12-10-2020

(2,3)-Cordial Digraphs

Jonathan Mousley
Utah State University, jonathanmousley@gmail.com

Manuel Santana
Utah State University, manuelarturosantana@gmail.com

(2,3)-Cordial Digraphs

Jonathan Mousley, Manuel Santana

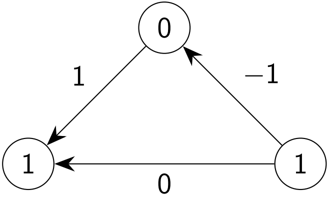
Utah State University

Fall Student Research Symposium, December 2020

What is a (2,3)-Cordial Labeling

Conditions

- Directed graph
- Friendly vertex labeling
- Head minus tail arc labeling
- Balance of arc labels



Application: Balanced Networks

Parallel programming

Breaking down computer program into discrete tasks, then assigned to multiple processors that execute simultaneously.

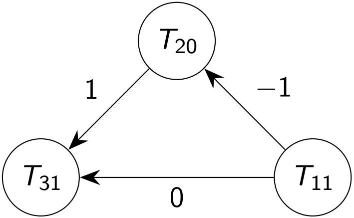
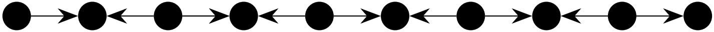


Figure: Parallel program

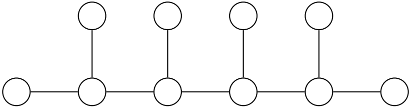
Strategy

- Balance workload across processors
- Balance internal communication within processors
- Minimize external communication within processors

Simple Cases



There are 256 unique ways to orient the arcs.



This graph is not (2,3)-orientable.

A Proof

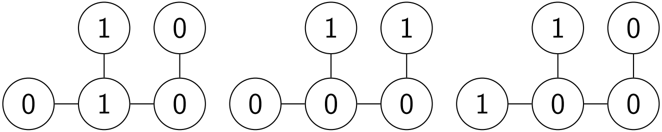


Figure: All labelings with one edge labeled 0

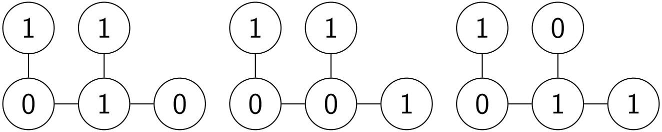
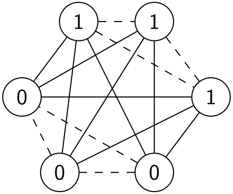


Figure: All labelings with two edges labeled 0

UtahStateUniversity
HONORS PROGRAM

An Important Theorem



Theorem

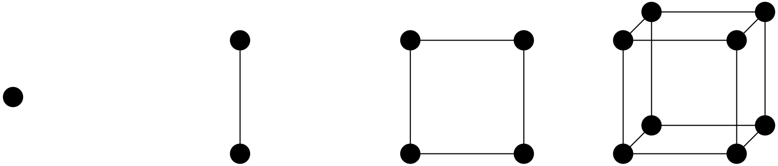
Given a directed graph $G = (V, E)$ with vertex set V and $n = |V|$ with $n \geq 6$, and edge set E. The maximum size of E such that G is $(2, 3)$-orientable for any given n is

$$|E|_{max} = \binom{n}{2} - Z + \left\lceil \frac{1}{2}\left(\binom{n}{2} - Z\right)\right\rceil$$

$$Z = \binom{\lceil \frac{n}{2} \rceil}{2} + \binom{\lfloor \frac{n}{2} \rfloor}{2}.$$

Utah State University
HONORS PROGRAM

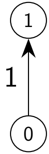
Hypercubes

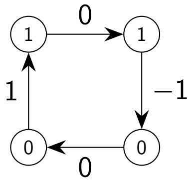


$k = 0$ $k = 1$ $k = 2$ $k = 3$

Figure: k-dimensional hypercubes for $k = 0, 1, 2, 3$

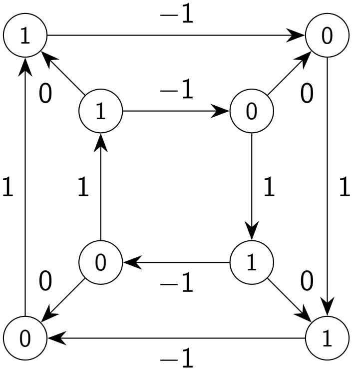
$(2,3)$-Cordial Oriented Hypercubes





$k = 1$ $\quad$ $k = 2$

(2, 3)-Cordial Oriented Hypercubes



$k = 3$

Proof by Induction

Theorem

All hypercubes of dimension $3k$ for $k \in \mathbb{N}$ are (2,3)-orientable.

Base Case

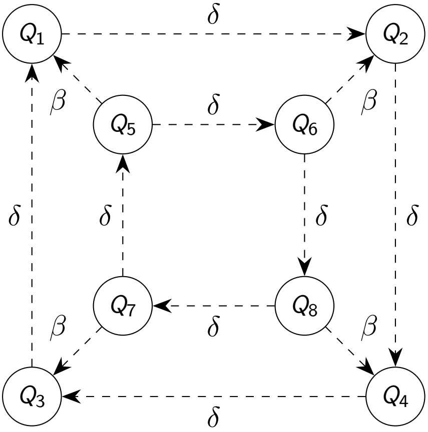
Dimension 3.

Induction Hypothesis

Some k-dimensional oriented hypercube is (2,3)-cordial.

Proof by Induction

Inductive Step, $k \implies k+3$



- Q_i: (2,3)-cordial k-dimensional oriented hypercube
- Dashed arc: represents 2^k arcs, one from each vertex
- δ: vertices of different labels connected
- β: vertices of like labels connected

UtahStateUniversity
HONORS PROGRAM

Other Results with Hypercubes

Theorem

All hypercubes of dimension $k \geq 1$ are (2,3)-orientable.

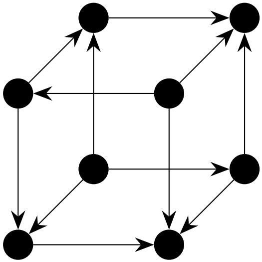
Theorem

Not all orientations of cubes are (2,3)-cordial.

3D Identification Problem

Cataloged several properties that guarantee (2,3)-cordiality in oriented cubes.

A 3D oriented hypercube, [illegible] (2,3)-cordial



Future Work on Hypercubes

- Continue study of properties that prevent (2,3)-cordiality for 3D case
- Generalize results from 3D case to k-dimensional case

UtahStateUniversity
HONORS PROGRAM

Bibliography

1. L. B. Beasley, **J. Mousley, M. Santana**, D. Brown, (2,3)-Cordial Digraphs, Submitted.
2. L. B. Beasley, Cordial Digraphs, In Press.
3. I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Comb. 23(1987) 201-208.
4. M. Hovey, A-cordial graphs, *Discrete Math.* 93 (1991) 183-194.