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**Abstract**

Some argue that intuitive judgements about mathematical statements lead us to believe in Mathematical Platonism. But the mathematical objects of platonistic theories are supposed to be non-spatiotemporal and disconnected from the world, specifically they are acausal. This has issues, if mathematical objects are disconnected from the world, then it seems like they make no difference to the world. The world would be the way it is even if mathematical objects did not exist. Additionally, people like Benacerraf (1973) argue that given a causal theory of knowledge, and given their acausal nature, we could never know about mathematical objects and so should not posit them. Elsewhere in the literature people offer indispensability arguments *for* mathematical objects (e.g. Baker, 2009). Our best scientific theories seem required to invoke explanations that quantify over mathematical objects, so we should believe in them in just the same way as we believe in the unobservable objects of science. We can then say we know that mathematical objects exist through an inference to the best explanation, based on our best scientific theories. But this does not tell us what mathematical objects are *like*, importantly it still does not answer "makes no difference" arguments. We may be forced to believe in them but this does not tell us whether or not they actually *do* anything. I want to discuss one way in which mathematical objects might *do* something. I think there is a useful and informative way we can talk about mathematical objects as being causal. I do this by discussing a case of mathematical constraint, as proposed by Marc Lange (2017). I will elaborate on the notion of mathematical constraint and talk about the constraint relation more generally. I then move on to discuss structural equation models and how they can be used to represent causal relationships. This fits particularly well with the Interventionist view of causation, which I will describe, and how it can be used as a test to determine which relationships are causal. I think mathematical constraint passes this test. With this framework in place, I will discuss a specific structural equation model which represents a constraint relation. This particular constraint relationship is also straightforwardly causal. I think the structure of this relationship naturally maps on to the structure of an archetypal example of mathematical constraint. Not only do they share a structure, but both relationships behave in the same ways under interventionist treatments. This should grant us reason to say that mathematical constraint is causal. For those interested in the epistemology of mathematics, this can allow a sidestepping of Benacerraf-style objections; it *would* be mysterious how we know about acausal mathematical objects, but given mathematical

objects are causal we can explain our knowledge. For those sympathetic to, or concerned about, makes no difference arguments, we again have a response, mathematical objects are causal so it *would* make a difference to the world if they did not exist.

**Keywords: Interventionism, Counterfactual, Structural Equation Model, Causation, Constraint**

## 1 Introduction

Philosophy of mathematics widely accepts that mathematical objects, if they were to exist, would be acausal. Indeed, this acausal nature presents the strongest challenge to mathematical Platonists, as it makes it difficult to explain how we can have knowledge of mathematical objects. Against this orthodoxy, this essay discusses a sense in which mathematical objects could be causal, this is intended as a move to not only tell us what kind of things mathematical objects are, bu to explain how it is that they can make a difference and we can gain knowledge of them. This is a controversial take on mathematical objects, but I think there is precedent in the literature to make this claim. To try and establish this claim, I will briefly describe the relation of mathematical constraint as proposed by Marc Lange (2017), and what it is supposed to *do*. To try and establish that mathematical constraint can be a causal relationship, I will aim to demonstrate that it fits with a typical causal pattern in a structural equation model, and that under an interventionist interpretation, we can view mathematical constraint as behaving in the same way as a causal relationship. This paper will build up a notion of causation in the tradition of interventionism and structural equation models. Having established this notion, it will then be applied to a case in mathematics to show that we can view the mathematical constraint relationship as a causal relationship. I will then go on to discuss, and initially respond to, some issues which are likely to be raised in response to this view.

## 2 Motivations

Given how we talk about numbers and certain intuitions, mathematical objects seem to be non-spatiotemporal, acausal entities. Benacerraf and others have raised issues with this (Benacerraf 1973 & Liggins, 2010). Given their non spatiotemporal and acausal nature, how we could know about mathematical objects is inexplicable. Given that knowledge of them seems impossible, our theories which posit them are left with a serious problem. This argument need not take this exact form and instead of referring to knowledge it may refer to the ability to form justified beliefs about mathematical objects, or be granted warrant to refer to them, and sometimes it is not even put in causal terms. The essence of any form of this objection is that without a connection to mathematical objects, positing them is suspect in some sense. If this connection is not causal then it is mysterious as to what it exactly is. I want to respond by saying there is a connection between mathematical objects and us, and furthermore this connection is a causal one so is not mysterious or ad hoc. I do agree that mathematical objects seem to be non-spatiotemporal, but I do not think this bars them from being causal. It seems plausible to suggest that mathematical objects ground the truth of certain statements and facts in the world. I will argue that this grounding is a kind of causation, even though the mathematical objects are non-spatiotemporal; because they operate on the world via constraint. To use a classic example, we cannot divide 23 strawberries (of equal size) equally between 3 people (without them cutting them up) *because* 23 is not divisible by 3 (into whole natural numbers)[1]. The mathematical fact constrains the way the world can be, it restricts the possible actions Mother can take in dividing the strawberries (Lange 2017).[2] It seems reasonable to say that this is a grounding claim. In some sense, Mother cannot divide the strawberries thusly *in virtue of the fact that* 23 is indivisible by 3. I think this is quite a natural reading of constraint, those tempted by Jessica Wilson's (2014) discussion of grounding may wish to say that constraint is not identical to big-G grounding but is instead a kind of small-g grounding relation. Alternatively, those who follow Karen Bennett

[1] Of course, 23 is divisible by 3, it just does not yield a whole natural number. For the purposes of this example and simplicity we restrict ourselves to division into natural numbers. This is also the reason we imagine the strawberries cannot be cut up in this scenario. Perhaps the children are unusually picky eaters and will only eat strawberries if they are whole, but still demand an even distribution. Hereafter all these caveats will be assumed and "23 is not divisible by 3" will be referring purely to division into whole natural numbers.

[2] This is a simple and restricted example but I think it serves to show the principles at play here and demonstrate how one might come to the conclusion that mathematical objects could be causal. There are many other examples of mathematical constraint, and many more complicated ones, these will obviously require slightly different and more complex treatments but those treatments will still be in the spirit of the account proposed in this paper.

(2017) could say that constraint, like grounding, is amongst the building relations. Whichever way one may wish to read it, I think it is appropriate and correct to place constraint in this region. My own preference is towards a Bennett-like picture. Grounding and causation are two kinds of broadly construed dependence relations and constraint represents a small section where they cross over. But I think that as long as one accepts grounding in the first place, constraint can be read as neutral across interpretations, and one need not accept Bennett's picture of building. Returning to the topic of discussion, I argue that the grounding by constraint present in mathematical cases like that above is a kind of causation because it shares a structure with straightforward causal relations. In structural equation models (SEMs), the constrainers in constraining relationships occupy the same place as causes do in casual relationships, on interventionist treatments of causation, and so we should be willing to accept them as causal. I will discuss a basic SEM, before moving on to more complicated ones and then digging into a final example which I believe demonstrates a structure shared by straightforwardly causal constraint relationships and mathematical ones.

## 3 Structural Equation Models & Interventionism

SEMs are models intended to represent causal relationships in a clear and straightforward way, through statistical data they allow us to infer causal relationships. They are used to great effect in statistical modelling, in areas like physics and economics, and are seen as a very useful heuristic. They also have many applications in philosophy, in particular counterfactual reasoning and the analysis of causation. In a SEM, one assigns values to variables and dictates some rules on how the variables interact with each other, in order to model the relationship at play.[3] For the illustrative causal SEMs which follow in this paper, C and E will generally stand for cause and effect, respectively. In the simple case below of a rock being thrown at a window, we can see how SEMs look and work:

Variables

[3] A more thorough exploration of SEMs, their use and applications, and their relevance to the topic of causation in Philosophy can be found in Hitchcock (2018).

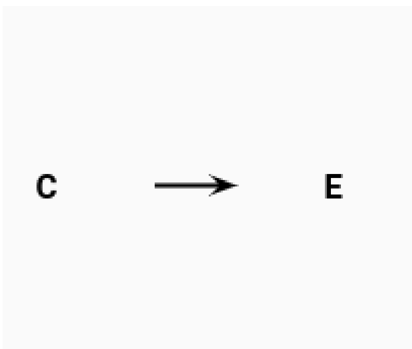
C: Whether Suzy throws the rock

E: Whether the window smashes

Structural Equations E=C

Assignment C=1; E=1

Graphical Representation




(Wilson 2018: 741)

*C* causes *E* if interventions changing the value of *C* affect the value of *E* in certain ways. This is the interventionist account of causation and is more clearly developed in a number of places, such as Woodward (2003). Specifically in relation to causation/grounding unity, this idea is picked up by Alastair Wilson (2018). Wilson proposes that the way interventionism is applied to show that a relationship is causal also works in cases of grounding. For reasons of theoretical unity amongst others he proposes that we treat grounding as simply a type of causation, a genus of the same species as it were (Wilson, 2018). I am quite sympathetic to this idea although I am not entirely sure it works in all cases. For my purposes, I merely want to show that one specific kind of grounding (by constraint) is identical to one specific type of causation (again, by constraint). I will nonetheless draw on Wilson's examples intended to show that grounding and causation are the same thing in order to build an understanding of interventionism and SEMs. Considering the model above under an interventionist treatment of causation, we can see how this relationship is causal. We determine if *C* causes *E* by performing interventions on *C*, for example preventing its occurrence, and see how this affects *E*. As Woodward states "We can explain what is for a relation between *X* and *Y* to be causal by appealing to facts about other causal relations involving *I*, *X* and *Y* and counterfactual claims involving the behaviour of *Y* under interventions on *X*" (Woodward 2003: 105).

Interventionist accounts of causation are a breed of counterfactual account which tend to state that a relationship between *X* and *Y* is causal based on counterfactuals like "if *X* had not happened, *Y* would not have happened". Specifically, interventionism specifies whether a relationship between *X* and *Y* is causal based upon what would happen if an intervention on *X* with respect to *Y* took place. An intervention is a technical notion which is generally defined by four criteria:

*I* is an intervention on *X* iff:

*I* causes *X*
*I* isolates *X* from previous causes of *X* so that the value of *X* is fixed by *I* alone.
Any path from *I* to some effect *Y* goes through *X*.
*I* is independent of any variable *Z* which causes *Y* and is on a path which does not go through *X*.
(Woodward 2003: 105)

If these criteria are met, then an action can appropriately be described as an intervention and the causal relationship can be tested. If an intervention on *X* with respect to *Y* causes *X* and proceeds to cause *Y* we can confirm that *X* causes *Y*.

We can see how this would work in the Suzy case. We see Suzy throw the rock, and we see the window break but she denies she caused the window to break. Let us imagine that Greg was also present next to the window at the time claiming to be casting a window breaking spell (Hitchcock, 2018). Now let us imagine that what actually happened is that Greg is in fact not casting a spell, and the window breaking is actually Suzy's fault. We can see how we might recreate the situation and perform interventions to determine this. We might hold fixed Suzy's throwing of the rock whilst varying Greg's casting of the window breaking spell, and vice versa. Soon we will start to see dependence patterns emerge between Suzy's throwing and the window smashing. If we are still uncertain, we could perform further interventions, varying the speed of

Suzy's throwing the rock or perhaps varying obstacles between Suzy and the window. Eventually though, we will be able to show that Suzy's relation to the window smashing is a genuine dependence relation but Greg's casting of the spell is not. Although this is a simple case and in such a situation we would not *need* to use the interventionist test to determine the causal relationship, this method can be adapted to more complex scenarios where it is not so clear. In particular, we will see this in the mathematics case to be described later. Interventions and SEMs allow us to determine what it is that the effect is dependent upon, what the cause is in a given situation. In the mathematics case we will see how it seems like this process yields the result that certain situations are causally dependent on mathematics.

## 4 Structural Equation Models & Grounding

Now that we have considered interventionism as it concerns straightforward causation and SEMs, we can now see how it can be used with respect to grounding relationships. Constructing an appropriate model allows us to highlight which are examples of a genuine dependence relation. I will not rigidly define appropriateness here as it will vary case to case; but as an example, if we take smoking to be the sole relevant causal factor of lung cancer, then on interventionist treatments of causation, under the above model, smoking would not cause lung cancer. This is because in some cases, people smoke and do not develop lung cancer, because of other relevant factors, e.g. genetic predisposition. Including all these things in a SEM will give us the correct result, this is where appropriateness comes in. One might query why I am modelling grounding relations with SEMs. What I want to show is that using an appropriate SEM and combining it with interventionism highlights a genuine dependence relation. I believe that this suffices for us to call something causal, but if people are resistant to this for various reasons they may at least be tempted to admit that constraint is a genuine dependence relation at the very least. I think such a concession would still allow us to answer the epistemic and "makes no difference" challenges because the constraint relation would be so similar to causation. So I think we would be licensed to use it to determine what exists in just the same way we use causation in the eleatic criterion.

As previously stated, SEMs also model different cases of grounding. Models with the structure of the Suzy case also describe straightforward grounding:

Simple: Singleton
Variables
C: Whether Socrates exists
E: Whether the singleton set {Socrates} exists
(Wilson 2018: 741).

Again, we can see how interventionism in this case will reveal the dependence relation to be genuine through counterfactuals like "if Socrates hadn't existed, {Socrates} wouldn't have existed" which are clearly true. In worlds in which there is no Socrates, intuition tells us that there would be no singleton set containing Socrates. Indeed, it seems difficult to make sense of a world in which there is a singleton set containing Socrates, but no Socrates. Clearly this shows some sort of dependence relation at play.

It is worth considering a potential problem here, as answering it will help to clarify why interventionism has been selected over other counterfactual accounts of causation. As has been mentioned, interventionism is a breed of counterfactual account. If it is true to say "if not-*X* then not-*Y*", then it is the case that *X* caused *Y*, but what about cases in which it is also true to say that if not-*Y* then not-*X*. For example in the Socrates case above. After all, it is true to say that if {Socrates} did not exist then Socrates would not have existed, but we do not think this is a causal or grounding relationship. We want to say there is a dependence relation at play in the first case but not in the second. The second case in some sense tracks the wrong relationship, Wilson (2018: 736) refers to such cases as wrong-trackers. Interventionism can allow us to avoid wrong trackers, as Wilson states "The distinction between right-tracking and wrong-tracking counterfactuals is then derived in the interventionist framework from a distinction between appropriate and inappropriate causal models. Right tracking counterfactuals are those with antecedents specifying some combination of interventions on model variables in some appropriate model, and with consequents specifying some values for other model variables in that model." (Wilson 2018: 738). I think this sort of response is linked to what we mean by an intervention, if we look back at the criteria which define an intervention above we can see why. There would be no method of causing (or producing) the non-existence of {Socrates} that does not go via eliminating Socrates from existence. We cannot fulfill the third criterion, given that in such a case the intervention on $X$ with respect to $Y$ would not go via $X$, there would be a simple causal dependence between the intervention and $Y$. Perhaps one way of spelling out this thought is in terms of conceivability. It is conceivable that sets do not exist, we can picture a world without sets. So there are worlds in which Socrates exists but {Socrates} does not exist. But one might argue that it is inconceivable to have a set existing without its members, so we cannot imagine an intervention on {Socrates} that leaves Socrates unchanged. It is important to recall that interventionism is not a reductive account of causation, it is just an account of when we can call a relation causal or test for this, and in this case no such test can be performed. Ultimately, interventionism combined with SEMs can reveal genuine dependence relations and distinguish them from false dependence relations.

As mentioned earlier, counterfactual accounts tend to state a relationship as causal if it is the case that had $X$ not happened, $Y$ would not have happened. However, sometimes the relationship between $X$ and $Y$ is causal, even though this counterfactual is false. Such as when there is a 'back-up' or pre-emptive cause, $Z$, that would have caused $Y$, had $X$ not. Cases like these tend to defeat counterfactual accounts of causation but this can be accommodated by interventionism and SEMs. This is where the notion of an appropriate model comes into play, because incorporating all relevant factors will allow us to see the genuine dependence relations at work in the model. A simple example of this is the case of **Assassination**. Two assassins, $A$ and $C$, are aiming rifles at a target, $B$. Both always hit their targets, and hitting their targets would definitely result in death. Also, if $A$ does not fire, for whatever reason, then $C$ will fire instead. As it happens, $A$ fires, and $B$ dies. It is clear that $A$ caused the death of $B$, but it isn't true to say that if $A$ hadn't fired, $B$ wouldn't have died, because $C$ would have fired instead. It seems a straightforward counterfactual account fails on this. But the Interventionist account can deal with this. Once you've established the notion of an appropriate model we can see that just taking $A$ and $B$ into account will lead to false causal reports. Interventions on an appropriate model which included $C$ would reveal the genuine causal dependence relations that were present.

Another such case is presented below:

Early Pre-emption: Marsupials

Variables

C: Whether Wombat bites into the plant

P: Whether Wombat swallows the plant

Q: Whether Kangaroo sees the plant

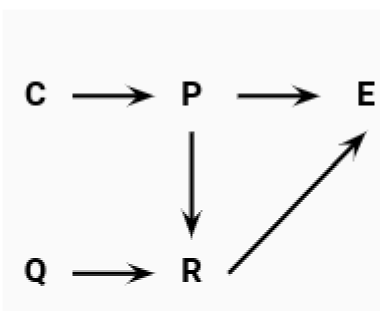
R: Whether Kangaroo eats the plant

E: Whether the plant is digested

Structural Equations P=C R=max(Q-C, 0) E=max(P, R)

Assignment C=1; P=1; Q=1; R=0; E=1

Graphical Representation




Wilson (2018: 744)

It is not the case that had the wombat not bitten the plant, the plant would not have been digested, because the kangaroo was there to eat the plant instead. Let us imagine this model without the *Q* and *R* variables taken into account. We might imagine an intervention performed on this model, such a muzzling wombat to see if the plant is digested. As it turns out, the wombat would be muzzled, thus unable to bite the plant and so unable to swallow the plant, but the plant would be digested, because the kangaroo would eat it. It would seem like we have shown that the wombat biting the plant did not cause the plant to be digested, but this is the wrong result. What has happened is that we have modelled this situation with an inappropriate model. The model is inappropriate because the kangaroo is also involved. So this is why it is critical to include the *Q* and *R* variables. We might imagine someone performing the above intervention and noticing kangaroo's involvement. Taking this into account, we might imagine someone muzzling the kangaroo as well as the wombat and seeing if the plant is digested, or letting the wombat free whilst the kangaroo is restrained and seeing what happens to the digestion variable. Performing interventions like this will allow us to establish the dependence relations at play, and so determine that the relationships are causal.

At this stage, we have seen how SEMs and the interventionist approach can be used to model and test relationships, allowing us to determine which of those are causal (or at least genuine dependence relations). Both these things are useful tools in assessing causation and, as we shall see, also useful for talking about constraint as a relationship.

## 5 Constraint Relationships

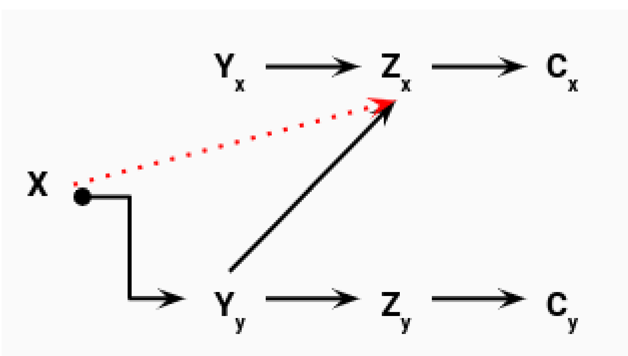
With this understanding of SEMs, we can move on to discuss the constraint relationship. I will explain a constraint relationship which intuition should hopefully tell us is causal, before showing that the same structure applies to cases of mathematical constraint. The example we will use is of a river. The river flows down a hill and comes to somewhat of a plateau. There are two potential distributaries (*A* and *B*) the river could then take from this plateau to continue descending. As it happens, the river only flows down one of these, *B* (perhaps because it is closer or lower). One day, a tree falls and blocks the path of distributary *B*, resulting in the river having to carve a new path through a different distributary (*A*). It seems pretty obvious to say that the fallen tree has caused the river to flow down distributary *A*. What it also seems obvious to say is that the fallen tree has made it impossible for water to flow down distributary *B*. This seems then, to be a constraining relationship. The tree has restricted the range of possible 'actions' the water can take, it has made certain 'options' impossible. In the strawberries example, the mathematical fact restricts the range of possible distributions, it results in certain distributions of the strawberries being impossible. The mathematical fact constrains the physical world. The cases are parallel and both seem to be equally constraining. It is worth noting as well that the case is causal and constraining in virtue of the same elements. The cause is the fact that the tree has fallen (or the event) and the effect is the fact the water flows the way it does (or the event). The constraint is the fact the tree has fallen, what that results in, the constrained, is the fact the water flows the way it does. Now of course one could say that rather than these relationships being identical, we instead have two at play but I see no strong arguments for this. As I will argue, these relationships behave in the same way under similar interventions and the patterns of dependence remain the same. Given the fact that they are between the same elements, it seems intuition and motivations from parsimony should compel us to conclude there is actually just one relation at play here, perhaps being described in different ways, and that relationship is causal and constraining.

Constraint: River

Structural equations: $Y_x=Y_y+1$, $Z_x=Y_x$, $C_x=Z_x$, $Y_y=X-1$, $Z_y=Y_y$, $C_y=Z_y$.

Assignment: $X=1$; $Y_x=1$; $Z_x=1$; $C_x=1$; $Y_y=0$; $Z_y=0$; $C_y=0$.

Graphical representation




The dashed line from $X$ to $Z_x$ represents the transitive dependence of $Z_x$ upon $X$, because $X$ ultimately rules out certain other possibilities ($Y_y$, ... $Y_L$); it is the ruling out of these other possibilities that causes $Z_x$. We can see how this abstract structure is applicable to the river case by assigning variables.

Variables:

X: Tree falling

$Y_x$: Distributary A open

$Z_x$: Water flows through distributary A

$C_x$: Consequences

$Y_y$: Distributary B open

$Z_y$: Water flows through distributary B

$C_y$: Consequences

In the river case, let us suppose we perform an intervention on *X*, changing its value to 0, i.e. the tree does not fall. We will see that the value of the $Z_y$ variable will change to a 1, i.e. water will flow through *B* and will not flow through *A*. For example, in this case it is conceivable that we could build a scale model of the tree/river case and perform interventions on this model, allowing us to scale-up those conclusions to the 'real' case. But let us also consider this case with the mathematical variables plugged in:

X: 23 is indivisible by 3.

$Y_x$: 23 objects divisible between 3 people non-evenly

$Z_x$: The strawberries divisible in a particular way

$C_x$: Consequences

$Y_y$: 23 objects divisible evenly into 3 groups

$Z_y$: 23 strawberries divisible evenly between 3 people

$C_y$: Consequences

Let us suppose that 23 had been divisible by 3, well then it would have been possible to divide 23 objects evenly into 3 groups, and furthermore Mother would have divided her 23 strawberries evenly between her 3 children. The $Z_y$ variable would have changed in exactly the same way. Likewise, if Mother had divided her strawberries evenly then she would not have divided them non-evenly, i.e. the $Z_x$ variable is also going to change in the same way as in the tree case. So on an interventionist treatment, 23 being indivisible by 3, caused 23 strawberries to be indivisible between 3 children. The distributary being blocked is the cause of the water flowing down the open one in the same way that 23 objects being indivisible into 3 groups is the cause of Mother distributing the strawberries in the way she ultimately does. An important point to bring up at this stage is that one may wish to resist here by placing the cause in this case outside of mathematics. Perhaps the cause in this situation is what resulted in Mother only buying 23 strawberries as opposed to one more, for example, (or only having 3 children), perhaps the cause is some sort of social state of affairs. I think this is too hasty though, the reason we point to mathematics as *the* explanation (and I want to say the cause) is because it explains why such a distribution is impossible in every case, whereas such social states of affairs would not. On top of that, the social facts may well form an important part of the background conditions which led to the distribution scenario, but what should matter is what the cause is within the scenario, so to speak. We can compare this with the earlier discussed **Assassination** case.The background conditions which led the assassins to be assigned a target are relevant in setting up the scenario, and we may wish to include them as part of the full causes of the scenario. But these background conditions do not thereby prevent the assassin's firing from being a cause of the death of the target. I propose we should treat the mathematical case in a similar way. Perhaps we may wish to include the background conditions as part of the relevant causes of the situation, but *in the situation* the cause we can arguably point to is still the mathematical fact, the social facts playing a role does not prevent this causal ascription.

## 6 Potential problems

### 6.1 Counterpossibles

#### 6.1.1 Intuitive counterpossibles

Before concluding, I want to address some potential problematic differences. In the tree case we are discussing straightforward contingent matters which might easily not have held. In the mathematics case we are dealing with necessary truths, things it seems could not have been otherwise. In this case it is less clear that we are dealing with causation rather than another relation. The response here is that we can help ourselves to counterpossible statements to take the place of the contingent statements in the tree case, such as "if the tree had not fallen". Counterpossibles like "if 23 had been divisible by 3" can be used in the mathematics case and we can see whether certain situations would have held if this was the case. Quite a lot of people view counterpossibles as merely trivially true, in virtue of their impossible antecedents, I think it is worth offering up some intuitive arguments about why that is hasty. We can do this by comparing two counterpossibles:

E. Had 23 been divisible by 3, then 3 would have been a factor of 23

F. Had 23 been divisible by 3, then 3 would not have been a factor of 23.

It seems clear that *E*. should strike us as true, whilst *F*. should strike us as false. These are not merely trivial statements. Now admittedly, this non-triviality may come from the fact that in *E*. the consequent is merely a rewording of the antecedent, and in *F*. it is a straightforward contradiction. That may well be a good point but we can still construct non-trivial counterpossibles:

G. Had[4] 23 been divisible by 3, then a calculator would have been able to perform the division operation on 23 and 3.

H. Had 23 been divisible by 3, then a calculator would not have been able to perform the division operation on 23 and 3.

Again, *G*. should strike us as intuitively true, whereas *H*. should strike us as intuitively false. Furthermore, compare both these with a further counterpossible:

J. Had 23 been divisible by 3, then Paris would be in France.

A counterpossible such as *J*. seems importantly vacuous in a way that *E-H* do not. This intuitive judgement should hopefully show that there is scope to think counterpossibles can possess non-trivial truth values. Now it is clearly true that in the mathematical cases, as opposed to the physical constraint case, the modality we are dealing with is different. But the abstract structure of these relations is the same, and how they behave under interventions is unchanged, this seems more important in determining a relation to be causal; the modality can be considered merely a difference in degree rather than kind.

[4] Perhaps the antecedent should be lengthened to include “and had mathematicians known this fact” to avoid makes-no-difference style complaints about the inefficacy of mathematical truths on our practice. This would still produce a counterpossible.

## 6.1.2 Counterpossibles in science

Some people may not be persuaded by so-called 'intuitive' counterpossibles like those we have discussed above. Simply put, one could just deny that these have differing truth values, they are one and all true and any appearance to the contrary is mere appearance. However, there are counterpossibles that are not so easy to dismiss. Many scientific theories and models appeal to counterpossibles in explanations and predictions. If these were all and only trivially true then we might worry that the scientific endeavour is threatened. Furthermore, the judgement that such scientific counterpossibles are non-trivial is far from pre-theoretic or just based on intuition. Instead, such a judgement is based on scientific reasoning. In other words, such examples provide a good amount of positive support to the non-triviality of counterpossibles. Not only do we have to reason through and work out that such counterpossibles are non-trivial with more than mere intuition, but we also need to judge them as non-trivial in order to make progress and predictions.

Numerous examples of such counterpossibles are offered and discussed by Tan (2019). Not only are there multiple examples of counterpossibles used in science, but they are used in different ways for different purposes. One area which Tan (2019) focusses on their use in is scientific explanation. He offers an archetypal example of a counterpossible and discusses why viewing it as counterpossible and as non-trivial is the correct verdict. The case offered is:

"If diamond had not been covalently bonded, then it would have been a better electrical conductor." (Tan, 2019: 40).

Tan claims that this is a scientific explanation of the fact that diamond cannot conduct electricity whereas solid carbon in some other forms can. The reason the covalent bonding explains this fact is because covalent bonds do not leave free electrons as they 'use up' all the electrons forming the strong bond. In other substances, free electrons allow for electrical conductivity (Tan, 2019: 40). The property of poor conductivity, that diamond has, is brought about as a result of these bonds, and so the microphysical structure. This counterfactual, then, provides an explanation in virtue of highlighting that dependence relation. But one might wonder if this is indeed a counterpossible, one may wonder if diamond could have been otherwise bonded, and so if this is a mere straightforward counterfactual. One can approach this in two ways, we might consider whether something is called diamond in virtue of its microphysical structure or in virtue of its theoretical role in science (Tan, 2019). Going the first route, one can easily see that this is a counterpossible, because if something is only diamond in virtue of its microphysical structure, then something which had a different microphysical structure would not be diamond. As a matter of metaphysical necessity, diamond has the structure that it does. So it is metaphysically impossible for diamond to be differently bonded.

Going the second way, one may think that we define diamond by its theoretical role, the diamond-stuff is the stuff which does x, y and z. But the reason diamond is distinguished from other substances and the reason it does the things it does, is because of its microphysical structure. In other words, nothing else could do the things diamond does without its microphysical structure. Nothing could fill the diamond role without actually being diamond. So again, in other words, it is metaphysically impossible that diamond could have been differently bonded than it in fact is. So it seems then, that "If diamond had not been covalently bonded... " is a counterpossible. Tan (2019) goes further than this, he insists that this is also counterpossible which is true, and non-vacuously so. This is because it describes an empirical fact, that the poor conductivity of diamond physically depends on its microphysical structure. So science relies on non-vacuous counterpossibles in scientific explanation (Tan, 2019: 42). One can easily see how this is not an isolated case, because many explanations of why substances have the properties they do, in the scientific context, will rely on a similar explanatory structure. It does then seem like we might well have to appeal to non-trivial counterpossibles in some cases so their use in this paper should hopefully not seem as controversial with that in mind.

## 6.2 Intelligibility of Interventions

Another potential issue is the intelligibility of interventions, we can make sense of what it would be to prevent the tree from falling but we cannot make sense of what it means for 23 to be divisible by 3. I think this objection is quite closely linked to the last one. It is easy enough to make sense of counterfactuals like "if the tree had not fallen", we know what that world would look like, and can imagine holding everything fixed except for the tree falling in that world. But in cases of counterpossibles, what we mean by "if 23 had been divisible by 3"[5] is less clear, and what that world would look like is mysterious. If we change that mathematical fact then people might naturally think we have to change more of mathematics, there will be a massive ripple of changes throughout the world and the objection will go that there is too much to make sense of to be able to make any statement about the truth or falsity of things at such a world non-trivial. So we could make any statement about imagined interventions on mathematical facts but they will not be meaningful and in any case would not allow us to make conclusions about what would lead from them because we do not know what else would be true or false at that world. To respond to these types of worries I would like to help myself to a response offered by Marc Lange (2019) in response to these issues concerning a different theory. As Lange sees it, the essence of the problem is that counterpossibles 'ripple out' into the world at such a magnitude that we cannot make sense of what is true and false in that world. His response is that this is the case with a lot of ordinary counterfactuals that people would use. For example, we tend to think of counterfactuals like "if Julius Caesar had been alive today. . . " and are happy to say we can hold this fixed without changing anything. But this is actually not the case, because we can ask how it is that Julius Caesar came to be alive today, did he time travel? That would require a lot of differences, was it a man-made time travel machine, did he fall through a wormhole in a sci-fi-esque accident? Perhaps instead of time travel it is just that the baby which was born and grew up to become Julius Caesar was actually born in 2007. But then unless he has the same upbringing (arguably impossible given how society has changed) he will in fact not be Julius Caesar. Lange's point is that although it might seem like ordinary counterfactuals hold with 'everything else fixed' this is in fact not the case (Lange, 2019). All counterfactuals ripple out because the world is fundamentally connected. So criticising counterpossibles for this is a weaker criticism than intended, given that it also applies to counterfactuals. We might well have to do some work in spelling out what exactly is held fixed in counterpossible worlds and what remains unchanged but the fact that this work needs to be done is not a problem. Alternatively, it seems in ordinarily rippling counterfactual cases we can merely stipulate that such

[5] Or more accurately, "if 23 had been divisible by 3 into whole, natural numbers".

counterfactuals are non-trivially true or false, so this should also be available to us in the counterpossible case (Lange, 2019). I think Lange has got this right, just because a counterfactual seems simple to understand does not mean it is and just because a counterpossible seems impossible to understand does not mean it is. Furthermore, the literature on impossible worlds is always expanding and a lot of work has been done in the area to show that impossible worlds have non-trivial truth conditions. It could very well turn out that all these accounts have it wrong but whilst the debate is still lively I do not think that we should jump to that conclusion and so in the interim we can help ourselves to such accounts to justify the sorts of counterpossibles and impossible interventions we want to.

## **7 Conclusion**

To conclude, the constraining relationship seems to be a genuine one which operates on the world. Furthermore it is at least arguable that this relationship shares a structure with relationships we would ordinarily wish to describe as causal. Given this, it seems like we should conclude that constraint is a causal relationship. This means that because mathematical objects are involved in constraint relationships, they are involved in causal relationships. Given this, there seems to be an open causal path available to us that could allow us to avoid the Benacerraf problem and gain knowledge of mathematical objects, and to also avoid "makes no difference" arguments, because mathematical objects do make a difference, thus allowing us to postulate them. In this essay we have seen one application of this thinking and how it can give us knowledge of the natural numbers. In a similar vein we can see how one might construct a mathematical version of the pre-emption case we discussed earlier. The fact that 17 does not have 2 as a factor is *because* it is a prime number, but equally even if it had not been prime, it is an odd number, and this also explains why it does not have 2 as a factor. This seems like some pre-emption/overdetermination is at play here (perhaps the explanation could be considered to be in the other direction, with the fact the number is odd being more important, but clearly there will be some sort of pre-emption/overdetermination). Perhaps such an explanation could be of use in a constraint explanation of a variation of the classic cicadas case (Baker, 2005). The reason that 17-year life cycle cicadas do not intersect with predators every 2 years is *because* 17 is prime, but even if 17 had not been prime, it is an odd number. So one might wish to say this non-intersection is overdetermined/pre-empted by mathematical facts. Of course so far we have only considered how natural numbers can be involved in constraint relations, and that is a small part of mathematics. Thankfully, the epistemic advantage of appealing to constraint could give us much more. Certain percolation phase transitions seem to exhibit constraint and this would ultimately grant us knowledge of the real numbers. Lange (2017:7-8) argues that the example of the Königsberg bridges is a constraint explanation. It is a constraint explanation because the arrangement of bridges and landmasses makes it impossible to traverse all the bridges exactly once in a single attempt, called an Eulerian path. To perform an Eulerian path, it must be the case that, considered as a network, either every vertex, or every vertex but two, is touched by an even number of edges, I will refer to this as the Eularian fact. We can see how we could plug this into the above model to produce a similar constraint explanation.

Variables:

X: Eulerian fact

$Y_x$: The Königsberg bridge arrangement can be crossed exactly once each in a single trip

$Z_x$: A given person, P, performs an Eulerian path over the bridges

$C_x$: Consequences

$Y_y$: The Königsberg bridge arrangement cannot be crossed exactly once each in a single trip

$Z_y$: A given person, P, performs a non-Eulerian path over the bridges

$C_y$: Consequences

What I hope this shows is that the constraint approach to mathematical explanation, treated as a causal relation can give us knowledge of, and a connection to, further mathematical objects. This provides us with a project and a process going forward, treating constraint as causal, or even as an important dependence relation in its own right, has the potential to give us a path to knowledge about many different areas of mathematics and go a large portion of the way to solving the epistemic problem. [6]

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[6]An earlier version of this article appeared in *Dialectic*. Available at: https://uoydialecticjournal.wordpress.com/

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