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# Stability analysis of a steady state in three time-delayed nonlinear oscillators coupled by a static connection

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## Introduction

A stabilization of a steady state in coupled nonlinear oscillators, amplitude death, is one of the important collective behaviors in coupled nonlinear systems. Recent literatures revealed that amplitude death can be induced by various types of connections, such as dynamic, conjugate, time-delay, partial time-delay, time-varying-delay, and gradient connections [1]. The death was experimentally verified by electronic circuits, living oscillators, thermo-optical oscillators, synthetic genetic networks, and laser systems [2].

Amplitude death in a pair of time-delayed nonlinear oscillators coupled by the static, the dynamic, and the delayed connections was investigated theoretically and experimentally [3]. The static connection has a simple structure; thus, it can be considered as the lowest-cost physical implementation in comparison to the other connections. However, it was shown that the dynamic and the delay connections can induce death, but the static connection never induces. In recent years, we reported that the amplitude death in a pair of time-delayed nonlinear oscillators can be induced even by the static connection when the oscillators include different delay times [4]. The present paper extends our previous results [4]: a pair of oscillators to three oscillators.

In order to investigate amplitude death, a characteristic equation of the linearized oscillators at a steady state has to be analyzed. For the three time-delayed oscillators, the characteristic equation includes the cross-talk terms of three time delays. It is difficult to deal with the cross-talk terms; therefore, the standard procedure for analyzing the characteristic equation [1, 3] is useless. To overcome this difficulty, this paper employs the two novel methodologies: Advanced Clustering with Frequency Sweeping (ACSF) [5] and Cluster Treatment of Characteristic Roots (CTCR) [6]. The combination of ACSF and CTCR methods as a powerful tool allows us to analyze the stability of the steady state and to obtain the stability boundary curves in a parameter space.

## Main results

Let us consider the three time-delayed nonlinear oscillators,

$$\dot{x}_i = f(x_{\tau_i}) - \alpha x_i + u_i, \quad (i = 1, 2, 3),$$

where $x_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote the system state and coupling signal, respectively. $x_{\tau_i} := x_i(t - \tau_i)$ is the delayed state, $\tau_i \geq 0$ is the delay time and $\alpha = 1$ is a parameter, the nonlinear function is

$$f(x) = \begin{cases} 0.95x + 1.4 & \text{if } x \leq 1.7 \\ -1.6x + 5.75 & \text{if } 1.7 < x \leq 4.3 \\ -1.15 & \text{if } x > 4.3 \end{cases}.$$

The oscillators are coupled by the static connection, with the coupling strength $k \in \mathbb{R}$:

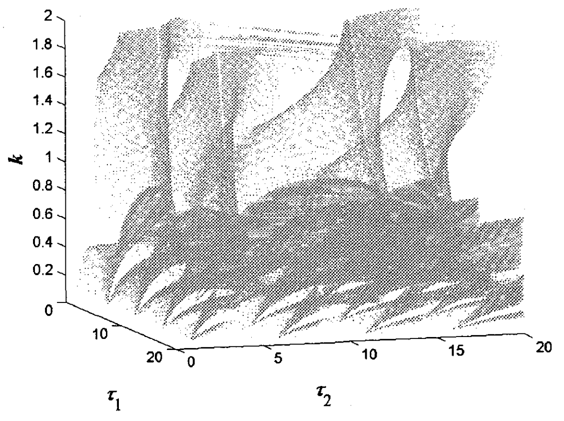
$$u_{1,2,3} = \frac{1}{2}k(x_{2,1,1} + x_{3,3,2} - 2x_{1,2,3}).$$

Each individual oscillator without coupling (i.e., $u_{1,2,3} \equiv 0$) has an unstable fixed point $x^* : 0 = f(x^*) - \alpha x^*$. The location of the fixed point $x^*$ never changes even by coupling; understandably, the static connection changes only its stability.

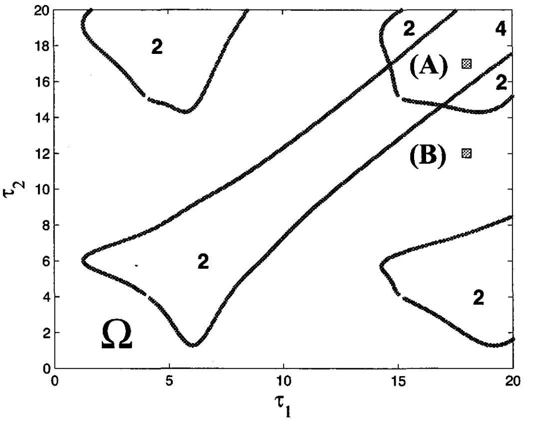
The combination of ACSF and CTCR methods allow us to obtain the boundary curves in three dimensional $(\tau_1, \tau_2, k)$ space with $\tau_3 = 6$ shown in Fig. 1 (a). The cross-section surface of the three-dimensional curves at $k = 2$ is illustrated in Fig. 1(b), where the amplitude death region is symbolized by $\Omega$. Every point on the curves corresponds to the purely imaginary root of the characteristic equation. The numbers of unstable roots are stated in several regions. Figure 2 shows the time-series data of the oscillators at points (A) and (B) in Fig. 1(b): they are outside and inside the amplitude death region, respectively. At point (A), as shown in Fig. 2(a), the state variables of the three oscillators, $[x_1(t), x_2(t), x_3(t)]$, do not converge on the steady state after coupling at $t = 500$. In contrast, the stabilization is induced at point (B) inside the amplitude death region as shown in Fig. 2(b). These time-series data agree with our boundary curves.

## Conclusion

This paper analyzed the stability of the steady state in three time-delayed nonlinear oscillators coupled by the static connection. The stability boundary curves in the parameter space are derived by using the combination of ACSF and CTCR methods.

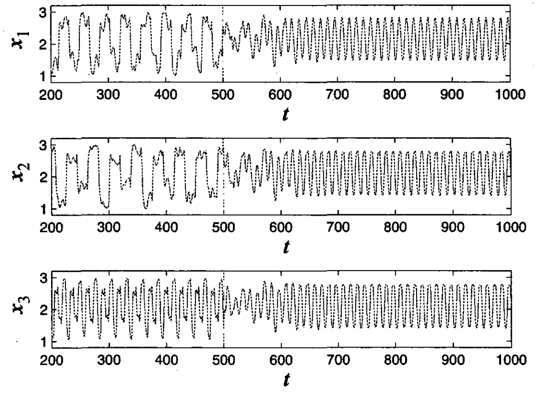


(a) Three dimensional $(\tau_1, \tau_2, k)$ space with $\tau_3 = 6$

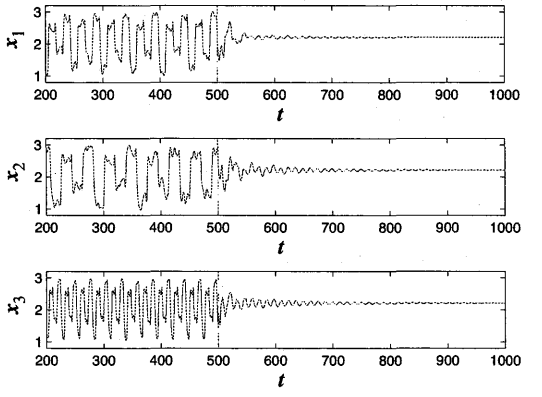



(b) Two dimensional $(\tau_1, \tau_2)$ space with $\tau_3 = 6$ and $k = 2$

Figure 1: Stability boundary curves for the steady state.



(a) $\tau_1 = 18, \tau_2 = 17, \tau_3 = 6, k = 2$ (A)



(b) $\tau_1 = 18, \tau_2 = 12, \tau_3 = 6, k = 2$ (B)

Figure 2: Time series data $x_{1,2,3}(t)$ before and after coupling.

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