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TIME-RESOLVED LUMINESCENCE FROM TWO-DIMENSIONAL ELECTRONS IN HIGH MAGNETIC FIELDS: A TOOL FOR STUDYING THE FRACTIONAL QUANTUM HALL EFFECT

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Abstract

Time-resolved magnetoluminescence investigations of specially doped AlGaAs/GaAs heterostructures allow the study of the electronic properties of two-dimensional electrons. The interactions of the two-dimensional electrons lead in high magnetic fields to the fractional quantum Hall effect. By analyzing the mean energy of the recombination line observed in the luminescence spectrum, the electronic properties of the fractional quantized states can be deduced. For negligible interaction between the charged acceptors and the interacting electron system, the derivative of the mean energy versus filling factor allows to obtain the jump of the chemical potential in crossing a fractional filling factor. From these jumps, the gaps in the excitation spectrum are deduced for different fractional filling factors and the hierarchy of the fractional quantum Hall effect is studied.

Key Words: Two-dimensional electrons, time-resolved luminescence, fractional quantum Hall effect.

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Introduction

The electronic properties of a two-dimensional (2D) electron gas (2DEG) have been a subject of intense research for almost 30 years. Nevertheless, many questions are still open, especially about the electronic structure in high magnetic fields. In a single particle description, the density of states of a 2DEG splits in a magnetic field into Landau- and spin levels (neglecting possible valley splittings which are unimportant for simple Fermi surfaces, like in GaAs).

$$E_{ns} = \hbar\omega_c\,(n + 1/2) + sg\mu_B B \qquad (1)$$

with B the magnetic field, $\hbar\omega_c = \hbar eB/m^*$ the cyclotron energy, g the g-factor, and μ_B Bohr's magneton. For a given carrier concentration n_s, only a certain number ν (called filling factor)

$$\nu = (hn_s/eB) \qquad (2)$$

of levels can be filled due to the degeneracy of individual levels given by eB/h. The interest into the physics of a 2DEG increased drastically after the discovery of the quantum Hall effect [37, 38], i.e., the observation of plateaus in the Hall resistance with values $R_{xy} = h/ie^2$ around integer filling factors $\nu = i$. Shortly after the quantum Hall effect, in a similar transport experiment, plateaus in the Hall resistance were also found around non-integer filling factors [6, 34]

$$\nu = (p/q) \qquad (3)$$

with p and q integers and q odd. Around this fractional filling factors, no gaps in the single particle density of states exist, in contrast to the quantum Hall effect with the cyclotron and spin gaps. Therefore, this observation, called fractional quantum Hall effect (FQHE), was assumed to originate from interaction effects. Laughlin [31] explained the occurrence of an excitation gap at fractional filling factors by the condensation of interacting 2D electrons into an incompressible quantum liquid.

In transport experiments, where these effects have been found, only indirect ways are available to deduce the gaps in the spectrum. In temperature dependent measurements, the activation energies can be deduced [3, 7, 18]. But, the results depend strongly on the disorder present in the sample and the method is especially unreliable in the case of small energy gaps, where the gap energy is comparable with the Landau level width. The measurements also have to be made over a certain range in temperature, i.e., at a fixed temperature or at lowest temperature, the value of the gap cannot be measured. Therefore, the application of other methods was necessary.

Several groups [see e.g., 12, 35], are now using luminescence experiments to study the electronic properties of 2DEG in high magnetic fields. But, in most of these experiments, the recombination between two-dimensional electrons and free holes is studied, and the results, especially about the FQHE regime, are very unclear. It was shown [32] that for coplanar electrons and holes, the photoluminescence spectrum in the FQHE regime does not exhibit anomalies associated with the FQHE, i.e., the photoluminescence is, in this case, not sensitive to interaction effects.

Another approach to study the magnetoluminescence of a 2DEG uses the recombination between the two-dimensional electrons and holes localized at acceptors [5]. These experiments are better suited for the study of the electronic properties than experiments studying the recombination of free holes. An acceptor and a hole form a small neutral complex, which does not perturb the local structure of a collective electronic state. Such experiments can be performed in a well-defined way due to the development in modern growth techniques. An important parameter here is the distance between the electrons and the holes located at the acceptors, as was shown in the theoretical work by Apal'cov and Rashba [1, 2]. Nowadays, due to the development of the δ-doping technique, acceptors can be placed at fixed distances to the 2DEG and with known concentrations [19, 33].

During recent years, time-resolved luminescence investigations have especially proven to be a sensitive tool to study the two-dimensional electronic systems in high magnetic fields. In the regimes of the integer and fractional quantum Hall effect, from such measurements, one is able to deduce cyclotron energies, spin splittings, and the gap energies of the FQHE. Such experiments will be discussed in the following sections with emphasis on the study of FQHE and the hierarchy of different states. By analyzing the center-of-mass (first moment) of the luminescence spectrum, the quasi-particle energy gaps of the FQHE can be deduced from the observed discontinuities in the chemical potential. The hierarchy of the FQHE states has also been investigated [28]. For several families of FQHE states, a universal scaling behavior has been found.

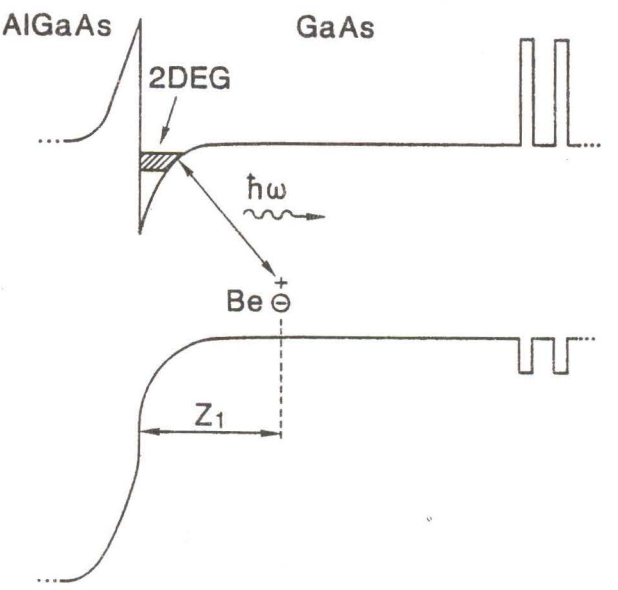

Figure 1. Band-structure of the special doped $Al_xGa_{1-x}As/GaAs$ heterostructures used for the luminescence experiments.

Samples

For our experiments, $Al_xGa_{1-x}As/GaAs$-heterostructures (with x about 0.3) grown by molecular beam epitaxy (MBE) were used. These heterostructures are the model system for investigations of the electronic properties of a 2DEG due to the simple band-structure of the GaAs, due to the good optical properties originating from the direct bandgap, and due to the high quality material achievable in the MBE growth process. The heterostructures used for our experiments not only have a 2DEG with a carrier concentration between 4×10^{10} cm^{-2} to 4×10^{11} cm^{-2} at the interface between the GaAs and the $Al_xGa_{1-x}As$ due to the Si-doping in the $Al_xGa_{1-x}As$, but they also contain a layer of acceptors (Be) with densities around 10^9 cm^{-2} in the GaAs. A schematic view of the band-structure of these devices is given in Figure 1. The layer of Be-acceptors has been incorporated during the MBE growth process by the δ-doping technique [33].

Such samples with a 2DEG and an additional layer of acceptors were first used in transport experiments to investigate the transport properties of two-dimensional electrons scattering mainly from known impurities [19]. When they are located near the 2DEG, the scattering is strong and the mobility greatly reduced due to the presence of the acceptors [20]. Therefore, these special

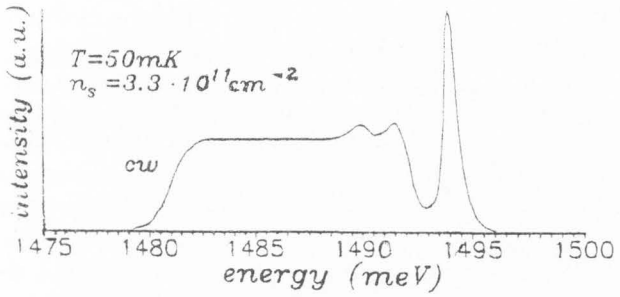

Figure 2. Typical luminescence spectrum (intensity in arbitrary units, a.u., versus energy in meV) measured under continuous illumination (cw) at low temperature (T = 50 mK) for a heterostructure with an additional layer of Be-acceptors at a distance of 40 nm from the interface and with an acceptor concentration of 5 x 10^{-9} cm^{-2}; the carrier concentration, n_s = 3.3 x 10^{11} cm^{-2}.

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doped samples can also be used as model systems to study disorder and localization properties [23, 24]. Similar samples are used here for the luminescence studies. The electronic properties of the 2DEG are investigated in the recombination of the electrons with holes localized at the acceptors. In contrast to the transport studies, where the acceptors were placed close to the 2DEG, in the luminescence studies, the acceptors are placed as far from the 2DEG as possible to increase the sensitivity to electron-electron interaction effects, but close enough to still have enough intensity and accuracy in the measurement. The increase of the distance between 2D electrons and the acceptor monolayer results in an exponential increase of the recombination time and hence in an exponential reduction of the intensity of the corresponding luminescence signal, so that bulk signals become more and more dominant. Due to the large distance between the acceptors and the 2DEG, the mobilities of the electrons are only slightly affected. Therefore, the heterostructures discussed here show electronic mobilities up to 2 x 10^6 cm^2/Vs.

In these structures, the luminescence from recombination between the two-dimensional electron gas and holes bound to the known Be-acceptors were studied in magnetic fields applied perpendicular to the 2DEG. The experiments were performed at temperatures to 23 mK, so the regime of the integer quantum Hall effect as well as the FQHE could be studied. To achieve a temperature of 23 mK, the samples were mounted in the mixing chamber of a ^{3}He/^{4}He dilution refrigerator. For the excitation of the system, an Ar^+ laser was used, coupling the light with a glass-fiber of 0.5 mm diameter to the sample. Peak powers up to 10^{-3} W/cm^2 were used. The luminescence light collected in the glass-fiber was detected by a 1 m double monochromator.

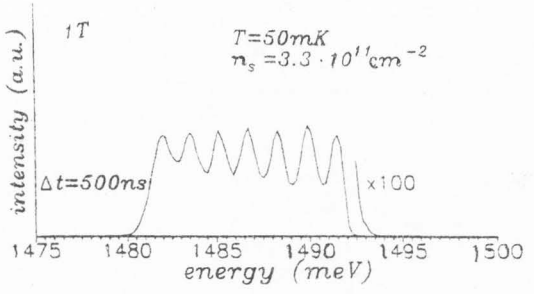

Figure 3. Luminescence spectrum taken at a magnetic field of B = 1 T and at a delay time (Δt = 500 ns) after the excitation pulse (same sample as in Fig. 2).

Figure 2 shows a typical spectrum measured at low temperatures and zero magnetic field under continuous illumination for a heterostructure with a layer of Be-acceptors at a distance of 40 nm to the interface and with a concentration of n_a = 5 x 10^9 cm^{-2}. Whereas the main part of the spectrum is dominated by the recombination of electrons in the lowest subband with holes located at the acceptors, some contribution of a bulk signal (in the spectral region of 1489-1492 meV) is observed due to the rather large distance between electrons and acceptors [25]. This bulk signal can be suppressed in time-resolved luminescence measurements due to its strong reduction for longer times. The sharp peak around 1495 meV originates from the recombination of excited electrons in the second size-quantized subband and holes located at the acceptors. This high population of the second subband originates from the excitation with the light and also shows the increased electron temperature of the system. The presence of this signal is associated with a rather slow inter-subband relaxation process and the corresponding signal is visible up 50 ns. Here, time-resolved luminescence is also of advantage. In addition, pulsed excitation and delayed detection allow for a cooling of the system and for the observation of energetic structures with a higher accuracy.

Time-Resolved Measurements

As mentioned above, the use of time-resolved luminescence should improve the results of such experiments. For our experiments, pulses with a duration of 20 ns and a repetition frequency of 0.1 MHz were generated by an acousto-optic modulator. For the detection, a gated photon counting system with a time resolution of about 30 ns was applied.

Figure 3 shows a typical luminescence spectrum of the same structure as shown in Figure 2 in a time-resolved luminescence experiment (for a delay time of 500

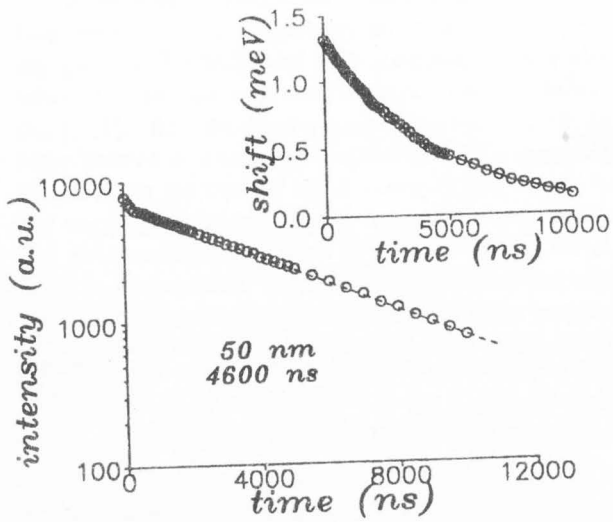

Figure 4. Time dependence of the luminescence intensity measured at $\nu = 1$ ($B = 4$ T) for a structure with a electron-hole separation (z_1) of 50 nm. A recombination time (t_o) of 4600 ns was deduced for this case (i.e., for $z_1 = 50$ nm). The inset shows the spectral shift of the luminescence line with time.

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ns and in a magnetic field of $B = 1$ T). The recombination from seven Landau-levels is observed; contributions from second subband and from bulk GaAs are not seen.

The density of active holes is much lower than the electron density n_s in these experiments due to the small concentration of acceptors (one to two orders of magnitude smaller). Therefore, the hole recombination rate t_0^{-1} governs the kinetics of the electron-hole luminescence intensity $I(t)$. For a homogeneous distribution of electrons in the two-dimensional layer, a single time constant t_o given mainly by the distance between the electrons and the holes is expected to be observed in the temporal evolution of the luminescence intensity $I(t)$.

$$I(t) = I_o exp\ (-t/t_o) \qquad (4)$$

Figure 4 gives an example of the measurement of the integral intensity as a function of time at a magnetic field corresponding to a filling factor of $\nu = 1$. One sees that the time evolution of the integral luminescence signal is described by a single exponential dependence (dashed line in Fig. 4). The recombination time t_o deduced from such measurements depends on the overlap between the electronic wavefunction and the neutral complex formed by the hole bound to the acceptor (with a spatial extent much smaller than those of the electrons). For the case shown in Figure 4 with a distance of $z_1 = 50$ nm, a time $t_o = 4600$ ns was deduced; for a sample with $z_1 = 30$ nm, t_o was found to be 260 ns, and for $z_1 = 40$ nm, t_o appeared to be 1200 ns (measured under similar conditions) [29]. The inset in Figure 4 shows that the decrease of the luminescence intensity with time is accompanied by a spectral shift of the line. This shift is caused by the change in band bending due to the charging of the acceptors during the recombination process. The amplitude of the spectral shift can be used in the simplest approximation as a rough measure of the concentration of acceptors $\Delta E \propto e n_a z_1$ [29]. The measurements presented in the following are taken always at a fixed delay time, i.e., the shifts in line position due to change in band bending are fixed and shifts in the spectral position of the luminescence line reflect changes in the electronic properties of the 2DEG.

Whereas for a homogeneous distribution of electrons in the plane, only a single exponent is expected in the time evolution, a spatial structure of the electronic system will influence the time evolution and lead to a non-exponential decay, since the overlap between electrons and holes not only in growth-direction (z-direction), but also in the plane are then of importance. At very low densities or very high magnetic fields, interacting electrons can form a lattice as their collective ground state, the so-called Wigner lattice. This local structure of the electronic system can be probed by analyzing the time-dependence of the luminescence signal [10, 30]. Whereas for an ideal 2DEG and for very small filling factors $\nu < 0.2$, the collective ground state should be a lattice and, for larger filling factors, the incompressible liquid of the FQHE should be the ground state, for a higher degree of disorder localization should dominate. The competition between these different effects states is also observable in such time-resolved experiments [4, 27].

In zinc-blende-type semiconductors such as GaAs, the top of the valence band is fourfold degenerate, i.e., holes have the spin quantum number 3/2. In a strong magnetic field and at low temperatures, the holes bound to the acceptors occupy the lowest spin-polarized acceptor level. Due to the angular momentum selection rules, the most effective electron-dipole radiative transition includes only electrons occupying the lowest spin level. Therefore, recombination tests also the spin configuration of the electrons. In the following, the main emphasis is on results for filling factors smaller than $\nu = 1$, where the electrons occupy mainly the lowest spin level.

Luminescence in the Regime of the FQHE

Energetical structures are observed in luminescence experiments either as shifts of the luminescence lines or splittings of the spectrum [8, 13, 36]. Whereas Landau-level splittings are nicely observable in our measurements (as seen in Fig. 3) for the recombination with acceptor bound holes, splittings in the regime of the FQHE

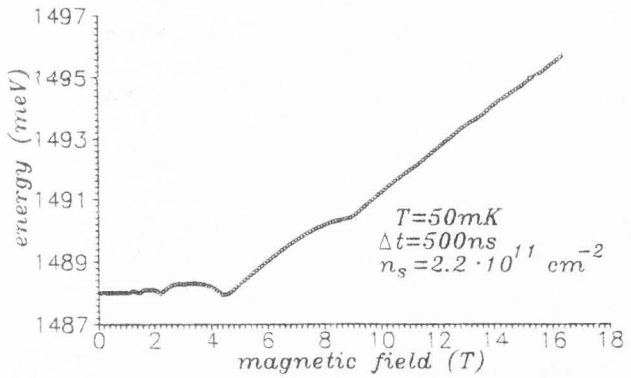

Figure 5. Magnetic field dependence of the center-of-mass (spectral position of the first moment, E_1) of the luminescence line measured at low temperature (T = 50 mK) for a time delay of Δt = 500 ns after the excitation pulse with a peak power of $W = 10^{-4}$ W/cm^2 for a carrier concentration of $n_s = 2.2 \times 10^{11}$ cm^{-2}.

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are not seen in such measurements. In order to check whether any shifts are observable in this regime, one has to define the spectral position of the luminescence line as accurate as possible. Therefore, the center-of-mass E_1 of the luminescence line (normalized first moment) has to be obtained:

$$E_1 = \{(\int I(E)EdE) / (\int I(E)dE)\} \qquad (5)$$

with I(E) the intensity of the luminescence light. One could also calculate higher moments to obtain the width (E_2) and the coefficient of asymmetry (E_3), but the degree of accuracy seem to decrease for the higher moments if the integrations are done numerically. Figure 5 shows the center-of-mass E_1 deduced from time-resolved luminescence measurements of a sample with a carrier concentration of $n_s = 2.2 \times 10^{11}$ cm^{-2} and an acceptor layer of $n_a = 5 \times 10^9$ cm^{-2} at a distance of 40 nm to the interface. The results are presented as a function of magnetic field. Clearly, some dips around integer filling factors are observed on a more or less flat background up to magnetic fields of B = 4.5 T corresponding to a filling factor of $\nu = 2$. For smaller filling factors and larger magnetic fields, the energy increases, since only the lowest Landau level is occupied and this level increases in energy with magnetic field by half of the cyclotron energy $\hbar\omega_c$, leading to an effective increase in band gap. The increase of energy is more or less linear in the magnetic field with few deviations {at filling factor of $\nu = 1$ (spin splitting) and at some fractional filling factors, e.g., $\nu = 2/3$ at B = 13.6 T}. To see the small shifts of the line at high magnetic fields more clearly, Figure 6 shows the time-resolved spectra measured for a sequence of magnetic fields around a filling factor of $\nu = 2/3$, where the linear change of the

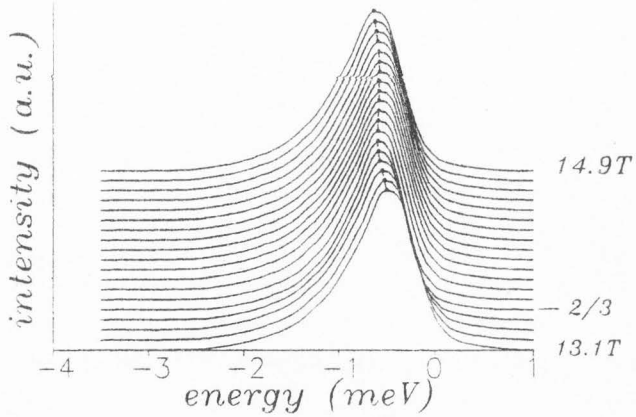

Figure 6. Sequence of spectra taken for magnetic fields varying from 13.1 T up to 14.9 T in steps of 0.1 T. The filling factor $\nu = 2/3$ is marked. In this presentation, the gap energy and the shift of the lowest Landau-level are subtracted from the photon energy ($E - E_{gap} - \hbar w_c/2$ with $m^* = 0.067\ m_o$).

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position of the line with magnetic field has already been accounted for. So, in these experiments, effects from the FQHE originating from the electron-electron interaction are observed. Also, the linewidth observed in Figure 6 is close to the broadening expected from electron-electron interaction [28], i.e., it seems to be not significantly affected by disorder.

Whereas in the first experiments of this kind [5] the shifts in the peak position have been taken as direct measurements of the gaps in the excitation spectrum of the fractional states, it turned out that one has to apply a different procedure. Apal'cov and Rashba [1, 2] showed that one has to take instead of the shift in peak position (or mean energy) the jump in the derivative of the center-of-mass of the recombination line versus filling factor or magnetic field to obtain the jump in the chemical potential. The jump in the chemical potential $\delta\mu$ in crossing a fractional filling factor $\nu = p/q$ is related to the gap in the excitation spectrum Δ by the charge $\pm\ qe$ of the elementary excitations (quasi-particles) [15, 31]:

$$\delta\mu = q\Delta \qquad (6)$$

Following this equation, the charge of the elementary excitations could be determined by measuring $\delta\mu$ and Δ in separate experiments. But, here, one is interested in the determination of the gap Δ in optical measurements. From eq. (6), Apal'cov and Rashba obtained for the discontinuity in the derivative of the center-of-mass of the recombination line the relation:

$$\{\delta(\partial E_1/\partial\nu)\} = \{(2q/\nu)\Delta - C\} \qquad (7)$$

where the second term C on the right side is a correction

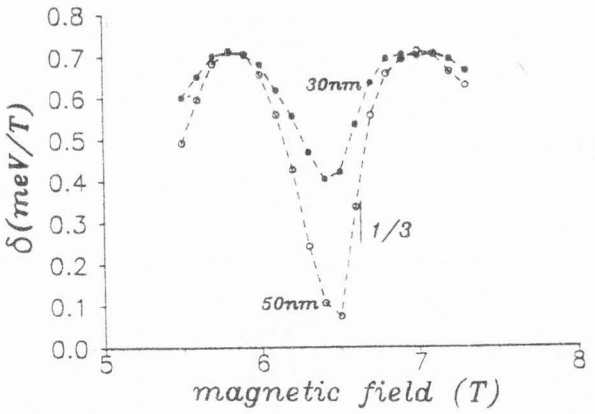

Figure 7. The dependence of the derivative of the center-of-mass δ of the luminescence line measured in the vicinity of $\nu = 1/3$ for different electron-hole spatial separations (30 nm and 50 nm) at a carrier concentration of 5.4 x 10^{10} cm^{-2}.

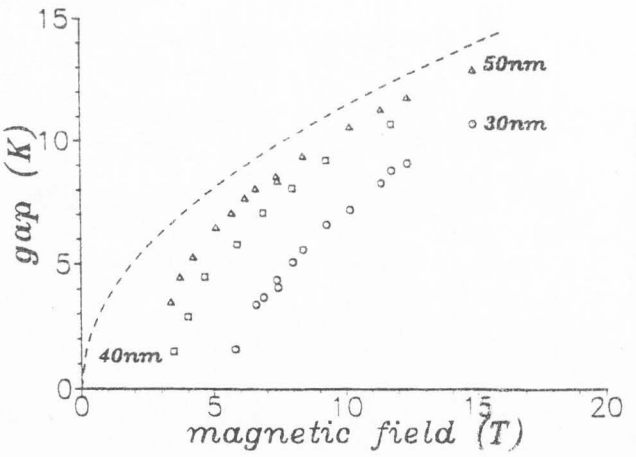

Figure 8. The magnetic field dependencies of the FQHE energy gap Δ_d measured at T = 50 mK (ν = 1/3 and 2/3) for different electron-hole spatial separations: 30 nm (circles), 40 nm (squares), and 50 nm (triangles).

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term due to the final state interaction, i.e., the interaction of the charged acceptor (after recombination) with the interacting electron system. This second term cancels and dominates the first term with the fractional gap for electrons and acceptors in the same plane. For such a coplanar arrangement, the interaction between the electrons is much weaker than the interaction between the electron and the hole (exciton). In contrast, holes placed at infinity will not disturb the final state of the electrons and the measured discontinuity will correspond directly to the jump in the chemical potential of the undisturbed interacting electron system. For a finite distance d between the electrons and holes bound to acceptors, the correction term should vanish with the third power of l_c/d for $d \gg l_c$ where l_c is the magnetic length given by $l_c = \sqrt{(\hbar/eB)}$:

$$C = c(l_c/d)^3 \qquad (8)$$

To study this correction term and to obtain the excitation gap Δ, samples with different distances z_1 between the interface and the acceptor layer were used [29]. In the experiments, the center of mass E_1 of the recombination line is obtained from numerical integrations of the spectra. In the second numerical treatment, the derivative δ of the center of mass E_1 with respect to the magnetic field B is calculated [25]. Since the derivative versus magnetic field is calculated, eq. (7) has to be slightly changed:

$$\delta(E_1/\partial B) = -(2q/B)\Delta + (\nu/B)C \qquad (9)$$

Figure 7 shows the derivative δ of E_1 versus magnetic field for two samples with different distances z_1 (30 nm and 50 nm) measured under similar conditions.

The magnetic field corresponding to a filling factor of ν = 1/3 is marked in the figure. Clearly, the discontinuity is stronger for the larger distance z_1, i.e., the correction due to the finite distance is smaller. In Figure 8, the apparent gap value deduced from such measurements for filling factors ν = 1/3 and 2/3 without any correction is plotted versus magnetic field for three different structures. One can see from this figure that the value of the measured effect strongly depends on the electron-hole spatial separation and it vanishes at different magnetic fields for the different separations. This disappearance of the optical fingerprint of the FQHE at small magnetic fields is due to the compensation of the electron-electron interaction by the electron-hole interaction, as mentioned above. For large magnetic fields, the determined values approach the value expected from theory for the gap originating from electron-electron interaction under inclusion of a finite channel width (dashed line in Fig. 8 given by $\Delta = 0.07e^2/l_c$). It was found that the coefficient c of the correction term seems to be a factor of 2 smaller [29] than determined in the extrapolation procedure by Apal'cov and Rashba [1, 2] for a filling factor of ν = 1/3. For electron-hole separations of 40 nm and 50 nm, the corrections are smaller than 10% for magnetic fields larger than 10 T. So, in these samples, reasonable values for the excitation gaps of the FQHE can be determined in the optical experiments at high magnetic fields. We also performed activated transport measurements for our samples (40 nm) to compare the gap energies deduced from the luminescence data with transport data and found in general consistent results (with larger deviations at smaller magnetic fields): e.g., ν = 1/3, B = 8.4 T: activation Δ_{act} = 4.1 K, optical

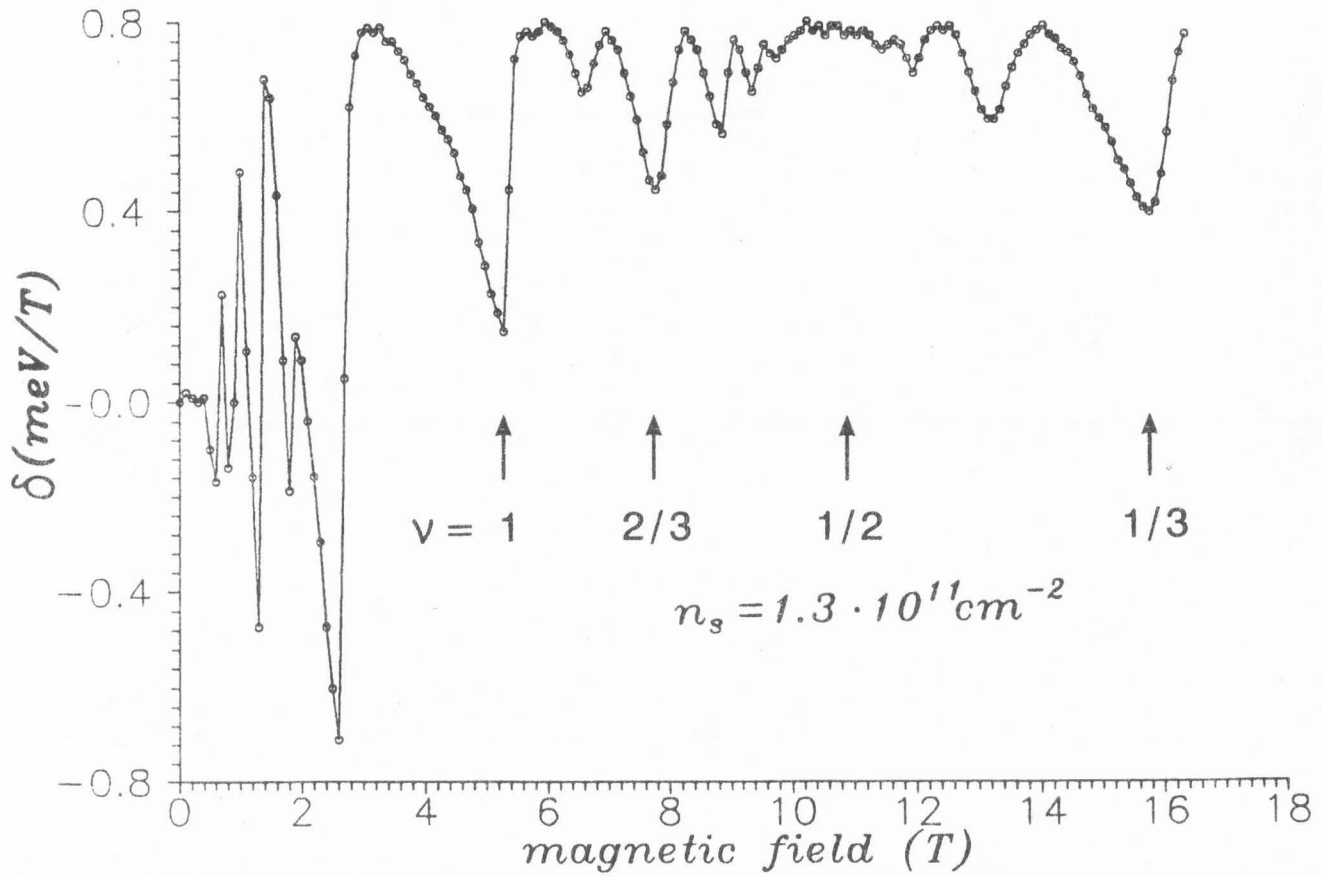

Figure 9. Derivative δ of the center of gravity with respect to the magnetic field plotted versus the magnetic field for a carrier concentration of n_s = 1.3 x 10^{11} cm^{-2}.

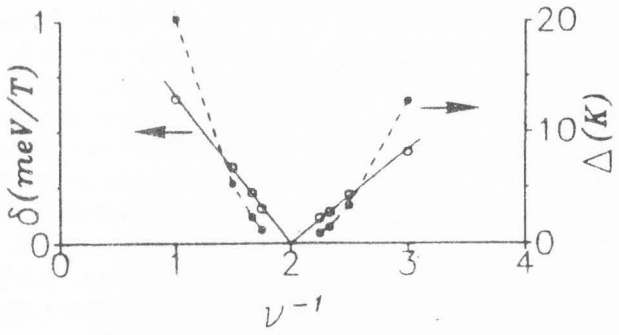

Figure 10. The discontinuities deduced from the measurements shown in Figure 9 and the derived energy gaps versus the inverse of the filling factor.

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gap from dE_1/dB Δ_{opt} = 6.2 K (without corrections); ν = 2/3, B = 13.5 T: Δ_{act} = 6.7 K, Δ_{opt} = 9.0 K. Interestingly, the optical values are always larger, which could originate from the different averaging process (over disorder, etc.) in the different methods.

In the optical experiments, upon increasing temperature, an abrupt thermal collapse is observed for those energy gaps which are associated with electron-electron interactions (enhanced spin gap and FQHE) [26]. A decrease in the gap value with temperature seems to result in an abrupt disappearance caused by the many-particle nature of the gap. The critical temperature for disappearance of the gap is also found to be much lower than the corresponding gap value [26], but below this critical temperature, no significant temperature dependence is observed, i.e., optical experiments at a fixed, reasonable low temperature are able to deliver gap values for a variety of fractional states (in contrast to transport measurements).

The Study of the Hierarchy of the FQHE

Having established the technique for analyzing the optical fingerprints of the FQHE, it is possible to study not only the strongest states like 1/3 or 2/3 but the whole hierarchy of the fractional states.

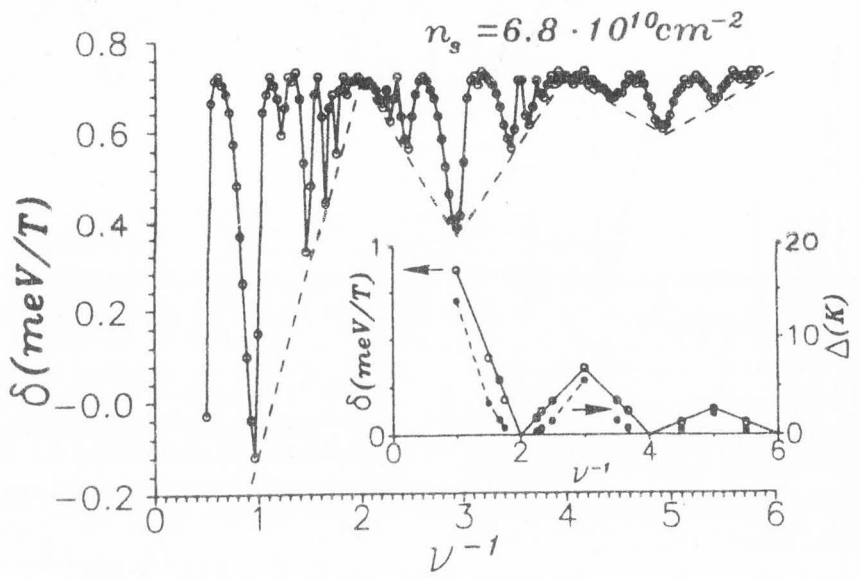

Figure 11. Derivative δ of the center-of-mass with respect to the magnetic field plotted versus the inverse of the filling factor for a carrier concentration of $n_s = 6.8 \times 10^{10}$ cm^{-2}. The inset shows the deduced discontinuities and the derived energy gaps versus the inverse of the filling factor.

After the explanation of the FQHE at filling factor $\nu = 1/3$ by Laughlin [31], Haldane [14] and Halperin [16] proposed a hierarchical scheme obtained by the condensation of the excitations of lower lying states to explain the observation of plateaus also at other fractional filling factors. Later on, Girvin [11] explicitly showed that the model of Laughlin is not only applicable to $\nu = 1/q$, but also due to the quasi-electron-quasi-hole symmetry to $\nu = 1\text{-}1/q$. Recently, an alternative attempt to extend Laughlin's theory was proposed by Jain [21, 22]. He describes the FQHE in terms of non-interacting quasiparticles (composite fermions), where the quasiparticles consist of an electron with an even number of flux quanta attached. He constructed trial wave functions for fractions of the form

$$\nu = \{p \,/\, (2np+1)\} \qquad (10)$$

with p a positive or negative integer. The integer number n counts different families of fractional states with $2n$ flux quanta attached to the electrons. Here, the transformed fermions are in an integer quantized Hall state with p Landau levels occupied. A well-defined Fermi surface for the composite fermions seems to exist at $\nu = 1/2$ [17] and, by extrapolation, for the other families at $\nu = 1/4$, $\nu = 1/6$, etc.

With time-resolved luminescence, it should be possible to also study this scheme of the hierarchy [28]. Figure 9 shows the derivative δ of the spectral position of the luminescence line E_1 for a carrier concentration of $n_s = 1.3 \times 10^{11}$ cm^{-2}, a separation $z_1 = 40$ nm and an acceptor concentration of $n_a = 5 \times 10^9$ cm^{-2}. Some filling factors are marked in the figure. All the discontinuities observed for filling factors $\nu \leq 2/3$ belong to the main family of the hierarchical scheme of Jain [21, 22] with $n = 1$ and the differences in amplitude of the observed discontinuities is consistent with his picture of the family which also coincides with the relation between different minima in the Shubnikov-de Haas oscillations observed in transport in this filling factor range [9]. The strongest states seem to be $\nu = 2/3$ and $\nu = 1/3$ and the amplitude of the dips decreases in approaching filling factor $\nu = 1/2$. The dip observed between $\nu = 1$ and $\nu = 2/3$ belongs to a different family centered around $\nu = 3/4$. Discontinuities corresponding to jumps in the chemical potential are also observed at integer filling factors, but with the exception of $\nu = 1$ and $\nu = 2$ their degree of accuracy is rather poor due to the emphasis of the measurements on the FQHE. Nevertheless, the question remains how accurate cyclotron gaps can be determined in this optical method [26] and what parameters influence the results (disorder, etc.).

In Figure 9, the amplitude of the dips seems to

increase linearly with magnetic field in going away from filling factor $\nu = 1/2$. To see this more clearly, in Figure 10, the discontinuities appearing in Figure 9 (depths of dips) are plotted versus the inverse of the filling factor (open circles connected by full lines). An astonishing linear dependence including filling factor $\nu = 1$ is obtained. Filling factor $\nu = 1$ corresponds to the spin splitting of the lowest Landau level and is not really a fractional state, although it can be included into the family with $n = 1$ by putting $p = -1$. The observed discontinuities are connected to the jumps in the chemical potential $\delta\mu$ of the interacting electron system by

$$\delta\mu = -(B/q)\delta(\partial E_1/\partial B) + C \qquad (11)$$

with C the above-mentioned corrections due to electron-hole interaction. In neglecting the correction term, the linear dependence would be destroyed for the jump in the chemical potential, but it seems to be unlikely to obtain a jump in the chemical potential independent of the parameter p as assumed in [17]. The measured discontinuities δ of Figure 10 are also transformed into gap values Δ by $\delta = 2q\Delta/B$ in neglecting the correction term and assuming that these corrections are rather small for a distance of 40 nm between the acceptor layer and the interface and magnetic fields of more than 5 T. The excitation gap values obtained in this way are shown in Figure 10 as full symbols connected by a dashed line. In contrast to some results of transport measurements [9], a linear dependence on the difference in magnetic field from filling factor $\nu = 1/2$ is not observed, it resembles more a parabolic dependence. Although the detailed dependencies seem to be not fully clear in the moment, the symmetry seen in Figure 10 is quite astonishing.

In using a sample with lower carrier concentration, more fractional states can be studied in the same range of magnetic fields [28]. Figure 11 shows an example for a carrier concentration of $n_s = 6.8 \times 10^{10}$ cm^{-2}. Fractional states with filling factors down to $\nu > 1/6$ are observed. The dashed line marks again the astonishing linear increase in amplitude of the discontinuities in going away from filling factors $\nu = 1/2$, $\nu = 1/4$ and $\nu = 1/6$. The obtained discontinuities and the deduced gap values are plotted in the inset. To make this apparent symmetry of the FQHE clearer, it is also possible to combine all values of the observed discontinuities in one plot [28]. This symmetry in the observed hierarchy of the fractional states could be one of the key ingredients leading to a full understanding of the FQHE.

Conclusions

The integer and FQHE can be measured on a two-dimensional electron gas in high magnetic fields. Although the first experimental observations have been in transport experiments, luminescence experiments can also be used for the investigations of the electronic properties of these systems. Studies of the optical recombination between electrons and acceptor-bound holes with the acceptors located at a large distance from the electronic layer were used to determine the excitation gaps of the fractional quantum Hall states. An astonishing symmetry has been observed for the hierarchy of the fractional states.

References

[1] Apal'cov VM, Rashba EI (1991) Magnetospectroscopy of 2D electron gas: Cusps in emission spectra and Coulomb gaps. Pis'ma Zh. Eksp. Teor. Fiz. **53**, 420-425 (Russian version).

[2] Apal'cov VM, Rashba EI (1991) Magnetospectroscopy of 2D electron gas: Cusps in emission spectra and Coulomb gaps. JETP Lett. **53**, 442-448 (Englisch translation of Ref. [1]).

[3] Boebinger GS, Störmer HL, Tsui DC, Chang AM, Hwang JCM, Cho AY, Tu CW, Weimann G (1987) Activation energies and localization in the fractional quantum Hall effect. Phys. Rev. B **36**, 7919-7929.

[4] Brown SA, Davies AG, Lindsay AC, Dunford RB, Clark RG, Simmonds PE, Voss HH, Harris JJ, Foxon CT (1994) Time-resolved magneto-photoluminescence measurements of the integer QHE, fractional QHE and extreme quantum limit. Surf. Sci. **305**, 42-49.

[5] Buhmann H, Joss W, von Klitzing K, Kukushkin IV, Martinez G, Plaut AS, Ploog K, Timofeev VB (1990) Magneto-optical evidence for fractional quantum Hall states down to filling factor 1/9. Phys. Rev. Lett. **65**, 1056-1059.

[6] Chakraborty T, Pietiläinen P (1988) The Fractional Quantum Hall Effect. Springer-Verlag, Berlin, Germany. pp. 11-89.

[7] Clark RG, Mallett JR, Haynes SR, Harris JJ, Foxon CT (1988) Experimental determination of fractional charge e/q for quasiparticle excitations in the fractional quantum Hall effect. Phys. Rev. Lett. **60**, 1747-1750.

[8] Dahl M, Heiman D, Pinczuk A, Goldberg B, Pfeiffer L, West K (1992) Suppression of ground state optical recombination in the quantum Hall regime. Phys. Rev. B **45**, 6957-6960.

[9] Du RR, Störmer HL, Tsui DC, Pfeiffer LN, West KW (1993) Experimental evidence for new particles in the fractional quantum Hall effect. Phys. Rev. Lett. **70**, 2944-2947.

[10] Fal'ko VI (1994) Recombination kinetics for

acceptor-bound holes in heterostructures: A probe of the local configuration of magnetically frozen electron insulators. Phys. Rev. B **49**, 2242-2245.

[11] Girvin SM (1984) Particle-hole symmetry in the anomalous quantum Hall effect. Phys. Rev. B **29**, 6012-6014.

[12] Goldberg BB, Heiman D, Pinczuk A, Pfeiffer L, West K (1990) Optical investigations of the integer and fractional quantum Hall effects: Energy plateaus, intensity minima and line splitting in band-gap emission. Phys. Rev. Lett. **65**, 641-644.

[13] Goldys M, Brown SA, Dunford R, Davies AG, Newbury R, Clark RG, Simmonds PE, Harris JJ, Foxon CT (1992) Magneto-optical probe of two-dimensional electron liquid and solid phases. Phys. Rev. B **46**, 7957-7960.

[14] Haldane FDM (1983) Fractional quantization of the Hall effect: A hierarchy of incompressible quantum fluid states. Phys. Rev. Lett. **51**, 605-608.

[15] Halperin BI (1983) Theory of the quantized Hall conductance. Helv. Phys. Act. **56**, 75-102.

[16] Halperin BI (1984) Statistics of quasiparticles and the hierarchy of fractional quantized states. Phys. Rev. Lett. **52**, 1583-1586.

[17] Halperin BI, Lee PA, Read N (1993) Theory of the half-filled Landau level. Phys. Rev. B **47**, 7312-7343.

[18] Haug RJ, von Klitzing K, Nicholas RJ, Maan JC, Weimann G (1987) Fractional quantum Hall effect in tilted fields. Phys. Rev. B **36**, 4528-4530.

[19] Haug RJ, Gerhardts RR, von Klitzing K, Ploog K (1987) Effect of repulsive and attractive scattering centers on the magnetotransport properties of a two-dimensional electron gas. Phys. Rev. Lett. **59**, 1349-1352.

[20] Haug RJ, von Klitzing K, Ploog K (1988) Transport in AlGaAs-GaAs Heterostructures with additional impurities. In: Proc. 19th Int. Conf. Physics of Semiconductors. Zawadzki W (ed.). Inst. Physics, Polish Academy of Science, Warsaw. pp. 307-310.

[21] Jain JK (1989) Composite-fermion approach for the fractional quantum Hall effect. Phys. Rev. Lett. **63**, 199-202.

[22] Jain JK, Kivelson SA, Trivedi N (1990) Scaling theory of the fractional quantum Hall effect. Phys. Rev. Lett. **64**, 1297-1300.

[23] Koch S, Haug RJ, von Klitzing K, Ploog K (1991) Experiments on scaling in AlGaAs/GaAs heterostructures under quantum Hall conditions. Phys. Rev. B **43**, 6828-6831.

[24] Koch S, Haug RJ, von Klitzing K, Ploog K (1991) Size-dependent analysis of the metal-insulator transition in the integral quantum Hall effect. Phys. Rev. Lett. **67**, 883-886.

[25] Kukushkin IV, Pulsford NJ, von Klitzing K, Ploog K, Haug RJ, Koch S, Timofeev VB (1992) Energy gaps of the fractional quantum Hall effect measured by magneto-optics. Europhys. Lett. **18**, 63-68.

[26] Kukushkin IV, Pulsford NJ, von Klitzing K, Haug RJ, Ploog K, Buhmann H, Potemski M, Martinez G, Timofeev VB (1993) Thermal collaps of the fractional-quantum-Hall-effect energy gaps. Europhys. Lett. **18**, 287-292.

[27] Kukushkin IV, Pulsford NJ, von Klitzing K, Haug RJ, Ploog K, Timofeev VB (1993) Wigner solid vs. incompressible Laughlin liquid: Phase diagram derived from time-resolved photoluminescence. Europhys. Lett. **23**, 211-216.

[28] Kukushkin IV, Haug RJ, von Klitzing K, Ploog K (1994) Hierarchy of the fractional quantum Hall effect states studied by time-resolved luminescence. Phys. Rev. Lett. **72**, 736-739.

[29] Kukushkin IV, Haug RJ, von Klitzing K, Eberl K, Tötemeyer K (1994) The influence of the electron-hole spatial separation on the magnetoluminescence measurements of the fractional-quantum-Hall-effect energy gaps. Phys. Rev. B **50**, 11259-11262.

[30] Kukushkin IV, Fal'ko VI, Haug RJ, von Klitzing K, Eberl K, Tötemeyer K (1994) Evidence of the triangular lattice of crystallized electrons from time resolved luminescence. Phys. Rev. Lett. **72**, 3594-3597.

[31] Laughlin RB (1983) Anomalous quantum Hall effect: An incompressible quantum fluid with fractionally charged excitations. Phys. Rev. Lett. **50**, 1395-1398.

[32] MacDonald AH, Rezai EH, Keller D (1992) Photoluminescence in the fractional quantum Hall regime. Phys. Rev. Lett. **68**, 1939-1942.

[33] Ploog K (1987) Delta-doping of GaAs with MBE. J. Cryst. Growth **81**, 304-315.

[34] Tsui DC, Störmer HL, Gossard AC (1982) Two-dimensional magnetotransport in the extreme quantum limit. Phys. Rev. Lett. **48**, 1559-1562.

[35] Turberfield AJ, Haynes SR, Wright PA, Ford RA, Clark RG, Ryan JF, Harris JJ, Foxon CT (1990) Optical detection of the integer and fractional quantum Hall effects in GaAs. Phys. Rev. Lett. **65**, 637-640.

[36] Turberfield AJ, Ford RA, Harris IN, Ryan JF, Foxon CT, Harris JJ (1993) Incompressible electron liquid states studied by optical spectroscopy. Phys. Rev. B **47**, 4794-4797.

[37] von Klitzing K, Dorda G, Pepper M (1980) New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance. Phys. Rev. Lett. **45**, 494-497.

[38] von Klitzing K, Dorda G, Pepper M (1990) In: The Quantum Hall Effect. Springer-Verlag, Berlin, Germany. pp. v-vi, 37-171.