le m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right) \\
&= 1 - F_\varpi\left(m^{-1}\left(\frac{S}{\xi \times UNC_i}\right)\right), \qquad (\because P^{\max} = S/UNC_k).
\end{aligned}
$$

$\square$