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New examples of oriented matroids with disconnected realization spaces

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Abstract

We construct oriented matroids of rank 3 on 13 points whose realization spaces are disconnected. They are defined on smaller points than the known examples with this property. Moreover, we construct the one on 13 points whose realization space is a connected and non-irreducible semialgebraic variety.

1 Oriented Matroids and Matrices

Throughout this section, we fix positive integers r and n.

Let $X = (x_1, \dots, x_n) \in \mathbb{R}^{rn}$ be a real (r, n) matrix of rank r, and $E = \{1, \dots, n\}$ be the set of labels of the columns of X. For such matrix X, a map χ_X can be defined as

$$\chi_X : E^r \to \{-1, 0, +1\}, \quad \chi_X(i_1, \dots, i_r) := \operatorname{sgn}\det(x_{i_1}, \dots, x_{i_r}).$$

The map χ_X is called the *chirotope* of X. The chirotope χ_X encodes the information on the combinatorial type which is called the *oriented matroid* of X. In this case, the oriented matroid determined by χ_X is of rank r on E.

We note for some properties which the chirotope χ_X of a matrix X satisfies.

1. χ_X is not identically zero.

2. χ_X is alternating, i.e. $\chi_X(i_{\sigma(1)}, \dots, i_{\sigma(r)}) = \operatorname{sgn}(\sigma)\chi_X(i_1, \dots, i_r)$ for all $i_1, \dots, i_r \in E$ and all permutation σ.

3. For all $i_1, \dots, i_r, j_1, \dots, j_r \in E$ such that $\chi_X(j_k, i_2, \dots, i_r) \cdot \chi_X(j_1, \dots, j_{k-1}, i_1, j_{k+1}, \dots, j_r) \geq 0$ for $k = 1, \dots, r$, we have $\chi_X(i_1, \dots, i_r) \cdot \chi_X(j_1, \dots, j_r) \geq 0$.

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The third property follows from the identity

$$\det(x_1,\ldots,x_r)\cdot\det(y_1,\ldots,y_r) \\ = \sum_{k=1}^{r}\det(y_k,x_2,\ldots,x_r)\cdot\det(y_1,\ldots,y_{k-1},x_1,y_{k+1},\ldots,y_r), \\ \text{for all } x_1,\ldots,x_r,y_1,\ldots,y_r\in\mathbb{R}^r.$$

Generally, an oriented matroid of rank r on E (n points) is defined by a map $\chi : E^r \to \{-1, 0, +1\}$, which satisfies the above three properties ([1]). The map χ is also called the chirotope of an oriented matroid. We use the notation $\mathcal{M}(E,\chi)$ for an oriented matroid which is on the set E and is defined by the chirotope χ.

An oriented matroid $\mathcal{M}(E,\chi)$ is called *realizable* or *constructible*, if there exists a matrix X such that $\chi = \chi_X$. Not all oriented matroids are realizable, but we don't consider non-realizable case in this paper.

Definition 1.1. *A realization of an oriented matroid $\mathcal{M} = \mathcal{M}(E,\chi)$ is a matrix X such that $\chi_X = \chi$ or $\chi_X = -\chi$.*

Two realizations X, X' of $\mathcal{M}$ are called linearly equivalent, if there exists a linear transformation $A \in GL(r,\mathbb{R})$ such that $X' = AX$. Here we have the equation $\chi_{X'} = \operatorname{sgn}(\det A)\cdot\chi_X$.

Definition 1.2. *The realization space $\mathcal{R}(\mathcal{M})$ of an oriented matroid $\mathcal{M}$ is the set of all linearly equivalent classes of realizations of $\mathcal{M}$, in the quotient topology induced from $\mathbb{R}^{rn}$.*

Our motivation is as follows: In 1956, Ringel asked whether the realization spaces $\mathcal{R}(\mathcal{M})$ are necessarily connected [6]. It is known that every oriented matroid on less than 9 points has a contractible realization space. In 1988, Mnëv showed that $\mathcal{R}(\mathcal{M})$ can be homotopy equivalent to an arbitrary semialgebraic variety [3]. His result implies that they can have arbitrary complicated topological types. In particular, there exist oriented matroids with disconnected realization spaces. Suvorov and Righter-Gebert constructed such examples of oriented matroids of rank 3 on 14 points, in 1988 and in 1996 respectively [7, 5]. However it is unknown which is the smallest number of points on which oriented matroids can have disconnected realization spaces. See [1] for more historical comments.

One of the main results of this paper is the following.

Theorem 1.3. *There exist oriented matroids of rank* 3 *on* 13 *points whose realization spaces are disconnected.*

Let d and p be positive integers. The solution of a finite number of polynomial equations and polynomial strict inequalities with integer coefficients on $\mathbb{R}^d$ is called an elementary semialgebraic set.

Let $f_1, \dots, f_p \in \mathbb{Z}[v_1, \dots, v_d]$ be polynomial functions on $\mathbb{R}^d$, and $V \subset \mathbb{R}^d$ be an elementary semialgebraic set. For a p-tuple $\epsilon = (\epsilon_1, \dots, \epsilon_p) \in \{-, 0, +\}^p$, let

$$V_\epsilon := \{v \in V \mid \operatorname{sgn}(f_i(v)) = \epsilon_i \quad \text{for } i = 1, \dots, p\}$$

denote the corresponding subset of V. The collection of the elementary semialgebraic sets $(V_\epsilon)_{\epsilon \in \{-,0,+\}^p}$ is called a *partition* of V.

In the case $r = 3$, a triple $(i, j, k) \in E^3$ is called a basis of χ if $\chi(i, j, k) \neq 0$. Let $B = (i, j, k)$ be a basis of χ such that $\chi(B) = +1$. The realization space of an oriented matroid $\mathcal{M} = \mathcal{M}(E, \chi)$ of rank 3 can be given by an elementary semialgebraic set

$$\mathcal{R}(\mathcal{M}, B) := \{X \in \mathbb{R}^{3n} \mid x_i = e_1,\, x_j = e_2,\, x_k = e_3,\ \chi_X = \chi\},$$

where e_1, e_2, e_3 are the fundamental vectors of $\mathbb{R}^3$. For another choice of a basis B' of χ, we have a rational isomorphism between $\mathcal{R}(\mathcal{M}, B)$ and $\mathcal{R}(\mathcal{M}, B')$. Therefore realization spaces of oriented matroids are semialgebraic varieties.

The universal partition theorem states that, for every partition $(V_\epsilon)_{\epsilon \in \{-,0,+\}^p}$ of $\mathbb{R}^d$, there exists a family of oriented matroids $(\mathcal{M}^\epsilon)_{\epsilon \in \{-,0,+\}^p}$ such that the collection of their realization spaces with a common basis $(\mathcal{R}(\mathcal{M}^\epsilon, B))_{\epsilon \in \{-,0,+\}^p}$ is stably equivalent to the family $(V_\epsilon)_{\epsilon \in \{-,0,+\}^p}$. See [2] or [4] for universal partition theorems.

We construct three oriented matroids $\mathcal{M}^\epsilon$ with $\epsilon \in \{-, 0, +\}$ of rank 3 on 13 points, whose chirotopes differ by a sign on a certain triple. These oriented matroids present a partial oriented matroid with the sign of a single base non-fixed, whose realization space is partitioned by fixing the sign of this base. Two spaces $\mathcal{R}(\mathcal{M}^-)$ and $\mathcal{R}(\mathcal{M}^+)$ are disconnected, and $\mathcal{R}(\mathcal{M}^0)$ which is a wall between the two is connected and non-irreducible. So we also have the following.

Theorem 1.4. *There exists an oriented matroid of rank* 3 *on* 13 *points whose realization space is connected and non-irreducible.*

Remark 1.5. An oriented matroid $\mathcal{M}(E, \chi)$ is called *uniform* if it satisfies $\chi(i_1, \dots, i_r) \neq 0$ for all $i_1 < \cdots < i_r \in E$. Suvorov's example on 14 points is uniform and the examples which we construct are non-uniform. It is still unknown whether there exists a uniform oriented matroid on less than 14 points with a disconnected realization space.

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2 Construction of the examples

Throughout this section, we set $E = \{1, \dots, 13\}$.

Let $X(s,t,u)$ be a real $(3,13)$ matrix with three parameters $s,t,u \in \mathbb{R}$ given by

$$
\begin{aligned}
X(s,t,u) &:= (x_1,\ldots,x_{13}) \\
&= \left(\begin{array}{cccccccccccc} 1 & 0 & 0 & 1 & s & s & 0 & 1 & 1 & st & s+t-u-st+su \\ 0 & 1 & 0 & 1 & 0 & 1 & t & t & u & t & t-u+su \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1-su & 1-u+su \end{array} \right. \\
&\qquad\qquad \left. \begin{array}{cc} s+t-st-s^2u & s(t-u+su) \\ t & t-u+su \\ 1-su & 1-u+su \end{array} \right).
\end{aligned}
$$

This is a consequence of the computation of the following construction sequence. Both operations "$\vee$" and "$\wedge$" can be computed in terms of the standard cross product "$\times$" in $\mathbb{R}^3$. The whole construction depends only on the choice of the three parameters $s,t,u \in \mathbb{R}$.

$$
\begin{aligned}
x_1 &= {}^t(1,0,0),\ x_2 = {}^t(0,1,0),\ x_3 = {}^t(0,0,1),\ x_4 = {}^t(1,1,1), \\
x_5 &= s \cdot x_1 + x_3, \\
x_6 &= (x_1 \vee x_4) \wedge (x_2 \vee x_5), \\
x_7 &= t \cdot x_2 + x_3, \\
x_8 &= (x_1 \vee x_7) \wedge (x_2 \vee x_4), \\
x_9 &= u \cdot x_2 + x_1, \\
x_{10} &= (x_7 \vee x_9) \wedge (x_3 \vee x_6), \\
x_{11} &= (x_4 \vee x_5) \wedge (x_8 \vee x_9), \\
x_{12} &= (x_1 \vee x_{10}) \wedge (x_4 \vee x_5), \\
x_{13} &= (x_3 \vee x_6) \wedge (x_1 \vee x_{11}).
\end{aligned}
$$

We set $X_0 = X\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right)$. The chirotope χ^ϵ is the alternating map such that

$$
\chi^\epsilon(i,j,k) = \begin{cases} \epsilon & \text{if } (i,j,k) = (9,12,13), \\ \chi_{X_0}(i,j,k) & \text{otherwise,} \end{cases} \qquad \text{for all } (i,j,k) \in E^3 (i<j<k),
$$

where $\epsilon \in \{-,0,+\}$.

The oriented matroid which we will study is $\mathcal{M}^\epsilon := \mathcal{M}(E, \chi^\epsilon)$.

Remark 2.1. We can replace X_0 with $X\left(\frac{1}{2}, \frac{1}{2}, u'\right)$ where u' is chosen from $\mathbb{R}\backslash\{-1, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, 3\}$. We will study the case $0 < u' < \frac{1}{2}$. If we choose u' otherwise, we can get other oriented matroids with disconnected realization spaces.

In the construction sequence, we need no assumption on the collinearity of x_9, x_{12}, x_{13}. Hence every realization of $\mathcal{M}^\epsilon$ is linearly equivalent to a matrix

$X(s,t,u)$ for certain s,t,u, up to multiplication on each column with positive scalar.

Moreover, we have the rational isomorphism

$$\mathcal{R}^*(\chi^\epsilon) \times (0,\infty)^{12} \cong \mathcal{R}(\mathcal{M}^\epsilon),$$

where $\mathcal{R}^*(\chi^\epsilon) := \{(s,t,u) \in \mathbb{R}^3 \mid \chi_{X(s,t,u)} = \chi^\epsilon\}$. Thus we have only to prove that the set $\mathcal{R}^*(\chi^\epsilon)$ is disconnected (resp. non-irreducible) to show that the realization space $\mathcal{R}(\mathcal{M}^\epsilon)$ is disconnected (resp. non-irreducible).

The equation $\chi_{X(s,t,u)} = \chi^\epsilon$ means that

$$\operatorname{sgn}\det(x_i,x_j,x_k) = \chi^\epsilon(i,j,k), \text{ for all } (i,j,k) \in E^3. \tag{1}$$

We write some of them which give the equations on the parameters s,t,u. Note that for all $(i,j,k) \in E^3(\{i,j,k\} \neq \{9,12,13\})$, the sign is given by

$$\chi^\epsilon(i,j,k) = \operatorname{sgn}\det(x_i,x_j,x_k)|_{s=t=1/2,u=1/3}.$$

From the equation $\operatorname{sgn}\det(x_2,x_3,x_5) = \operatorname{sgn}(s) = \operatorname{sgn}(1/2) = +1$, we get $s > 0$. Similarly, we get $\det(x_2,x_5,x_4) = 1-s > 0$, therefore

$$0 < s < 1. \tag{2}$$

From the equations $\det(x_1,x_7,x_3) = t > 0, \det(x_1,x_4,x_7) = 1-t > 0$, we get

$$0 < t < 1. \tag{3}$$

Moreover, we have the inequalities

$$\det(x_1,x_9,x_3) = u > 0, \tag{4}$$
$$\det(x_4,x_7,x_9) = 1-t-u > 0, \tag{5}$$
$$\det(x_3,x_9,x_8) = t-u > 0, \tag{6}$$
$$\det(x_5,x_{13},x_7) = s\bigl(t^2-(1-s)u\bigr) > 0, \tag{7}$$
$$\det(x_6,x_{12},x_8) = (1-s)\bigl((1-t)^2-su\bigr) > 0. \tag{8}$$

From the equation $\det(x_9,x_{12},x_{13}) = u(1-2s)(1-2t+tu-su)$, we get

$$\operatorname{sgn}\bigl(u(1-2s)(1-2t+tu-su)\bigr) = \epsilon. \tag{9}$$

Conversely, if we have Eqs. (2) - (9), then we get (1).

We can interpret a $(3,13)$ matrix as the set of vectors $\{x_1,\ldots,x_{13}\} \subset \mathbb{R}^3$. After we normalize the last coordinate for x_i $(i \in E\backslash\{1,2,9\})$, we can visualize the matrix on the affine plane $\{(x,y,1) \in \mathbb{R}^3\} \cong \mathbb{R}^2$. Figure 1 shows the affine image of X_0. See Figures 2, 3 for realizations of $\mathcal{M}^\epsilon$.

Proof of Theorem 1.3. We prove that $\mathcal{R}^*(\chi^-)$ and $\mathcal{R}^*(\chi^+)$ are disconnected. From Eqs. (2) - (9), we obtain

$$\mathcal{R}^*(\chi^-) = \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 0 < s < 1,\, 0 < u < t < 1-u, \\ (1-t)^2 - su > 0,\, t^2 - (1-s)u > 0, \\ (1-2s)(1-2t+tu-su) < 0 \end{array} \right\},$$

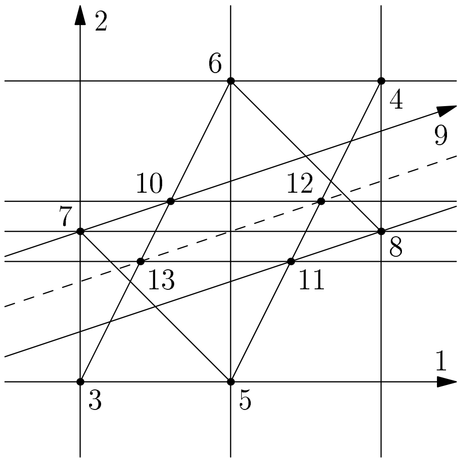

Figure 1: Column vectors of X_0.

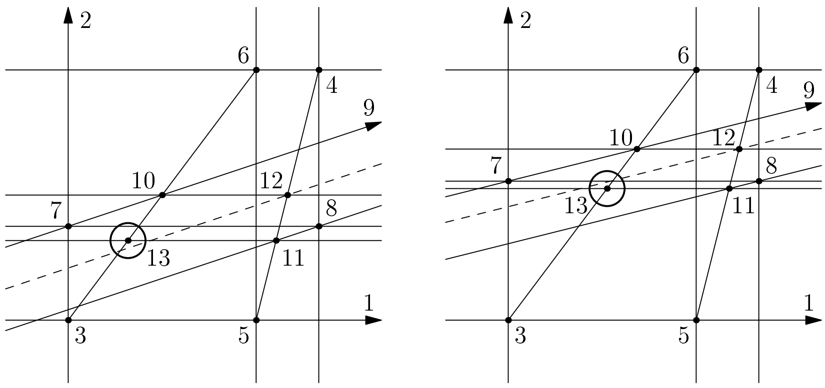

Figure 2: Realization of $\mathcal{M}^-$(on the left) and that of $\mathcal{M}^+$(on the right).

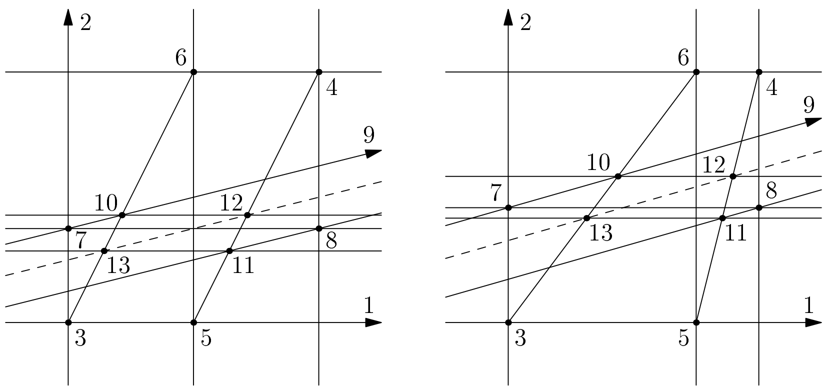

Figure 3: Realizations of $\mathcal{M}^0$.

$$\mathcal{R}^*(\chi^+) = \left\{ (s,t,u) \in \mathbb{R}^3 \,\middle|\, \begin{array}{c} 0 < s < 1,\, 0 < u < t < 1-u, \\ (1-t)^2 - su > 0,\, t^2 - (1-s)u > 0, \\ (1-2s)(1-2t+tu-su) > 0 \end{array} \right\}.$$

First, we show that $\mathcal{R}^*(\chi^-)$ is disconnected, more precisely, consisting of two connected components, by proving the next proposition.

Proposition 2.2.

$$\begin{aligned} \mathcal{R}^*(\chi^-) = & \left\{ (s,t,u) \in \mathbb{R}^3 \,\middle|\, \begin{array}{c} 0<s<1/2 \\ 1/2<t<1 \end{array},\, 0<u<\min\left\{1-t,\, \frac{(1-t)^2}{s},\, \frac{2t-1}{t-s}\right\} \right\} \\ & \cup \left\{ (s,t,u) \in \mathbb{R}^3 \,\middle|\, \begin{array}{c} 1/2 < s < 1 \\ 0 < t < 1/2 \end{array},\, 0 < u < \min\left\{t,\, \frac{t^2}{1-s},\, \frac{1-2t}{s-t}\right\} \right\}. \end{aligned}$$

Proof. There are two cases

$$(1-2s)(1-2t+tu-su) < 0 \Leftrightarrow \left\{ \begin{array}{c} 1-2s > 0,\, 1-2t+tu-su < 0, \\ \text{or} \\ 1-2s < 0,\, 1-2t+tu-su > 0. \end{array} \right.$$

Note that

$$(2-u)(2t-1) = -2(1-2t+tu-su) + u(1-2s), \tag{10}$$

$$t^2 - (1-s)u = -(1-2t+tu-su) + (1-t)(1-t-u), \tag{11}$$

$$(1-t)^2 - su = (1-2t+tu-su) + t(t-u). \tag{12}$$

($\subset$) For the case $1-2s > 0$ and $1-2t+tu-su < 0$, the inequality $2t-1 > 0$ follows from Eq. (10). Since we have $0 < s < 1/2 < t < 1$, we get

$$\left\{ \begin{array}{l} 1-2t+tu-su < 0, \\ (1-t)^2 - su > 0, \\ 1-t-u > 0 \end{array} \right. \Leftrightarrow u < \min\left\{1-t,\, \frac{(1-t)^2}{s},\, \frac{2t-1}{t-s}\right\}. \tag{13}$$

For the other case $1-2s < 0$, similarly, we get $1-2t > 0$ from Eq. (10). Since we have $0 < t < 1/2 < s < 1$, we get

$$\left\{ \begin{array}{l} 1-2t+tu-su > 0, \\ t^2 - (1-s)u > 0, \\ t-u > 0 \end{array} \right. \Leftrightarrow u < \min\left\{t,\, \frac{t^2}{1-s},\, \frac{1-2t}{s-t}\right\}. \tag{14}$$

($\supset$) For the component $0 < s < 1/2 < t < 1$, the inequalities $1-2t+tu-su < 0$, $(1-t)^2 - su > 0$, $1-t-u > 0$ follow from (13). Thus we get $t^2 - (1-s)u > 0$ from Eq. (11). The inequality $u < t$ holds because $t > 1/2$ and $u < 1-t$.

For the other component $0 < t < 1/2 < s < 1$, similarly, we get the inequalities $1-2t+tu-su > 0$, $t^2 - (1-s)u > 0$, $t-u > 0$ from (14), and $(1-t)^2 - su > 0$ from Eq. (12). Last, we get $u < 1-t$ from $t < 1/2$ and $u < t$. $\square$

For the set $\mathcal{R}^*(\chi^+)$, we have the following proposition.

Proposition 2.3.

$$\mathcal{R}^*(\chi^+) = \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 0 < s < 1/2, 0 < u < 1/2, \\ (1-u)^2 - (1-s)u > 0, \end{array} \sqrt{(1-s)u} < t < \frac{1-su}{2-u} \right\}$$
$$\cup \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 1/2 < s < 1, 0 < u < 1/2, \\ (1-u)^2 - su > 0, \end{array} \frac{1-su}{2-u} < t < 1 - \sqrt{su} \right\}.$$

The proof is similar to that of Proposition 2.2 and omitted.

Proof of Theorem 1.4. We show that $\mathcal{R}^*(\chi^0)$ consists of two irreducible components whose intersection is not empty. From Eqs. (2) - (9), we get

$$\mathcal{R}^*(\chi^0) = \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 0 < s < 1,\ 0 < u < t < 1-u, \\ (1-t)^2 - su > 0,\ t^2 - (1-s)u > 0, \\ (1-2s)(1-2t+tu-su) = 0 \end{array} \right\}.$$

Here we have the decomposition

$$\mathcal{R}^*(\chi^0) = \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; 0 < t < 1,\ 0 < u < 2t^2,\ u < 2(1-t)^2,\ 1-2s = 0 \right\}$$
$$\cup \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 0 < s < 1,\ 0 < u < 1/2,\ (1-u)^2 - su > 0, \\ (1-u)^2 - (1-s)u > 0,\ 1-2t+tu-su = 0 \end{array} \right\}.$$

The intersection of the two irreducible components is the set

$$\left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; s = t = \tfrac{1}{2},\ 0 < u < \tfrac{1}{2} \right\} \cong \left\{ X\left(\tfrac{1}{2}, \tfrac{1}{2}, u\right) \;\middle|\; 0 < u < \tfrac{1}{2} \right\}.$$

The proof is also similar to that of Proposition 2.2 and omitted.

Figure 3 shows two realizations of $\mathcal{M}^0$. On the left, it shows the affine image of $X\left(\frac{1}{2}, \frac{3}{8}, \frac{1}{4}\right)$, on the irreducible component $1-2s = 0$. On the right, the image of $X\left(\frac{3}{4}, \frac{11}{24}, \frac{2}{7}\right)$, so it is on the other component $1-2t+tu-su = 0$. They can be deformed continuously to each other via $X\left(\frac{1}{2}, \frac{1}{2}, u\right)$ $(0 < u < \frac{1}{2})$.

We set

$$\mathcal{R}^* := \left\{ (s,t,u) \in \mathbb{R}^3 \;\middle|\; \begin{array}{c} 0 < s < 1,\ 0 < u < t < 1-u, \\ (1-t)^2 - su > 0,\ t^2 - (1-s)u > 0 \end{array} \right\}.$$

The set $\mathcal{R}^* \times (0,\infty)^{12}$ is rationally isomorphic to a realization space of a partial oriented matroid with the sign $\chi(9,12,13)$ non-fixed. The collection of the semialgebraic sets $(\mathcal{R}^*(\chi^\epsilon))_{\epsilon \in \{-,0,+\}}$ is a partition of $\mathcal{R}^*$. Figure 4 illustrates this partition in 3-space.

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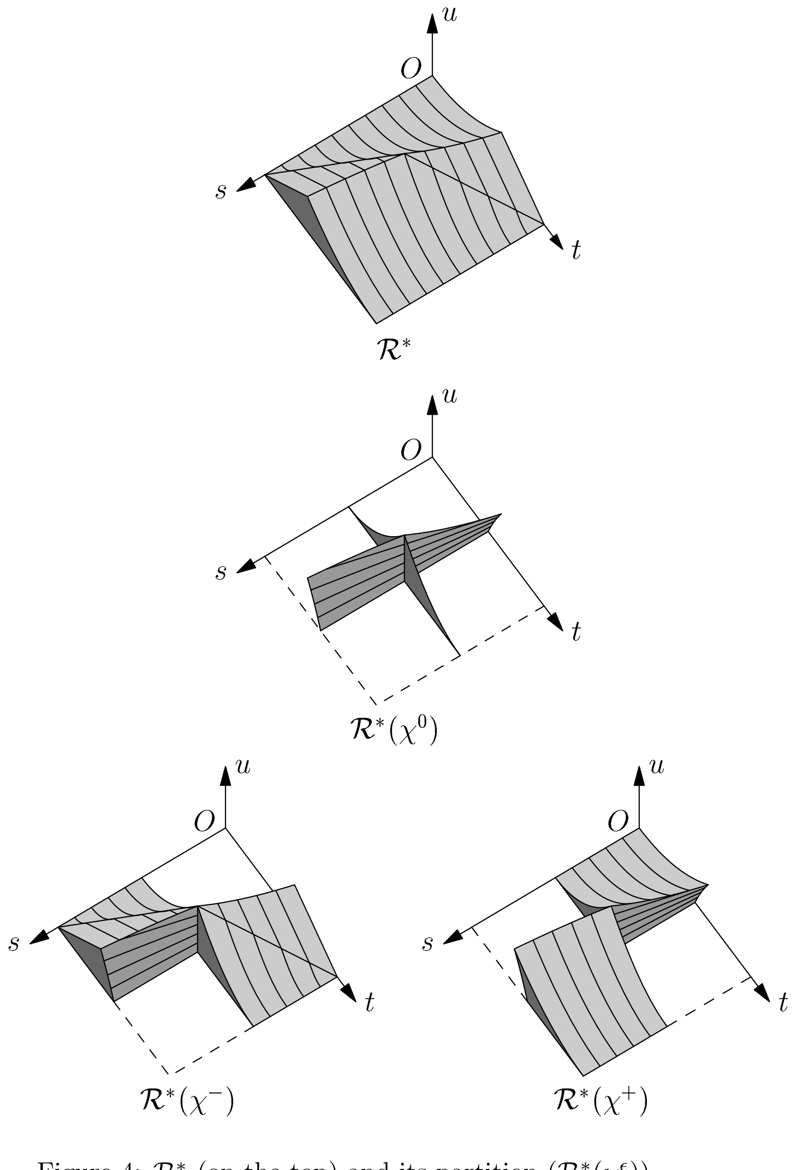

Figure 4: $\mathcal{R}^*$ (on the top) and its partition $(\mathcal{R}^*(\chi^\epsilon))_{\epsilon\in\{-,0,+\}}$.

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