Scanning Microscopy

Volume 1992
Number 6 *Signal and Image Processing in Microscopy and Microanalysis*

Article 11

1992

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Scanning Microscopy Supplement 6, 1992 (pages 121-128) 0892-953X/92$5.00+.00
Scanning Microscopy International, Chicago (AMF O'Hare), IL 60666 USA

RANDOM IMAGE MODELS FOR MICROSTRUCTURE ANALYSIS AND SIMULATION

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Abstract

A wide spectrum of Random Function models, based on the theory of Random sets, are introduced to simulate single or multivariate signals as encountered in the common practice of electron probe microscopy.

These models are built in three steps, combining the choice of a family of primary random functions and of Poisson varieties in the n−dimensional space for their implantation. For electron microscopy images, they can describe the following situations:

− topography (as obtained from stereo pair images in fractography) simulated by Boolean and by alternate sequential random functions;

− thick slices (as in the case of Transmission Electron Microscope (TEM) and Scanning Transmission Electron Microscope (STEM) specimens) for the dilution random functions;

− perspective views (e.g. secondary electron images in the SEM from non planar samples, such as powder samples) for the Dead Leaves model;

− multispectral mappings on polished sections. Their main strength is to enable the estimation of parameters (namely the statistical properties of the structural unit made of primary random functions, and the density of its implantation in space) from simple operations and measurements on grey level images based on Mathematical Morphology, without any segmentation of images. This purpose is illustrated by the main properties of the models with reference to electron microscopy and microprobe situations, and by simulations.

Key Words : Random Image Modeling, Image simulation, Poisson process, Random Function models.

Introduction

When working on electron probe imaging, various scales of the structure of specimens can be examined: high resolution images concern the atomic or the molecular scale, with the purpose to identify a lattice of atoms, an interface between two materials, or biological molecules; on a larger scale (or so called mesoscale), the arrangement of components of a structure is no more regular, and requires a different description. In this paper, we are concerned by modeling structures from this mesoscale up to a macroscopic scale. In fact the range of potential applications of this kind of approach is rather wide, since it can be used as well for chemical mappings obtained in an alloy from a microprobe, secondary electron fractographic images, porous media or natural textures simulation, remote sensing from satellite images, etc.

The main purposes of random structure models are to sum up the microstructure in some parameters, to predict morphological properties that are not directly accessible, and to provide means of simulations. We developed a wide spectrum of Random Function (R.F.) models based on the Random Sets theory [13, 15], that are of interest for microstructure modeling.

They can be either scalar models (to simulate a single signal), or multivariate models (to simulate multispectral images such as color images, or encountered from Energy Dispersive Spectrometry (EDS), Wavelength Dispersive Spectrometry (WDS), or Electron Energy−Loss Spectrometry (EELS) spectra), where the correlations between components may be changed from a statistical independence to a functional dependence [9, 10, 11].

In this paper, we briefly review and illustrate the construction and some properties of the following models, inviting the interested reader to study the given references for more details and for the mathematical aspects: the Boolean random set model, the Boolean R.F., the Dead Leaves R.F., the Alternate Sequential R.F., some Diffusion−Reaction R.F., and the Dilution R.F..

Probabilistic Properties of Random Structure Models

We considered the following random structures: random closed sets (single or multi−components) and semi−continuous random functions (scalar or multivariate), with limitation to the stationary case. In the frame of the theory of random sets [13,15], these structures and their models are fully characterized by a functional called their capacity of CHOQUET: − A closed random set

A (e.g. a two−phase medium like a porous medium) is known from the functional T(K) on the compact sets K:

$$T(K) = P\{K \bigcap A \neq \varnothing\} \quad (1)$$

In equation (1), K may be chosen as a single point {x}, a pair of points $\{x_1, x_2\}$, or even a non countable set of points, like a ball of radius r, B(r). With these choices, equation (1) represents for a stationary model the volume fraction, and the covariance of A; using B(r) enables to know the distribution of distances to A of a point in the complementary set of A, A^c. This distribution separates two sets A and A' differing from a random point process, while the criteria involving a countable set of points are blind to this difference.

If we consider now a family of components A_i (i = 1, 2,...,m) and of compact sets K_i (i = 1, 2,...,m), the A_i are characterized by the multivariate CHOQUET capacity T(K), defined as:

$$1 - T(K) = Q(K) =$$

$$P\left\{x \in \left((A_1 \oplus \overset{\vee}{K}_1) \bigcup (A_2 \oplus \overset{\vee}{K}_2) \bigcup \ldots (A_m \oplus \overset{\vee}{K}_m)\right)^C\right\} \quad (2)$$

In equation (2), K is a short notation for $(K_1, K_2,...,K_m)$, and the symbol $\oplus$ stands for the Minkowski addition: $A \oplus K = \bigcap \{a + k; a \in A, k \in K\}$. From this addition is derived the operation of dilation defined in Mathematical Morphology [6, 17] by:

$$A \oplus \overset{\vee}{K} = \{x; K_x \bigcap A \neq \varnothing\}$$

with $\overset{\vee}{K} = \{-x; x \in X\}$ and K_x is obtained by translating K to point x.

We consider now a random function Z (to describe a X−ray map or an electron image), or more generally a multivariate random function Z, with components Z_1, Z_2,...,Z_m. When each component is an upper semi continuous function (USC) with support in $\mathbb{R}^n$, its subgraph Γ^{Z_i} ($\Gamma^{Z_i} = \{(x, z), x \in \bar{\mathbb{R}}$, with $z \leq Z_i(x)\}$), is a closed set in $\mathbb{R}^n \times \mathbb{R}$. Z is characterized by means of lower semi continuous (LSC) test functions $g_1, g_2,..., g_m$ with compact supports $K_1, K_2,..., K_m$, defining the (multivariate) CHOQUET Capacity T(g):

$$1 - T(g) = 1 - T(g_1, g_2, \ldots, g_m)$$
$$= Q(g_1, g_2, \ldots, g_m) = P(A_1, A_2, \ldots, A_m) \quad (3)$$

where

$$A_i = \{Z_i; Z_i(x) < g_i(x), \forall x \in K_i\} \quad (4)$$

T(g) is equivalent to the CHOQUET Capacity defined in the frame of the random sets theory by G. Matheron [15]. If the functions g_i with compact supports are translated to $x \in \mathbb{R}^n$, we call $D_Z^c(g)$ the set of points x for which (4) is satisfied. We have

$$D_Z(g) = \bigcup_i D_{Z_i}(g_i),$$

where $D_{Z_i}(g_i) = \cup \left\{A_{Z_i}(z) \oplus \overset{\vee}{K}_{g_i}(z); z \in \bar{\mathbb{R}}\right\}$,
$A_{Z_i}(z) = \{x; Z_i(x) \geq z\}$ and $K_{g_i}(z) = \{x; g_i(x) \leq z\}$.

With these notations, $T(g) = P\{x \in D_Z(g)\}$. As a particular case, consider for the scalar R.F. Z the following test functions g :

* $g(x_i) = z_i$ (i=1,2,...,n) ;

we have $T(g) = 1 - P\{Z(x_1) < z_1,...,Z(x_n) < z_n\}$ which is the multivariate distribution, namely the spatial law. When restricting it to the points x_1 and x_2, we get the bivariate distribution, from which is deduced the two−point correlation function, or covariance.

* $g(x) = z, \forall x \in K$;

we have $1 - T(g) = P\{Z_V(K) < z\}$
with $Z_V(K) = \vee \{Z(x); x \in K\}$ where $\vee$ means the supremum value. For a given model, this enables to know the probabilistic law of the change of support of the data by this operator $\vee$.

The functionals T(K) and T(g) are the connection between theory and experience: for appropriate models, T is calculated as a function of the parameters and of the construction of the model. From real data or from simulations, T is estimated, and it is therefore possible to test the validity of the model and to estimate its parameters.

Most models introduced below are built in three steps:
− we start from a family of primary random sets or random functions;
− we use Poisson varieties in the n−dimensional space for their implantation.
− a local rule of combination of the implanted random sets or random functions is applied in each point of the space.

The Boolean Random Models

The Boolean random closed set [13, 15] was initially proposed to simulate complex porous media. In fact this is a prototype of binary model, from which others can be derived, as illustrated in the further sections. Many examples of applications are given in [3, 10, 17]. We recall here its construction and its main properties in the multicomponent case (imagine a set of several binary images for each examined field), as introduced in [9−11]. Its construction is as follows: we consider a Poisson point process in $\mathbb{R}^n$ with intensity (average number of points per unit of volume) θ. In each point of the process, an independent realization of a multicomponent random compact set $\{A'_1, A'_2, \ldots, A'_m\}$ is implanted, each component of the multicomponent random set $A = \{A_1, A_2, \ldots, A_m\}$ being a Boolean random closed set. The CHOQUET capacity of this model is (μ being the Lebesgue measure in $\mathbb{R}^n$, i. e. the length for n=1, the area for n=2 and the volume for n=3):

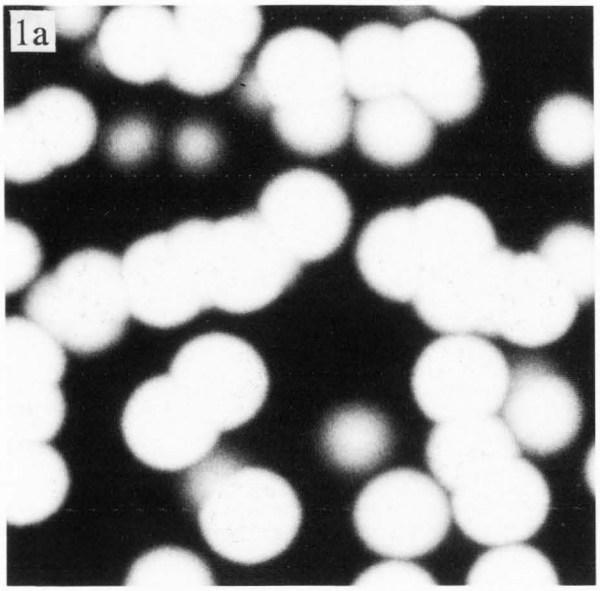
$$1 - T(K) = Q(K) = \exp\{-\theta\bar{\mu}(A' \oplus K)\} \quad (5)$$

with $A' \oplus K =$

$$= (A'_1 \oplus K_1) \bigcup (A'_2 \oplus K_2) \bigcup \ldots \bigcup (A'_m \oplus K_m)$$

Replacing the constant θ by a space dependant intensity $\theta(x)$ allows us to build non stationary Boolean random set. It is one of the numerous examples of random aggregates models developed in [10].

The Boolean scalar R.F. model was introduced to simulate rough surfaces [5, 17, 18]. Generalizations were proposed [9, 10]: sequential construction, use of Poisson varieties (see for instance Poisson lines in the plane shown



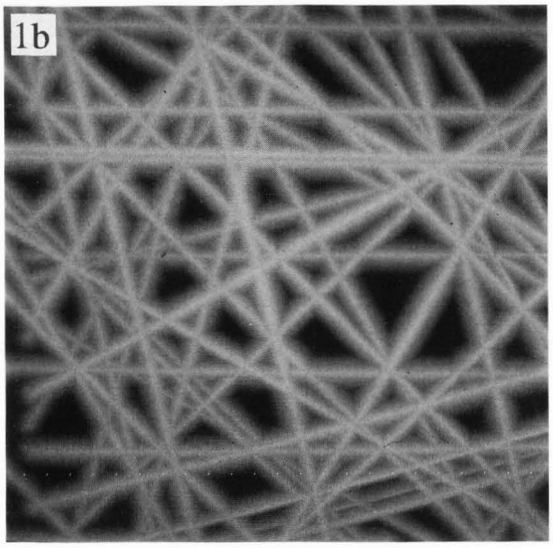


Figure 1. Simulation of Boolean R. F. in $\mathbb{R}^2$ (512x512 pixels); (a) primary function: distance function on disks with radius 45 pixels. (b) Boolean variety on Poisson lines in the plane; same primary function as in (a) with uniform radii between 15 and 25 pixels.

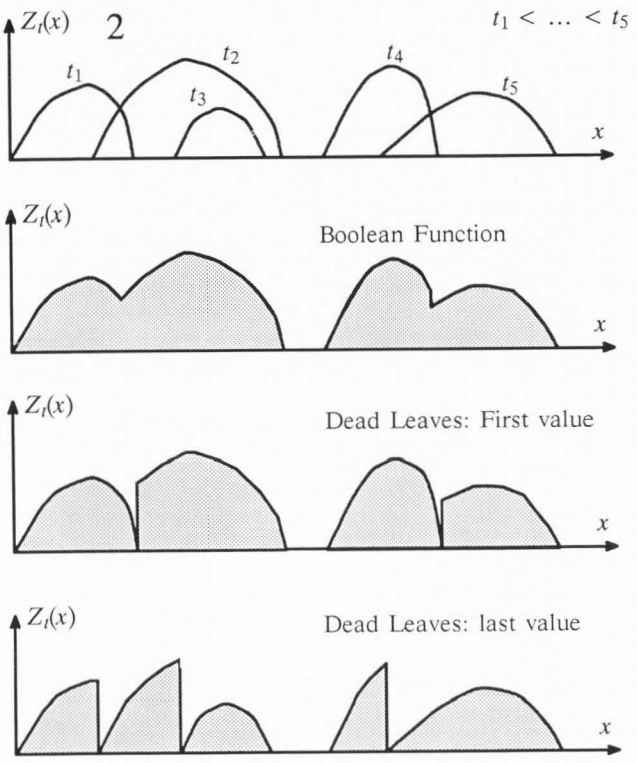


Figure 2. Construction of a Boolean R.F. and a Dead Leaves R.F. from the same sequence of primary functions.

in Figure 1 b) instead of Poisson points, and multivariate version, as presented now.

We consider on a time interval [0,t]: i) a Poisson point process $\mathcal{P}$ in $\mathbb{R}^n \times \mathbb{R}$ with density measure $\mu(dx) \otimes \theta(dt)$, θ being a σ finite measure in $\mathbb{R}^n$; here a stationary process, resulting in a stationary R.F. is considered; ii) independent realizations of a family of Upper Semi–Continuous multivariate primary random functions with m components $Z'_{it}(x)$, and with closed subgraphs $\Gamma^{Z'_i} = A'_i(t)$ having almost surely compact sections.

Definition: a multivariate R.F. Z is a Boolean R.F. with primary function Z' and with intensity $\mu(dx) \otimes \theta(dt)$ when its components are scalar Boolean random functions obtained by (Figure 2):

$$Z_{it}(x) = \vee \{Z'_i t_k(x - x_k), (t_k, x_k) \in \mathcal{P}\} \qquad (6)$$

A dual version of this model is obtained from the $\wedge$ (infimum) operator instead of $\vee$ and Lower Semi–Continuous primary random functions.

Theorem: the CHOQUET Capacity of the multivariate Boolean random function Z is given by:

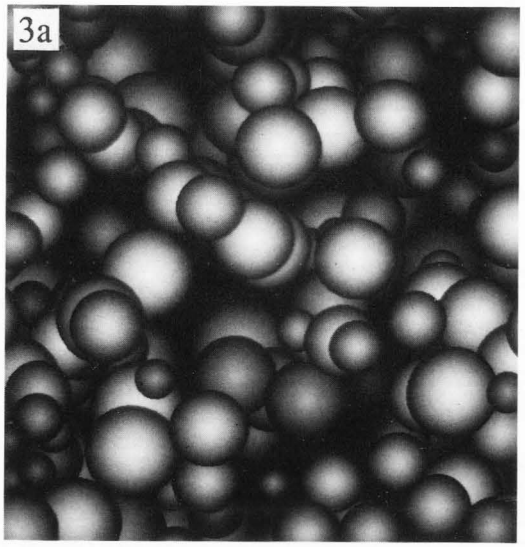
$$1 - T(g) = Q(g) = \exp\left\{ - \int_0^t \theta(du)\mu(D_{Z'_u}(g)) \right\} \qquad (7)$$

where $D_{Z't}(g) = D_{Z't_1}(g_1) \bigcup D_{Z't_2}(g_2) \bigcup \ldots D_{Z't_m}(g_m)$. As a particular case, we get the spatial law of Z for $g_i(x_i) = z_i$, else 0:

$$F(x,z) = P\{Z_1(x_1) < z_1, \ldots, Z_m(x_m) < z_m\}$$

$$= \exp\left\{ - \int_0^t \theta(du)\, \bar{\mu}(A'_1(u)_{z_1 x_1} \bigcup \ldots \bigcup A'_m(u)_{z_m x_m}) \right\} \qquad (8)$$

In equation 8 the set A'(u) is cut at the level z and translated at the point x. The main properties of the multivariate Boolean R.F. are the following: in the case of a single primary random function for each Poisson point



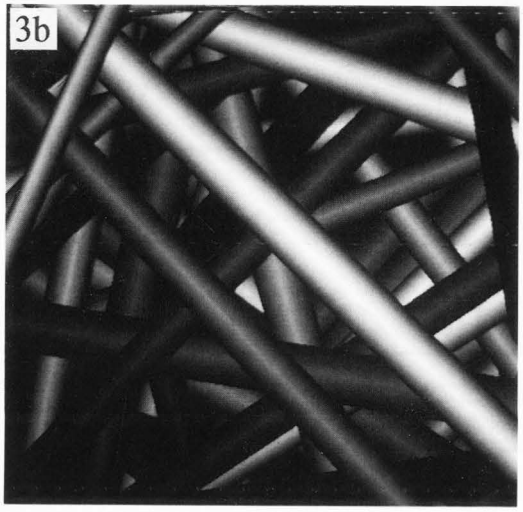


Figure 3. Simulation of Dead Leaves R.F. in $\mathbb{R}^2$ (512x512 pixels); (a) primary function: distance function on disks with a random radius. (b) Dead Leaves variety on Poisson lines in the plane; same primary function as in (a) with uniform radii between 16 and 36 pixels.

(Z'_k with probability p_k), the components are independent Boolean random functions. Otherwise, the components are correlated. As for the Boolean model in the binary case, this type of model is well suited for the simulation of interlocking media (Figure 1). In addition the two functions Z and $-Z$ are not equivalent, as for gaussian R.F. In an application to the roughness of steel plates, this was used to simulate the print of the roll mill on the steel [5].

Dead Leaves Random Functions

This type of sequential Random Functions was introduced [7,8] in order to simulate either the evolution of a microstructure (such as a sequence of crystallization of components) or perspective views (as aggregates seen on the scanning electron microscope). It generalizes earlier

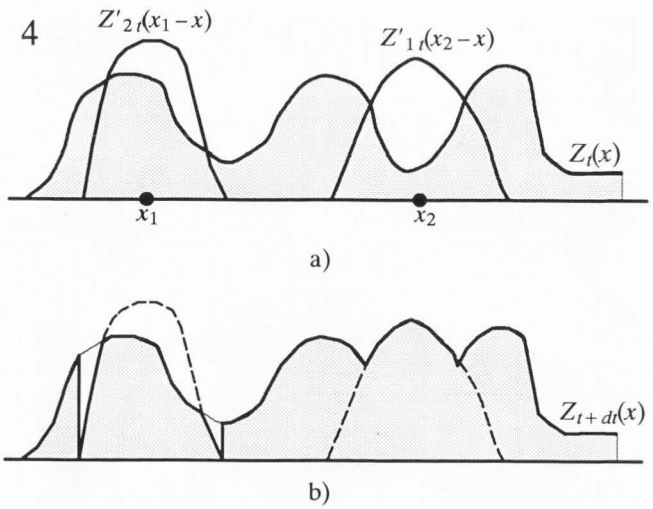


Figure 4. Construction of Alternate Sequential R. F. models.

models: the Dead Leaves tessellation [14] and the multi−components models for multiphase media introduced in [3,4].

Starting as for the multivariate Boolean random functions, we can use the following constructions, illustrated in Figure 2, where a comparison to the Boolean R. F. is made:

i) $Z_{it}(x)$ = { first observed value in x on a primary function $Z'_{it}(x)$}

ii) $Z_{it}(x)$ = { last observed value in x on a primary function $Z'_{it}(x)$}

Examples of simulations are provided in Figure 3, very similar to a spherical powder and to fiber aggregates seen in secondary electrons on the scanning electron microscope. The main known properties of these models are the following, and are given in [7−11]:

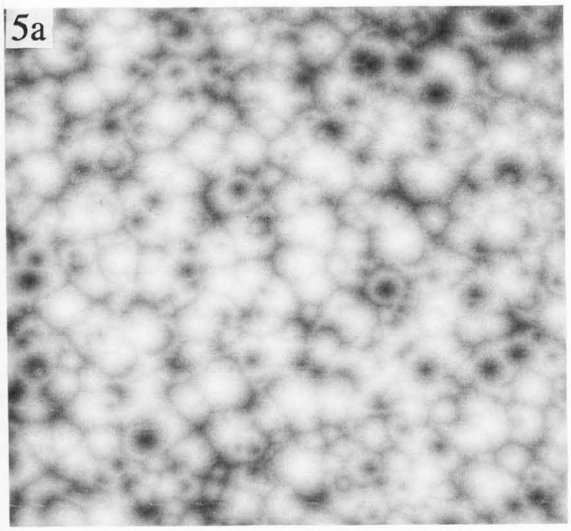
− multivariate distribution function at point x, for multiple primary functions Z'_{it} with a common support A'(t));

− the bivariate distributions in the general case.

− for a particular class of Dead Leaves models, the probabilistic properties of the R.F. after a change of support by the infimum operator; this property is used for applications to the morphological analysis of powders by image analysis [12]: estimation, without any segmentation of images, of the composition of a mixture of components with a different morphology; estimation of the unbiased size distribution of the grains of a powder.

Alternate Sequential Random Functions

These models [10, 11] are intermediary between the Boolean and the Dead Leaves R.F. models. They may be useful for the simulation of processes appearing in tribology, as well as for geology (sequence of abrasions and of depositions on a relief). In particular the symmetry between summits and valleys, which is broken for the Boolean R. F., can be recovered for special versions of the model. We give now the construction of the multivariate case: one starts with two families of primary functions



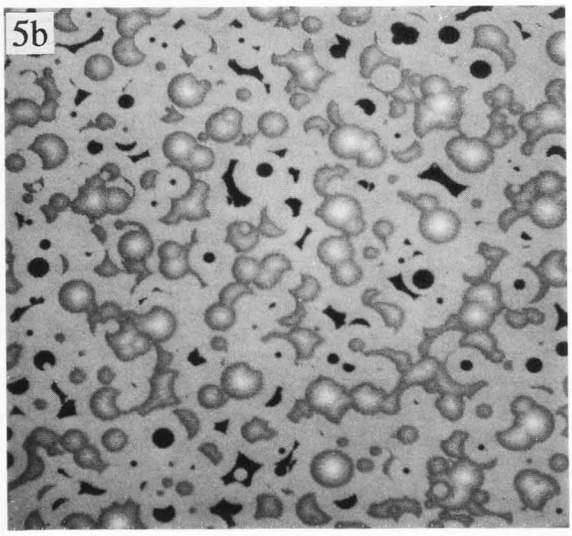
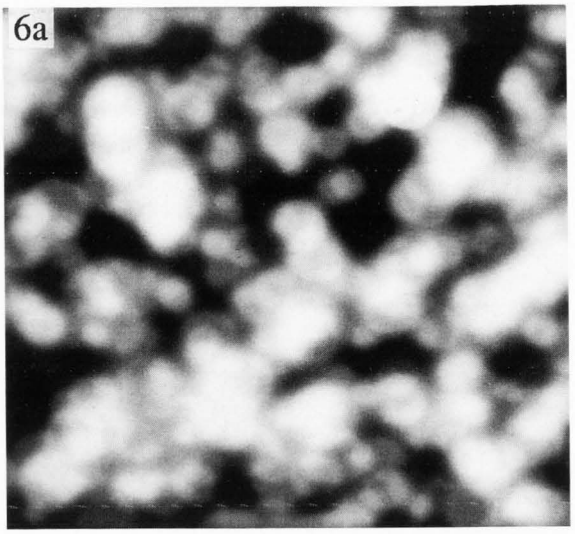


Figure 5. Simulation of an Alternate Sequential R.F. in $\mathbb{R}^2$ (512x512 pixels); (a) primary function: distance function on disks with a random radius (uniform from 15 to 30 pixels). (b) binary image obtained by thresholding (a) between the grey levels 81 and 136 resulting into binary dead leaves with circular primary grains.



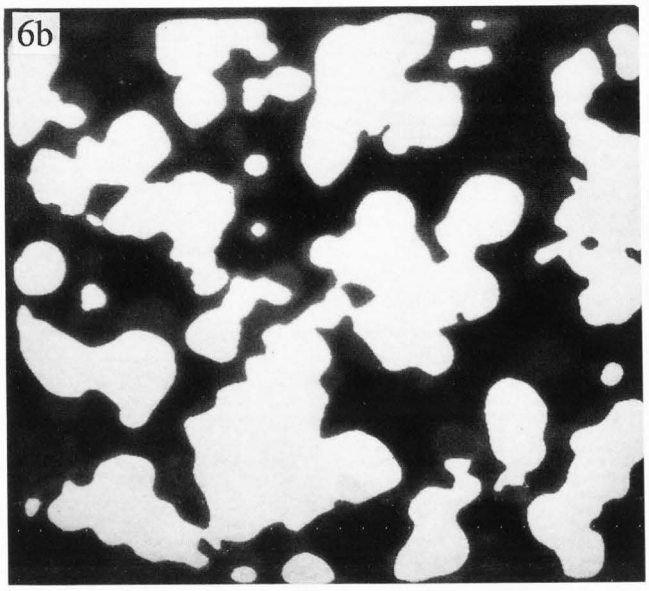


Figure 6. Simulation of a Dilution R.F. in $\mathbb{R}^2$ (512x512 pixels): linear diffusion of a sequential input of random disks with a constant grey level (30) and random radii (uniform between 10 and 30 pixels)); (a) R.F. for a coefficient of diffusion D = 6. (b) binary image obtained by thresholding (a) between the levels 109 and 255.

(Z'_{i1t} USC and Z'_{i2t} LSC ($i = 1, 2, \ldots, m$)) with subgraph $A'_{i1}(t)$ and overgraph $B'_{i2}(t) = \{(x, z); z \geq Z'_{i2}(x)\}$. Each component of the multivariate A.S.R.F. is driven by the evolution equation (9) given below. We consider two Poisson point processes $\mathcal{P}_1$ and $\mathcal{P}_2$ in $\mathbb{R}^n \times \mathbb{R}$, with intensity measures $\mu(dx) \otimes \theta_1(dt)$ and $\mu(dx) \otimes \theta_2(dt)$. From $\mathcal{P}_1$ and $\mathcal{P}_2$ are built two infinitesimal Boolean R.F. $Z_t^1(dt)$ and $Z_t^2(dt)$ according to:

$Z_t^1(x, dt) = \vee \{Z'_{1t}(x - x_{1k}), x_{1k} \in \mathcal{P}_1(dt)\}$ and
$Z_t^2(x, dt) = \wedge \{Z'_{2t}(x - x_{2k}), x_{2k} \in \mathcal{P}_2(dt)\}$, we have (Figure 4) :

$$Z_{t+dt}(x) = Z_1(x) \vee Z_t^1(x, dt) \wedge Z_t^2(x, dt) \qquad (9)$$

A simulation is given in Figure 5 a with a single primary function $Z'_{1t} = Z'_{2t}$. In addition, the function Z' does not depend on the time t (homogeneous model). From equation 9, the relationship with the Boolean R.F. is apparent, but here, there is a combination (in alternance) of the two operators $\wedge$ and $\vee$. By thresholding one component at the level z, one obtains a two component color Dead Leave model [3,4] with primary grains $A'_1(t)_z$ and $B'_2(t)_z$, and intensity $\theta_1(dt)$ and $\theta_2(dt)$ (Figure 5 b).

The main probabilistic properties of this model (univariate and bivariate distribution functions, distribution of apparent minima and maxima) are given in [10, 11].

Reaction–Diffusion Random Functions

This class of spatio–temporal models is rather wide, and still in development. We have introduced here some results given in [10]. Its main domains of potential applications are the following: to simulate microstructures obtained from chemical reactions between various species, chemical segregations occurring during a solidification process, competition between populations (such as in ecological models). Many complex microstructures studied in quantitative microscopy are relevant of such processes.

The Reaction–Diffusion R. F. models are solution of stochastic evolution equations involving interactions between variables. In the general case, these equations are non–linear parabolic partial differential equations, such as studied in [1] for the deterministic case. For the random model, we introduced a random spatio–temporal source in the evolution equations.

Linear Reaction–Diffusion R. F. models were studied in a particular case for two species [2], and generalized in [10]. They are solutions of stochastic linear parabolic partial differential equations with a random source. As a consequence, they involve a product of convolution by a gaussian kernel, resulting into very smooth functions for appropriate random sources. This is in particular, the case for the dilution random functions introduced in the next section, and illustrated in Figure 6.

Dilution Random Functions

In the construction of the multivariate Boolean R.F., we replace the operation $\vee$ by the addition +, we define Dilution R.F. [16]. This kind of process simulates the observation of thick slices on the electron microscope, with addition of the mass of each component along the thickness. In [10], such processes were studied as particular (linear) Reaction–Diffusion R.F. models. An example of simulation is given in Figure 6. Thresholding such a function gives a random set, with very smooth boundaries (Figure 6b). The probabilistic properties of this model are derived from the characteristic function $\Phi_t(ZU,X)$ of its multivariate distribution (spatial law).

Theorem: using multiple primary functions Z'_i with characteristic function $\phi_i(ZU,X))$ and noting $X = \{x_1,x_2,..., x_m\}$ and $ZU = \sum U_k Z_k (x_k)$, $X-y = \{x-y\}$, we have:

$$\Phi_t(ZU,X) = \exp\left\{\int_0^t \theta(dv) \int_{\mathbb{R}^n} (\phi_v(ZU, X-y) - 1)dy\right\} \tag{10}$$

The centered covariances are obtained by second order derivation of Log Φ at u = 0 in the case of a pair of points $X = \{ x, x+h \}$:

$$\overline{C}_{ij}(h)) = \int_0^t \theta(du) \int_{\mathbb{R}^n} E\left[Z'_{iu}(x-y)Z'_{ju}(x+h-y)\right]dy)$$

$$= \int_0^t \theta(du) g_{uij}(h) \tag{11}$$

The components Z_{it} are independent when the multivariate primary functions are single grains (Z'_k with probability p_k). In addition, if $p_i(x)$ is a family of weighting functions, and if $Z(x)$ is a dilution R.F., $Z^* \overset{\vee}{p}$ with components $Z_i * \overset{\vee}{p}_i$ (where * denotes the convolution product) is still a Dilution R.F., with primary functions $Z'_i * \overset{\vee}{p}_i$. This enables us to predict the probabilistic properties of the R.F. after a change of support by convolution, such as for unfocussed images.

General Properties of the Models

The introduced models share various interesting properties for the applications: they are defined in the Euclidean space $\mathbb{R}^n$, and not on a discrete grid of points; they can depend on a low number of parameters, which are accessible from experimental data. Some theoretical calculations or their main probabilistic properties (the bivariate distributions, and in some cases the CHOQUET capacity) are available. Their estimation on data is obtained from elementary transformations (erosions, dilations), without requiring complex segmentation procedures. In their sequential construction, they all make use of a primitive, namely a Primary function and of a local (punctual) rule of implantation and combination with already present data. They possess an interesting stereological invariance, due to the Poisson process: their lower dimension sections, or projections as obtained in transmission examination, are models of the same kind in a lower dimensional spaces, with primary functions and density induced by the model defined in $\mathbb{R}^n$. From this property, it results that a stereological reconstruction of the model is made possible in many cases (such as for instance when operating on primary functions with a random spherical support). Replacing the Poisson point process by Poisson varieties gives the facility to introduce stratified and fibrous media in $\mathbb{R}^3$.

If we consider the case of electron microscopy images, the introduced models can describe the following practical situations:

– topographic images simulations and description (as obtained from stereo pair images in fractography or on aggregates) by Boolean and by alternate sequential random functions;

– thick slices (as in the case of TEM and STEM specimens) for the dilution random functions;

– perspective views (e.g. secondary electron images in the SEM from non planar samples, such as powder samples) for the Dead Leaves model;

– multispectral mappings (like multi–element X–ray maps obtained with the electron microprobe) on polished sections.

Conclusion

A large spectrum of random function models (scalar or multivariate) was introduced. They have potential applications in many fields, even outside the field of imaging and of microscopy: color images, random vectors or tensors (mainly for physical applications). They present a high versatility for the construction of textures, and can be easily simulated. They also can achieve various degrees of the correlation of the components: from the functional dependence to the probabilistic independence.

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Discussion with Reviewers

N. Bonnet : *Your paper really convinces the reader that many experimental images in the field of electron microscopy and microanalysis can be simlulated by random models. However, your method will be even more appreciated if you are able :*

a) to check the validity of the model to really describe the experimental situation (or to choose between several variants of a model)

b) to estimate some useful parameters of the model from experimental images.

Could you give some indications concerning the parameters which can be attained and the practical procedures you can use in order to obtain these results ?

Author : The estimation of the parameters of a model, and the tests to check its validity are very important topics, and too wide to be developed in this paper (details are given for each model in the references). We can briefly illustrate these points for the Boolean models : a very practical test can be performed when the primary grains are convex sets since in that case, the expression $\mu(A' \oplus \lambda K)$ in equation (5) is a polynomial of degree n in λ, if λK (with size λ) and A' are convex sets in $\mathbb{R}^n$. If this assumption is accepted, we need to identify the model from the estimation of the intensity θ and of the statistical properties of the primary grains A'. Useful indications are often obtained from the geometrical covariogram of A' ($\overline{\mu}(A' \cap A'_h)$), which can be estimated from the covariance Q(h) of the random set A^c. For instance, if A' is a sphere, with a random radius R following the distribution f(r), it can be shown that θ and f(r) can be, in principle, estimated from Q(h) alone. In practice, it is wise to use a priori models of distributions f(r), and to estimate its parameters.

In applications, we generally use models with as few parameters as possible (from 2 to 4–5 at most).

From these parameters and different compacts λK (segment, disc, couple of points etc.) in equations (1), (2) or function g in equation (3), we calculate the theoretical expressions corresponding to the possible models. For each kind of compact set, we get a full curve (depending on size λ) that can be calculated theoretically, and estimated on real images. The identification of the model is done by curve fitting (usually by means of a least square criterion) on part of the data. Separate estimations of the parameters can be done from different curves, to perform a cross–validation of the model.

N. Bonnet : *You mention several times the possible application of this kind of method to multispectral imaging (like multi–element X–ray maps). Could you comment on the applications you have in mind in this specific case and the kind of models which could be used for this purpose ?*

Author : Multispectral images occur very often in microscopy, and help account for chemical information on a small scale. Our multivariate models can be used to solve the following problems, among others :

– Combined morphological and chemical analysis of powders (nonplanar samples seen in the SEM), using the Dead Leaves models.

– Use of chemical data for quantitative fractographic analysis, study of the deformations or wear of a rough surface (Boolean, and alternate sequential models)

– Modeling microsegregations during the solidification of industrial metallic alloys (as for instance the images presented in the paper "Nonlinear statistical filtering and applications to segregations in steels from microprobe images" by C. Daly, D. Jeulin, and D. Benoît, in these Proceedings) by the Reaction–Diffusion models;

– Modeling multiphase microstructures observed at a high magnification on bulk specimens with a microprobe, prior to improving their spatial resolution by an appropriate deconvolution filter.

– Outside the field of image analysis, such models (Boolean R.F.) are used to study the fracture statistics of materials when several fracture mechanisms are in competition (see text reference [10])

Reviewer : You write that one of the strong points of using Random Function models is the ability to estimate parameters describing the statistical properties of the structural unit under investigation from simple operations and measurements. Data obtained from an electron microscope is normally noisy and may also be nonstationary. Can the random function model approach be applied to noisy data, and if so, is it possible to derive by mathematics or simulations the relation between bias/precision as a function of the noise level ?

Author : Nonstationary versions of the models presented are given in text reference [10]. They are useful for the simulation and the description of structures such as random aggregates. For their implementation, images containing different realizations of full aggregates must be available.

When working with noisy images, two types of problems can be solved :
– estimation of the parameters of the underlying model ;
– restoration of the images.

They both require the use of a model (usually a probabilistic model) for the generation of the noise (usually addition of an uncorrelated random function, or a randomization of the signal by a Poisson variable, etc.). In any case, the resulting images are still a realization of a Random Function model (built by the combination of the underlying structural model and of the random noise), and it is usually possible to estimate the parameters derived from the structure and from the noise separately, although it requires specific developments. For instance, the covariance of a noisy image differs from the covariance of the pure signal by a discontinuity at the origin. It can be recovered from a cross–correlation between two separate acquisitions, or from the "noisy" covariance if the underlying structure is assumed to be mean square continuous.

Similarly, the point distribution (or more generally a multivariate distribution) of a noisy image can be, in principle, recovered, but usually generates ill–posed problems (regularization techniques were developed by C. Daly (Thesis, Paris School of Mines, 1991) for the general estimation of the univariate distribution function from noisy images). Estimating an underlying model of histogram is much simpler, as few parameters are involved.

Efficient filters can improve noisy images (see the references mentioned in the previous question and C. Daly's thesis). They require the estimation of the underlying covariance (Wiener type linear filters) or of the underlying bivariate distribution (disjunctive kriging filters), accessible from the noisy images using appropriate assumptions.

Further estimations of the data required for a full specification of the model (for instance T(K) in equation (1), or T(g) in equation (3) can be obtained on filtered images, if the final SNR is not too low. In any case, it is possible to give some interval of confidence of the estimated parameters and to select the most appropriate estimator from simulations of the full model (structure, and structure + noise) to decide whether a model can be accepted, or even cannot be tested in the presence of a given noise level. This is a great advantage in using a model since such simulations are not available without modeling, so that it is often difficult to assess the validity of a filtering procedure when its use is limited to real data.