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"Auction versus Dealership Markets: Impact of
Proprietary Trading with Transaction Fees"

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Auction versus Dealership Markets: Impact of Proprietary Trading with Transaction Fees*

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Abstract. In this study, we consider a one-period financial market with a monopolistic dealer/broker and an infinite number of investors. While the dealer who trades on his own account (with proprietary trading) simultaneously sets both the transaction fee and the asset price, the broker who brings investors' orders to the market (with no proprietary trading) sets only the transaction fee, given that the price is determined according to the market-clearing condition among investors. We analyze the impact of proprietary trading on the asset price, transaction fee, trading volume, and the welfare of investors. Results show that proprietary trading increases both the trading volume and the transaction fee, and improves social welfare. Our study effectively demonstrates how proprietary trading affects market equilibrium and welfare of investors.

Keywords: Proprietary trading, dealer market, brokered market, transaction fees.

JEL classification: D53, G12, D42.

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1 Introduction

Recently, the global financial crisis has triggered a reassessment of the economic costs and benefits of banks' involvement in proprietary trading and other activities in financial markets. In response to the crisis, financial authorities in several countries have either adopted or are considering adopting regulatory measures on investment banking, including the Volcker rule in the US, the Liikanen Report to the EU, and the proposals of the Vickers Commission for the UK.

Table 1: Comparison of selected structural reform proposals related to proprietary trading

Approach	Volcker	Liikanen	Vickers
Deal as principal in securities and derivatives	No	No	No
Engage in market making	Yes	No	No

Source: Gambacorta and van Rixtel (2013)

As seen in Table 1, which concerns proprietary trading, while there is a consensus on prohibiting principal trading (trading for banks' own profit rather than on behalf of customers), the proposals differ in whether or not to allow market-making activities.[1] The question of whether to ban proprietary trading as a market-making activity[2] is intrinsically related to the two types of trading systems: the brokered market and the dealer market.[3] In a brokered market, brokers present information in the form of the market price to potential buyers and sellers and collect transaction fees in return. There are no special agents who trade assets on their own account to make a market (i.e., proprietary trading is prohibited). On the other hand, in a dealer market, dealers play the role of market makers and determine the price at which they are willing to buy and sell an asset (i.e., proprietary trading is allowed). A natural question arises: which system is better from the viewpoint of market liquidity and welfare? To answer this question, we need to examine the impact of proprietary trading on the asset price, the transaction fee, the trading volume, and welfare of investors.

[1] Although there are other differences among these proposals, we focus on the impact of proprietary trading as market making activities.

[2] We use the term "proprietary trading" for "proprietary trading as market-making activities".

[3] In the real world, investors trade financial assets in mainly three types of market organizations: an auction market, a brokered market, and a dealer market. In an auction market, buyers and sellers directly confront each other when bargaining their price. When there are insufficient participants in an auction market, investors need to trade through brokers (e.g., Ritter et al., 2008).

As for the two trading systems, several theoretical studies are related to the current study, including Röell (1990), Fishman and Longstaff (1992), Sarkar (1995), and Bernhardt and Taub (2010). The models presented in these studies have examined the impact of dual trading (trading on the broker/dealer's own account in addition to executing customers' orders). In particular, they focus on how the dealer strategically uses the private information, extracted from orders, to make profits. The standard models assume a risk-neutral and perfectly competitive market maker, which is distinguished from a dealer. Under this assumption, the market price is equal to the conditional expectation of the asset payoff, given the market maker's information, as in Kyle (1985). However, in reality, in many cases, one agent plays the role of the dealer and the market maker simultaneously. Moreover, in actual financial markets, competition among market makers is not perfect. Although obvious conflicts of interest exist that are inherent in determining prices when dealers execute customers' orders against their own account, market makers may have the market power to determine or at least to affect asset prices. Also, unlike many of the previous studies mentioned, the broker/dealer and the investors are not necessarily risk neutral.

Since existing models focus on informed trading, transaction fees in imperfectly competitive situations are rarely considered. However, as the literature of market microstructure grows, the effect of transaction fees on market participants becomes an important issue (e.g., Kyle and Obizhaeva, 2013). One exception is Sarkar (1995), who incorporates a commission fee as an extension to his basic dual trading model. In his study, the transaction fee is considered as a charge for market access, which is independent of the order size and dependent only on whether or not traders make a trade. The dealer determines the fee by the zero-profit condition that his expected trading profits plus his income from the expected fee equal the total costs of the brokerage activities. Notably, the costs are exogenously given, and the commission fee is determined irrespective of the asset price.

The objective of this study is to examine the impact of proprietary trading with transaction fees. Unlike in Sarkar (1995), the transaction fee is assumed to be proportional to the order amount. We construct a one-period CARA-Normal model with a monopolistic dealer/broker and an infinite number of investors. The risk-averse investors in this study have heterogeneous initial endowments and a heterogeneous belief about the liquidation value of the asset.[4] While the broker who brings the investors' orders to the market (with no proprietary trading) sets the transaction fees based on the market-clearing price, the

[4]We do not explicitly describe the information structure among agents; information asymmetry, as assumed in Hellwig (1980), is easily incorporated into our model.

dealer who trades on his own account (with proprietary trading) simultaneously determines both the transaction fee and the asset price. This study attempts to address the following research questions: (i) What is the impact of proprietary trading on the asset price, the transaction fee, the trading volume, and social welfare? (ii) How do the results depend on the investors' belief about the liquidation value of the asset? (iii) How does risk aversion of the investors or the dealer affect the equilibrium solutions?

The main contribution of this study is its effective demonstration of how proprietary trading with transaction fees affects market equilibrium solutions. It also provides several new testable implications for empirical studies. Regarding question (i) above, we find that proprietary trading enables the dealer to set a more favorable price for investors, even though the dealer monopolistically seeks his own profits. Proprietary trading is also found to induce a larger trading volume and a higher transaction fee. The larger trading volume is a result of its role as a liquidity provider for the market. The result of higher transaction fees is consistent with Huang and Stoll (1996), who reported that the cost of executing transactions is higher on NASDAQ (dealer market) than on the NYSE (auction market) by every measure they calculated. We also find that social welfare (the expected utilities of both the dealer and average investors) is improved. The dealer obviously benefits from proprietary trading because he can set the price in addition to the transaction fee. Furthermore, the average expected utility of the investors also increases by virtue of a more favorable price. These results are consistent with Fishman and Longstaff (1992), who examined dual trading in futures markets and empirically found that dual traders earn higher profits than non-dual traders, and that customers of dual-trading dealers do better than customers of non-dual-trading brokers.

Next, regarding question (ii), we find that in a brokered market, the equilibrium price is set to be equal to the risk-adjusted mean of investors' belief about the asset value. On the other hand, the transaction fee depends on the deviation of the investors' belief. The more divergent the belief among investors, the higher the transaction fee is set. In a dealer market, when both the dealer and average investors have the same mean in the belief about the asset value, the transaction fee with proprietary trading coincides with the one with no proprietary trading. The larger the divergence between the belief of the dealer and average investors, the more actively investors trade to seek profits. As a result, the transaction fee is set higher in a dealer market than in a brokered market. The asset price is less sensitive to the investors' belief in a dealer market with no proprietary trading, because a fraction of the price adjustment is mitigated by the increase in the transaction fee.

Finally, regarding question (iii), we find that as investors become more risk averse, the transaction fee increases in both markets due to the higher demand for risk hedging by investors. Moreover, if the total net supply of the asset in a market is positive, the price is set lower since the sell order for risk hedging increases. On the other hand, when the dealer becomes more risk averse, both the price and the transaction fee are set lower to induce investors to hold the asset, while the risk aversion of the broker does not affect the equilibrium. In particular, our analysis demonstrates that when the dealer is infinitely risk-averse, the equilibrium solution with proprietary trading converges with the one with no proprietary trading.

The remainder of this study is organized as follows. In Section 2, we set up our model. In Section 3, we solve and derive the equilibriums with and without proprietary trading. The numerical analysis is conducted in Section 4 to examine how proprietary trading with transaction fees affects market equilibrium and investors' welfare. We discuss in Section 5 the results of the current paper in comparison with those of the previous studies. Section 6 presents the conclusions.

2 Model

Consider a one-shot financial market in which two types of assets, risk-free and risky assets, are traded. The two assets are traded among agents at time 0. The risk-free asset plays the role of storage technology in that the interest rate is zero. The payoff of the risky asset, denoted by v, is random at $t = 0$ and is realized at time 1.

There are two types of market participants: investors who are price takers and the monopolistic agent (dealer or broker) who collects transaction fees from investors and clears their orders. The difference between a dealer and a broker is that while the dealer trades with investors on his own account (i.e., with proprietary trading), the broker only clear investors' orders (i.e., with no proprietary trading).

Both the transaction fee c and the asset price p are determined at time 0. The transaction fee c is assumed to be proportional to the trading amount; that is, investors have to pay c to either buy or sell a unit of the asset. Therefore, $p + c$ is regarded as the so-called "ask" price, while $p - c$ is the "bid" price.

Let $\mathcal{F}_M$ denote the information set of the dealer or broker. The dealer/broker's utility function is given by the exponential utility as

$$U_M(p, c) = -\frac{1}{\gamma} \log \left(\mathbb{E} \left[\mathrm{e}^{-\gamma R(p,c)} \middle| \mathcal{F}_M \right] \right), \tag{2.1}$$

where $R(p, c)$ is the dealer/broker's final wealth and γ represents the coefficient of his absolute risk aversion.

We denote the set of investors by $\mathcal{I}$ and index each investor by $i \in \mathcal{I}$. Investor i trades the asset to maximize the utility given by

$$U_i(p, c) = -\frac{1}{a} \log \left(\mathbb{E} \left[\mathrm{e}^{-aY_i(p,c)} \middle| \mathcal{F}_i \right] \right), \tag{2.2}$$

where $Y_i(p, c)$ is the final wealth and $\mathcal{F}_i$ is the information set of investor i. For simplicity, the coefficient of absolute risk aversion, a, is assumed to be common among investors.

2.1 Investor's optimization problem

Following Kim and Verrecchia (1991) and other extant studies, we suppose that investor i is endowed with an amount ω_i of the risky asset and $\{\omega_i\}_{i \in \mathcal{I}}$ follows IID $N(\bar{\omega}, \sigma_\omega^2)$. In addition, the payoff of the risky asset is assumed to follow a normal distribution with respect to the information of investor i, i.e.,

$$v\big|_{\mathcal{F}_i} \sim N(\mu_i, \sigma_v^2).$$

The assumption that the heterogeneity lies in the expected value of v and the variance σ_v^2 is common makes the problem simple. We do not explicitly consider the information structure $\{\mathcal{F}_i\}_{i \in \mathcal{I}}$. It is worth noting that the information asymmetry assumed in many microstructure studies, such as Hellwig (1980) and Admati (1985), are easily incorporated into our model.

Let x_i be the trading amount of the risky asset by investor i, where a positive (negative, respectively) value of x_i indicates that investor i buys (sells, respectively) $|x_i|$ units of the risky asset. Given the above assumptions, the final wealth Y_i is expressed as

$$Y_i(p, c) = v\omega_i + (v - p)x_i - \mathrm{sgn}[x_i] \cdot cx_i, \tag{2.3}$$

where $\mathrm{sgn}[x] = 1_{\{x>0\}} - 1_{\{x<0\}}$. Since v is the only random variable in (2.3), the optimal trading volume of investor i is easily obtained by maximizing $U_i(p, c)$ in (2.2). We thus have

$$x_i^*(p, c) = -\omega_i + \frac{\mu_i - p - \mathrm{sgn}[x_i^*(p, c)]c}{a\sigma_v^2}, \tag{2.4}$$

where the first term represents the hedging motivation for the risk inherent in the initial endowment, and the second term describes the profit-seeking motivation.

Let $\zeta_i := \mu_i - a\sigma_v^2\omega_i$, which can be interpreted as the risk-adjusted mean of investor i's belief about the asset value. Then, (2.4) can be rewritten as

$$x_i^*(p,c) = 1_{\{\zeta_i > p+c\}}\frac{\zeta_i - (p+c)}{a\sigma_v^2} + 1_{\{\zeta_i < p-c\}}\frac{\zeta_i - (p-c)}{a\sigma_v^2}. \tag{2.5}$$

Equation (2.5) clarifies how investor i optimally trades the asset; that is, if the parameter ζ_i is strictly higher (lower, respectively) than the ask price $p+c$ (the bid price $p-c$, respectively), then investor i buys (sells, respectively) the asset. If $p - c \leq \zeta_i \leq p + c$, then investor i does not trade the asset due to the presence of the transaction fee. Note that the subscript i only appears in ζ_i. In other words, investors' heterogeneity is fully characterized by the parameter ζ_i.

2.2 Broker's or Dealer's optimization problem

The payoff of the risky asset v is also assumed to be a normal with respect to the broker's/dealer's information set $\mathcal{F}_M$, i.e.,

$$v\big|_{\mathcal{F}_M} \sim N(\mu_M, \sigma_M^2). \tag{2.6}$$

First, we define the optimization problem of the monopolistic broker. As explained before, the broker charges a proportional transaction fee denoted by c, but is not allowed to trade the risky asset on his own account. Hence, we can define the broker's optimization problem as follows:

Assumption 1. *In a brokered market, the price $p = p_{\mathrm{b}}$ is determined by the market-clearing condition*

$$\sum_{i\in\mathcal{I}} x_i^*(p,c) = 0$$

and the fee $c = c_{\mathrm{b}}$ to maximize (2.1)*, where*

$$R(p,c) = \sum_{i\in\mathcal{I}} c \times |x_i^*(p,c)|. \tag{2.7}$$

On the other hand, the dealer who plays the role of a market maker determines the price at which he is willing to buy and sell the asset. Therefore, the dealer's optimization problem can be described as follows:

Assumption 2. *In a dealer market, the dealer sets the price $p = p_{\mathrm{d}}$ and the fee $c = c_{\mathrm{d}}$ to maximize* (2.1)*, where*

$$R(p,c) = (v-p)\sum_{i\in\mathcal{I}} \times(-x_i^*(p,c)) + c\sum_{i\in\mathcal{I}} |x_i^*(p,c)|. \tag{2.8}$$

The first term of (2.8) represents the profit earned from proprietary trading. Note that this term can be negative. In other words, depending on the realization of the payoff v, the dealer may suffer losses from his or her proprietary trading. The second term describes the fee revenue earned from investors. Unlike the first term, this term is always positive.

It should be emphasized that the research focus of our paper is different from that of Sarkar (1995), who mainly examined the impact of dual trading (trading on the broker-dealer's own account in addition to executing customers' orders) on both informed and uninformed traders. He also extended the model to include a commission fee that was independent of the order size and dependent only on whether or not an investor traded with the dealer/broker. In this study, we focus on how proprietary trading as a market-making activity affects the asset price and the proportional transaction fee; thus, the transaction fee is endogenously determined in equilibrium.

We should also mention that the differences in the research focus generate differences in the assumptions. In Sarkar (1995), to concentrate on how the broker-dealer can make a profit from mimicking informed investors' trades, it is assumed that there is another risk-neutral and perfectly competitive agent, the market maker, whose role is to exclusively fix the asset price at which he will execute the total orders. On the other hand, we suppose a risk-averse and monopolistic broker (with no proprietary trading) or a dealer (with proprietary trading) each of whom simultaneously sets the market price of the asset and collects brokerage fees. Figure 1 illustrates the difference between Sarkar (1995) and the current study in the model.

[Figure 1 is inserted around here.]

We make this assumption because we are interested in the effect of the proprietary trading by a monopolistic dealer/broker on the price, the trading volume, social welfare, etc. in equilibrium.

3 Equilibrium Solutions

In this section, we derive the market equilibrium solutions. To simplify the problem and make it mathematically tractable, we assume that there are an infinite number of investors. More concretely, we let $\mathcal{I} = \boldsymbol{R}$. Furthermore, μ_i, the mean of investor i's subjective belief about the payoff v, is assumed to follow

$$\mu_i \sim N(\mu_I, \sigma_I^2) \tag{3.1}$$

in $\mathcal{I}$ and be independent of $\{\omega_i\}$.[5] In this situation, the parameter $\zeta_i = \mu_i - a\sigma_v^2\omega_i$ follows a normal as

$$\zeta_i \sim N(\mu_I - a\sigma_v^2\bar{\omega}, \sigma_I^2 + a^2\sigma_v^4\sigma_\omega^2), \tag{3.2}$$

because of the independence between μ_i and ω_i.

We write $\mu_\zeta := \mu_I - a\sigma_v^2\bar{\omega}$ and $\sigma_\zeta^2 := \sigma_I^2 + a^2\sigma_v^4\sigma_\omega^2$, and let

$$\begin{aligned} q_I(\zeta) =& \frac{1}{\sqrt{2\pi\sigma_\zeta^2}} \mathrm{e}^{-\frac{(\zeta-\mu_\zeta)^2}{2\sigma_\zeta^2}}, \\ X_\pm(p,c) =& \pm \int_{p\pm c}^{\pm\infty} \frac{\zeta - (p \pm c)}{a\sigma_v^2} q_I(\zeta)\mathrm{d}\zeta. \end{aligned}$$

Note that $X_+(p,c)$ and $X_-(p,c)$ represent buy orders (which are positive) and sell orders (which are negative), respectively.

Under assumption (3.1), the net amount of orders from all investors is not random and is given by

$$\begin{aligned} \sum_{i\in\mathcal{I}} x_i^*(p,c) =& X_+(p,c) + X_-(p,c) \\ =& \frac{\sigma_\zeta}{a\sigma_v^2}\Big[(\phi(d_+) + d_+\Phi(d_+)) - (\phi(d_-) + d_-\Phi(d_-))\Big], \end{aligned} \tag{3.3}$$

where Φ and ϕ are the distribution and density functions of a standard normal, respectively, and

$$d_\pm := \pm\frac{\mu_\zeta - (p \pm c)}{\sigma_\zeta}.$$

We also derive the trading volume, denoted by X, as

$$\begin{aligned} X(p,c) =& \sum_{i\in\mathcal{I}} |x_i^*(p,c)| = X_+(p,c) - X_-(p,c) \\ =& \frac{\sigma_\zeta}{a\sigma_v^2}\Big[(\phi(d_+) + d_+\Phi(d_+)) + (\phi(d_-) + d_-\Phi(d_-))\Big]. \end{aligned} \tag{3.4}$$

Neither (3.3) or (3.4) is random, thus facilitating the mathematical tractability of the problem.

3.1 A brokered market

First, we show the equilibrium solution for a brokered market. From (3.4), the market-clearing condition $X(p,c) = 0$ becomes

$$(\phi(d_+) + d_+\Phi(d_+)) - (\phi(d_-) + d_-\Phi(d_-)) = 0. \tag{3.5}$$

[5] The assumption (3.1) is justified by the central limit theorem if the subjective mean is decomposed into a common term and an idiosyncratic IID term as in Hellwig (1980).

It is easily verified that the price p_{b} is equal to μ_ζ, regardless of the transaction fee and the broker's information. We should also notice that there is no randomness in the broker's optimization problem. Thus, the utility function is equal to

$$U_M(p,c) = R(p,c) = -\frac{2\sigma_\zeta^2}{a\sigma_v^2}\Big(\bar{d}\phi(\bar{d}) + \bar{d}^2\Phi(\bar{d})\Big), \tag{3.6}$$

where $\bar{d} = -c/\sigma_\zeta$.

We obtain the following proposition by maximizing (3.6).

Proposition 1. *The equilibrium price in a brokered market, p_{b}, is given by*

$$p_{\mathrm{b}} = \mu_\zeta$$

and the proportional transaction fee by

$$c_{\mathrm{b}} = -\sigma_\zeta z, \tag{3.7}$$

where $z < 0$ is the solution of the equation

$$z + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}z}\log\Phi(z) = 0. \tag{3.8}$$

It is easily verified that the negative root of (3.8) is uniquely determined. We find that, with no proprietary trading, the equilibrium price is set equal to the mean of the investor's belief about the asset value. On the other hand, the transaction fee depends on the variance of the investor's belief. The more divergent the belief among investors, the higher the transaction fee is set.

3.2 A dealer market

Now consider the problem of the dealer. Given p and c, the final wealth of the dealer is obtained by substituting (3.3) and (3.4) into (2.8):

$$\begin{aligned} R(p,c) =& (v-p)\sum_{i\in\mathcal{I}}(-x_i^*(p,c)) + c\sum_{i\in\mathcal{I}}|x_i^*(p,c)| \\ =& -(v-p-c)\frac{\sigma_\zeta}{a\sigma_v^2}[\phi(d_+) + d_+\Phi(d_+)] \\ &+ (v-p+c)\frac{\sigma_\zeta}{a\sigma_v^2}[\phi(d_-) + d_-\Phi(d_-)]. \end{aligned} \tag{3.9}$$

From the fact that v is the only random variable in (3.9) and follows a normal as (2.6), we can calculate (2.1) as

$$\begin{aligned} U_M(p,c) =& \frac{\sigma_\zeta}{a\sigma_v^2}(\mu_\zeta - \mu_M)\Big[\Big(\phi(d_+) + d_+\Phi(d_+)\Big) - \Big(\phi(d_-) + d_-\Phi(d_-)\Big)\Big] \\ & - \frac{\sigma_\zeta^2}{a\sigma_v^2}\Big[\Big(d_+\phi(d_+) + d_+^2\Phi(d_+)\Big) + \Big(d_-\phi(d_-) + d_-^2\Phi(d_-)\Big)\Big] \\ & - \frac{\gamma}{2}\frac{\sigma_\zeta^2\sigma_M^2}{a^2\sigma_v^4}\Big[\Big(\phi(d_+) + d_+\Phi(d_+)\Big) - \Big(\phi(d_-) + d_-\Phi(d_-)\Big)\Big]^2. \end{aligned} \tag{3.10}$$

The first two terms represent the expected profit from the sell orders and buy orders of the proprietary trading, respectively. The last term of (3.10) reflects the risk-aversion effect that stems from the proprietary trading, the profit from which is random.

Maximizing (3.10) with respect to p and c leads to the following proposition:

Proposition 2. *The equilibrium price and the proportional transaction fee in an dealer market, p_{d} and c_{d}, satisfy the simultaneous equation system*

$$\begin{aligned} & (\mu_\zeta - \mu_M)\Phi(\hat{d}_\pm) \mp \sigma_\zeta[\phi(\hat{d}_\pm) + 2\hat{d}_\pm\Phi(\hat{d}_\pm)] \\ & - \frac{\gamma\sigma_\zeta\sigma_M^2}{a\sigma_v^2}\Phi(\hat{d}_\pm)\Big[\Big(\phi(\hat{d}_+) + \hat{d}_+\Phi(\hat{d}_+)\Big) - \Big(\phi(\hat{d}_-) + \hat{d}_-\Phi(\hat{d}_-)\Big)\Big] = 0, \end{aligned} \tag{3.11}$$

where

$$\hat{d}_\pm = \pm\frac{\mu_I - a\sigma_v^2\bar{\omega} - (p_{\mathrm{d}} \pm c_{\mathrm{d}})}{\sigma_I^2 + a^2\sigma_v^4\sigma_\omega^2}.$$

Although analytical solutions seem difficult to obtain, we conduct numerical analysis in the next section to analyze how the two markets differ, especially from the viewpoint of investors' welfare.

3.3 Relationship between the two markets

We find an important analytical property that connects the two markets.

Corollary 1. *The market equilibrium in the limiting case of an infinite risk-averse dealer is equal to the one in the broker's case; that is,*

$$\begin{pmatrix} p_{\mathrm{d}} \\ c_{\mathrm{d}} \end{pmatrix} \xrightarrow{\gamma\to\infty} \begin{pmatrix} p_{\mathrm{b}} \\ c_{\mathrm{b}} \end{pmatrix}.$$

Proof. If $\gamma \to \infty$, then $\mathbb{V}[R(p,c)]$ must be zero. Since v is the only random variable in (3.9), the part of $\sum_{i\in\mathcal{I}}(-x_i^*(p,c))$ must be zero, which is exactly the market-clearing condition in a brokered market. Moreover, $R(p,c)$ is reduced from (2.8) to (2.7), which is exactly the final wealth of the broker. Therefore, we have $p_{\mathrm{d}} = p_{\mathrm{b}}$ and $c_{\mathrm{d}} = c_{\mathrm{b}}$. □

Intuitively, when $\gamma \to \infty$, the dealer would never take any risk and would, thus, avoid trading on his own account. Since the profit from transaction fees is not random and is always positive, maximizing the dealer's wealth is equivalent to maximizing the fee revenue, leading to the same equilibrium as is achieved in a brokered market.

4 Numerical Analysis

This section conducts numerical analysis to investigate the effect of proprietary trading on equilibrium. The base case parameter values are given in the next table:

Table 2: Base case parameter values in the numerical analysis.

a	σ_ω	σ_v	μ_I	σ_I	μ_M	$\bar{\omega}$	γ	σ_M
1	1	0.5	1	1	1	1	0.5	0.25

Note that we have $\mu_\zeta < \mu_M$ in this parameter setting, implying that the risk-adjusted mean of the average investors' belief is lower than that of the dealer's/broker's belief.

4.1 Effect of mean of average investors' belief

First, we examine the effect of μ_I, the average expectation about the asset value v among investors. The parameter μ_I affects the equilibrium only through μ_ζ, which is equal to $\mu_I - a\sigma_v^2\bar{\omega}$. The average initial endowment among investors, $\bar{\omega}$, also appears only in μ_ζ. Therefore, we omit the analysis of $\bar{\omega}$.

Figure 2 depicts the effect of μ_I on the proportional transaction fee c.

[Figure 2 is inserted around here.]

We find that the transaction fee in a brokered market is constant, regardless of μ_I, reconciling result (3.7) in Proposition 1. On the other hand, the transaction fee set by the dealer varies, depending on the expectation of the average investors' belief about the asset value v. A key observation is that the graph is of U-shape and the minimum is achieved at $\mu_I = \mu_M + a\sigma_v^2\bar{\omega}$, or equivalently $\mu_\zeta = \mu_M$. The first line of (3.11) disappears when $\mu_\zeta = \mu_M$. Then, the first-order condition of the dealer's maximization problem reduces to (3.5). Therefore, we have $c_\mathrm{d} = c_\mathrm{b}$. Intuitively, if the belief of both the dealer and average investors coincides, the result in the two markets is the same and is independent of proprietary trading.

At first glance, a brokered market is more favorable to investors because the transaction fee is lower. However, with the figures below, we will find that the opposite is true: on average, a dealer market is more favorable to investors.

Second, we analyze the effect of μ_I on p, the market price of the asset.

[Figure 3 is inserted around here.]

Figure 3 shows that the asset price increases with μ_I. Intuitively, the higher the mean of the average investors' belief about the asset value, the higher the price is set because more investors want to buy the asset at the same price. We also find that the asset price p_{d} set by the dealer is lower than the one set by the dealer for $\mu_\zeta > \mu_M$ (on the right side of the intersection of the solid blue and red lines), and higher for $\mu_\zeta < \mu_M$ (on the left side of the intersection). In other words, the slope of the asset price with proprietary trading is more moderate compared to the price with no proprietary trading. The reason can be interpreted as follows: The larger the divergence between the beliefs of the dealer and average investors, the higher will be the revenue from transaction fees. Therefore, it is not necessary to adjust the price drastically.

Third, we consider the effect of μ_I on the trading volume defined by (3.4).

[Figure 4 is inserted around here.]

Figure 4 indicates that the trading volume in the two markets coincides at $\mu_I = \mu_M + a\sigma_v^2\bar{\omega}$, or equivalently $\mu_\zeta = \mu_M$. The larger the divergence between the belief of the dealer and average investors, the higher is the trading volume in a dealer market. This is because proprietary trading enables investors to trade more actively to seek profits. Consider the case $\mu_\zeta > \mu_M$. Average investors do not trade in a brokered market due to the presence of the transaction fee ($\zeta_i \in (p_{\mathrm{b}} - c_{\mathrm{b}}, p_{\mathrm{b}} + c_{\mathrm{d}})$). In a dealer market, however, the average investors may trade, depending on the relationship between μ_ζ and $p_{\mathrm{d}} \pm c_{\mathrm{d}}$. As we have seen from Figure 3, in this parameter setting, the price in a dealer market is more favorable to the average investors, and they are willing to buy the asset if $\mu_\zeta > p_{\mathrm{d}} + c_{\mathrm{d}}$. The fact that the density function $q_I(\zeta)$ takes the highest value at μ_ζ results in a higher trading volume in a dealer market.

Fourth, we examine the effect of μ_I on investors' welfare. Here, the welfare of the total investors is naturally defined by

$$U_I = \int_{\mathcal{I}} U_i(\zeta) q_I(\zeta) \mathrm{d}\zeta. \tag{4.1}$$

Figure 5 shows the effect of μ_I in the two markets.

[Figure 5 is inserted around here.]

From Figure 5, we find that the welfare of the total number of investors is higher in a dealer market. An intuitive explanation for this result is similar to that given for Figure 4: that is, the trading price is more favorable to average investors, and therefore, their expected profit becomes higher.

Fifth, we study the effect of μ_I on the utility U_M of the dealer/broker.

[Figure 6 is inserted around here.]

Figure 6 shows that the utility of the dealer is always higher than that of the broker, except for the case where $\mu_I = \mu_M + a\sigma_v^2\bar{\omega}$: here, the utility coincides in the two markets. The reason for this result can be intuitively explained from the fact that the dealer has two control variables p and c, while the broker can only optimally choose the proportional transaction fee c.

Figures 5 and 6 indicate that although the dealer has monopolistic power to set the price and the transaction fee, on average, a dealer market is desirable for investors. This observation is new in the literature, especially in studies that compare the two major trading systems in actual financial markets.

4.2 Effect of deviation in average investors' belief

In this analysis, we investigate the effect of σ_I, the standard deviation of investors' subjective belief $\{\mu_i\}$.

Figure 7 shows how this deviation of belief affects the transaction fee c.

[Figure 7 is inserted around here.]

We find that the transaction fee is an increasing function of the deviation of investors' belief in both cases: that is, the more divergent the belief among investors, the higher the transaction fee is set. Also note that the fee in a dealer market is always higher than that in a brokered market. However, we should consider not only the transaction fee but also the price as in the analysis of μ_I.

Figure 8 describes the effect of the deviation in average investors' belief on price p in the two-market organization.

[Figure 8 is inserted around here.]

It is worth mentioning that $p_{\mathrm{b}} < p_{\mathrm{d}}$ in the figure. In our parameter setting, $\mu_\zeta < \mu_M$ and $\mu_\zeta < p_m$ for $m = \mathrm{b}, \mathrm{d}$. Thus, as in Figure 3, the price is more desirable to average investors in a dealer market than in a brokered market.

The next figure gives us the effect on X, the total trading volume in the market.

[Figure 9 is inserted around here.]

As seen from the figure, the trading volume monotonically increases with the deviation in belief. An economic explanation is given in a similar way to that in Figure 7 as follows: suppose that σ_I becomes higher. Then, for more investors, the risk-adjusted mean is far from the price, or $\zeta_i \notin (p - c, p + c)$. Consequently, they are more willing to trade the risky asset. Again, the trading volume is higher in a dealer market than in a brokered market because the price is more favorable to average investors, leading to more active trading by investors.

Figure 10 plots the effect on investors' welfare.

[Figure 10 is inserted around here.]

We observe that a higher value of σ_I leads to a higher value of welfare. As seen in Figures 7 to 9, more investors are motivated to trade the asset when σ_I is high because the risk-adjusted mean and the price substantially differ. The difference between μ_ζ and p results in an increase in welfare. Moreover, welfare in a dealer market is higher than in a brokered market because the price is more favorable to average investors, as shown in the previous analyses.

Lastly, we present Figure 11 to describe how the deviation in belief affects the utility of the dealer/broker.

[Figure 11 is inserted around here.]

As with investors' welfare, the utility of the dealer/broker is increasing in σ_I. We omit a detailed explanation of the figure because it is simply a restatement of that for Figure 6.

4.3 Effect of risk aversion

The effect of a, the risk-aversion coefficient of investors, is described in Figures 12 and 13.

[Figures 12 and 13 are inserted around here.]

As investors become more risk averse, their demand for hedging grows. Thus, the dealer is able to set a higher transaction fee. Since sell orders increase, the price is set lower.

In parallel, Figures 14 and 15 describe the effect of γ, the risk aversion coefficient of the dealer/broker.

[Figures 14 and 15 are inserted around here.]

When the dealer becomes more risk averse, he is not willing to hold the asset. Thus, if the total net supply of the asset is positive, both the price and the transaction fee are set lower to induce investors to buy the asset. As we proved in Proposition 2, as γ increases, the equilibrium solutions p and c in a dealer market converge to those in a brokered market.

The effects of other parameters are also obtained by numerical calculations. However, the qualitative results and the economic explanations are similar to those in the previous cases. Therefore, we omit the illustrations, which are available from the authors on request.

5 Discussions

From the numerical results above, we can summarize that a dealer market is more desirable to investors compared to a brokered market, even though the dealer has a monopolistic power to set the price and the fee. In other words, proprietary trading leads to a favorable consequence for financial markets.

This finding can be explained as follows: Recall that the dealer's profit is given by (2.8), and the first term expresses the profit from proprietary trading. A more favorable price for investors attracts more orders, leading to a higher value of $X(p, c)$. Hence, even though the dealer is monopolistic and risk-averse in our model, he has an incentive to set a more favorable price to earn a higher profit from proprietary trading.

In a brokered market, the broker always set the price $p_{\mathrm{b}} = \mu_\zeta$ to clear the investors' orders. Consequently, investors with $\zeta_i \in (p_{\mathrm{b}} - c_{\mathrm{b}}, p_{\mathrm{b}} + c_{\mathrm{b}})$ do not trade the asset because the transaction fee discourages them from doing so. Put differently, investors whose belief is close to the average belief in the market always decide not to trade in a brokered market.

Suppose $\mu_\zeta > \mu_M$. In this situation, the dealer has an incentive to set the price lower since it attracts buy orders and makes the dealer earn more profits from proprietary trading. On the other hand, a lower price is also more favorable to average investors because they are willing to trade the asset if $p_{\mathrm{d}} + c_{\mathrm{d}} < \zeta_i$. The same situation applies to the case where $\mu_\zeta < \mu_M$. Consequently, a dealer market is, on the whole, better for investors than a brokered market thanks to proprietary trading.

Now suppose that γ is large; equivalently, the dealer is quite risk averse. The dealer is unwilling to take the risk of proprietary trading but is willing to earn profits from the second term of (2.8), revenue from transaction fees, because this revenue source is deterministic.[6] In this case, the equilibrium of the limiting case converges to that in a brokered market.

To clarify the contribution of this study, we present the following table that briefly compares the results of our model with those in Sarkar (1995), who studied the impact of dual trading with brokerage fees.

Table 3: Comparison between our results with those of Sarkar (1995). Here, "d.t." and "p.t." represent dual trading and proprietary trading, respectively.

	Sarkar (1995)	our study
Transaction fee	lower with d.t.	higher with p.t.
Trading volume	lower with d.t.	higher with p.t.
Welfare	lower with d.t. for informed	higher with p.t. on average
	higher with d.t. for uninformed	

In Sarkar (1995), the commission fee is chosen by the condition of the broker's zero profit condition, or that the broker's expected trading profit plus expected fee revenue equals the total costs of brokerage. Since the broker makes a positive profit from mimicking the informed investors' trades, the commission fee is lower with dual trading. The trading volume of informed investors as well as their welfare decreases. On the other hand, the welfare of the uninformed investor increases because their trading volume is unaffected and the commission fee is lower.

In our study, our research interest is in proprietary trading as a market-making activity wherein a dealer/broker has monopolistic power over the price and the fee. The main result is that since proprietary trading enables the dealer to set a more favorable price for investors, trading volume and the transaction fee increase, and the welfare of average investors is improved. Sarkar (1995) assumed that the price is set by a perfectly competitive market maker, indicating that the price has no effect on the transaction fee. The effect of proprietary trading by a monopolistic dealer who sets both the price and fee is not fully investigated in the literature. To the best of the authors' knowledge, the present study is the first to investigate an equilibrium in which the market price and transaction fee are set by one agent under the condition of imperfect competition.

[6] Although the trading volumes in actual markets are not deterministic, the fee revenues earned by the dealer/broker are always positive. Therefore, the implication of our model is robust and applicable to real-world markets.

6 Conclusion

In this study, we construct a simple financial market model in which a monopolistic market arranger and an infinite number of investors participate in trading a risky asset. While the broker who brings investors' orders to the market (with no proprietary trading) sets only the transaction fee, given that the price is determined according to the market-clearing condition among investors, the dealer who trades on his own account (with proprietary trading) simultaneously sets both the transaction fee and the market price of the asset. We find that proprietary trading enables the dealer to set a more favorable price not only for the dealer himself but also for investors. As a result, the trading volume and the transaction fee both increase, and social welfare improves.

Finally, we should point out an interesting but challenging question. Our model could be extended to an intermediate oligopolistic case between the extremes of a perfectly competitive market arranger in the standard models and monopolistic one in our model. The study of such an oligopolistic equilibrium can be taken up in future research.

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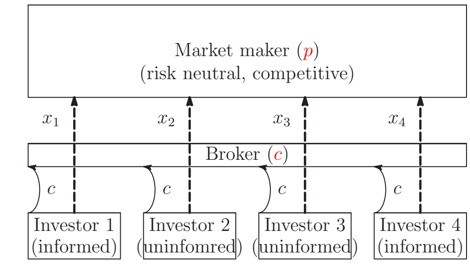
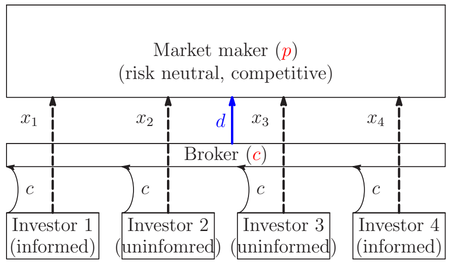
Figures

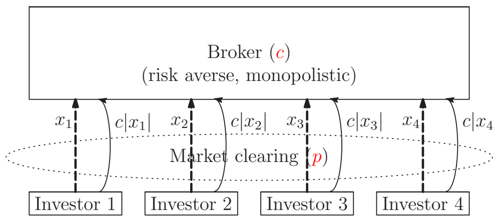
Figure 1: Illustration of the difference between Sarkar (1995) and the current study in the model. In Sarkar (1995), the risk-neutral and competitive market maker sets the price and is different from the broker. On the other hand, we assume a risk-averse and monopolistic dealer/broker who collects the transaction fee.



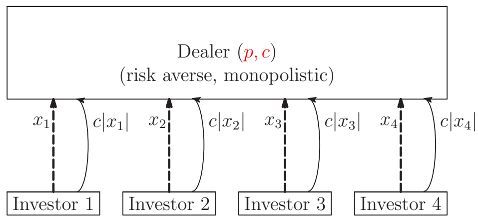
(a) Broker without dual trading in Sarkar (1995)



(b) Broker with dual trading in Sarkar (1995)



(c) Broker without proprietary trading in this study



(d) Dealer with proprietary trading in this study

Figure 2: The effect of μ_I on fee c

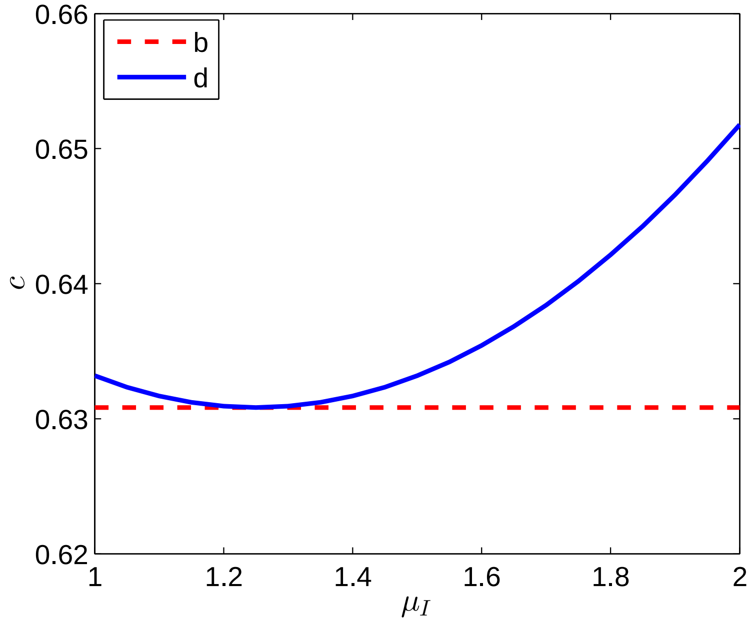


Figure 3: The effect of μ_I on price p

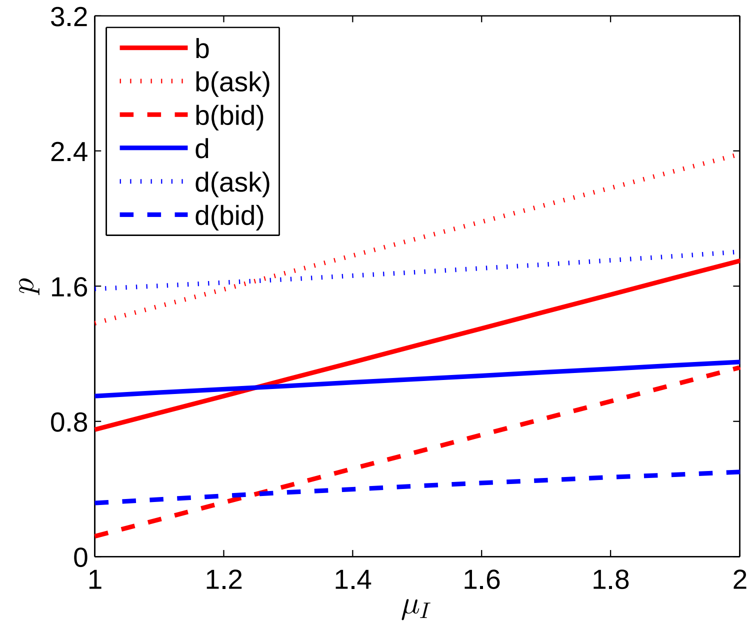


Figure 4: The effect of μ_I on trading volume X

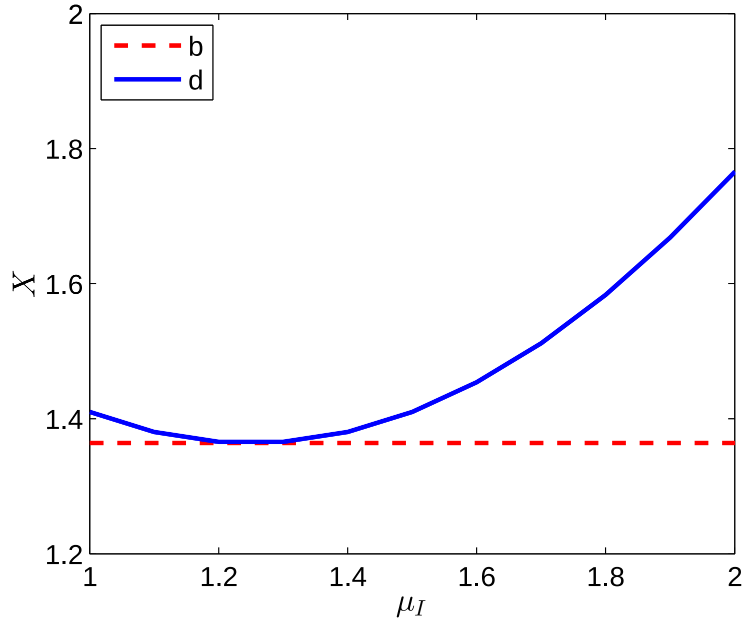


Figure 5: The effect of μ_I on welfare U_I

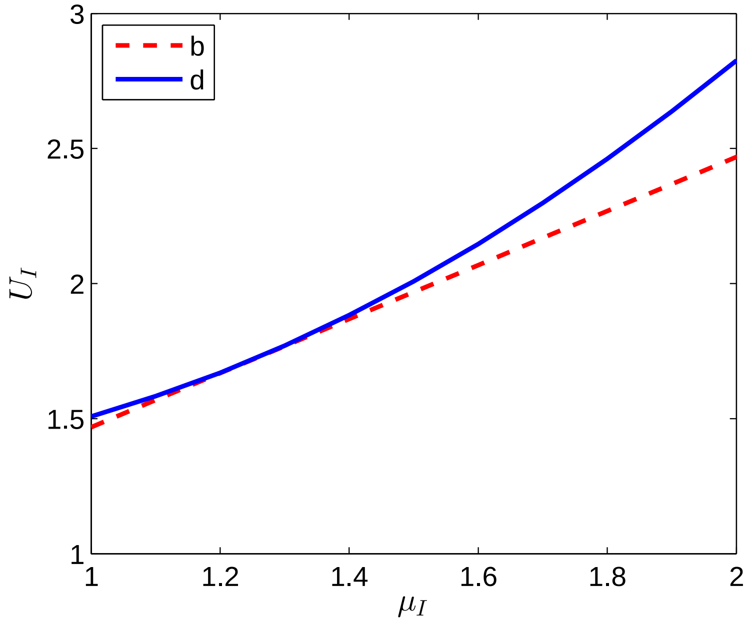


Figure 6: The effect of μ_I on utility of the dealer/broker U_M

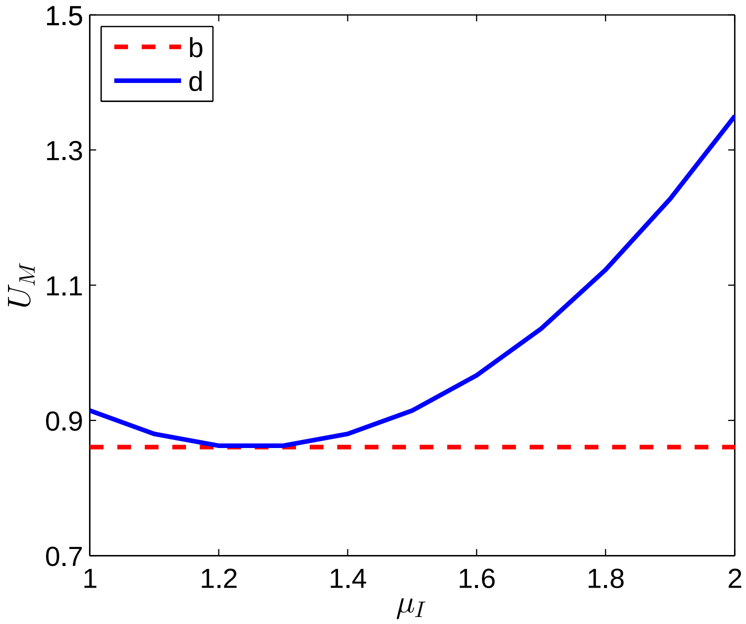


Figure 7: The effect of σ_I on transaction fee c

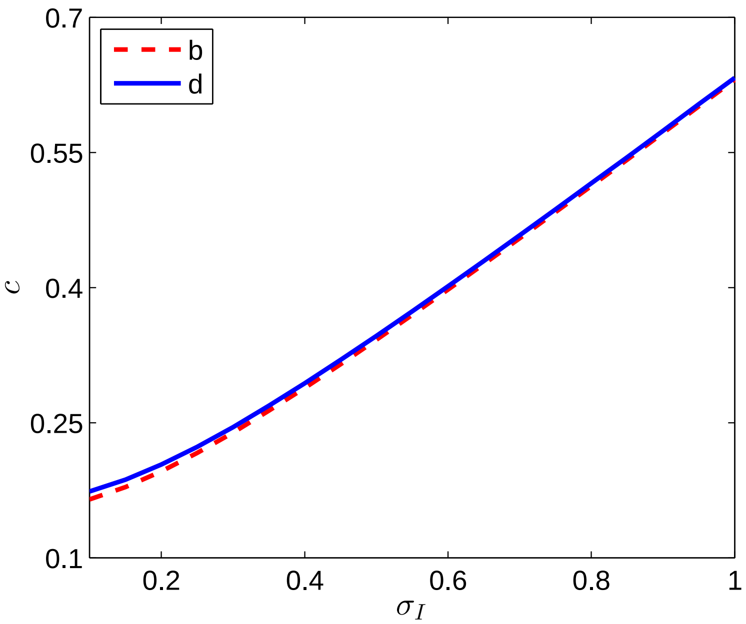


Figure 8: The effect of σ_I on price p

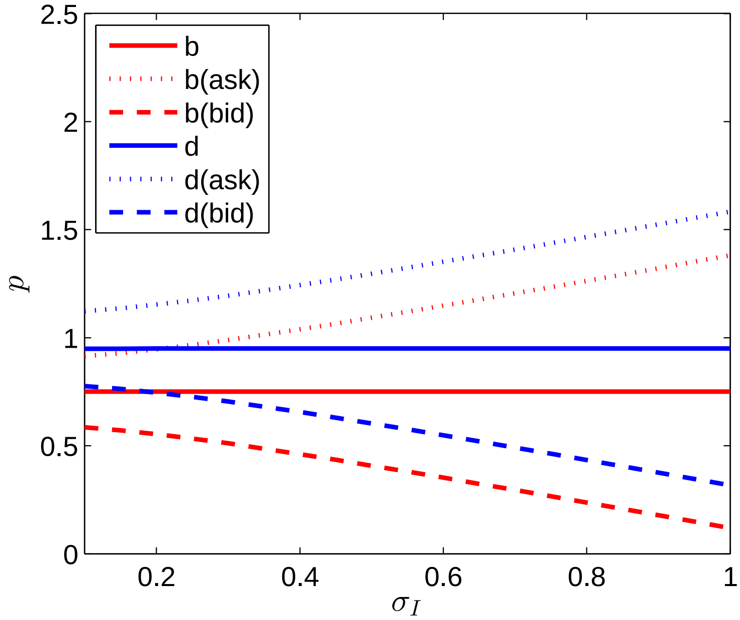


Figure 9: The effect of σ_I on trading volume X

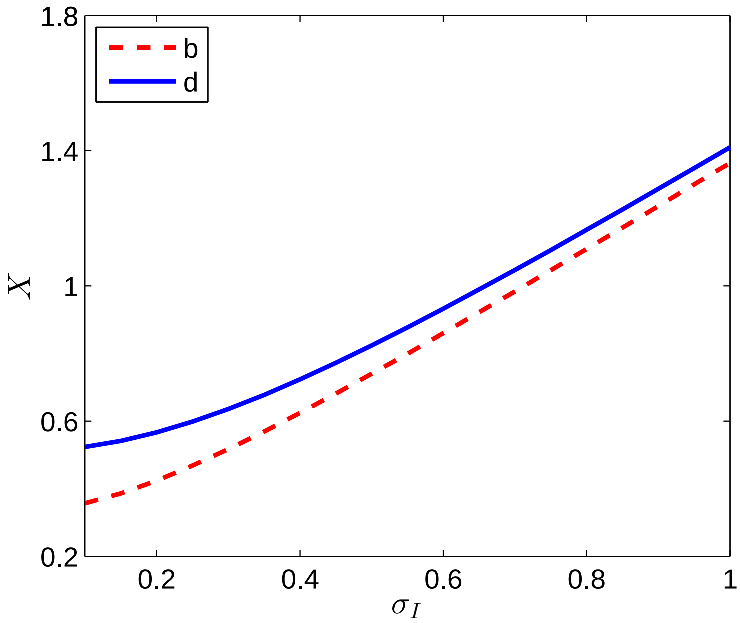


Figure 10: The effect of σ_I on welfare U_I

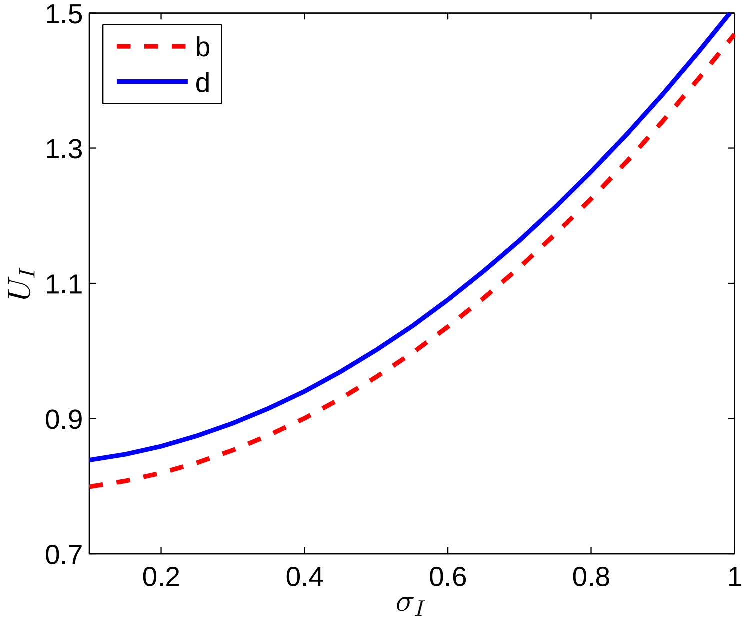


Figure 11: The effect of σ_I on utility of the dealer/broker U_M

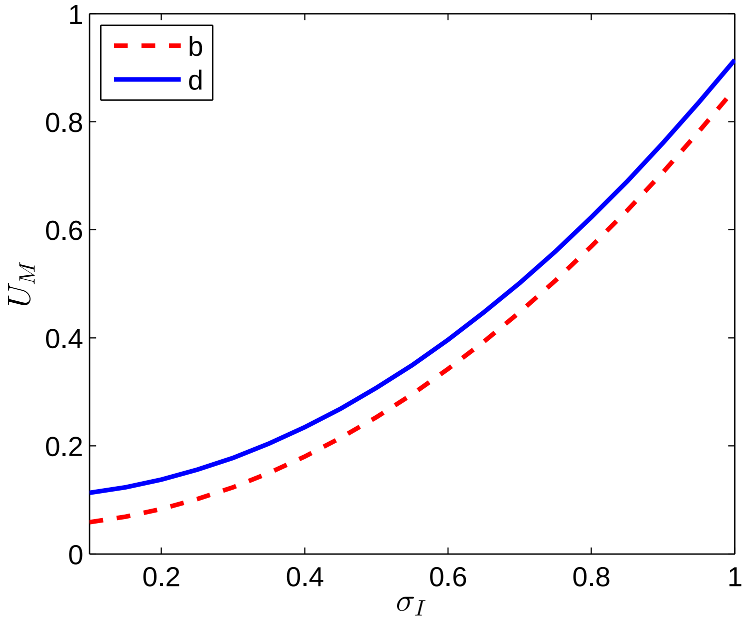


Figure 12: The effect of a on fee c

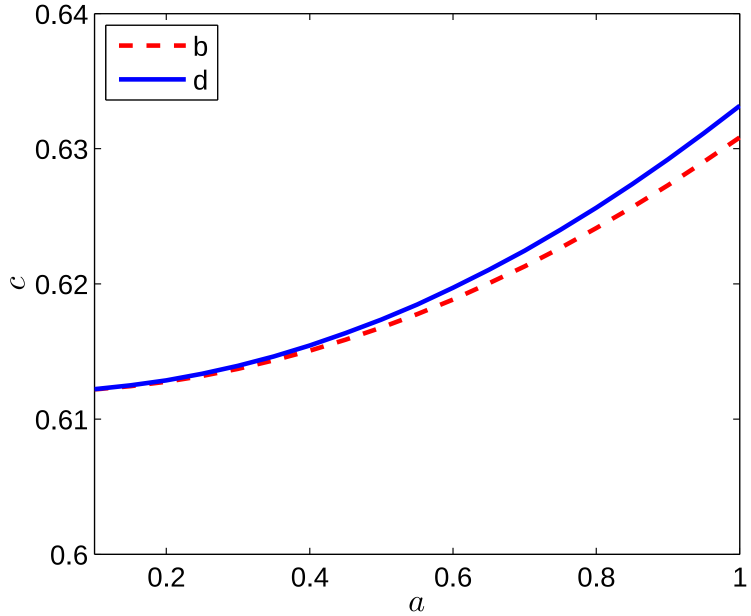


Figure 13: The effect of a on price p

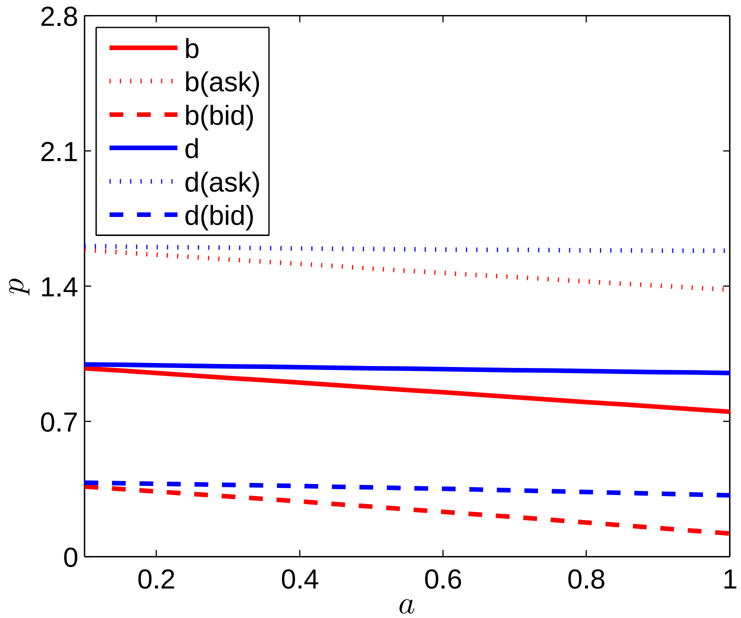


Figure 14: The effect of γ on fee c

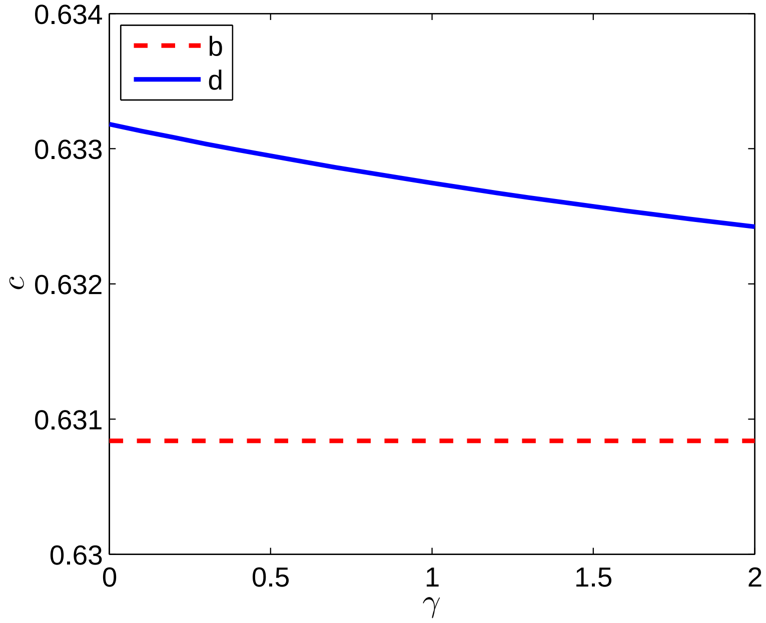


Figure 15: The effect of γ on price p