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A Sub-band Filter Design Approach for Sound Field Reproduction

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Content

- Introduction
- Methodology
- Results
- Conclusions

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Content

- Introduction
- Methodology
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Statement of research problem

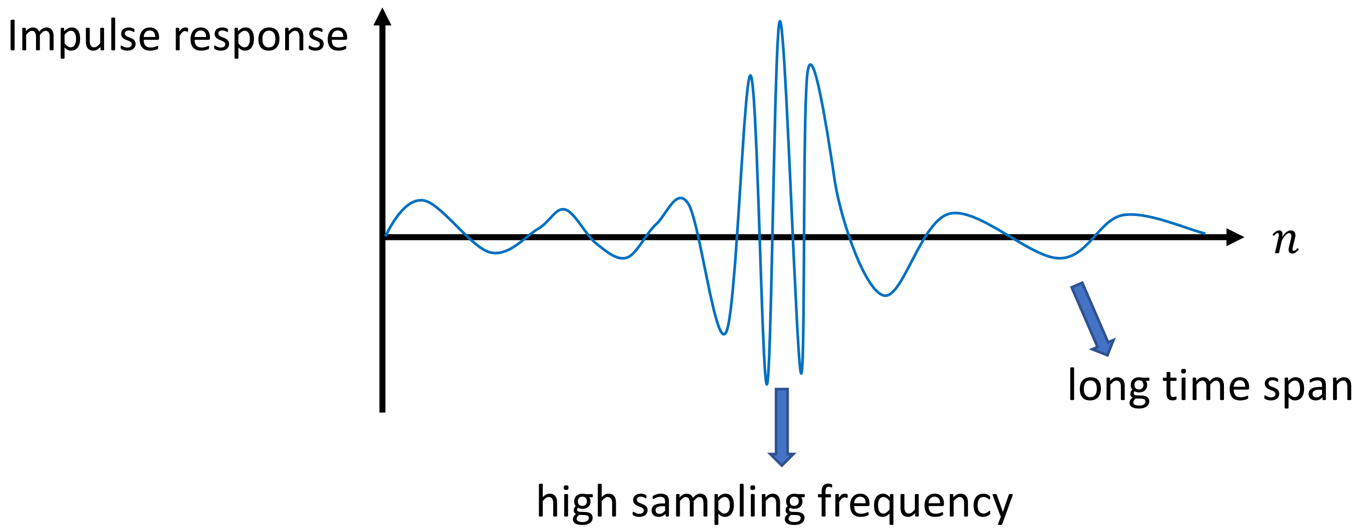
- ❑ Sound field reproduction uses loudspeakers to produce desired sound at locations.

- ❑ When designing filter for sound that spans a wide frequency range:

Low frequency band → longer time span

High frequency band → higher sampling frequency

→ Large number of filter coefficients



Statement of research problem

An approach is proposed to design filter in a sub-band form:

- Design all sub-band filters directly in one optimization problem:

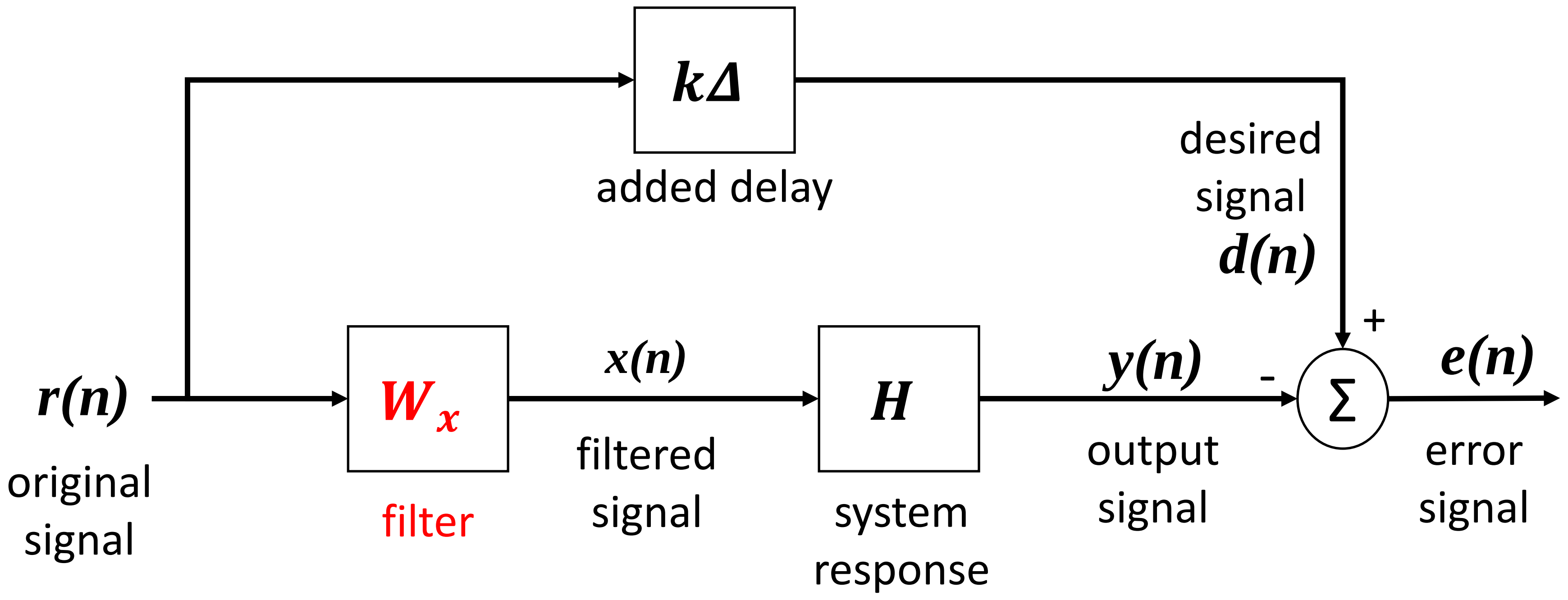
  The **transition region** between two sub-band filters can be designed conveniently

- The **computational load** can be reduced even if sub-band filters structure is not required

Content

- Introduction
- Methodology
- Results
- Conclusions

Designing filter directly



- **Example**:

  Use loudspeaker to produce desired sound at certain locations

- **Cost function**:

  Minimizing the power of error signal $\boldsymbol{e}$

- **Constraints:**

  Filter response $\boldsymbol{W}_x$(f)

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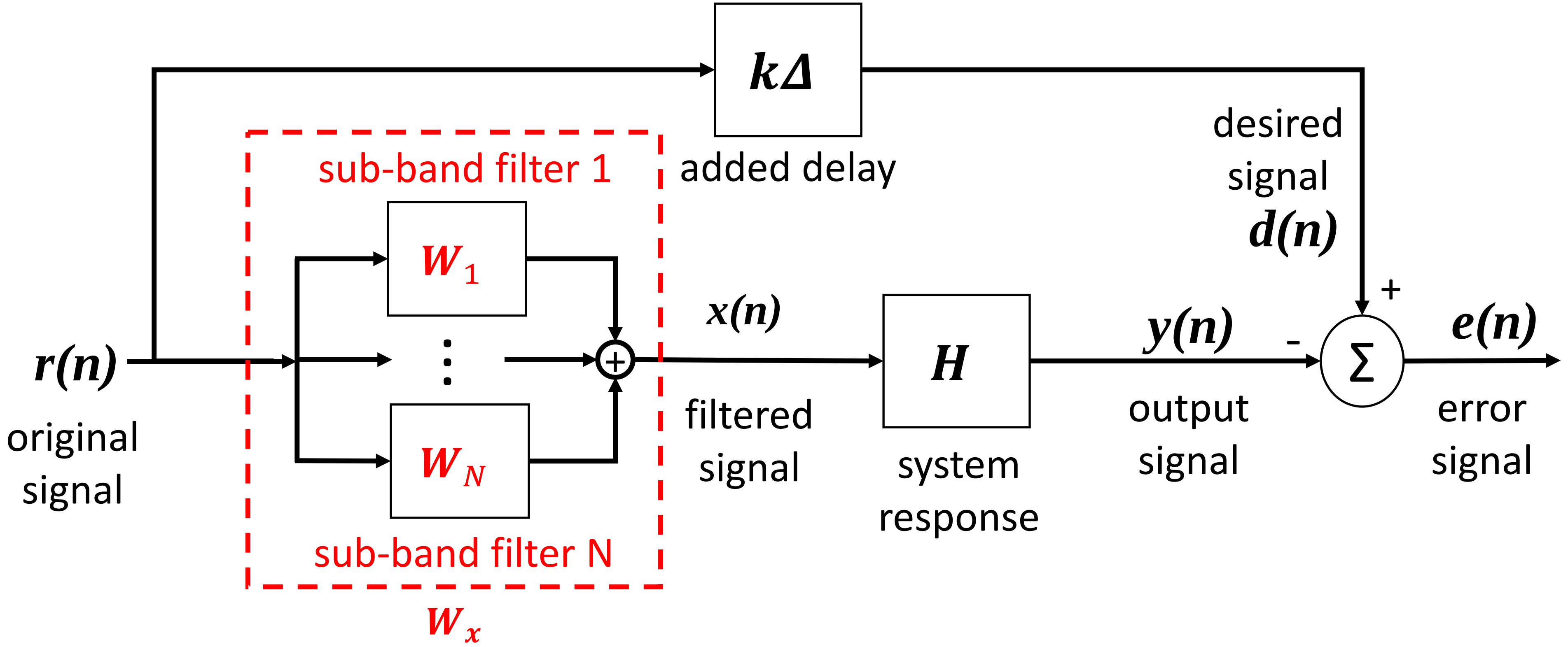
Designing filter when sub-band technique is used



- **Example**:

  Use loudspeaker to produce desired sound at certain locations

- **Cost function**:

  Minimizing the power of error signal ***e***

- **Constraints:**

  Filter response $\boldsymbol{W}_i$(f)

Expressing sub-band filters as one equivalent filter

- **Conventional method (one single filter)**

frequency response of designed filter at frequency f_k :

$$W_x(f_k) = F(f_k, f_s, N_t)\,\vec{w}_x \;, \qquad F(f_k, f_s, N_t) = \begin{bmatrix} 1 & e^{-\frac{j2\pi f_k}{f_s}} & \cdots & e^{-\frac{j2\pi f_k(N_t-1)}{f_s}} \end{bmatrix}$$

f_s is the sampling frequency,
N_t is the number of filter coefficients,
$\vec{w}_x$ is the filter coefficients

- **Sub-band structure**

frequency response of designed filter at frequency f_k:

$$\sum_{i=1}^{N} W_i(f_k) = \sum_{i=1}^{N} \begin{bmatrix} 1 & e^{-\frac{j2\pi f_k}{f_{s_i}}} & \cdots & e^{-\frac{j2\pi f_k(N_t-1)}{f_{s_i}}} \end{bmatrix} \vec{w}_i$$

Expressing sub-band filters as one equivalent filter

So designing sub-band filters can be treated as:

designing one filter $\vec{\tilde{w}}_x$ with modified Fourier matrix $\tilde{F}(f_k)$

$$\widetilde{W}_x(f_k) = \sum_{i=1}^{N} W_i(f_k) = \sum_{i=1}^{N} \left[1 \quad e^{-\frac{j2\pi f_k}{f_{s_i}}} \quad \cdots \quad e^{-\frac{j2\pi f_k (N_t - 1)}{f_{s_i}}} \right] \vec{w}_i = \tilde{F}(f_k)\, \vec{\tilde{w}}_x \,,$$

$$\tilde{F}(f_k) = \left[F\left(f_k, f_{s_1}, N_{t_1}\right) \quad \cdots \quad F\left(f_k, f_{s_N}, N_{t_N}\right) \right],$$

$$\vec{\tilde{w}}_x = \begin{bmatrix} \vec{w}_1 \\ \vdots \\ \vec{w}_N \end{bmatrix}$$

So all the sub-band filters can be designed in one optimization problem if designed in the frequency domain.

The transition region can be designed more conveniently.

Overview of proposed design process

Design Problem → (Conventional method) → **General Nonlinear algorithm (e.g., sequential quadratic programming)**

Design Problem → (Proposed method) → **Convex Optimization** → **Cone Programming** → **Primal-dual Interior-point**

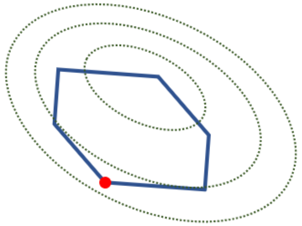
Design Problem

Cost function:
Power of ***e***

Constraints:
Filter response

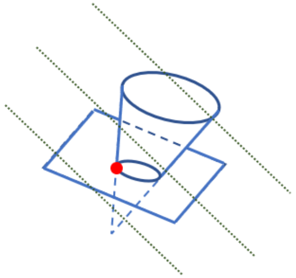
Convex Optimization

Global minimum usually guaranteed



Cone Programming

Possible to apply efficient algorithms



Primal-dual Interior-point

- Efficient
- Upper-bound for iteration times

Problem formulation

Design Problem Expressed in Convex Problem

Cost function:

Total power of e:

$$\sum_{k=k_1}^{k_2} |E(f_k)|^2 , \quad \Rightarrow \quad \vec{\widetilde{w}}_x^{\,\mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\widetilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\widetilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Convex ✓

- Quadratic
- $A_J(f_k)$ p.s.d

Constraints:

Filter response:

The magnitude of frequency response:

$$|W_i(f_k)| \leq C_i(f_k) \quad \Rightarrow \quad \|F(f_k, f_{s_1}, N_{t_1})\, \vec{w}_i\|_2 - C_i(f_k) \leq 0$$

→ Vector norm **Convex** ✓

Cone Programming Reformulation

Convex Problem

Cost function:

$$\vec{\tilde{w}}_x^{\mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\tilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Constraints:

$$\|F(f_k, f_{s_1}, N_{t_1})\, \vec{w}_i\|_2 - C_i(f_k) \leq 0$$

?

Standard Cone Programming

Cost function: $c^{\mathrm{T}}x$

Constraints: $x \in K_i, \qquad i = 1, 2, 3\ \ldots$

$$Ax = b$$

c **to be a constant vector**

K_i **to be a convex cone**

A, b **to be a constant matrix and vector**

Cone Programming Reformulation

Convex Problem ➡ **Cone Programming**

- **Reformulate quadratic cost function**

Cost function: $x^{\mathrm{T}} A\, x + b^{\mathrm{T}} x \; + c$

⇕

Cost function: $t_0 + b^{\mathrm{T}} x$ ➡ Linear cost function

Constraints: $\|\sqrt{A}\, x\|_2 \leq \sqrt{t_0\, \tilde{t}_0}$ ➡ Rotated second-order cone

$\tilde{t}_0 = 1$ ➡ Linear constraint

- **The vector norm constraint**

Constraints: $\|x\|_2 - c \leq 0$

⇕

Constraints: $\|x\|_2 \leq t$ ➡ Second-order cone

$t = c$ ➡ Linear constraint

Cone Programming Reformulation

Convex Problem

Cost function:

$$\vec{\tilde{w}}_x^{\,\mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\tilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Constraints:

$$\|F(f_k, f_{s_1}, N_{t_1})\,\vec{w}_i\|_2 - C_i(f_k) \le 0$$

Cone Programming

Cost function:

$$t_0 + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x$$

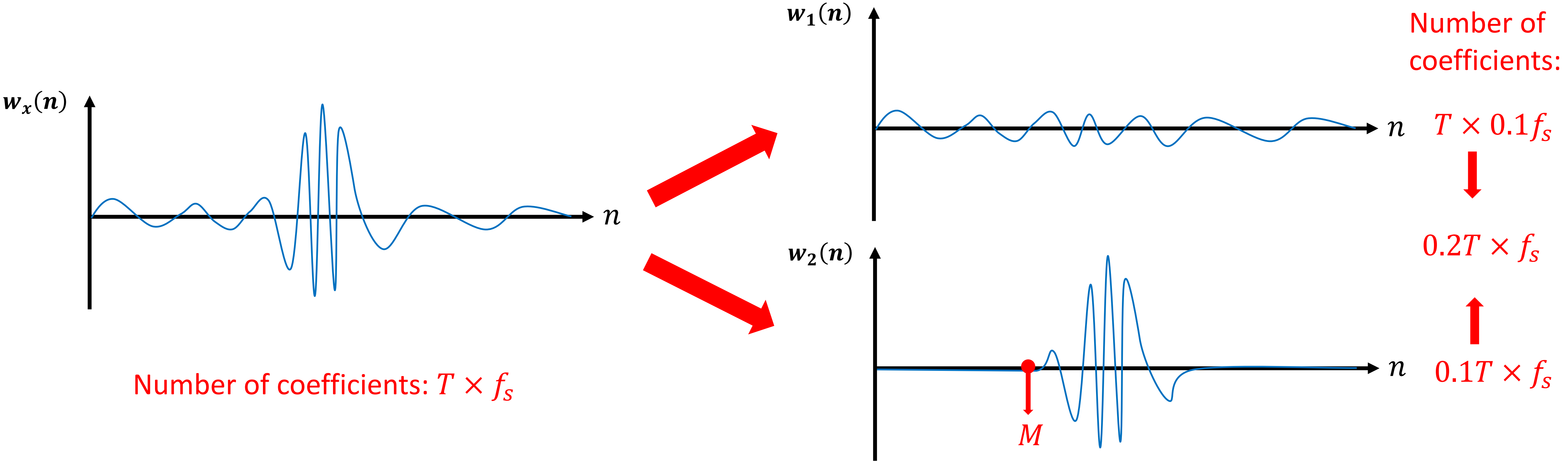
Constraints:

$$\|F(f_k, f_{s_1}, N_{t_1})\,\vec{w}_i\|_2 \le t_{3,k}\,,$$

$$t_{3,k} = C(f_k)$$

A reduced order technique

Sometimes, the designed filter has high frequency response concentrated in small time span:



In this case, $\vec{w}_i$ (with higher sampling frequency) can be chosen to start with $t = M\Delta$, where $M > 0$, then we have:

$$F_r(f_k, f_s, N_t) = \left[e^{-\frac{j2\pi f_k M}{f_s}} \quad e^{-\frac{j2\pi f_k (M+1)}{f_s}} \quad \cdots \quad e^{-\frac{j2\pi f_k (N_t - 1)}{f_s}} \right]$$

Content

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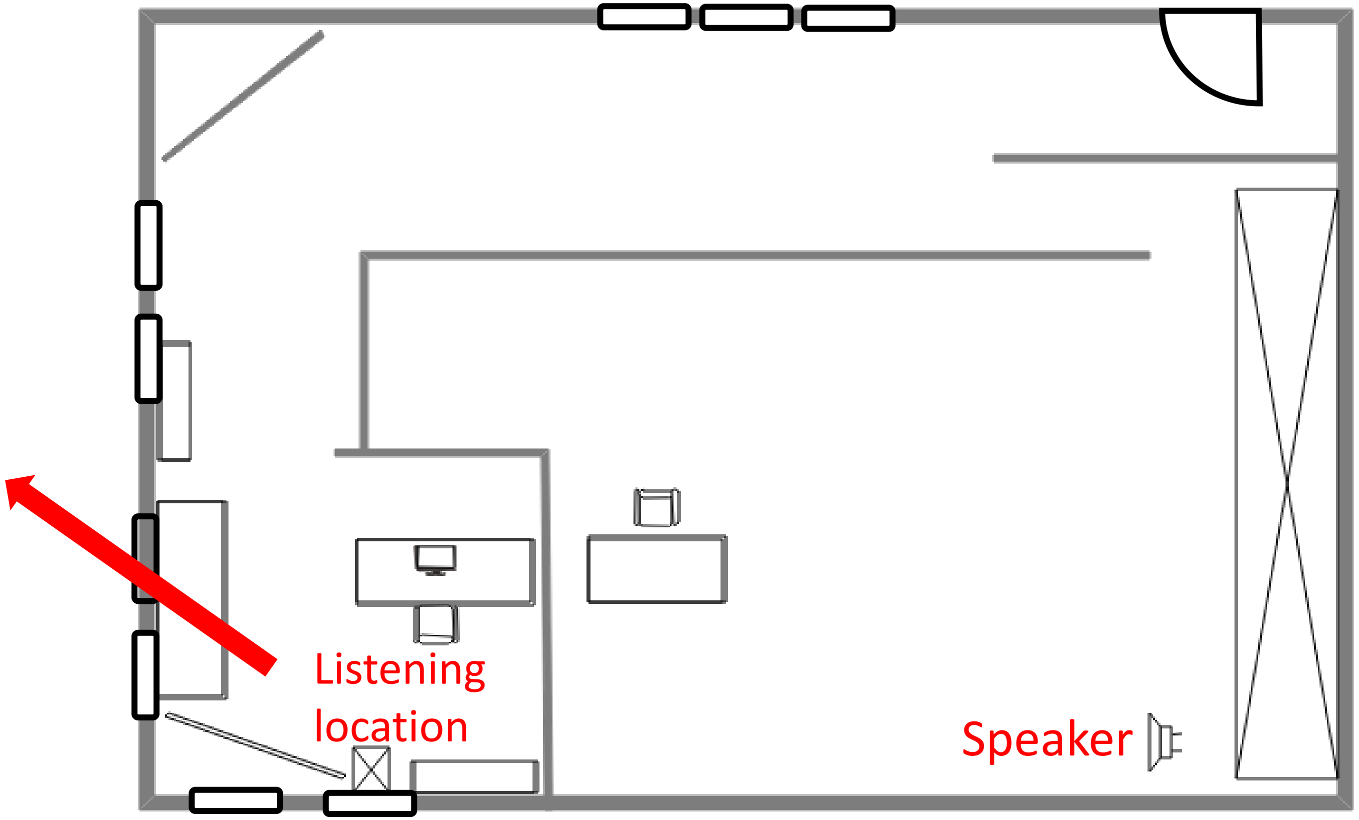
Experimental setup

- An experimental setup for psychoacoustic listening test
- Speaker should produce desired sound at listening location



Experimental setup

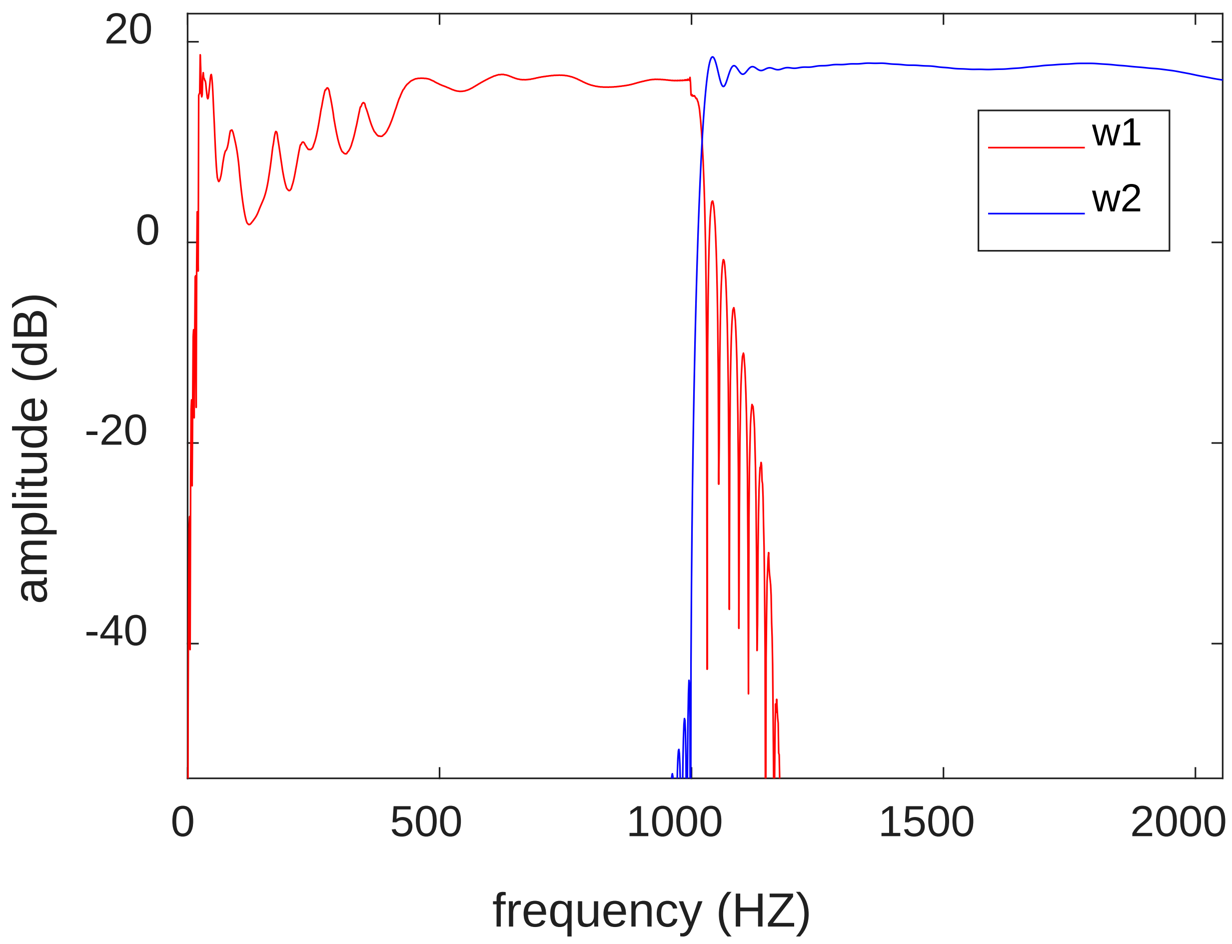
- Required sampling frequency: **48 kHz (Δ =20.83 us)**
- Desired delay: **19200 Δ**
- Two sub-band filters:

	Sampling frequency	**Filter coefficients**	**Starting time**
Filter 1	2.4 kHz	1920	0
Filter 2	48 kHz	3000	17700 **Δ**

- SeDuMi is used to solve the reformulated cone programming problem

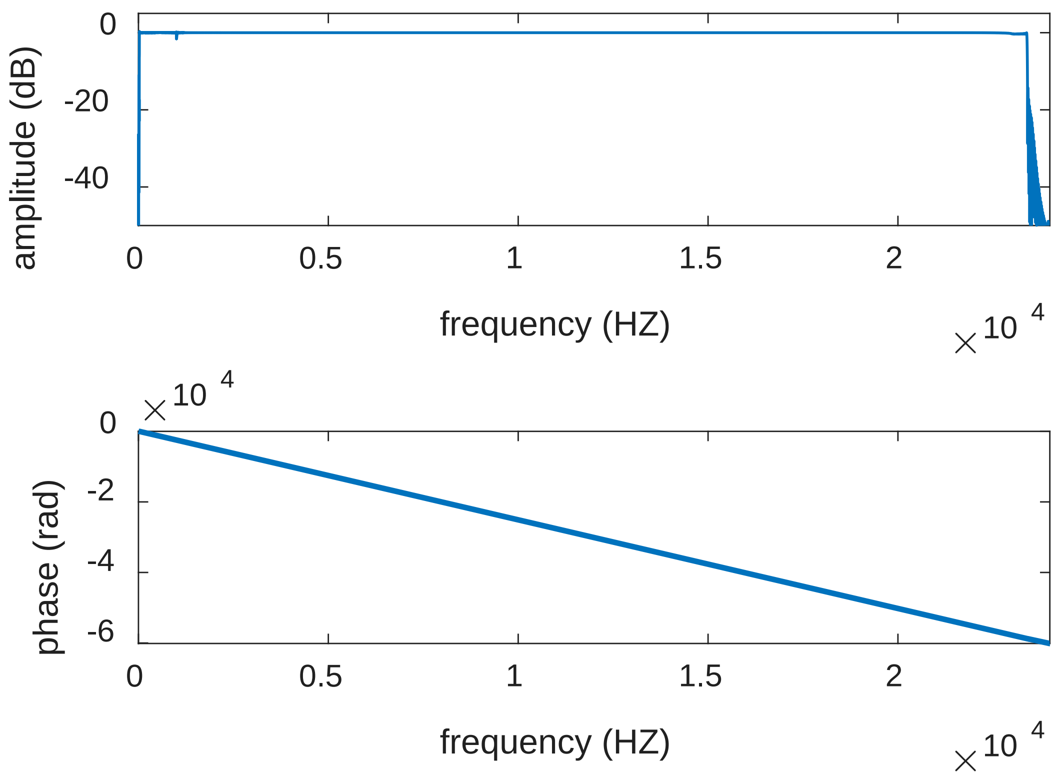
Result

The frequency response of both filter around 1200 Hz

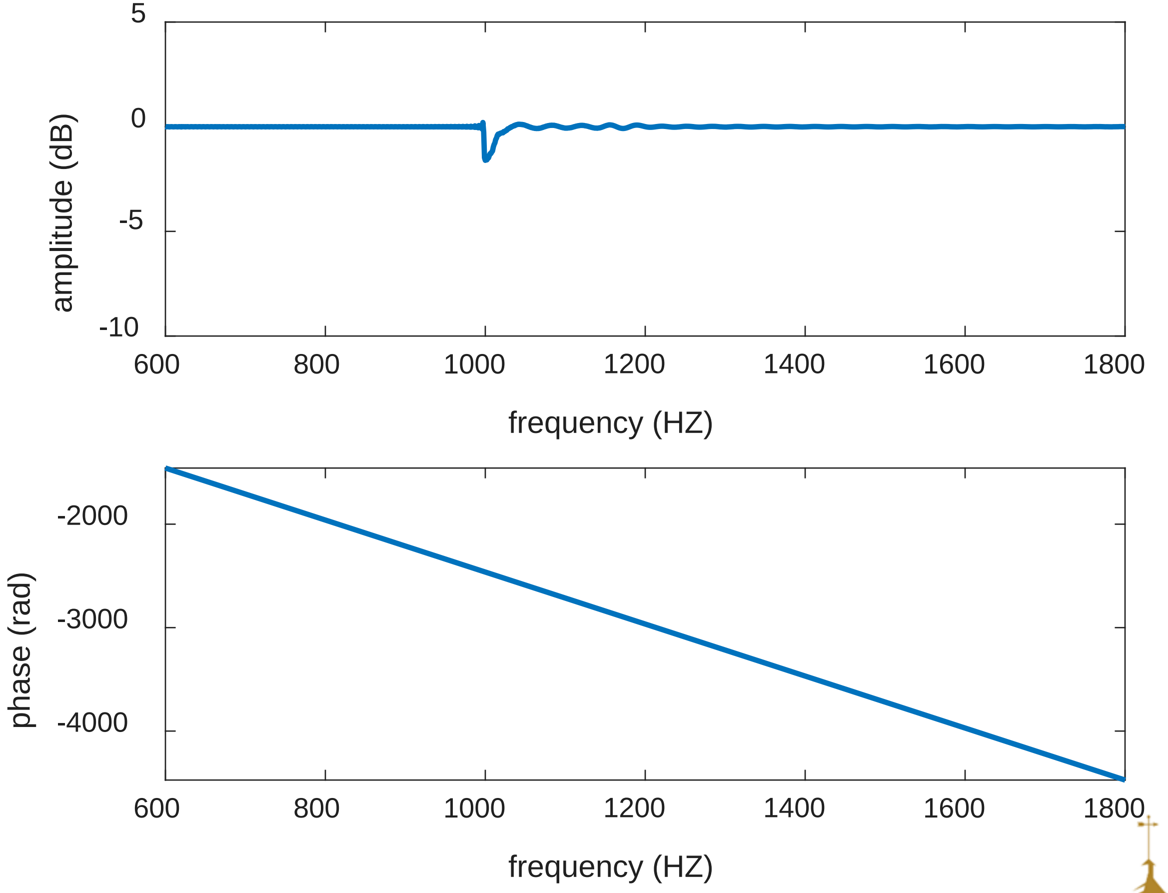


Result

The frequency response of $\mathrm{H(f)}\widetilde{W}_x(f)$



$\mathrm{H(f)}\widetilde{W}_x(f)$ around 1200 Hz

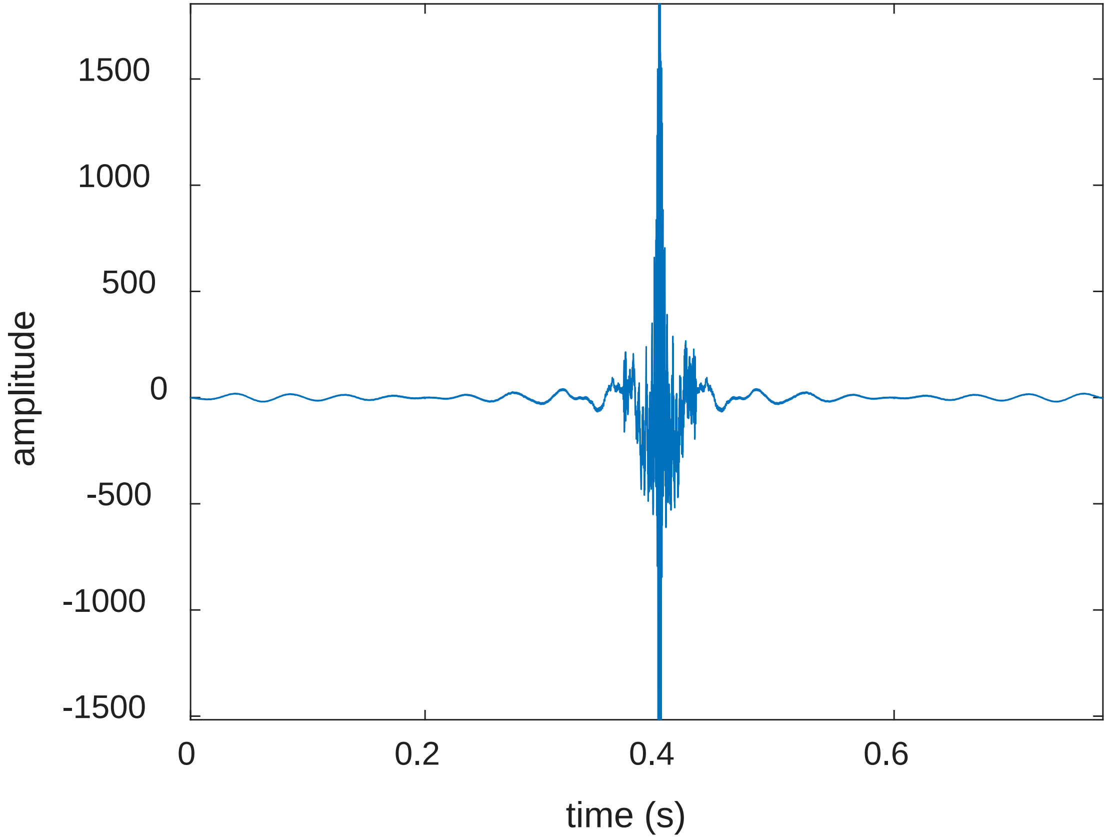


Result

Combining two sub-band filters together in time domain

The combination is done by:

- Upsampling the sub-band filter 1 with lower sampling frequency
- Adds the upsampled filter 1 with filter 2



The designed filter coefficients are 1920+3000 = 4920, which is much smaller than $48000 \times \frac{1920}{2400} = 38400$

Content

- Introduction
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Conclusions

- The proposed method can design sub-band filters for sound field reconstruction in one optimization problem, so designing transition region is more convenient.

- The optimization problem can be reformulated to a convex problem, then further reformulated to a cone programming problem. These guarantees the global optimal solution can be found in an efficient way.

- A reduced-order technique can be used to reduce the variables in filter design problem if different frequency bands of required filter have impulse response concentrated in different time intervals.

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Q&A