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# Inflationary gravitational waves in the effective field theory of modified gravity

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In the approach of the effective field theory of modified gravity, we derive the second-order action and the equation of motion for tensor perturbations on the flat isotropic cosmological background. This analysis accommodates a wide range of gravitational theories including Horndeski theories, its generalization, and the theories with spatial derivatives higher than second order (e.g., Hořava–Lifshitz gravity). We obtain the inflationary power spectrum of tensor modes by taking into account corrections induced by higher-order spatial derivatives and slow-roll corrections to the de Sitter background. We also show that the leading-order spectrum in concrete modified gravitational theories can be mapped on to that in General Relativity under a disformal transformation. Our general formula will be useful to constrain inflationary models from the future precise measurement of the B-mode polarization in the cosmic microwave background.

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## I. INTRODUCTION

The detection of primordial gravitational waves in the cosmic microwave background (CMB) offers an exciting possibility for probing the physics around the grand unified theory scale. In particular, the inflationary paradigm [1–3] predicts the generation of nearly scale-invariant primordial tensor and scalar perturbations with the tensor-to-scalar ratio $r$ less than the order of 0.1 [4,5]. Since the spectral index $n_s$ of scalar perturbations was measured by the Planck satellite in high precision ($n_s = 0.9603 \pm 0.0073$ at 68% C.L. [6]), the precise bounds on $r$ from the ongoing and upcoming CMB B-mode polarization experiments [7–10] are the next important step for approaching the origin of inflation.

Many of the single-field inflationary models proposed so far belong to a class of Horndeski theories [11]—the most general Lorentz-invariant scalar-tensor theories with second-order equations of motion. In fact, the leading-order power spectra of tensor and scalar perturbations were derived for inflationary models in the framework of Horndeski theories [12,13]. These results were employed to place observational constraints on individual models (such as slow-roll inflation [3], k-inflation [14], Starobinsky inflation [1], and Higgs inflation [15,16]) from the WMAP and Planck data [17,18].

There exist more general modified gravitational theories beyond the Horndeski domain. Choosing the so-called unitary gauge in which the perturbation of a scalar field $\phi$ on the flat Friedmann–Lemaître–Robertson–Walker (FLRW) background vanishes, the Horndeski Lagrangian can be expressed in terms of geometric scalar variables appearing in the $3+1$ Arnowitt–Deser–Misner (ADM) decomposition of space-time [19].

In Horndeski theories the coefficients in front of such ADM scalars have two particular relations, but it is possible to consider extended theories with arbitrary coefficients: Gleyzes–Langlois–Piazza–Vernizzi (GLPV) theories [20]. In general space-time, the equations of motion in GLPV theories should contain derivatives higher than second order, but the Hamiltonian analysis based on linear perturbations on the flat FLRW background shows that GLPV theories have one scalar propagating degree of freedom without ghostlike Ostrogradski instabilities [20–23]. This second-order property also holds for the odd-type perturbations on the spherically symmetric background [24].

The full action of GLPV theories cannot be generally mapped to that of Horndeski theories [22,25] under the so-called disformal transformation [26,27], so the two theories are not equivalent to each other. It is possible, however, to derive the two non-Horndeski pieces in the GLPV Lagrangian separately from a subset of the Horndeski Lagrangian under the disformal transformation.

Another example outside the Horndeski domain is Hořava–Lifshitz gravity [28], in which an anisotropic scaling in time and space plays a role for the realization of a power-counting renormalizable theory. In this case there are spatial derivatives up to sixth order, with which Lorentz invariance is explicitly broken. The building blocks of Hořava–Lifshitz gravity are the three-dimensional ADM geometric scalars invariant under a foliation-preserving diffeomorphism.

The effective field theory (EFT) of cosmological perturbations is a powerful framework to deal with low-energy degrees of freedom in a systematic and unified way [29–31]. This approach is not only useful to parametrize higher-order correlation functions of curvature perturbations generated during inflation [6,32,33] but also to perform a systematic study for the physics of a late-time cosmic acceleration induced by the modification of gravity [34–47]. In fact, recent studies [19,20,40,44–46] showed that the EFT approach can encompass a wide range of theories including Horndeski/GLPV theories and Hořava–Lifshitz gravity.

The EFT approach of Ref. [19] is based upon a general Lagrangian $L$ in unitary gauge that depends on the lapse $N$ and several ADM geometric scalars constructed from the extrinsic curvature $K_{\mu\nu}$ and the three-dimensional intrinsic curvature $\mathcal{R}_{\mu\nu}$. The action expanded up to second order in scalar metric perturbations shows that the theory has one scalar degree of freedom with spatial derivatives higher than second order in general, while the time derivatives are of second order. Both Horndeski and GLPV theories satisfy conditions for the absence of such higher-order spatial derivatives [19,43,44].

The original projectable version of Hořava–Lifshitz gravity, in which the lapse $N$ depends on the time $t$ alone, is plagued by the strong coupling problem [48,49]. In the nonprojectable version where $N$ depends on both time and space, the acceleration vector $a_i = \nabla_i \ln N$ does not vanish, and hence several scalar quantities like $\eta a_i a^i$ can be present in the Lagrangian ($\eta$ is a constant) [50]. In this case there are some parameter spaces of $\eta$ in which the strong coupling problem in the original theory can be alleviated. This strong coupling still remains an open issue without realizing a truly renormalizable and UV complete theory [51,52].

The nonprojectable version of Hořava–Lifshitz gravity can be incorporated in the EFT approach of Ref. [19] by taking into account additional geometric scalar quantities (associated with spatial derivatives up to sixth order) to the Lagrangian [45,46]. In Ref. [47] the second-order action for scalar perturbations was derived for the generic EFT Lagrangian encompassing Horndeski/GLPV theories and Hořava–Lifshitz gravity. This result can be useful for the computation of the primordial scalar power spectrum generated during inflation and for discussing conditions under which the ghosts and instabilities are absent (see Ref. [43] for a review).

In this paper we employ such a general EFT approach for the study of tensor perturbations on the flat FLRW background. Our analysis is more generic than those of Refs. [43,45,53] in that higher-order spatial derivatives appearing in Hořava–Lifshitz gravity are explicitly taken into account for the computation of the second-order action of tensor perturbations. Unlike Ref. [54], we do not consider the terms associated with the broken spatial diffeomorphism. We provide a general formula for the inflationary power spectrum of tensor modes by taking into account slow-roll corrections to the leading-order spectrum.

This paper is organized as follows. In Sec. II we present the action of our EFT approach and briefly review how several modified gravitational theories are incorporated in our general framework. In Sec. III we derive the second-order action and the equation of motion for tensor perturbations. In Sec. IV we obtain the spectrum of gravitational waves generated during inflation, and in Sec. V we apply the results to concrete modified gravitational theories. Section VI is devoted to conclusions.

## II. GENERAL EFT ACTION OF MODIFIED GRAVITY

The EFT of cosmological perturbations is based upon the $3+1$ decomposition of space-time described by the line element [55]

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \end{aligned} \tag{2.1}$$

where $N$ is the lapse function, $N^i$ is the shift vector, and $h_{ij}$ is the three-dimensional spatial metric. Introducing a unit vector $n_\mu = (-N, 0, 0, 0)$ orthogonal to the constant $t$ hypersurfaces $\Sigma_t$, the induced metric $h_{\mu\nu}$ on $\Sigma_t$ can be expressed of the form $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$.

The extrinsic curvature is defined by $K_{\mu\nu} = h^\lambda_\mu n_{\nu;\lambda} = n_{\nu;\mu} + n_\mu a_\nu$, where a semicolon represents a covariant derivative and $a_\nu \equiv n^\lambda n_{\nu;\lambda}$ is the acceleration vector. The scalar quantities that can be constructed from the extrinsic curvature are the trace of $K_{\mu\nu}$ and the square of $K_{\mu\nu}$, i.e.,

$$K \equiv K^\mu{}_\mu, \qquad \mathcal{S} \equiv K_{\mu\nu} K^{\mu\nu}. \tag{2.2}$$

The internal geometry of $\Sigma_t$ is characterized by the three-dimensional Ricci tensor $\mathcal{R}_{\mu\nu} = {}^{(3)}R_{\mu\nu}$, which is dubbed the intrinsic curvature. From $\mathcal{R}_{\mu\nu}$ we can construct the following scalar quantities:

$$\mathcal{R} \equiv \mathcal{R}^\mu{}_\mu, \qquad \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \qquad \mathcal{U} \equiv \mathcal{R}_{\mu\nu} K^{\mu\nu}. \tag{2.3}$$

Since it is possible to express the Riemann tensor $R_{\mu\nu\lambda\sigma}$ in terms of the Ricci tensor and scalar in three dimensions, we do not need to consider scalar combinations associated with $R_{\mu\nu\lambda\sigma}$.

In Hořava–Lifshitz gravity, there are other scalar quantities that generate spatial derivatives up to sixth order:

$$\mathcal{Z}_1 \equiv \nabla_i \mathcal{R} \nabla^i \mathcal{R}, \qquad \mathcal{Z}_2 \equiv \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk}. \tag{2.4}$$

We can also take into account the terms like $\mathcal{R}^j_i \mathcal{R}^k_j \mathcal{R}^i_k$ and $\mathcal{R}\mathcal{R}^j_i \mathcal{R}^i_j$, but they are irrelevant to the dynamics of linear scalar perturbations on the flat FLRW background. Hence, we do not incorporate those terms in the following analysis.

In the original Hořava–Lifshitz gravity [28], the space-time foliation is preserved by the space-independent reparametrization $t \to t'(t)$, so the lapse $N$ is assumed to be a function of time $t$ alone (which is called the projectability condition). This can be extended to a non-projectable version in which the lapse depends on both the spatial coordinate $x^i$ $(i = 1, 2, 3)$ and $t$ [50]. Since the acceleration $a_i = \nabla_i \ln N$ does not vanish in this case, we can also consider the scalar combinations

$$\alpha_1 \equiv a_i a^i, \qquad \alpha_2 \equiv a_i \Delta a^i, \qquad \alpha_3 \equiv \mathcal{R}\nabla_i a^i,$$
$$\alpha_4 \equiv a_i \Delta^2 a^i, \qquad \alpha_5 \equiv \Delta\mathcal{R}\nabla_i a^i, \tag{2.5}$$

where $\Delta \equiv \nabla_i \nabla^i$.

The action of general modified gravitational theories that depends on the above-mentioned scalar quantities is given by

$$S = \int d^4x \sqrt{-g} L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \ldots, \alpha_5; t), \tag{2.6}$$

where $g$ is a determinant of the metric $g_{\mu\nu}$ and $L$ is a Lagrangian. The action (2.6) encompasses Horndeski/GLPV theories and Hořava–Lifshitz gravity. In the following we will present explicit forms of the Lagrangians in these theories.

First of all, Horndeski theories are described by the Lagrangian

$$\begin{aligned} L = {} & G_2(\phi, X) + G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R - 2G_{4,X}(\phi, X)[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \\ & + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 \\ & - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}], \end{aligned} \tag{2.7}$$

where $\Box\phi \equiv (g^{\mu\nu}\phi_{;\nu})_{;\mu}$ and $G_j$ $(j = 2, \ldots, 5)$ are functions in terms of a scalar field $\phi$ and its kinetic energy $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $R$ and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor in four dimensions, respectively. Here and in the following, a lower index of $L$ denotes the partial derivatives with respect to the scalar quantities represented in the index, e.g., $G_{j,X} \equiv \partial G_j/\partial X$. In unitary gauge we have $\phi = \phi(t)$ and $X = -\dot{\phi}(t)^2/N^2$, where a dot represents a derivative with respect to $t$. Hence, the dependence of $\phi$ and $X$ in the action (2.7) is interpreted as that of the lapse $N$ and the time $t$. In fact, we can express the Lagrangian (2.7) of the form [19,20,44]

$$\begin{aligned} L = {} & A_2(N, t) + A_3(N, t)K + A_4(N, t)(K^2 - \mathcal{S}) \\ & + B_4(N, t)\mathcal{R} + A_5(N, t)K_3 \\ & + B_5(N, t)(\mathcal{U} - K\mathcal{R}/2), \end{aligned} \tag{2.8}$$

where $K_3 = K^3 - 3KK_{\mu\nu}K^{\mu\nu} + 2K_{\mu\nu}K^{\mu\lambda}K^{\nu}{}_{\lambda}$. Horndeski theories have the correspondence

$$A_2 = G_2 - XF_{3,\phi}, \tag{2.9}$$

$$A_3 = 2(-X)^{3/2}F_{3,X} - 2\sqrt{-X}G_{4,\phi}, \tag{2.10}$$

$$A_4 = -G_4 + 2XG_{4,X} + XG_{5,\phi}/2, \tag{2.11}$$

$$B_4 = G_4 + X(G_{5,\phi} - F_{5,\phi})/2, \tag{2.12}$$

$$A_5 = -(-X)^{3/2}G_{5,X}/3, \tag{2.13}$$

$$B_5 = -\sqrt{-X}F_5, \tag{2.14}$$

where $F_3$ and $F_5$ are auxiliary functions satisfying $G_3 = F_3 + 2XF_{3,X}$ and $G_{5,X} = F_5/(2X) + F_{5,X}$. From Eqs. (2.11)–(2.14), the two relations

$$A_4 = 2XB_{4,X} - B_4, \qquad A_5 = -XB_{5,X}/3 \tag{2.15}$$

hold, under which the number of six independent functions reduces to 4.

GLPV [20] generalized Horndeski theories in such a way that the coefficients $A_4$, $B_4$, $A_5$, and $B_5$ are not necessarily related to each other. The general action (2.6) can incorporate both Horndeski and GLPV theories described by the Lagrangian (2.8).

The action (2.6) also covers Hořava–Lifshitz gravity given by the Lagrangian

$$\begin{aligned} L = {} & \frac{M_{\rm pl}^2}{2}(\mathcal{S} - \lambda K^2 + \mathcal{R} + \eta_1\alpha_1) \\ & - \frac{1}{2}(g_2\mathcal{R}^2 + g_3\mathcal{Z} + \eta_2\alpha_2 + \eta_3\alpha_3) \\ & - \frac{1}{2M_{\rm pl}^2}(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2 + \eta_4\alpha_4 + \eta_5\alpha_5), \end{aligned} \tag{2.16}$$

where $M_{\rm pl} = 2.435 \times 10^{18}$ GeV is the reduced Planck mass and $\lambda, \eta_1, \ldots, \eta_5, g_2, \ldots, g_5$ are constants. The original Hořava–Lifshitz gravity [28] corresponds to the case $\eta_1 = \cdots = \eta_5 = 0$, whereas its healthy extension [50] involves the dependence of acceleration.

## III. SECOND-ORDER ACTION FOR TENSOR PERTURBATIONS

### A. Cosmological perturbations

The perturbed line element involving the four scalar perturbations $\delta N$, $\psi$, $\zeta$, $E$, and tensor perturbations $\gamma_{ij}$ can be written of the form

$$\begin{aligned} ds^2 = {} & -(1 + 2\delta N)dt^2 + 2\partial_i\psi dx^i dt \\ & + a^2(t)[(1 + 2\zeta)\hat{h}_{ij} + 2\partial_i\partial_j E]dx^i dx^j, \end{aligned} \tag{3.1}$$

where $a(t)$ is the scale factor and

$$\hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{mk}\gamma_{im}\gamma_{kj}, \tag{3.2}$$

with $\det \hat{h} = 1$. The tensor perturbation $\gamma_{ij}$ is traceless and divergence free, i.e., $\gamma_{ii} = \partial_i\gamma_{ij} = 0$. The last term on the rhs of Eq. (3.2) was introduced for the simplification of calculations, but it does not affect the second-order action of tensor modes [56].

Under the infinitesimal coordinate transformation $t \to t + \delta t$ and $x^i \to x^i + \delta^{ij}\partial_j\delta x$, the metric perturbation $E$ transforms as $E \to E - \delta x$. Throughout the paper we choose the gauge $E = 0$ to fix the spatial threading $\delta x$.

The field perturbation $\delta\phi$ transforms as $\delta\phi \to \delta\phi - \dot{\phi}\,\delta t$ under the gauge transformation. In Horndeski and GLPV theories, the unitary gauge $\delta\phi = 0$ is chosen to fix the time slicing $\delta t$. In the projectable version of Hořava–Lifshitz gravity [28], the lapse $N$ is a function of $t$ alone, and hence $\delta N = 0$. In the nonprojectable Hořava–Lifshitz gravity [50], there is no such restriction for the gauge choice. The different gauge choices associated with the temporal coordinate transformation do not affect the second-order action of tensor perturbations presented later.

On the flat FLRW background described by the line element $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, the extrinsic curvature and the intrinsic curvature are given, respectively, by $\bar{K}_{ij} = H\bar{h}_{ij}$ and $\bar{\mathcal{R}}_{ij} = 0$, where a bar represents the background values and $H = \dot{a}/a$ is the Hubble parameter. Then, the scalar quantities appearing in the Lagrangian $L$ of Eq. (2.6) are $\bar{N} = 1$, $\bar{K} = 3H$, $\bar{\mathcal{S}} = 3H^2$, $\bar{\mathcal{R}} = \bar{\mathcal{Z}} = \bar{\mathcal{U}} = 0$, $\bar{\mathcal{Z}}_1 = \bar{\mathcal{Z}}_2 = 0$, and $\bar{\alpha}_1 = \bar{\alpha}_2 = \cdots = \bar{\alpha}_5 = 0$.

Expanding the action (2.6) up to second order in scalar perturbations for the spatial gauge choice $E = 0$, we can obtain the equations of motion for the background and linear scalar perturbations without fixing the temporal gauge. Varying the first-order perturbed action with respect to $\delta N$ and $\delta\sqrt{h}$, respectively, the background equations are given by [19,44,47]

$$\bar{L} + L_{,N} - 3H\mathcal{F} = 0, \tag{3.3}$$

$$\bar{L} - \dot{\mathcal{F}} - 3H\mathcal{F} = 0, \tag{3.4}$$

where

$$\mathcal{F} \equiv L_{,K} + 2HL_{,\mathcal{S}}. \tag{3.5}$$

The linear scalar perturbation equations derived by varying the second-order action in terms of $\delta N$, $\psi$, and $\zeta$ are presented in Ref. [47].

### B. Second-order tensor action

Let us derive the second-order action of Eq. (2.6) for tensor perturbations. Regarding the extrinsic curvature, tensor modes satisfy the relations $K = 3H$ and $\delta K^i_j = \delta^{ik}\dot{\gamma}_{kj}/2$. Up to first order, the three-dimensional Ricci tensor reads

$$\mathcal{R}_{ij} = -\frac{1}{2}a^2\Delta\gamma_{ij}. \tag{3.6}$$

The three-dimensional Ricci scalar from tensor perturbations is a second-order quantity, which is given by

$$\mathcal{R} = \frac{1}{4}\delta^{ik}\delta^{jl}\gamma_{ij}\Delta\gamma_{kl}. \tag{3.7}$$

Then the quantity $\mathcal{Z}_1$ is fourth order in perturbations.

On using the above relations, the second-order action for tensor modes reduces to $S_h^{(2)} = \int d^4x a^3 L_h^{(2)}$, where $L_h^{(2)} = L_{,\mathcal{S}}\delta K^i_j\delta K^j_i + \mathcal{E}\mathcal{R} + L_{,\mathcal{Z}}\mathcal{R}^i_j\mathcal{R}^j_i + L_{,\mathcal{Z}_2}\mathcal{Z}_2$ with

$$\mathcal{E} \equiv L_{,\mathcal{R}} + \frac{1}{2}\dot{L}_{,\mathcal{U}} + \frac{3}{2}HL_{,\mathcal{U}}. \tag{3.8}$$

More explicitly, it is given by

$$S_h^{(2)} = \int d^4x \frac{a^3}{4}\delta^{ik}\delta^{jl}(L_{,\mathcal{S}}\dot{\gamma}_{ij}\dot{\gamma}_{kl} + \gamma_{ij}\mathcal{O}_t\gamma_{kl}), \tag{3.9}$$

where

$$\mathcal{O}_t \equiv \mathcal{E}\Delta + L_{,\mathcal{Z}}\Delta^2 - L_{,\mathcal{Z}_2}\Delta^3. \tag{3.10}$$

Note that there are no contributions to $S_h^{(2)}$ from the scalars (2.5). The condition for avoiding the tensor ghost corresponds to $L_{,\mathcal{S}} > 0$.

Varying the action (3.9) with respect to $\gamma_{ij}$, we obtain the equation of motion

$$\ddot{\gamma}_{ij} + \left(3H + \frac{\dot{L}_{,\mathcal{S}}}{L_{,\mathcal{S}}}\right)\dot{\gamma}_{ij} - c_t^2\Delta\gamma_{ij} - \frac{L_{,\mathcal{Z}}}{L_{,\mathcal{S}}}\Delta^2\gamma_{ij} + \frac{L_{,\mathcal{Z}_2}}{L_{,\mathcal{S}}}\Delta^3\gamma_{ij} = 0, \tag{3.11}$$

where

$$c_t^2 \equiv \frac{\mathcal{E}}{L_{,\mathcal{S}}}. \tag{3.12}$$

In the absence of spatial derivatives higher than second order, $c_t$ exactly corresponds to the propagation speed of gravitational waves. To avoid the small-scale Laplacian instability in this case, we require that $c_t^2 > 0$.

General Relativity corresponds to $G_4 = M_{\rm pl}^2/2$ and $G_5 = 0$ in the Horndeski Lagrangian (2.7), i.e., $-A_4 = B_4 = M_{\rm pl}^2/2$ and $A_5 = B_5 = 0$ in Eq. (2.8). In this case we have $L_{,\mathcal{S}} = M_{\rm pl}^2/2$, $\mathcal{E} = M_{\rm pl}^2/2$, $c_t^2 = 1$, $L_{,\mathcal{Z}} = 0$, and $L_{,\mathcal{Z}_2} = 0$, so Eq. (3.11) reduces to $\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \Delta\gamma_{ij} = 0$.

## IV. INFLATIONARY TENSOR MODES

In this section we derive the power spectrum of tensor perturbations generated during inflation.

### A. Power spectrum in Fourier space

We expand the tensor perturbation $\gamma_{ij}(\boldsymbol{x},\tau)$ into the Fourier series as $\gamma_{ij}(\boldsymbol{x},\tau)=\int\frac{d^3k}{(2\pi)^{3/2}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\hat{\gamma}_{ij}(\boldsymbol{k},\tau)$, where

$$\hat{\gamma}_{ij}(\boldsymbol{k},\tau)=\sum_{\lambda=+,\times}\hat{h}_\lambda(\boldsymbol{k},\tau)e^{(\lambda)}_{ij}(\boldsymbol{k}). \tag{4.1}$$

Here, $\boldsymbol{k}$ is a comoving wave number, $\tau\equiv\int a^{-1}dt$ is the conformal time, and $e^{(\lambda)}_{ij}$ ($\lambda=+,\times$) are symmetric polarization tensors. The polarization tensors are transverse ($k_je^{(\lambda)}_{ij}=0$) and traceless ($e^{(\lambda)}_{ii}=0$) with the normalization satisfying $e^{(\lambda)}_{ij}(\boldsymbol{k})e^{*(\lambda')}_{ij}(\boldsymbol{k})=\delta_{\lambda\lambda'}$. We write the Fourier mode $\hat{h}_\lambda(\boldsymbol{k},\tau)$ of the form

$$\hat{h}_\lambda(\boldsymbol{k},\tau)=h_\lambda(k,\tau)a_\lambda(\boldsymbol{k})+h^*_\lambda(k,\tau)a^\dagger_\lambda(-\boldsymbol{k}), \tag{4.2}$$

where the annihilation and creation operators $a_\lambda(\boldsymbol{k})$ and $a^\dagger_\lambda(\boldsymbol{k}')$ obey the commutation relation $[a_\lambda(\boldsymbol{k}),a^\dagger_{\lambda'}(\boldsymbol{k}')]=\delta_{\lambda\lambda'}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$.

On the quasi-de Sitter background, the conformal time is given by $\tau\simeq-1/(aH)$, so that the asymptotic past and future correspond to $\tau\to-\infty$ and $\tau\to0$, respectively. The tensor power spectrum $\mathcal{P}_h(k)$ is defined by the vacuum expectation value of $\hat{\gamma}_{ij}$ in the $\tau\to0$ limit, as $\langle0|\hat{\gamma}_{ij}(\boldsymbol{k}_1,0)\hat{\gamma}_{ij}(\boldsymbol{k}_2,0)|0\rangle=(2\pi^2/k_1^3)\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)\mathcal{P}_h(k_1)$. On using Eqs. (4.1) and (4.2), it follows that

$$\mathcal{P}_h(k)=\frac{k^3}{2\pi^2}(|h_+(k,0)|^2+|h_\times(k,0)|^2). \tag{4.3}$$

### B. Equation of motion for a canonical field

A canonically normalized field $v_\lambda(k,\tau)$ is defined by

$$v_\lambda(k,\tau)\equiv zh_\lambda(k,\tau),\qquad z\equiv a\sqrt{L_{,\mathcal{S}}/2}. \tag{4.4}$$

Then the kinetic term in the action (3.9) can be expressed as $S_K=\int d\tau d^3x\sum_{\lambda=+,\times}|v'_\lambda|^2/2$, where a prime represents a derivative with respect to $\tau$. From Eq. (3.11) each Fourier component $v_\lambda(k,\tau)$ obeys the equation of motion

$$v''_\lambda+\left[\mathcal{K}(k,\tau)-\frac{z''}{z}\right]v_\lambda=0, \tag{4.5}$$

where the function $\mathcal{K}(k,\tau)$ is defined as

$$\mathcal{K}(k,\tau)\equiv c_t^2k^2\left(1+c_1\frac{k^2}{a^2M_{\rm pl}^2}+c_2\frac{k^4}{a^4M_{\rm pl}^4}\right), \tag{4.6}$$

and

$$c_1\equiv-\frac{L_{,\mathcal{Z}}M_{\rm pl}^2}{\mathcal{E}},\qquad c_2\equiv-\frac{L_{,\mathcal{Z}_2}M_{\rm pl}^4}{\mathcal{E}}. \tag{4.7}$$

In the context of low-energy effective field theories, we will discuss the case where $\mathcal{K}(k,\tau)\simeq c_t^2k^2$, such that the linear form of the dispersion relation, $\omega=c_tk$, is not modified by the nonlinear terms in Eq. (4.6) well below the cutoff of the theories. Otherwise, we would need to know the UV completion of the theories, or our treatment would break down. In fact, the nonlinear terms are suppressed for the physical wave number $k_{\rm phys}=k/a$ much below the cutoff value $k_{\rm phys}^{\rm max}\equiv M_{\rm pl}/|c_1|^{1/2}$ or $M_{\rm pl}/|c_2|^{1/4}$. In Hořava–Lifshitz gravity [28] and in the trans-Planckian physics studied in Refs. [57–61], $k_{\rm phys}^{\rm max}$ is close to $M_{\rm pl}$, i.e., $|c_1|\sim|c_2|\sim O(1)$. In the EFT approach to inflation advocated by Weinberg [31], the cutoff scale is slightly smaller than $M_{\rm pl}$, say $\sqrt{\epsilon}M_{\rm pl}$, where $\epsilon=-\dot{H}/H^2$ is the slow-roll parameter typically of the order of 0.01.

Since we have the application to Hořava–Lifshitz gravity and the trans-Planckian physics in mind, we shall focus on the situation in which the cutoff scale $k_{\rm phys}^{\rm max}$ is much larger than the Hubble parameter $H$ during inflation. In this case the Hubble radius crossing occurs in the linear regime of the dispersion relation (i.e., $\mathcal{K}\simeq c_t^2k^2$), so that the second and third terms in the parentheses of Eq. (4.6) are regarded as small corrections to the first term. In other words, the parameter defined by

$$\sigma\equiv\frac{c_1H_k^2}{M_{\rm pl}^2} \tag{4.8}$$

is much smaller than 1, where $H_k$ is the Hubble parameter at $c_tk=aH$. Under this condition the EFT approach to inflation can be justified.

According to the previous discussion, we will solve Eq. (4.5) iteratively and write its solution in the form

$$v_\lambda=v_\lambda^{(0)}+v_\lambda^{(1)}, \tag{4.9}$$

where the leading-order perturbation $v_\lambda^{(0)}$ obeys the equation of motion

$$v_\lambda^{(0)\prime\prime}+\left(c_t^2k^2-\frac{z''}{z}\right)v_\lambda^{(0)}=0. \tag{4.10}$$

The field $v_\lambda^{(1)}$ induced by the nonlinear corrections to Eq. (4.6) satisfies

$$v_\lambda^{(1)\prime\prime}+\left(c_t^2k^2-\frac{z''}{z}\right)v_\lambda^{(1)} = -c_t^2\frac{k^4}{a^2M_{\rm pl}^2}\left(c_1+c_2\frac{k^2}{a^2M_{\rm pl}^2}\right)v_\lambda^{(0)}. \tag{4.11}$$

To solve Eq. (4.10), we take into account the slow-roll inflationary corrections to the leading-order solution on the de Sitter background [62]. We then substitute the leading-order solution into Eq. (4.11) to obtain an iterative solution of $v_\lambda^{(1)}$.

### C. Solutions to the tensor equations of motion

In the following we consider the situation in which the parameters defined by

$$\epsilon\equiv-\frac{\dot H}{H^2},\qquad \epsilon_{\mathcal S}\equiv\frac{\dot{L_{,\mathcal S}}}{HL_{,\mathcal S}},\qquad s\equiv\frac{\dot c_t}{Hc_t} \tag{4.12}$$

are much smaller than unity during inflation. The smallness of $\epsilon$ comes from the quasi de Sitter background. Dividing Eq. (3.4) by $2H^2L_{,\mathcal S}$, the term $\epsilon_{\mathcal S}$ appears in addition to $\epsilon$. Hence, $\epsilon_{\mathcal S}$ is at most the same order as $\epsilon$.

The tensor propagation speed square in Horndeski theories can be estimated as $c_t^2=1+O(\epsilon)$, so the parameter $s$ is of the order of $\epsilon^2$ (see Sec. V B). In GLPV theories, $c_t^2$ generally differs from 1. As we will see in Sec. V C, it is possible to obtain the Einstein frame with $c_t^2$ equivalent to 1 under the so-called disformal transformation. Provided that the cosmological background in the Einstein frame is quasi-de Sitter, we will show that the variation of $c_t^2$ in the original frame is small, i.e., $|s|\ll 1$.

The quantity $z''/z$, up to next-to-leading-order corrections, can be estimated as

$$\frac{z''}{z}=2(aH)^2\left(1-\frac{1}{2}\epsilon+\frac{3}{4}\epsilon_{\mathcal S}\right). \tag{4.13}$$

Introducing a dimensionless variable

$$y\equiv\frac{c_tk}{aH}, \tag{4.14}$$

its time derivative obeys $y'=-aHy(1-\epsilon-s)$. Then, Eq. (4.10) can be expressed as

$$(1-2\epsilon-2s)y^2\frac{d^2v_\lambda^{(0)}}{dy^2}-sy\frac{dv_\lambda^{(0)}}{dy} +\left(y^2-2+\epsilon-\frac{3}{2}\epsilon_{\mathcal S}\right)v_\lambda^{(0)}=0. \tag{4.15}$$

Here and in the following, we drop contributions of the slow-roll corrections of the order of $\epsilon^2$. In other words, we deal with the first-order slow-roll parameters as constants.

The solution to Eq. (4.15), after neglecting nonlinear terms in the slow-roll parameters, is given by

$$v_\lambda^{(0)}(y)=y^{(1+s)/2}\{\alpha_kH_\nu^{(1)}[(1+\epsilon+s)y] +\beta_kH_\nu^{(2)}[(1+\epsilon+s)y]\}, \tag{4.16}$$

where $\alpha_k$ and $\beta_k$ are integration constants, $H_\nu^{(1)}(x)$ and $H_\nu^{(2)}(x)$ are Hankel functions of the first and second kinds respectively, and

$$\nu=\frac{3}{2}+\epsilon+\frac{1}{2}\epsilon_{\mathcal S}+\frac{3}{2}s. \tag{4.17}$$

The Bunch–Davies vacuum corresponds to the choice $\beta_k=0$. On using the property $H_\nu^{(1)}(x\gg 1)\simeq -\sqrt{2/(\pi x)}e^{i[x+(3-2\nu)\pi/4]}$, the solution in the asymptotic past reads

$$v_\lambda^{(0)}(y\gg 1)\simeq-\alpha_k\sqrt{\frac{2}{\pi}}\frac{y^{s/2}}{\sqrt{1+\epsilon+s}}e^{i[(1+\epsilon+s)y+(3-2\nu)\pi/4]}. \tag{4.18}$$

The coefficient $\alpha_k$ is determined by the Wronskian condition $v_\lambda^{(0)}v_\lambda^{(0)*\prime}-v_\lambda^{(0)*}v_\lambda^{(0)\prime}=i$, such that (up to second order in the slow-roll parameters)

$$\alpha_k=-\frac{1}{4}\sqrt{\frac{\pi}{c_{tk}k}}(2+\epsilon+s), \tag{4.19}$$

where $c_{tk}$ is the value of $c_t$ at $c_tk=aH$ (i.e., at $y=1$). For the derivation of Eq. (4.19), we used the property that any time-dependent function $f(\tau)$ on the quasi-de Sitter background can be expanded around $y=1$ (denoted by the subscript $k$), as $f(\tau)=f(\tau_k)-(\dot f/H_k)\ln(\tau/\tau_k)$ [63]. For $\mu$ much smaller than 1, the quantity $y$ is also expanded as $y^\mu\simeq 1+\mu\ln(\tau/\tau_k)$, so the variation of $c_t$, $H$, and $L_{,\mathcal S}$ can be quantified as

$$c_t=c_{tk}y^{-s},\qquad H=H_ky^{\epsilon},\qquad L_{,\mathcal S}=L_{,\mathcal Sk}y^{-\epsilon_{\mathcal S}}. \tag{4.20}$$

Substituting Eq. (4.19) and $\beta_k=0$ into Eq. (4.16), we obtain

$$v_\lambda^{(0)}(y)=-\frac{\sqrt{\pi}}{2}\frac{aH}{(c_tk)^{3/2}}\left(1+\frac{1}{2}\epsilon+\frac{1}{2}s\right) \times y^{3/2}H_\nu^{(1)}[(1+\epsilon+s)y]. \tag{4.21}$$

Using the property $H_\nu^{(1)}(x\to 0)=-(i/\pi)\Gamma(\nu)(x/2)^{-\nu}$ and the relations (4.20), the solution $h_\lambda^{(0)}(y)=v_\lambda^{(0)}(y)/z$ long after the Hubble radius crossing ($y\to 0$) reduces to

$$h_\lambda^{(0)}(0) = i\frac{H_k}{\sqrt{2\pi L_{,\mathcal{S}k}}}\frac{2^\nu\Gamma(\nu)}{(c_{tk}k)^{3/2}}(1-\epsilon-s). \tag{4.22}$$

Expanding the function $2^\nu\Gamma(\nu)$ around $\nu = 3/2$, it follows that

$$h_\lambda^{(0)}(0) = i\frac{H_k}{\sqrt{L_{,\mathcal{S}k}}}\frac{1}{(c_{tk}k)^{3/2}}\left[1+(1-\gamma-\ln 2)\epsilon + \frac{1}{2}(2-\gamma-\ln 2)\epsilon_{\mathcal{S}} + \frac{1}{2}(4-3\gamma-3\ln 2)s\right], \tag{4.23}$$

where $\gamma = 0.5772\ldots$ is the Euler–Mascheroni constant.

The next step is to derive the solution to Eq. (4.11) by using the leading-order solution of Eq. (4.21) on the de Sitter background [obtained by setting $\epsilon = \epsilon_{\mathcal{S}} = s = 0$ and $a = -1/(H\tau)$ with $H = \text{constant}$], i.e., $v_\lambda^{(0)}(\tau) = -i(1+ic_tk\tau)e^{-ic_tk\tau}/[\sqrt{2}\tau(c_tk)^{3/2}]$. The speed of propagation for this mode, for large $k$'s, coincides, by construction, with $c_t$, such that this choice is consistent with the assumption that the corrections do not modify the standard propagation of tensor modes. Integrating Eq. (4.11) after substitution of the leading-order solution of $v_\lambda^{(0)}$, the resulting particular solution is given by

$$\begin{aligned} v_\lambda^{(1)}(\tau) &= \frac{e^{-ic_tk\tau}}{120\sqrt{2}c_t^4(c_tk)^{3/2}\tau}\frac{H^2}{M_{\rm pl}^2} \\ &\times\left[5\left(5c_1c_t^2 - 7c_2\frac{H^2}{M_{\rm pl}^2}\right)(3i-3c_tk\tau-2c_t^3k^3\tau^3)\right. \\ &-10i\left(2c_1c_t^2-7c_2\frac{H^2}{M_{\rm pl}^2}\right)c_t^4k^4\tau^4 \\ &\left.-6c_2\frac{H^2}{M_{\rm pl}^2}(7+2ic_tk\tau)c_t^5k^5\tau^5\right]. \end{aligned} \tag{4.24}$$

The correction $v_\lambda^{(1)}(\tau)$ has an oscillatory part $e^{-ic_tk\tau}$, which by construction follows the oscillations of the dominant contribution, $v_\lambda^{(0)}(\tau)$. Long after the Hubble radius crossing ($\tau \to 0$), the perturbation $h_\lambda^{(1)}(\tau) = v_\lambda^{(1)}(\tau)/z$ approaches

$$h_\lambda^{(1)}(0) = -i\frac{H}{8\sqrt{L_{,\mathcal{S}}}(c_tk)^{3/2}}\left(5c_1\frac{H^2}{c_t^2M_{\rm pl}^2} - 7c_2\frac{H^4}{c_t^4M_{\rm pl}^4}\right). \tag{4.25}$$

Since we are not interested in the next-order solution to Eq. (4.25), we can replace $H$, $c_t$, and $L_{,\mathcal{S}}$ for $H_k$, $c_{tk}$, and $L_{,\mathcal{S}k}$, respectively.

### D. Spectrum of inflationary tensor modes

The tensor power spectrum is known by substituting $h_\lambda(0) = h_\lambda^{(0)}(0) + h_\lambda^{(1)}(0)$ into Eq. (4.3), as

$$\mathcal{P}_h(k) = \frac{H_k^2}{\pi^2L_{,\mathcal{S}k}c_{tk}^3}\left[1-2(C+1)\epsilon - C\epsilon_{\mathcal{S}} - (3C+2)s - \frac{5}{4}\frac{\sigma}{c_{tk}^2} + \frac{7c_2}{4c_1^2}\frac{\sigma^2}{c_{tk}^4}\right], \tag{4.26}$$

where $C = \gamma - 2 + \ln 2 = -0.729637\ldots$ and $\sigma$ is defined by Eq. (4.8). The leading-order power spectrum is given by $\mathcal{P}_h^{\rm lead}(k) = H_k^2/(\pi^2L_{,\mathcal{S}k}c_{tk}^3)$.

The last two terms in the square bracket of Eq. (4.26), which correspond to the corrections induced by spatial derivatives higher than second order, are suppressed by the factor $\sigma \approx H_k^2/(k_{\rm phys}^{\rm max})^2$. Provided that $\sigma/c_{tk}^2 \ll \epsilon$, these terms are smaller than the slow-roll corrections.

We introduce the tensor spectral index $n_t$ as

$$n_t \equiv \left.\frac{d\ln\mathcal{P}_h(k)}{d\ln k}\right|_{c_tk=aH}. \tag{4.27}$$

On using the property $d\ln k/dt|_{c_tk=aH} = H(1-\epsilon-s)$ and defining the following slow-roll parameters

$$\eta \equiv \frac{\dot\epsilon}{H\epsilon}, \qquad \eta_{\mathcal{S}} \equiv \frac{\dot\epsilon_{\mathcal{S}}}{H\epsilon_{\mathcal{S}}}, \qquad \delta_s \equiv \frac{\dot s}{Hs}, \tag{4.28}$$

it follows that

$$\begin{aligned} n_t &= -2\epsilon - \epsilon_{\mathcal{S}} - 3s - 2\epsilon^2 - 5\epsilon s - \epsilon\epsilon_{\mathcal{S}} - \epsilon_{\mathcal{S}}s - 3s^2 \\ &\quad - 2(C+1)\epsilon\eta - C\epsilon_{\mathcal{S}}\eta_{\mathcal{S}} - (3C+2)s\delta_s \\ &\quad + \frac{5}{2c_{tk}^2}\sigma(\epsilon+s) - \frac{7c_2}{4c_1^2}\frac{1}{c_{tk}^4}\sigma^2(\epsilon+s), \end{aligned} \tag{4.29}$$

which should be evaluated at $c_tk = aH$. The leading-order spectral index is given by $n_t^{\rm lead} = -2\epsilon - \epsilon_{\mathcal{S}} - 3s$.

## V. APPLICATION TO CONCRETE THEORIES

We estimate the inflationary tensor power spectrum and its spectral index in concrete modified gravitational theories by using the general results derived in Sec. IV.

### A. Theories with higher-order spatial derivatives

Let us consider the theories described by the Lagrangian

$$\begin{aligned} L &= \frac{M_{\rm pl}^2}{2}(\mathcal{S} - \lambda K^2 + \mathcal{R}) + A_2(N,t) + A_3(N,t)K \\ &\quad + \frac{M_{\rm pl}^2}{2}\eta_1\alpha_1 - \frac{1}{2}(g_2\mathcal{R}^2 + g_3\mathcal{Z} + \eta_2\alpha_2 + \eta_3\alpha_3) \\ &\quad - \frac{1}{2M_{\rm pl}^2}(g_4\mathcal{Z}_1 + g_5\mathcal{Z}_2 + \eta_4\alpha_4 + \eta_5\alpha_5). \end{aligned} \tag{5.1}$$

For $A_2 = A_3 = 0$ this corresponds to the Lagrangian (2.16) of Hořava–Lifshitz gravity, including both the projectable ($\alpha_i = 0$) and nonprojectable ($\alpha_i \neq 0$) versions.

We take into account the terms $A_2(N,t)$ and $A_3(N,t)K$ in Eq. (5.1) to realize inflation by a scalar degree of freedom. In fact, the Lagrangian $L = (M_{\rm pl}^2/2)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi$ of the kinetic braiding theories [64] reduces to Eq. (5.1) with $\lambda = 1$, $A_2 = G_2 - XF_{3,\phi}$, $A_3 = 2(-X)^{3/2}F_{3,X}$, $\eta_1 = \cdots = \eta_5 = 0$ and $g_2 = \cdots = g_5 = 0$ in the unitary gauge, where we used the fact that the four-dimensional Ricci scalar is expressed as $R = \mathcal{S} - K^2 + \mathcal{R}$ up to a boundary term. The field $\phi$ is responsible for the cosmic acceleration as it happens for k-inflation ($G_3 = 0$) and potential-driven slow-roll inflation [$G_3 = 0$ and $G_2 = -X/2 - V(\phi)$].

Since $L_{,\mathcal{S}} = \mathcal{E} = M_{\rm pl}^2/2$, $c_1 = g_3$, and $c_2 = g_5$, Eqs. (4.26) and (4.29) read

$$\mathcal{P}_h(k) = \frac{2H_k^2}{\pi^2 M_{\rm pl}^2}\left[1 - 2(C+1)\epsilon - \frac{5}{4}\sigma + \frac{7g_5}{4g_3^2}\sigma^2\right], \tag{5.2}$$

$$n_t = -2\epsilon - 2\epsilon^2 - 2(C+1)\epsilon\eta + \frac{5}{2}\sigma\epsilon - \frac{7g_5}{4g_3^2}\sigma^2\epsilon, \tag{5.3}$$

where $\sigma = g_3 H_k^2/M_{\rm pl}^2$. If $g_3 = g_5 = 0$, then the last two terms in Eqs. (5.2) and (5.3) vanish. In this case, the above tensor power spectrum reduces to the one in standard slow-roll inflation [62].

The contributions from the terms $A_2(N,t)$ and $A_3(N,t)K$ do not directly appear in Eqs. (5.2) and (5.3), but they affect the tensor power spectrum indirectly through the background equations of motion (3.3) and (3.4).

Since the leading-order spectrum is $\mathcal{P}_h^{\rm lead}(k) = 2H_k^2/(\pi^2 M_{\rm pl}^2)$, the energy scale of inflation is directly known from the measurement of primordial gravitational waves. More concretely, we have $H_k/M_{\rm pl} \simeq \pi\sqrt{r\mathcal{P}_s(k)/2}$, where $\mathcal{P}_s(k) \simeq 2.2 \times 10^{-9}$ is the observed scalar power spectrum [6] and $r = \mathcal{P}_h^{\rm lead}(k)/\mathcal{P}_s(k)$ is the tensor-to-scalar ratio. On using the observational bound $r \lesssim 0.2$ [6], we have that $H_k/M_{\rm pl} \lesssim 4 \times 10^{-5}$. Hence, for $|g_3|, |g_5| \lesssim 1$, the corrections induced by spatial derivatives higher than second order are suppressed compared to the slow-roll corrections (typically of the order of 0.01).

Provided that $H$ decreases during inflation, the tensor spectrum is red tilted ($n_t \simeq -2\epsilon < 0$). From the background Eqs. (3.3) and (3.4), we obtain $M_{\rm pl}^2(3\lambda - 1)\dot{H} = A_{2,N} + 3HA_{3,N}$. If $\lambda > 1/3$, then the condition $\dot{H} < 0$ translates to $A_{2,N} + 3HA_{3,N} < 0$. In the unitary gauge, the field kinetic energy is given by $X = -N^{-2}\dot{\phi}^2$, so the Hubble parameter decreases for $A_{2,X} + 3HA_{3,X} < 0$.

## B. Horndeski theories

In the unitary gauge, the Lagrangian (2.7) of Horndeski theories is equivalent to Eq. (2.8) with the relations (2.9)–(2.14). On using the fact that the term $K_3$ is given by $K_3 = 3H(2H^2 - 2KH + K^2 - \mathcal{S})$ up to quadratic order in the perturbations on the flat FLRW background, we have $L_{,\mathcal{S}} = G_4(1+\epsilon_1)$ and $\mathcal{E} = G_4(1+\epsilon_2)$, where

$$\epsilon_1 \equiv -\frac{2XG_{4,X}}{G_4} - \frac{XG_{5,\phi}}{2G_4} + \frac{H(-X)^{3/2}G_{5,X}}{G_4}, \tag{5.4}$$

$$\epsilon_2 \equiv \frac{XG_{5,\phi}}{2G_4} - \frac{XG_{5,X}\ddot{\phi}}{G_4}. \tag{5.5}$$

The terms $\epsilon_1$ and $\epsilon_2$, which involve $X$, work as the slow-roll corrections to the leading-order contribution $G_4$. In fact, all these terms appear on the rhs of the background equation for $\epsilon$ (Eq. (9) of Ref. [65]), so they are the same order as $\epsilon$. The tensor propagation speed square is given by $c_t^2 \simeq 1 - \epsilon_1 + \epsilon_2 + O(\epsilon^2)$, and hence $s = \epsilon_2\eta_2/2 - \epsilon_1\eta_1/2 + O(\epsilon^3)$, where $\eta_j \equiv \dot{\epsilon}_j/(H\epsilon_j)$ with $j = 1, 2$. In the following, we set $G_4 = (M_{\rm pl}^2/2)F(\phi, X)$, where $F(\phi, X)$ is a dimensionless function with respect to $\phi$ and $X$. Then, the slow-roll parameter $\epsilon_{\mathcal{S}}$ can be expressed as $\epsilon_{\mathcal{S}} = \epsilon_F + \epsilon_1\eta_1 + O(\epsilon^3)$, where $\epsilon_F \equiv \dot{F}/(HF)$.

The tensor power spectrum and its spectral index, up to next-to-leading-order terms, read

$$\mathcal{P}_h(k) = \frac{2H_k^2}{\pi^2 M_{\rm pl}^2 F}\left[1 - 2(C+1)\epsilon - C\epsilon_F + \frac{\epsilon_1}{2} - \frac{3\epsilon_2}{2}\right], \tag{5.6}$$

$$\begin{aligned} n_t = {} & -2\epsilon - \epsilon_F - 2\epsilon^2 - \epsilon\epsilon_F + \frac{1}{2}\epsilon_1\eta_1 - \frac{3}{2}\epsilon_2\eta_2 \\ & - 2(C+1)\epsilon\eta - C\epsilon_F\eta_F, \end{aligned} \tag{5.7}$$

where $\eta_F \equiv \dot{\epsilon}_F/(H\epsilon_F)$.

Compared to Eq. (5.2), the leading-order power spectrum $\mathcal{P}_h^{\rm lead}(k) = 2H_k^2/(\pi^2 M_{\rm pl}^2 F)$ of Eq. (5.6) is divided by the term $F$. This term is associated with the conformal factor $\Omega^2$ under the transformation $\hat{g}_{\mu\nu} = \Omega^2(\phi, X)g_{\mu\nu}$. In the following, we study the case in which the conformal factor depends on $\phi$ alone, i.e., on $t$ in unitary gauge. This assumption is justified provided that the $X$ dependence in $\Omega^2$ works only as slow-roll corrections to the leading-order $\phi$-dependent term. Under the conformal transformation $\hat{g}_{\mu\nu} = \Omega^2(t)g_{\mu\nu}$, the coefficients $A_4$ and $B_4$ in Eq. (2.8) transform, respectively, as [22]

$$\hat{A}_4 = \Omega^{-2}A_4\left(1 - \frac{3\dot{\Omega}}{N\Omega}\frac{A_5}{A_4}\right), \tag{5.8}$$

$$\hat{B}_4 = \Omega^{-2}B_4\left(1 + \frac{\dot{\Omega}}{2N\Omega}\frac{B_5}{B_4}\right), \tag{5.9}$$

where $A_4 = -G_4[1 + O(\epsilon)]$ and $B_4 = G_4[1 + O(\epsilon)]$ from Eqs. (2.11) and (2.12). Since the second terms in the parentheses of Eqs. (5.8) and (5.9) can be regarded as slow-roll corrections, we have $\hat{A}_4 = -\Omega^{-2}G_4[1 + O(\epsilon)]$ and $\hat{B}_4 = \Omega^{-2}G_4[1 + O(\epsilon)]$. Choosing the conformal factor $\Omega^2 = 2G_4/M_{\rm pl}^2 = F$, it follows that $\hat{A}_4 = -(M_{\rm pl}^2/2)[1 + O(\epsilon)]$ and $\hat{B}_4 = (M_{\rm pl}^2/2)[1 + O(\epsilon)]$.

Under the conformal transformation $\hat{g}_{\mu\nu} = \Omega^2(t)g_{\mu\nu}$, the structure of the Lagrangian (2.8) is preserved with the modified leading-order coefficients $\hat{A}_2 = \Omega^{-4}A_2$, $\hat{A}_3 = \Omega^{-3}A_3$, $\hat{A}_5 = \Omega^{-1}A_5$, and $\hat{B}_5 = \Omega^{-1}B_5$ in the presence of slow-roll corrections [involving the derivative $\dot{\Omega}/(N\Omega)$] [22]. This means that, for the choice $\Omega^2 = F$, the leading-order tensor spectrum in the transformed (Einstein) frame can be derived by setting $L_{,\mathcal{S}} = M_{\rm pl}^2/2$ and $c_t = 1$ in Eq. (4.26), i.e., $\hat{\mathcal{P}}_h^{\rm lead}(k) = 2\hat{H}_k^2/(\pi^2 M_{\rm pl}^2)$. Since the Hubble parameters in two frames are related to each other as $\hat{H} = [H + \dot{F}/(2NF)]/\sqrt{F}$, the spectrum $\hat{\mathcal{P}}_h^{\rm lead}(k)$ is equivalent to $\mathcal{P}_h^{\rm lead}(k) = 2H_k^2/(\pi^2 M_{\rm pl}^2 F)$ at leading order in slow roll. Provided that the null energy condition is not violated in the Einstein frame, the Hubble parameter $\hat{H}$ decreases, in which case the tensor power spectrum is red tilted.

The above properties can be notably seen in the Higgs inflationary scenario with the scalar-field potential $V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$ and the function $F = 1 + \zeta\phi^2/M_{\rm pl}^2$, where $\zeta$ is a nonminimal coupling [15] (see also Refs. [66]). To realize the self-coupling $\lambda$ of the order of 0.1, the nonminimal coupling is constrained to be $\zeta = O(10^4)$ from the CMB normalization. For $\zeta \gg 1$, the quantity $F$ is related to the number of $e$-foldings $N_e$ from the end of inflation, as $F \simeq 4N_e/3$ [67], which is much larger than 1 on scales relevant to the CMB anisotropies. The action in the Einstein frame is characterized by a canonically normalized field with the potential $\hat{V} = V(\phi)/F^2$ [68], in which case the tensor spectrum $\mathcal{P}_h^{\rm lead}(k) = 2\hat{H}_k^2/(\pi^2 M_{\rm pl}^2)$ is red tilted due to the decrease of $\hat{H}$.

### C. GLPV theories

Let us proceed to the GLPV theories in the unitary gauge, i.e., the Lagrangian (2.8). In this case, the functions $L_{,\mathcal{S}}$ and $\mathcal{E}$ are given by $L_{,\mathcal{S}} = -A_4(1 + \epsilon_1)$ and $\mathcal{E} = B_4(1 + \epsilon_2)$, respectively, where

$$\epsilon_1 = \frac{3HA_5}{A_4}, \qquad \epsilon_2 = \frac{\dot{B}_5}{2B_4}. \tag{5.10}$$

Provided that $\epsilon_1$ and $\epsilon_2$ are regarded as slow-roll corrections to the leading-order terms of $L_{,\mathcal{S}}$ and $\mathcal{E}$, we have $c_t^2 = -(B_4/A_4)(1 - \epsilon_1 + \epsilon_2)$. The difference from Horndeski theories is that $A_4$ and $B_4$ are not related with each other, so $c_t^2$ generally differs from 1. Then, the leading-order tensor spectrum is given by

$$\mathcal{P}_h^{\rm lead}(k) = \frac{H_k^2}{\pi^2 |A_4| c_{tk,{\rm lead}}^3}, \tag{5.11}$$

where $c_{tk,{\rm lead}}^2 = -B_4/A_4$.

We perform the disformal transformation given by $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, where $\Gamma(\phi, X)$ is a function in terms of $\phi$ and $X$ [26,27]. In Ref. [22], it was shown that the structure of the GLPV action is preserved under this transformation.[1] The coefficients $A_4$ and $B_4$ in the Lagrangian (2.8) are transformed as

$$\tilde{A}_4 = \sqrt{1 + \Gamma X} A_4, \qquad \tilde{B}_4 = \frac{B_4}{\sqrt{1 + \Gamma X}}. \tag{5.12}$$

In the new frame, the tensor propagation speed square is given by $\tilde{c}_{t,{\rm lead}}^2 = -\tilde{B}_4/\tilde{A}_4 = c_{t,{\rm lead}}^2/(1 + \Gamma X)$. If we choose the function

$$\Gamma = -\frac{1 - c_{t,{\rm lead}}^2}{X}, \tag{5.13}$$

then it follows that $\tilde{c}_{t,{\rm lead}}^2 = 1$. In this case, the coefficients in Eq. (2.8) are transformed as $\tilde{A}_2 = A_2/c_{t,{\rm lead}}$, $\tilde{A}_3 = A_3$, $\tilde{A}_4 = c_{t,{\rm lead}}A_4$, $\tilde{B}_4 = B_4/c_{t,{\rm lead}}$, $\tilde{A}_5 = c_{t,{\rm lead}}^2 A_5$, and $\tilde{B}_5 = B_5$. Since $\tilde{c}_{t,{\rm lead}}^2 = 1$ in the new frame, the leading-order spectrum becomes $\mathcal{P}_h^{\rm lead}(k) = \tilde{H}_k^2/(\pi^2|\tilde{A}_4|)$. If we make the conformal transformation $\hat{g}_{\mu\nu} = \Omega^2(t)\tilde{g}_{\mu\nu}$ further with $\Omega^2 = 2|\tilde{A}_4|/M_{\rm pl}^2$, the resulting leading-order spectrum reduces to $\mathcal{P}_h^{\rm lead}(k) = 2\hat{H}_k^2/(\pi^2 M_{\rm pl}^2)$.

Under the disformal transformation $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, the lapse function $N$ is generally transformed to $\tilde{N} = N\sqrt{1 + \Gamma X}$ [22,25]. Setting $N = 1$ for the background, the choice of $\Gamma$ in Eq. (5.13) can be interpreted as $\tilde{N} = c_{t,{\rm lead}}$. The Hubble parameters in the Einstein and original frames are related with each other as $\tilde{H} = H/\tilde{N} = H/c_{t,{\rm lead}}$. This leads to the relation $\tilde{\epsilon} = \epsilon + s$, where $\tilde{\epsilon} = -\dot{\tilde{H}}/(\tilde{N}\tilde{H}^2)$ and $s = \dot{c}_{t,{\rm lead}}/(Hc_{t,{\rm lead}})$. Provided that the cosmological background in the Einstein frame is quasi-de Sitter, we have that $\tilde{\epsilon} \ll 1$, and hence $|s| \ll 1$. Thus, the assumption $|s| \ll 1$ used to derive the tensor power spectrum (4.26) is justified.

[1] In the presence of an additional matter, there is a mixing between the sound speeds of the scalar field $\phi$ and matter in GLPV theories even for the metric frame minimally coupled to matter [20,44]. The disformal transformation gives rise to a kinetic-type coupling of the scalar field with matter in the transformed frame [22,27], which helps us to understand the origin of such a nontrivial mixing. Here, we do not take into account an additional matter, as we are interested in the application to single-field inflation.

The above discussion shows that the combination of the disformal and conformal transformations, $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, can lead to a metric frame in which the leading-order tensor power spectrum is of the standard form that depends on the Hubble parameter $\hat{H}_k$ alone. This conclusion is consistent with the recent results of Ref. [53] in which the authors took the EFT approach without having the direct connection to particular modified gravitational theories.

## VI. CONCLUSIONS

We have studied tensor perturbations on the flat FLRW background for the general action (2.6) that encompasses most of the modified gravitational theories proposed in the literature—including Horndeski theories, GLPV theories, and Hořava–Lifshitz gravity. The equation of motion (3.11), which follows from the second-order action (3.9), involves the spatial derivatives higher than second order for the theories where the Lagrangian $L$ depends on $\mathcal{Z}$ or $\mathcal{Z}_2$.

We derived the inflationary power spectrum of tensor modes under the condition that the cutoff scale $k_{\rm phys}^{\rm max}$ associated with the nonlinear terms of Eq. (4.6) is much larger than the Hubble parameter $H_k$ at $c_t k = aH$ during inflation. On using the small parameter $\sigma$ of the order of $H_k^2/(k_{\rm phys}^{\rm max})^2$, the solution to Eq. (4.5) is obtained iteratively on the de Sitter background. Taking into account the slow-roll corrections to the leading-order solution as well, the resulting tensor power spectrum is given by Eq. (4.26) with the spectral index (4.29).

The corrections from the higher-order spatial derivatives to the leading-order power spectrum are suppressed by the factor $\sigma/c_{tk}^2$. This conclusion is consistent with the effect of modified trans-Planckian dispersion relations on the inflationary power spectrum [58–61]. For $k_{\rm phys}^{\rm max}$ close to $M_{\rm pl}$ and for $c_{tk}$ not very much smaller than 1, the corrections induced by the spatial derivatives higher than second order are smaller than the slow-roll corrections arising from the deviation from the de Sitter background.

We applied our general formula of the inflationary tensor power spectrum to a number of concrete modified gravitational theories. For the Lagrangian (5.1), which encompasses kinetic braiding models and Hořava–Lifshitz gravity, the leading-order spectrum is directly related to $H_k$, as $\mathcal{P}_h^{\rm lead}(k) = 2H_k^2/(\pi^2 M_{\rm pl}^2)$.

In Horndeski theories, where the tensor propagation speed is 1 at leading order in slow roll, $\mathcal{P}_h^{\rm lead}(k)$ involves a dimensionless factor $F = 2G_4/M_{\rm pl}^2$ in the denominator. Under the conformal transformation $\hat{g}_{\mu\nu} = F g_{\mu\nu}$, the spectrum in the Einstein frame simply reduces to $\mathcal{P}_h^{\rm lead}(k) = 2\hat{H}_k^2/(\pi^2 M_{\rm pl}^2)$.

In GLPV theories, the leading-order tensor spectrum (5.11) involves the terms $A_4$ and $c_{tk,\rm lead}^2 = -B_4/A_4$. We showed that, under the disformal transformation $\hat{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu} + \Gamma(\phi, X)\partial_\mu\phi\partial_\nu\phi$, it is possible to find a frame in which $\hat{c}_{tk,\rm lead}^2 = 1$ and $\hat{A}_4 = -M_{\rm pl}^2/2$ up to slow-roll corrections. Thus, the prediction of inflationary tensor modes is robust in that there exists the metric frame in which the leading-order spectrum is simply proportional to $\hat{H}_k^2$ in a vast class of modified gravitational theories.

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