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STUDY OF SEICHE IN LAKE BIWA-KO [I] —ON THE NUMERICAL CALCULATION BY DEFANT'S METHOD—

By

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Abstract

One-dimensional numerical calculation by Defant's method is performed in order to ascertain the physical mechanism of the longitudinal seiche motions in Lake Biwa-ko. The possible period of seiche of the fundamental mode is found to be 212.0 minutes, and the periods of its higher harmonics are 71.3, 61.0, 39.0, 31.7 minutes and so on. The profiles of the vertical displacements of water surface and the mean horizontal velocity of these seiche motions are shown. It is of interest that the first harmonics and the second have fairly close periods. That these two components really exist is verified from an analysis of the phase shift seen in the observed oscillation with a period of 60 to 70 minutes. These two oscillations of periods of 71.3 and 61.0 minutes as computed are respectively the bi- and tri-nodal seiche in the whole basin.

1. Introduction

Over a continental shelf area, or in a bay, proper oscillation developes according to its geometrical shape. These oscillations are well known as continental shelf seiches and as bay oscillations respectively, and have been studied from many points of view for a long time. This seiche motion, which was originally defined as an oscillation of water in a lake, has an important effect on the water motion in a lake as well as the drift of water by wind and the internal seiche. For instance, it is reported that the amplitude of the seiche motion in Lake Biwa-ko might be over 20 cm or so. Therefore, it is important to grasp the nature of the seiche motion in order to understand the dynamics of the water motion in the lake and also to plan lake bank retaining works.

Since Nakamura and Honda [1902] observed for the first time the seiche motion in Lake Biwa-ko, several investigators have reported that seiches with periods from several minutes to four hours are observable in Lake Biwa-ko. Table 1 is a summary of these previous papers, and the seiche in Lake Biwa-ko may be divided into three groups shown in the columns named mode-1, mode-2

Table 1. List of studies concerning seiches in Lake Biwa-ko

Year	Authors	Method	Period (min)			Explanations
			Mode-1	Mode-2	Mode-3	
1902	Nakamura S. and Honda K.	Observation	231.0			Oscillation in south basin
				72.0		
					30.0	
		Model experiment		72.0		Uni-nodal oscillation in north basin
					31.0	Bi-nodal oscillation in north basin
1926	Suda K. *et al.*	Observation		70.0		Uni-nodal oscillation in whole basin
					30.0	Oscillation in north basin
1931	Takatani S.	Observation	236.2			Oscillation in south basin
1935	Nomitsu T.	Observation	250.0			Bay oscillation in south basin
				70.0		Uni-nodal oscillation between Nagahama and Katada
1938	Takahashi T. and Namekawa T.	Observation	220.0			Oscillation in south basin
				66.0		
1938	Toyohara Y. and Habu K.	Observation		66.0		Uni-nodal oscillation between Nagahama and Katada
					32.0	First harmonics of oscillation of mode-2 and oscillation over lake shelf of east side
		Mean	234.3	69.3	31.0	

and mode-3.

As to the oscillation of mode-1, on the basis of their observation all investigators interpreted this mode as a bay oscillation in the south basin. The mean period of the oscillation of this mode is 234.3 minutes.

The oscillation of mode-2 was considered to be a seiche in the south basin by Nakamura and Honda [1902] as a result of their observations. From their model experiment, however, they explained it as a uni-nodal seiche in the north basin. Again, it was interpreted as a uni-nodal seiche in the whole basin by Suda *et al.* [1926], and as a uni-nodal oscillation between Nagahama and Katada by Nomitsu [1935] and Toyohara and Habu [1938]. The mean period of the oscillation of this mode is 69.3 minutes.

The oscillation of mode-3 which has a mean period of 31.0 minutes was interpreted as a bi-nodal seiche in the north basin by Nakamura and Honda [1902] from the model experiment, as an oscillation in the north basin by Suda *et al.* [1926] from their observations and as an oscillation composed of a shelf oscillation in the east side of Lake Biwa-ko and the first harmonics of the oscillation of mode-2 by Toyohara and Habu [1938].

Thus, it may be concluded that seiches with periods of about 234, 69 and 31 minutes occur in Lake Biwa-ko, and that all investigators regard the seiche of mode-1 as a free oscillation in the south basin, but that concerning the seiches of mode-2 and mode-3 their opinions diverge widely.

Most of the paper referred to above were concerned only with the period analysis of seiche and did not mention such problems as the distribution of the vertical displacement of the water surface, quantitative analysis and the prediction of the amplitude distribution of each seiche component, or the relation of the surface seiche to the internal seiche and the current in the lake. We have begun to study these problems in order to understand the dynamics of water motion in Lake Biwa-ko through field observations and nonlinear two dimensional numerical experiments, the technique of which has been established through studies of tidal currents or storm surges.

Before proceeding to the study described above, as a preliminary numerical experiment and as a useful guide in selecting the positions of the field observations we performed one dimensional numerical calculations about the surface seiches which occur on a comprehensive scale in Lake Biwa-ko by means of the well-known Defant's method. This paper is the report on this work and the calculated results are compared with some results obtained by observation.

2. Results of the numerical calculation

Assuming that the seiche oscillates back and forth predominantly in the x-

direction, $x=0$ to $x=l$, chosen along the "Talweg", the whole basin is divided into segments with thickness Δx and cross-section $S_i(x)$, where $i=1, 2, \ldots, n$ and $l=n\cdot\Delta x$. Then, the well-known Defant equations which give the solutions of the possible seiches in a lake with variable cross-sections are obtained as

$$\left.\begin{aligned} &\eta_{i+1}=\eta_i+(\xi_i+\xi_{i+1})\cdot\alpha/2, \\ &\xi_{i+1}=-[q_i+(\eta_i+\alpha\cdot\xi_i/4)\cdot v_{i+1}]/S_{i+1}\cdot(1+\alpha v_{i+1}/4S_{i+1}), \\ &q_{i+1}=q_i+v_{i+1}\cdot(\eta_i+\eta_{i+1})/2, \\ &\alpha=4\pi^2/gT^2 \end{aligned}\right\} \quad (1)$$

where T is the period of a seiche, η is the vertical displacement of water surface, ξ the horizontal displacement and v_i is the surface area of the lake between the section $i-1$ and i. By definition of the seiche motion $\xi_1=0$ and $q_1=0$ are required as the boundary conditions at one end of the lake, $x=0$. Therefore from eq. (1) with arbitrary values of η_1, at the end of the lake and the adequate value of the period T, the other end of the lake is reached at η_n,

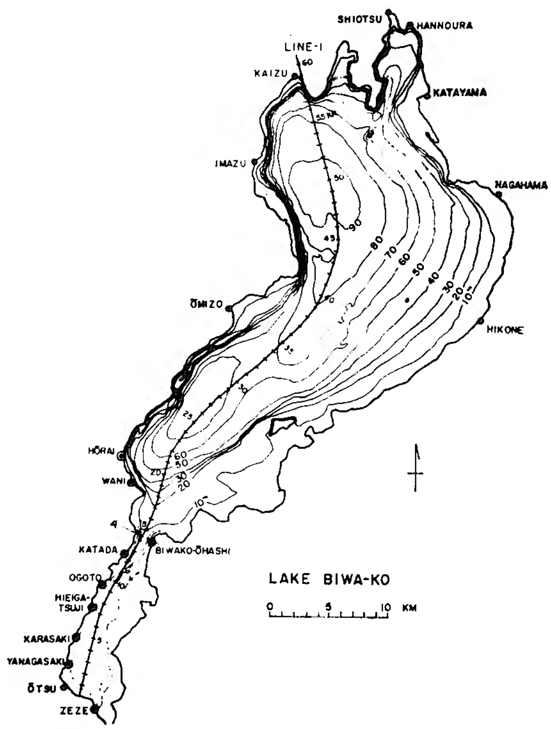

Fig. 1. Bottom topography of Lake Biwa-ko. Longitudinal seiche oscillations are assumed to occur along the solid thick line. Numerals along this line are the length in kilometers from the southern end of the lake. Double black circles show the observation stations.

ξ_n and q_n. If the period T of the oscillation is chosen correctly, the computation must give the value $\xi_n=0$ in order to fulfill the boundary condition at the other end of the lake. If $\xi_n=0$, the computation process is repeated by selecting another value of the period T.

In this report the arbitrary quantity η_1 is set to be equal to 2 cm, and Lake Biwa-ko is divided into 61 segments with the thickness 1 km along the line-1 shown by the thick solid line in Fig. 1, and the area of the cross-section and the surface area of each segment are calculated from the chart with bottom topography (Fig. 1). The solid line in Fig. 2 shows the relation between ξ_n and T computed at intervals of 0.1 minutes from 20 to 220 minutes. In this figure the values of T at which the line intersects the axis $\xi_n=0$ will give the period of the longitudinal seiche oscillation in the whole basin.

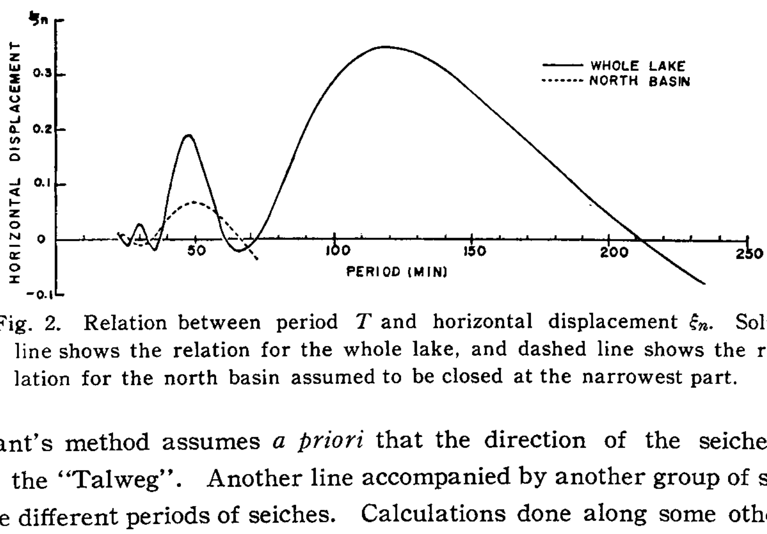

Fig. 2. Relation between period T and horizontal displacement ξ_n. Solid line shows the relation for the whole lake, and dashed line shows the relation for the north basin assumed to be closed at the narrowest part.

Defant's method assumes *a priori* that the direction of the seiche motion is along the "Talweg". Another line accompanied by another group of segments may give different periods of seiches. Calculations done along some other lines, however, revealed the difference in period to be small. Therefore the following discussion will be based on the result along the line-1 which is chosen so as to go nearly along "Talweg".

From the calculation possible periods of seiche are 212.0, 71.3, 61.0, 37.0 and 31.7 minutes, and these oscillations have the nodal lines of 1, 2, 3, 4 and 5 in number respectively. These periods of seiche are not in agreement with the Merian's relation in a rectangular basin with a constant depth. In particular, the periods of 71.3 and 61.0 minutes are too close. This result will be explained by the fact that Lake Biwa-ko is a connecting system of a shallow and small south basin and a deep and large north basin. In fact analytical solutions for a connecting system of two rectangular basins with constant depths and constant widths to be set equal to the mean values of the south and north basins of Lake Biwa-ko calculated by the author resemble the present calculated result.

The distributions of the vertical displacement of water surface with these

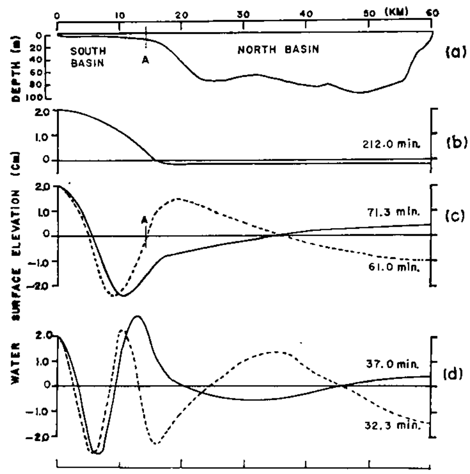

Fig. 3. Distributions of depth and vertical displacements of water surface along the line-1: a) Depth profile; b) Calculated water surface profile of the seiche with the period of 212.0 minutes; c) that of 71.3 and 61.0 minutes; and d) that of 37.0 and 32.3 minutes.

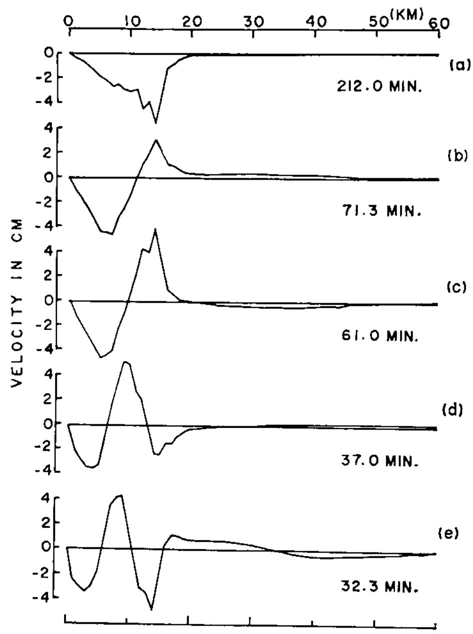

Fig. 4. Distributions of calculated velocity for each mode of the seiche.

periods are shown in Fig. 3, the distributions of the mean velocity in Fig. 4, and the distributions of the potential energy of each segment in Fig. 5.

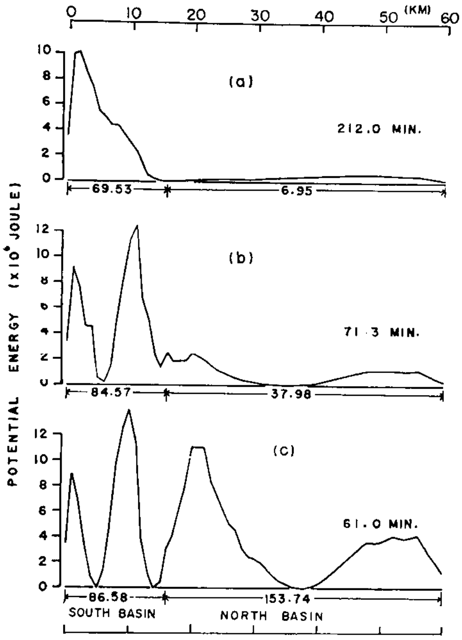

Fig. 5. Distributions of potential energy for each mode of the seiche.

Uni-nodal seiche oscillation

The oscillation of this mode has a period of 212.0 minutes and one nodal line at $x=16$ km which is about 1.5 km north of the mouth of the south basin. Total amounts of potential energy of the segment of the south side and of the north side of the nodal line are shown under the abscissas in Fig. 5. The ratio of total potential energy of the north side of the nodal line to that of the south side is about 0.1, and the ratio of the vertical displacement at the north end $x=61$ km, to that at the south end, $x=0$ km is also about 0.1. Energy concentration in the south basin seems to have formed the basis of the opinions in the previous papers where it is regarded as an oscillation only in the south basin. The calculated period agrees with the Merian's period after the mouth correction as the bay oscillation in the south basin. This oscillation should, however, be considered substantially to occur over the whole lake area.

Bi- and tri-nodal seiche oscillations

The bi- and tri-nodal seiche oscillations have periods of 71.3 and 61.0 minutes respectively. The difference in period between the two is only about 10 minutes.

It is easy to imagine that for the previous investigators it would have been very difficult to distinguish them clearly, using their simple technique of the period analysis of recording data. The period of 66.0 minutes which Takahashi and Namekawa [1938] and Toyohara and Habu [1938] presented as the mean period of the bi-nodal oscillation agrees with the period of an oscillation composed of two seiches with calculated periods of 71.3 and 61.0 minutes. It may be inferred from these points that their observed records would have contained these two components of seiches without separation.

From the distribution of the mean velocity of seiche motions in Fig. 4 it can be seen that components have maximum values of velocity at $x=14$ km, and their values are 3.08 cm/sec for the bi-nodal seiche and 5.93 cm/sec for the tri-nodal seiche. From the distribution of the vertical displacement in Fig. 3 it is found that both of these oscillations make uni-nodal seiches in the north basin. Assuming Lake Biwa-ko to be closed at the narrowest part at $x=14.5$ km (point A in Fig. 1), a one dimensional numerical calculation after Defant along the line-1 in the north basin gives a relation between period T and horizontal displacement ξ_n, which is shown by the dashed line in Fig. 2. The solutions are 66.0 and 34.2 minutes, and the former lies between 71.3 and 61.0 minutes and gives the uni-nodal oscillation in the north basin, while the latter lies between 37.0 and 31.7 minutes and gives the bi-nodal oscillation in the north basin. This result is quite interesting and our coming two or three dimensional numerical experiments may yield much more information about this problem.

It is shown from the distribution of potential energy in Fig. 5 that the ratio of the energy in the north basin to that in the south basin is 0.5 for the bi-nodal seiche and 2.0 for the tri-nodal seiche. Therefore, if these two oscillations have the same amplitude at $x=0$, the tri-nodal seiche will exceed the bi-nodal one in amplitude in the north basin.

The characters of other multi-nodal seiches are similar to those of the oscillations mentioned above.

3. Comparison with observations

It is found from the one dimensional numerical calculation that there is a uni-nodal seiche with a period of 212.0 minutes, and other multi-nodal seiches with periods of 71.3, 61.0 minutes etc. in Lake Biwa-ko.

In order to check the reality of the calculated results it is necessary to compare these calculated results with those of the precise spectrum analysis of the observed data over a fairly long period. There is, however, hardly any reliable observation providing adequate information about the space distribution of vertical displacement of water surface in Lake Biwa-ko. Some comparison is made by the use of the digital data which were measured with the eye for about six hours on Oct. 28, 1968 with simple devices at the nine points in Lake Biwa-ko shown by the double black circles in Fig. 1, although the degree of accuracy is not high. The observed data at each station are separated into three time-series with periods of about 240, 66 and 30 minutes by the running mean method, because the period of the observation is too short to adapt the spectrum analysis or Fourier analysis, and also because two time-series with such close periods 71.3 and 61.0 minutes are inseparable by the running mean method.

In the first place the oscillation with the period of 240 minutes is discussed. The space distribution of vertical displacement of water surface is shown in

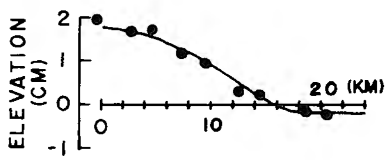

Fig. 6. Vertical displacement of water surface of the uni-nodal seiche. Solid line shows the calculated vertical displacement, and black dots show the observed values at 13^h10^m on 28 October, 1968.

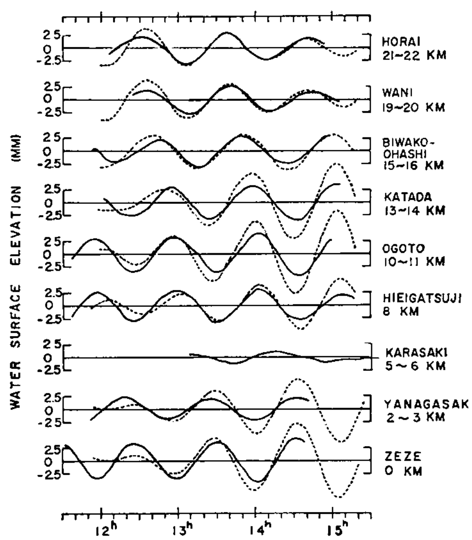

Fig. 7. Vertical displacements of the oscillation with the period of 60~70 minutes. Solid lines show the observed displacements with the period of 66.0 minutes, and dashed lines show the composite oscillation of the calculated bi- and tri-nodal seiches, whose amplitudes are equal at the southern end of the lake.

Fig. 6, where the black circles indicate the observed displacement at 13:10 and the solid line indicates that computed, with the adequate value of η_1, selected equal to 1.9 cm. From this figure, it may be concluded that the seiche of this mode is a uni-nodal seiche in the whole basin.

The solid curves in Fig. 7 show the oscillations with the period of about 66.0 minutes separated from the observed data of the seiche by the running mean method. It is strange that as seen in this figure the phase of the oscillation shifts gradually as the observation point moves to the south. The phase lag between Wani and Ogoto reaches 24 minutes. This oscillation can be considered to consist of two oscillations with periods of 71.3 and 61.0 minutes.

If $\eta_2(i)$ and $\eta_3(i)$ denote the computed vertical displacement of the bi- and tri-nodal oscillation in the i-th segment, an oscillation in this segment, $\zeta(i)$, composed from $\eta_2(i)$ and $\eta_3(i)$ may be expressed in the form of

$$\zeta(i)=\eta_2(i)\cdot\cos\omega_2 t+\eta_3(i)\cdot\cos\omega_3 t. \tag{2}$$

$\zeta(i)$ computed for any value of the ratio of $\eta_2(0)$ to $\eta_3(0)$ will indicate some phase lag. In this case the ratio $\eta_2(0)/\eta_3(0)=1$ gives the most remarkable agreement between the observed phase lag and the calculated one. The composed oscillations ζ for the ratio $\eta_2(0)/\eta_3(0)=1$ are shown by dashed curves in Fig. 7.

Hence it may be concluded that the phase lag in the observed record is derived from the coexistence of two oscillations with different but close periods. At the same time this conjecture yields the information at least in this case, that the amplitude of the bi-nodal seiche is nearly equal to that of the tri-nodal seiche in the south basin, and that the former would be nearly half of the latter in the north basin.

4. Conclusion

In this paper one dimensional numerical calculation of seiche by Defant's method is performed as a first step in approaching the whole dynamics of water motion in Lake Biwa-ko. Some results of this calculation are as follows:

(1) The possible periods of the longitudinal seiches in Lake Biwa-ko are 212.0, 71.3, 61.0, 39.0, 31.7 minutes and so on.

(2) The oscillation with the period of 212.0 minutes is a uni-nodal seiche with a nodal line at $x=16$ km and its potential energy is concentrated in the south basin. The space distribution of the computed vertical displacement of water surface agrees with that of the observed one.

(3) The oscillations with the period of 71.3 and 61.0 minutes are the bi- and tri-nodal seiche respectively, and in the north basin both have only one

node. The observation supports the existence of these two components of seiche which has not been known up to the present. During the period of the observation the ratio of the vertical displacement of the bi-nodal seiche to that of the tri-nodal seiche at the southern end of the lake is considered to have been nearly equal to unity.

(4) Other multi-nodal seiches with periods of 37.0 and 31.7 minutes have similar characters to the bi- and tri-nodal seiches.

As mentioned above, we obtained some information about the behavior of the longitudinal seiche motions in Lake Biwa-ko. In order to have more precise information through numerical experiments the two or three dimensional problem has to be treated with considerations of bottom friction, surface stress of wind, Coriolis' force etc. It will also be necessary to perform limnological observations such as the water surface elevation or the distribution of water velocity at some points for a fairly long period as well as to obtain meteorological information such as the time changes and the space distributions of wind speed and direction on Lake Biwa-ko. These problems are left open for further research.

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