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Invited Article

TURBULENCE MODULATION OF THE UPWARD TURBULENT BUBBLY FLOW IN VERTICAL DUCTS

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ABSTRACT

The present paper aims at improving the modeling of turbulence for the upward turbulent bubbly flow through the use of experimental databases that contain data on small and large vertical ducts. First, the role of bubble-induced turbulence was analyzed, which indicated the dominant role of the bubble-induced turbulence in the duct center for relatively high void fraction cases. Therefore, the turbulence therein was mainly focused on, which indicated that the stronger turbulence could be induced by bubbles in large ducts with similar void fractions as compared to that in small ducts. Next, the turbulence of upward turbulent bubbly flow near the wall is discussed to understand the interaction between the wall-induced and bubble-induced turbulence. It showed that the existence of a wall could suppress the bubble-induced turbulence given the same void fraction, and the existence of bubbles could also suppress the solely wall-induced turbulence as compared to the single-phase turbulent flow, even though the total turbulence is enhanced. The above characteristics indicated that the current turbulence modeling method needs to be modified, especially when the bubble-induced turbulence plays a dominant role.

1. Introduction

Upward turbulent bubbly flow (UTBF) plays an important role in the optimum design and the safe operation of nuclear facilities, such as light water reactors. Closure of turbulence is one of the key problems in the modeling of turbulent bubbly flow that could affect the other parameters' predictions such as mean flow, bubble distribution, and also rates for bubble breakup and coalescence. Compared to single-phase turbulent flow, the mechanisms of turbulence generation are more complex in the UTBF, which includes the shear-induced turbulence and bubble-induced turbulence as well as the interactions between these two mechanisms. In order to improve the closure of turbulence in UTBF, understanding and modeling of the turbulence modulation by the bubbles are necessary. Several studies have been carried out by comparing the results of the turbulence characteristics in UTBFs to those observed in single-phase turbulent flows. According to the

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experimental measurements of the UTBFs in vertical ducts, the presence of bubbles can lead to both turbulence enhancement and reduction as compared to single-phase turbulent flows [1–13] especially near a wall. However, the physical process of turbulence modulation has not yet been clarified, and the predictions for turbulence enhancement and reduction by the bubbles are still difficult. Lance and Bataille [7] studied bubble-induced pseudo-turbulence in uniform bubbly flow and showed that the bubble-induced pseudo-turbulence could be estimated in the potential theory under the critical void fraction, and above the critical void fraction the turbulence was strongly amplified by the hydrodynamic interactions between the bubbles and the background turbulence. However, the relationship between the bubble-induced turbulence and the void fraction was only focused on a very low void fraction of less than 3%. As for the modeling of turbulence, Theofanous and Sullivan [14] and van Wijngaarden [15] discussed on bubble-induced turbulence by comparing the turbulent intensities prior to and after the bubbles' introduction. van Wijngaarden [15] derived the pseudo-turbulent Reynolds stress for the rising bubbles in the laminar and turbulent flows. Theofanous and Sullivan [14] estimated the total turbulence of the bubbly flow in the pipes based on the mean flow velocity and the wall shear stress. With the eddy viscosity assumption, the turbulent shear stress for the turbulent bubbly flow was modeled by a zero or one equation derived from the single-phase turbulence, where the eddy viscosity and the mixing length were focused on [16,17]. More accurate turbulent models for turbulent bubbly flow have also been developed on the basis of the two-equation model or the Reynolds stress model of the single-phase turbulent flow [18–27]. For example, Kataoka and Serizawa [18] derived the transport equations for the turbulent Reynolds stress and dissipation of the two-phase flow. Lahey et al [20,21] and Lopez de Bertodano et al [22,23] developed the $\tau-\varepsilon$ model for the turbulent bubbly flow. Among these models, the attention was mainly paid to source terms induced by bubbles. However, it is notable that most of the previous numerical validations was carried out for a single model and focused on the small ducts and low void fractions. Brief overviews of the existing models of the bubble-induced turbulence were done by Rzehak and Krepper [28,29]. Moreover, Rzehak and Krepper [28,29] carried out a detailed comparison of the different bubble-induced source terms to qualify their validity after selecting the existing experimental databases in the upward vertical circular pipes, in which the experimental databases in

large ducts established by Shawkat et al [12] were also considered. Unlike those in small ducts, the predictions of turbulent kinetic energy in large ducts tend to be smaller than the experimental measurements for cases with higher void fractions. However,due to the lack of experimental databases in large ducts, the question as to whether the above models are suitable for analysis of the UTBF in large ducts still needs further investigation. Moreover, only the turbulence enhancement near the wall could be predicted because of the superposition of the wall-induced and bubble-induced turbulence and by neglecting the interaction between these two mechanisms. To improve the turbulence modeling of the turbulent bubbly flow, it is necessary to study the interaction between the wall-induced and bubble-induced turbulence.

The present study aims at understanding the turbulence modulation in the UTBF in vertical ducts. The paper is arranged as follows: in Section 2, the experimental databases of turbulence for the turbulent bubbly flow is first collected and described; Section 3 presents the analytical results including the comparison of the turbulence in different ducts and the interaction between the wall-induced and bubble-induced turbulence. Their interaction is studied by analyzing the bubble effect on solely wall-induced turbulence and the wall effect on solely bubble-induced turbulence. Here, in the UTBF, solely wall-induced turbulence is defined as the strength of the wall-induced turbulence if there is no effect from the bubbles, and solely bubble-induced turbulence is defined as the strength of the bubble-induced turbulence if there is no effect from the wall. Finally, the improvement of turbulence modeling is discussed. The conclusions are given in Section 4.

2. Experimental databases

As mentioned previously, turbulence generation in UTBFs is very complicated and could be affected by many factors such as void fraction, bubble size and bubble deformation, initial turbulence, and liquid phase velocity. To understand turbulence modulation, such as bubble-induced turbulence and its interaction with the wall-induced turbulence, a large number of experimental databases from various ducts will be necessary. In addition, the experimental databases should also include several flow quantities simultaneously, such as turbulent kinetic energy, void fraction, and bubble size, and satisfy certain requirements. However, it is rather difficult to find enough effective experimental databases to conduct a

Table 1 – Experimental databases and flow conditions in different duct geometries.

Duct shape	Researchers	Duct size D_H (mm)	Flow conditions	Bubble size D_B (mm)
Small circular pipe	Serizawa [1]	60	$J_l = 0.37-1.03$ m/s	3–4.5
	Wang et al [5]	57.15	$J_g = 0-0.321$ m/s	
	Liu and Bankoff [6]	57.2		
	Hibiki and Ishii [9]	50.8		
	Hosokawa and Tomiyama [11]	25		
Large circular pipe	Shawkat et al [12]	200	$J_l = 0.45-0.68$ m/s	3–5.5
			$J_g = 0-0.18$ m/s	
Large square duct	Sun [13]	136	$J_l = 0.5-1.25$ m/s	3–5
			$J_g = 0-0.27$ m/s	

thorough analysis on turbulence modulation. Table 1 shows the collected experimental databases and the flow conditions in different ducts used in this study. More details can be found in the corresponding references and also in the work of Rzehak and Krepper [29], which summarized several databases. According to duct size (D_H) and shape, ducts can be classified as small circular pipes, small noncircular pipes, large circular pipes, and large noncircular pipes. If the duct size, D_H, is larger than the maximum bubble size under the test conditions, $D_{B,max}$, it is defined as a large duct; otherwise, it is defined as a small duct. The experimental databases established by Sun in the large non-circular ducts [13] are chosen as the main databases for the later analysis. Moreover, to show the bubble effect or wall effect on the turbulence modulation in UTBF the present paper simplifies the problem as that given by the liquid and gas superficial velocities, J_l and J_g, and the bubble size, D_B, in that the generation of local turbulence is determined by the bubble properties shown by the void fraction and the wall effect shown by the distance from the wall.

3.   Results and discussions

Before considering the bubble effect on solely wall-induced turbulence and the wall effect on solely bubble-induced turbulence, a sound knowledge of each is required. As for solely wall-induced turbulence, extensive research has been done on single-phase turbulent flow [30]. However, for solely bubble-induced turbulence, very few studies exist, the most notable of which are the experiments performed by Lance and Bataille [7] in uniform bubbly flow for small void fractions of less than 3%. For a high void fraction, the strong bubble—bubble interaction will be important to the generation of turbulence. To understand the characteristics of solely bubble-induced turbulence, comprehensive experimental databases in a wide range of void fractions are necessary. The present paper firstly studies the role of bubble-induced turbulence for different cases. Then, the wall effect on solely bubble-induced turbulence and the bubble effect on solely wall-induced turbulence are discussed.

3.1.   Role of bubble-induced turbulence in UTBF

The role of bubble-induced turbulence is shown by the ratio of the turbulent intensities k_t in UTBF to that in the single-phase flow k_s under the same liquid superficial velocity J_l. A larger k_t/k_s ratio corresponds to the stronger role of bubble-induced turbulence. Fig. 1 shows the distributions of the k_t/k_s ratio together with the void fraction distribution α for the existing experimental databases in different ducts. As shown in Fig. 1, the k_t/k_s ratio for the turbulent bubbly flow decreases from the duct center to the wall even with a much higher void fraction near the wall, indicating the increasing role of bubble-induced turbulence from the wall to the duct center. Moreover, in the duct center, the k_t/k_s ratio is usually much larger than 1, especially in the large ducts even under small void fraction cases. For example, according to Shawkat et al's [12] and Sun's [13] databases, the k_t/k_s ratio is shown to be larger than 10, even for void fractions of less that 5%, indicating the more

dominant role of bubble-induced turbulence therein. Considering the dominant role of the bubble-induced turbulence in the duct center, the turbulence therein is used as a reference for solely bubble-induced turbulence. Fig. 2 summarizes the flow conditions for different ducts together with the contour lines corresponding to the flow conditions when the $k_{t,c}/k_{s,c}$ ratio in the duct center is about 5, 10, and 25, respectively. The contour lines here were obtained by interpolating the existing flow conditions in the corresponding references. It was found that the contour lines for the small circular pipes from Hibiki and Ishii's database [9] and Hosokawa and Tomiyama's database [11] lie above that of the large ducts in Shawkat et al's [12] and Sun's databases [13], i.e., under the same flow condition, in larger ducts $k_{t,c}/k_{s,c}$ is larger than that in small ducts. For example, in the case of $J_l \approx 0.5$ m/s and $J_g \approx 0.05$ m/s as marked in Fig. 2, $k_{t,c}/k_{s,c} \approx 25$ in large ducts, whereas $k_{t,c}/k_{s,c} \approx 10$ in small ducts. This indicates that bubble-induced turbulence played a much more dominant role in larger ducts especially in the core region under similar flow conditions. The following two factors could be responsible for the above observations: (1) for large ducts, there exist more space for the bubble's horizontal deformation and wake development behind the bubbles, which induced stronger turbulence; and (b) there exists less wall effect on turbulence in the core region for large ducts.

3.2.   Turbulence modulation near the wall

In this section, the turbulence of the UTBF near the wall is discussed to understand the interaction between the bubble-induced and wall-induced turbulence. Owing to the lack of models of solely bubble-induced turbulence and solely wall-induced turbulence in the UTBF, the following discussions will be made in general terms.

3.3.   Wall effect on the solely bubble-induced turbulence

To show the wall effect on the generation of local turbulence, the ratio of the local turbulent kinetic energy k to that in the duct center k_c is defined and shown in Fig. 3 for different ducts. It was observed that for the single-phase turbulent flow, because the turbulence is solely induced by the wall, the $k_s/k_{s,c}$ ratio increases from the duct center to the near-wall region with an increase in the wall effect. Here, the near-wall boundary layer was not considered because of the lack of experimental databases. Comparing the single-phase turbulent flow to UTBF, it was found that the $k_{t,w}/k_{t,c}$ ratio for UTBF for the given flow conditions is much less than that for the single-phase turbulent flow near the wall even with a higher void fraction, which can be expressed as

$$\frac{k_{s,w}}{k_{s,c}} > \frac{k_{t,w}}{k_{t,c}}, \tag{1}$$

where $k_{s,w}$ and $k_{s,c}$ denote the turbulence intensity near the wall and in the duct center for the single-phase turbulent flow, respectively; and $k_{t,w}$ and $k_{t,c}$ are the turbulence near the wall and in the duct center for UTBF, respectively. This means that there is less wall effect on total turbulence near the wall in UTBF as compared to single-phase turbulent flow. Then, we consider the wall effect on solely bubble-induced turbulence as described below.
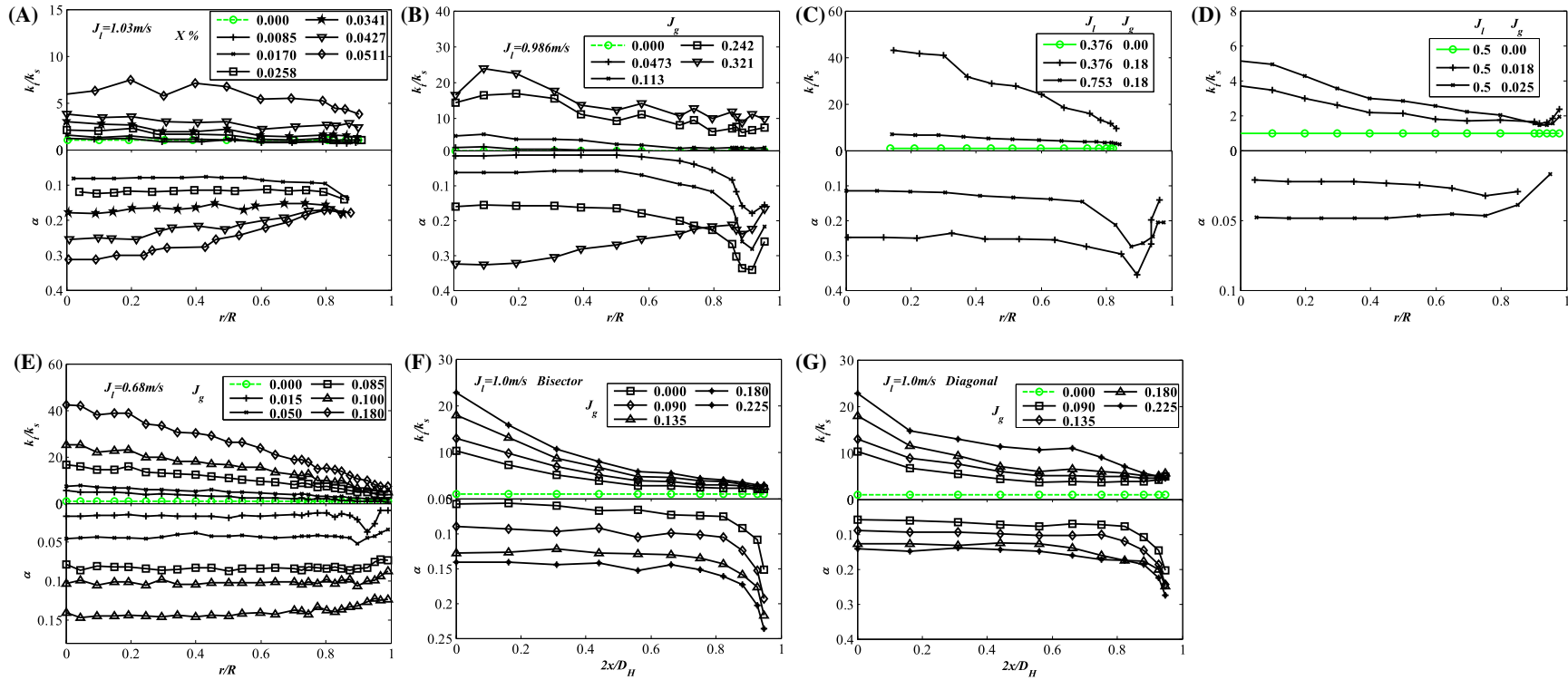
Fig. 1 – Lateral distributions of the k_t/k_s ratio based on existing databases in different ducts. (A) In the small circular pipe by Serizawa [1]. (B) In the small circular pipe by Hibiki and Ishii [9]. (C) In the small circular pipe by Liu and Bankoff [6]. (D) In the small circular pipe by Hosokawa and Tomiyama [11]. (E) In the large circular pipe by Shawkat et al [12]. (F) In the large square duct by Sun [13] along the bisector line. (G) In the large square duct by Sun [13] along the diagonal line.
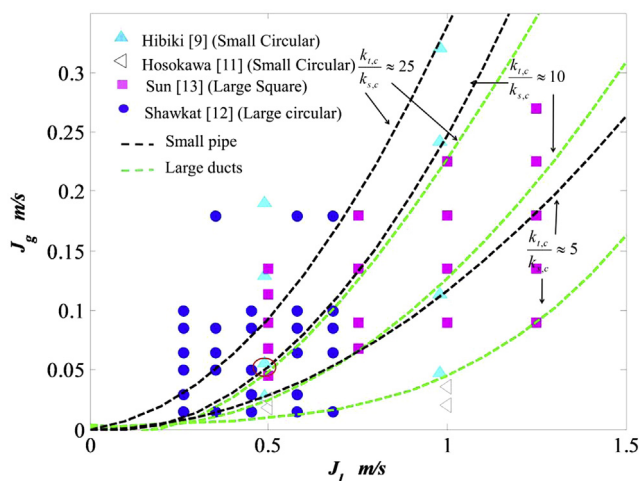
Fig. 2 – **Flow conditions for different ducts according to the $k_{t,c}/k_{s,c}$ ratio in the duct center when it is about 5, 10, and 25, respectively.**

In UTBF, because turbulence generation includes the contributions from both bubbles and the wall, which are defined as k_t^B and k_t^W and considering the weakening of the wall effect from the wall to the duct center, it could be ascertained that

$$\frac{k_{t,w}}{k_{t,c}} = \frac{k_{t,w}^B + k_{t,w}^W}{k_{t,c}^B + k_{t,c}^W} > \frac{k_{t,w}^B}{k_{t,c}^B}, \tag{2}$$

where $k_{t,w}^B$ and $k_{t,c}^B$ are the contribution to the total turbulence caused by bubbles near the wall and in the duct center for the turbulent bubbly flow, respectively; and $k_{t,w}^W$ and $k_{t,w}^W$ are the contributions to the total turbulence by the wall, near the wall, and in the duct center.

Assuming a linear or concave relationship between turbulence intensity and void fraction based on the experimental databases in the duct center, the following relations could be ascertained:

$$\frac{k_{t,w}^B}{k_{t,c}^B} > \frac{\alpha_w}{\alpha_c} \quad \text{and} \quad \frac{k_{t,w}}{k_{t,c}} > \frac{\alpha_w}{\alpha_c}, \tag{3}$$

where α_w and α_c are the void fraction near the wall and in the duct center, respectively. Here, to show the wall effect on solely bubble-induced turbulence, the experimental databases in large ducts were mainly used considering the dominant effect of bubble-induced turbulence therein. Fig. 4 compares the left- and right-hand sides of Eq. (3) based on Sun's experimental database [13], for the locations near the wall and near the corner. In contrast, it is observed that

$$\frac{k_{t,w}}{k_{t,c}} < \frac{\alpha_w}{\alpha_c}. \tag{4}$$

This inconsistency between Eq. (2) and the experimental databases indicates that, as compared to the core region under the same void fraction, the bubble-induced turbulence is reduced by the existence of the wall. The turbulence-reducing effect could result from the elongation of the bubble and the suppression of bubble wake by the wall. This also explains the phenomenon of the stronger bubble-induced turbulence being generated in the duct center of larger

ducts, as compared to that in small ducts, as shown in Fig. 2, which is attributable to a stronger turbulence-reducing effect on bubble-induced turbulence in the duct center by the existence of the wall in small ducts.

3.4. Bubble effect on solely wall-induced turbulence

The bubble effect on wall-induced turbulence has been considered by several researchers, including turbulence-reducing effects under a very low void fraction and small bubble size [3,4]. The turbulence reduction phenomenon by the bubbles is described as

$$k_{t,w}^W + k_{t,w}^B(\alpha_w) < k_{s,w}. \tag{5}$$

Under higher void fraction or larger bubble size, the total turbulence increases more than in single-phase turbulence flow. However, for these cases, turbulence is generated by both the wall and bubbles, and bubble-induced turbulence usually plays a more important role. It is still uncertain whether solely wall-induced turbulence increases. Hereafter, the bubble effect on solely wall-induced turbulence for these cases is considered by comparing the solely wall-induced turbulence near the wall for the single-phase flow $k_{s,w}$ to that for the upward bubbly flow $k_{t,w}^W$ under the same liquid superficial velocities. The key point is to estimate the portion of the solely wall-induced turbulence near the wall, which is very difficult owing to the coupling of the two turbulence generation mechanisms. By contrast, although the prediction of the solely bubble-induced part is currently difficult, it should be stronger than that predicted by the potential theory in which bubble-induced wake is not considered, i.e.,

$$k_{t,w}^B(\alpha_w) > \alpha_w W_r^2/2, \tag{6}$$

where W_r is the terminal velocity of the bubbles given the bubble size.

After substituting Eq. (6) to Eq. (5) and rearranging, it was noticed that the turbulent enhancement will certainly occur if $\alpha_w > 2k_{s,w}/W_r^2$ and the turbulent reduction can occur only if $\alpha_w < 2k_{s,w}^W/W_r^2$. Fig. 5 shows the comparison between the experimentally measured void fraction and the calculated void fraction based on $\alpha_{calc} = 2k_{s,w}^W/W_r^2$ near the wall. Because there is a lack of experimental databases showing void fractions and near-wall turbulence, only the data from Shawkat et al [12] and Sun [13] are shown, based on which the above considerations are confirmed. Moreover, in Eq. (6) the estimation of solely bubble-induced turbulence neglects the bubble wake. Therefore, if there is a better estimation of solely bubble-induced turbulence, it is believed that a better agreement could be reached.

Fig. 6 compares the wall-induced turbulence $k_{s,w}$ in the single-phase flow to $k_{t,w} - 0.5W_r^2$ in UTBF based on Sun's experimental databases in [13]. Though for these cases the measured total turbulence $k_{t,w}$ are enhanced by the bubbles, i.e., $k_{t,w} > k_{s,w}$, the cases satisfying the following conditions are observed,

$$k_{t,w} > k_{s,w} \quad \text{and} \quad k_{t,w} - \alpha_w W_r^2/2 \le k_{s,w}. \tag{7}$$

The number of cases satisfying the conditions of Eq. (7) will be higher with a better prediction of solely bubble-induced
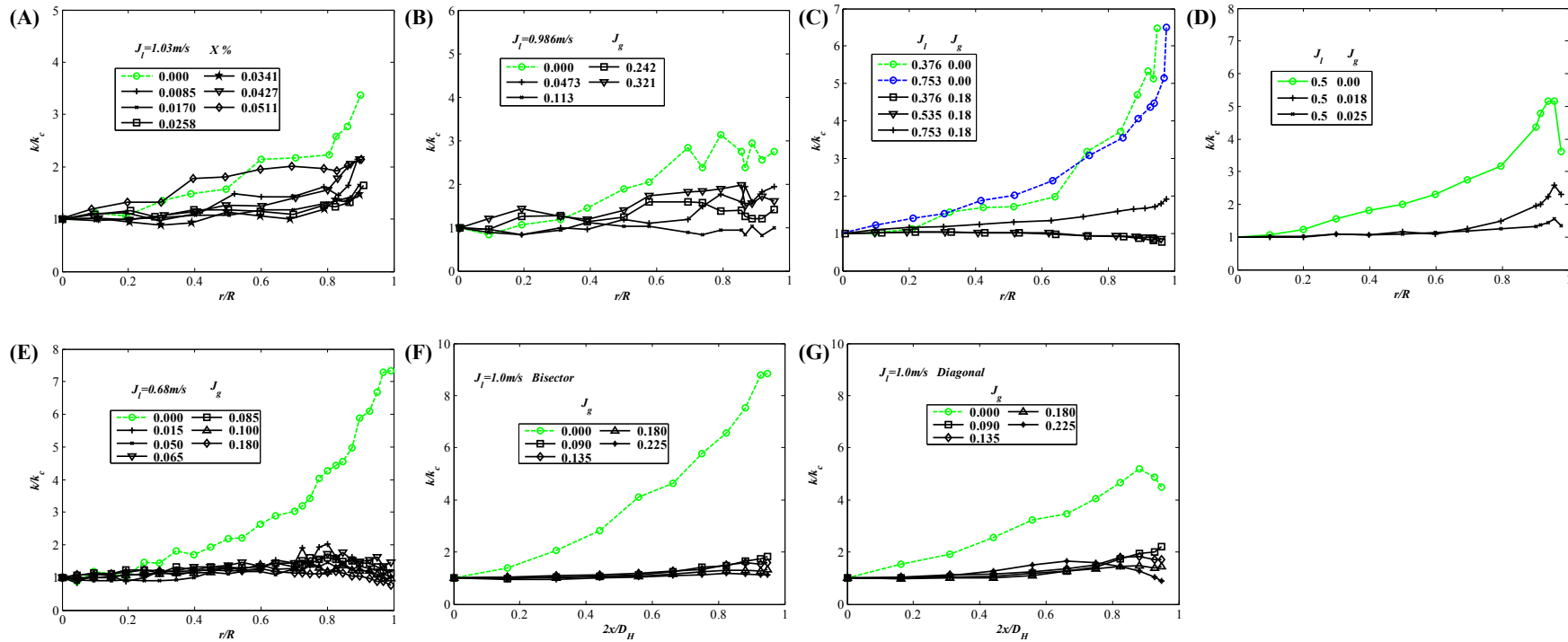
Fig. 3 — Lateral distributions of the k/k_c ratio based on existing databases in different ducts. (A) In the small circular pipe by Serizawa [1]. (B) In the small circular pipe by Hibiki and Ishii [9]. (C) In the small circular pipe by Liu and Bankoff [6]. (D) In the small circular pipe by Hosokawa and Tomiyama [11]. (E) In the large circular pipe by Shawkat et al [12]. (F) In the large square duct by Sun [13] along the bisector line. (G) In the large square duct by Sun [13] along the diagonal line.
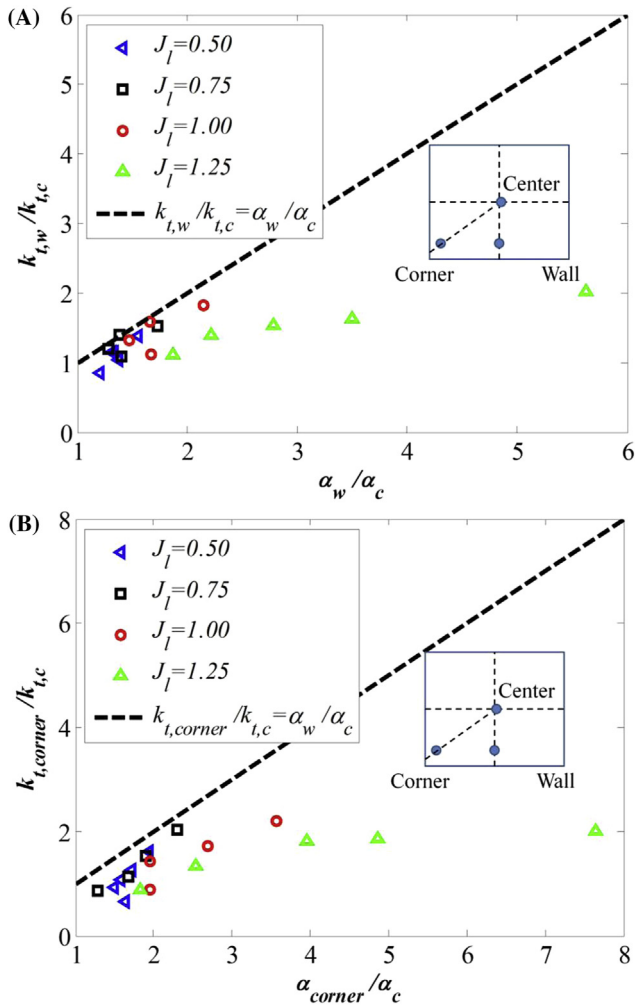
Fig. 4 − Comparison between $k_{t,w}/k_{t,c}$ and α_w/α_c based on Sun's experimental database [13].
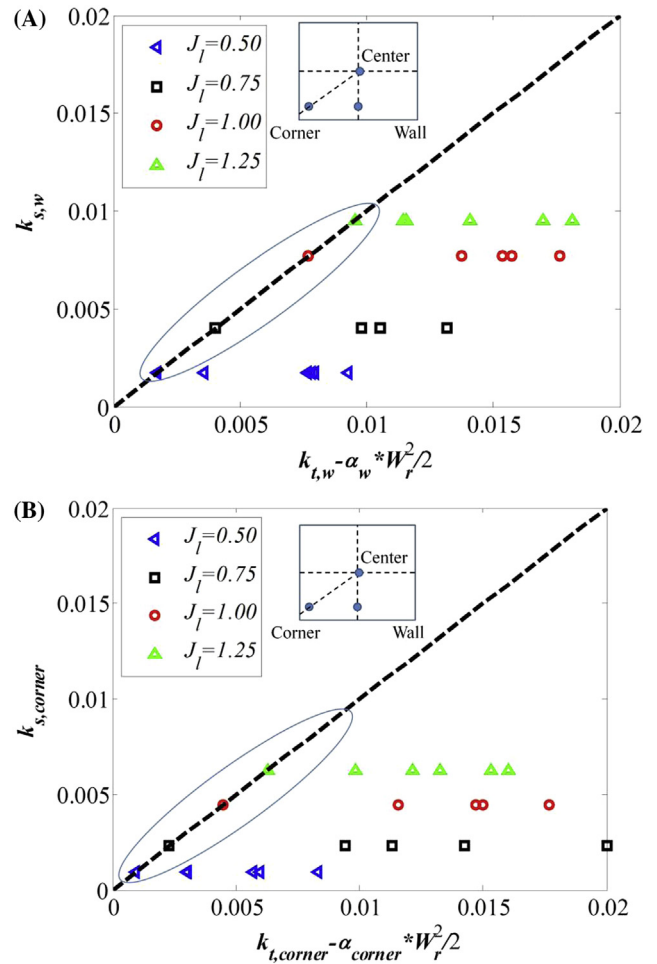

Fig. 6 − Comparison of wall-induced turbulence $k_{s,w}$ in the single-phase flow to $k_{t,w} - 0.5W_r^2$ in UTBF based on Sun's experimental database [13].
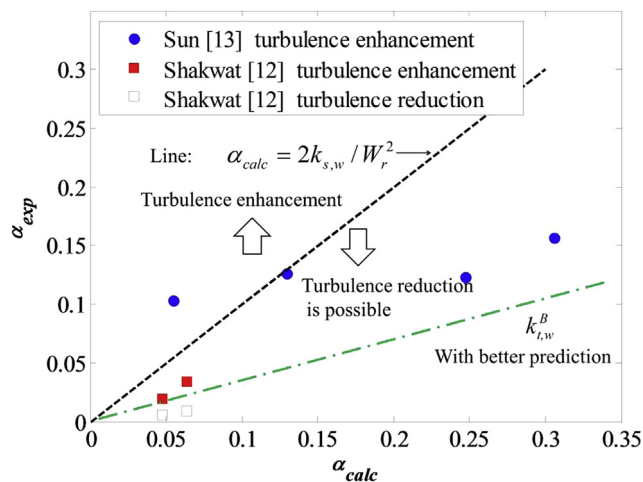

Fig. 5 − Comparison of the experimental measured void fraction and the calculated void fraction based on $\alpha_{calc} = 2k_{s,w}^W/W_r^2$ near the wall.

turbulence. It means that even with the total turbulence enhancement as compared to the single-phase turbulent flow, the underlying solely wall-induced part could still be reduced. In other words, the existence of bubbles in these cases also suppressed the generation of wall-induced turbulence. As for the mechanism of the turbulence reduction near the wall for small bubbles or low void fraction, Serizawa and Kataoka [4] attributed the following two effects to the bubbles: (1) large velocity fluctuations gradient near the bubble interface, which increases the turbulent energy dissipation; (2) an energy dumping effect due to the bubble deformation. For the cases with relatively larger bubble size and high void fractions, the two mechanisms shown in Fig. 7 might be responsible, which are the flow acceleration laminarization by bubbles and the suppression of the coherent structures. For UTBF, the lateral migration of bubbles formed the wall peak of the void fraction or the bubble layer near the wall [1,2]. As compared to the single-phase turbulent flow, the liquid flow near the wall that is confined between the wall and the bubble layer could be laminarized owing to the limited horizontal space and the acceleration of the liquid flow caused by the bubbles. In
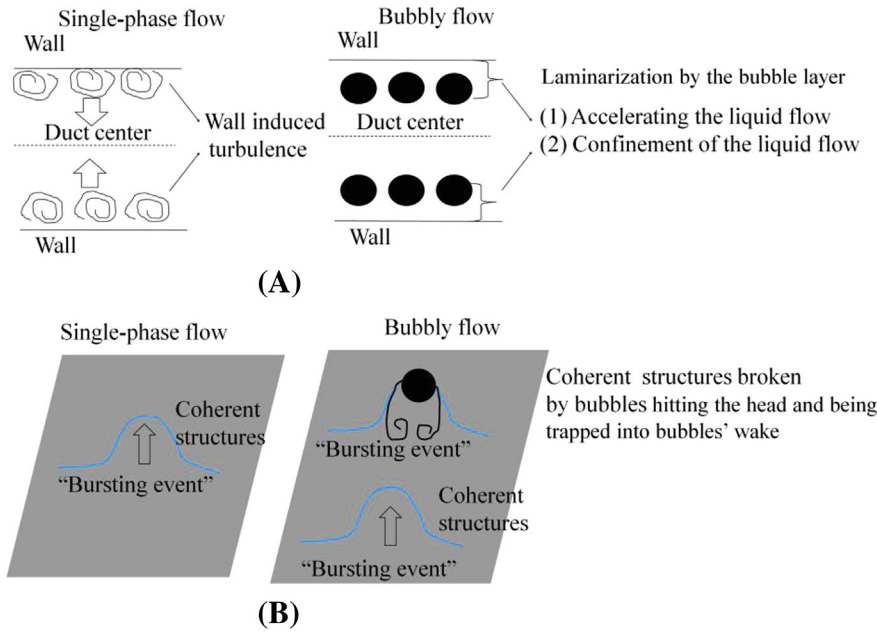
Fig. 7 − Schematic diagram of the mechanism for turbulence reduction by bubbles. (A) Flow acceleration. (b) Breaking the coherent structures.

addition, as is known, for the single-phase flow, the "bursting event" of the coherent structures [30] is the key process for the turbulence energy generation near the wall as shown in Fig. 7B. For UTBF, the passing-by of the bubbles whose size is similar to that of the coherent structures is able to break these structures by hitting their heads or trapping them in the bubbles' wake, and then suppressing the source of the wall-induced turbulence.

3.5.   Improvement of the turbulence modeling in UTBF

The above analysis indicates that for bubble-induced turbulence in the UTBF, the existence of a wall could suppress solely bubble-induced turbulence if we are comparing the turbulence near the wall to that in the duct center where there is less wall effect. Moreover, for wall-induced turbulence, the existence of bubbles, by contrast, could also suppress solely wall-induced turbulence in the UTBF for a wider range of the void fraction, even when the total turbulence is enhanced, as compared to the turbulence in the single-phase flow. These effects are not considered in the existing turbulence models of the UTBF. Among them, total turbulence is usually modeled by adding the bubble-induced turbulence to the wall-induced turbulence (such as $k-\varepsilon$ model), as shown in Fig. 8A. The starting point of this method is actually the wall-induced turbulence. For the UTBF with a small void fraction or in small ducts, the prediction could be acceptable because the wall-induced turbulence therein plays a more dominant role than that of the bubble-induced turbulence. However, when the bubble-induced turbulence becomes dominant, such as under a high void fraction or in a large duct, the above models will be inappropriate. In the present study, to improve the turbulence modeling of the UTBF, a new method is proposed by changing the starting point when modeling the turbulence for the turbulent bubbly flow under higher

void fractions or large bubble size or in a larger duct, as shown in Fig. 8B. Considering the dominant role of bubble-induced turbulence and bubble suppression on solely wall-induced turbulence, the starting point is changed from wall-induced turbulence to bubble-induced turbulence, by first modeling solely bubble-induced turbulence, such as in a
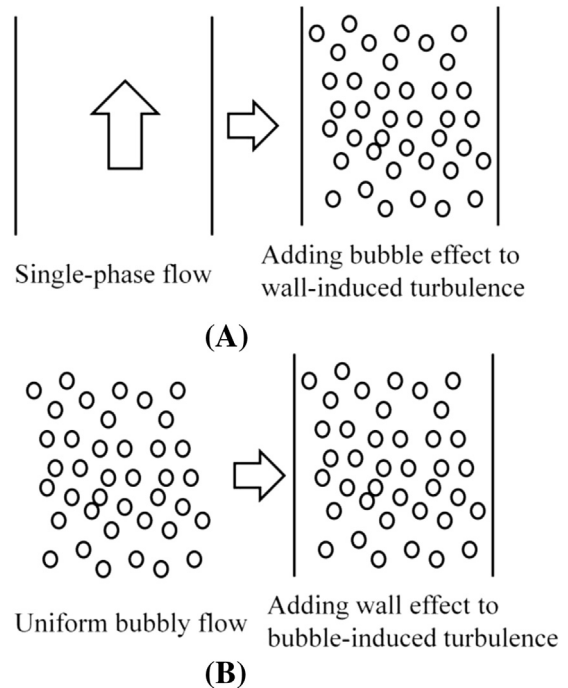
Fig. 8 − Schematic diagram of turbulence modeling method. (A) Previous method by correcting the bubble effect on wall-induced turbulence. (B) New method by correcting the wall effect on bubble-induced turbulence.

uniformly bubbly flow without wall effect. The turbulence in ducts is then corrected by adding the wall effect on bubble-induced turbulence, i.e., the wall suppression on solely bubble-induced turbulence. In this way, both the suppression effects of bubble-induced turbulence and wall-induced turbulence can be predicted.

4.   Conclusion

In summary, the present study discussed the turbulence modulation of the turbulent bubbly flow, focusing in particular on the wall effect on solely bubble-induced turbulence and the bubble effect on solely wall-induced turbulence according to the existing experimental databases. It was determined that, for the turbulence of the UTBF in ducts that could be induced by bubbles and the wall, (1) the existence of a wall could suppress solely bubble-induced turbulence; and (2) the existence of bubbles could also suppress solely wall-induced turbulence, even with the enhancement of total turbulence including both bubble-induced and wall-induced turbulence. Considering these characteristics, the current turbulence modeling method needs to be modified, especially when the bubble-induced turbulence plays a dominant role. In the present study, changing the starting point from wall-induced turbulence to bubble-induced turbulence is suggested for future turbulence modeling. To develop the new model, more studies on solely bubble-induced turbulence for UTBF and the wall effect are suggested based on the greater volume of experimental databases that will be derived from future work.

Acknowledgments

Conflicts of interest

The authors declare that there is no conflict of interest.

Nomenclature

D_B	Bubble size [m]
D_H	Duct hydraulic radius [m]
J	Superficial velocity [m^2/s^2]
k	Turbulent kinetic energy [m^2/s^2]
W_r	Relative velocity [m/s]
X	Quality [—]
$<ww>$	Axial turbulent intensity [m^2/s^2]

Greek letters

α	Void fraction

Subscripts

c	In the duct center
corner	Near the corner
g	Gas phase
l	Liquid phase
s	Single phase flow
t	Turbulent bubbly flow
w	Near the wall center

Subscripts

B	Bubble-induced
W	Wall-induced

REFERENCES

[1] A. Serizawa, Fluid Dynamics Characteristics of Two-phase Flow, Doctoral thesis, Kyoto University, 1974.

[2] A. Serizawa, I. Kataoka, I. Michiyoshi, Turbulence structure of air—water bubbly flow—II. local properties, Int. J. Multiphase Flow 2 (1975) 235—246.

[3] I. Michiyoshi, A. Serizawa, Turbulence in two-phase bubble flow, Nucl. Eng. Des. 95 (1986) 253—267.

[4] A. Serizawa, I. Kataoka, Turbulence suppression in bubbly two-phase flow, Nucl. Eng. Des. 122 (1990) 1—16.

[5] S. Wang, S. Lee, O. Jones, R. Lahey, 3-D turbulence structure and phase distribution measurements in bubbly two-phase flows, Int. J. Multiphase Flow 13 (1987) 327—343.

[6] T.J. Liu, S.G. Bankoff, Structure of air—water bubbly flow in a vertical pipe—I. liquid mean velocity and turbulence measurements, Int. J. Heat Mass Transfer 36 (1993) 1049—1060.

[7] M. Lance, J. Bataille, Turbulence in the liquid phase of a uniform bubbly air—water flow, J. Fluid Mech. 222 (1991) 95—118.

[8] M. Lance, J. Bataille, Homogeneous turbulence in bubbly flows, J. Fluids Eng. 113 (1991) 295—300.

[9] T. Hibiki, M. Ishii, Experimental study on interfacial area transport in bubbly two-phase flows, Int. J. Heat Mass Transfer 42 (1999) 3019—3035.

[10] S. Hosokawa, A. Tomiyama, Turbulence modification in gas—liquid and solid—liquid dispersed two-phase pipe flows, Int. J. Heat Mass Transfer 25 (2004) 489—498.

[11] S. Hosokawa, A. Tomiyama, Multi-fluid simulation of turbulent bubbly pipe flows, Chem. Eng. Sci. 64 (2009) 5308—5318.

[12] M.E. Shawkat, C.Y. Ching, M. Shoukri, Bubble and liquid turbulence characteristics of bubbly flow in a large diameter vertical pipe, Int. J. Multiphase Flow 34 (2008) 767—785.

[13] H.M. Sun, Study on Upward Air—water Two-phase Flow Characteristics in a Vertical Large Square Duct, Doctoral thesis, Kyoto University, 2014.

[14] T.G. Theofanous, J. Sullivan, Turbulence in two-phase dispersed flows, J. Fluid Mech. 116 (1982) 343—362.

[15] L. van Wijngaarden, On pseudo turbulence, Theor. Comput. Fluid Dyn. 10 (1998) 449—458.

[16] Y. Sato, M. Sadatomi, Momentum and heat transfer in two-phase bubble flow—I. Theory, Int. J. Multiphase Flow 7 (1981) 167—177.

[17] D.A. Drew, R.T. Lahey Jr., Phase distribution mechanisms in turbulent low-quality two-phase flow in a circular pipe, J. Fluid Mech. 117 (1982) 91—106.

[18] I. Kataoka, A. Serizawa, Modeling and Prediction of Turbulence in Bubbly Two-phase Flow, Proc. 2nd Int. Conf. Multiphase Flow, Kyoto, MO2, 1995, pp. 11—16.

[19] I. Kataoka, A. Serizawa, Basic equations of turbulence in gas—liquid two-phase flow, Int. Multiphase Flow 5 (1989) 843—855.

[20] R.T. Lahey Jr., The analysis of phase separation and phase distribution phenomena using two-fluid models, Nucl. Eng. Des. 122 (1990) 17—40.

[21] R.T. Lahey Jr., M. Lopez de Bertodano, O.C. Jones Jr., Phase distribution in complex geometry conduits, Nucl. Eng., Des. 141 (1993) 177—201.

[22] M. Lopez de Bertodano, Turbulent Bubbly Two-phase Flow in a Triangular Duct, Doctoral thesis, Resselaer Polytechnic Institute, 1992.

[23] M. Lopez de Bertodano Jr., R.T. Lahey Jr., O.C. Jones, Development of a k—ε model for bubbly two-phase flow, J. Fluids Eng. 116 (1994) 128—134.

[24] C. Morel, Turbulence modeling and first simulations in turbulent two-phase flows, CEA Grenoble, France.

[25] A. Troshko, Y.A. Hassan, A two-equation turbulence model of turbulent bubbly flows, Int. J. Multiphase Flow 27 (2001) 1965—2000.

[26] M.S. Politano, P.M. Carrica, J. Converti, A model for turbulent polydisperse two-phase flow in vertical channels, Int. J. Multiphase Flow 19 (2003) 1153—1182.

[27] J. Chahed, V. Roig, L. Masbernat, Eulerian—Eulerian two-fluid model for turbulent gas—liquid bubbly flows, Int. J. Multiphase Flow 29 (2003) 23—49.

[28] R. Rzehak, E. Krepper, Bubble-induced turbulence: comparison of CFD models, Nucl. Eng. Des. 258 (2013) 57—65.

[29] R. Rzehak, E. Krepper, CFD modeling of bubble-induced turbulence, Int. J. Multiphase Flow 55 (2013) 138—155.

[30] S.B. Pope, Turbulent flows, Cambridge University Press, Cambridge, 2000, pp. 267—322 (Chapter 7).