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Free boundary problems for a viscous incompressible fluid

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1 A review of free boundary problems

In this section we review free boundary problems for a viscous incompressible fluid. In writing the review we are indebted to the works due to Zadrzyńska [54], Solonnikov [36] and Nishida [21] that we consulted.

The domain $\Omega_t \subset \mathbb{R}^d$ occupied by the fluid is given only on the initial time $t = 0$, while for $t > 0$ it is to be determined. The velocity vector filed $v(x, t) = (v_1, \ldots, v_d)$ and the pressure $p(x, t)$ satisfy the Navier-Stokes equations

$$v_t + (v \cdot \nabla)v - \nu\Delta v + \nabla p = f, \quad \nabla \cdot v = 0 \qquad x \in \Omega_t, \quad t > 0 \tag{1.1}$$

and suitable initial and boundary conditions, where $\nu > 0$ is a constant coefficient of viscosity and f is a vector field of external forces. We classify three kinds of free boundary problems with respect to the geometry of the domain Ω_t.

1.1 The motion of an isolated mass of a viscous fluid

This is the problem of describing the motion of an isolated mass of viscous fluid bounded by a free boundary. In the problem Ω_t is a bounded domain, a free surface $\partial\Omega_t = \Gamma_t$ is a compact, and initial and boundary conditions are given by

$$\begin{aligned} &v|_{t=0} = v_0(x) \qquad x \in \Omega_0 \\ &Sn - \sigma H n|_{x\in\Gamma_t} = 0. \end{aligned} \tag{1.2}$$

Here $\sigma \geqq 0$ is a constant coefficient of the surface tension. n is the unit outward normal vector to Γ_t. $S = \nu D(v) - pI$ is the stress tensor, $D(v)$ is the deformation tensor with the elements

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$\{D(v)\}_{ij} = \partial v_i/\partial x_j + \partial v_j/\partial x_i$, I is the identity matrix, $H/(d-1)$ is the mean curvature of Γ_t. The sign of H is chosen in such a way that $Hn = \Delta(t)x$, where $\Delta(t)$ is the Laplace-Beltrami operator on Γ_t.

The most of the existence results are obtained after transforming the free boundary problem written by Eulerian coordinates x to a problem in a fixed domain written by Lagrangean coordinates ξ. Let $x(\xi,t)$ be a solution of the Cauchy problem

$$\frac{dx}{dt} = v(x,t), \qquad x(0) = \xi. \tag{1.3}$$

Integrating (1.3), we connect Eulerian coordinates x with Lagrangean coordinates ξ by the formula

$$x = \xi + \int_0^t u(\xi,\tau)\,d\tau = X_u(\xi,t), \tag{1.4}$$

where $u(\xi,t) = v(X_u(\xi,t),t)$. A kinematic boundary condition at the free surface $\Gamma_t = \{x = x(\xi,t) \mid \xi \in \Gamma\}$ is assumed. This expresses the fact that the free surface Γ_t consists for all $t > 0$ of the same fluid particles, which do not leave it and are not incident on it from inside Ω_t. Passing to Lagrangean coordinates $x \in \Omega$ in (1.1) and (1.2), and setting $p(X_u(\xi,t),t) = q(\xi,t)$, we obtain

$$\begin{aligned} &u_t - \nu\nabla_u^2 u + \nabla_u q = f(X_u,t), \quad \nabla_u \cdot u = 0 \qquad \xi \in \Omega, \quad t > 0 \\ &u|_{t=0} = v_0(\xi), \quad S_u n - \sigma\Delta(t)X_u|_{\xi\in\Gamma} = 0. \end{aligned} \tag{1.5}$$

Here

$$\nabla_u = \mathcal{A}\nabla = \{\sum_m A_{im}\frac{\partial}{\partial\xi_m}\}_{i=1,\dots,d}$$

$$S_u = \nu D_u(u) - qI, \qquad \{D_u(u)\}_{ij} = \sum_m\Big(A_{im}\frac{\partial u_j}{\partial\xi_m} + A_{jm}\frac{\partial u_i}{\partial\xi_m}\Big).$$

$\mathcal{A}$ is the matrix with elements $A_{im} = \partial\xi_m/\partial x_i|_{x=X_u(\xi,t)}$, $n = n(X_u) = \mathcal{A}n_0/|\mathcal{A}n_0|$, where $n_0(\xi)$ is the unit outward normal to $\partial\Omega = \Gamma$.

1.1.1 The case of $\sigma = 0$

In this case the effect of surface tension on Γ_t is excluded. The pioneer work was done by Solonnikov [33] in 1977, he proved the local in time unique solvability of (1.5) in the framework of Hölder spaces $C^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2},1)$ by using the results of the linearized problem [32]. Later on, Solonnikov proved in [37] the local in time unique solvability of (1.5) for arbitrary initial data and the global in time unique solvability of (1.5) for $f = 0$ and sufficiently small initial data in the class of isotropic Sobolev spaces $W^{2,1}_p$ with $d < p < \infty$ when $d = 2$, 3. Solonnikov used the technique of the hydrodynamical potentials with the estimates of the kernels of the corresponding singular integrals. Recently, we extended the Solonnikov results in [37] in the class of anisotropic Sobolev spaces $W^{2,1}_{q,p}$ with $d < q < \infty$ and $2 < p < \infty$ of functions whose L_q norm with respect to the space variable together with the corresponding norms of the first and second spatial derivatives and the first time derivative are integrable with respect to time with the exponent p [1].

The novelty of our results consists of two moments. First, we extend the results of Solonnikov by unbinding of the exponents of integrability with respect to the space and time variables.

[1]The results were announced in [29].

Second, we allow the exponent of integrability with respect to time p go down up to 2 providing the weaker setting than one that was allowed by Solonnikov whose case was $p = q > d$. We develop the semigroup approach which consists on the following. The linear operator obtained under the linearization of the free boundary problem generates the analytic semigroup on the functional space of q-integrable divergence free functions which was known due to Grubb and Solonnikov [16] (cf. Theorem 2.1 in § 2). This should allow us to apply some facts of the general analytic semigroup theory to obtain two coercive estimates (Theorems 2.3 and 2.6 in § 2) which are crucial for the study of the nonlinear problem. The core of our approach is to show the L_p-L_q maximal regularity of the linearized problem global in time with exponential decay, which is stated in Theorem 2.3. One of our main issues to prove Theorem 2.3 is to use $\mathcal{R}$-boundedness and operator valued Fourier multiplier theorem which are recently developed by Weis [53], Denk, Hieber and Prüss [14] and Amann [4]. Methodologically, our approach seems to be simpler and more demonstrative than the estimates of kernel of singular integrals in the anisotropic Sobolev spaces. Thanks to the global in time L_p-L_q maximal regularity of the linearized problem on the whole time interval $(0, \infty)$ with exponential decay (Theorem 2.3), our proof of the global in time existence theorem (Theorem 2.5 in § 2) for the nonlinear problem (1.5) is much simpler than Solonnikov's proof [37]. In fact, we can show Theorem 2.5 simply by contraction mapping principle (cf. [31]). In the next section we state our results precisely.

Mucha and Zajączkowski considered the case where the self-gravitational force exists, namely $f = \kappa \nabla U$, where κ is the gravitational constant and U is the Newtonian potential. They proved in [20] the local in time unique solvability of (1.5) in $W^{2,1}_p$ with $d = 3$ and $3 < p < \infty$ for arbitrary initial data. They used the local in time unique solvability result of the linearized problem of (1.5) which was proved in [19].

1.1.2 The case of $\sigma > 0$

In this case the effect of surface tension on Γ_t is included. Solonnikov formulated the local in time solvability of (1.5) in the Sobolev-Slobodetskii space $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $f = 0$ and arbitrary initial data in [34]. In [35], Solonnikov proved the global in time solvability of (1.5) in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $f = 0$ provided that initial data are sufficiently small and the initial domain Ω_0 is sufficiently close to a ball. In [38] and [41], Solonnikov considered the case where the self-gravitational force exists. He proved in [41] the local in time unique solvability of (1.5) in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for arbitrary initial data, and in [38] the global in time unique solvability of (1.5) in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $f = \kappa \nabla U$, where κ is the gravitational constant and U is the Newtonian potential, provided that initial data are sufficiently small and the initial domain is sufficiently close to a ball. In [34, 35, 41, 38], he used the local in time unique solvability result of the linearized problem of (1.5) which was proved in [39].

Moglilevskiĭ and Solonnikov [18] proved the local in time solvability of (1.5) in Hölder spaces. Schweizer [27] proved the local in time unique existence of (1.5) for small initial data by using the semigroup approach. Padula and Solonnikov [26] proved the global in time unique solvability of (1.1) and (1.2) in Hölder spaces by using the mapping of Ω_t on a ball instead of Lagrangean coordinates.

1.1.3 The case of σ dependent on the temperature

This is an evolution problem of thermocapillary convection. Besides v and p satisfy (1.1), the temperature of the fluid $\theta(x,t)$ satisfies

$$\theta_t + (v\cdot\nabla)\theta - \kappa\Delta\theta = \lambda|D(v)|^2 \qquad x\in\Omega_t, \quad t>0$$

and they satisfy the initial and boundary conditions, where κ and λ are positive constants or small positive functions of θ.

Solonnikov proved in [40] the local in time unique solvability in $C^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha\in(\frac{1}{2},1)$, and proved in [42] the global in time unique solvability in $C^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha\in(\frac{1}{2},1)$ provided that the initial velocity and the initial temperature are sufficiently small and the domain is close to a ball. In the case $\lambda=0$, Lagnova and Solonnikov [17] obtained the local in time unique solvability in Hölder spaces, and Wagner [52] obtained the local in time unique solvability in Sobolev spaces.

1.2 Two phase problems

This problem describes the motion of two liquids separated by free interface. Let Ω be a bounded domain in $\mathbb{R}^d$ ($d\geqq 2$) or the whole space. Let $\Omega_t^+\subset\Omega$ be occupied by the fluid of viscosity $\nu^+>0$ and Γ_t be a boundary of Ω_t^+ which is strictly contained in Ω. Put $\Omega_t^- = \Omega\setminus(\Omega_t^+\cup\Gamma_t)$. Ω_t^- is occupied by the fluid of viscosity $\nu^->0$. Given functions $w^\pm$ defined on $\Omega_t^\pm$, we put

$$w = \begin{cases} w^+ & x\in\Omega_t^+,\ t>0 \\ w^- & x\in\Omega_t^-,\ t>0. \end{cases}$$

Moreover given function w defined on Ω, $w^\pm$ denote the restriction of w to $\Omega_t^\pm$. The velocity vector field $v^\pm$ and the pressure $p^\pm$ satisfy the Navier-Stokes equations

$$\partial_t v + (v\cdot\nabla)v - \operatorname{Div} S^\pm(v,p) = f, \quad \nabla\cdot v = 0 \quad \text{in } \Omega_t^\pm, \quad t>0 \tag{1.6}$$

and the boundary and the initial conditions

$$\begin{aligned}
&\left[\lim_{x\to x_0\in\Gamma_t,\ x\in\Omega_t^+} S^+(v,p) - \lim_{x\to x_0\in\Gamma_t,\ x\in\Omega_t^-} S^-(v,p)\right] n|_{\Gamma_t} = \sigma H n|_{\Gamma_t} \\
&\lim_{x\to x_0\in\Gamma_t,\ x\in\Omega_t^+} v|_{\Gamma_t} = \lim_{x\to x_0\in\Gamma_t,\ x\in\Omega_t^-} v|_{\Gamma_t} \\
&v|_{\partial\Omega} = 0, \qquad v|_{t=0} = v_0,
\end{aligned} \tag{1.7}$$

where $\sigma\geqq 0$ is coefficient of the surface tension, n is the unit outward normal to Γ_t of Ω_t^+, and $S^\pm(v,p)$ are stress tensors defined by $S^\pm(v,p)=\nu^\pm D(v)-pI$. A kinematic boundary condition at the free interface Γ_t is assumed. When Ω is the whole space, additional conditions are necessary.

In the case of a bounded Ω, Tanaka [46] proved the global in time solvability of (1.6) and (1.7) in $W_2^{2+\alpha}$ with $\alpha\in(\frac{1}{2},1)$ for $\sigma>0$, $d=3$ and sufficiently small data with discontinuity of densities. Giga and Takahashi [15] and Takahashi [45] proved the global in time existence of weak solutions of (1.6) and (1.7) in the spaces such that the first derivative of the velocity in L_p with $p>2(d+1)$ with respect to time and space for $\sigma=0$ provided that ν^+ is close to ν^-. Nouri and Poupand [25] proved the local in time existence of a weak solution of the Navier-Stokes equation describing a multi fluid flow for arbitrary initial data for $\sigma=0$. Recently, one of the authors Shimizu obtained the global in time unique solvability of (1.6) and (1.7) for $f = 0$ and sufficiently small initial data, and the local in time unique solvability of (1.6) and (1.7) for arbitrary initial data in $W^{2,1}_{q,p}$ with $2 < p < \infty$ and $d < q < \infty$ when $\sigma = 0$. The proofs of the result will be given in a forthcoming paper.

In the case of the whole space Ω, Denisova [10] proved the local in time unique solvability for arbitrary initial data in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $\sigma \geqq 0$ and $d = 3$ with discontinuity of densities by using the local in time unique solvability result of the linearized problem in [8] and [11]. Denisova and Solonnikov [13] proved the local in time unique solvability for arbitrary initial data in the Hölder spaces with a power-like weight for $\sigma > 0$ and $d = 3$ with discontinuity of densities by using the local in time unique solvability result of the linearized problem in [9] and [12]. Recently, Abels [2] proved the existence of varifold and measure-valued varifold solutions for singular free interfaces for $\sigma \geqq 0$.

1.3 Surface wave problems

This problem describes the motion of a fluid which occupies a semi-infinite domain in $\mathbb{R}^d$ ($d = 2, 3$) between the moving upper surface and a fixed bottom. Let

$$\Omega_t = \{x = (x', x_d) \in \mathbb{R}^d \mid \ x' \in \mathbb{R}^{d-1}, \ -b(x') < x_d < \eta(x', t)\}.$$

The velocity vector v and the pressure p satisfy the Navier-Stokes equation (1.1). The upper free surface $S_F : \ x_d = \eta(x', t)$ satisfies the kinematic boundary condition

$$\eta_t = u_d - \sum_{k=1}^{d-1} (\partial_k \eta) u_k \quad \text{on} \quad S_F. \tag{1.8}$$

The boundary condition on S_F is given by

$$p n_i - \nu D_{ij}(v) n_j = [g\eta - \sigma \nabla \cdot \{(1 + |\nabla \eta|^2)^{-\frac{1}{2}} \nabla \eta\}] n_i \quad \text{on} \quad S_F, \tag{1.9}$$

where $n = (n_1, \ldots, n_d)$ is the outward normal to S_F, g is the gravitation constant, and σ is the coefficient of surface tension. On the bottom surface $S_B : \ x_d = -b(x')$ the boundary is impenetrable

$$v = 0 \quad \text{on} \quad S_B. \tag{1.10}$$

The initial condition is the following

$$\eta = \eta_0(x') \ \ x \in \mathbb{R}^{d-1}, \quad u = u_0(x) \ \ x \in \Omega_0 \qquad \text{at} \ \ t = 0. \tag{1.11}$$

The pioneer work of this problem was done by Beale [5] in 1980. Beale proved the local in time unique solvability for $\sigma = 0$ and $d = 3$ in the Bessel potential spaces $H_2^{\ell,\frac{\ell}{2}}$ with $3 < \ell < \frac{7}{2}$. In [6], Beale proved the global in time unique solvability in $H_2^{\ell,\frac{\ell}{2}}$ with $3 < \ell < \frac{7}{2}$ for $\sigma > 0$, $d = 3$ and $f = 0$ provided that the initial data η_0 and u_0 are sufficiently small. Beale and Nishida [7] obtained the asymptotic power-like in time decay of the global solutions. The local existence theorem for $\sigma > 0$ and $d = 2$ was established by Allain [3]. Tani [47] proved the local in time unique solvability for $\sigma > 0$ and $d = 3$ in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$. Sylvester [44] showed the global in time solvability in $H_2^{\ell,\frac{\ell}{2}}$ with $\frac{9}{2} < \ell < 5$ for $\sigma = 0$ and $d = 3$ provided that initial data are sufficiently small by using Beale's method. Tani and Tanaka [48] proved the global in time solvability in $W_2^{2+\alpha}$ with $\alpha \in (\frac{1}{2}, 1)$ for $\sigma \geqq 0$ and $d = 3$ provided that initial data are sufficiently small by using Solonnikov's method. Recently, Abels [1] proved the local in time unique solvability in the isotropic Sobolev spaces $W_p^{2,1}$ with $d < p < \infty$.

Nishida, Teramoto and Yoshihara [22] considered this problem under the assumption that the motion of fluid is horizontally periodic and that spatial mean of the motion of unknown free surface over the space period is equal to zero. They proved the global in time unique solvability and exponential stability in $H_2^{\ell,\frac{\ell}{2}}$ with $3 < \ell < \frac{7}{2}$ for sufficiently small initial data.

Teramoto has studied the motion of a viscous incompressible fluid which flows down an inclined plane under the effect of gravity. The fluid is bounded from below by a fixed plane which is inclined at an angle $0 < \phi < \pi/2$ to the horizontal plane. He proved the local in time unique solvability for $\sigma = 0$ and $d = 3$ in [49], for $\sigma > 0$ and $d = 3$ in [50] by using Beale's idea. Nishida, Teramoto and Win [24] proved the global in time unique existence and stability for $d = 2$ and sufficiently small initial data. Nishida, Teramoto and Yoshihara provided the Hopf bifurcation theorem in [23].

2 Results

As mentioned in the previous section, in this section we state our results precisely. We consider a time dependent problem with free surface for the Navier-Stokes equations which describes the motion of an isolated finite volume of viscous incompressible fluid without taking surface tension into account. The region $\Omega_t \subset \mathbb{R}^d$, $d \geqq 2$, occupied by the fluid is given only on the initial time $t = 0$, while for $t > 0$ it is to be determined. The velocity vector field $v(x,t) = (v_1, \ldots, v_d)^*$ [2] and the pressure $\theta(x,t)$ for $x \in \Omega_t$ satisfy the Navier-Stokes equations:

$$\begin{aligned}
&v_t + (v \cdot \nabla)v - \mathrm{Div}\, S(v,\theta) = f(x,t) && \text{in } \Omega_t,\ t > 0\\
&\mathrm{div}\, v = 0 && \text{in } \Omega_t,\ t > 0\\
&S(v,\theta)n_t + \theta_0(x,t)n_t = 0 && \text{in } \Gamma_t,\ t > 0\\
&v|_{t=0} = v_0 && \text{on } \Omega.
\end{aligned} \tag{2.1}$$

Here, Γ_t denotes the boundary of Ω_t, $n_t(x)$ is the unit outward normal to Γ_t at the point $x \in \Gamma_t$, $\nabla = (\partial_1, \ldots, \partial_d)$ with $\partial_i = \partial/\partial x_i$, and $S(v,\theta)$ is the stress tensor defined by $S(v,\theta) = D(v) - \theta I$ where $D(v)$ is the deformation tensor of the velocities with elements $D_{ij}(v) = \partial_i v_j + \partial_j v_i$ and I is the $d \times d$ identity matrix. The external force $f(x,t)$ and the pressure $\theta_0(x,t)$ are functions defined on the whole space. In what follows, we may always assume that $\theta_0(x,t) = 0$, since we can arrive at this case by replacing $\theta(x,t)$ by $\theta + \theta_0$.

Aside from the dynamical boundary condition, a further kinematic condition for Γ_t is satisfied. We write $\Omega = \Omega_0$ and $\Gamma = \Gamma_0$ and we assume that Γ is a $C^{2,1}$ compact hypersurface.

Passing to Lagrangean coordinates in (2.1) and setting $\theta(X_u(\xi,t),t) = \pi(\xi,t)$, we obtain

$$\begin{aligned}
&u_t - \mathrm{Div}\,[S(u,\pi) + U(u,\pi)] = f(X_u(\xi,t),t) && \text{in } \Omega \times (0,T)\\
&\mathrm{div}\, u + E(u) = \mathrm{div}\,[u + \tilde{E}(u)] = 0 && \text{in } \Omega \times (0,T)\\
&[S(u,\pi) + U(u,\pi)]n = 0 && \text{on } \Gamma \times (0,T)\\
&u|_{t=0} = u_0 && \text{in } \Omega,
\end{aligned} \tag{2.2}$$

where $u_0(\xi) = v_0(x)$. Here and hereafter, n denotes the unit outward normal to Γ, and $U(u,\pi)$,

[2] M^* denotes the transpose of M

$E(u)$ and $\tilde{E}(u)$ are nonlinear terms of the following forms:

$$U(u,\pi) = V_1(\int_0^t \nabla u\, d\tau)\nabla u + V_2(\int_0^t \nabla u\, d\tau)\pi$$
$$E(u) = V_3(\int_0^t \nabla u\, d\tau)\nabla u, \quad \tilde{E}(u) = V_4(\int_0^t \nabla u\, d\tau)u$$

with some polynomials $V_j(\cdot)$ of $\int_0^t \nabla u\, d\tau$, $j = 1, 2, 3, 4$, such as $V_j(0) = 0$. As a linearized problem of (2.2), we obtain the following Stokes equation with Neumann boundary condition:

$$\begin{aligned} &u_t - \mathrm{Div}\, S(u,\pi) = f && \text{in } \Omega \times (0,T) \\ &\mathrm{div}\, u = g = \mathrm{div}\, \tilde{g} && \text{in } \Omega \times (0,T) \\ &S(u,\pi)n|_\Gamma = h, \quad u|_{t=0} = u_0. \end{aligned} \tag{2.3}$$

In order to state our main results precisely, we introduce the function spaces and some symbols which will be used throughout the paper. For any domain D in $\mathbb{R}^d$, integer m and $1 \leqq q \leqq \infty$, $L_q(D)$ and $W_q^m(D)$ denote the usual Lebesgue space and Sobolev space of functions defined on D with norms: $\|\cdot\|_{L_q(D)}$ and $\|\cdot\|_{W_q^m(D)}$, respectively. And also, for any Banach space X, interval I, integer ℓ and $1 \leqq p \leqq \infty$, $L_p(I,X)$ and $W_p^\ell(I,X)$ denote the usual Lebesgue space and Sobolev space of the X - valued functions defined on I with norms: $\|\cdot\|_{L_p(I,X)}$ and $\|\cdot\|_{W_p^\ell(I,X)}$, respectively. Set

$$\begin{aligned} &W_{q,p}^{m,\ell}(D\times I) = L_p(I, W_q^m(D)) \cap W_p^\ell(I, L_q(D)) \\ &\|u\|_{W_{q,p}^{m,\ell}(D\times I)} = \|u\|_{L_p(I,W_q^m(D))} + \|u\|_{W_p^\ell(I,L_q(D))} \\ &W_{p,0}^\ell((0,T),X) = \{u \in W_p^\ell((-\infty,T),X) \mid u = 0 \text{ for } t < 0\} \\ &W_q^0(D) = L_q(D),\;\; W_p^0(I,X) = L_p(I,X),\;\; W_{p,0}^0((0,T),X) = L_{p,0}((0,T),X). \end{aligned}$$

Given $\alpha \geqq 0$, we set

$$\begin{aligned} &< D_t >^\alpha u(t) = \mathcal{F}^{-1}[(1+s^2)^{\alpha/2}\mathcal{F}u(s)](t) \\ &H_p^\alpha(\mathbb{R},X) = \{u \in L_p(\mathbb{R},X) \mid < D_t >^\alpha u \in L_p(\mathbb{R},X)\} \\ &\|u\|_{H_p^\alpha(\mathbb{R},X)} = \| < D_t >^\alpha u\|_{L_p(\mathbb{R},X)} + \|u\|_{L_p(\mathbb{R},X)}. \end{aligned}$$

Here and hereafter, $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse formula, respectively. Set

$$\begin{aligned} &H_{q,p}^{1,1/2}(D\times\mathbb{R}) = H_p^{1/2}(\mathbb{R}, L_q(D)) \cap L_p(\mathbb{R}, W_q^1(D)) \\ &H_{q,p,0}^{1,1/2}(D\times\mathbb{R}_+) = \{u \in H_{q,p}^{1,1/2}(D\times\mathbb{R}) \mid u = 0 \text{ for } t < 0\} \\ &\|u\|_{H_{q,p}^{1,1/2}(D\times\mathbb{R})} = \|u\|_{H_p^{1/2}(\mathbb{R},L_q(D))} + \|u\|_{L_p(\mathbb{R},W_q^1(D))}. \end{aligned}$$

Finally, given $0 < T < \infty$ we set

$$\begin{aligned} &H_{q,p}^{1,1/2}(D\times(0,T)) = \{u \mid {}^\exists v \in H_{q,p}^{1,1/2}(D\times\mathbb{R}),\;\; u = v \text{ on } D\times(0,T)\} \\ &\|u\|_{H_{q,p}^{1,1/2}(D\times(0,T))} = \inf\{\|v\|_{H_{q,p}^{1,1/2}(D\times\mathbb{R})} \mid {}^\forall v \in H_{q,p}^{1,1/2}(D\times\mathbb{R}),\;\; v = u \text{ on } D\times(0,T)\} \\ &H_{q,p,0}^{1,1/2}(D\times(0,T)) = \{u \mid {}^\exists v \in H_{q,p,0}^{1,1/2}(D\times\mathbb{R}_+),\;\; u = v \text{ on } D\times(0,T)\} \end{aligned}$$

$$\|u\|_{H^{1,1/2}_{q,p,0}(D\times(0,T))} = \inf\{\|v\|_{H^{1,1/2}_{q,p}(D\times\mathbb{R})} \mid {}^\forall v \in H^{1,1/2}_{q,p,0}(D\times\mathbb{R}_+),\ \ v = u \text{ on } D\times(0,T)\}.$$

Given Banach space X with norm $\|\cdot\|_X$, we set $X^d = \{v = (v_1, \ldots, v_d)^* \mid v_j \in X\}$, $\|v\|_X = \sum_{j=1}^d \|v_j\|_X$. The dot $\cdot$ denotes the inner-product of $\mathbb{R}^d$. $F = (F_{ij})$ means an $d\times d$ matrix whose i-th column and j-th row component is F_{ij}. For the differentiation of an $d\times d$ matrix of functions $F = (F_{ij})$, an d-vector of functions $u = (u_1, \ldots, u_d)^*$ and a scalar function θ, we use the following symbols: $\theta_t = \partial_t\theta = \partial\theta/\partial t$, $\partial_j\theta = \partial\theta/\partial x_j$,

$$\nabla\theta = (\partial_1\theta, \ldots, \partial_d\theta)^*,\ \ \nabla^k\theta = (\partial_x^\alpha\theta \mid |\alpha| = k), u_t = \partial_t u = (\partial_t u_1, \ldots, \partial_t u_d), \nabla u = (\partial_i u_j),$$
$$\nabla^k u = (\partial_x^\alpha u_i, |\alpha| = k, i = 1, \ldots, d),\ \ \operatorname{div} u = \sum_{j=1}^d \partial_j u_j,\ \ \operatorname{Div} F = (\sum_{j=1}^d \partial_j F_{1j}, \ldots, \sum_{j=1}^d \partial_j F_{dj})^*.$$

The inner products $(\cdot,\cdot)_\Omega$ and $(\cdot,\cdot)_\Gamma$ are defined by $(u, v)_\Omega = \int_\Omega u(x)\cdot v(x)\,dx$ and $(u, v)_\Gamma = \int_\Gamma u(x)\cdot v(x)\,d\sigma$ where $d\sigma$ denotes the surface element of Γ. By C we denote a generic constant and $C_{a,b,\cdots}$ denotes the constant depending on the quantities a, b, $\cdots$. The constants C and $C_{a,b,\cdots}$ may change from line to line.

In order to introduce our class of initial data for (2.2), we discuss an analytic semigroup approach to the initial boundary value problem:

$$\begin{aligned} u_t - \operatorname{Div} S(u,\pi) = 0, \quad \operatorname{div} u = 0 \quad \text{in } \Omega\times(0,\infty) \\ S(u,\pi)n|_\Gamma = 0, \quad u|_{t=0} = u_0. \end{aligned} \tag{2.4}$$

Since the time derivative of π is missing in (2.4), to obtain the evolution equation for u we have to eliminate π from (2.4). To do this, for a while instead of (2.4) we shall consider the resolvent problem:

$$\lambda u - \operatorname{Div} S(u,\pi) = f, \quad \operatorname{div} u = 0 \quad \text{in } \Omega\times(0,\infty), \quad S(u,\pi)n|_\Gamma = 0 \tag{2.5}$$

and we shall discuss how to eliminate π from (2.5). We introduce the second Helmholtz decomposition corresponding to (2.4). Set

$$\begin{aligned} J_q(\Omega) &= \{u = (u_1, \ldots, u_d)^* \in L_q(\Omega)^d \mid \operatorname{div} u = 0 \ \ \text{in } \Omega\} \\ G_q(\Omega) &= \{\nabla\pi \mid \pi \in W^1_q(\Omega),\ \ \pi|_\Gamma = 0\}. \end{aligned}$$

Then, by Grubb and Solonnikov [16] (cf. also Shibata and Shimizu [28]) we know that

$$L_q(\Omega)^d = J_q(\Omega) \oplus G_q(\Omega)$$

for $1 < q < \infty$, where $\oplus$ denotes the direct sum. Let P_q be the solenoidal projection: $L_q(\Omega)^d \to J_q(\Omega)$ along $G_q(\Omega)$. Then, substituting the 2nd Helmholtz decomposition of f: $f = P_q f + \nabla\theta$, where $\theta \in W^1_q(\Omega)$ and $\theta|_\Gamma = 0$, into (2.5), we have

$$\lambda v - \operatorname{Div} S(v, \pi - \theta) = P_q f,\ \ \operatorname{div} v = 0 \quad \text{in } \Omega, \quad S(v, \pi-\theta)n|_\Gamma = 0. \tag{2.6}$$

Denoting $\pi - \theta$ by π in (2.6) again, from now on we consider (2.5) under the condition that $\operatorname{div} f = 0$. Then, applying the divergence to (2.5) and multiplying the boundary condition by n, we have

$$\Delta\theta = 0 \quad \text{in } \Omega, \quad \theta|_\Gamma = [D(v)n]\cdot n - \operatorname{div} v|_\Gamma, \tag{2.7}$$

where we have used the facts that $\operatorname{div} v = 0$ in Ω and $n\cdot n = 1$ on Γ. We know that given $v \in W^2_q(\Omega)^d$ there exists a unique $\theta \in W^1_q(\Omega)$ which solves (2.7) and enjoys the estimate:

$$\|\theta\|_{W^1_q(\Omega)} \leqq C\|v\|_{W^2_q(\Omega)}.$$

From this point of view, let us define the map $K : W_q^2(\Omega)^d \to W_q^1(\Omega)$ by $\theta = K(v)$ for $v \in W_q^2(\Omega)^d$. By using this symbol, the equation (2.5) is rewritten in the form:

$$\lambda v - \mathrm{Div}\, S(v, K(v)) = f \quad \text{in } \Omega, \quad S(v, K(v))n|_\Gamma = 0 \tag{2.8}$$

for $f \in J_q(\Omega)$. We set

$$A_q u = -\mathrm{Div}\, S(u, K(u)) \quad \text{for } u \in \mathcal{D}(A_q)$$
$$\mathcal{D}(A_q) = \{u \in J_q(\Omega) \cap W_q^2(\Omega)^d \mid S(u, K(u))n|_\Gamma = 0\}.$$

From Grubb and Solonnikov [16] and Shibata and Shimizu [28], we know the following theorem.

Theorem 2.1. *Let $1 < q < \infty$. Then, A_q generates an analytic semigroup $\{e^{-A_q t}\}_{t \geqq 0}$ on $J_q(\Omega)$.*

Remark 2.2. The function $e^{-A_q t} u_0$ is an initial flow for (2.2).

Now, we shall state our results. For the initial data, we introduce the following space:

$$\mathcal{D}_{q,p}(\Omega) = [J_q(\Omega), \mathcal{D}(A_q)]_{1-1/p,p}.$$

Here and hereafter, $[\cdot,\cdot]_{\theta,p}$ denotes the real interpolation functor. In order to state a global in time existence theorem for (2.2), we introduce the rigid space $\mathcal{R}_{gd}$ which is defined by the relation:

$$\mathcal{R}_{gd} = \{Ax + b \mid A : d \times d \text{ anti-symmetric matrix}, b \in \mathbb{R}^d\}.$$

In what follows, we denote the basis of $\mathcal{R}_{gd}$ by $\{p_\ell\}_{\ell=1}^M$, which are normalized such as $(p_\ell, p_m)_\Omega = \delta_{\ell m}$ $(\ell, m = 1, \ldots, M)$, where $\delta_{\ell m}$ are Kronecker's delta symbols.

The first theorem is the main result which shows the global in time L_p-L_q maximal regularity of (2.3) with exponential stability.

Theorem 2.3. *Let $1 < p, q < \infty$. Then, there exists a $\gamma_0 > 0$ such that if u_0, f, g, $\tilde g$ and h satisfy the following conditions:*

$$u_0 \in \mathcal{D}_{q,p}(\Omega), \ e^{\gamma t} f \in L_p((0,\infty), L_q(\Omega))^d, \quad e^{\gamma t} g \in L_{p,0}((0,\infty), W_q^1(\Omega))$$
$$e^{\gamma t}\tilde g \in W_{p,0}^1((0,\infty), L_q(\Omega))^d, \quad e^{\gamma t} h \in H_{q,p,0}^{1,1/2}(\Omega \times (0,\infty))^d$$

for some $\gamma \in [0, \gamma_0]$, and

$$(u_0, p_\ell)_\Omega = 0, \quad (f(\cdot,t), p_\ell)_\Omega + (h(\cdot,t), g_\ell)_\Gamma = 0$$

for $t \geqq 0$ and $\ell = 1, \ldots, M$, then the equation (2.3) with $T = \infty$ admits a unique solution

$$(u, \pi) \in W_{q,p}^{2,1}(\Omega \times (0,\infty))^d \times L_p((0,\infty), W_q^1(\Omega)).$$

Moreover there exists $\tilde\pi \in H_{q,p}^{1,1/2}(\Omega \times (0,\infty))$ such that $\tilde\pi = \pi$ on $\Gamma \times (0,\infty)$. The solution satisfies the estimates:

$$\|e^{\gamma t} u\|_{W_{q,p}^{2,1}(\Omega\times(0,\infty))} + \|e^{\gamma t}\pi\|_{L_p((0,\infty),W_q^1(\Omega))} + \|e^{\gamma t}\tilde\pi\|_{H_{q,p}^{1,1/2}(\Omega\times(0,\infty))} \leqq C\{\|u_0\|_{\mathcal{D}_{q,p}(\Omega)}$$
$$+ \|e^{\gamma t} f\|_{L_p((0,\infty),L_q(\Omega))} + \|e^{\gamma t} g\|_{L_p(\mathbb{R},W_q^1(\Omega))} + \|e^{\gamma t}\tilde g\|_{W_p^1(\mathbb{R},L_q(\Omega))} + \|e^{\gamma t} h\|_{H_{q,p,0}^{1,1/2}(\Omega\times(0,\infty))}\}$$

and the condition: $(u(\cdot,t), p_\ell)_\Omega = 0$ for $t \geqq 0$ and $\ell = 1, \ldots, M$.

Remark 2.4. Let us define the Besov space $B^{2(1-1/p)}_{q,p}(\Omega)$ by the real interpolation:

$$B^{2(1-1/p)}_{q,p}(\Omega) = [L_q(\Omega), W^2_q(\Omega)]_{1-1/p,p}$$

and set

$$JB^{2(1-1/p)}_{q,p}(\Omega) = \{u \in B^{2(1-1/p)}_{q,p}(\Omega)^d \mid \operatorname{div} u = 0 \text{ in } \Omega\}.$$

Then, we see that $\mathcal{D}_{q,p}(\Omega) \subset JB^{2(1-1/p)}_{q,p}(\Omega)$. Moreover, from Proposition 2.13 combined with Remarks 2.7 (c) in Steiger [43] (cf. also Triebel [51]) it follows that

$$\mathcal{D}_{q,p}(\Omega) = \begin{cases} \{v \in JB^{2(1-1/p)}_{q,p}(\Omega) \mid S(v, K(v))n|_\Gamma = 0\} & 2(1-1/p) > 1 + 1/q \\ JB^{2(1-1/p)}_{q,p}(\Omega) & 2(1-1/p) < 1 + 1/q. \end{cases}$$

The next theorem shows the global in time unique solvability of (2.2) for $f = 0$ and sufficiently small initial data which are orthogonal to the rigid space, which is proved by using the contraction mapping principle based on Theorem 2.3.

Theorem 2.5. *Let $2 < p < \infty$ and $d < q < \infty$. We consider the case where $T = \infty$ and $f = 0$ in (2.2). Then, there exist positive numbers ϵ and γ such that if $u_0 \in \mathcal{D}_{q,p}(\Omega)$, $\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} \leqq \epsilon$ and $(u_0, p_\ell)_\Omega = 0$ for $\ell = 1, \ldots, M$, then the equation (2.2) with $T = \infty$ and $f = 0$ admits a unique solution*

$$(u, \pi) \in W^{2,1}_{q,p}(\Omega \times (0, \infty))^d \times L_p((0, \infty), W^1_q(\Omega)).$$

Moreover there exists $\tilde\pi \in H^{1,1/2}_{q,p}(\Omega \times (0, \infty))$ such that $\tilde\pi = \pi$ on $\Gamma \times (0, \infty)$. The solution satisfies the estimate:

$$\|e^{\gamma t} u\|_{W^{2,1}_{q,p}(\Omega\times(0,\infty))} + \|e^{\gamma t}\pi\|_{L_p((0,\infty),W^1_q(\Omega))} + \|e^{\gamma t}\tilde\pi\|_{H^{1,1/2}_{q,p}(\Omega\times(0,\infty))} \leqq C\epsilon$$

for some $\gamma > 0$ and the condition:

$$(u(\cdot, t), p_\ell)_\Omega = 0 \quad \textit{for } \ell = 1, \ldots, M \textit{ and } t \geqq 0.$$

The next theorem shows the L_p-L_q maximal regularity of (2.3) local in time.

Theorem 2.6. *Let $1 < p, q < \infty$ and $0 < T < \infty$. If u_0, f, g, $\tilde g$ and h satisfy the condition:*

$$u_0 \in \mathcal{D}_{q,p}(\Omega), \quad f \in L_p((0,T), L_q(\Omega))^d, \quad g \in L_{p,0}((0,T), W^1_q(\Omega))$$
$$\tilde g \in W^1_{p,0}((0,T), L_q(\Omega))^d, \quad h \in H^{1,1/2}_{q,p,0}(\Omega \times (0,T))^d$$

then the equation (2.3) admits a unique solution

$$(u, \pi) \in W^{2,1}_{q,p}(\Omega \times (0,T))^d \times L_p((0,T), W^1_q(\Omega)).$$

Moreover there exists $\tilde\pi \in H^{1,1/2}_{q,p}(\Omega\times(0,T))$ such that $\tilde\pi = \pi$ on $\Gamma\times(0,T)$. The solution satisfies the estimate:

$$\begin{aligned} \|u\|_{W^{2,1}_{q,p}(\Omega\times(0,T))} + \|\pi\|_{L_p((0,T),W^1_q(\Omega))} + \|\tilde\pi\|_{H^{1,1/2}_{q,p}(\Omega\times(0,T))} \leqq C(1+T)\{\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} \\ + \|f\|_{L_p((0,T),L_q(\Omega))} + \|g\|_{L_p((0,T),W^1_q(\Omega))} + \|\tilde g\|_{W^1_p((0,T),L_q(\Omega))} + \|h\|_{H^{1,1/2}_{q,p,0}(\Omega\times[0,T))}\} \end{aligned} \tag{2.9}$$

where the constant C is independent of T, u, π, u_0, f, g, $\tilde g$ and h.

The next theorem shows the local in time unique solvability of (2.2) for any initial data and right member of f, which is proved by using the contraction mapping principle based on Theorem 2.6.

Theorem 2.7. *Let $2 < p < \infty$ and $d < q < \infty$. Then, for any $R > 0$ and $R' > 0$ there exists a time $T > 0$ depending on R and R' such that the equation* (2.2) *admits a unique solution*

$$(u, \pi) \in W^{2,1}_{q,p}(\Omega \times (0,T))^d \times L_p((0,T), W^1_q(\Omega)).$$

Moreover there exists $\tilde\pi \in H^{1,1/2}_{q,p}(\Omega \times (0,T))$ such that $\tilde\pi = \pi$ on $\Gamma \times (0,T)$. The solution satisfies the estimate:

$$\|u\|_{W^{2,1}_{q,p}(\Omega\times(0,T))} + \|\pi\|_{L_p((0,T),W^1_q(\Omega))} + \|\tilde\pi\|_{H^{1,1/2}_{q,p}(\Omega\times(0,T))} \leqq CR$$

for some constant C depending essentially only on p and q provided that $u_0 \in \mathcal{D}_{q,p}(\Omega)$, $f \in L_p(\mathbb{R}_+, L_q(\mathbb{R}^d))$, $\nabla f \in L_\infty(\mathbb{R}^d \times \mathbb{R}_+)$ and

$$\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} + \|f\|_{L_p(\mathbb{R}_+,L_q(\mathbb{R}^d))} \leqq R, \quad \|\nabla f\|_{L_\infty(\mathbb{R}^d\times\mathbb{R}_+)} \leqq R'.$$

The proofs of Theorems 2.3 and 2.6 are given in [30] and those of Theorems 2.5 and 2.7 are given in [31].

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