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On a characteristic property of the tent map

Yoshihito Ogasawara and Shin'ichi Oishi
Faculty of Science and Engineering
Waseda University

As a generalization of a characteristic property of fundamental chaotic maps such as the tent map, a primitive chaos is defined by $(X,\ \{X_\lambda,\ \lambda\in\Lambda\},\ \{f_{X_\lambda},\ \lambda\in\Lambda\}))$ which satisfies the following property, where X is a set, $\{X_\lambda,\ \lambda\in\Lambda\}$ is a family of subsets of X which contains no empty set, and $\{f_{X_\lambda},\ \lambda\in\Lambda\}$ is a family of maps $f_{X_\lambda}:X_\lambda\to X$ [1].

(P) For any infinite sequence $\omega_0,\ \omega_1,\ \omega_2,\ldots$ of sets, there exists an initial point $x_0\in\omega_0$ such that $f_{\omega_0}(x_0)\in\omega_1,\ f_{\omega_1}(f_{\omega_0}(x_0))\in\omega_2,\ldots$, where $\omega_i\in\{X_\lambda,\ \lambda\in\Lambda\}$ for each i.

The primitive chaos is closely related to general problems about determinism, causality, free will, and irreversibility [1], which are vital problems in science (e.g., refs. [2-26]).

Then, the following propositions present sufficient conditions for guaranteeing the primitive chaos.

Proposition 1 [1]. *If X is a nondegenerate Peano continuum, for any $\varepsilon>0$, there exist finitely many nondegenerate Peano subcontinua $X_1,\ldots,X_n$ covering X such that dia $X_i<\varepsilon,\ i=1,\ldots,n$. Then, for each i, for any positive integer n^i, for any n^i points $x_1^i,\ldots,x_{n^i}^i\in X_i$ and $y_1^i,\ldots,y_{n^i}^i\in X$, there exists a continuous surjection $f_{X_i}:X_i\to X$ such that $f_{X_i}(x_1^i)=y_1^i,\ldots,f_{X_i}(x_{n^i}^i)=y_{n^i}^i$, and they satisfy the property* (P).

Proposition 2 [27]. *If X is a Cantor set, for any positive integer n, there exist a partition $\{X_1, \ldots, X_n\}$ of X and maps $f_{X_i} : X_i \to X$, $i = 1, \ldots, n$, and they satisfy the property (P).*

Here, a nondegenerate space means the space that consists of more than one point, a Peano continuum is a locally connected continuum, and a continuum is a nonempty connected compact metric space. A Cantor set is any space that is homeomorphic to the Cantor middle-third set, and a space is a Cantor set if and only if it is a zero-dimensional, perfect compact metric space [28, Theorem 8.1].

Proposition 1 explains the reason why we are surrounded by diverse chaotic behaviors [29], and Proposition 2 implies the possibility of the Cantor set for a new recognition of natural phenomena.

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Faculty of Science and Engineering
Waseda University
Ohkubo, Shinjuku-ku, Tokyo 169-8555 Japan
E-mail address: ogasawara@aoni.waseda.jp

早稲田大学・理工学術院　小笠原 義仁，大石 進一