代数幾何学シンポジウム記録
2010年度 pp.119-119

$K3$ surfaces and log del Pezzo surfaces of index three

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We want to classify log del Pezzo surfaces of index k.

History of classification

- $k=1$: classical result
- $k=2$: Alexeev and Nikulin, Nakayama

Generalize the idea of [AN] to the $k=3$ case!

Review of [AN] ($k=2$ case)

- Smooth Divisor Theorem
  $\exists C \in |-2K_Z|$ s.t. C : smooth curve and $C \not\ni$ singularities.
- Right resolution
  In general, we get the following dual graph by the minimal resolution.

$$(-3)\text{—}(-2)\text{—}\cdots\text{—}(-2)\text{—}(-3)$$

$\uparrow$: blow up at all intersection points

$$(-4)\text{—}(-1)\text{—}(-4)\text{—}\cdots\text{—}(-1)\text{—}(-4)$$

- Classification of non-symplectic involutions on $K3$ surfaces by Nikulin

We get a correspondence between $K3$ surfaces with a non-symplectic involution and log del Pezzo surfaces of index 2.

Main Theorem ($k=3$ case)

There exists a correspondence between $K3$ surfaces with a non-symplectic automorphisms of order 3 and log del Pezzo surfaces of index 3.

- Multiple Smooth Divisor Property
  $\exists 2C \in |-3K_Z|$ s.t. C : smooth curve and $C \not\ni$ singularities.
- Right resolution
  It is a successive union of the unit chain

$$(-3)\text{—}(-1)\text{—}(-6).$$

- Classification of non-symplectic automorphisms of order 3 on $K3$ surfaces by Artebani and Sarti, Taki (independently)

There exists a log del Pezzo surface of index 3 which does not satisfy MSDP. (ex. $\mathbb{P}(1,1,3)$) Thus the observation does not give the complete classification.

Example