5-2021

Effects of Flow Path Factors in the Permeability of Natural and Man-made Granular Soils Using the Kozeny-Carman Equation

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Abstract

The research presented in this paper serves to observe how the flow path factors relate to the void ratio, the effective diameter of a soil particle, and the permeability of a given sample assuming the behavior is consistent across natural and artificial soils. The basis of the comparison is the derived Kozeny-Carman Equation for permeability with focus on the factors C_s and C_l where C_s represents the shape of the flow path and C_l represents the length of the flow path that a single water molecule must travel through a soil sample. Permeameter tests were conducted for six types of material including three natural sands and two man-made samples to compare data. Man-made or unnatural samples included stainless steel pins and ceramic spheres. Four other natural sands were tested at both 50% and 90% relative density to compare data. Natural sands included C33 sand, Quartz sand, Fine Ottawa sand and Coarse Ottawa sand. It was observed that while the permeability of the samples was impacted by the geometry of the flow paths, as shown in the flow path factors C_sC_l, as well as the void ratios, it appears that the effective diameter D_s has a larger impact on permeability.

Introduction

The Kozeny–Carman equation is used in engineering to calculate the pressure drop for laminar flow through a given soil sample. The equation was developed in 1927 by Josef Kozeny, using the simplified model in which he used numerous parallel capillary tubes of equal length and diameter to describe the packed bed and later modified by Carman in 1937 (McCabe et al. 2005). The below equation (Equation 1) has been derived from the Kozeny-Carman equation to include the variables for effective diameter (D_s), void ratio of a given sample (e), and the dynamic viscosity (μ) as well as the specific weight of water (γ_w) at a measured temperature (see water properties in Table A1). It also includes the flow path factors C_s and C_l where C_s represents the shape of the flow path and C_l represents the length of the flow path that a single water molecule must travel through a soil sample.

Equation 1. Kozeny-Carman Equation for Permeability

$$\boldsymbol{k = \frac{1}{36} c_s c_l \frac{\gamma_w}{\mu} D_s^2 \frac{e^3}{(1+e)}}$$

$Where\ c_s c_l = flow\ path\ factors,\ D_s = effective\ diameter,\ e = void\ ratio,$
$\gamma_w = specific\ weight\ water$, and $\mu = dynamic\ viscosity\ water$

Little research has been done on the product of C_s and C_l, which this paper will refer to as the flow path factors. This research serves to observe how the flow path factors relate to the void ratio, the effective diameter of a soil particle and the permeability of a given sample assuming the behavior is consistent across natural and artificial soils.

Test Procedures

Permeameter tests were conducted for six types of material including three natural sands and two man-made samples to compare data. Man-made or unnatural samples included stainless steel pins and ceramic spheres. Four other natural sands were tested at both 50% and 90% relative density to compare data. Natural sands included C33 sand, Quartz sand, Fine Ottawa sand and Coarse Ottawa sand. The exact procedure conducted for each material will follow.

C33 Sand

The C33 Sand was obtained from the USU Engineering Soils Lab. The sample was prepped by washing through a 200 sieve and then dried in an oven for 24 hours to remove all moisture. A relative density test was conducted in accordance to ASTM standards to determine the minimum and maximum void ratios of the sample (see Equation A5). The sample was then processed through a sieve stack ranging from a #10 to #100 US standard sieve sizes to provide grain size analysis. Samples were collected from the retained material in each sieve, including the pan, and photographs were taken with a stereograph microscope in order determine sphericity (see Figure A3). The effective diameter was calculated from the sphericity using Equation A1 (see Table A2 for results). The samples were divided then compacted using a 50% and 90% relative density. For each relative density, a permeability test was conducted using a constant head permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each gradient at 50% and 90% relative density for a total of 16 trials per sample.

Quartz Sand

The Quartz Sand was obtained from the USU Engineering Soils Lab. The sample was prepped by washing through a 200 sieve and then dried in an oven for 24 hours to remove all moisture. A relative density test was conducted in accordance to ASTM standards to determine the minimum and maximum void ratios of the sample (see Equation A5). The sample was then processed through a sieve stack ranging from a #10 to #100 US standard sieve sizes to provide grain size analysis. Samples were collected from the retained material in each sieve, including the pan, and photographs were taken with a stereograph microscope in order determine sphericity (see Figure A4). The effective diameter was calculated from the sphericity using Equation A1 (see Table A2 for results). For each relative density, a permeability test was conducted using a constant head permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each gradient at 50% and 90% relative density for a total of 16 trials per sample.

Fine Ottawa Sand

The Fine Ottawa Sand was obtained from the USU Engineering Soils Lab. The sample was prepped by washing through a 200 sieve and then dried in an oven for 24 hours to remove all moisture. A relative density test was conducted in accordance to ASTM standards to determine the minimum and maximum void ratios of the sample (see Equation A5). The sample was then processed through a sieve stack ranging from a #10 to #100 US standard sieve sizes to provide grain size analysis. Samples were collected from the retained material in each sieve, including the pan, and photographs were taken with a stereograph microscope in order determine sphericity (see Figure A5). The effective diameter was calculated from the sphericity using Equation A1 (see Table A2 for results). For each relative density, a permeability test was conducted using a constant head permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each gradient at 50% and 90% relative density for a total of 16 trials per sample.

Coarse Ottawa Sand

The Coarse Ottawa Sand was obtained from the USU Engineering Soils Lab. The sample was prepped by washing through a 200 sieve and then dried in an oven for 24 hours to remove all moisture. A relative density test was conducted in accordance to ASTM standards to determine the minimum and maximum void ratios of the sample (see Equation A5). The sample was then processed through a sieve stack ranging from a #10 to #100 US standard sieve sizes to provide grain size analysis. Samples were collected from the retained material in each sieve, including the pan, and photographs were taken with a stereograph microscope in order determine sphericity (see Figure A6). The effective diameter was calculated from the sphericity using Equation A1 (see Table A2 for results). For each relative density, a permeability test was conducted using a constant head permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each gradient at 50% and 90% relative density for a total of 16 trials per sample.

Stainless Steel Pins

The Guntap Stainless Steel Tumbling Media Pins were purchased online. Twenty-five sample pins were hand measured with a caliper to confirm the average length and diameter which was measured at a diameter of 1.198mm and a length of 6.558mm with an average volume of 7.386mm^3. The volume was used to determine the void ration using Equation A4. Photographs were taken with a stereograph microscope (see Figure A7). The Stainless-steel pins were tested in two orientations including one that was predominately horizontal and one with a random orientation. For each orientation, a permeameter test was conducted with a constant head in a permeability cell, also called a permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each gradient orientation for a total of 16 trials per sample.

Ceramic Spheres

The Raytech 41210R Microbrite Ceramic Sphere Balls were purchased online. The sample was prepared by sorting through a lab manufactured screen with 3mm drilled holes. One hundred sample spheres were hand measured with a caliper from the spheres which were retained in the screen to confirm the average diameter which was measured at a diameter of 3.18mm with an average volume of 16.93mm^3. One hundred sample spheres were also hand measured with a caliper from the spheres which were filtered out in the screen to confirm the average diameter which was measured at a diameter of 1.02mm with an average volume of 0.57mm^3. The volume was used to determine the void ration using Equation A4. Photographs were taken with a stereograph microscope (see Figure A8). Ceramic spheres were separated into two samples, the first included only spheres that were 1mm in diameter and the second consisted of 80% spheres at 3mm and 20% at 1mm. For each mixture, a permeameter test was conducted with a constant head in a permeability cell, also called a permeameter (see Figure A1 for test set up). Prior to filling the permeameter cell with water, the cell was saturated with Co2 gas to prevent oxygen bubbles from filling the voids in the sample. Water was then filled into the cell using a vacuum and left on the sample until it no longer produced bubbles to de-air the sample (see Figure A2). The manometer tube was set to the appropriate gradient being tested. Permeameter tests were run at 4 different gradients: 0.25, 0.50, 1.00 and 1.50 gradient. Water temperature was checked and recorded as well as the tail water elevation height. A stopwatch was used to time each trial. Each test began by pulling the stopper on the permeameter tube and the minimum and maximum nominal bubble tube elevations were measured to determine the elevation change. Two trials were duplicated for each mixture for a total of 16 trials per sample.

Results and Analyses

Permeameter tests provided the permeability results k in cm/s as shown in Table 1 below. The measured void ratios, effective diameter, and tested permeabilities were used in the Kozeny-Carman equation to solve for flow path factors C_sC_l. Table 1 below shows the results of the findings for each sample.

Table 1. Results comparing Flow Path Factors C_sC_l to other variables, including Void Ratio, Permeability and Effective Diameter.

Soil	Flow Path Factors, C_lC_s	Void Ratio, e	Permeability, k (cm/s)	Ds (cm)
Ceramic Sphere (1mm)	0.117	0.711	0.730	0.10
Ceramic Sphere (80% 3mm + 20% 1mm)	0.052	0.630	1.669	0.28
Stainless Steel Pins (Horizontal)	0.064	0.818	1.421	0.17
Stainless Steel Pins (Random)	0.043	1.201	2.508	0.17
C33 Sand (Dr=50%)	0.249	0.466	0.049	0.032
C33 Sand (Dr=90%)	0.292	0.362	0.029	0.032
Quartz Sand (Dr=50%)	0.210	0.531	0.052	0.016
Quartz Sand (Dr=90%)	0.230	0.449	0.035	0.016
Coarse Ottawa Sand (Dr=50%)	0.242	0.597	0.292	0.058
Coarse Ottawa Sand (Dr=90%)	0.266	0.535	0.241	0.058
Fine Ottawa Sand (Dr=50%)	0.258	0.582	0.047	0.023
Fine Ottawa Sand (Dr=90%)	0.426	0.476	0.045	0.023

Conclusions

It was observed that while the permeability of the samples was impacted by the geometry of the flow paths, as shown in the flow path factors C_sC_l, as well as the void ratios, it appears that the effective diameter D_s has a larger impact on permeability. Further research is recommended to expand comparisons with other artificial sands. For example, disc shaped particles or cube particles may be obtained and tested to further study the effects of C_sC_l in the Kozeny-Carmen permeability equation.

Acknowledgments

Academic guidance was provided by Dr. James Bay, associate professor in the Civil and Environmental Engineering department at Utah State University. Gracious support was also given from John Garner P.E. of Garner Engineering Inc. who also provided detailed feedback and constructive criticism. Grateful acknowledgement also goes out to the University of North Carolina Asheville for allowing access to cameras and equipment.

References

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Krumbein, C. and Sloss, L. 1963. *Stratigraphy and Sedimentation,* 2nd edition, San Francisco: Freeman and Company.

Appendix

Table A1 - Water Properties

Temperature - *t* - (°C)	Dynamic Viscosity - *μ* - ($N s/m^2$)	Specific Weight - *γ* - (N/m^3)
19	1.03E-03	9790.8
19.5	1.02E-03	9789.9
20	1.00E-03	9789
20.5	9.91E-04	9787.9
21	9.80E-04	9786.8

Table A2- Effective Diameter and Sphericity of each sieve for C33.

Effective Diameter Calculations			C33					
Sieve Number	Diameter (mm)	Mass Retained	Dmax, mm	Dmin, mm	Deff, mm	Avg. Shericity, S	Vs (cm^3)	SA (cm^2)
#4	4.76	0	9.50	4.76	6.86	1	0.00	0.00
#8	2.38	97.99	4.76	2.38	3.43	0.7	36.63	914.42
#10	2.00	22.26	2.38	2.00	2.18	0.7	8.32	326.50
#16	1.18	68.53	2.00	1.18	1.55	0.7	25.62	1412.90
#20	0.84	44.79	1.18	0.84	1.00	0.7	16.74	1433.78
#30	0.60	40.35	0.84	0.60	0.71	0.7	15.08	1818.60
#40	0.42	54.94	0.60	0.42	0.50	0.7	20.54	3503.70
#50	0.30	63.03	0.42	0.30	0.35	0.7	23.56	5689.68
#60	0.25	29.42	0.30	0.25	0.27	0.7	11.00	3455.19
#100	0.15	54.55	0.25	0.15	0.20	0.7	20.39	8955.91
#200	0.07	23.19	0.15	0.07	0.11	0.7	8.67	6933.90
		Ds (cm)	0.03250				186.55	34444.57

Table A3- Effective Diameter and Sphericity of each sieve for Quartz.

Effective Diameter Calculations			Quartz					
Sieve Number	Diameter (mm)	Mass Retained	Dmax	Dmin	Deff	Avg. Shericity, S	Vs	SA
#4	4.76	0	9.50	4.76	6.86	1	0.00	0.00
#8	2.38	0	4.76	2.38	3.43	1	0.00	0.00
#10	2.00	0	2.38	2.00	2.18	1	0.00	0.00
#16	1.18	0	2.00	1.18	1.55	1	0.00	0.00
#20	0.84	0.02	1.18	0.84	1.00	0.7	0.01	0.64
#30	0.60	0.03	0.84	0.60	0.71	0.7	0.01	1.35
#40	0.42	40.97	0.60	0.42	0.50	0.7	15.32	2612.78
#50	0.30	202.9	0.42	0.30	0.35	0.7	75.85	18315.65
#60	0.25	88.17	0.30	0.25	0.27	0.7	32.96	10354.99
#100	0.15	144.23	0.25	0.15	0.20	0.7	53.92	23679.40
#200	0.07	23.57	0.15	0.07	0.11	0.7	8.81	7047.52
		Ds (cm)	0.01808				186.87	62012.34

Table A4- Effective Diameter and Sphericity of each sieve for Fine Ottowa sand.

Effective Diameter Calculations			FOS					
Sieve Number	Diameter (mm)	Mass Retained	Dmax	Dmin	Deff	Avg. Shericity, S	Vs	SA
#4	4.76	0	9.50	4.76	6.86	1	0.0	0.0
#8	2.38	0	4.76	2.38	3.43	1	0.0	0.0
#10	2.00	0	2.38	2.00	2.18	1	0.0	0.0
#16	1.18	0	2.00	1.18	1.55	1	0.0	0.0
#20	0.84	0.32	1.18	0.84	1.00	0.7	0.1	10.2
#30	0.60	7.4	0.84	0.60	0.71	0.7	2.8	333.5
#40	0.42	136.09	0.60	0.42	0.50	0.7	50.9	8678.9
#50	0.30	231.05	0.42	0.30	0.35	0.7	86.4	20856.7
#60	0.25	60.88	0.30	0.25	0.27	0.7	22.8	7150.0
#100	0.15	59.36	0.25	0.15	0.20	0.7	22.2	9745.6
#200	0.07	5.79	0.15	0.07	0.11	0.7	2.2	1731.2
		Ds (cm)	0.02316				187.24	48506.18

Table A5- Effective Diameter and Sphericity of each sieve for Coarse Ottowa sand.

Effective Diameter Calculations			COS					
Sieve Number	Diameter (mm)	Mass Retained	Dmax	Dmin	Deff	Avg. Shericity, S	Vs	SA
#4	4.76	0	9.50	4.76	6.86	1	0.00	0.00
#8	2.38	0	4.76	2.38	3.43	1	0.00	0.00
#10	2.00	0	2.38	2.00	2.18	1	0.00	0.00
#16	1.18	1.87	2.00	1.18	1.55	0.7	0.70	38.55
#20	0.84	35.76	1.18	0.84	1.00	0.9	13.37	890.34
#30	0.60	452.68	0.84	0.60	0.71	0.8	169.22	17852.22
#40	0.42	7.39	0.60	0.42	0.50	0.9	2.76	366.55
#50	0.30	1.23	0.42	0.30	0.35	0.8	0.46	97.15
#60	0.25	0.32	0.30	0.25	0.27	0.8	0.12	32.88
#100	0.15	0.66	0.25	0.15	0.20	0.7	0.25	108.36
#200	0.07	0.26	0.15	0.07	0.11	0.7	0.10	77.74
		Ds (cm)	0.05764				186.97	19463.80

Equation A1 - Effective Diameter Equation, D_{eff}

$$D_{eff} = \frac{(D_{max} - D_{min})}{\ln\left(\frac{D_{max}}{D_{min}}\right)}$$

Equation B2 – Volume of Solids Equation, Vs

$$V_s = \frac{m}{G_s \cdot \rho_w}$$

Equation C3 – Surface Area Equation, S.A.

$$S.A. = \frac{V_s \cdot \frac{6}{D_{eff}}}{S}$$

Equation D4 – Void Ratio Equation for artificial soils, where actual volumes were measured, e

$$e = \frac{V_v}{V_s}$$

Equation E5 – Void Ratio Equation for natural soils based on relative density, e_{Dr}

$$e_{Dr} = \left(1 - \frac{Dr}{100}\right)(e_{max} - e_{min}) + e_{min}$$

Equation F6 – Average sphericity, S

$$S = \frac{S.A._D}{S.A._{D_{min}}}$$

Equation G7 – Grain size diameter, D_s

$$Ds = 6 \cdot \frac{\Sigma V_i \cdot S_i}{\Sigma SA_i}$$

Figure H1 – Test set-up

Figure I2 – De-aired water used in testing

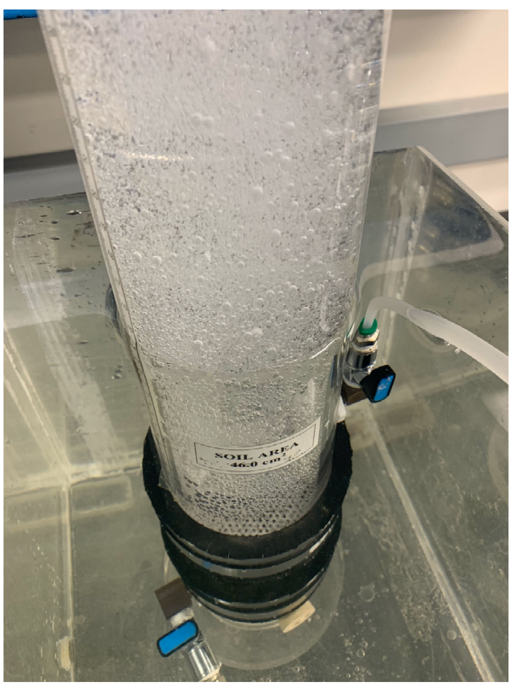
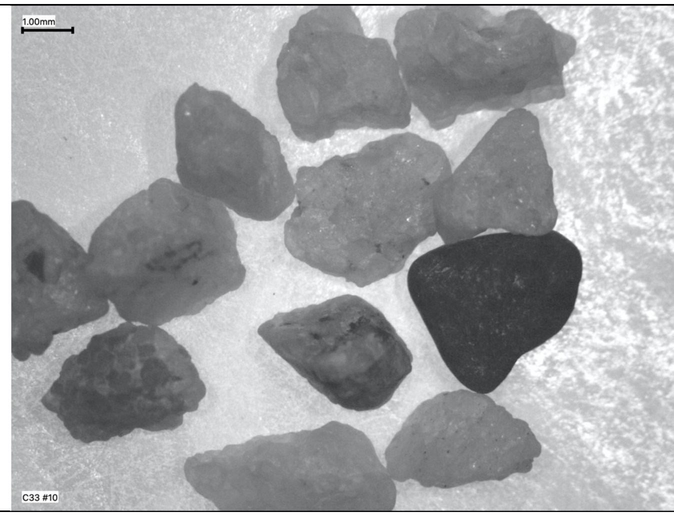
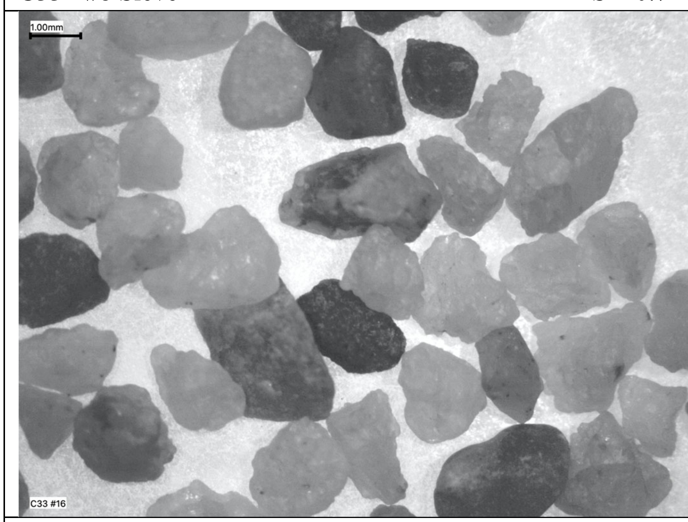


Figure A3. Stereograph photos of C33 Sand

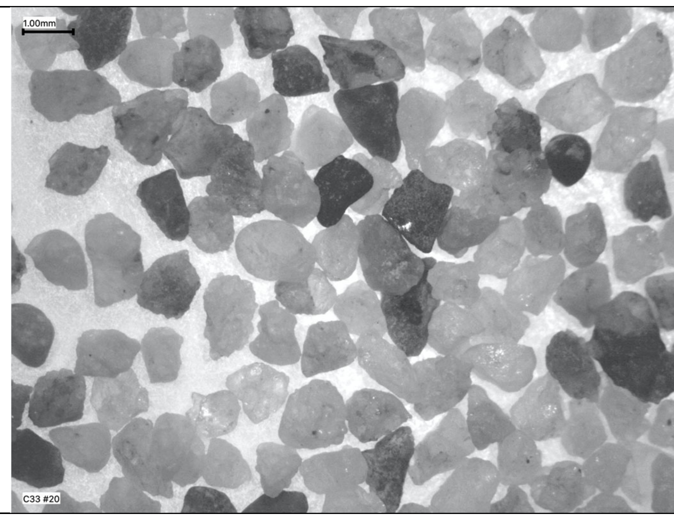
C33 - #8 Sieve S = 0.7



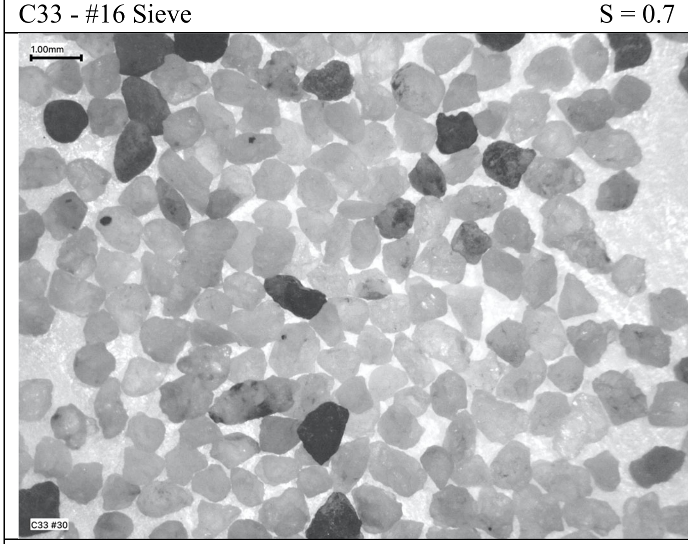
C33 - #10 Sieve S = 0.7



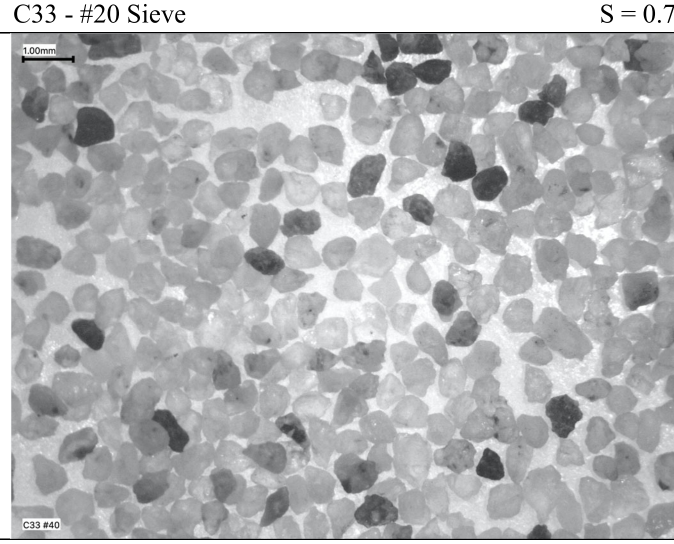
C33 - #16 Sieve S = 0.7



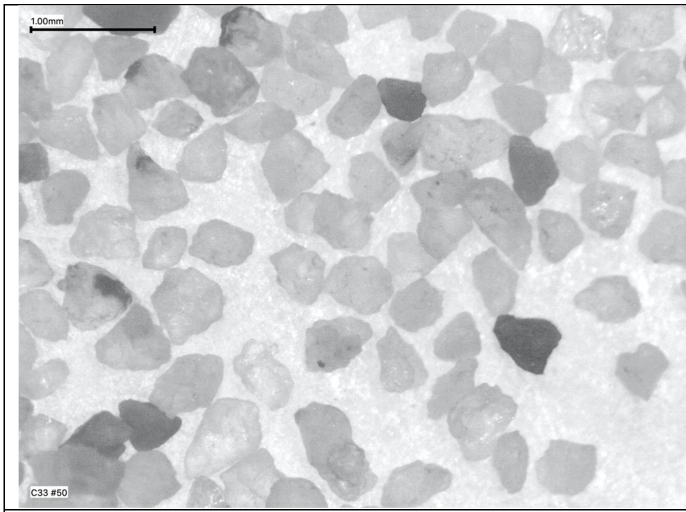
C33 - #20 Sieve S = 0.7



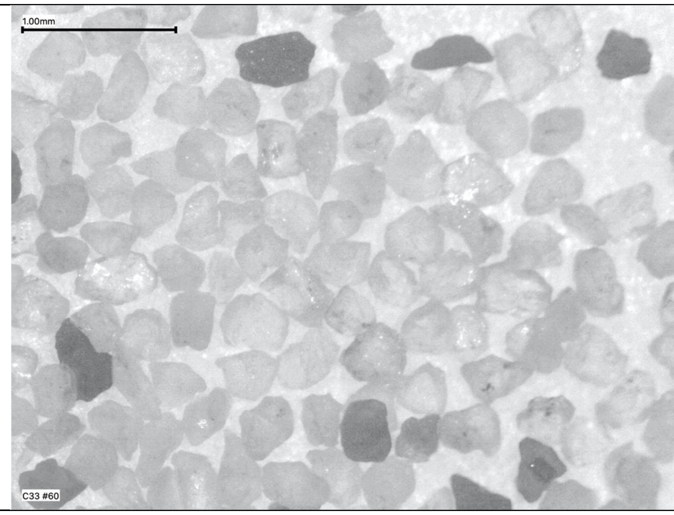
C33 - #30 Sieve S = 0.7



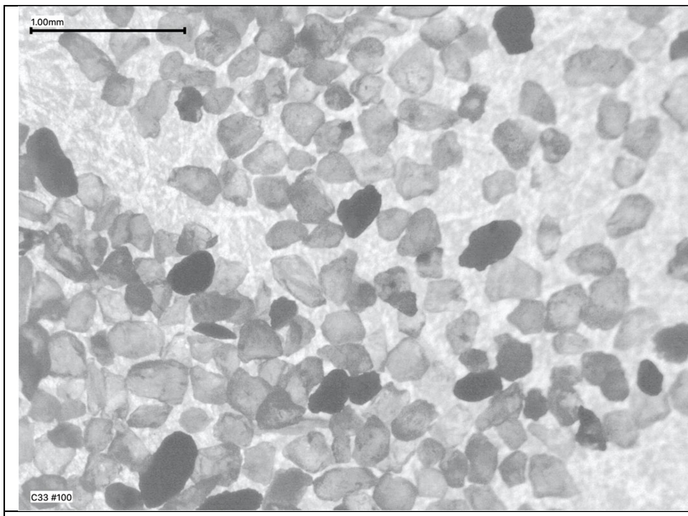
C33 - #40 Sieve S = 0.7



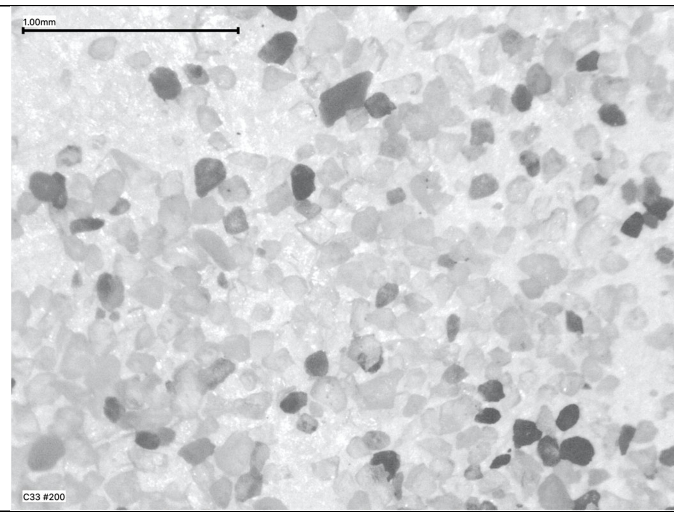
C33 - #50 Sieve S = 0.7



C33 - #60 Sieve S = 0.7

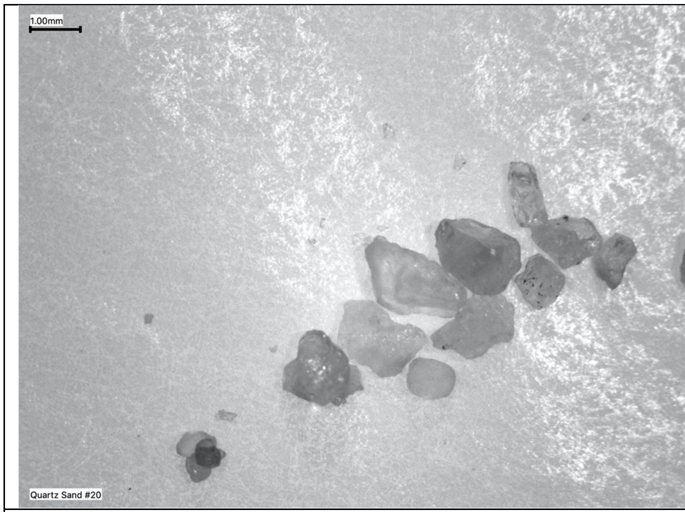


C33 - #100 Sieve S = 0.7

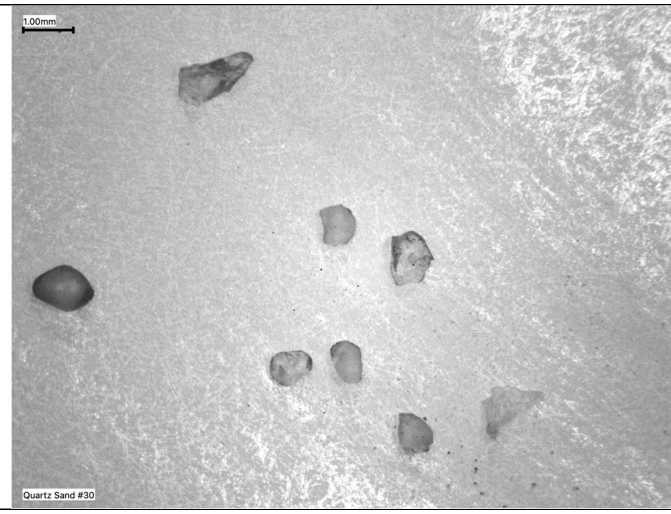


C33 - #200 Sieve S = 0.7

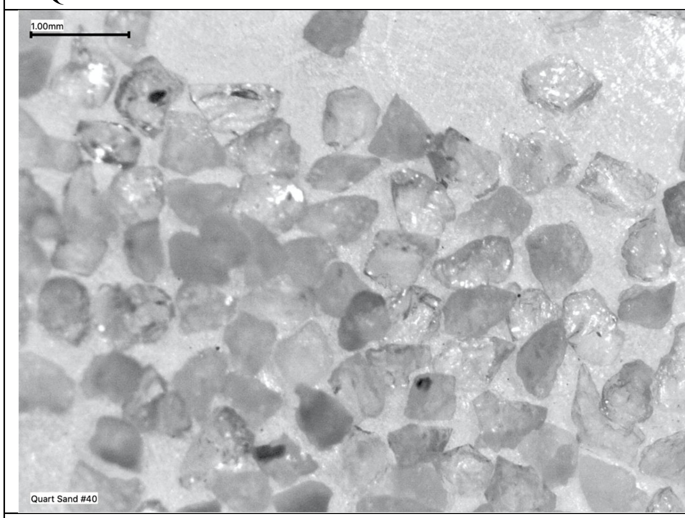
Figure A4. Stereograph photos of Quartz Sand



Quartz Sand - #20 Sieve S = 0.7



Quartz Sand - #30 Sieve S = 0.7



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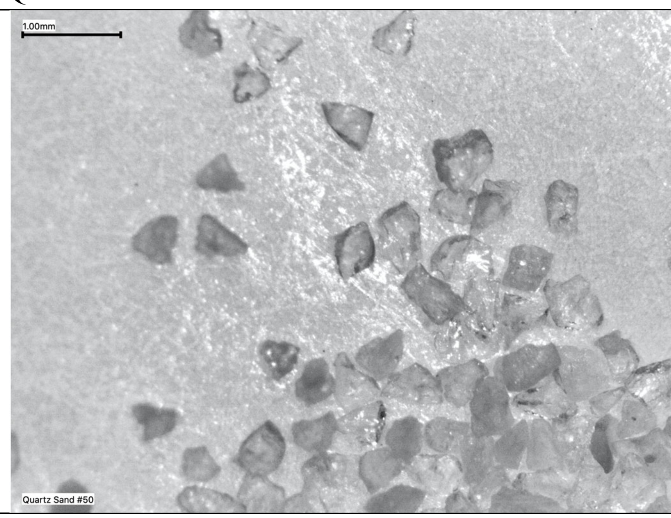
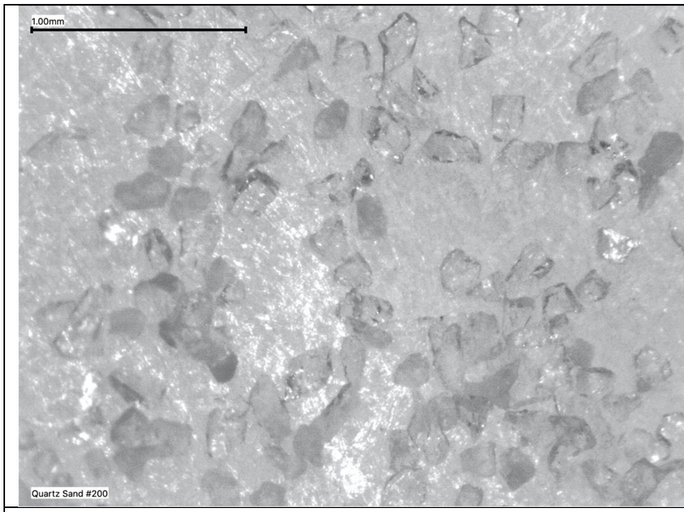


Quartz Sand - #50 Sieve S = 0.6

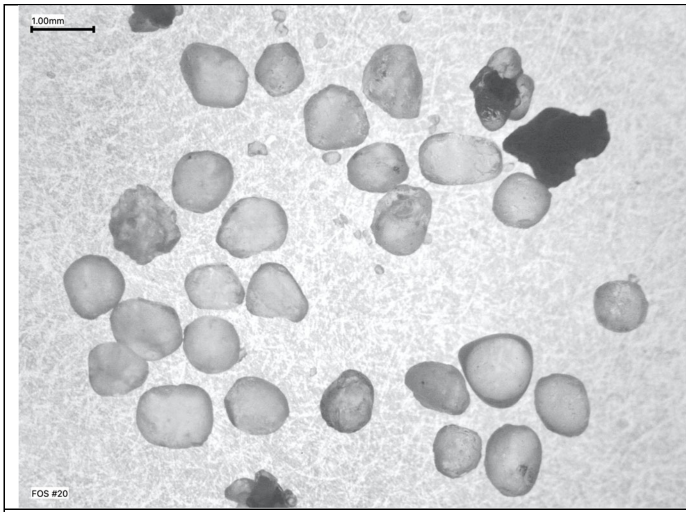
Quartz Sand - #60 Sieve S = 0.6

Quartz Sand - #100 Sieve S = 0.6

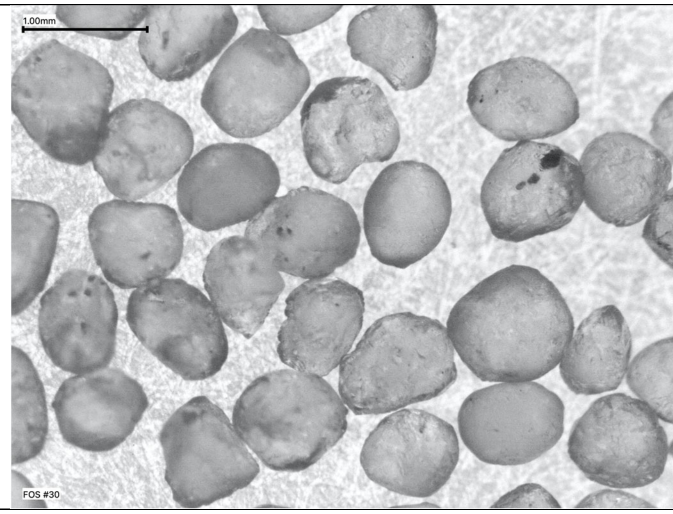


Quartz Sand - #200 Sieve S = 0.6

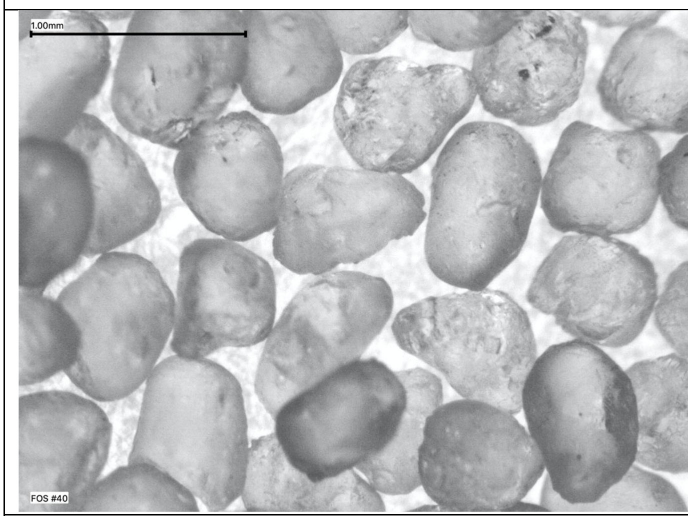
Figure A5. Stereograph photos of Fine Ottawa Sand



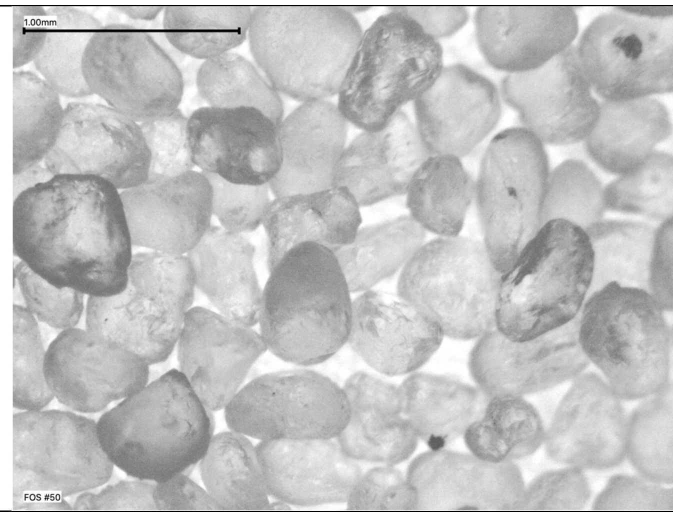
FOS - #20 Sieve S = 0.8



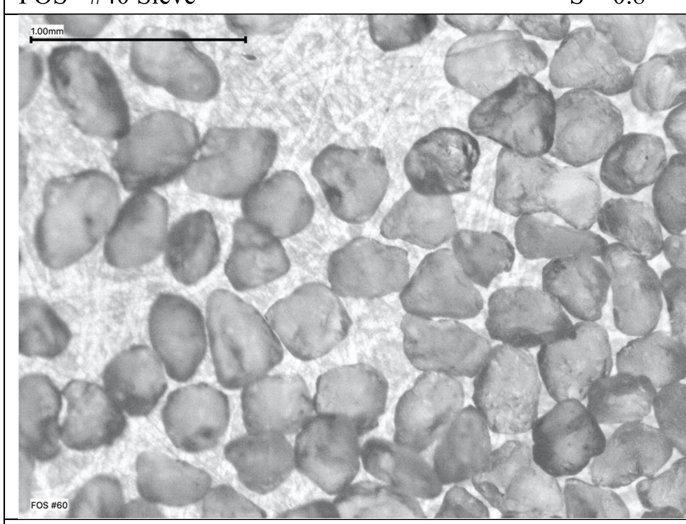
FOS - #30 Sieve S = 0.8



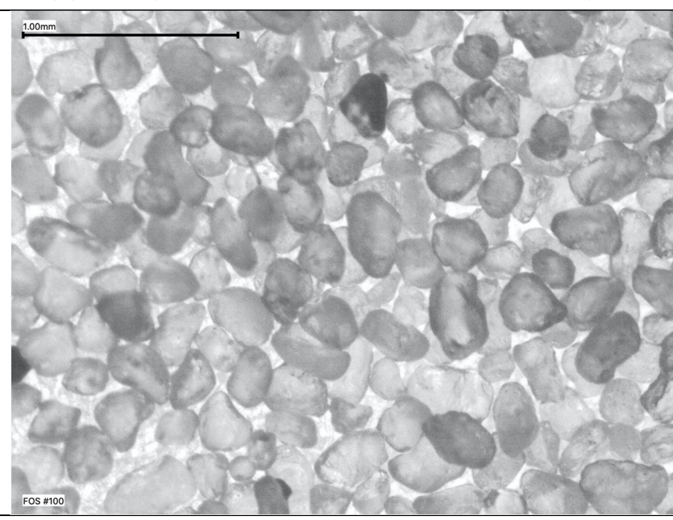
FOS - #40 Sieve S = 0.8



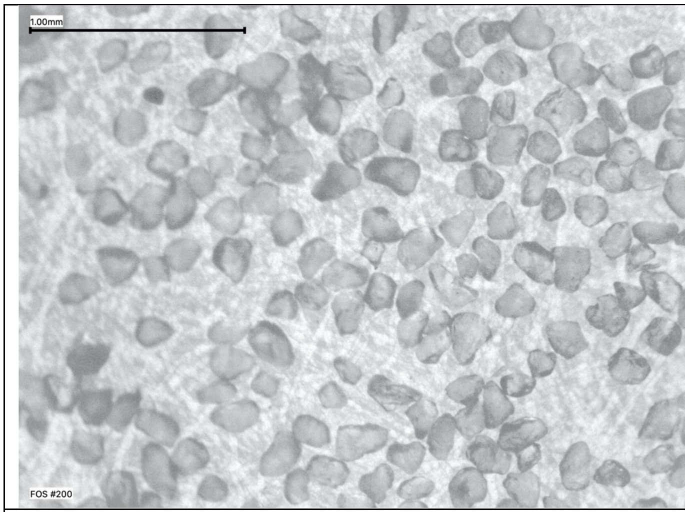
FOS - #50 Sieve S = 0.8



FOS - #60 Sieve S = 0.7

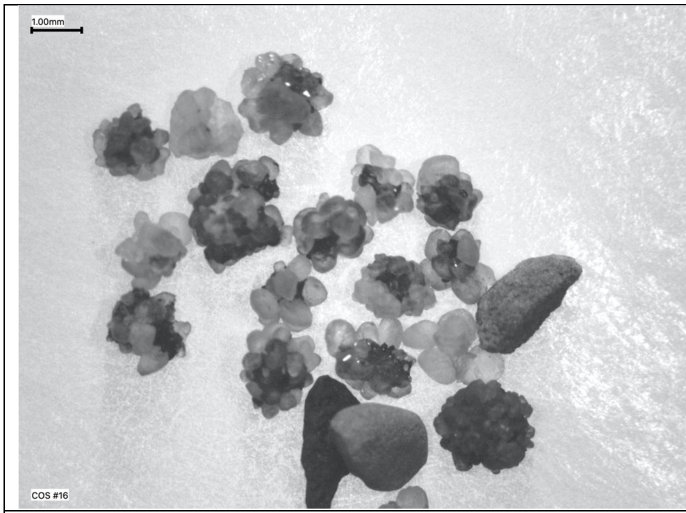


FOS - #100 Sieve S = 0.7

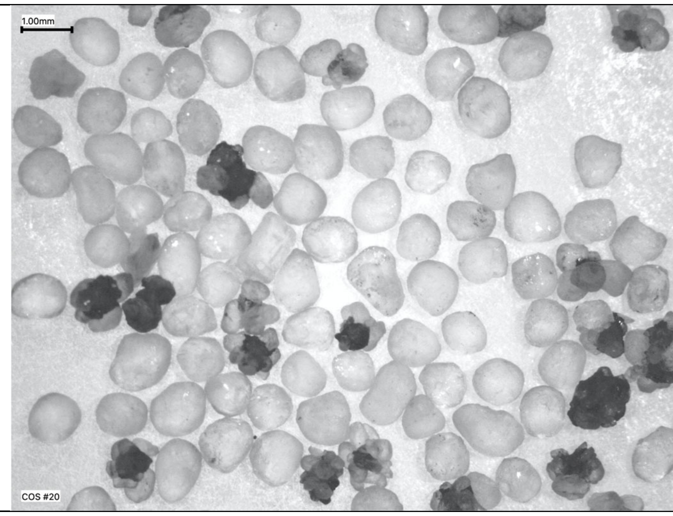


FOS - #200 Sieve S = 0.7

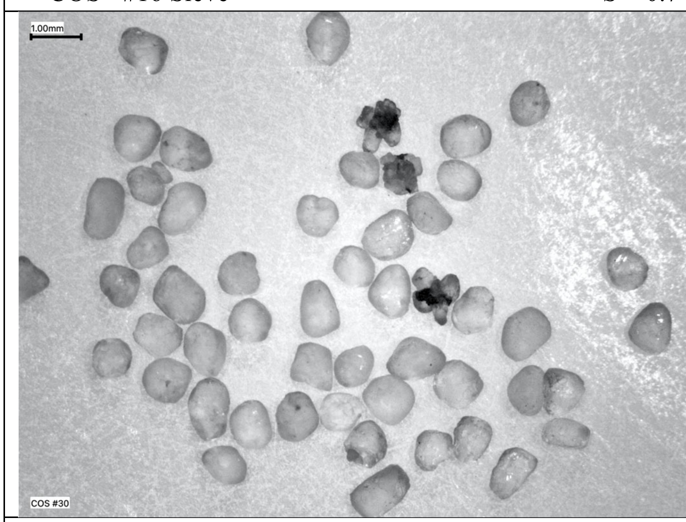
Figure A6. Stereograph photos of Coarse Ottawa Sand



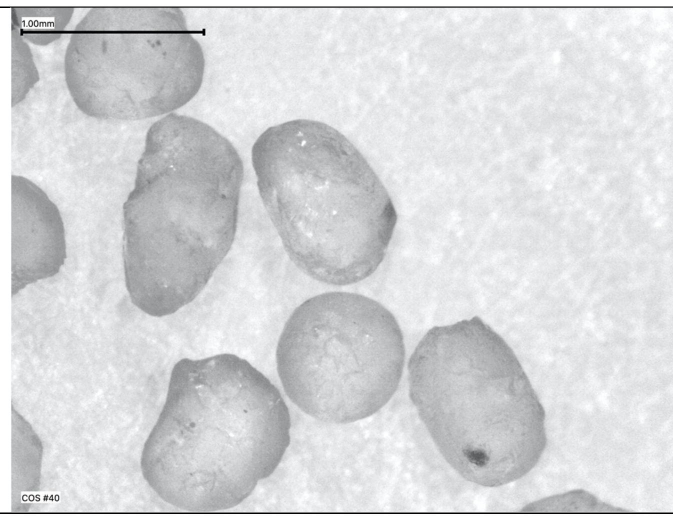
COS - #16 Sieve S = 0.7



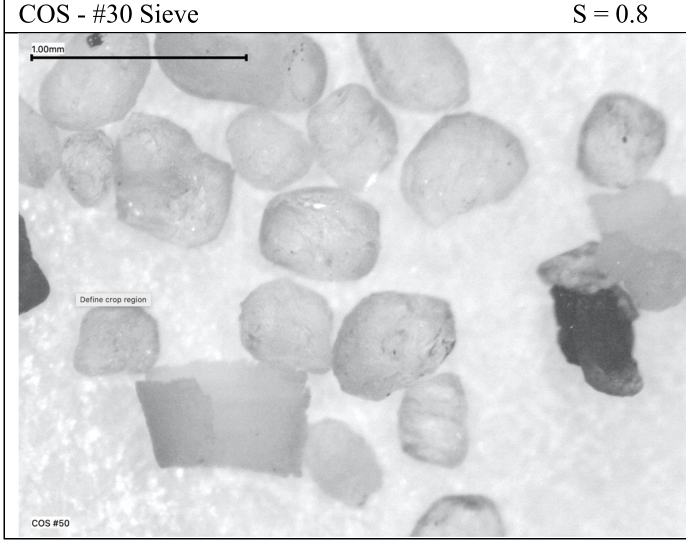
COS - #20 Sieve S = 0.9

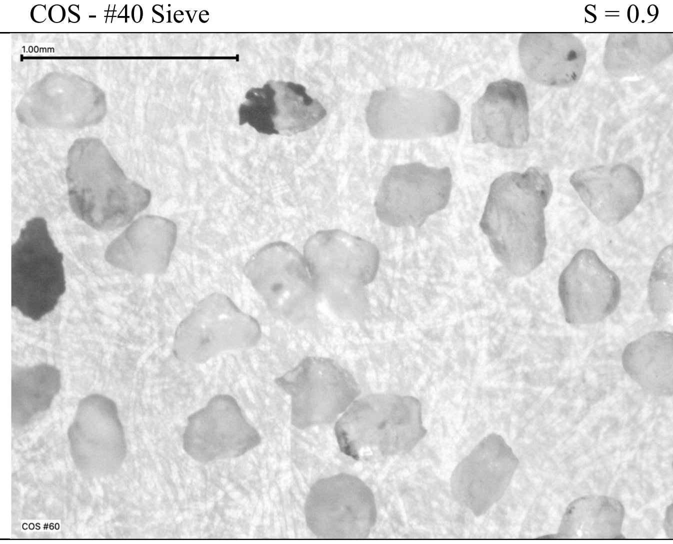


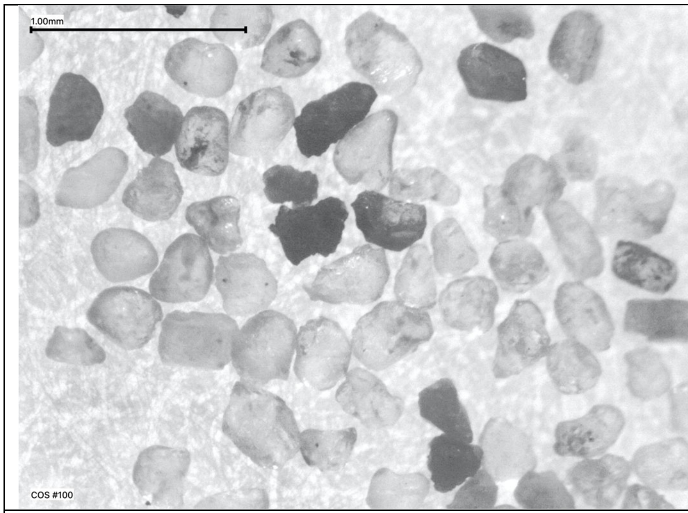
COS - #30 Sieve S = 0.8



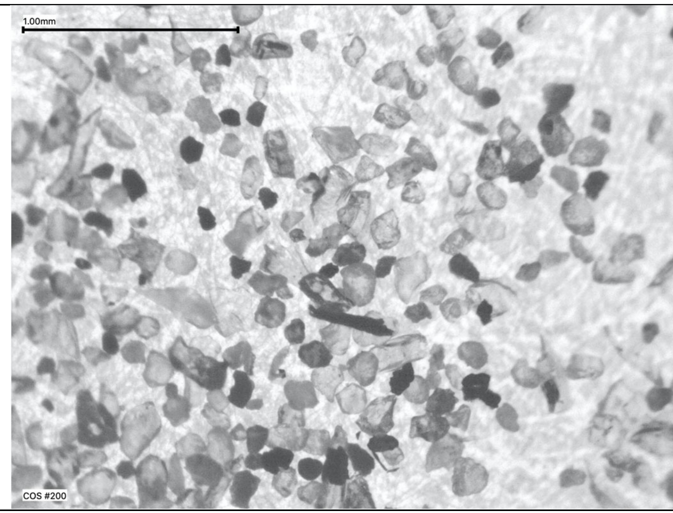
COS - #40 Sieve S = 0.9





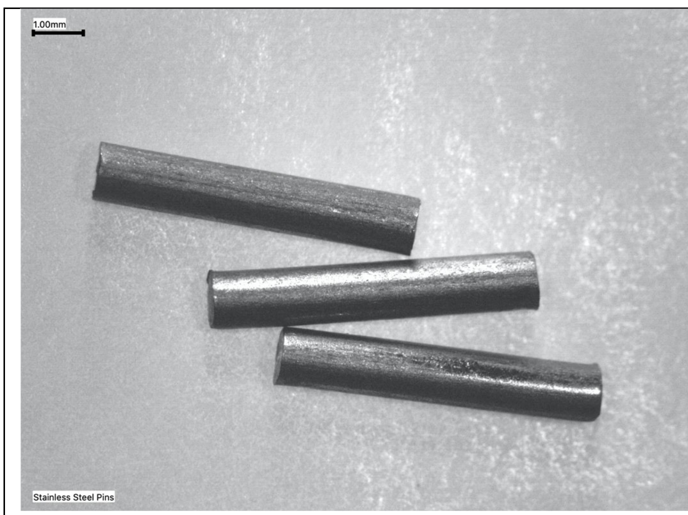


COS - #100 Sieve S = 0.7



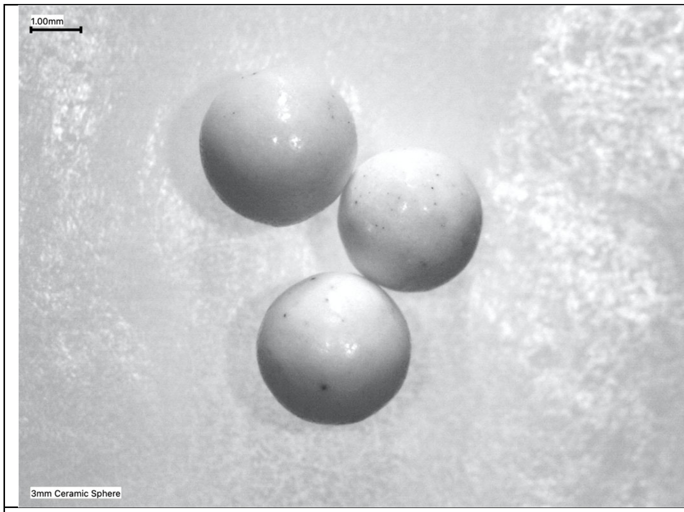
COS - #200 Sieve S = 0.7

Figure A7. Stereograph photos of Stainless Steel Pins and sample shown in permeameter tube

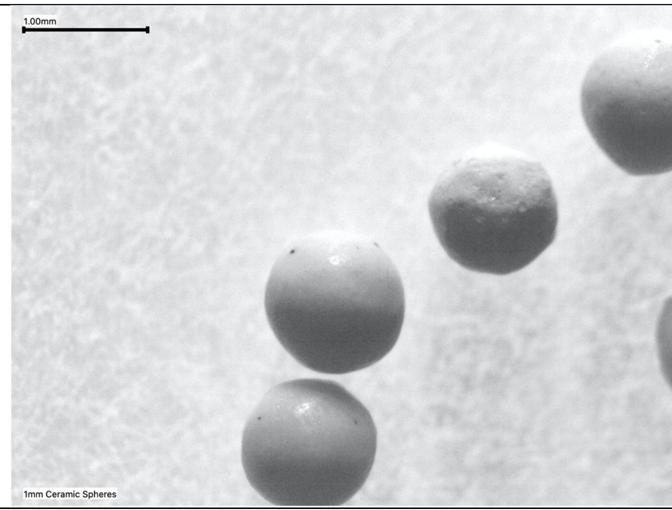


Stainless Steel Pins – 1mm x 3mm

Figure A8. Stereograph photos of Ceramic Spheres with samples shown in permeameter tube showing 100% 1mm test sample and 80% spheres at 3mm and 20% at 1mm.



Ceramic Spheres – 3mm



Ceramic Spheres – 1mm

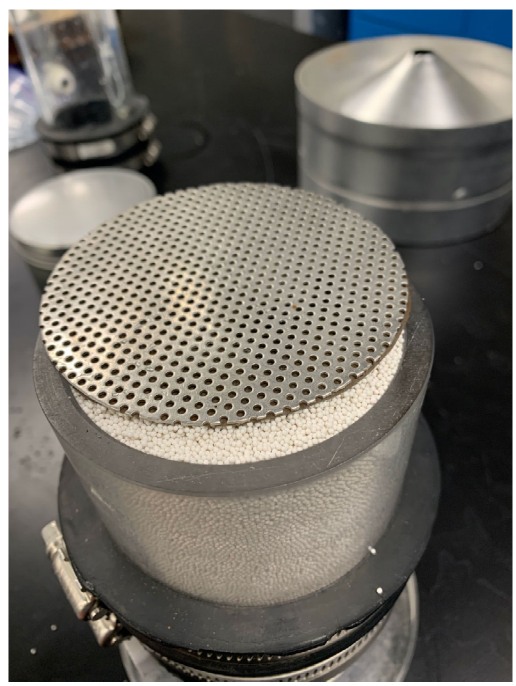
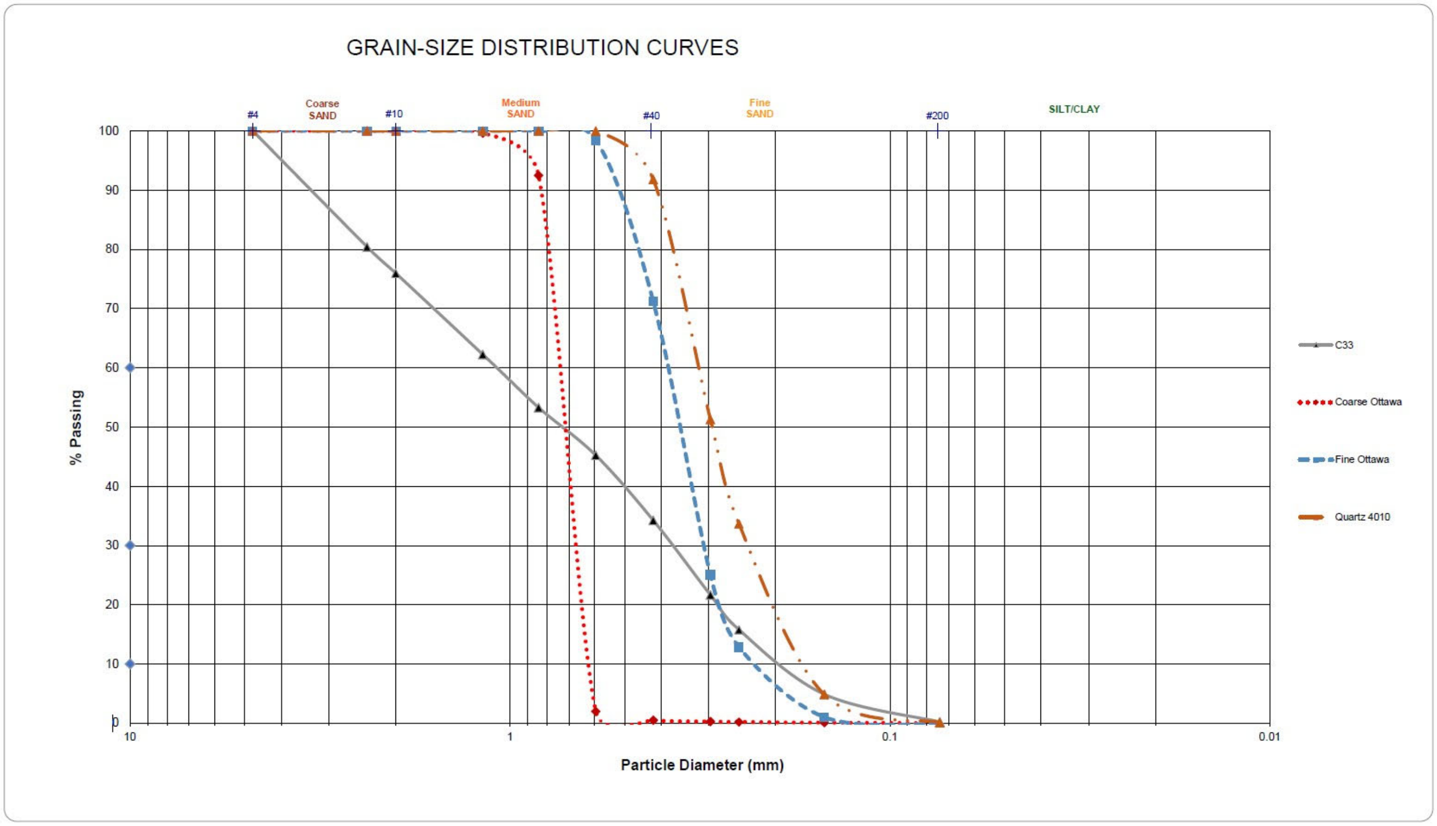


Figure A9. Grain-Size Distribution Curves showing all samples.



		Percent Passing by Weight			
		C33	Coarse Ottawa	Fine Ottawa	Quartz 4010
Sieve Number	**Diameter (mm)**	**Soil Passing (%)**	**Soil Passing (%)**	**Soil Passing (%)**	**Soil Passing (%)**
#4	4.76	100	100	100	100
#8	2.38	80	100	100	100
#10	2.00	76	100	100	100
#16	1.18	62	100	100	100
#20	0.84	53	92	100	100
#30	0.60	45	2	98	100
#40	0.42	34	0	71	92
#50	0.30	22	0	25	51
#60	0.25	16	0	13	34
#100	0.15	5	0	1	5
#200	0.07	0	0	0	0