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MINERAL RECOGNITION AND LIBERATION DEGREE MEASUREMENTS IN INDUSTRIAL ORE PROCESSING

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Abstract

This study aims at identifying calculation procedures and representation methods for identifying the mineral species present in a rock. The goal of the study is to identify: the characteristic spectrum for each mineral species identified in the ore; the main shape of the individual species; the preferential arrangement of the individual mineral species; the grain-size distribution of the individual mineral species and of the grains resulting from comminution processes; the degree of liberation for each mineral; and the degree of liberation of the comminution products obtained from the ores.

Key words: Backscattered Electronic Image (BEI) Analysis, Image Processing, Particle Shape, Mineral Arrangement, Grain Size Distribution, Degree of Liberation, Comminution Process.

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Introduction

Controlling an ore process entails an evaluation of how the mineral species in the feed are distributed among the products of the process.

In general, three types of evaluation can be made:

–where the application of a specific process on an ore is to be evaluated, the *feasibility study* must provide for the characterization of the ore as a function of the texture and of the relations existing between the in situ mineral species. This latter type of evaluation can be made also for the intermediate products of treatment processes so as to identify the possibilities of separating the elementary components of the ore (liberation degree).

–where the treatment process is a comminution and classification process, the *evaluation for controlling* the process must serve to recognize particle shapes (and in particular their *sizes*) as they are produced by the various phases of the process;

–where the treatment process is a separation process, the *evaluation for control* must serve to recognize the various mineral species and to measure their distribution among the products.

Starting from the ore samples or from the products issuing from a treatment process, it is therefore necessary to define a series of quantificative characteristics of the mineral species in the ore or in the products. This means that the following operations are to be carried out:

–identify the various species present;

–identify the shapes of the minerals present which make up the host rock of the ore and the shapes of the particles occurring in the output of a treatment process;

–identify the preferential arrangement of the minerals inside the ore;

–evaluate the size distribution of the minerals in the ore and of the particles occurring in the products of the treatment processes;

–evaluate the relationship between the various minerals in the ore or in its products (grades);

–evaluate the relationship between the particles consisting of a single mineral species and the overall amount of that same mineral in all the products (liberation degree);

–evaluate the distribution of the ratio between volume (or mass) of the part of each particle consisting of a certain mineral species, and the volume (or mass) of the same particle (liberation function).

The latter is the ultimate and most exhaustive goal, pursued by a *feasibility or control evaluation* for treatment processes because it is a means for measuring not only the results, but also for predicting what is to be done in order to apply the process or in order to modify it.

In the light of these remarks, image-processing procedures (Serra, 1980) have been re-examined and re-elaborated so as to obtain the information required for evaluating specific parameters of the treatment processes, for example:

– identification of the shapes and comparison with the reference shapes;

– definition of the principal axes of inertia of the bodies;

– definition of the points of contact, of lines and areas between different mineralogical phases;

– discretization of the geometric parameters of the grains and of their associations for the simulation of the characteristics of the products of the treatment processes;

– simulation of the dividing of a body into grains;

– simulation of the distribution of a species in complex association as a consequence of divisions.

For this purpose, research work done in the fields of geology and mineralogy was taken into account, which had developed methodologies characterized by objectivity in the collection of information (Gateau, 1978), which entailed measuring linear and/or surface parameters (Gateau and Prevosteau, 1978) for deriving quantitative type of information about the structure of the various mineralogical phases which make up the sample (Barbery and Prevosteau, 1976).

Thus, on the basis of the foregoing parameters, some procedures have been developed whereby the characteristic functions of the treatment processes can be worked out.

Image processing system and types of images analyzed

The elaborations discussed in this paper were carried out by means of a system consisting of the following units:

– a TESAK* VDC 501/A videographic terminal

– a color monitor (resolution = 512 × 512 pixels)

– a B/W camera for loading images from the outside

– a DIGITAL PDP 11/23 computer

– a driver for an 8" floppy-disk

– a TESAK VD 1001 terminal

– a DIGITAL LA 100 printer.

The images stored by the TESAK VDC 501/A graphic terminal (*controller*) can be schematized as a square matrix, measuring 512 × 512 elements. Each location is associated with a given value. The point-to-point image, which is stored, reaches a total of 262144 *pixels (picture elements)*. The VDC 501/A does not process pictures, but it does allow this to be done by a host computer.

Each pixel is characterized by three values, two of which provide the coordinates of the pixel (integer values from 0 to 511); the third is a numerical value which identifies the level of grey or color of the element involved.

The image memory can be considered as consisting of twelve layers each being of 512 × 512 × 1 bits.

The possible color levels (value which can be attributed to a pixel) are $2^{12}=4096$, and the values that a pixel may take on vary between 0 and 4095. Each layer contributes to the forming of the color associated with a given pixel with a certain amount – differing according to the layer – of one of the three fundamental colors (red, blue and green).

Appropriate marks can be used so as to *write* on only some of the layers, and not all of them altogether.

During the acquisition stage, by using the "controller" (TESAK VDC 501/A), the image transmitted by the telecamera – from a videorecorder or a slide reader – can be monitored in real time.

The telecamera may be connected to a light microscope, to an electron microscope, or to a microprobe (Backscattered Electronic Images – B.E.I.).

The use of videorecorder or a slide reader makes image processing much more flexible because it allows one to use image analysis on samples prepared and studied "far" from the processing center. Moreover, this type of solution makes available a file of images which can be easily managed and which does not take up room in the mass storage of the processor (host computer).

Acquisition through the videorecorder or slide reader however does entail the disadvantage of working on "filtered" images, in that the conversion is responsible for some reduction or alteration of the image.

Once the optimal conditions for acquisition have been attained the image can be "frozen" in the image storage or memory (TESAK VDC 501/A) and is later stored on a disk (mass storage of the host computer: DIGITAL PDP 11/23).

The elaboration of this research work were carried out on B/W images stored on the first eight image memory layers. The remaining four overlaying layers were used to superimpose histograms onto the graphical image or to emphasize in color some of the characteristics which had emerged through the analysis.

The device which converts the analogical signal generated by the telecamera or videorecorder (TV signal) into a digital signal does so by associating each picture element with 8 bits ($2^8=256$ levels of grey); as a consequence, only the first 8 layers are used for storage out of the 12 layers which are available for storing the images.

For developing the algorithms and programs described in this paper, photographs (taken by means of a telecamera) of samples of a calaminar ore were used; an ore consisting of a complex association of oxidized, semi-oxydized minerals and zinc and lead sulphides in a carbonatic gangue, from the Buggerru Caitas deposit (Sardinia, Italy).

The photographs refer to Backscattered Electronic Image analyses (B.E.I.) obtained by means of a JEOL JXA-50A microprobe based on wavelength scatter (W.D.S.).

B.E.I. analyses were used (photo) because they can be used to establish a sort of two-way correspondence between level of grey of the image and the presence of a given element, and consequently of a given mineralogical species, in the image being analyzed; the density of the backscattered electrons (B.E.I.) analaysis depends in fact on the nature of the elements which are present.

Recognition of the various mineralogical phases of an ore and of a product

The surface of a body projected onto a plane may be detected by means of a color, or a black and white picture. A filter system

*TESAK is an Italian company, based in Florence, which is active in the design and construction of advanced information systems, particularly in the area of graphics and image processing.

can be used to pass from one to the other, so that any analysis of a *color picture* may be cut down to the analysis of at least three black and white pictures. Thus, the topic may be dealt with by referring only to a black and white picture.

A black and white picture (Ip), with regard to its analysis by means of *picture processing*, may be considered as a continuous two-dimensional functions $f(x,y)$ of two planar variables x and y which are capable of providing for each pair of coordinates, a representative value of *luminance (l)*

Thus, any operation on a picture (lp) corresponds to a mathematical operation on the function $f(x,y)$, representative of a given distribution of *luminance* in a plane domain fully connected without single points, as pointed out by Andrews et al., (1972).

The function involved is actually a discrete function of x and y, and may assume a finite number of values in relation to a given scale of greys; indeed, even though the starting picture (Ip) can be defined by a continuous function, it is stored by the analysis device as if it were matrix of elements. Each element (pixel) will contribute through the value of grey associated with it to the forming of a digitalized image (Id). It will be more similar to the picture (Ip).

The smaller the distortions produced by the conversion system, the greater the number of pixels used per surface unit by the analyzing system for storage.

The conversion process from (Ip) to (Id) can be schematized as follows:

– sampling of function $f(x,y)$ to obtain from it a square mesh sample matrix;

– quantification of space samples;

– coding in a binary system.

The individual space samples (*pixels*) are represented by an integer value, g, which identifies the luminance value on an appropriate scale.

Since during a conversion process of an image (Ip), a certain number of errors are produced linked to the intrinsic characteristics of the devices themselves, the picture (Ip), described by the above-mentioned function $f(x,y)$, and stored as (Id), may be concisely described through:

$$f^*(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(u,v,x,y) \cdot f(x,y)\, du\, dv \quad (1)$$

where:

u, v are the space coordinates of the real image;

$h(u,v,x,y)$ is a function which keeps account of the distortions produced by the conversion device during the generation of the image (Id).

In the case of an ore consisting of a number of mineral species (and obviously, in the case of products obtained from the treatment of the same ore) the function $f(x,y)$ is not, as a rule, a continuous function of the whole image, whereas it can vary steadily inside the portions of the image which represent given mineral species. This means that for an image which is given in black and white, by means of suitable preliminary color filterings and if appropriately digitalized, it is possible to associate each i-th mineral species with $F_i(g)$ where $0 \leqslant GL_i < g < GL_{i+K}+K \leqslant G^*$ where

$F_i(g)$ = the function representing the distribution of the grey levels in the i-th mineral species

GL_i, GL_{i+K} = respectively the lower or the upper extreme of the range of greys for which the function is defined;

K = the amplitude of the range.

The characteristic distribution function $F_i(g)$ of an i^{th} mineral species can be obtained by referring to standards (and in this case also the procedure is automated), or by having the operator identify the minerals. The second procedure was adopted owing to the difficulties in defining a distribution function $f_i(g)$ independent from the origin of the mineral and from any possible defects of inclusions in the crystalline lattice, which may produce differential behaviors during the acquisition phase of the image, even though the basic chemical structure may be the same.

In general, by assuming the hypothesis that an image can be considered as a discrete two-dimensional function $\pi(x,y)$, of the discrete variable x and y, if the following

$$\pi_i^1(x,y),\ \pi_i^2(x,y),\ \ldots\ \pi_i^j(x,y),\ \ldots,\ \pi_i^r(x,y) \quad (2)$$

are the distribution functions of the levels of grey relative to each of the investigations ($j=1 \ldots r$) carried out in the different sampling zones consisting of the i^{th} mineralogical species, it is possible, by means of appropriate calculations algorithms, to obtain – starting from the r functions – the function $F_i(g)$ defined previously, which will then be taken as being representative of the i^{th} species.

For the automatic evaluation of the spectral interval of the levels of grey characteristics of a given mineral species, of their frequency and recurrence of the spectral interval, with respect to the surface of the image, a processing program was developed and it is illustrated by the scheme in Fig. la.

The result of the elaboration in the case of an ore consisting of several minerals is shown in Figs. lb and lc.

Geometric characterization of the mineral individuals in an ore and of the particles produced by comminution

The evaluations connected with the geometric characterization of the mineral individuals in an ore and of the particles resulting from comminution entail a *simplification* of the image, so that each mineral species (of an ore) and each association of minerals (of a particle) can be associated with a given value GL_f, while all the other mineral species (of an ore), namely the *background* (in the case of particles), must be associated with a value $GL_f \neq GL_m$.

Thus, by assuming that it is possible to consider the image as a discrete two dimensional function $\pi(x,y)$, of the variables x and y (they, too, being discrete), a simplification can be made by applying relations of the following type:

$$\forall (x,y)\ GL_1 < \pi(x,y) < GL_i \rightarrow \pi(x,y) = GL_f = constant \quad (3)$$

$$\forall (x,y)\ GL_i < \pi(x,y) < GL_j \rightarrow \pi(x,y) = GL_m = constant \quad (4)$$

$$\forall (x,y)\ GL_j < \pi(x,y) < GL_G \rightarrow \pi(x,y) = GL_f = constant \quad (5)$$

where

x,y are discrete spatial variables which identify a *pixel;*

*In the hardware used for this study G=255. This means that a memorized image (Id) can be considered as a matrix ($m \times n$) whose elements may assume any value between 0 (black) and 255 (white).

(1a)

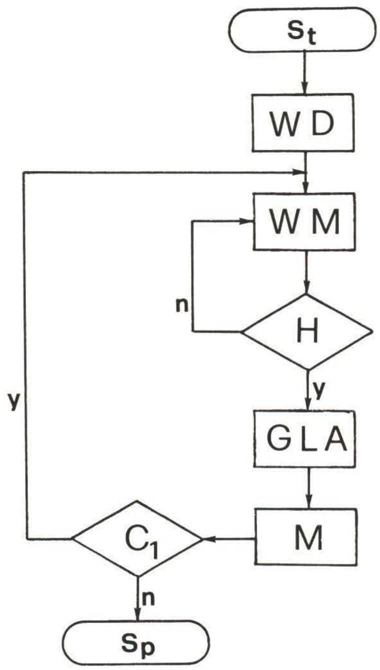


Fig. 1a. Block scheme illustrating the analyses procedures used in the program for determining the spectra of greys which are characteristic for each mineral species present in the image.

St = START; Sp = STOP; y = YES; n = NO

WD = Choice of the size of the window within which the levels of grey are to be analysed

WM = Interactive management of the window previously defined through the keyboard. The window may vary in size and be positioned in any zone of the image

H = Option for continuing with the phase described in WM or for going on to the next phase (GLA)

GLA = Analysis, calculation and visualization (Fig. 1 b and 1c) of the distribution of the levels of grey present inside the window described above

M = Storage in the files of the values of grey levels determined in phase GLA

C_1 = Option for either continuing with the analysis after having defined a new window or end of the program.

Fig. 1(b) and (c): Procedure for determining the distribution of the levels of grey relative to the portion of image delimited by a *window* whose shape, size and position can be defined by the user.

An analysis carried out on different zones of a representative image of the same i^{th} mineralogical species has led to the definition of a *"characteristic spectrum"* of the distribution of the levels of grey of that species $f_i(g)$.

A histogram is represented on the left side of the image relative to the distribution of the levels of grey $|f_i(g)|$ resulting from the analysis carried out inside the window which can be seen in the photograph.

In the right-hand side of the image a histogram is shown which is the outcome of the summation of all the distribution histograms of the levels of grey relative to the analyses carried out on the mineral species present:

$$F(g) = \sum_{i=1}^{n} f_i(g)$$

1b – On the left: the spectrum of characteristic grey levels $f_n(g)$ of the mineral species bounded by the portion of the image defined by the last sampling *window* (upper left).

On the right: the spectrum of the grey levels resulting from all the sampling operations:

$$F(g) = \sum_{i=1}^{n} f_i(g)$$

1c – On the left: the spectrum of grey levels relating to the last of two sampling operations:

On the right: the spectrum of grey levels resulting from the *two* sampling operations:

$$F(g) = \sum_{i=1}^{2} f_i(g)$$

The conversion system from picture to image has been clearly very highly selective.

$\pi(x,y)$ is the already mentioned discrete function which synthetically represents the whole image;

GL_i *(Grey Level)* represents the *i*-th value ($0 \leqslant i \leqslant G$) of the level of grey that the function $\pi(x,y)$ provides for the pixel involved;

GL_f and GL_m are the values of grey that the function $\pi(x,y)$ assumes depending on the luminance value that the very function $\pi(x,y)$ provides for each pixel of the image, where $0 \leqslant f \leqslant m \leqslant G$ or $0 \leqslant m \leqslant f \leqslant G$.

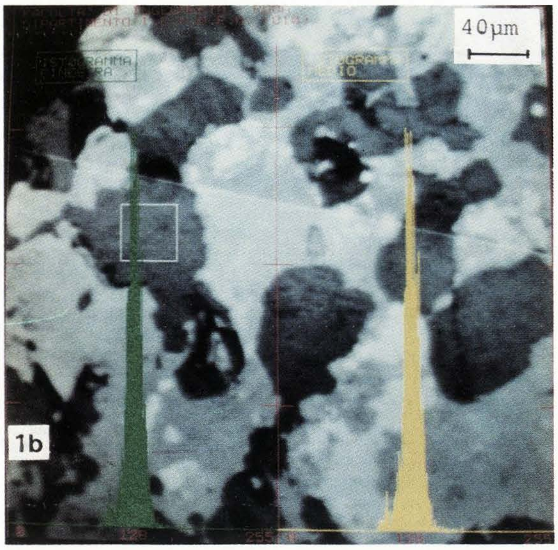
Once a mineral individual has been identified with respect to the ore, or a particle (consisting of several minerals) has been discriminated with respect to the background, the procedure shown in Fig. 2 was adopted to obtain *equivalent sizes,* characteristic of each mineral individual or particle.

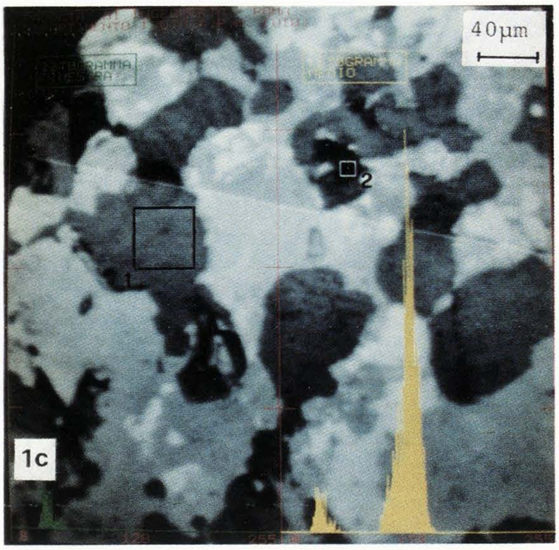
On the basis of this procedure, the surface corresponding to the image of a particle or an individual mineral is replaced with the surface of a regular geometric figure, carefully selected and characterized by the same barycentre and by the same principal axes of inertia.

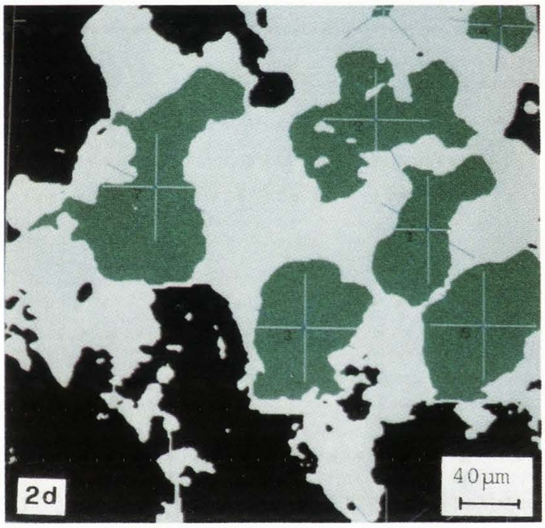
The program provides two vectors resulting from the composition (direction and module) of the axes of inertia which characterize all the particles or mineral individuals present in the image.

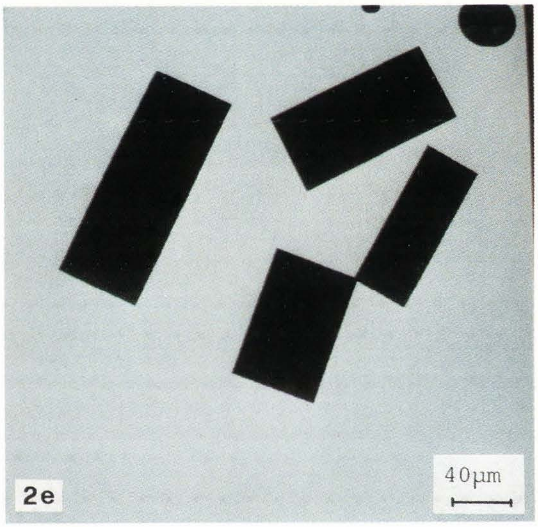
By using this type of elaboration, the orientations of the mineral individuals in the rock can be quantified, thus providing parameters which are useful in evaluating texture. The ratio between the modules of the two vectors and the angle which they form represent a simplified quantitative evaluation of the shape of the particles in the products.

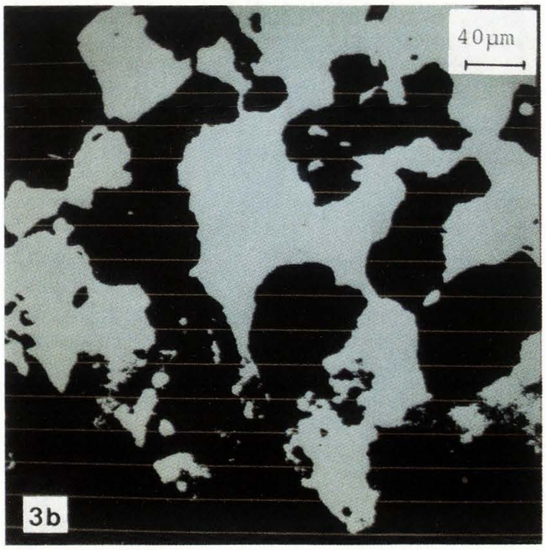
This evaluation, even if approximate, is however, particularly useful in identifying the possibilities of classifying the particles and the preceding procedure can be used to evaluate a certain number of factors affecting screening (size, average size and shape) (Kelly and Spottiswood, 1982).

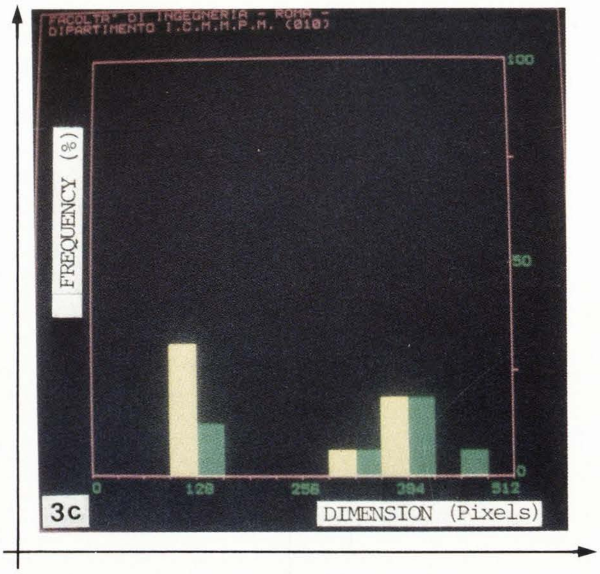












For legends of Figures 2 and 3 see pages 122 and 123, respectively.

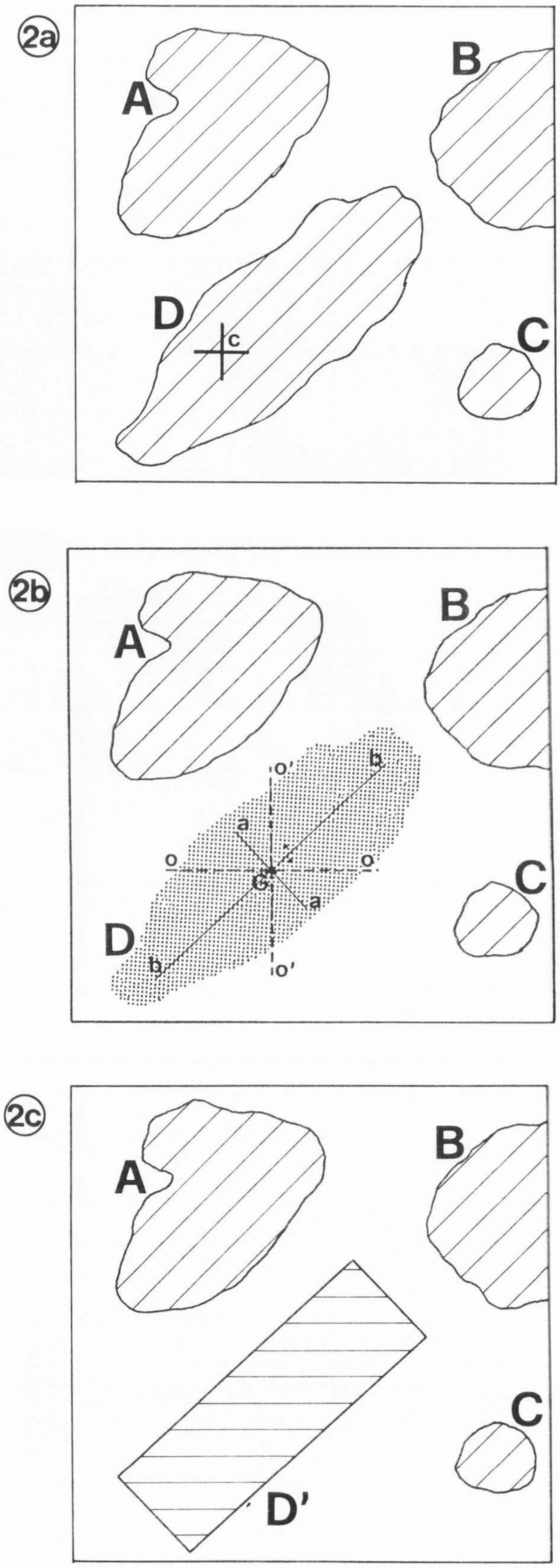


Fig. 2. **Sequence illustrating the procedure adopted to obtain *equivalent sizes* of a particle. The program is designed to process data both automatically and interactively. The interactive procedure is realized by the following sequence of operations:**

(a) Selection of the particle to which the procedure to obtain the *equivalent sizes* is to be applied; in this case particle *D*, identified by cursor *c* is selected.

(b) The program calculates the centre of gravity *(G)*, the perimeter, the area of the particle *D* (previously identified), the central axes of inertia (a-a, b-b) and the module of the principal moments of inertia.

(c) Replacing the particle *D* with the particle *D′* having equivalent sizes (i.e. same surface areas, same direction of principal inertia axes and the same module of principal moments of inertia as particle *D*.

(d) The operation described in (b) as it appears on the videographic screen of the terminal after the program has been performed (images relating to the particles of a calamine ore obtained by means of a microprobe).

(e) The operation described in (c) as it appears on the videographic screen of the terminal (images relative to the case presented in (d)).

Evaluation of the size distribution of mineral individuals or particles

Image processing of a particle or of a mineral individual is possible by associating each particle or mineral individual with a maximum equivalent size. The ensuing statistical elaboration of the data obtained identifies an *equivalent size distribution.*

This procedure is an alternative to the procedure usually used in quantitative mineralogy for analyzing microscopic preparations, based on the measurement of intercepts according to equidistanced alignments (Figs. 3a and 3b).

The data-processing scheme and the results obtained from the examination of size distribution of the mineral individuals in a rock are schematically presented in Fig. 3c.

By means of the procedure schematically described in Fig. 3a, it is possible to determine the frequency of the recurrence of a mineral species along equidistant alignments in the image, which is different from the grain size distribution of the mineral individuals or of the particles present in an ore. The results are the less reliable, the larger the scanning interval and, in any case, they are not representative of the sizes of the examined particles.

On the contrary, with the new methodology presented here, it is possible to define a truly representative function of the true size distribution of the mineral individuals considered. It is also possible to define the size distribution of the particles which can be obtained from the comminution process by means of an image processing procedure which attributed to the *background* (for example, the matrix) a given level of grey, and another level of grey is to be attributed to the particles independently of their mineralogical composition.

Further progress in analyzing particle sizes was achieved by developing a program which allows to ascribe parameters to the boundaries of each single particle following the procedure described in Fig. 4. An example of application of the above procedure is provided in Fig. 5 with reference to an isolated particle.

(3a)

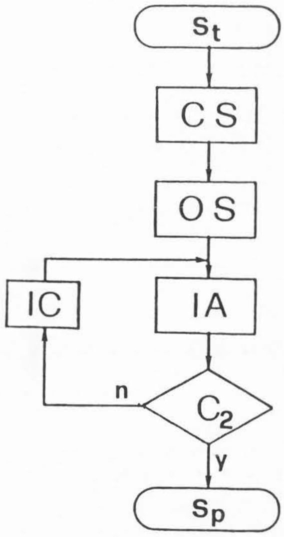


Fig. 3a: Block scheme illustrating the analysis procedure followed by the program for determining the value of the intercepts along predetermined alignments with respect to the boundary of a given mineral species. The program conducts the analysis along directions parallel to one of the two sides of the screen (vertically and horizontally).

The algorithm operates on *reduced images*, i.e. images in which the mineralogical species of interest was *emphasized* (black in the photograph) by means of appropriate calculation algorithms, as opposed to the other mineral species (white in the photograph) (Cappellini et al., 1978; Murray, 1982).

St = START; Sp = STOP; y = YES; n = NO

CS = Selection of the length of step *p* of the scans used to determine the value of the intercept of the mineral species of interest

OS = Assigning of a value to the initial scanning coordinate

IA = Analysis of the intercepts along a pre-established alignment

Being (x_0, y) the initial scanning coordinate values, in the case of analyses along alignments parallel to the *y* axis, and being *p* the above-mentioned scan step, the following may be obtained:

1st scan $x = x_0 = constant$

$0 \leqslant y \leqslant 511$

2nd scan $x = x_0 + p = constant$

$0 \leqslant y \leqslant 511$

n-th scan $x = x_0 + np = constant$

$0 \leqslant y \leqslant 511$

provided that the following expression holds:

$x_n = (x_0 + n_p) \leqslant 511$

The analysis ends when the following condition is fulfilled:

$x_{n+1} = | x_0 + (n + 1)_p | \leqslant 511$

C_2 = Option for:

continuing the analysis along another alignment (the *(i+1)*th), provided that the initial scanning coordinate relating to the *i*-th alignment of the step *p* has increased

exit from the program

IC = Increment of the initial scanning coordinate on the basis of the pre-established value of the step *p*

Fig. 3b: Operation for determining and calculating the value of the intercepts identified along pre-determined alignments. Once the scanning direction and step have been set, the analysis proceeds automatically. By means of this program the occurrence of a given mineralogical species along an alignment can be evaluated. For an investigation step equal to one the area percentage of each species present in the sample being analysed is obtained. Selecting an investigation step other than one is decided on the basis of the accuracy required. In this example where the ore is Calamine (black) the step is equal to 32 pixels.

Fig. 3c: On the x axis:

–occurrence expressed in pixels of the black species (Calamine) along the selected alignments

On the y axis:

–frequency of the values of occurrence measured along the selected alignments: the alternation of colors (yellow and green) makes the reading of the results easier.

By means of the previous procedure, the boundary, which in the case of a *plane particle* is represented by a curve, may be described with polar coordinates $(\rho(\theta)$, suitably choosing a point inside the particle as pole (Scharcz and Shane, 1969).

In the case of a convex figure, the boundary can be easily identified: indeed, the expression of Fourier's series of a particle's boundary can be expressed as:

$$p(\theta) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i \, cos_i\theta + \sum_{i=1}^{\infty} b_i \, sin_i\theta \tag{6}$$

where the constants a_0, a_i, and b_i are respectively the following expressions:

$$a_o = \frac{1}{\pi} \int_{-\pi}^{+\pi} \rho(\theta) \, d\theta \tag{7}$$

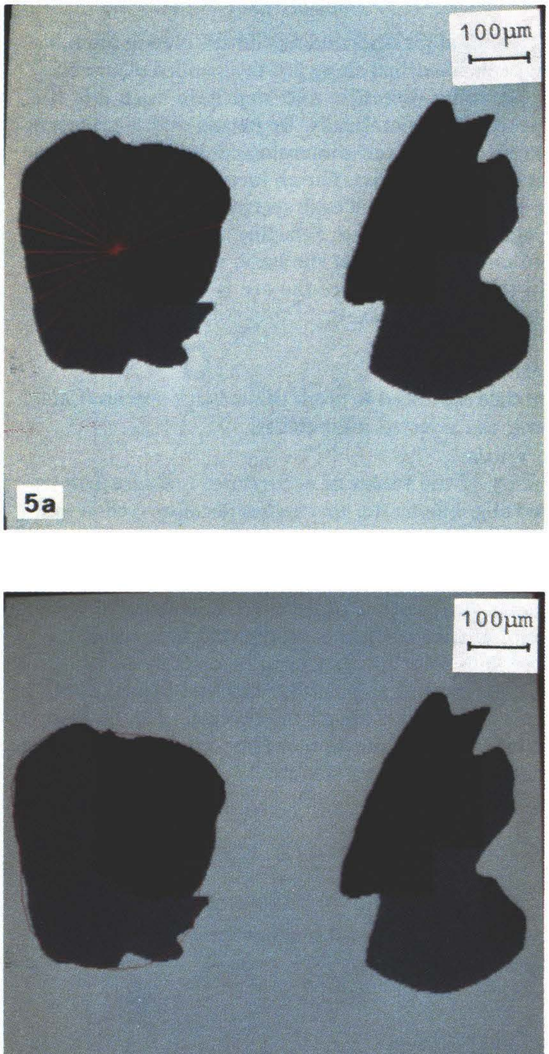
$$a_i = \frac{1}{\pi} \int_{-\pi}^{+\pi} \rho(\theta) \, cos_i\theta \, d\theta \tag{8}$$

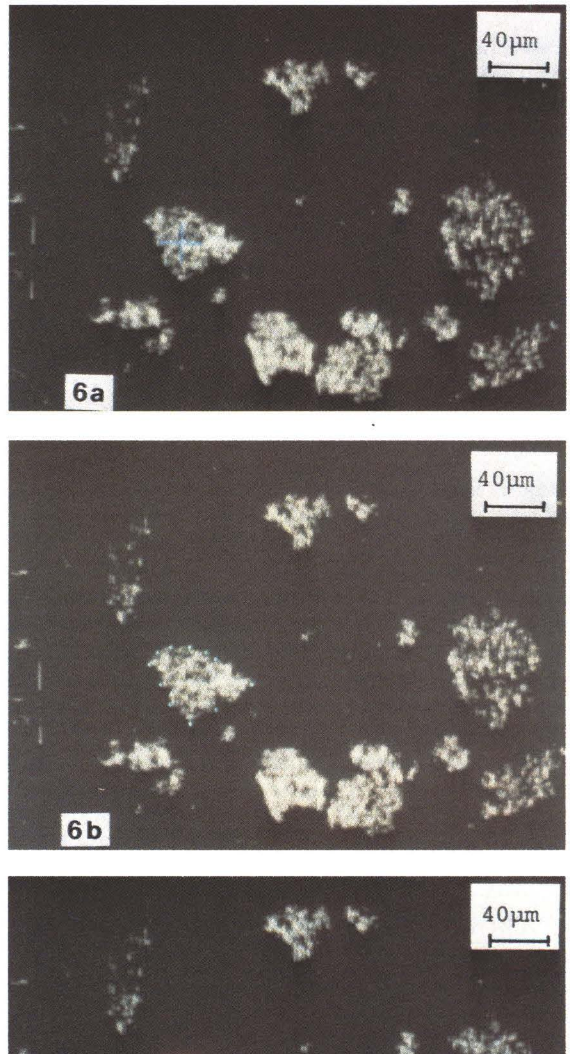
$$b_i = \frac{1}{\pi} \int_{-\pi}^{+\rho} \rho(\theta) \, sin_i\theta \, d\theta \tag{9}$$

The numerical method used to determine those coefficients requires that ρ can be sampled for constant values of θ. If N is the total number of sampled points, θ will turn out to be equal to $2\pi/N$.

Having selected a sampling interval, having analyzed the boundaries of a certain number (n) of particles and having obtained for each the values of a_{ik} and $b_{ik}(k=1, \ldots, m)$ of the coefficients of Fourier's serial development for each boundary, it is therefore possible to make statistical processing of the same parameters (e.g. by working out a_i and b_i and a_i and b_i) so as to provide an evaluation of the characteristic shapes of the system of particles.

For routine assessments of shape parameters of a single particle or of a system of particles, the *characterization* of the time of analysis is extremely important; this factor depends on the sampling density and on the number of elements examined, as a function of the precision and sensitivity of the analysis and of the reliability of the samples. For example, a complete analysis





For legends of Figures 5 and 6 see pages 126 and 127, respectively.

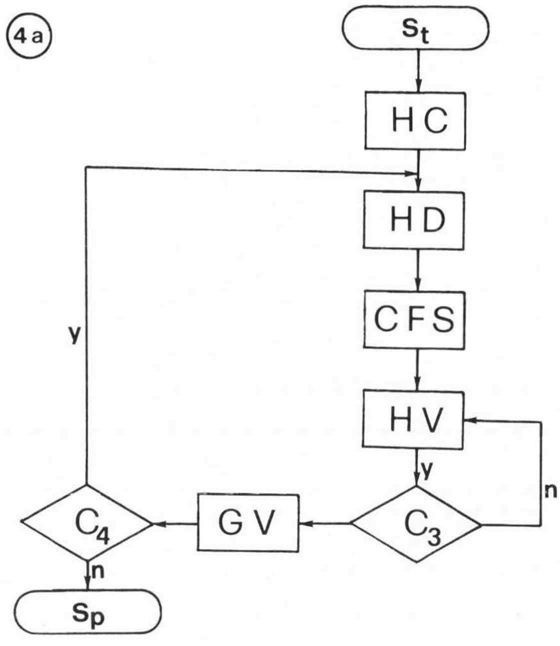


Fig. 4a: Block scheme illustrating the calculation procedure followed in the program for analysing the boundary of a particle according to a Fourier series development of the boundary (Scharcz and Shane, 1969)

St = START; Sp = STOP; y = YES; n = NO

HC = Selection of the number of harmonics by developing the boundary of the particle with a Fourier series

HD = Procedure for identifying the particle to be analysed and subsequent sampling of the boundary

CFS = Calculation of the coefficients of the series

HV = Visualization of the boundary of the particle on the videograph as a result of the described processing procedure

C_3 = Option for choosing the number of harmonics to be used in representing the boundary or for moving into phase GV

GV = Visualization, on the videograph, of the calculated boundary of the particle analysed according to the number of imposed harmonics (as in HC),

Visualization of the modules of the sampling vectors

Visualization of the value of the selected harmonics

C_4 = Option for continuing the analysis by taking into account another particle, or end of the program.

An example of the application of the above procedure is provided in Fig. 5 with reference to an isolated particle.

Notice that the first harmonic provides an indication of roundness, the second provides an indication of elongation, the subsequent ones give further indications relating to shape: but the modules of the first 8 harmonics are themselves sufficient for discriminating the shape of the particles more frequently analysed.

of a particle whose boundary sampling was carried out at every 9°, requires that the development of Fourier's series deveopment should stop at the 20th harmonic (in this case, considering the equipment used for the tests, the analysis time is about 20 seconds).

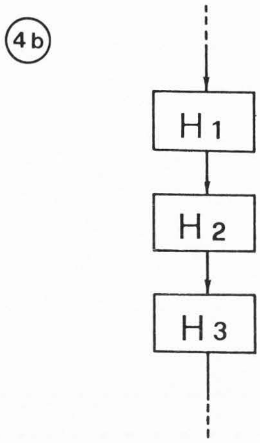


Fig. 4b: Block diagram giving a more detailed illustration of the procedure described in Fig. 4a (phase HD).

H_1 – Identification of the particle by means of a cursor driven by the user

H_2 – Calculation of the geometric parameters (perimeter, area, center of gravity) of the previously identified particle

H_3 – Sampling, starting from the center of gravity, of the boundary of the particle with a sampling step 8 which proves to be equal to $2\pi/N$, where N is the selected number of harmonics (as in HC, Fig. 4a).

In case an early classification of the particle was required, by adopting parameters referring to simplified shapes (triangles, circles, rectangles), it is sufficient to analyze the first 8 harmonics.

An analysis of the first harmonics allows to evaluate the *average size* of the particles.

An analysis of the amplitude of the second harmonic is already sufficient to classify the particle on the basis of its lengthening.

Finally, by associating the procedure shown in Fig. 2 (whereby the barycentre of a particle's image can be identified) with the above-described procedure, the *maximum size* for each particle can be determined. This size can be obtained as value of the maximum intercept calculated among the boundaries of the particle crossed by the straight lines passing through the barycentre.

The procedure described in Fig. 5 can be used to infer the size distribution of *average size* and of the *maximum equivalent size* of the particles under examination.

Notice that the first harmonic provides an indication of roundness, the second provides an indication of elongation, the subsequent ones give further indications relating to shape: but the modules of the first 8 harmonics are in themselves sufficient for discriminating the shape of the particles more frequently analyzed.

Evaluation of the degree of liberation of comminution products from ores

The evaluation of the degree of liberation of each of the mineral species present in the ore particles has been studied with reference to the information which can be inferred from an image of the particles. In this stage of the study, therefore, first of all, a *degree of mineralization referred to the surfaces* was considered, namely the ratio between the surface of the part

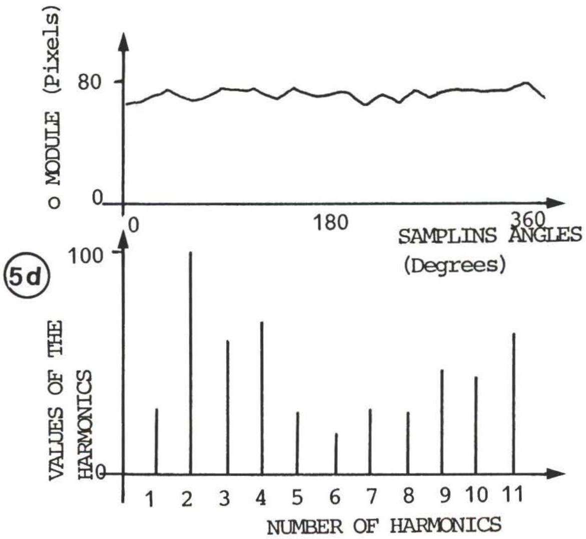


Fig. 5: Sequence illustrating the procedure for analyzing the boundary of a particle, on the left, by developing it in a Fourier series.

The program operates on images for which the grey levels have faded to white *(gL = 255)* for the matrix or for minerals other than the one being examined.

(a) Visualization on the videographic terminal of the sampling phase of the boundary points of the particle taken as a basis for the following determinations of the characteristic coefficients of the Fourier series

(b) Visualization of the boundary of the particle considering four harmonics of the Fourier series

(c) Top diagram:

Boundary of a particle as it results from the sampling phase (a). The angle values of the sampling angle are given on the x axis and the sampling vector module is given on the y axis.

Bottom diagram:

Values of the selected harmonics. The number of harmonics is given on the x axis, their respective values are given on the y axis

(d) Superimposition of the image of the particle of the two graphs shown above, relative to the shape characteristics of the particle on the left as they appear on the videographic screen of the terminal

of the image of a particle consisting of a single species and the surface of the image of the whole particle.

A *degree of liberation referred to the surface* has been defined, namely, the ratio between the surface of the image of all the particles consisting of single species and the surface of the image of the given species in all the particles.

In order to reach to this evaluation, first of all, the boundaries of the particles were identified (by using the processing technique described formerly, thus identifying the surfaces relating to the images. The intervals of the levels of grey characterizing the different mineral species were then recognized and a reference level of grey was attributed to the background (i.e., matrix housing the particles).

The surface of the image of a given mineral recurring in all the particles was calculated on the *reduced surface* obtained after the fading out to white of all the levels of grey external to the characteristic interval of the examined mineral.

Thus, with reference to each mineral species, an automatic calculation procedure of the parameters already defined was developed: mineralization degree of each particle and liberation degree of the examined system of particles.

Ongoing studies tend to expand the calculation results to the case of a tridimensional examination of the particles by introducing procedures for evaluating coefficients of shape, not only of the particles but also of the minerals inside the particles.

Conclusion

The application of image analysis techniques to the study of the textural and structural characteristics of ores in order to design appropriate treatment processes and to control such production processes, provides a number of advantages in determining the parameters of interest, provided the right type of equipment is available to develop algorithms and to implement the specifically designed calculation programs.

This present study has permitted development of a series of programs which, applied to the images of the samples of an ore or images of the products of a comminution process, allow:

– to characterize precisely a certain species on the basis of the spectrum of grey associated with it;

– to associate a series of parameters (perimeter, area, center of gravity, principal axes of inertia) with each particle of an ore so as to provide the means for attaining a thorough geometrical characterization;

– to characterize the sample being analyzed from a textural and structural point of view;

– to provide an evaluation of the degree of liberation, referring to the surface, of the mineral species associated in the ore.

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Discussion with Reviewers

Reviewer I: Please elaborate on the processing of x-ray maps.
Authors: In this preliminary stage of our study, the image processing technique was applied to X-Ray Maps (XRM): photos taken during WDS (Wavelength Dispersive Spectrometres) analyses with a JEOL JXA-50A MICROPROBE were examined. In these analyses, the secondary radiation of the element of interest was transferred from the rate-meter to a photographic plate as a result of the camera's being assembled in parallel with the integrating circuit (Fig. 6a).

The plate exposure index was selected on the basis of the number of photons which could be detected within a time unit. A constant scanning rate, attained through baffle plates, resulted in the impression on the photographic plate of a trace of the signal which had an intensity proportional to that of the signal itself.

Subsequently, an algorithm was applied to analyze the signal density distribution (the trace of the X photons on the photo).

Fig. 6b shows the image portion on which the photon trace density is comprised within a certain interval. Following the definition of the particle outline (Fig. 6c), this is enhanced by processing the image with conventional colors (Fig. 6d).

As a source of information for an investigation, a photo has great limitations due to the "filtration" effect which conceals part of the relevant information. Therefore, a direct signal transfer, through a photomultiplier, from the microprobe to the computer mass storage would undoubtedly result in improved signal processing accuracy.

Progress is being made in working out computer programs to carry out direct XRM image processing leading to:

– visual area display of the various signal intensities in different conventional colors;

– quantitative assessment of the distribution of the various elements on the image portion (XRM) analyzed, on the basis of the signal intensity in each pixel.

J.S. Walker: How many different minerals having different $F_i(g)$ can be resolved in a frame? What is done if the functions overlap, and how much overlap is permissible?
Authors: The analysis procedure shown on Fig. 1a was used to resolve up to a maximum of eight mineral species having different $F_i(g)$ (with $i = 1, 2, \ldots 8$) without overlap. This limitation to 8 different $F_i(g)$ is due to the limitations of the hardware used in our investigation, that is, to an adequate storage capacity for the histograms generated through a progressive gating (Figs. 1b and 1c).

With a view to overcoming the problems arising from function overlap, our image analysis laboratory is carrying out trials aimed at establishing the following:

– procedures of removal of any overlap of the characterizing spectra of two or more mineral species;

– the maximum percentage of overlap between two functions which would permit to resolve the two spectra through a subsequent image processing.

Overlap suppression is being attempted through two different approaches:

– image filtration during its stage acquisition (image transfer and storage);

– numerical filtration of the digitalized images previously acquired.

The first approach required a fairly simple procedure as compared with the second, however, the latter method, though proving rather efficient with small amounts of overlap, failed to yield satisfactory results when overlap was much greater, at least according to the calculation algorithms worked out so far.

It is worthwhile pointing out that the permissible overlap is related to the image resolution and the fidelity of the digitalized image discretization.

G. Remond: Have the authors performed quantative image analyses based on color (line) discrimination rather than using luminosity for a particular incident or analyzed wavelength?
Authors: So far the quantitative image analysis was based on the discrimination of the luminosity distribution in each pixel, that is, within one pixel matrix.

However, attempts have been made to deal with the quantitative image assessment by starting from the distribution of the wavelengths (color spectra) which are typical of a pixel or of a pixel system.

The difficulties encountered in attempting to work out a method of this type are virtually of a technological nature, in that most equivalent units which, by starting from a TV signal generated from the camera, convert the analogic signal into a digital one, are endowed with conversion systems of a maximum of 8 bits, each image being characterized therefore by a maximum of $2^8 = 256$ luminosity levels.

Color takes require the use of conversion systems provided with a higher number of bits, namely 12 ($12^{12} = 4096$ luminosity levels), or 16 ($2^{16} = 65356$ luminosity levels), which entails,

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Fig. 6: A sequence of images as it is visualized on the terminal videograph screen during the application of the procedure of evaluation of the signal density distribution (photon X trace on the photograph) in an XRM analysis of a Calamine ore sample (Fig. 6a).

The element Zn mapping is shown, the radiation used being $K\alpha_1$ and the crystal being the LIF 200.

The program includes a preliminary calculation of the surface density of the photon X traces, and considers as a reference area a pixel matrix whose size and position can be established by the user. The density value of the signals falling within this area is assumed as representative of the sample's zones containing the element to be investigated.

Subsequently, starting from the zone on which the preliminary investigation has been accomplished, the algorithm carries out a whole set of density determinations through a totally automatic procedure in various directions ($0° \leqslant \alpha_i \leqslant 360°$) established previously by the user.

The program proceeds with the investigation in each direction until the difference between the density value detected and the reference value is within the allowance established by the user.

At this stage, the investigation along the *i-th* direction stops, the last position of the analysis matrix central element is stored and the investigation proceeds along the $(i + 1)^{th}$ direction. Processing will be completed when the whole round angle has been covered (Fig. 6b).

Figures 6c and 6d show with conventional colors the profile and surface of the particles identified.

Figures 5 and 6 are on color plate page 124.

in addition to the use of greater storage capacity image analysis equipment, the aid of acquisition equipment (TV camera) of a resolution degree of the color levels on the image such as to be appreciated by a more sophisticated conversion system (with 12 or 16 bits).

Considering that the equipment used to carry out the analysis illustrated in our note was provided with twelve image storage planes its digitalized image processing capacity could not exceed 12 bits.

To overcome this limitation, our image analysis laboratory is in the process of investigating into two different methods of color image acquisition and processing:

–"direct" acquisition of color images through a 12 bit conversion system (images with 4096 luminosity levels), though presenting the drawbacks inherent in this method as mentioned previously;

– acquisition, through the utilization of 3 different screens (cyan, magenta and blue, for example), of 3 images (I_{ph}, h=s, . . . , 3) of the same image (Ip).

Both procedures would provide images containing a greater wealth of information to work with; the first permitting the resolution of luminosity values comprised between 0 and 4025 in each pixel or pixel set, the second would provide a triad of values, each related to a wavelength interval (due to the color screening) and characterized by the integral of the luminosity of that interval, in each pixel or pixel system.

G. Remond: What is the time-data acquisition and data processing for a complete determination of the texture or the degree of liberation in the case of a 1000 grains ore specimen?

Authors: The time required for a complete determination of the texture or degree of liberation in the case of multiple grains, depends virtually on 2 factors:

– the number of images to be analyzed to investigate a number of grains sufficiently high to make the analysis significant;

– the number and type of parameters (texture and degree of liberation) related to each grain.

The number of images to be analyzed determines the total duration of the determinations, in that the images are acquired sequentially and, to be processed, they must first be in the memory of the host computer (DIGITAL PDP 11/23) and from there, they must be transferred sequentially to the image memory (TESAK VDC 501/A) and then processed.

The time required for the acquisition and transfer of each image from the host memory to the controller is about 2 seconds. And the time the algorithm takes to carry out a full analysis (shape and liberation degree) of a particle is about 5 seconds.

Assuming that 10 pictures are to be analyzed, each consisting of about 100 grains (for a total of 1000 grains), the processing time required when using the current hardware and software configurations will be slightly less than 90 minutes.

Processing time can be shortened considerably (by one order of magnitude) as follows:

a) by restricting the determination to the assessment of one parameter only (the liberation degree, for instance);

b) by ruling out all graphic processing (drawing outlines, centre of gravity, major inertia axes, discrimination enhancement of the various species contained in the specimen through conventional coloring) which is carried out of each particle as its analysis is being completed;

c) by utilizing programs in PDP ASSEMBLER language instead of the PDP FORTRAN IV language programs which are being currently used.

With the restrictions under a) and b), processing time can be shortened by an order of magnitude of 50%.