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LIST OF SYMBOLS

d, q	: direct and quadrature axes of reference frame of individual synchronous machine
D, Q	: direct and quadrature axes of rotating common reference frame
ω_0	: synchronous angular velocity ($=2\pi f_0$) (rad./sec.)
ω	: instantaneous angular velocity (rad./sec.)
f_0	: nominal frequency (hz)
δ	: angular displacement between d,q axes and D,Q axes (rad.)
ψ_{fd}	: field flux linkage
ψ_d, ψ_q	: direct and quadrature axes armature flux linkages
ψ_{kd}, ψ_{kq}	: d- and q-axis damper circuits flux linkages
$x_{ffd}, x_{kkd}, x_{kkq}$	: total reactances in field and d- and q-axis damper circuits
x_{fkd}	: mutual reactance between field and d-axis damper circuits
x_{ad}, x_{aq}	: stator-rotor mutual reactances in d- and q-axis circuits
$x_{f\ell}$	: field leakage reactance
$x_{a\ell}$	: armature leakage reactance
$x_{kd\ell}, x_{kq\ell}$	: leakage reactances in d- and q-axis damper circuits
x_d, x_q	: d- and q-axis synchronous reactances
x_d', x_q'	: d- and q-axis transient reactances
r_{fd}	: field resistance
r	: armature resistance
r_{kd}, r_{kq}	: d- and q-axis damper circuits resistances
v_{fd}, i_{fd}	: field circuit voltage and current
i_{kd}, i_{kq}	: d- and q-axis damper circuits currents
v_d, v_q	: d- and q-axis terminal voltages
i_d, i_q	: d- and q-axis terminal currents
v_D, v_Q	: D- and Q-axis terminal voltages
i_D, i_Q	: D- and Q-axis terminal currents

v_t, i_t : terminal voltage and current

i'_t : load current at generator terminal

E_{qd} : voltage behind q-axis reactance

E_d, E_q : d- and q-axis internal induced voltages in steady state

E'_d, E'_q : d- and q-axis internal induced voltages in transient state

T'_{do}, T'_{qo} : d- and q-axis transient time constants (sec.)

E_{fd} : excitation voltage refered to armature circuit $(= v_{fd} \cdot x_{ad} / r_{fd})$

J : inertia constant $(= \omega_o \cdot M)$ (sec.)

M : angular momentum

P : active power output at generator terminal

Q : reactive power output at generator terminal

P_g : air gap torque

P_e : electrical output of generator

P_d : damping coefficient

P_t : mechanical input to rotor

K_f : voltage regulator open loop gain

T_f : voltage regulator open loop time constant (sec.)

K_e : parameter of exciter

T_e : exciter time constant (sec.)

K_s : derivative stabilizing gain of voltage regulator

T_s : derivative stabilizing loop time constant (sec.)

V_r : reference voltage

K_g : governor gain

T_g : governor time constant (sec.)

T_h : time constant representing turbine delay (sec.)

ΔE_{fd} : deviation of excitation voltage

V_a : voltage regulator output

V_s : derivative stabilizing signal of voltage regulator

ΔP_v : deviation of governor opening position

$\mathbb{Y}_{jk}$	: short circuit transfer admittance matrix between the j-th bus and the k-th bus
G_{jk}	: short circuit transfer conductance between the j-th bus and the k-th bus
B_{jk}	: short circuit transfer suseptance between the j-th bus and the k-th bus
G, B	: conductance and suseptance of shunt impedance load at generator terminal
t	: time (sec.)
p	: differential operator (d/dt)
$\mathfrak{x}$	: state variable vector
$\mathfrak{u}$	: control signal vector
$\mathfrak{w}$	: output variable vector
$\mathfrak{f}(\mathfrak{x})$	: non-linear functional vector
$\mathbb{A}, \mathbb{B}, \mathbb{C}$	: coefficient matrices
$\mathbb{Q}, \mathbb{N}$	: positive or positive semi-definite matrices
$\mathbb{R}, \mathbb{P}$	: positive definite matrices
$\mathbb{K}, \mathbb{L}$	: solution matrices of matrix Riccati equation or Lyapunov's matrix equation
$\mathbb{F}$	: feedback gain matrix
$\mathbb{J}$	: Jacobian matrix of non-linear functional vector
$\mathbb{I}$	: unit matrix
$Tr(\cdot)$	: sum of diagonal elements of matrix $(\cdot)$

In this thesis, the subscript j denotes the j-th machine, the subscript 0 denotes the steady state value and Δ denotes the deviation from the steady state value.

CONTENTS

CHAPTER 1 INTRODUCTION

Section 1-1. General

The dynamic stability and the transient stability of electrical power system have been major subjects of theoretical and practical interests for some twenty years, and they continue to grow in importance today as generation and transmission equipments are being applied with high reactances and correspondingly lower stability margin. In view of the increasing complexity of present-day power systems, their design and operation require a more detailed analysis of possible performance mode that may achieved by available computer programs. Owing to the progress of digital computers with great memory capacity and quick processing ability , many excellent works on dynamic and transient stability of electrical power systems have been done recently.

Furthermore, extensive growth of electrical power systems and the developement of high-voltage long distance transmission systems separating from loads have accentuated the importance of increasing the dynamic and transient stability limits of generators. Due to the decrease of stability margin inherent in the design of present-day generations and increasing tendency toward higher power factor operating conditions, more emphasis is being placed recently on the developement of compensating controllers for the required system stabilization.

Section 1-2. Scope of Studies

In this thesis, the developement of compensating controllers for the required system stabilization will be mainly considered.

In chapter 2, the mathematical representations of synchronous machine, excitation control system, speed governing control system and transmission network are described in the form of a mathematical model on a general purpose digital computer in order to simulate the electrical power systems.

The model system contains an arbitrary number of synchronous machines and transmission networks of arbitrary topological form including impedance loads. The whole system is expressed with Park's quantities using the mathematical representations. Furthermore, the large signal performance of the power systems can be described by the non-linear first order differential equations and the small signal performance of the power systems can be described by the linearized equations of the non-linear equations around the steady state operating point.

In chapter 3, various mathematical methods will be shown in order to analyze the power system performance, i.e. the dynamic stability and the transient stability. And the applications of these methods to the model power systems will also shown.

In chapter 4, the problem of optimization of the synchronous machine performance by minimizing the quadratic performance index both the output variables and the control signals is considered for the case of transient involving small disturbances. In this approach, the linearized equations are considered and the control law consisting of constant feedback coefficients of all the state variables of the system. And the improvement of the dynamic stability of the model system applied with the above controller is also investigated.

In chapter 5, the controller using only the directly measurable output variables is derived using the techniques of the model reduction. Furthermore an application of the direct method of Lyapunov to the control problem of the linear system is also considered. And the improvement of the dynamic stability of the model system applied with the above controller is investigated.

In chapter 6, an application of Lyapunov's direct method to the control problem of the non-linear system is considered. The controller of the of the model system is determinied using the energy function as the Lyapunov function of the model system under the several assumptions. And the possibility of obtaining the controller of the non-linear system will be demonstrated.

In chapter 7, in order to improve the overall stability of the system, i.e. the dynamic stability and the transient stability, the controller of the system is derived using the Lyapunov function of Krasovskii under consideration of theoretical results in former chapters. Furthermore, a method to construct the controller using the only measurable states of the system is also considered. And the effectiveness of the controller will be shown by the numerical analysis of the model systems.

In chapter 8, the results of the work described in this thesis are summarized.

CHAPTER 2 REPRESENTATION OF POWER SYSTEM

Section 2-1. Description of Synchronous Machine (I)

Complete description of the dynamic behaviour of a synchronous machine requires consideration of its electrical and mechanical characteristics as well as those of associated control systems. The necessary mathematical statements are summarized here. Throughout this paper, only those modes of operation that do not require zero-phase variables are considered.

The equations describing the balanced 3-phase performance of a synchronous machine are derived in several references and are summarized by Shackshaft in the following form:[1],[2]

Direct-axis flux linkages

$$\psi_{fd} = x_{ffd} \cdot i_{fd} - x_{ad} \cdot i_d + x_{fkd} \cdot i_{kd} \quad (2\text{-}1)$$

$$\psi_d = x_{ad} \cdot i_{fd} - x_d \cdot i_d + x_{ad} \cdot i_{kd} \quad (2\text{-}2)$$

$$\psi_{kd} = x_{fkd} \cdot i_{fd} - x_{ad} \cdot i_d + x_{kkd} \cdot i_{kd} \quad (2\text{-}3)$$

Quadrature-axis flux linkages

$$\psi_q = - x_q \cdot i_q + x_{aq} \cdot i_{kq} \quad (2\text{-}4)$$

$$\psi_{kq} = - x_{aq} \cdot i_q + x_{kkq} \cdot i_{kq} \quad (2\text{-}5)$$

Direct-axis voltages

$$v_{fd} = \frac{1}{\omega_0} \cdot p\psi_{fd} + r_{fd} \cdot i_{fd} \;, \quad E_{fd} = x_{ad} \cdot v_{fd} / r_{fd} \quad (2\text{-}6)$$

$$v_d = \frac{1}{\omega_0} \cdot p\psi_d - r \cdot i_d - \frac{\omega}{\omega_0} \cdot \psi_q \quad (2\text{-}7)$$

$$0 = \frac{1}{\omega_0} \cdot p\psi_{kd} + r_{kd} \cdot i_{kd} \quad (2\text{-}8)$$

Quadrature-axis voltages

$$v_q = \frac{1}{\omega_0} \cdot p\psi_q - r \cdot i_q + \frac{\omega}{\omega_0} \cdot \psi_d \quad (2\text{-}9)$$

$$0 = \frac{1}{\omega_0} \cdot p\psi_{kq} + r_{kq} \cdot i_{kq} \quad (2\text{-}10)$$

These equations are of per-unit form.[3],[4] In the per-unit system voltages, fluxes, currents and impedances are all expressed as the ratio of their actual values to the selected base values. The base values chosen are such that all per-unit mutual inductances between rotor and stator circuit in each axis are equal to one another. On this basis the following relations between self-, mutual-, and leakage-reactances pertain:

$$x_{ffd} = x_{ad} + x_{f\ell}$$

$$x_d = x_{ad} + x_{a\ell} , \quad x_q = x_{aq} + x_{a\ell} \qquad (2\text{-}11)$$

$$x_{KKd} = x_{ad} + x_{Kd\ell} , \quad x_{KKq} = x_{aq} + x_{Kq\ell}$$

In addition, time is scaled in real time.

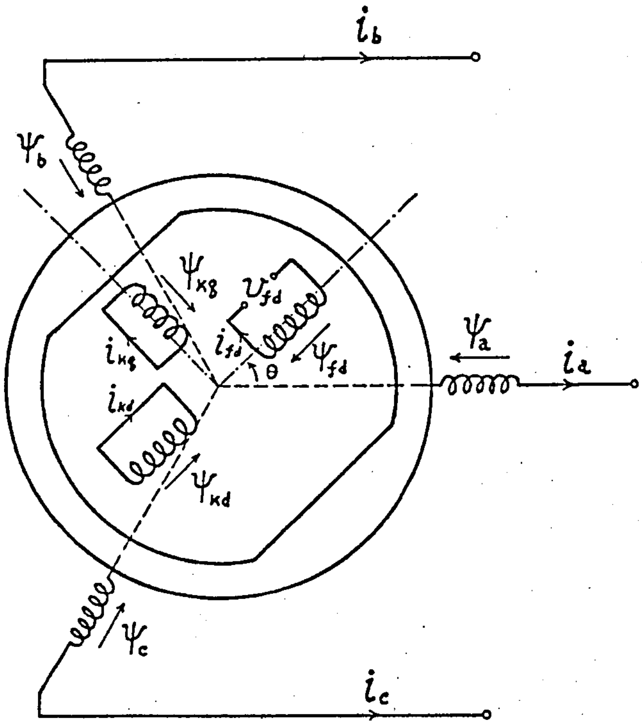


Fig.2-1-1 Schematic layout of the windings of a synchronous machine

In eqns.(2-1)-(2-5) the self- and mutual-reactances are dependent on the fluxes in the machine due to the saturation of iron circuits. A statement of this relationship is therefore necessary. The following assumptions are made in order to obtain an expression for the variation of the machine reactances with iron saturation:

(1) The leakage reactances of all windings are independent of the state of iron.

(2) The leakage fluxes do not contribute to the iron saturation, which

is therefore determined by the mutual flux.

(3) The mutual reactance between the two direct-axis rotor circuits is equal to the mutual reactance between these circuits and the armature.

As a consequence of this first assumption, only the mutual reactances change with saturation, and leakage reactances are not significantly affected. The three assumptions indicate that x_{ad} need only be replaced by $K_s^* \cdot x_{ado}$, and x_{aq} by $K_s^* \cdot x_{aqo}$ in the developed machine equations:

$$x_{ad} = K_s^* \cdot x_{ado} \quad , \quad x_{aq} = K_s^* \cdot x_{aqo} \tag{2-12}$$

where x_{ado} and x_{aqo} are the unsaturated values of x_{ad} and x_{aq} , respectively

Knowing the open-circuit magnetization curve, the saturation factor K_s^* may be determined.

In this thesis, the saturation factor K_s^* is asummed to be equal to unity, namely the unsaturated values x_{ado} and x_{aqo} are used as x_{ad} and x_{aq} , but the saturation factor K_s^* , which may be obtained experimentally, can be easily introduced into the representation of the mutual reactances x_{ad} and x_{aq} as shown in eqn.(2-12).

The following equations are necessary to complete the description of a synchronous machine.

Electrical torque at air gap

$$T_g = \psi_d \cdot i_d - \psi_q \cdot i_q \tag{2-13}$$

Reactive power at terminal

$$Q = v_q \cdot i_d - v_d \cdot i_q \tag{2-14}$$

Terminal voltage

$$v_t = (v_d^2 + v_q^2)^{1/2} \tag{2-15}$$

Mechanical equations of motion

$$p\,\delta = \Delta\omega \tag{2-16}$$

$$M \cdot p\Delta\omega = P_t - T_g - P_d \cdot \Delta\omega \tag{2-17}$$

Section 2-2. Description of Synchronous Machine (II)

In order to obtain the simplified equations of a synchronous machine shown in section 2-1, further assumptions[5] are made;

(1) Direct-axis damper circuit is neglected, i.e. $\psi_{kd} = i_{kd} = 0.0$

(2) The armature electrical transients created by terms $p\psi_d$ and $p\psi_q$ are neglected, assuming that the electrical transients are much faster than the electromechanical transients, i.e. $p\psi_d = p\psi_q = 0.0$

(3) The term ω/ω_0 is equal to unity, assuming that the deviation of angular velocity from synchronous angular velocity is very small.

By these assumptions the equations of synchronous machine described in section 2-1 are rewritten as follows:

Direct-axis flux linkages

$$\psi_{fd} = x_{ffd} \cdot i_{fd} - x_{ad} \cdot i_d \qquad (2\text{-}18)$$

$$\psi_d = x_{ad} \cdot i_{fd} - x_d \cdot i_d \qquad (2\text{-}19)$$

Quadrature-axis flux linkages

$$\psi_{kq} = x_{kkq} \cdot i_{kq} - x_{aq} \cdot i_q \qquad (2\text{-}20)$$

$$\psi_q = x_{aq} \cdot i_{kq} - x_q \cdot i_q \qquad (2\text{-}21)$$

Direct-axis voltages

$$v_{fd} = \frac{1}{\omega_0} \cdot p\psi_{fd} + r_{fd} \cdot i_{fd} \qquad (2\text{-}22)$$

$$v_d = - r \cdot i_d - \psi_q \qquad (2\text{-}23)$$

Quadrature-axis voltages

$$v_q = - r \cdot i_q + \psi_d \qquad (2\text{-}24)$$

$$0 = \frac{1}{\omega_0} \cdot p\psi_{kq} + r_{kq} \cdot i_{kq} \qquad (2\text{-}25)$$

From eqn.(2-18)-eqn.(2-25), the time rate of the flux variations in direct- and quadrature-axis rotor circuits are expressed by the following two equations:[5]

$$pE'_q = \{E_{fd} - E'_q - (x_d - x'_d) \cdot i_d\} / T'_{do} \qquad (2\text{-}26)$$

$$pE'_d = \{ \quad - E'_d + (x_q - x'_q) \cdot i_q\} / T'_{qo} \qquad (2\text{-}27)$$

Furthermore, the direct- and quadrature-axis voltages become as follows:

$$v_d = -r\cdot i_d + x_q'\cdot i_q + E_d' \qquad v_q = -r\cdot i_q - x_d'\cdot i_d + E_q' \tag{2-28}$$

where

$$\begin{aligned} E_d' &= E_d + (x_q - x_q')\cdot i_q, & E_q' &= E_q + (x_d - x_d')\cdot i_d \\ E_d &= -x_{aq}\cdot i_{kq}, & E_q &= x_{ad}\cdot i_{fd} \\ E_d' &= -x_{aq}\cdot \Psi_{kq}/x_{kkq}, & E_q' &= x_{ad}\cdot \Psi_{fd}/x_{ffd} \\ x_d' &= x_d - x_{ad}^2/x_{ffd}, & x_q' &= x_q - x_{aq}^2/x_{kkq} \\ T_{do}' &= x_{ffd}/r_{fd}\cdot \omega_o, & T_{qo}' &= x_{kkq}/r_{kq}\cdot \omega_o \\ E_{fd} &= x_{ad}\cdot v_{fd}/r_{fd} \end{aligned}$$

If the machine is a salient-pole type machine, it has no quadrature-axis circuit, so that Ψ_{kq}, i_{kq}, E_d and E_d' are all equal to zero, and eqn.(2-26) need not be considered. To complete the description of a synchronous machine the following equations are also necessary:

Active power at terminal

$$P_e = v_d\cdot i_d + v_q\cdot i_q \tag{2-29}$$

Reactive power at terminal

$$Q = v_q\cdot i_d - v_d\cdot i_q \tag{2-30}$$

Terminal voltage

$$v_t = (v_d^2 + v_q^2)^{1/2} \tag{2-31}$$

Mechanical equations of motion

$$p\delta = \Delta\omega \tag{2-32}$$

$$M\cdot p\Delta\omega = P_t - P_e - P_d\cdot\Delta\omega \tag{2-33}$$

Section 2-3. Description of Excitation Control System

The effect of AVR (automatic voltage regulator) must be included[6] in the study of dynamic behaviour of power system. Simple models of AVR and their mathematical representations are given in this section.

The voltage regulator and excitation system fitted to a usual synchronous generator may be classified into three types, i.e. (1) the magnetic amplifier type or rotary amplifier type, (2) the differential type, (3) the SCR type.

The mathematical model of type (1) or type (2) regulator must be expressed by the differential equations of 6 to 7 order. A typical block diagram of this type is shown in Fig.2-3-1

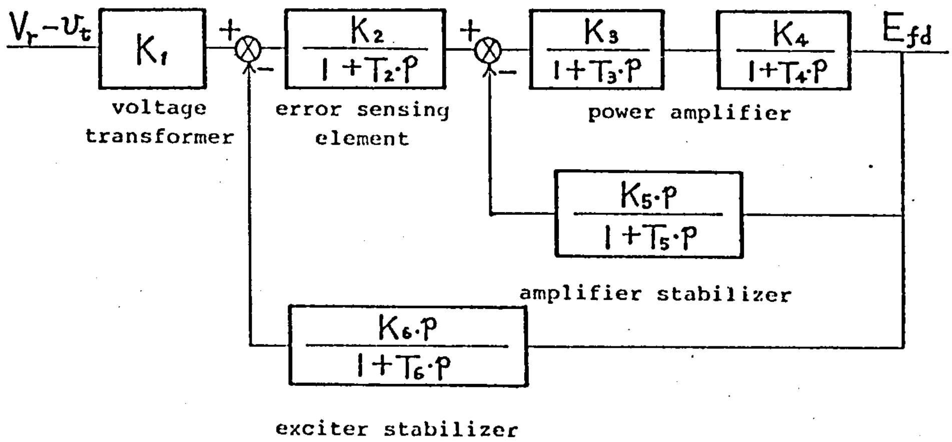


Fig.2-3-1 Block diagram of automatic voltage regulator

It may therefore be rather inadequate to represent such regulators with a simple time lag. However, in the case of dynamic stability studies, it is possible because of the following restrictions are generally satisfied: (1) The variation of the signal to AVR is small enough not to cause any saturation in excitation system. (2) The frequency of the signal is low enough. Then the models can be simplified as shown in Fig.2-3-2.

The type (3) regulator consists of semiconductive elements and does not have any rotating parts. Therefore, the time lag is so small as some 10 ms.. This type of regulator may be expressed by a simple time lag as shown in Fig.2-3-2, assuming that the saturation of rectifier is negliegible.

Hereafter, AVR is expressed by the simplified models shown in Fig.2-3-2 in this thesis. The mathematical representations of these simplified models become as follows:

$$\text{(a)}\quad \Delta E_{fd} = \frac{K_f}{1 + T_f \cdot p} \cdot (V_r - v_t + u_1) \qquad \text{(2-34)}$$

$$\text{(b)}\quad \Delta E_{fd} = \frac{K_f}{1 + T_f \cdot p} \cdot (V_r - v_t - V_s + u_1) \qquad \text{(2-35)}$$

$$V_s = \frac{K_s \cdot p}{1 + T_s \cdot p} \cdot \Delta E_{fd}$$

$$\text{(c)}\quad V_a = \frac{K_f}{1 + T_f \cdot p} \cdot \{ K \cdot (V_r - v_t) - V_s + u_1 \} \qquad \text{(2-36)}$$

$$\Delta E_{fd} = \frac{1}{1 + T_e \cdot p} \cdot V_a \ , \quad V_s = \frac{K_s \cdot p}{1 + T_s \cdot p} \cdot \Delta E_{fd}$$

In eqn.(2-34)-eqn.(2-36), u_1 is an additional control signal to AVR.

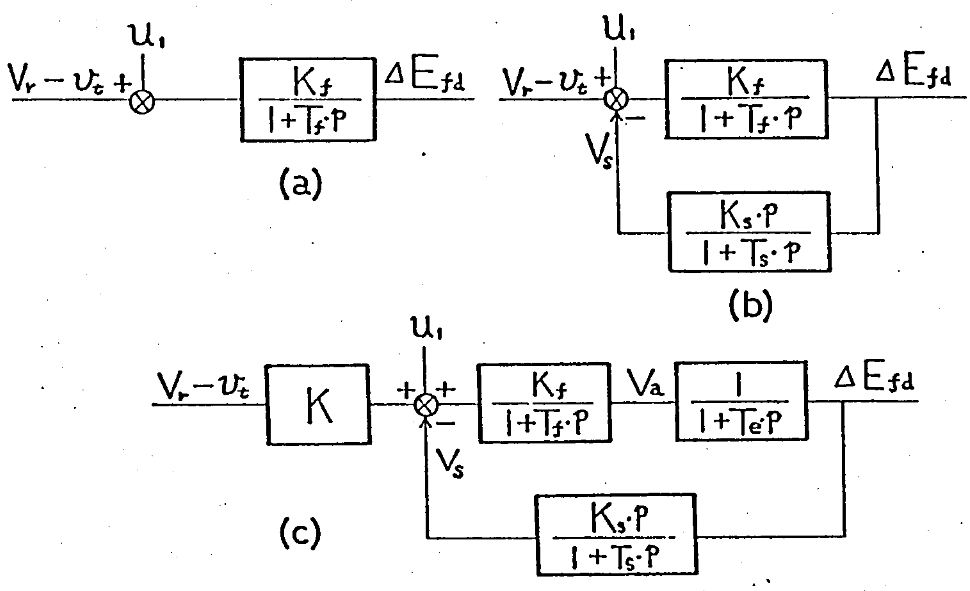


Fig.2-3-2 Simplified model of voltage regulator

Section 2-4. Description of Speed Governing Control System

A typical block diagram of the hydro governor and that of the steam governor are shown in Fig.2-4-1 under the following assumptions:[7],[8]

(1) There are no time lag among the movement of all elements, such that between the main shaft and the governor sleeve movement.

(2) Under steady conditions, the operation of the governor is such that the steam or the water admitted to the turbine is linear function of the speed of the turbine.

(3) Every amplifier can be expressed by a simple time lag.

(4) The condition of the steam source or the water source never change.

Under the similar restrictions to those of AVR, the simplified models as shown in Fig.2-4-2 are used in this thesis. The mathematical explanations of those models are described as follows:

$$\text{(a)}\quad \Delta P_t = \frac{K_g}{1 + T_g \cdot p} \cdot \left(-\frac{\Delta\omega}{\omega_o} + u_2 \right) \qquad \text{(2-37)}$$

$$\text{(b)}\quad \Delta P_v = \frac{K_g}{1 + T_g \cdot p} \cdot \left(-\frac{\Delta\omega}{\omega_o} + u_2 \right) \qquad \text{(2-38)}$$

$$\Delta P_t = \frac{1}{1 + T_h \cdot p} \cdot \Delta P_v$$

In eqn.(2-37) and eqn.(2-38), u_2 is an additional control signal to the governor.

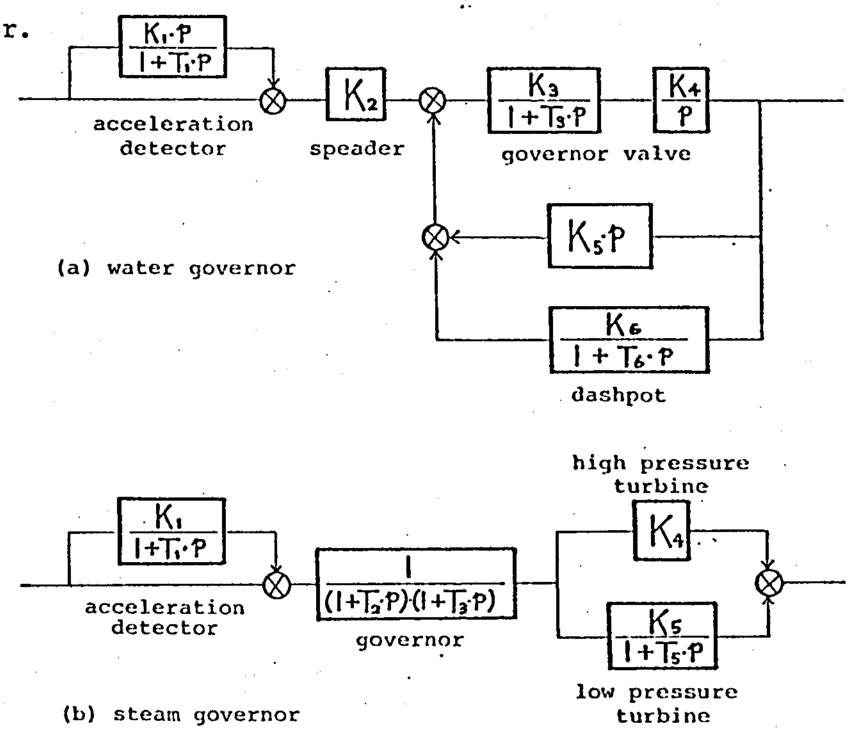


Fig.2-4-1 Block diagram of governor

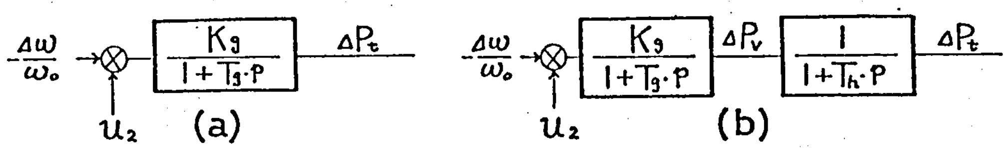


Fig.2-4-2 Simplified model of governor

Section 2-5. Description of Transmission System (I)

In this section, the interconnecting network and the local impedance loads are stated in terms of the three-phase quantities with respect to a stationary reference frame. But each individual synchronous machine is described by Park's quantities in the frame fixed to its rotor as shown in section 2-1 and section 2-2. Then, at the nodes where the synchronous machines are connected to the transmission network, the three-phase quantities must be related to Park's quantities by axis transformation in order to describe the whole system using Park's quantities.

Axis transformation[9],[10],[11],[12]

The axis transformation used here is mainly based on Park's transformation[13] and its inverse transformation. The angular position of the j-th machine rotor with respect to a stationary reference frame is expressed by θ_j as follows:

$$\theta_j = \omega_o \cdot t + \delta_j \tag{2-39}$$

$$\omega_j = p\theta_j = \omega_o + p\delta_j \tag{2-40}$$

$$\delta_{ij} = \theta_i - \theta_j = \delta_i - \delta_j \tag{2-41}$$

where ω_o is the synchronous angular velocity.

Let $\mathbb{W}_{aj}$ be the column vector of the three-phase quantities at the j-th bus and $\mathbb{W}_{dij}$ be the column vector of their Park's quantities with respect to the rotating frame fixed to the i-th machine rotor. Then the relationships between $\mathbb{W}_{aj}$ and $\mathbb{W}_{djj}$ become:

$$\mathbb{W}_{djj} = \mathbb{P}(\theta_j) \cdot \mathbb{W}_{aj} \tag{2-42}$$

$$\mathbb{W}_{aj} = \mathbb{P}^{-1}(\theta_j) \cdot \mathbb{W}_{djj} \tag{2-43}$$

A combined transformation matrix and its derivative are introduced as:

$$\mathbb{T}(\delta_{ij}) = \mathbb{P}(\theta_i) \cdot \mathbb{P}^{-1}(\theta_j) \tag{2-44}$$

$$\mathbb{T}'(\delta_{ij}) = -\frac{\partial}{\partial \delta_{ij}} \mathbb{T}(\delta_{ij}) = \mathbb{P}(\theta_i) \cdot \frac{\partial}{\partial \theta_j} \mathbb{P}^{-1}(\theta_j) \qquad (2\text{-}45)$$

The elements of the foregoing transformation matrices are shown in Table 2-5-1. The application of the transformation matrix $\mathbb{T}(\delta_{ij})$ to $\mathbb{W}_{djj}$ gives:

$$\mathbb{W}_{dij} = \mathbb{T}(\delta_{ij}) \cdot \mathbb{W}_{djj} \qquad (2\text{-}46)$$

By using this set of axis transformations, the three-phase quantities are projected onto the every rotating frames.

Table 2-5-1 Transformation matrices

$\mathbb{P}(\theta_j) = 2/3$

$\cos\theta_j$	$\cos(\theta_j - 120°)$	$\cos(\theta_j + 120°)$
$-\sin\theta_j$	$-\sin(\theta_j - 120°)$	$-\sin(\theta_j + 120°)$
1/2	1/2	1/2

$\mathbb{P}^{-1}(\theta_j) =$

$\cos\theta_j$	$-\sin\theta_j$	1
$\cos(\theta_j - 120)$	$-\sin(\theta_j - 120)$	1
$\cos(\theta_j + 120)$	$-\sin(\theta_j + 120)$	1

$\mathbb{T}(\delta_{ij}) =$

$\cos\delta_{ij}$	$\sin\delta_{ij}$	0
$\sin\delta_{ij}$	$\cos\delta_{ij}$	0
0	0	1

$\mathbb{T}'(\delta_{ij}) =$

$\sin\delta_{ij}$	$-\cos\delta_{ij}$	0
$-\cos\delta_{ij}$	$\sin\delta_{ij}$	0
0	0	1

Network equations

Any interconnecting network can be transformed into the equivalent circuit that has the simplest form of the Lagrangian tree, as shown in Fig. 2-5-1. All nodes, to which no shunt loads nor power sources are connected may be eliminated, and only those nodes that should be formulated in necessary set of equations may remain in the equivalent circuit.

A shunt load consists of a resistor, an inductor and a capacitor, as shown in Fig.2-5-2, and conventionally includes the capacitance between the transmission lines and the ground.

Choosing the n-th node as a voltage reference node, the equivalent circuit of Fig.2-5-1 yields the following set of equations:

$$\mathbb{v}_{aj} - \mathbb{v}_{an} = \sum_{k=1}^{n-1} (R_{jk} + L_{jk} \cdot p) \cdot \mathring{\mathbb{i}}'_{ak} \qquad (2\text{-}47)$$

$$\sum_{j=1}^{n} \mathring{\mathbb{i}}'_{aj} = \mathbb{0} \qquad (2\text{-}48)$$

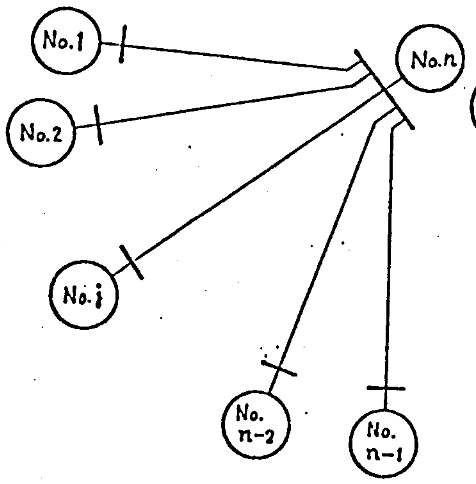


Fig.2-5-1 Equivalent circuit of Lagrangian tree form

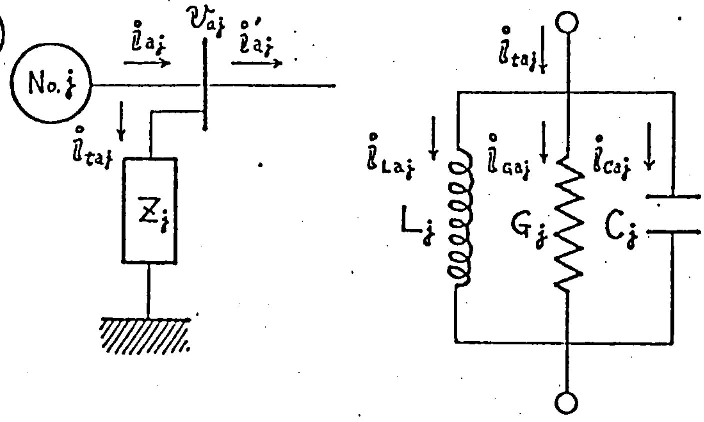


Fig.2-5-2 Local shunt load

An additional shunt load at each bus may be expressed as:

$$\mathring{i}_{aj} = \mathring{i}'_{aj} + \mathring{i}_{Laj} + \mathring{i}_{Caj} + \mathring{i}_{Gaj} \qquad (2\text{-}49)$$

where $L_j \cdot p\, \mathring{i}_{Laj} = v_{aj}$, $\mathring{i}_{Caj} = C_j \cdot p\, v_{aj}$, $\mathring{i}_{Gaj} = G_j \cdot v_{aj}$

The application of the axis transformation above described to eqn.(2-47)-eqn.(2-49) gives:

$$v_{dj} - \mathbb{T}(\delta_{jn}) \cdot v_{dn} = \sum_{K=1}^{n-1} [\, \{ R_{jK} \cdot \mathbb{T}(\delta_{jK}) + \omega_K \cdot L_{jK} \cdot \mathbb{T}'(\delta_{jK}) \} \cdot \mathring{i}'_{dK} + L_{jK} \cdot \mathbb{T}(\delta_{jK}) \cdot p\, \mathring{i}'_{dK} \,] \qquad (2\text{-}50)$$

$$\sum_{j=1}^{n} \mathbb{T}(\delta_{nj}) \cdot \mathring{i}'_{dj} = 0 \qquad (2\text{-}51)$$

$$\mathring{i}_{dj} = \mathring{i}'_{dj} + \mathring{i}_{Ldj} + \mathring{i}_{Cdj} + \mathring{i}_{Gdj} \qquad (2\text{-}52)$$

where $\omega_j \cdot L_j \cdot \mathbb{T}'(0) \cdot \mathring{i}_{Ldj} + L_j \cdot \mathbb{T}(0) \cdot p\, \mathring{i}_{Ldj} = v_{dj}$

$\mathring{i}_{Cdj} = \omega_j \cdot C_j \cdot \mathbb{T}'(0) \cdot v_{dj} + C_j \cdot \mathbb{T}(0) \cdot p\, v_{dj}$

$\mathring{i}_{Gdj} = G_j \cdot v_{dj}$

Now eqn.(2-50)-eqn.(2-52) form a set of first order differential equations describing the behaviour of the balanced phase transmission system. Their transient solution, by the definition of ω_j as the angular velocity of the j-th machine, depends on the transient performance of the rotor.

These equations contain zero-phase equations, but because the model system is restricted to the case of balanced phase condition, their extra complexity is omitted from the analysis. Therefore the order of all vectors and transformation matrices $\mathbb{T}(\delta_{ij})$ and $\mathbb{T}'(\delta_{ij})$ are reduced by one. Further, the transient occurring on the transmission system and on the shunt loads is much faster than the electromechanical transient of synchronous machine and their duration is short in comparison with even the shortest lived transients occurring in the machine windings. Therefore, their transient solution may be neglected[(14),(15)] for the purpose of obtaining a boundary condition here and the transmission system equations (2-50)-(2-52) become as follows:

$$v_{dj} - \mathbb{T}(\delta_{jn})\cdot v_{dn} = \sum_{k=1}^{n-1}\left\{R_{jk}\cdot\mathbb{T}(\delta_{jk}) + \omega_k\cdot L_{jk}\mathbb{T}'(\delta_{jk})\right\}\mathring{i}_{dk}' \qquad (2\text{-}53)$$

$$\sum_{j=1}^{n} \mathbb{T}(\delta_{nj})\cdot \mathring{i}_{dj}' = \mathbb{0} \qquad (2\text{-}54)$$

$$\mathring{i}_{dj} = \mathring{i}_{dj}' + \mathbb{Y}_j(\omega_j)\cdot v_{dj} \qquad (2\text{-}55)$$

where $$\mathbb{Y}_j(\omega_j) = \begin{bmatrix} G_j , & 1/\omega_j\cdot L_j - \omega_j\cdot C_j \\ -1/\omega_j\cdot L_j + \omega_j\cdot C_j , & G_j \end{bmatrix}$$

It is evident that some of the transmission system parameters equal to zero allow eqn.(2-53)-eqn.(2-55) to be used to describe the performance of several transmission systems of simpler form. Let $n = 2$, $R_{jk} = G_j = 1/L_j = C_j = 0$, and the 2-nd node be the infinite bus, then the familiar equations for a one machine infinite bus system may be given:

$$\begin{bmatrix} v_{d1} \\ v_{q1} \end{bmatrix} = \begin{bmatrix} \cos\delta_{12} , & \sin\delta_{12} \\ -\sin\delta_{12} , & \cos\delta_{12} \end{bmatrix}\cdot\begin{bmatrix} 0 \\ v_{q2} \end{bmatrix} + \omega_1\cdot L_{11}\begin{bmatrix} 0 , & -1 \\ 1 , & 0 \end{bmatrix}\cdot\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \qquad (2\text{-}56)$$

For the convinience in later manipulation of equations, let $\mathbb{Y}'(\omega_j)$ be introduced as:

$$\mathbb{Y}_j'(\omega_j) = \frac{\partial}{\partial \omega_j} \cdot \mathbb{Y}_j(\omega_j) = \begin{bmatrix} 0 & , -1/\omega_j^2 \cdot L_j - C_j \\ -1/\omega_j^2 \cdot L_j + C_j \ , & \end{bmatrix} \quad (2\text{-}57)$$

Section 2-6. Description of Transmission System (II)

In this section, the interconnecting network and the local impedance loads are expressed in the relation between the busbar voltages and busbar currents in the common reference frame fixed to the rotor of an imaginary machine rotating with synchronous angular velocity ω_0 (= $2\pi f_0$), then at the nodes where the synchronous machines are connected to the transmission network, these quantities in the common reference frame must be related to Park's quantities by axis transformation in order to describe the entire system using Park's quantities.

Axis transformation[16],[17]

The axis transformation used in this section is almost same to that described in section 2-5.

Let $\mathbb{W}_{Dj}$ be the column vector of quantities at the j-th bus expressed in the common reference frame rotating with synchronous speed, and $\mathbb{W}_{dj}$ be the column vector of their Park's quantities with respect to the rotating reference frame of the j-th machine. Balanced conditions exist in the system, allowing the zero-sequence quantities to be neglected, so that the vector $\mathbb{W}_{Dj}$ and $\mathbb{W}_{dj}$ become second order column vectors. The phasor relation between two reference frames is shown in Fig.2-6-1. Then the relationship between $\mathbb{W}_{Dj}$ and $\mathbb{W}_{dj}$ becomes:

$$\mathbb{W}_{dj} = \mathbb{P}(\delta_j) \cdot \mathbb{W}_{Dj} \quad (2\text{-}58)$$

$$\mathbb{W}_{Dj} = \mathbb{P}^{-1}(\delta_j) \cdot \mathbb{W}_{dj} \quad (2\text{-}59)$$

where, δ_j is the displacement angle of the j-th machine rotor from the rotating common reference frame, and $\mathbb{P}(\delta_j)$ and $\mathbb{P}^{-1}(\delta_j)$ are the transformation matrices whose elements are shown in Table 2-6-1.

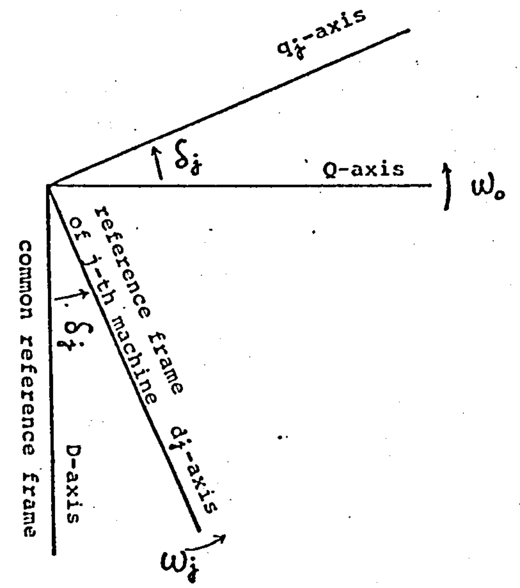


Fig.2-6-1 Phasor relation between two reference frames

Table 2-6-1 Transformation matrices

$\mathbb{P}(\delta_j)=$

$\cos\delta_j$	$\sin\delta_j$
$-\sin\delta_j$	$\cos\delta_j$

$\mathbb{P}^{-1}(\delta_j)=$

$\cos\delta_j$	$-\sin\delta_j$
$\sin\delta_j$	$\cos\delta_j$

The displacement angle δ_j is expressed as follows:

$$\delta_j = \int_0^t (\omega_j - \omega_o)\,d\tau + \delta_{jo} = \int_0^t \Delta\omega_j\,d\tau + \delta_{jo} \qquad (2\text{-}60)$$

where δ_{jo} is the initial value of δ_j .

In the transient state ω_j , $\Delta\omega_j$, and δ_j all change owing to the unbalanced torque, and:

$$p\,\delta_j = (\omega_j - \omega_o) = \Delta\omega_j \qquad (2\text{-}61)$$

Network equations[16],[17]

The interconnected network and the impedance loads are stated in the relation between the busbar voltages and currents in the common reference frame. The relationship between the voltages and the currents becomes:[18]

$$\mathring{i}_{Dj} = \sum_{K=1}^{n} \mathbb{Y}_{jK}\cdot v_{DK} \qquad (j = 1 \sim n) \qquad (2\text{-}62)$$

where, the vector v_{Dj} and $\mathring{i}_{Dj}$, respectively, express the voltage and current of the j-th bus and $\mathbb{Y}_{jK}$ is a second order matrix consisting of a short circuit transfer conductance G_{jk} and suseptance B_{jk} between the j-th bus and the k-th bus, and $\mathring{\mathfrak{i}}_{Dj}$, $\mathfrak{v}_{Dj}$ and $\mathbb{Y}_{jk}$ are described as follows:

$$\mathring{\mathfrak{i}}_{Dj} = \begin{bmatrix} i_{Dj} \\ i_{Qj} \end{bmatrix} , \quad \mathfrak{v}_{Dj} = \begin{bmatrix} v_{Dj} \\ v_{Qj} \end{bmatrix} , \quad \mathbb{Y}_{jk} = \begin{bmatrix} G_{jk} , & -B_{jk} \\ B_{jk} , & G_{jk} \end{bmatrix} \qquad (2\text{-}63)$$

Because of the balanced condition of the system, the zero-sequence quantities is neglected here. By the axis transformation described above, eqn.(5-62) becomes as follows:

$$\mathring{\mathfrak{i}}_{dj} = \sum_{k=1}^{n} \mathbb{Y}'_{jk} \cdot \mathfrak{v}_{dk} \qquad (2\text{-}64)$$

where, the vector $\mathfrak{v}_{dj}$ and $\mathring{\mathfrak{i}}_{dj}$, respectively express the voltage and current of the j-th bus in the frame fixed to the j-th machine rotor, and $\mathbb{Y}'_{jk}$ is a second order matrix, and they become:

$$\mathring{\mathfrak{i}}_{dj} = \begin{bmatrix} i_{dj} \\ i_{qj} \end{bmatrix} , \quad \mathfrak{v}_{dj} = \begin{bmatrix} v_{dj} \\ v_{qj} \end{bmatrix} , \quad \mathbb{Y}'_{jk} = \mathbb{P}(\delta_j) \cdot \mathbb{Y}_{jk} \mathbb{P}^{-1}(\delta_k) \qquad (2\text{-}65)$$

The equivalent circuit of the transmission system need not be altered if the frequency diviation of the system is only affected by the system disturbance. For convenience in later manipulation, it is assumed that the equivalent circuit of the transmission system is not affected by the system frequency deviations.

Section 2-7. Description of Entire Power System

After both sets of equations, i.e. synchronous machine equations connected with controllers equations and transmission system equations, are obtained, the quantities of the transmission network are projected into the frames fixed to the machines rotors as shown in section 2-5 and section 2-6. This axis transformation enables the entire power system to be expressed by Park's quantities.

The entire power system is descrived in vector form as follows:

$$p\mathbb{x} = \mathbb{f}(\mathbb{x}) + \mathbb{B}\cdot\mathbb{u} \qquad (2\text{-}66)$$

where,

$\mathbb{x}$: vector of state variables of the system

$\mathbb{f}(\mathbb{x})$: nonlinear functional vector

$\mathbb{u}$: vector of additional control signals to automatic voltage regulators and governors

$\mathbb{B}$: coefficient matrix

In the small signal dynamic stability analysis, the system disturbances are assumed sufficiently small, so the eqn.(2-66) can be rewritten by the following linearized equation around the operating point.

$$p\Delta\mathbb{x} = \mathbb{A}\cdot\Delta\mathbb{x} + \mathbb{B}\cdot\mathbb{u} \qquad (2\text{-}67)$$

where,

$\mathbb{A}$: Jacobian matrix of $\mathbb{f}(\mathbb{x})$ at the operating point ($=\partial\mathbb{f}/\partial\mathbb{x}^T|_{\mathbb{x}=\mathbb{x}_0}$)

$\mathbb{x} = \mathbb{x}_0 + \Delta\mathbb{x}$, $\mathbb{f}(\mathbb{x}_0) = \mathbb{0}$, $\mathbb{u}|_{\mathbb{x}=\mathbb{x}_0} = \mathbb{0}$

$\mathbb{x}_0$: the value of state variables vector at the operating point

CHAPTER 3 MATHEMATICAL METHOD OF STABILITY ANALYSIS

The dynamic stability and transient stability analysis of electrical power have been subjects of major theoretical and practical interests for some twenty years, and they continue to grow in importance today as generation and transmission equipments are being applied with high reactances and correspondingly lower stability margin. In view of the increasing complexity of present-day power systems, their design and operation requires a more detailed stability analysis that may be achieved by available computer programs. Owing to the progress of digital computers with great memory capacity and quick processing ability, many excellent works[19],[20] on dynamic and transient stability of electrical power systems have been done.

In this chapter, the mathematical methods of stability analysis used in this thesis are summarized, and the applications to model systems are shown, where the additional control signals to automatic voltage regulators and governors are not considered.

Section 3-1. Mathematical Method of Dynamic Stability Analysis

From eqn.(2-67), the small signal performance of the entire power system is described by a set of linearized differential equations of the form:

$$p\Delta\mathcal{X} = \mathbb{A}\cdot\Delta\mathcal{X} \qquad (3\text{-}1)$$

where, the additional control signals are not considered, i.e. $\mathcal{U} = \mathbb{O}$.

The construction of matrix $\mathbb{A}$ involves an equivalent circuit of a transmission network, some reference frames and an axis transformation. And it also involves power flow calculation for initial conditions. Once the matrix $\mathbb{A}$ is obtained, standard computer programs may be used for dynamic stability analysis of power system.

3-1-1. Root-locus Analysis

After forming the matrix $\mathbb{A}$, the characteristic equation of the system is described as follows:[21],[22]

$$\det \left| \mathbb{A} - p\mathbb{I} \right| = 0 \qquad (3\text{-}2)$$

And the eigenvalues of the system described by eqn.(3-1) may be found by solving the characteristic equation (3-2).

The eigenvalues of a linear dynamical system correspond to its natural mode of response, with each real part giving the reciprocal decay time constant or damping coefficient of a mode, and each real pair of imaginary parts giving natural frequency.

The necessary and sufficient condition for dynamic stable is that all the eigenvalues have negative real parts. Thus, the dynamic stability may be directly checked with the real parts of the eigenvalues. Further, a form of quantitative information on the relative stability of the system may be obtained by plotting the variation of the eigenvalues as system conditions , for instance bus voltages, power factors, are varied.

3-1-2. Time Domain Analysis

The system is described in the time domain by the state space equation (3-1) with a constant matrix $\mathbb{A}$, then the state space formulation can be used to caluculate the numerical solution of eqn.(3-1).

The exact solution of eqn.(3-1) is represented as follows:[23]

$$\Delta \mathcal{X}(t) = \mathcal{E}^{At} \cdot \Delta \mathcal{X}(0) \qquad (3\text{-}3)$$

where, $\mathcal{E}^{At}$ is the state transition matrix of the system.

The recursive formulas for digital computation may be derived from eqn. (3-3) as follows:

$$\Delta \mathcal{X}\left[(n+1)\cdot \Delta t\right] = \mathcal{E}^{A\cdot\Delta t} \cdot \Delta \mathcal{X}(n\cdot \Delta t) \qquad (3\text{-}4)$$

where, Δt is an increment of time.

The state transition matrix $\varepsilon^{\mathbb{A}\cdot\Delta t}$ may be obtained by the following infinite matrix series:

$$\varepsilon^{\mathbb{A}\cdot\Delta t} = \sum_{K=0}^{\infty} \frac{\mathbb{A}^K\cdot\Delta t^K}{K!} \quad , \quad \mathbb{A}^0 = \mathbb{I} : \text{unit matrix} \qquad (3\text{-}5)$$

Since the matrix series of eqn.(3-5) is uniformly convergent in any finite interval, the transition matrix $\varepsilon^{\mathbb{A}\cdot\Delta t}$ can be evaluated within prescribed accuracy from eqn.(3-5).

The system stability is checked by the numerical solution of eqn.(3-1), i.e. if the solution curves converge to zero, the system becomes stable, and the convergence is faster, the system becomes more stable.

3-1-3. Lyapunov's Direct Method

The basis of Lyapunov's direct method used in this thesis is the solution of the following Lyapunov's matrix equation[23] of the system (3-1):

$$\mathbb{A}^T\cdot\mathbb{K} + \mathbb{K}\cdot\mathbb{A} = -\mathbb{Q} \qquad (3\text{-}6)$$

where, matrix $\mathbb{Q}$ is a positive definite or positive semi-definite symmetric matrix and matrix $\mathbb{K}$ is the symmetric solution matrix of eqn.(3-6).

It is known that;

(1) When the matrix $\mathbb{Q}$ is positive definite, eqn(3-6) will yield a positive definite matrix $\mathbb{K}$, if, only if, the system described by eqn.(3-1) is asymptotically stable.

(2) If the system is asymptotically stable and the matrix $\mathbb{K}$ is positive definite or positive semi-definite, then the following relationship is satisfied:

$$I = \int_0^{\infty} \Delta\mathbb{X}^T\cdot\mathbb{Q}\cdot\Delta\mathbb{X}\, dt = \Delta\mathbb{X}^T\cdot\mathbb{K}\cdot\Delta\mathbb{X}\Big|_{t=0.0} \qquad (3\text{-}7)$$

In eqn.(3-7), the value of I can be considered as some kind of performance index[24] of the system (3-1). Eqn.(3-7) emphasizes the dependence of the value of I on both the solution of Lyapunov's matrix equation (3-6) and the initial values of the state variables $\Delta\mathbb{X}(0)$.

In order to use this performance index to represent the stability measure of the system, it is usually necessary to eliminate this dependence on the initial state $\Delta \mathcal{X}(0)$. Mathematically, a simple way to eliminate the dependence on the initial state is to average the performance index I obtained for a linearly independent set of initial states. This is equivalent to assuming the initial state $\Delta \mathcal{X}(0)$ to be random variables uniformly distributed on the surface of the unit sphere. In this case, the averaging value of I , which is designated the expected value of I , becomes:

$$\hat{I} = \frac{1}{n} \cdot \mathrm{tr}(\mathbb{K}) \tag{3-8}$$

where, $\hat{I}$: expected value of I , n : order of state variables

Tr(.) : sum of the diagonal elements of the matrix contained in (.)

From eqn.(3-8), Tr($\mathbb{K}$) can be considered as the stability measure of the system described by eqn.(3-1), and for the smaller value of Tr($\mathbb{K}$), the system become more stable.

Section 3-2. Application to a 3-machine Problem

A multi-machine power system contains an extraordinary amount of system parameters. It would be confusing to study all these effects. Hence, a simple model of a 3-machine system as shown in Fig.3-2-1 has been studied to demonstrate the effects of the load flow and the local shunt loads on the dynamic stability.

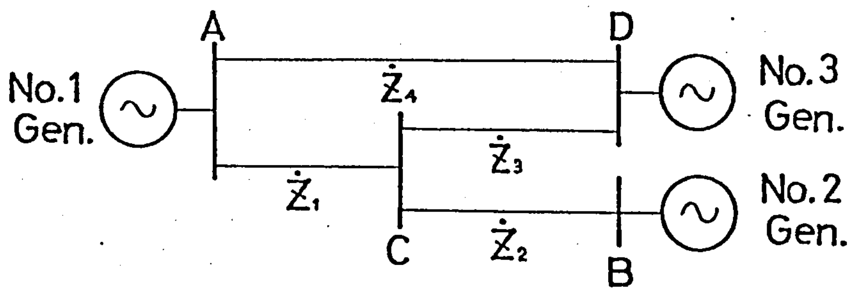


Fig.3-2-1 Model of 3-machine system

In the model system, No.3 machine is equipped with neither voltage regulator nor governor. The simplified models shown in Fig.2-3-2(a) and Fig.2-4-2(a) are used for the control systems of the other machine, but the additional control signals u_1 and u_2 are not considered here.

3-2-1. Linearized Equations of Model System

In this section, the mathematical representations of synchronous machines and transmission network described in section 2-2 and section 2-5 have been used in order to obtain the linearized equations of the model system.

The machine equations are rewritten for small perturbations for the case of the j-th machine described below.

From eqns.(2-26),(2-27),(2-29),(2-32) and (2-33):

$$p\Delta E'_{qj} = \{\Delta E_{fdj} - \Delta E'_{qj} - (x_{dj} - x'_{dj})\cdot\Delta i_{dj}\}/T'_{doj} \qquad (3\text{-}9)$$

$$p\Delta E'_{dj} = \{-\Delta E'_{dj} + (x_{qj} - x'_{qj})\cdot\Delta i_{qj}\}/T'_{qoj} \qquad (3\text{-}10)$$

$$p\Delta\delta_{jn} = \Delta\omega_j - \Delta\omega_n \qquad (3\text{-}11)$$

$$p\Delta\omega_j = (\Delta P_{tj} - P_{dj}\cdot\Delta\omega_j - v_{djo}\cdot\Delta i_{dj} - v_{qjo}\cdot\Delta i_{qj} - i_{djo}\cdot\Delta v_{dj} - i_{qjo}\cdot\Delta v_{qj})/M_j \qquad (3\text{-}12)$$

From eqn..(2-28):

$$\Delta v_{dj} = -r_j\cdot\Delta i_{dj} + x'_{qj}\cdot\Delta i_{qj} + \Delta E'_{dj} \qquad (3\text{-}13)$$

$$\Delta v_{qj} = -r_j\cdot\Delta i_{qj} - x'_{dj}\cdot\Delta i_{dj} + \Delta E'_{qj} \qquad (3\text{-}14)$$

Similarly, the control systems equations are rewritten as follows.

From eqn.(2-31), eqn.(2-34) and eqn.(2-37):

$$p\Delta E_{fdj} = -(\Delta E_{fdj} + K_{fj}\cdot\Delta v_{tj})/T_{fj} \qquad (3\text{-}15)$$

$$p\Delta P_{tj} = -(\Delta P_{tj} + K_{gj}\cdot\Delta\omega_j/\omega_o)/T_{gj} \qquad (3\text{-}16)$$

where, $\Delta v_{tj} = (v_{djo}\cdot\Delta v_{dj} + v_{qjo}\cdot\Delta v_{qj})/v_{tjo}$, $u_1 = u_2 = 0.0$

Further, if desired, any other model for voltage regulator or speed governor can be easily introduced.

From eqn.(2-53)-eqn.(2-55) the network equations for the perturbed motions become as follows:

$$\Delta \mathbb{V}_{dj} - \mathbb{T}(\delta_{jno})\cdot \Delta \mathbb{V}_{dn} + \mathbb{T}'(\delta_{jno})\cdot \mathbb{V}_{dno}\cdot \Delta\delta_{jn}$$
$$= \sum_{K=1}^{n-1}\Big[\{R_{jK}\cdot \mathbb{T}(\delta_{jKo}) + \omega_o\cdot L_{jK}\cdot \mathbb{T}'(\delta_{jKo})\}\cdot \Delta \mathring{\mathbb{i}}'_{dK} + \{ \qquad (3\text{-}17)$$
$$-R_{jK}\cdot \mathbb{T}'(\delta_{jKo}) + \omega_o\cdot L_{jK}\cdot \mathbb{T}(\delta_{jKo})\}\cdot \mathring{\mathbb{i}}'_{dKo}\cdot \Delta\delta_{jK} + L_{jK}\cdot \mathbb{T}'(\delta_{jKo})\cdot \mathring{\mathbb{i}}'_{dKo}\cdot \Delta\omega_j\Big]$$

$$\sum_{j=1}^{n}\{\mathbb{T}(\delta_{njo})\cdot \Delta \mathring{\mathbb{i}}'_{dj} - \mathbb{T}'(\delta_{njo})\cdot \mathring{\mathbb{i}}'_{djo}\cdot \Delta\delta_{nj}\} = \mathbb{0} \qquad (3\text{-}18)$$

$$\Delta \mathring{\mathbb{i}}_{dj} = \Delta \mathring{\mathbb{i}}'_{dj} + \mathbb{Y}_j(\omega_o)\cdot \Delta \mathbb{V}_{dj} + \mathbb{Y}'_j(\omega_o)\cdot \mathbb{V}_{djo}\cdot \Delta\omega_j \qquad (3\text{-}19)$$

where, $\Delta \mathring{\mathbb{i}}_{dj} = \begin{bmatrix}\Delta i_{dj}\\ \Delta i_{qj}\end{bmatrix}$, $\Delta \mathring{\mathbb{i}}'_{dj} = \begin{bmatrix}\Delta i'_{dj}\\ \Delta i'_{qj}\end{bmatrix}$, $\Delta \mathbb{V}_{dj} = \begin{bmatrix}\Delta v_{dj}\\ \Delta v_{qj}\end{bmatrix}$

Finally a set of equations (3-9)-(3-19) are rearranged to give the matrix equation of the system. Let a pair of vectors $\Delta\mathbb{X}$ and $\Delta\mathbb{Y}$ be defined as:

$$\Delta\mathbb{X} = [\Delta\delta_{13}, \Delta E'_{q1}, \Delta E'_{d1}, \Delta E_{fd1}, \Delta\omega_1, \Delta P_{t1}, \Delta\delta_{23}, \Delta E'_{q2}, \qquad (3\text{-}20)$$
$$\Delta E'_{d2}, \Delta E_{fd2}, \Delta\omega_2, \Delta P_{t2}, \Delta E'_{q3}, \Delta E'_{d3}, \Delta\omega_3]^T$$

$$\Delta\mathbb{Y} = [\Delta v_{d1}, \Delta v_{q1}, \Delta i_{d1}, \Delta i_{q1}, \ldots, \Delta v_{d3}, \Delta v_{q3}, \Delta i_{d3}, \Delta i_{q3}]^T \qquad (3\text{-}21)$$

where, the order of $\Delta\mathbb{X}$ becomes 15 and the order of $\Delta\mathbb{Y}$ becomes 12 for the given model system.

Then from eqn.(3-9)-eqn.(3-12) and eqn.(3-15)-eqn.(3-16):

$$p\Delta\mathbb{X} = \mathbb{A}_1\cdot\Delta\mathbb{X} + \mathbb{A}_2\cdot\Delta\mathbb{Y} \qquad (3\text{-}22)$$

From eqn.(3-13), eqn.(3-14) and eqn.(3-17)-eqn.(3-19):

$$\mathbb{A}_3\cdot\Delta\mathbb{X} = \mathbb{A}_4\cdot\Delta\mathbb{Y} \qquad (3\text{-}23)$$

From above two equations in vector form, the linearized equation of the model system becomes of the form shown in eqn.(3-1), and the matrix $\mathbb{A}$ becomes as follows:

$$\mathbb{A} = \mathbb{A}_1 + \mathbb{A}_2\cdot\mathbb{A}_4^{-1}\cdot\mathbb{A}_3 \qquad (3\text{-}24)$$

where, the components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$ and $\mathbb{A}_4$ are shown in Table A-1. (see appendix)

3-2-2. System Parameters and Initial Conditions

The parameters of the model system are shown in Table 3-2-1. The data for the machines are taken from Kimbark.[5] The equivalent circuit of the transmission network yields the impedance matrix of the order 2×2 as shown in Table 3-2-2, where the bus D of the model system is considered as the voltage reference.

Table 3-2-1 System data

machine No.	No.1	No.2	No.3
x_d	1.10	1.10	1.10
x_d'	0.23	0.23	0.23
x_q	1.08	1.08	1.08
x_q'	0.23	0.23	0.23
r	0.05	0.05	0.05
T_{do}'	9.5	9.5	9.5
T_{qo}'	1.7	1.7	1.7
$J\,(=\omega_o M)$	5.0	5.0	5.0
P_d	10.0^{-6}	10.0^{-6}	10.0^{-6}
K_f	40.0	40.0	-
T_f	4.0	4.0	-
K_g	3.0	3.0	-
T_g	1.0	1.0	-

$\dot{Z}_1 = 0.0 + j0.25$, $\dot{Z}_2 = 0.0 + j0.25$
$\dot{Z}_3 = 0.02 + j0.252$, $\dot{Z}_4 = 0.02 + j0.502$

Table 3-2-2 Impedance matrix of equivalent circuit

	Bus A	Bus B
Bus A	0.01 + j0.261	0.01 + j0.126
Bus B	0.01 + j0.126	0.01 + j0.439

Before the dynamic stability of the model system is studied, it is necessary to find the initial values of the pertinent variables. Prior to a disturbance, either the active power output and the terminal voltage, or the reactive power output are known for each machine of the model system. After load flow calculation,[25] the operation angle δ_{jo} is determined according to the phaser diagram as shown in Fig.3-2-2. Once the angle δ_{jo} is known, the initial values of the other variables may be determined and transformed into the rotor-pole axis of every machine in the model system by the axis transformation described in section 2-5.

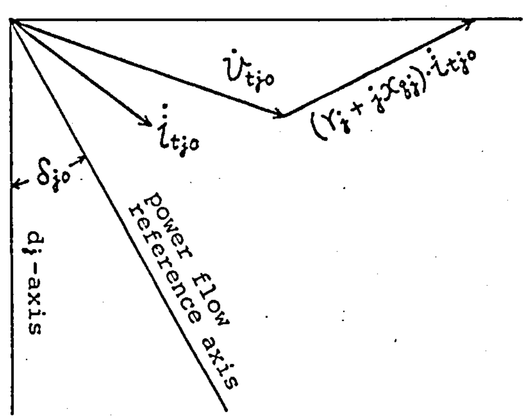


Fig.3-2-2 Phasor diagram for initial values

3-2-3. Stability Margin

For the numerical calculation of the dynamic stability analysis of the model system, the root-locus analysis described in section 3-1-1 has been used . For the inclusion of a stability margin in the analysis , the eigenvalues are restricted so that all of them may lie on the left half plane apart from the imaginary axis, which corresponds to the dominant mode in the performance of the disturbed system is forced to fall within the left half domain restricted by the line $\alpha = 1/T_D$ [11],[12], as shown in Fig.3-2-3.

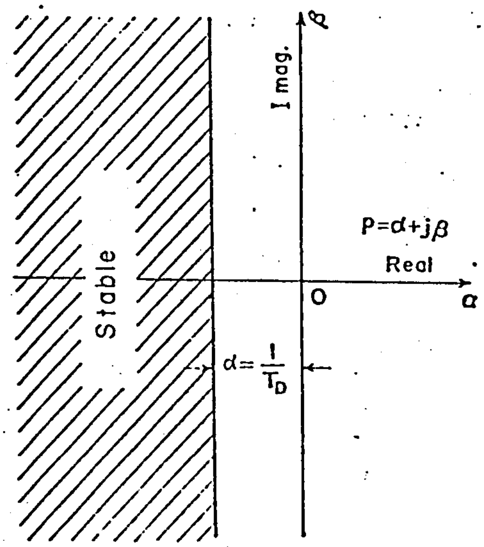


Fig.3-2-3 Domain of eigenvalues restricted by $\alpha = 1/T_D$

By the restriction of the area of the eigenvalues, the critical condition of the operation contains the dominant mode whose decay time is T_D sec. at longest.

3-2-4. Numerical Results

Here, the underexcited or leading power factor operating points have been chosen for the operating conditions of No.1 machine. They are the conditions where the small signal performance are of most interest.

Table 3-2-4 shows a typical listing of the eigenvalues for the model system and their corresponding values in second or in hz are shown in brackets. The eigenvalues associated with the slow permanent droop action of the governors and with the rotor oscillations appear first in this list. The other group of the rapidly damped high frequency modes is associated with the electrical circuits.

Fig.3-2-4 shows the loci of two dominant eigenvalues as the power output of No.1 machine, $W_1 = P_1 + jQ_1$, is varied. Either of the two eigenvalues approaches to the imaginary axis as $-Q_1$ increases, consequently the model system becomes less stable. On the otherhand, its frequency rises as P_1 increases.

The domains of operation allowing for the margin proposed in section 3-2-3 have been obtained against the various values of T_D as shown in Fig.3-2-5. The boundary of the domain takes rather diverse shapes depending on the value of T_D. This fact points out the difficulty of finding some physical meaning in the widely used margin, which is specified only with the critical power output P_{max} and the operating point output P_o as follows[26]; $K_p = (P_{max} - P_o)/P_{max}$. Fig.3-2-6 shows the similar domains of operation while the active power output of No.2 machine is varied.

Holding all the parameters fixed, only the shunt load at the terminal of No.1 machine is varied to know its effect on the stability.

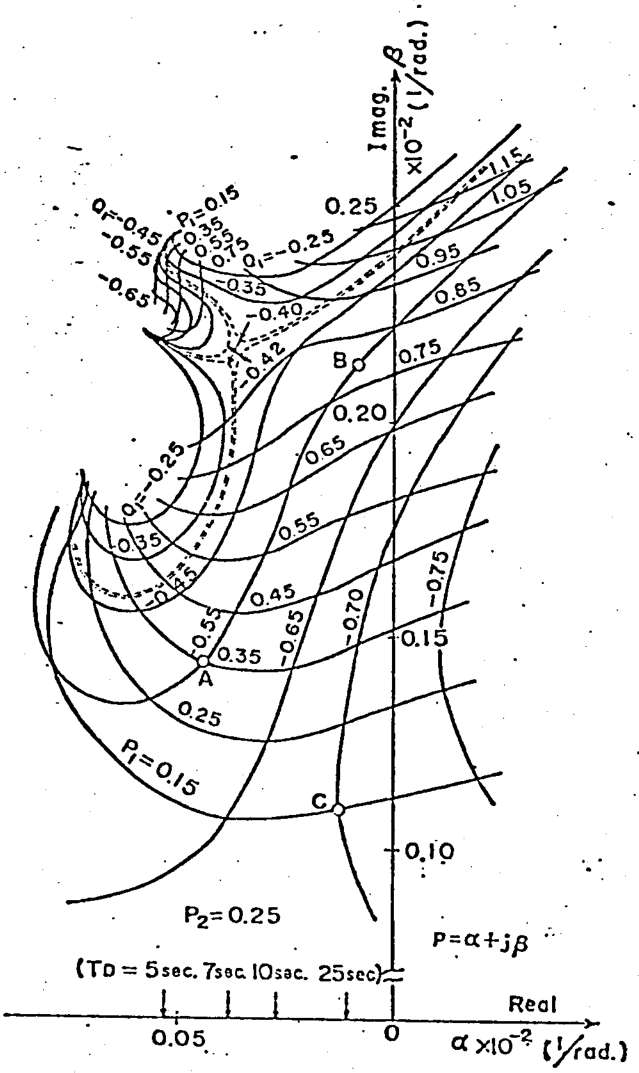


Fig.3-2-4 Loci of two dominant eigenvalues as W_1 varies ($W_1 = P_1 + jQ_1$)

Table 3-2-3 Typical listing of eigenvalues for model system

Real roots	Complex conjugate roots	
-0.1961 (-5.10 sec.)	-0.1699 + j0.5402	(-5.89 sec. , 0.08 hz)
-0.7742 (-1.29 sec.)	-0.2006 + j0.8696	(-4.99 sec. , 0.14 hz)
-0.9647 (-1.04 sec.)	-0.2836 + j12.6485	(-3.53 sec. , 2.02 hz)
-1.0140 (-0.99 sec.)	-0.3983 + j13.7333	(-2.51 sec. , 2.19 hz)
-1.3884 (-0.72 sec.)	-0.4987 + j0.3844	(-2.00 sec. , 0.06 hz)

Operating point: point A in Fig.3-2-4

machine No.	No.1	No.2	No.3
Active power (P)	0.35	0.25	-0.593
Reactive power (Q)	-0.55	0.456	0.285
Terminal voltage (V_t)	0.89	0.12	1.0

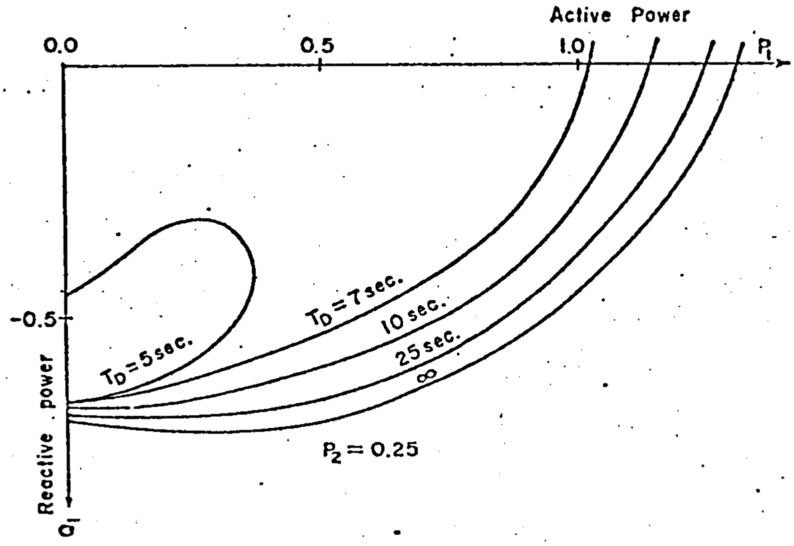


Fig.3-2-5 Domain of operation as margin T_D varies

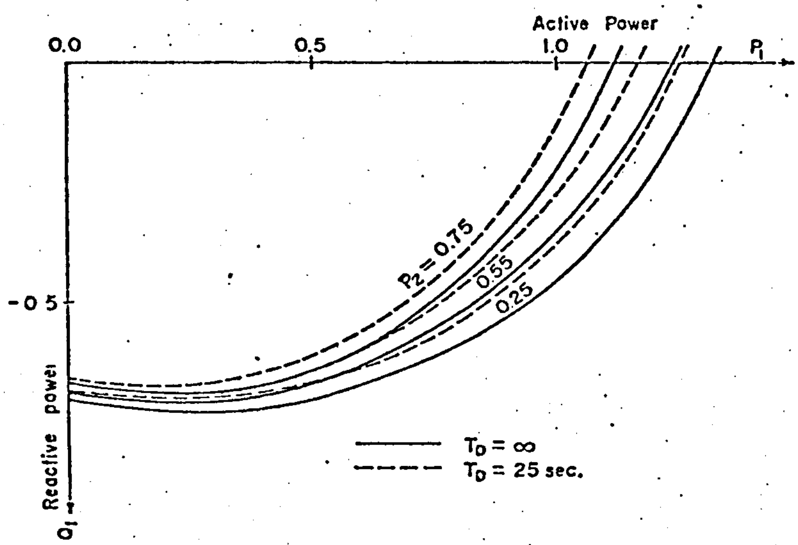


Fig.3-2-6 Domain of operation as P varies

The power factor (cosφ) and the percent consumption (p.c.) of the load with respect to the absolute value of the power output from No.1 machine are varied at the operating point A, B and C in Fig.3-2-4. Fig.3-2-7 shows the loci of the two dominant eigenvalues as the shunt load varies as described above. It is clear that the leading power consumption at No.1 machine terminal, which is feeding the leading power to the system, makes the system less stable, and that the excess lagging reactive power consumption also makes the system less stable.

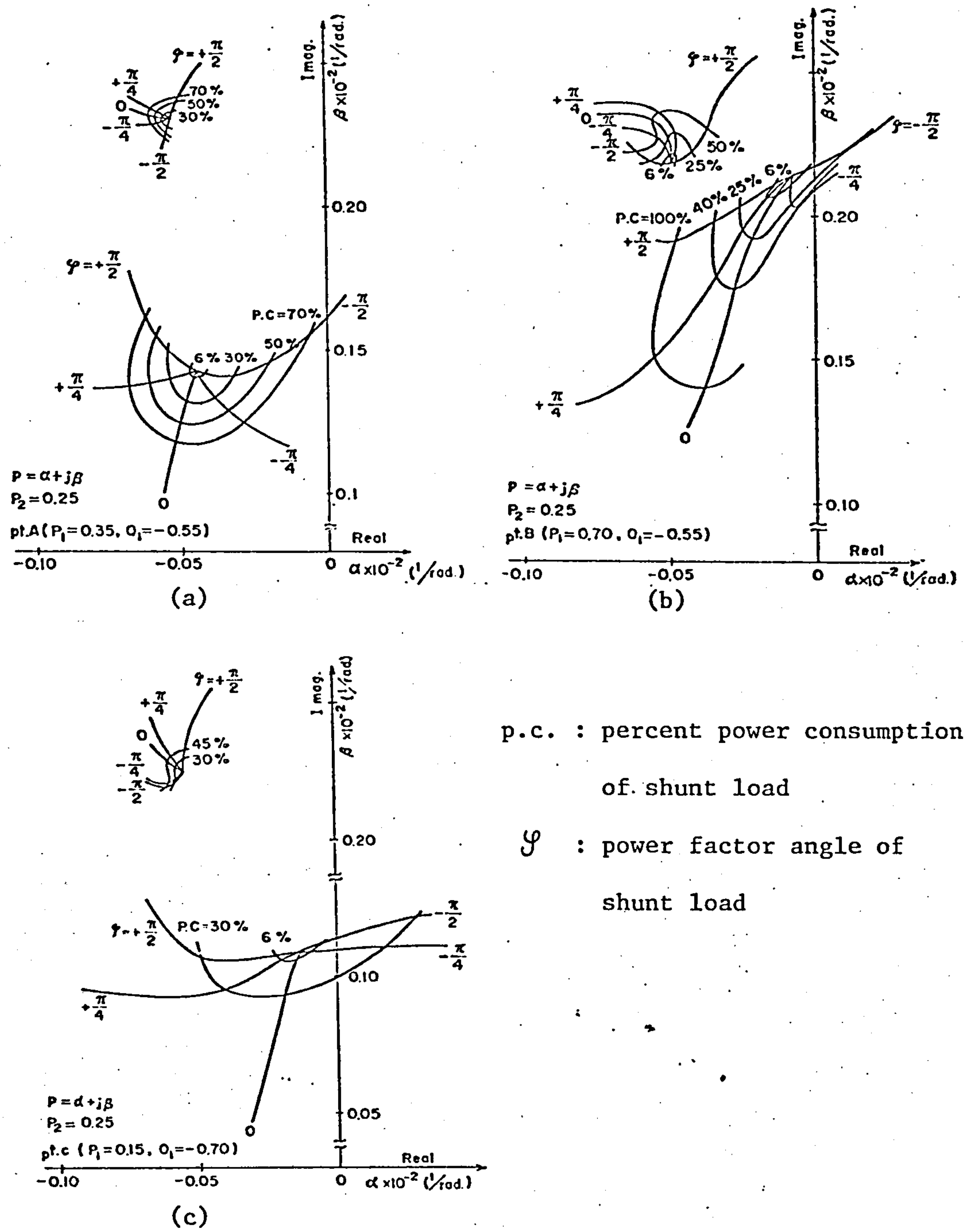


Fig.3-2-7 Loci of two dominant eigenvalues as shunt impedance load varies

Here, the alternative methods described in section 3-1-2 or in section 3-1-3 are not used to investigate the dynamic stability of the model system, but these methods are used in later chapters to investigate the small signal performance of power systems.

Section 3-3. Mathematical Method of Transient Stability Analysis

From eqn.(2-66), the large signal performance of the entire power system is described by a set of non-linear differential equations of the form:

$$p\mathfrak{X} = \mathfrak{f}(\mathfrak{X}) \qquad (3\text{-}25)$$

where, the additional control signals are not considered, i.e. $\mathfrak{U} = \mathbf{0}$.

The construction of the non-linear functional vector $\mathfrak{f}(\mathfrak{X})$ also involves an equivalent circuit of transmission network, some reference frames, an axis transformation and power flow calculation for the initial conditions described in section 3-1.

The transient stability of the system may be investigated by solving[27] the eqn.(3-25) of the system for a given initial condition, and the solution curves converge to their steady state values, then the system becomes transient stable for the given initial condition. On the other hand, the solution curves diverge, then the system is transient unstable.

The integration method of eqn.(3-25) adopted in this thesis is a fourth order Runge Kutta Gill procedure,[28] in which four evaluations of the rate of change of eqch differential variables are made at specific interval within the time duration of the integration step, giving a truncation error that is approximately proportional to the fifth power of the step intervals.

Section 3-4. Application to a 3-machine Problem

A simple model of a 3-machine system as shown in Fig.3-4-1 has been used in order to investigate the transient stability of this model system. In this model, No.3 machine is conventionary used to represent a large scale power system: it represents a machine which is equivalent to about 10 machines connected to bus No.3. Also, it is assumed that the internal power consumption of this large scale power system is equivalent to the power which is consumed at the shunt impedance load at bus No.3. For the control systems of each machine of the model system, the simplified models shown in Fig.2-3-2(c) and Fig.2-4-2(b) are used.

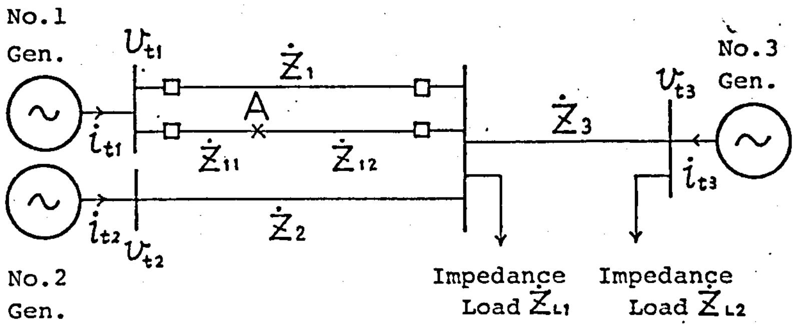


Fig.3-4-1 Model of 3-machine system

3-4-1. Non-linear First Order Differential Equations of Model System

In this section, the mathematical representations of synchronous machines and transmission network described in section 2-2 and section 2-6 have been used in order to obtain the non-linear differential equations of the model system. For the case of the j-th machine, the machine and control systems equations which represent the large signal performance are given as follows. From eqn.(2-26), eqn.(2-27), eqn.(2-29), eqn.(2-32) and eqn.(2-33):

$$pE'_{qj} = \{ E_{fdj} - E'_{qj} - (x_{dj} - x'_{dj}) \cdot i_{dj} \} / T'_{doj} \tag{3-26}$$

$$pE'_{dj} = \{ \qquad - E'_{dj} + (x_{qj} - x'_{qj}) \cdot i_{qj} \} / T'_{qoj} \tag{3-27}$$

$$p\delta_j = \Delta\omega_j \tag{3-28}$$

$$p\Delta\omega_j = (P_{tj} - P_{ej} - P_{dj} \cdot \Delta\omega_j) / M_j \tag{3-29}$$

where, $E_{fdj} = E_{fdjo} + \Delta E_{fdj}$, $P_{tj} = P_{tjo} + \Delta P_{tj}$, $P_{ej} = v_{dj} \cdot i_{dj} + v_{qj} \cdot i_{qj}$

From eqn.(2-36), the voltage regulator action of j-th machine becomes:

$$pV_{aj} = -K_{fj} \cdot (K_j \cdot \Delta v_{tj} + V_{sj}) / T_{fj} - V_{aj} / T_{fj} \tag{3-30}$$

$$p\Delta E_{fdj} = (V_{aj} - \Delta E_{fdj}) / T_{ej} \tag{3-31}$$

$$pV_{sj} = (K_{sj} \cdot p\Delta E_{fdj} - V_{sj})/T_{sj} \tag{3-32}$$

where, $v_{tj} = \sqrt{v_{dj}^2 + v_{qj}^2}$, $\Delta v_{tj} = v_{tj} - V_{rj}$, $u_{1j} = 0$

From eqn.(2-38), the governor action of j-th machine becomes:

$$p\Delta P_{vj} = (-K_{gj} \cdot \Delta\omega_j/\omega_o - \Delta P_{vj})/T_{gj} \tag{3-33}$$

$$p\Delta P_{tj} = (\Delta P_{vj} - \Delta P_{tj})/T_{hj} \tag{3-34}$$

The terminal voltages and currents required for the connection with the network are described in vector form from eqn.(2-28) as follows:

$$\mathbb{E}'_{dj} = \mathbb{V}_{dj} - \mathbb{X}'_{qj} \cdot \mathring{\mathbb{I}}_{dj} \tag{3-35}$$

where, $\mathbb{V}_{dj} = \begin{bmatrix} v_{dj} \\ v_{qj} \end{bmatrix}$, $\mathring{\mathbb{I}}_{dj} = \begin{bmatrix} i_{dj} \\ i_{qj} \end{bmatrix}$, $\mathbb{X}'_{qj} = \begin{bmatrix} r_j , & x'_{qj} \\ -x'_{dj} , & r_j \end{bmatrix}$

The behaviour of the entire power system is expressed by one such set of equations described above for each machine together with the terminal constraints imposed by the interconnected network. The interconnected network is described as follows from eqn.(2-64).

$$\mathring{\mathbb{I}}_{dj} = \sum_{K=1}^{n} \mathbb{Y}'_{jK} \cdot \mathbb{V}_{dK} \tag{3-36}$$

From eqn.(3-26)-eqn.(3-36), the behaviour of the model system is described in vector form shown in eqn.(3-25), where the state variable vector $\mathfrak{X}$ and the non-linear functional vector $\mathfrak{f}(\mathfrak{X})$ become as follows:

$$\mathfrak{X} = [\delta_1, \Delta\omega_1, E'_{q1}, E'_{d1}, V_{a1}, E_{fd1}, V_{s1}, \Delta P_{v1}, \Delta P_{t1}, \cdots, \delta_3, \Delta\omega_3, E'_{q3}, E'_{d3}, V_{a3}, E_{fd3}, V_{s3}, \Delta P_{v3}, \Delta P_{t3}]^T \tag{3-37}$$

$$\mathfrak{f}(\mathfrak{X}) = [f_1, f_2, f_3, \cdots\cdots, f_{25}, f_{26}, f_{27}]^T \tag{3-38}$$

$f_1 = p\delta_1$, $f_2 = p\Delta\omega_1$, $f_3 = pE'_{q1}$, $f_4 = pE'_{d1}$

$f_5 = pV_{a1}$, $f_6 = pE_{fd1}$, $f_7 = pV_{s1}$, $f_8 = p\Delta P_{v1}$

$f_9 = p\Delta P_{t1}$, $f_{10} = p\delta_2$, $f_{11} = p\Delta\omega_2$, $f_{12} = pE'_{q2}$

$$f_{13} = pE'_{d2}\ ,\ f_{14} = pV_{a2}\ ,\ f_{15} = p\Delta E_{fd2}\ ,\ f_{16} = pV_{s2}$$
$$f_{17} = p\Delta P_{v2}\ ,\ f_{18} = p\Delta P_{t2}\ ,\ f_{19} = p\delta_3\ ,\ f_{20} = p\Delta\omega_3$$
$$f_{21} = pE'_{q3}\ ,\ f_{22} = pE'_{d3}\ ,\ f_{23} = pV_{a3}\ ,\ f_{24} = p\Delta E_{fd3}$$
$$f_{25} = pV_{s3}\ ,\ f_{26} = p\Delta P_{v3}\ ,\ f_{27} = p\Delta P_{t3}$$

In the computation process, the induced voltages E'_{qj} and E'_{dj} and the difference angle δ_j which result from the machine equations and appear as the integrable variables in the machine equations are considered as the input quantities for the solutions of the transmission network equations (3-35) and (3-36), whereas the terminal voltages v_{dj} and v_{qj} , and terminal currents i_{dj} and i_{qj} are determined as their output quantities from the transmission network.

3-4-2. System Parameters and Initial Condition

The data of the machines, the transmission network and the impedance loads are shown in Table 3-4-1. The data for the machines are taken from Kimbark[(5)] and the typical values are used for the control systems parameters. The equivalent circuit of the model system yields the admittance matrices of the order under the several conditions of the model system described below ; (1) steady state,(2) three-phase to earth fault at the point A in the model system,(3) isolation of the faulted line by circuit breakers. The admittance matrix of each situation is shown in Table 3-4-2.

After load flow calculation of the model system for the given operating condition, the initial conditions of the model system have been determined by the phaser diagram shown in Fig.3-4-2. The initial conditions of the model system are shown in Table 3-4-3.

Table 3-4-1 System data

Machine No.	No.1	No.2	No.3
x_d	1.15	1.15	0.115
x'_d	0.37	0.37	0.037
x''_d	0.24	0.24	0.024
x_q	0.75	0.75	0.075
x'_q	0.75	0.75	0.075
x''_q	0.34	0.34	0.034
T'_{do}	5.60	5.60	5.60
$J(=\omega_0 M)$	5.60	5.60	56.0
K_g	25.0	25.0	95.0
T_g	0.10	0.10	0.10
T_h	0.30	0.30	0.30
K	1.00	1.00	1.00
K_f	5.00	5.00	5.00
K_s	0.007	0.007	0.007
T_f	0.20	0.20	0.20
T_e	0.20	0.20	0.20
T_s	0.30	0.30	0.30

$\dot{Z}_1 = 0.0 + j0.2$, $\dot{Z}_2 = 0.0 + j0.1$
$\dot{Z}_3 = 0.0 + j0.1$, $\dot{Z}_{11} = 0.0 + j0.1$
$\dot{Z}_{12} = 0.0 + j0.3$, $\dot{Z}_{L1} = 0.3 + j0.1$
$\dot{Z}_{L2} = 0.1 + j0.02$

Table 3-4-3 Initial conditions

Machine No.	No.1	No.2	No.3
P_{ej0}	1.00	1.00	9.77
Q_{j0}	0.40	0.58	2.15
v_{tj0}	1.00	0.99	0.97
v_{Dj0}	0.99	0.99	0.97
v_{Qj0}	0.14	0.03	0.00
i_{tj0}	1.08	1.16	10.28
i_{Dj0}	1.05	1.03	10.04
i_{Qj0}	-0.25	-0.55	-2.21
δ_{j0}	-0.91	-1.05	-0.99
E_{fdj0}	1.84	2.01	1.66
E'_{qj0}	1.18	1.24	1.09
E'_{dj0}	0.00	0.00	0.00

Table 3-4-2 Admittance matrices under several system conditions

Admittance matrix in steady state

0.1095 - j4.0511	0.2190 + j1.8978	0.2190 + j1.8978
0.2190 + j1.8978	0.4380 - j6.2043	0.4380 + j3.7956
0.2190 + j1.8978	0.4380 + j3.7956	10.0533 - j8.1275

Admittance matrix during fault

0.0257 - j12.2700	0.1029 + j0.9202	0.1029 + j0.9202
0.1029 + j 0.9202	0.4115 - j6.3193	0.4115 + j3.6807
0.1029 + j 0.9202	0.4115 + j3.6807	10.0269 - j8.2424

Admittance matrices when faulted line is isolated

0.0334 - j2.2383	0.1336 + j1.0468	0.1336 + j1.0468
0.1336 + j1.0468	0.5345 - j5.8129	0.5345 + j4.1871
0.1336 + j1.0468	0.5345 + j4.1871	10.1499 - j7.7360

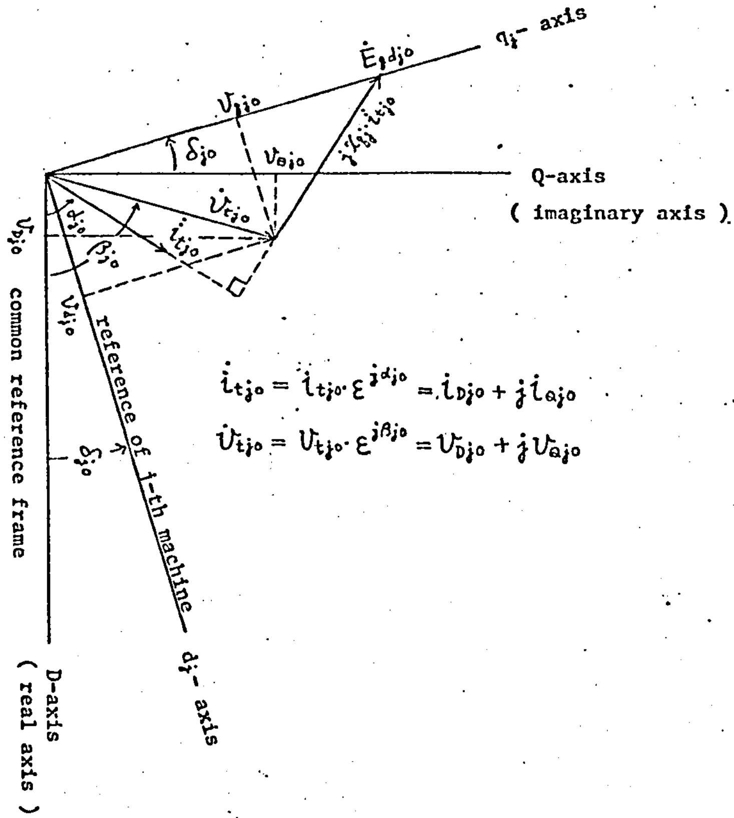


Fig.3-4-2 Phasor diagram for computation of initial conditions

3-4-3. Numerical Results

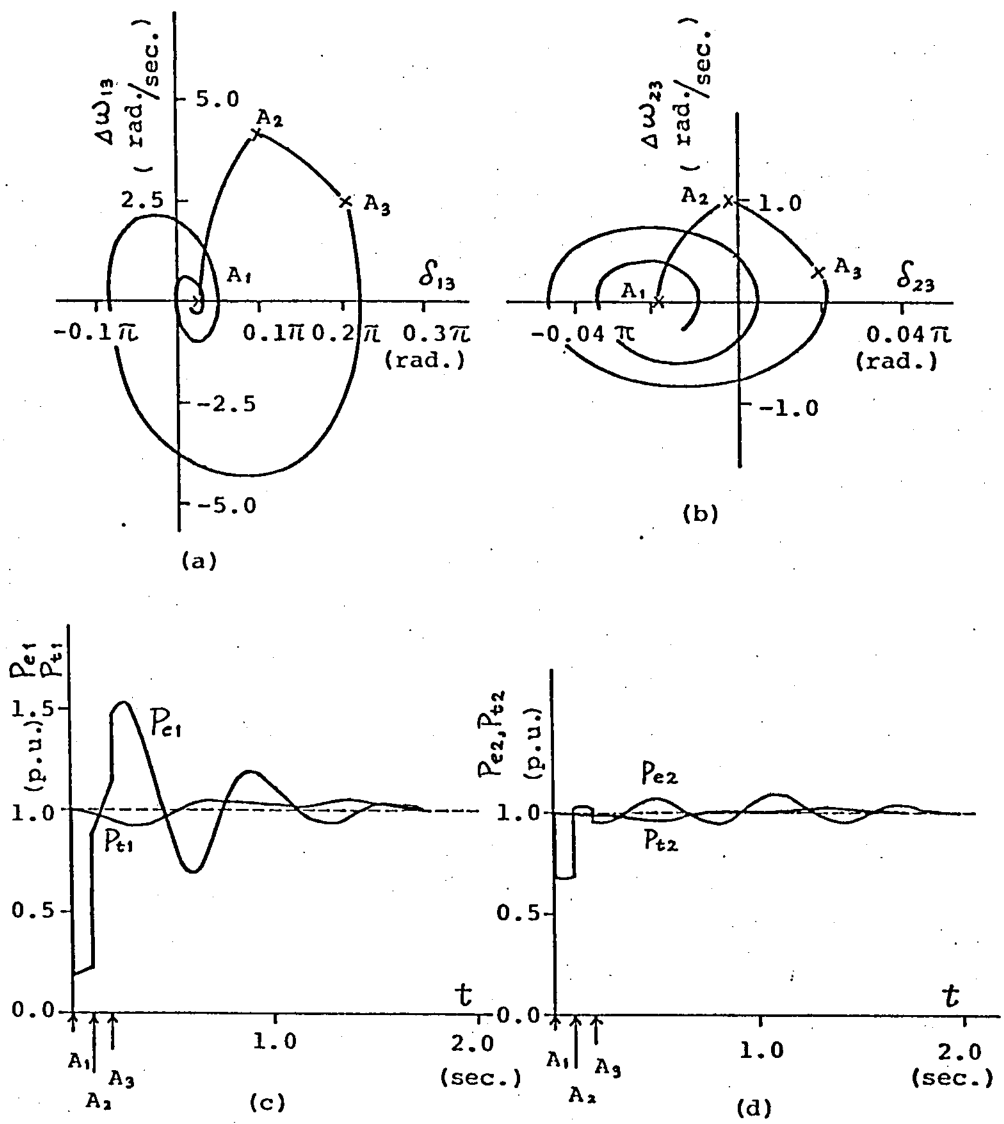
Fig.3-4-3 shows the case that the model system is transient stable under the following system conditions;

(1) The three-phase to ground fault of 0.1 sec. duration occurrs at the point A in the model system at t=0.0 sec..

(2) The faulted line is isolated by the circuit breakers at t=0.1 sec..

(3) The faulted line is reclosed at t=0.2 sec. after clearing the fault from the system.

The admittance matrices of the transmission network have already been shown in Table 3-4-2 for the above three system conditions. In this case, all the system variables converge to their steady state values after clearing the fault as shown in Fig.3-4-3, then the model system is transient stable for the above conditions.



A_1: t = 0.0 sec. , A_2: t = 0.1 sec. , A_3: t = 0.2 sec.

Fig.3-4-3 Transient stable case of model system

Fig.3-4-4 shows the case that the model system is transient unstable under the following three system conditions;

(1) The three-phase to ground fault of 0.3 sec. duration occurrs at the point A in the model system at t=0.0 sec..

(2) The faulted line is isolated from the system by the circuit breakers at t=0.3 sec..

(3) The faulted line is reclosed after clearing the fault at t=0.4 sec..

In this case, all the system variables pulsate after clearing the fault as shown in Fig.3-4-4, then the system is transient unstable for the above conditions.

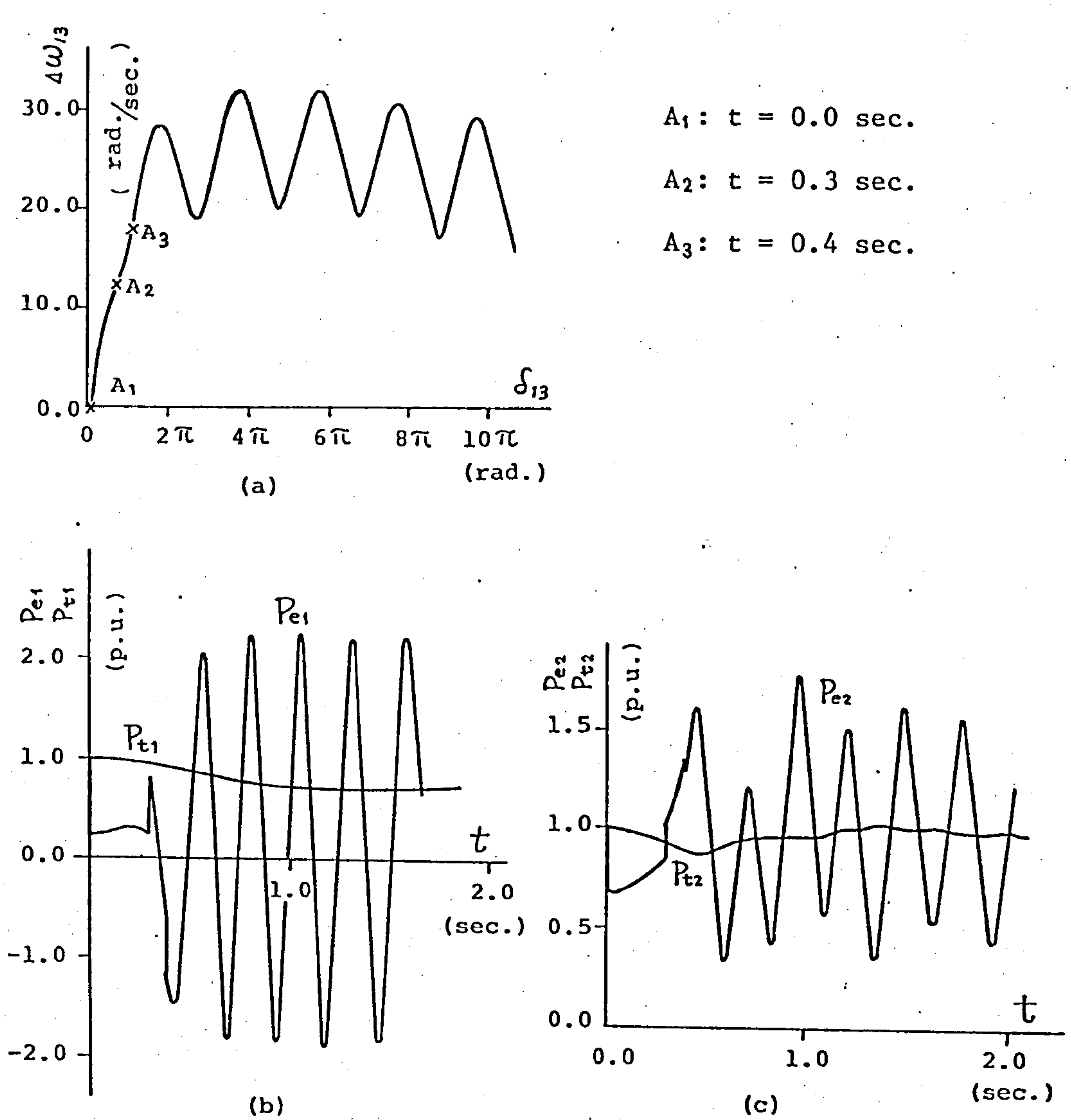


Fig.3-4-4 Transient unstable case of model system

Section 3-5. Summary

In this chapter, the mathematical methods of stability analysis of the power system and the applications of these methods to the model system have been represented.

The methods have the following advantages.

(1) The methods are not limited to one machine or pairs of machines, but can handle a number of machines connected to a transmission network of any form. The methods are limited only by the memory capacity of the digital computer used in those implementation.

(2) The methods use the model of a round-rotor machine or the model of a salient-pole machine and include the governors and the voltage regulators actions. Further, the methods allow the inclusion of any alternative governor or voltage regulator that acts continuously.

(3) For the case of the dynamic stability analysis, the state space of eqn.(3-1) enables the use of any technique of modern multivariable linear system theory.

In later chapters, the methods represented in this chapter are used in order to investigate the performance of the given system.

CHAPTER 4 IMPROVEMENT OF DYNAMIC STABILITY BY STATE FEEDBACK CONTROL

In the most general case, the dynamical system can be represented by the non-linear differential equations and the implementation of optimal controls, determined directly from this non-linear model through a standard optimizing procedure, is extremely difficult. However, for the case of transient involving small disturbances, it is possible to linearize the original non-linear system about the operating point.

In this chapter, the disturbances in the system are assumed to be sufficiently small, then the optimization of synchronous machine performance has been considered by minimizing the quadratic performance index in both system variables and control variables. In this approach, the linearized equations of machine are considered and the control law consisting of constant feedback coefficients of the state variables of the system has been derived.[29],[30],[31] Satisfactory performance of the machine around the selected operating point has thereby been obtained.

Section 4-1. Determination of Optimal State Feedback Controller for Linearized System

In this chapter, it is assumed that the system disturbances are sufficiently small and all the state variables of the system are measurable.

Here, we consider the linearized system described by the following equation :

$$p\Delta \mathbb{X} = \mathbb{A}\cdot\Delta \mathbb{X} + \mathbb{B}\cdot \mathbb{u} \tag{4-1}$$

$$\Delta \mathbb{w} = \mathbb{C}\cdot\Delta \mathbb{X} \tag{4-2}$$

where, $\Delta \mathbb{X}$: n-th order state variables vector

$\Delta \mathbb{w}$: m-th order output variables vector

$\mathbb{u}$: r-th order control variables vector

$\mathbb{A}, \mathbb{B}, \mathbb{C}$: $(n \times n)$, $(n \times r)$ and $(m \times n)$ coefficient matrices of the system

As the cost functional of the system described by eqn.(4-1) and eqn.(4-2), the following quadratic performance index J is chosen:[32]

$$J = \frac{1}{2}\int_0^\infty (\Delta \mathbb{w}^T \cdot \mathbb{Q}_w \cdot \Delta \mathbb{w} + \mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u})\,dt \tag{4-3}$$

From eqn.(4-2), the above equation is rewritten as:

$$J = \frac{1}{2}\int_0^\infty (\Delta \mathbb{x}^T \cdot \mathbb{Q} \cdot \Delta \mathbb{x} + \mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u})\,dt \tag{4-4}$$

where, $\mathbb{Q} = \mathbb{C}^T \cdot \mathbb{Q}_w \cdot \mathbb{C}$

In the above two equations, the matrix $\mathbb{Q}_w$ and the matrix $\mathbb{Q}$ are positive definite or positive semi-definite and the matrix $\mathbb{R}$ is positive definite.

The optimal control vector $\mathbb{u}$, which minimizes the quadratic performance index described by eqn.(4-3) or eqn.(4-4), becomes:[33],[34]

$$\mathbb{u} = -\mathbb{F} \cdot \Delta \mathbb{x} \quad , \quad \mathbb{F} = \mathbb{R}^{-1} \cdot \mathbb{B}^T \cdot \mathbb{K} \tag{4-5}$$

where, the matrix $\mathbb{K}$ is the solution matrix of the following matrix Riccati equation and becomes positive definite symmetric, if the original system described by eqn.(4-1) is completely controllable.

$$\mathbb{A}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{A} - \mathbb{K} \cdot \mathbb{B} \cdot \mathbb{R}^{-1} \cdot \mathbb{B}^T \cdot \mathbb{K} + \mathbb{Q} = \mathbb{0} \tag{4-6}$$

By the optimal control described by eqn.(4-5), the value of the cost functional J becomes:

$$J = \frac{1}{2} \Delta \mathbb{x}^T \cdot \mathbb{K} \cdot \Delta \mathbb{x} \Big|_{t=0} \tag{4-7}$$

Section 4-2. Stability of Closed-loop System

By the above optimal control described by eqn.(4-5), the original system described by eqn.(4-1) becomes:

$$p\Delta\mathbb{X} = (\mathbb{A} - \mathbb{B}\cdot\mathbb{F})\cdot\Delta\mathbb{X} \qquad (4\text{-}8)$$

The stability of this closed-loop system is determined by directly computing the characteristic roots of the closed-loop system matrix ($\mathbb{A} - \mathbb{B}\cdot\mathbb{F}$) as described in section 3-1-1. Furthermore, the stability of this closed-loop system is also determined by the solution matrix $\mathbb{L}$ of the following Lyapunov's matrix equation[23] of the closed-loop system.

$$(\mathbb{A} - \mathbb{B}\cdot\mathbb{F})^T\cdot\mathbb{L} + \mathbb{L}\cdot(\mathbb{A} - \mathbb{B}\cdot\mathbb{F}) = -\mathbb{N} \qquad (4\text{-}9)$$

As described in section 3-1-3, if the closed-loop system described by eqn.(4-8) is asymptotically stable, the matrix $\mathbb{L}$ becomes positive definite with the matrix $\mathbb{N}$ being arbitrary chosen to be positive definite, and the following relationship is satisfied.

$$I = \int_0^\infty \Delta\mathbb{X}^T\cdot\mathbb{N}\cdot\Delta\mathbb{X}\,dt = \Delta\mathbb{X}^T\cdot\mathbb{L}\cdot\Delta\mathbb{X}\Big|_{t=0} \qquad (4\text{-}10)$$

Furthermore, the expected value of I becomes:

$$\hat{I} = \frac{1}{n}\,Tr(\mathbb{L}) \qquad (4\text{-}11)$$

where, n is the order of state variables.

From eqn.(4-11), for the smaller value of Tr($\mathbb{L}$), the closed-loop system is more stable.

As described in section 3-1-2, the exact solution of the closed-loop system described by eqn.(4-8) becomes:

$$\Delta\mathbb{X}(t) = \mathcal{E}^{(\mathbb{A} - \mathbb{B}\cdot\mathbb{F})t}\cdot\Delta\mathbb{X}(0) \qquad (4\text{-}12)$$

where, $\mathcal{E}^{(\mathbb{A} - \mathbb{B}\cdot\mathbb{F})t}$ is the state transition matrix of the closed-loop system. The system responses are obtained by solving the eqn.(4-12) by the recursive formulas for digital computer.

Section 4-3. Application to a One-machine Problem

The power system under investigation consists of a synchronous machine unit connected to an infinite bus through a transmission line as shown in Fig.4-3-1.

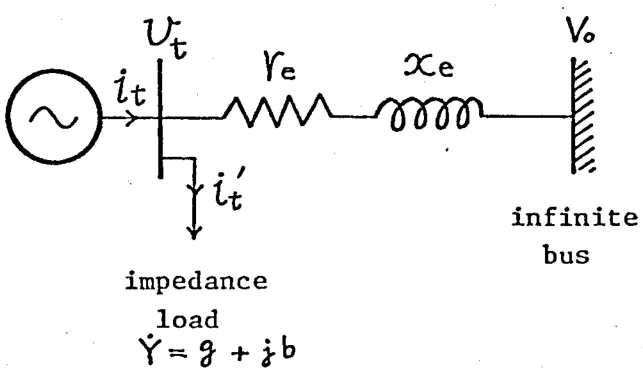


Fig.4-3-1 Model of one machine system

It has both voltage regulator and speed governor. The models shown in Fig. 2-3-2(b) and Fig.2-4-2(b) are used for these control systems, and the additional control signals u_1 and u_2 are determined to minimize the given performance index of the model system.

4-3-1. Linearized Equations of Model System

The mathematical representation of synchronous machine described in section 2-1 is used in order to obtain the linearized equation of the model system. At first, the synchronous machine equations (2-1)-(2-10) are re-arranged and linearized as follows around the operating point.

$$p\Delta\Psi_{fd} = \omega_o \cdot \left\{ \frac{r_{fd}}{x_{ad}} \cdot \Delta E_{fd} + \frac{r_{fd}}{x_{f\ell}} \cdot (\Delta\Psi_{ad} - \Delta\Psi_{fd}) \right\} \quad (4\text{-}13)$$

$$p\Delta\Psi_d = \omega_o \cdot \left\{ \Delta v_d + \Delta\Psi_q + \Psi_{qo} \cdot \frac{\Delta\omega}{\omega_o} + \frac{r}{x_{a\ell}} \cdot (\Delta\Psi_{ad} - \Delta\Psi_d) \right\} \quad (4\text{-}14)$$

$$p\Delta\Psi_{kd} = \omega_o \cdot \frac{r_{kd}}{x_{kd\ell}} \cdot (\Delta\Psi_{ad} - \Delta\Psi_{kd}) \quad (4\text{-}15)$$

$$p\Delta\Psi_q = \omega_o \cdot \left\{ \Delta v_q - \Delta\Psi_d - \Psi_{do} \cdot \frac{\Delta\omega}{\omega_o} + \frac{r}{x_{a\ell}} \cdot (\Delta\Psi_{aq} - \Delta\Psi_q) \right\} \quad (4\text{-}16)$$

$$p\Delta\Psi_{kq} = \omega_o \cdot \frac{r_{kq}}{x_{kq\ell}} \cdot (\Delta\Psi_{aq} - \Delta\Psi_{kq}) \quad (4\text{-}17)$$

where, $\Delta i_d = (\Delta\Psi_{ad} - \Delta\Psi_d)/x_{a\ell}$, $\Delta i_q = (\Delta\Psi_{aq} - \Delta\Psi_q)/x_{a\ell}$

$$\Delta\Psi_{ad} = \left(\frac{\Delta\Psi_d}{x_{al}} + \frac{\Delta\Psi_{fd}}{x_{fl}} + \frac{\Delta\Psi_{kd}}{x_{kdl}}\right)/K_1 , \quad \Delta\Psi_{aq} = \left(\frac{\Delta\Psi_q}{x_{al}} + \frac{\Delta\Psi_{kq}}{x_{kql}}\right)/K_2$$

$$K_1 = \frac{1}{x_{ad}} + \frac{1}{x_{al}} + \frac{1}{x_{fl}} + \frac{1}{x_{kdl}} , \quad K_2 = \frac{1}{x_{aq}} + \frac{1}{x_{al}} + \frac{1}{x_{kql}}$$

From eqn.(2-13), eqn(2-16) and eqn.(2-17):

$$p\Delta\delta = \Delta\omega \tag{4-18}$$

$$p\Delta\omega = (\Delta P_t - P_d\cdot\Delta\omega - \Delta P_q)/M \tag{4-19}$$

where, $\Delta P_q = \Psi_{do}\cdot\Delta i_q - \Psi_{qo}\cdot\Delta i_d + i_{qo}\cdot\Delta\Psi_d - i_{do}\cdot\Delta\Psi_q$

From eqn.(2-35) and eqn.(2-38) the control systems equations are rewritten as follows:

$$p\Delta E_{fd} = \frac{K_f}{T_f}\cdot(-\Delta v_t - V_s) - \frac{1}{T_f}\cdot\Delta E_{fd} + \frac{K_f}{T_f}\cdot u_1 \tag{4-20}$$

$$pV_s = \frac{K_s}{T_s}\cdot p\Delta E_{fd} - \frac{1}{T_s}\cdot V_s \tag{4-21}$$

$$p\Delta P_v = -\frac{K_g}{T_g}\cdot\frac{\Delta\omega}{\omega_o} - \frac{1}{T_g}\cdot\Delta P_v + \frac{K_g}{T_g}\cdot u_2 \tag{4-22}$$

$$p\Delta P_t = \frac{1}{T_h}\cdot\Delta P_v - \frac{1}{T_h}\cdot\Delta P_t \tag{4-23}$$

where, from eqn.(2-31), $\Delta v_t = \frac{v_{do}}{v_{to}}\cdot\Delta v_d + \frac{v_{qo}}{v_{to}}\cdot\Delta v_q$

Furthermore, the transmission network is represented as follows:[11],[12]

$$\mathring{\mathbb{i}}_d = \mathring{\mathbb{i}}_d' + \mathbb{Y}\cdot\mathbb{v}_d \tag{4-24}$$

$$\mathbb{v}_d = \mathbb{T}(\delta)\cdot\mathbb{V}_o + \mathbb{Z}_e\cdot\mathring{\mathbb{i}}_d' \tag{4-25}$$

where, $\mathbb{v}_d = \begin{bmatrix} v_d \\ v_q \end{bmatrix}$, $\mathbb{V}_o = \begin{bmatrix} 0 \\ V_o \end{bmatrix}$, $\mathring{\mathbb{i}}_d = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$, $\mathring{\mathbb{i}}_d' = \begin{bmatrix} i_d' \\ i_q' \end{bmatrix}$

$\mathbb{Y} = \begin{bmatrix} g, & -b \\ b, & g \end{bmatrix}$, $\mathbb{Z}_e = \begin{bmatrix} r_e, & -x_e \\ x_e, & r_e \end{bmatrix}$, $\mathbb{T}(\delta) = \begin{bmatrix} \cos\delta, & \sin\delta \\ -\sin\delta, & \cos\delta \end{bmatrix}$

For a small perturbation from a fixed operating point, eqn.(4-24) and eqn.(4-25) become:

$$\Delta\mathring{\mathbb{i}}_d = \Delta\mathring{\mathbb{i}}_d' + \mathbb{Y}\cdot\Delta\mathbb{v}_d \tag{4-26}$$

$$\Delta\mathbb{v}_d = \mathbb{T}'(\delta_o)\cdot\mathbb{V}_o\cdot\Delta\delta + \mathbb{Z}_e\cdot\Delta\mathring{\mathbb{i}}_d' \tag{4-27}$$

where, $\Delta \mathcal{V}_d = \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix}$, $\Delta \mathring{i}_d = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}$, $\Delta \mathring{i}_d' = \begin{bmatrix} \Delta i_d' \\ \Delta i_q' \end{bmatrix}$, $\mathbb{T}'(\delta_o) = \begin{bmatrix} -\sin\delta_o, \cos\delta_o \\ -\cos\delta_o, -\sin\delta_o \end{bmatrix}$

In the above equations, the subscript o denotes the steady state value.

Let vectors $\Delta \mathcal{X}$, $\Delta \mathcal{Y}$ and $\mathcal{U}$ be defined as:

$$\Delta \mathcal{X} = [\Delta\Psi_{fd}, \Delta\Psi_d, \Delta\Psi_{kd}, \Delta\Psi_q, \Delta\Psi_{kq}, \Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v, \Delta P_t]^T \quad (4\text{-}28)$$

$$\Delta \mathcal{Y} = [\Delta\Psi_{ad}, \Delta\Psi_{aq}, \Delta i_d, \Delta i_q, \Delta v_d, \Delta v_q]^T \quad (4\text{-}29)$$

$$\mathcal{U} = [u_1, u_2]^T \quad (4\text{-}30)$$

Then, the linearized equation of the model system becomes:

$$p\Delta\mathcal{X} = \mathbb{A}_1 \cdot \Delta\mathcal{X} + \mathbb{A}_2 \cdot \Delta\mathcal{Y} + \mathbb{B} \cdot \mathcal{U} \quad (4\text{-}31)$$

$$\Delta\mathcal{Y} = \mathbb{A}_3 \cdot \Delta\mathcal{X} \quad (4\text{-}32)$$

From above two equations, the linearized equation of the model system becomes of the form shown in eqn.(4-1), and the matrix $\mathbb{A}$ becomes:

$$\mathbb{A} = \mathbb{A}_1 + \mathbb{A}_2 \cdot \mathbb{A}_3 \quad (4\text{-}33)$$

The output vector $\Delta\mathcal{W}$ of the model system is selected as:

$$\Delta\mathcal{W} = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v, \Delta v_t, \Delta i_t]^T \quad (4\text{-}34)$$

Then, $\Delta\mathcal{W} = \mathbb{C}_1 \cdot \Delta\mathcal{X} + \mathbb{C}_2 \cdot \Delta\mathcal{Y}$ (4-35)

From eqn.(4-32) and eqn.(4-35), the matrix $\mathbb{C}$ in eqn.(4-2) becomes:

$$\mathbb{C} = \mathbb{C}_1 + \mathbb{C}_2 \cdot \mathbb{A}_3 \quad (4\text{-}36)$$

The components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$, $\mathbb{B}$ and $\mathbb{C}$ are shown in Table A-2 in appendix.

4-3-2. System Parameters and Initial Conditions

The parameters of the model system are shown in Table 4-3-1. Before the construction of matrix $\mathbb{A}$, it is necessary to find the steady state values of the system variables. After the load flow calculation, the operation angle δ_o is determined by the phasor diagram of Fig.4-3-2. The initial conditions of the model system are shown in Table 4-3-2, where the operating point of the synchronous machine is selected as follows: P_o (active power output or electrical output)=1.0 p.u., Q_o (reactive power output)=-0.5 p.u. and v_{to} (terminal voltage)=1.1 p.u..

Table 4-3-1 System parameters

Machine constants			
r_{fl} =0.00107	r_{Kd} =0.0035	r_{Kq} =0.0035	r =0.002
x_{fl} =0.14	x_{Kdl} =0.04	x_{Kql} =0.04	x_{al} =0.14
x_{ad} =1.86	x_{aq} =1.86	P_d =0.005	
J (= $\omega_o \cdot M$) =6.0 (sec.)			
Control systems constants			
K_f =20.0	T_f =1.0 (sec.)	K_s =0.05	T_s =0.5 (sec.)
K_g =10.0	T_g =0.8 (sec.)	T_h =0.25 (sec.)	
Line constants			
g =0.1	b =-0.05	r_e =0.01	x_e =0.8

Table 4-3-2 Initial conditions

$P_o(P_{eo})$ =1.0	Q_o =-0.5	v_{to} =1.1	θ_o =0.4058 (rad.)
V_o =1.6322	v_{do} =1.0939	v_{qo} =0.1160	i_{do} =0.8561
i_{qo} =0.5478	ψ_{do} =0.1171	ψ_{qo} =-1.0956	δ_o =1.8710 (rad.)
i_{fdo} =0.9835	ψ_{Kdo} =0.2369	ψ_{Kqo} =-1.0189	P_{to} =1.0020
ψ_{fdo} =0.3746	E_{fdo} =1.8292		

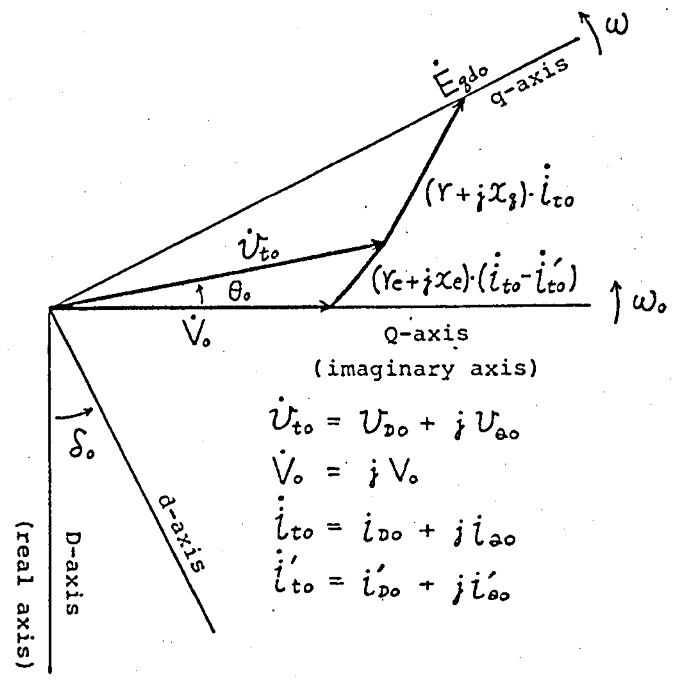


Fig.4-3-2 Phasor diagram for initial values

4-3-3. Numerical Results

For various weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$ shown in Table 4-3-3, the optimal state feedback gain matrix $\mathbb{F}$ of the model system has been determined by the method described in section 4-1.

Table 4-3-3 Weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$

Case	matrix $\mathbb{Q}_w$	matrix $\mathbb{R}$
Case 1	diag.(1,1,1,1,1,1,1)	diag.(1,1)
Case 2	diag.(1,1,1,1,1,1,1)	diag.(0.001,0.001)
Case 3	diag.(10,1,1,1,1,10,10)	diag.(1,1)
Case 4	diag.(10,1,1,1,1,10,10)	diag.(0.1,0.1)
Case 5	diag.(10,1,1,1,1,10,10)	diag.(0.01,0.01)
Case 6	diag.(10,1,1,1,1,10,10)	diag.(0.001,0.001)
Case 7	diag.(100,1,1,1,1,100,100)	diag.(1,1)
Case 8	diag.(100,1,1,1,1,100,100)	diag.(0.1,0.1)
Case 9	diag.(100,1,1,1,1,100,100)	diag.(0.001,0.001)
Case 10	diag.(10,1,1,1,1,10,1)	diag.(0.001,0.001)
Case 11	diag.(1,1,1,1,1,10,1)	diag.(0.001,0.001)
Case 12	diag.(10,1,1,1,1,1,1)	diag.(0.001,0.001)
case 13	without control	

For the model system, the matrix $\mathbb{F}$ becomes a (11×2) matrix as shown in Table 4-3-4.

The closed-loop stability of the model system applied with these optimal controllers has been investigated by various methods described in section 4-2.

The characteristic roots of the closed-loop matrix ($\mathbb{A} - \mathbb{B} \cdot \mathbb{F}$) of the model system are shown in Table 4-3-5.

In this table, the case 13 shows the chracteristic roots of the original uncontrolled system, i.e. the characteristic roots of the matrix $\mathbb{A}$. In this case all the characteristic roots lie on the left half plane of the complex field, namely all the real parts of the characteristic roots are negative, so the original system is stable. The original system is governed by two dominant modes of oscillations; the low frequency rotor oscillation induced by the excitation system ($-0.0934 \pm j0.874$) and the natural rotor oscillation ($-0.761 \pm j9.35$). By the state feedback optimal control the original system is much stabilized, because all the characteristic roots are shifted to the left side apart from the imaginary axis of the complex plane and the low frequency rotor oscillation induced by the excitation system does not appear.

The values of $Tr(\mathbb{L})$ of the closed-loop system are shown in Table 4-3-6 for the given $\mathbb{Q}_w$ and $\mathbb{R}$ matrices. As shown in this table, the original system is much stabilized by the state feedback optimal control, and for the same $\mathbb{Q}_w$ matrix the smaller values of the matrix $\mathbb{R}$ make the system more stable. Furthermore, the case 11 makes the system much more stable among the various cases. In this case, the weight of ΔV_t in the matrix $\mathbb{Q}_w$ is emphasized.

For the above calculation, the positive definite matrix $\mathbb{N}$ in the Lyapunov's matrix equation (4-9) of the model system is selected as the (11×11) unit matrix.

Table 4-3-4 Feedback gain matrices $\mathbb{F}^T$

	Case 1		Case 2		Case 3		Case 4	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\Psi_{fd}$	0.468	-4.971	13.181	-90.995	0.951	-5.389	2.929	-11.825
$\Delta\Psi_{d}$	0.036	-0.031	1.271	-1.223	0.036	-0.047	0.129	-0.159
$\Delta\Psi_{kd}$	0.183	-5.052	6.084	-138.785	0.335	-6.216	1.119	-17.511
$\Delta\Psi_{q}$	0.005	0.060	-0.294	4.046	0.005	0.109	0.008	0.473
$\Delta\Psi_{kq}$	0.480	3.850	-2.721	91.948	0.554	5.396	0.563	14.708
$\Delta\delta$	0.474	-7.560	8.337	-167.893	0.482	-6.812	1.155	-15.385
$\Delta\omega$	-0.0004	0.411	-0.409	24.470	0.0006	0.700	-0.028	2.878
ΔE_{fd}	0.955	-0.029	31.582	-0.033	0.962	-0.031	3.124	-0.031
V_s	-0.772	-0.048	0.532	0.028	-0.782	0.025	-0.798	-0.006
ΔP_v	-0.054	1.965	-0.047	32.825	-0.054	2.163	-0.051	4.518
ΔP_t	-0.194	5.089	-4.818	131.317	-0.193	6.430	-0.530	17.679

	Case 5		Case 6		Case 7		Case 8	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\Psi_{fd}$	9.130	-30.042	28.772	-86.773	3.577	-5.985	11.326	-10.210
$\Delta\Psi_{d}$	0.420	-0.506	1.170	-1.598	0.034	-0.100	0.119	-0.301
$\Delta\Psi_{kd}$	3.534	-50.244	11.268	-152.071	1.172	-9.490	4.091	-22.569
$\Delta\Psi_{q}$	-0.024	1.637	-0.495	5.312	0.012	0.284	0.012	0.961
$\Delta\Psi_{kq}$	0.430	42.277	0.223	128.542	1.107	10.518	2.365	28.143
$\Delta\delta$	3.208	-37.836	9.714	-106.571	0.716	-3.580	2.366	2.680
$\Delta\omega$	-0.096	9.787	-0.298	31.593	0.034	1.707	0.083	5.622
ΔE_{fd}	9.964	-0.030	31.594	-0.031	0.993	-0.026	3.156	-0.022
V_s	-0.522	0.005	0.465	0.024	-0.823	-0.010	-0.837	-0.003
ΔP_v	-0.048	11.401	-0.046	33.031	-0.044	2.686	-0.036	5.067
ΔP_t	-1.499	50.428	-4.554	152.537	-0.095	10.549	-0.265	26.082

	Case 9		Case 10		Case 11		Case 12	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\Psi_{fd}$	113.052	-54.255	15.853	-107.986	15.198	-98.706	14.972	-105.057
$\Delta\Psi_{d}$	0.200	-2.828	1.245	-1.438	1.239	-1.295	1.280	-1.396
$\Delta\Psi_{kd}$	43.793	-170.377	7.677	-155.972	7.053	-142.848	7.238	-154.353
$\Delta\Psi_{q}$	-2.600	9.592	-0.434	4.813	-0.407	4.158	-0.334	4.769
$\Delta\Psi_{kq}$	20.008	248.473	-4.897	108.391	-4.878	105.198	-3.120	99.154
$\Delta\delta$	24.382	116.936	5.661	-128.778	7.024	-161.632	6.237	-131.330
$\Delta\omega$	0.687	55.215	-0.706	28.965	-0.547	25.140	-0.650	28.703
ΔE_{fd}	31.651	-0.018	31.584	-0.042	31.584	-0.039	31.583	-0.040
V_s	0.184	0.008	0.522	0.058	0.523	0.055	0.526	0.049
ΔP_v	-0.028	33.588	-0.058	32.957	-0.053	32.845	-0.056	32.950
ΔP_t	-2.607	210.712	-5.758	144.947	-5.251	133.404	-5.594	144.186

Table 4-3-5 Characteristic roots

Case 1	Case 2	Case 3
-0.335 $-0.196 \times 10 \pm j0.197$ -0.197×10 $-0.640 \times 10 \pm j0.130 \times 10^{2}$ -0.103×10^{2} -0.157×10^{2} -0.205×10^{2} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.330 $-0.197 \times 10 \pm j0.126$ -0.199×10 $-0.952 \times 10 \pm j0.132 \times 10^{2}$ -0.103×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.496 -0.195×10 -0.224×10 -0.431×10 $-0.636 \times 10 \pm j0.133 \times 10^{2}$ -0.104×10^{2} -0.155×10^{2} -0.205×10^{2} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$
Case 4	**Case 5**	**Case 6**
-0.491 -0.199×10 -0.224×10 -0.431×10 $-0.890 \times 10 \pm j0.137 \times 10^{2}$ -0.103×10^{2} -0.400×10^{2} -0.637×10^{2} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.490 -0.199×10 -0.223×10 -0.431×10 $-0.941 \times 10 \pm j0.135 \times 10^{2}$ -0.103×10^{2} -0.125×10^{3} -0.201×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.490 -0.199×10 -0.223×10 -0.431×10 $-0.946 \times 10 \pm j0.135 \times 10^{2}$ -0.103×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$
Case 7	**Case 8**	**Case 9**
-0.111×10 -0.192×10 -0.221×10 $-0.646 \times 10 \pm j0.147 \times 10^{2}$ -0.964×10 $-0.135 \times 10^{2} \pm j0.251 \times 10$ -0.205×10^{2} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.109×10 -0.198×10 -0.222×10 $-0.889 \times 10 \pm j0.156 \times 10^{2}$ -0.960×10 -0.119×10^{2} -0.399×10^{3} -0.637×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.109×10 -0.199×10 -0.222×10 $-0.954 \times 10 \pm j0.154 \times 10^{2}$ -0.959×10 -0.118×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$
Case 10	**Case 11**	**Case 12**
-0.503 -0.199×10 -0.218×10 -0.333×10 $-0.948 \times 10 \pm j0.134 \times 10^{2}$ -0.103×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.447 -0.199×10 $-0.204 \times 10 \pm j0.518$ $-0.952 \times 10 \pm j0.132 \times 10^{2}$ -0.103×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	-0.477 -0.199×10 -0.204×10 -0.339×10 $-0.948 \times 10 \pm j0.134 \times 10^{2}$ -0.103×10^{2} -0.395×10^{3} -0.636×10^{3} $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$
Case 13		
$-0.934 \times 10^{-1} \pm j0.874$ $-0.761 \pm j0.935 \times 10$ -0.123×10 -0.187×10 -0.409×10 -0.457×10 -0.999×10 $-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$		

Table 4-3-6 Values of Tr($\mathbb{L}$)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case10	Case11	Case12	Case13
L(1,1)	5.5526	4.9470	4.8762	4.5271	4.4282	4.3966	7.6988	7.7833	7.8071	4.4710	4.5412	4.5218	73.521
L(2,2)	0.0036	0.0035	0.0036	0.0035	0.0035	0.0035	0.0036	0.0036	0.0036	0.0035	0.0035	0.0035	0.0045
L(3,3)	2.6673	1.7134	2.4187	1.8639	1.7039	1.6531	2.5121	2.2081	2.0320	1.6795	1.6609	1.6762	17.764
L(4,4)	0.0063	0.0058	0.0064	0.0061	0.0060	0.0060	0.0068	0.0068	0.0070	0.0059	0.0058	0.0059	0.0143
L(5,5)	1.2851	0.9614	1.3641	1.2071	1.1844	1.1180	1.7759	1.9379	2.1362	1.0345	1.0407	0.9856	53.154
L(6,6)	4.8021	3.5330	5.0004	4.1007	3.8537	3.7782	5.6332	5.5718	5.9760	3.6350	3.4893	3.6106	30.476
L(7,7)	0.1066	0.0875	0.1070	0.0961	0.0924	0.0913	0.1184	0.1185	0.1238	0.0891	0.0870	0.0888	0.3619
L(8,8)	0.0284	0.0033	0.0279	0.0104	0.0049	0.0033	0.0265	0.0103	0.0033	0.0033	0.0033	0.0033	2.4891
L(9,9)	0.2596	0.2495	0.2564	0.2496	0.2495	0.2495	0.2519	0.2495	0.2496	0.2495	0.2495	0.2495	56.946
L(10,10)	0.0517	0.0013	0.0406	0.0120	0.0039	0.0013	0.0270	0.0104	0.0012	0.0012	0.0013	0.0013	17.747
L(11,11)	1.9154	1.0737	1.7668	1.2832	1.1152	1.1112	1.7033	1.4801	1.3781	1.0883	1.0683	1.0854	14.935
Tr($\mathbb{L}$)	16.679	12.579	15.868	13.360	12.646	12.412	19.758	19.380	19.718	12.261	12.151	12.231	267.41

The responses of the model system variables have been obtained by solving the eqn.(4-12) of the model system. In Fig.4-3-4 the responses of the system applied with the optimal controllers and the uncontrolled system are shown for the given initial deviations $\Delta\delta|_{t=0}= 0.5$ and $\Delta\Psi_{fd}|_{t=0}= 0.5$.

The damping characteristic of the system is much improved by the state feedback optimal controllers, and the emphasis of the weights of ΔV_t and Δi_t of the matrix $\mathbb{Q}_w$ restrains the variations of these variables. This fact is much useful in choosing the weighting matrix $\mathbb{Q}_w$, namely the variations of the much weighted variables are restrained by the control.

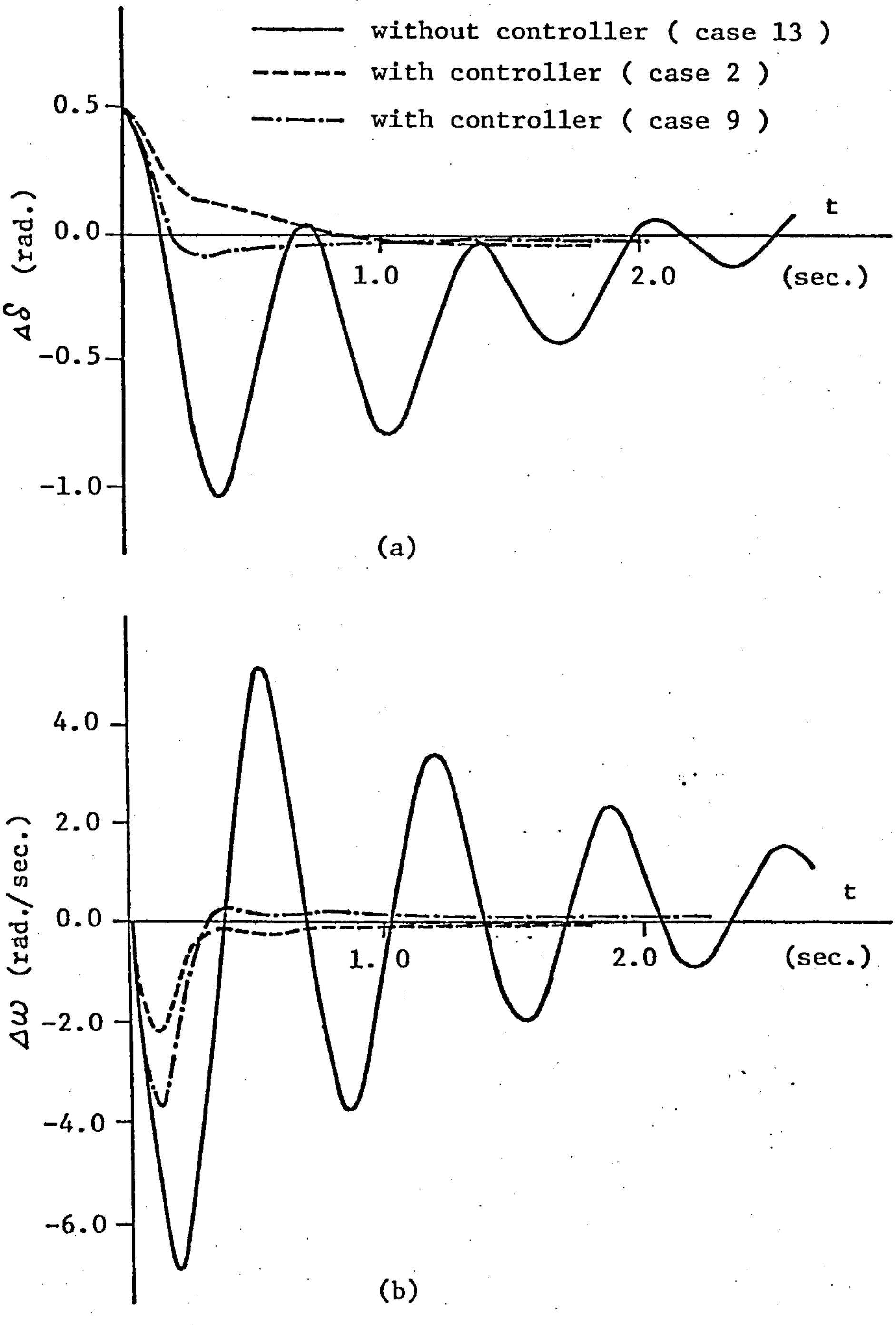
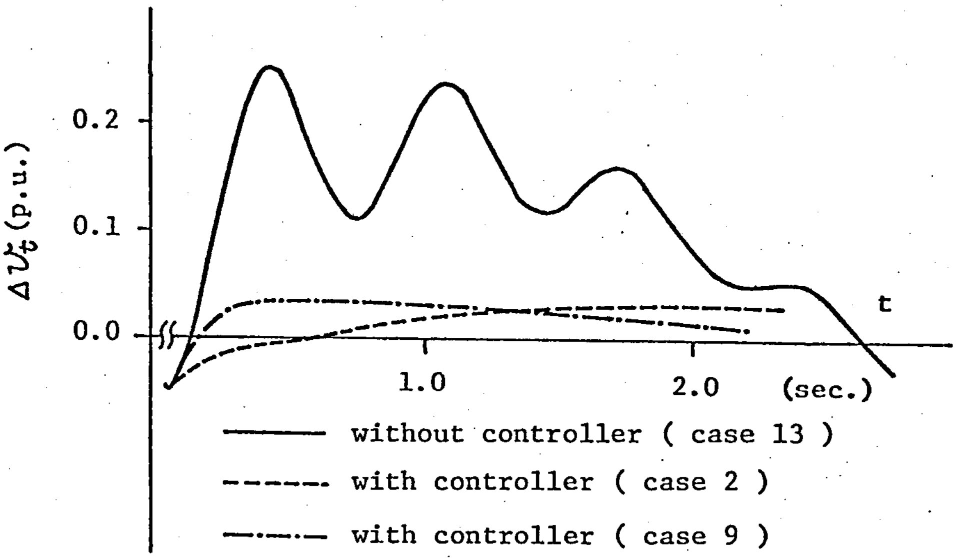
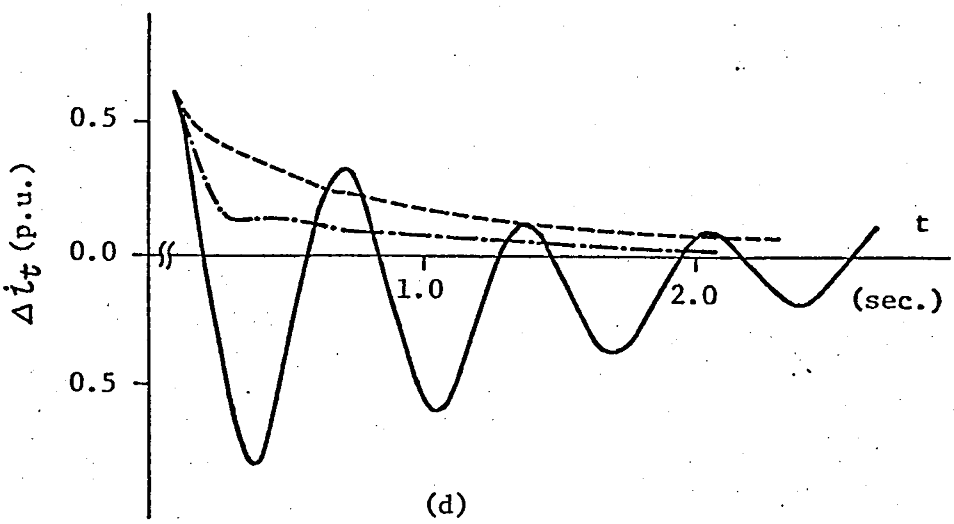


Fig.4-3-3 System responses for the initial deviations

$\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$



(c)



(d)

Fig.4-3-3 System responses for the initial deviations

$\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$

Section 4-4. Summary

The optimal control theory of linear system has been applied to a model one machine infinite bus system. The original uncontrolled system can be much stabilized by the state feedback controller, and the appropriate selection of the weighting matrices of the cost functional makes the system more stable. Furthermore, the variations of much weighted variables are restrained by the controller.

There are more works remaining to be done in applying the optimal control theory to the power systems with system non-linearities and control constraints and so on. The studies about these problems will be shown in later chapters. Furthermore, in this chapter, the control signals are represented by the linear function of all state variables, namely the state feedback controller obtained in this chapter requires the complete measurement of the system states, and for especially large scale power systems it is almost impossible to have all the informations about system states, so it is also necessary to design the controllers applied with the only measurable states of the system.

CHAPTER 5 IMPROVEMENT OF DYNAMIC STABILITY BY OUTPUT FEEDBACK CONTROL

The optimal control of power system dynamics has become a popular subjects since Yu, Vongsuriya and Wedman first introduced the optimal control theory to the power system stability problem, however it has not yet been used in practical power systems. The main difficulty is that all the state variables required for the controller are not directly measurable.

Luenberger's observer[35],[36],[37],[38] can be constructed to estimate the unmeasurable states from informations available but the addition of a dynamical observer of high order will make the overall controlled system more complex and unduly sensitive to system disturbances and changes of system parameters.

An alternative method is to design a output feedback controller using only the directly measurable states of the system, but it will never be as good as all-state feedback optimal control, namely the controller becomes a suboptimal controller for the system.

In this chapter, in order to construct a output feedback controller for the power system in terms of the directly measurable output variables of the system, the model reduction techniques[37] are applied. In other words, the output feedback controller obtained is physically realizable and can easily implemented.

Section 5-1. Determination of Output Feedback Controller for Linearized System using Matrix Riccati Equation

In this chapter, it is assumed that the system disturbances are sufficiently small, and the original non-linear system is linearized around the operating point. As shown in eqn.(4-1) and eqn.(4-2), the linearized system equation becomes:

$$p\Delta x = \mathbb{A} \cdot \Delta x + \mathbb{B} \cdot u \qquad (5\text{-}1)$$

$$\Delta \mathbb{w} = \mathbb{C} \cdot \Delta \mathbb{x} \qquad (5\text{-}2)$$

where, $\Delta \mathbb{x}$: n-th order state variables vector

$\Delta \mathbb{w}$: m-th order output variables vector

$\mathbb{u}$: r-th order control signals vector

$\mathbb{A}, \mathbb{B}, \mathbb{C}$: $(n \times n)$, $(n \times r)$ and $(m \times n)$ coefficient matrices of the system

Eqn.(5-1) and eqn.(5-2) describe the n-th order linear system. In general the number of output variables m is smaller than the number of state variables n , then the inverse matrix of matrix $\mathbb{C}$ does not exist.

In order to construct the output feedback controller of the system, the system order is reduced to the same order of the output variables by the model reduction techniques described later as follows:

$$p \Delta \mathbb{x}_s = \mathbb{A}_s \cdot \Delta \mathbb{x}_s + \mathbb{B}_s \cdot \mathbb{u} \qquad (5\text{-}3)$$

$$\Delta \mathbb{w} = \mathbb{C}_s \cdot \Delta \mathbb{x}_s \qquad (5\text{-}4)$$

where, $\Delta \mathbb{x}_s$: m-th order state variables vector of the reduced system

$\Delta \mathbb{w}$: m-th order output variables vector

$\mathbb{u}$: r-th order control signals vector

$\mathbb{A}_s, \mathbb{B}_s, \mathbb{C}_s$: $(m \times m)$, $(m \times r)$ and $(m \times m)$ coefficient matrices of the reduced system

In the reduced order system, the inverse matrix of matrix $\mathbb{C}_s$ exists.

As the cost functional of the reduced system described by eqn.(5-3) and eqn.(5-4), the same quadratic performance index J shown in eqn.(4-3) is chosen:

$$J = \frac{1}{2}\int_0^\infty (\Delta \mathbb{w}^T \cdot \mathbb{Q}_w \cdot \Delta \mathbb{w} + \mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u})\, dt \qquad (5\text{-}5)$$

From eqn.(5-4), the above equation becomes:

$$J = \frac{1}{2}\int_0^\infty (\Delta x_s^T \cdot Q_s \cdot \Delta x_s + u^T \cdot R \cdot u)\,dt \qquad (5\text{-}6)$$

where, $Q_s = C_s^T \cdot Q_w \cdot C_s$

In the above two equations, the matrix Q_w and the matrix Q_s are positive definite or positive semi-definite ($m \times m$) matrices and the matrix R is a positive definite ($r \times r$) matrix.

The optimal control vector u for the reduced system, which minimizes the performance index described by eqn.(5-5) or eqn.(5-6), becomes:

$$u = -F_s \cdot \Delta x_s \quad , \quad F_s = R^{-1} \cdot B_s^T \cdot K_s \qquad (5\text{-}7)$$

As described in section 4-1, the matrix K_s is the m-th order solution matrix of the following matrix Riccati equation for the reduced system.

$$A_s^T \cdot K_s + K_s \cdot A_s - K_s \cdot B_s \cdot R^{-1} \cdot B_s^T \cdot K_s + Q_s = 0 \qquad (5\text{-}8)$$

From eqn.(5-4) and eqn.(5-7), the output feedback controller becomes:

$$u = -F_w \cdot \Delta w \quad , \quad F_w = F_s \cdot C_s^{-1} \qquad (5\text{-}9)$$

By the control described by eqn.(5-7) or eqn.(5-9), the value of the cost functional J for the reduced system becomes:

$$J = \frac{1}{2} \Delta x_s^T \cdot K_s \cdot \Delta x_s \Big|_{t=0} \qquad (5\text{-}10)$$

Section 5-2. Determination of Output Feedback Controller for Linearized System using Lyapunov's Matrix Equation

In this section, instead of the solution of the matrix Riccati equation, the solution of the Lyapunov's matrix equation of the reduced system is used to determine the output feedback controller of the system.

It is well known that besides providing stability information, Lyapunov's direct method is also effective[23],[40] in formulating the solution to the control problem. The value of the Lyapunov function for the system is an important measure of the system stability and represents the distance from the steady state equilibrium point, and by minimizing the ratio of the derivative of the Lyapunov function with respect to time to the Lyapunov function, the system performance is improved.

The system to be considered is described by eqn.(5-3) and eqn.(5-4). Here it is assumed that, under a control $\mathbb{U} = \mathbb{0}$, the transient process of the system will be asymptotically stable. The vector $\mathbb{U}$ is selected as to decrease the value of the functional:

$$I(\mathbb{u}) = \int_0^{\infty} (\Delta \mathbb{X}_s^T \cdot \mathbb{Q}_s \cdot \Delta \mathbb{X}_s)\, dt \qquad (5\text{-}11)$$

This functional becomes a performance index for the reduced system described by eqn.(5-3) and eqn.(5-4), and the matrix $\mathbb{Q}_s$ is a positive definite or positive semi-definite $(m \times m)$ matrix. The Lyapunov function can be chosen for the reduced system as follows:

$$V(\Delta \mathbb{X}_s) = \Delta \mathbb{X}_s^T \cdot \mathbb{K}_s \cdot \Delta \mathbb{X}_s \qquad (5\text{-}12)$$

where, the matrix $\mathbb{K}_s$ is the positive definite $(m \times m)$ solution matrix of the following Lyapunov's matrix equation for the reduced system.

$$\mathbb{A}_s^T \cdot \mathbb{K}_s + \mathbb{K}_s \cdot \mathbb{A}_s = -\mathbb{Q}_s \qquad (5\text{-}13)$$

The time derivative of the above Lyapunov function along the trajectory represented by eqn.(5-3) becomes:

$$\frac{dV}{dt} = -\Delta \mathbb{X}_s^T \cdot \mathbb{Q}_s \cdot \Delta \mathbb{X}_s + 2 \cdot \mathbb{u}^T \cdot \mathbb{B}_s^T \cdot \mathbb{K}_s \cdot \Delta \mathbb{X}_s \qquad (5\text{-}14)$$

And the following relationship is satisfied.

$$I(\mathbb{u}) = V(\Delta \mathbb{X}_s)\Big|_{t=0} + 2 \int_0^{\infty} \mathbb{u}^T \cdot \mathbb{B}_s^T \cdot \mathbb{K}_s \cdot \Delta \mathbb{X}_s\, dt \qquad (5\text{-}15)$$

The control vector $\mathbb{U}$ is required to decrease the value of the performance index $I(\mathbb{u})$, the control vector becomes:

$$\mathbb{U} = -\mathbb{F}_s \cdot \Delta \mathbb{X}_s \quad , \quad \mathbb{F}_s = 2 \cdot \mathbb{P}^{-1} \cdot \mathbb{B}_s^T \cdot \mathbb{K}_s \qquad (5\text{-}16)$$

where, the matrix $\mathbb{P}$ is a positive definite matrix.

From eqn.(5-4) and eqn.(5-16), the output feedback controller becomes:

$$\mathbb{U} = -\mathbb{F}_w \cdot \Delta w \quad , \quad \mathbb{F}_w = \mathbb{F}_s \cdot \mathbb{C}_s^{-1} \tag{5-17}$$

From eqn.(5-14), eqn.(5-15) and eqn.(5-16), the performance index $I(\mathbb{U})$ and the time derivative of the Lyapunov function dV/dt become:

$$I(\mathbb{U}) = V(\Delta \mathbb{X}_s)\big|_{t=0} - \int_0^\infty (\mathbb{U}^T \cdot \mathbb{P} \cdot \mathbb{U})\,dt \tag{5-18}$$

$$\frac{dV}{dt} = -\Delta \mathbb{X}_s^T \cdot \mathbb{Q}_s \cdot \Delta \mathbb{X}_s - \mathbb{U}^T \cdot \mathbb{P} \cdot \mathbb{U} \tag{5-19}$$

As described in the above two equations, by the control represented by eqn.(5-16) or eqn.(5-17) the equilibrium point is approached faster since the performance index $I(\mathbb{U})$ is smaller due to the additional negative control term $-\int_0^\infty \mathbb{U}^T \cdot \mathbb{P} \cdot \mathbb{U}\,dt$, and it is also assured because the time derivative of Lyapunov function dV/dt is smaller due to the additional negative control term $-\mathbb{U}^T \cdot \mathbb{P} \cdot \mathbb{U}$.

In order to minimize the performance index $I(\mathbb{U})$ of the reduced system, following recursive formula is also possible.

$$\mathbb{U} = \sum_{K=1}^{\ell} \mathbb{U}_K \tag{5-20}$$

$$\mathbb{U}_K = -\mathbb{F}_s^{(K)} \cdot \Delta \mathbb{X}_s \quad , \quad \mathbb{F}_s^{(K)} = 2 \cdot \mathbb{P}_K^{-1} \cdot \mathbb{B}_s^T \cdot \mathbb{K}_s^{(K)} \tag{5-21}$$

where, ℓ is anarbitrary positive integer and the matrix $\mathbb{K}_s^{(k)}$ is a $(m \times m)$ solution matrix of the following Lyapunov's matrix equation.

$$\mathbb{A}_s^{(K)T} \cdot \mathbb{K}_s^{(K)} + \mathbb{K}_s^{(K)} \cdot \mathbb{A}_s^{(K)} = -\mathbb{Q}_s \tag{5-22}$$

where, $\mathbb{A}_s^{(K)} = \mathbb{A}_s^{(K-1)} - \mathbb{B}_s \cdot \mathbb{F}_s^{(K-1)}$, $\mathbb{A}_s^{(0)} = \mathbb{A}_s$, $\mathbb{F}_s^{(0)} = \mathbb{0}$, $\mathbb{K}_s^{(1)} = \mathbb{K}_s$

In this case, the performance index $I(\mathbb{U})$ of the reduced system becomes:

$$I(\mathbb{U}) = V(\Delta \mathbb{X}_s)\big|_{t=0} - \sum_{K=1}^{\ell} \int_0^\infty (\mathbb{U}_K^T \cdot \mathbb{P}_K \cdot \mathbb{U}_K)\,dt \tag{5-23}$$

where, $V(\Delta \mathbb{X}_s) = \Delta \mathbb{X}_s^T \cdot \mathbb{K}_s \cdot \Delta \mathbb{X}_s$

From eqn.(5-4), eqn.(5-20) and eqn.(5-21), the output feedback controller becomes:

$$\mathbb{U} = -\mathbb{F}_w \cdot \Delta w \quad , \quad \mathbb{F}_w = \left(\sum_{K=1}^{\ell} \mathbb{F}_s^{(K)}\right) \cdot \mathbb{C}_s^{-1} \tag{5-24}$$

It is also possible to determine the control signal as follows:

$$u_j = -u_{vj}\cdot sgn[\mathbb{B}_s^T\cdot \mathbb{K}_s\cdot \Delta \mathbb{X}_s]_j \quad , \quad j = 1 \sim r \qquad (5\text{-}25)$$

where, u_{vj} is a positive constant, and the control signal u_j is bounded, e.g. by $|u_j| \leqslant u_{vj}$ for $j = 1 \sim r$. The symbol $[\quad]_j$ refers to the j-th component of the column vector [ ] .

Introducing the vector function with vector argument $\mathbb{S}gn$, we obtain:

$$\mathbb{U}^T = -\mathbb{U}_v^T\cdot \mathbb{S}gn[\mathbb{B}_s^T\cdot \mathbb{K}_s\,\Delta \mathbb{X}_s] \qquad (5\text{-}26)$$

where, $\mathbb{U}_v = [u_{v1}, u_{v2}, \cdots\cdots, u_{vr}]^T$

$$\mathbb{S}gn[\quad] = \text{diag.}[\cdot\cdot, sgn[\quad]_j, \cdots\cdots]^T$$

By the control expressed by eqn.(5-25) or eqn.(5-26), the performance index $I(\mathbb{U})$ becomes:

$$I(\mathbb{U}) = V(\Delta \mathbb{X}_s)|_{t=0} - 2\int_0^\infty |\Delta \mathbb{X}_s^T\cdot \mathbb{K}_s\cdot \mathbb{B}_s|\cdot \mathbb{U}_v\,dt \qquad (5\text{-}27)$$

As shown in the above equation, by the control described by eqn.(5-25) or eqn.(5-26) the equilibrium point of the system is also approached faster since the performance index $I(\mathbb{U})$ is smaller due to the additional negative control term $-2\int_0^\infty|\Delta \mathbb{X}_s^T\mathbb{K}_s\cdot \mathbb{B}_s|\cdot \mathbb{U}_v\,dt$.

The controller described by eqns.(5-7),(5-9),(5-16),(5-17), and (5-24) is a proportional type controller and the controller described by eqns.(5-25), and (5-26) becomes a bang-bang type controller.

Section 5-3. Stability of Closed-loop System

The above controllers for the reduced system could serve as suboptimal controllers for the original system described by eqn.(5-1) and eqn.(5-2), and the original system is governed by the following closed-loop equation.

$$p\Delta \mathbb{X} = (\mathbb{A} - \mathbb{B}\cdot \mathbb{F}_w\cdot \mathbb{C})\cdot \Delta \mathbb{X} \qquad (5\text{-}28)$$

Here, it is noted that the stability of the original system applied with the above output feedback controllers is not assured.

The stability of this closed-loop system is determined directly computing the characteristic roots of the closed-loop system matrix ($\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C}$) as already described in section 3-1-1.

The stability of this closed-loop system is also examined by the solution matrix $\mathbb{L}$ of the following Lyapunov's matrix equation of the closed-loop system.

$$(\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C})^T\cdot\mathbb{L} + \mathbb{L}\cdot(\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C}) = -\mathbb{N} \qquad (5\text{-}29)$$

As described in section 3-1-3, if the closed-loop system expressed by eqn.(5-28) is asymptotically stable, the solution matrix $\mathbb{L}$ of eqn.(5-29) becomes positive definite, and the following relationship is satisfied.

$$I = \int_0^\infty \Delta x^T\cdot\mathbb{N}\cdot\Delta x\, dt = \Delta x^T\cdot\mathbb{L}\cdot\Delta x\,\Big|_{t=0} \qquad (5\text{-}30)$$

Furthermore, the expected value of I becomes:

$$\hat{I} = \frac{1}{n}\,Tr(\mathbb{L}) \qquad (5\text{-}31)$$

For the smaller value of Tr($\mathbb{L}$), the closed-loop system becomes much stable.

The exact solution of the closed-loop system becomes:

$$\Delta x(t) = \varepsilon^{(\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C})t}\cdot\Delta x(o) \qquad (5\text{-}32)$$

where, $\varepsilon^{(\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C})t}$ is the state transition matrix of the closed-loop system. The responses of the closed-loop system are obtained by solving the eqn.(5-32) by the recursive formula described in section 3-1-2.

Section 5-4. Model Reduction Techniques

5-4-1. State Variables Grouping Technique[41),(42]

The n state variables of the original system described by eqn.(5-1) and eqn.(5-2) are classified into two groups; Δx_1 of m state variables and Δx_2 of (n−m) state variables, each group being associated with large and small time constants of the system, respectively. Thus, eqn.(5-1) and eqn.(5-2) can be rewritten in the following partitioned form.

$$p \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} \mathbb{A}_{11} & \mathbb{A}_{12} \\ \mathbb{A}_{21} & \mathbb{A}_{22} \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} \mathbb{B}_1 \\ \mathbb{B}_2 \end{bmatrix} \cdot \mathbb{u} \qquad (5\text{-}33)$$

$$\Delta w = \begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} \qquad (5\text{-}34)$$

where, the matrices $\mathbb{A}_{11}$, $\mathbb{A}_{12}$, $\mathbb{A}_{21}$, $\mathbb{A}_{22}$, $\mathbb{B}_1$, $\mathbb{B}_2$, $\mathbb{C}_1$, and $\mathbb{C}_2$ are respectively $(m \times m)$, $(m \times n-m)$, $(n-m \times m)$, $(n-m \times n-m)$, $(m \times r)$, $(n-m \times r)$, $(m \times m)$, and $(m \times n-m)$ coefficient matrices.

The transients due to the small time constants would have decayed fast and the Δx_2 variables closely follow the Δx_1 variables. This justifies the omission of $p\Delta x_2$ term in eqn.(5-33). Then the reduced system is described in the form as shown in eqn.(5-3) and eqn.(5-4), and the matrices $\mathbb{A}_s$, $\mathbb{B}_s$, and $\mathbb{C}_s$ become:

$$\mathbb{A}_s = \mathbb{A}_{11} - \mathbb{A}_{12} \cdot \mathbb{A}_{22}^{-1} \cdot \mathbb{A}_{21} \qquad (5\text{-}35)$$

$$\mathbb{B}_s = \mathbb{B}_1 - \mathbb{A}_{12} \cdot \mathbb{A}_{22}^{-1} \cdot \mathbb{B}_2 \qquad (5\text{-}36)$$

$$\mathbb{C}_s = \mathbb{C}_1 \qquad (\Delta x_2 \cong 0) \qquad (5\text{-}37)$$

Thus, the n-th order original system is reduced to a m-th order simplified model. The reduced model may represent the long-lived transients of the original system, and the dominant eigenvalues of the original system may be approximately determined by the characteristic roots of the matrix $\mathbb{A}_s$ $(= \mathbb{A}_{11} - \mathbb{A}_{12} \cdot \mathbb{A}_{22}^{-1} \cdot \mathbb{A}_{21})$.

5-4-2. Eigenvalues Grouping Technique[41],[43]

If Δz represents the n modes of the system described by eqn.(5-1) and eqn.(5-2), then the n state variables Δx are related to Δz by:

$$\Delta x = \mathbb{M} \cdot \Delta z \qquad (5\text{-}38)$$

where, the matrix $\mathbb{M}$ is the $(n \times n)$ modal matrix of the $(n \times n)$ matrix $\mathbb{A}$ and becomes:

$$\mathbb{M} = [\mathbb{M}_1, \mathbb{M}_2, \mathbb{M}_3, \cdots\cdots, \mathbb{M}_n] \qquad (5\text{-}39)$$

where, the j-th column vector $\mathbb{M}_j$ of the matrix $\mathbb{M}$ is the eigenvector of the j-th eigenvalue λ_j of the matrix $\mathbb{A}$. By the transformation described by eqn.(5-38), the system equations (5-1) and (5-2) can be rewritten in the following partitioned form.

$$p \begin{bmatrix} \Delta\mathbb{Z}_1 \\ \Delta\mathbb{Z}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{\Lambda}_1 & \mathbb{0} \\ \mathbb{0} & \mathbb{\Lambda}_2 \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbb{Z}_1 \\ \Delta\mathbb{Z}_2 \end{bmatrix} + \begin{bmatrix} \mathbb{\Gamma}_1 \\ \mathbb{\Gamma}_2 \end{bmatrix} \cdot \mathbb{u} \qquad (5\text{-}40)$$

$$\Delta\mathbb{W} = \begin{bmatrix} \mathbb{D}_1 & \mathbb{D}_2 \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbb{Z}_1 \\ \Delta\mathbb{Z}_2 \end{bmatrix} \qquad (5\text{-}41)$$

where, $\text{diag.}[\mathbb{\Lambda}_1, \mathbb{\Lambda}_2] = \mathbb{M}^{-1} \cdot \mathbb{A} \cdot \mathbb{M}$

$\mathbb{\Lambda}_1 = \text{diag.}(\lambda_1, \lambda_2, \lambda_3, \cdots\cdots, \lambda_m)$

$\mathbb{\Lambda}_2 = \text{diag.}(\lambda_{m+1}, \lambda_{m+2}, \cdots\cdots, \lambda_n)$

$\mathbb{\Gamma}_1$ = top $(m \times r)$ submatrix of $\mathbb{M}^{-1} \cdot \mathbb{B}$

$\mathbb{\Gamma}_2$ = bottom $(n-m \times r)$ submatrix of $\mathbb{M}^{-1} \cdot \mathbb{B}$

$\mathbb{D}_1$ = front $(m \times m)$ submatrix of $\mathbb{C} \cdot \mathbb{M}$

$\mathbb{D}_2$ = rear $(m \times n-m)$ submatrix of $\mathbb{C} \cdot \mathbb{M}$

If the matrix $\mathbb{A}$ has only distinct eigenvalues this transformation will yield a diagonal matrix whose elements are the eigenvalues of the matrix $\mathbb{A}$. However, if the matrix $\mathbb{A}$ has only repeated eigenvalues a transformation to a Jordan canonic form will be obtained. Assuming, for simplicity, the system has distinct eigenvalues such that $|\lambda_1| < |\lambda_2| < \cdots < |\lambda_n|$.

In general, the eigenvalues of the system may be devided into two groups; those that are farther from the imaginary axis $\mathbb{\Lambda}_2$, and those that are nearer to the imaginary axis $\mathbb{\Lambda}_1$. Then, the variables vector $\Delta\mathbb{Z}_1$ may represent the long-lived transients of the system, and the variables vector $\Delta\mathbb{Z}_2$ may represent the short-lived transients of the system. Therefore, we can assume $\Delta\mathbb{Z}_2 \cong \mathbb{0}$.

In this case, the reduced system can be described in the form shown in eqn.(5-3) and eqn.(5-4), and the matrices $\mathbb{A}_s$, $\mathbb{B}_s$ and $\mathbb{C}_s$ become:

$$\mathbb{A}_s = \mathbb{A}_1 \qquad (5\text{-}42)$$

$$\mathbb{B}_s = \mathbb{\Gamma}_1 \qquad (5\text{-}43)$$

$$\mathbb{C}_s = \mathbb{D}_1 \qquad (5\text{-}44)$$

Section 5-5. Application to a One-machine Problem

The same one-machine infinite-bus system shown in section 4-3 has been used in this section in order to investigate the control effects of the output feedback controller described above and also to compare the control effects of the output feedback controller with those of the state feedback optimal controller shown in section 4-3.

The system configulation and the block diagrams of the associated control systems have already been described in section 4-3. Furthermore, the parameters and the initial conditions of the model one-machine system have been shown in section 4-3.

5-5-1. Numerical Results (I)

The output feedback controller for the model system has been determined by the methods described in section 5-1 and section 5-4-1.

In order to obtain the simplified model of the original system, the state variables $\Delta \mathbb{X}$ represented by eqn.(4-28) have been devided into following two groups.

$$\Delta \mathbb{X}_1 = [\Delta\Psi_{fd}, \Delta\Psi_{kq}, \Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v]^T$$

$$\Delta \mathbb{X}_2 = [\Delta\Psi_d, \Delta\Psi_{kd}, \Delta\Psi_q, \Delta P_t]$$

In this case, the reduced system is 7-th order, and the 7-th order measurable output variables $\Delta \mathbb{W}$ are defined as shown in section 4-3 as follows:

$$\Delta \mathbb{W} = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v, \Delta v_t, \Delta i_t]^T$$

In the model system, the matrix $\mathbb{B}_2$ becomes equal to $\mathbb{0}$, so the matrix

$\mathbb{C}_s$ in eqn.(5-37) of the reduced system becomes as follows without the assumption $\Delta x_2 \cong 0$: $\mathbb{C}_s = \mathbb{C}_1 - \mathbb{C}_2 \cdot \mathbb{A}_{22}^{-1} \cdot \mathbb{A}_{21}$.

The characteristic roots of the original and the reduced system are shown in Table 5-5-1 for the given operating point of the model system; P_o (active power output)=1.0 p.u., Q_o (reactive power output)=-0.5 p.u., and V_{to} (terminal voltage)=1.1 p.u..

The dominant eigenvalues of the original system are retained in the simplified model by the model reduction described in section 5-4-1. So the effectiveness of this model reduction is assured.

Table 5-5-1. Characteristic roots of original and reduced system

Original model	Reduced model
$-0.934 \times 10^{-1} \pm j0.874$	$-0.953 \times 10^{-1} \pm j0.101 \times 10$
$-0.761 \pm j0.935 \times 10$	$-0.630 \pm j0.885 \times 10$
-0.123×10	-0.124×10
-0.187×10	-0.187×10
-0.457×10	-0.455×10
-0.409×10	
-0.999×10	
$-0.156 \times 10^{3} \pm j0.202 \times 10^{4}$	

For the various weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$ shown in Table 5-5-2, the output feedback controller of the model system has been determined using the solution of the matrix Riccati equation of the reduced system as described in section 5-1. The feedback gain matrices $\mathbb{F}_w$ are shown in Table 5-5-3.

Table 5-5-2. Weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$

Case No.	matrix $\mathbb{Q}_w$	matrix $\mathbb{R}$
1	diag.(1,1,1,1,1,1,1)	diag.(1,1)
2	diag.(1,1,1,1,1,1,1)	diag.(0.1,0.1)
3	diag.(1,1,1,1,1,1,1)	diag.(0.001,0.001)
4	diag.(10,1,1,1,1,10,10)	diag.(1,1)
5	diag.(10,1,1,1,1,10,10)	diag.(0.1,0.1)
6	diag.(10,1,1,1,1,10,10)	diag.(0.001,0.001)
7	diag.(10,10,1,1,1,1,1)	diag.(1,1)
8	diag.(1,1,0,0,0,1,1)	diag.(1,1)

Table 5-5-3. Feedback gain matrices $\mathbb{F}_w^T$

	Case 1		Case 2		Case 3		Case 4	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\delta$	-0.525	2.534	-1.910	7.439	-19.249	72.877	-1.676	6.263
$\Delta\omega$	0.007	1.930	-0.014	6.319	-0.198	63.596	0.020	2.215
ΔE_{fd}	1.909	-0.011	6.235	-0.021	63.166	-0.018	1.925	-0.006
V_s	-1.544	-0.138	-1.580	-0.047	1.048	-0.018	-1.566	-0.079
ΔP_v	-0.042	6.323	-0.041	12.704	-0.032	72.433	-0.022	6.740
Δv_t	-0.867	-10.694	0.484	-20.297	21.240	104.961	-0.243	-12.259
Δi_t	1.007	-8.841	3.161	-15.985	31.349	72.584	2.536	-7.637
	Case 5		**Case 6**		**Case 7**		**Case 8**	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\delta$	-5.456	19.395	-54.750	191.822	-0.571	6.590	-0.227	2.204
$\Delta\omega$	0.018	6.898	-0.014	66.922	-0.002	6.282	0.033	1.927
ΔE_{fd}	6.251	-0.009	63.196	-0.002	1.911	-0.033	0.134	0.007
V_s	-1.601	-0.029	0.891	-0.030	-1.552	-0.054	-0.547	-0.172
ΔP_v	-0.019	13.136	-0.008	72.871	-0.061	11.194	-0.017	6.003
Δv_t	2.571	-24.128	42.471	-138.417	-0.782	-19.688	-0.104	-10.862
Δi_t	8.232	-10.878	82.915	-15.052	1.133	-17.131	0.955	-8.834

The dynamic stability of the model system applied with these output feedback controllers has been investigated by various methods described in section 5-3.

The characteristic roots of the closed-loop matrix ($\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C}$) of the model system are shown in Table 5-5-4, and the characteristic roots of the original uncontrolled system are shown in Table 5-5-1. All the characteristic roots of the original uncontrolled system are shifted to the left side apart from the imaginary axis, therefore the dynamic stability of the model system is much improved by these controllers.

The values of Tr($\mathbb{L}$) of the model system applied with these controllers are shown in Table 5-5-5. As shown in this Table, the value of Tr($\mathbb{L}$) of the model system is much decreased by these controllers, so it is evident that the system performance is much improved by these controllers. But, in comparison with the value of Tr($\mathbb{L}$) of the model system applied with optimal state feedback controllers shown in Table 4-3-6, the value of Tr($\mathbb{L}$) of the

Table 5-5-4 Characteristic roots of closed-loop system

Case 1	Case 2	Case 3
$-0.579 \pm j0.180$	-0.418	-0.371
$-0.165\times10 \pm j0.127\times10^2$	-0.105×10	-0.148×10
-0.180×10	$-0.157\times10 \pm j0.149\times10^2$	$-0.163\times10 \pm j0.175\times10^2$
-0.207×10	-0.179×10	-0.174×10
-0.996×10	-0.202×10	-0.199×10
-0.380×10^2	-0.100×10^2	-0.101×10^2
-0.813×10^2	-0.124×10^3	-0.907×10^3
$-0.156\times10^3 \pm j0.202\times10^4$	-0.161×10^3	-0.127×10^4
	$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$

Case 4	Case 5	Case 6
-0.571	-0.541	-0.525
$-0.105\times10 \pm j0.128\times10^2$	$-0.772 \pm j0.147\times10^2$	$-0.636 \pm j0.178\times10^2$
$-0.186\times10 \pm j0.691$	-0.202×10	-0.199×10
-0.208×10	$-0.221\times10 \pm j0.679$	$-0.258\times10 \pm j0.432$
-0.984×10	-0.995×10	-0.101×10^2
-0.383×10^2	-0.125×10^3	-0.912×10^3
-0.865×10^2	-0.166×10^3	-0.127×10^4
$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$

Case 7	Case 8
$-0.513 \pm j0.318$	$-0.596 \pm j0.776$
$-0.162\times10 \pm j0.150\times10^2$	-0.151×10
-0.189×10	$-0.167\times10 \pm j0.129\times10^2$
-0.207×10	-0.173×10
-0.100×10^2	-0.457×10
-0.380×10^2	-0.104×10^2
-0.142×10^3	-0.774×10^2
$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$

Table 5-5-5 Value of Tr($\mathbb{L}$)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	without controller
L(1,1)	7.797	6.877	6.152	8.331	8.241	8.055	6.510	9.665	73.521
L(2,2)	0.004	0.005	0.022	0.005	0.007	0.052	0.005	0.005	0.005
L(3,3)	4.562	4.514	4.626	6.880	8.708	11.440	4.434	4.794	17.764
L(4,4)	0.010	0.012	0.031	0.014	0.020	0.070	0.013	0.011	0.014
L(5,5)	2.335	2.167	2.068	3.489	4.021	4.715	2.114	2.481	53.154
L(6,6)	9.638	9.159	8.970	13.910	17.574	25.253	9.316	10.277	30.476
L(7,7)	0.185	0.217	0.291	0.289	0.441	0.768	0.219	0.186	0.362
L(8,8)	0.015	0.007	0.003	0.015	0.007	0.003	0.015	0.171	2.489
L(9,9)	0.278	0.255	0.250	0.277	0.255	0.250	0.279	1.095	56.946
L(10,10)	0.017	0.006	0.001	0.019	0.007	0.001	0.007	0.020	17.747
L(11,11)	4.531	3.944	3.595	1.321	7.520	9.097	3.807	5.073	14.935
Tr($\mathbb{L}$)	29.375	27.161	26.009	34.548	47.801	59.703	26.718	33.784	267.413

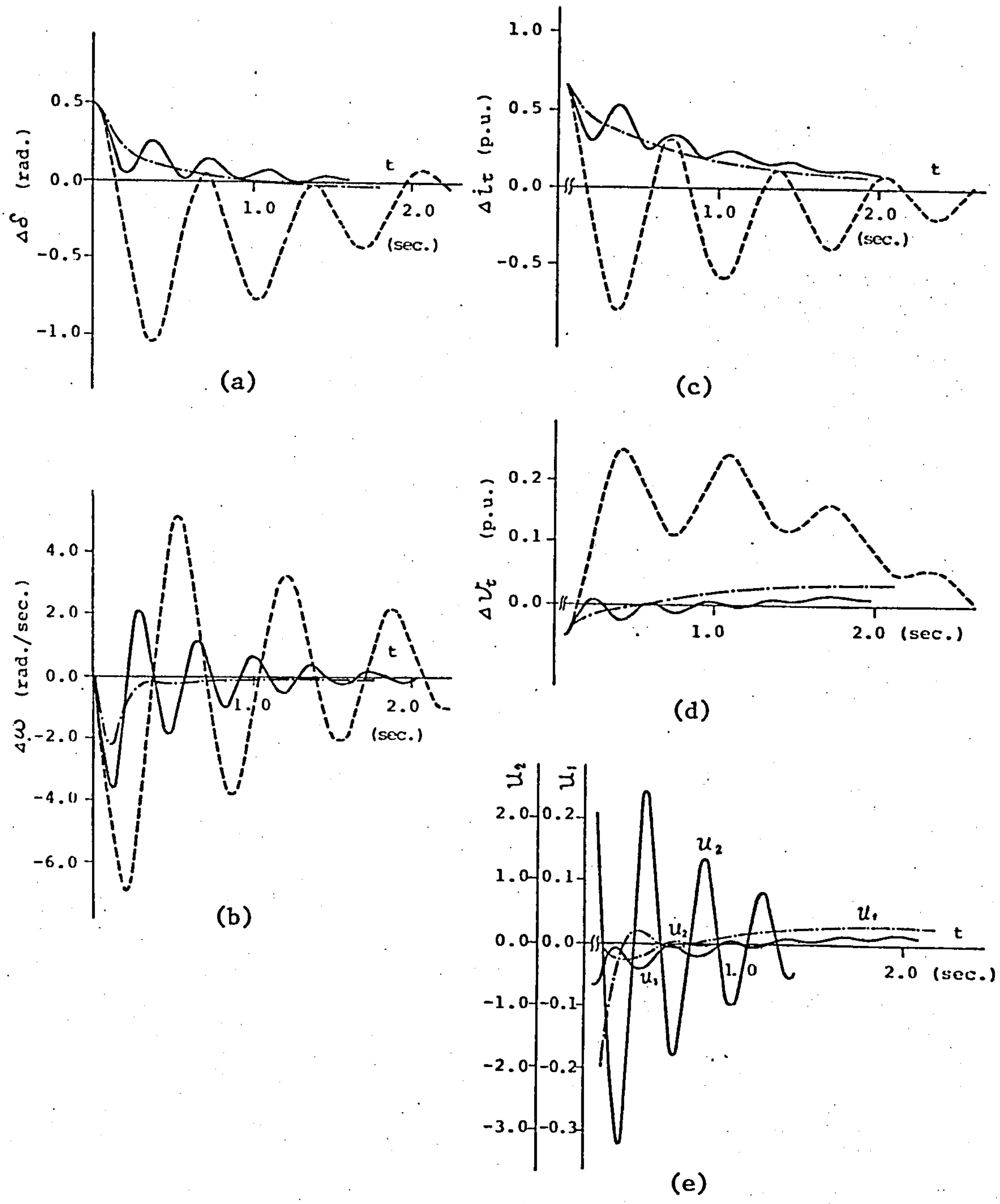
model system applied with the output feedback controllers takes a little greater value for the same weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$. Therefore, the output feedback controllers obtained become suboptimal controllers for the model system.

For the model system applied with the optimal state feedback controllers, the smaller value of the matrix $\mathbb{R}$ makes the system more stable for the same $\mathbb{Q}_w$ matrix, but for the model system applied with the above output feedback controllers, the relation above described is not necessarily satisfied, since the feedback gains have been determined through the reduced model.

In the above numerical calculations, the (11×11) unit matrix has been selected as the matrix $\mathbb{N}$ in eqn.(5-29).

The responses of the model system variables have been obtained by solving the closed-loop equation of the model system for the given initial deviations $\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$.

The typical responses are shown in Fig.5-5-1, and Fig.5-5-2. As shown in Fig.5-5-1, the damping characteristic of the system is much improved by the output feedback controller, but in comparison with that of the system applied with the optimal state feedback controller the improvement is a little smaller. As shown in Fig.5-5-2, the originary unstable system is stabilized by the output feedback controller, therefore it is evident that the dynamic stable region of the model system may be expanded by the output feedback controller.

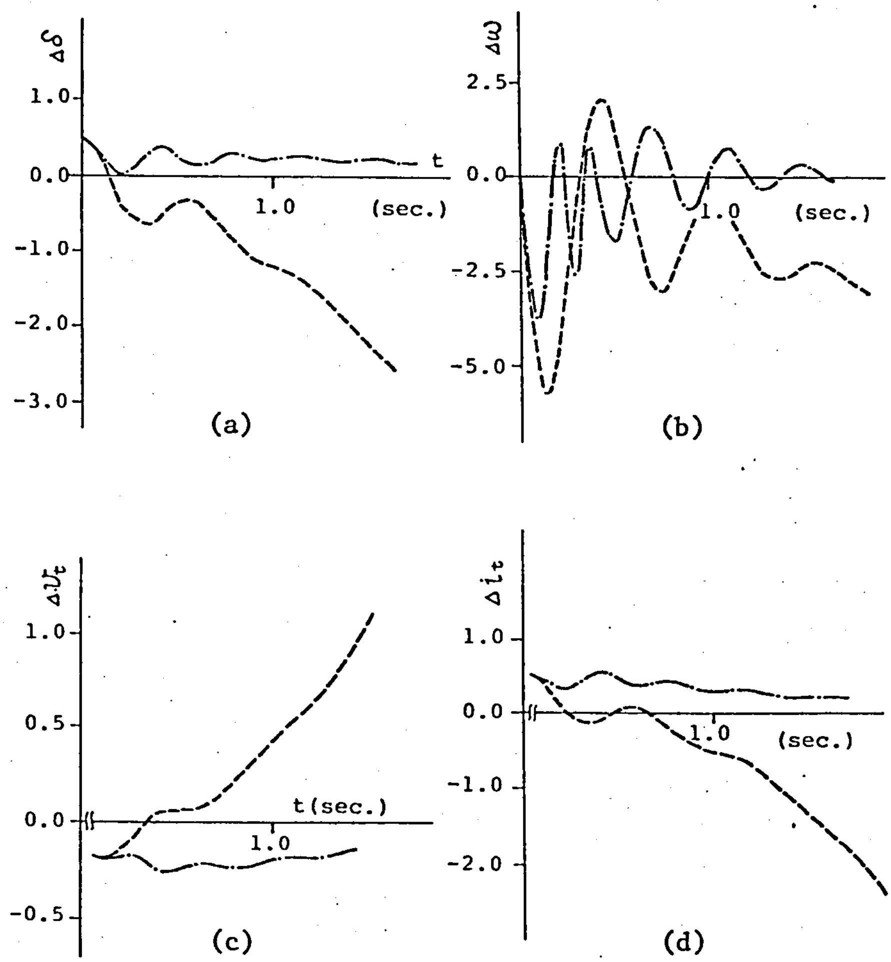


——— : with output feedback controller (case 3)

—·—· : with state feedback optimal controller (case 2 in chapter 4)

----- : without controller

Fig.5-5-1 Responses of the model system for the initial deviations $\Delta\delta|_{t=0}=$ 0.5 and $\Delta\psi_{fd}|_{t=0}=$ 0.5



operating point: P_o = 1.0 (p.u.), Q_o = -1.1 (p.u.), v_{to} = 1.1 (p.u.)

—·—· : with output feedback controller (case 3)

----- : without controller

Fig.5-5-2 Responses of the originaly unstable system for the initial deviations $\Delta\delta|_{t=0}$ = 0.5 and $\Delta\Psi_{fd}|_{t=0}$ = 0.5

5-5-2. Numerical Results (II)

The output feedback controller for the model system has been determined by the methods described in section 5-1 and section 5-4-2.

In order to obtain the reduced model, the modal matrix $\mathbb{M}$ of the model system has been determined as described below. As shown in Table 5-5-1, the characteristic roots of the original system; λ_1 , λ_2 , ... , λ_{11} become:

$$\lambda_1 = \alpha_1 + j\beta_1 = -0.934 \times 10^{-1} + j0.874, \quad \lambda_2 = \alpha_1 - j\beta_1$$

$$\lambda_3 = \alpha_2 + j\beta_2 = -0.761 + j0.935 \times 10, \quad \lambda_4 = \alpha_2 - j\beta_2$$

$$\lambda_5 = -0.123 \times 10, \quad \lambda_6 = -0.187 \times 10, \quad \lambda_7 = -0.409 \times 10$$

$$\lambda_8 = -0.457 \times 10, \quad \lambda_9 = -0.999 \times 10,$$

$$\lambda_{10} = \alpha_3 + j\beta_3 = -0.156 \times 10^3 + j0.202 \times 10^4, \quad \lambda_{11} = \alpha_3 - j\beta_3$$

Instead of the transformation matrix $\mathbb{M}$ described in eqn.(5-39), a modified transformation matrix is used in order to avoid the complex arithmetic, and the modified matrix[43] becomes:

$$\mathbb{M} = [\mathbb{M}_1^{(R)}, \mathbb{M}_1^{(I)}, \mathbb{M}_3^{(R)}, \mathbb{M}_3^{(I)}, \mathbb{M}_4, \cdots, \mathbb{M}_9, \mathbb{M}_{10}^{(R)}, \mathbb{M}_{10}^{(I)}]$$

where, $\mathbb{M}_1 = \mathbb{M}_1^{(R)} + j\mathbb{M}_1^{(I)}, \quad \mathbb{M}_3 = \mathbb{M}_3^{(R)} + j\mathbb{M}_3^{(I)}, \quad \mathbb{M}_{10} = \mathbb{M}_{10}^{(R)} + j\mathbb{M}_{10}^{(I)}$

$\mathbb{M}_j$: eigenvector of j-th eigenvalue λ_j

$\mathbb{M}_j^{(R)}$: real part of eigenvector $\mathbb{M}_j$

$\mathbb{M}_j^{(I)}$: imaginary part of eigenvector $\mathbb{M}_j$

Then, the matrices $\mathbb{\Lambda}_1$ and $\mathbb{\Lambda}_2$ becomes:

$$\mathbb{\Lambda}_1 = \begin{bmatrix} \alpha_1 & \beta_1 & & & & & \\ -\beta_1 & \alpha_1 & & & & \mathbb{0} & \\ & & \alpha_2 & \beta_2 & & & \\ & & -\beta_2 & \alpha_2 & & & \\ & & & & \lambda_5 & & \\ & \mathbb{0} & & & & \lambda_6 & \\ & & & & & & \lambda_7 \end{bmatrix} \quad \mathbb{\Lambda}_2 = \begin{bmatrix} \lambda_8 & & & \\ & \lambda_9 & & \mathbb{0} \\ & & \alpha_3 & \beta_3 \\ \mathbb{0} & & -\beta_3 & \alpha_3 \end{bmatrix}$$

Furthermore, the matrices $\mathbb{\Gamma}_1$, $\mathbb{\Gamma}_2$, $\mathbb{D}_1$ and $\mathbb{D}_2$ have also been determined by matrices $\mathbb{B}$, $\mathbb{C}$ of the model system and the above transformation matrix. And, the coefficient matrices $\mathbb{A}_s$, $\mathbb{B}_s$ and $\mathbb{C}_s$ of the reduced system have been determined by eqn.(5-42), eqn.(5-43) and eqn.(5-44). In this case, the reduced system becomes a 7-th order system, and the same 7-th order measurable output variables $\Delta \mathbb{w}$ as shown in section 5-1-1 have been considered.

For various weighting matrices shown in Table 5-5-6, the output feedback controller has been determined using the solution matrix of the matrix Riccati equation of the reduced system as described in section 5-1. The feedback gain matrices $\mathbb{F}_s$ and $\mathbb{F}_w$ for the model system are shown in Table 5-5-7.

Table 5-5-6 Weighting matrices $\mathbb{Q}_w$, $\mathbb{R}$ and value of Tr($\mathbb{L}$)

Case No.	matrix $\mathbb{Q}_w$	matrix $\mathbb{R}$	Tr($\mathbb{L}$)
1	diag.(1,1,1,1,1,1,1)	diag.(1,1)	94.498
2	diag.(1,1,1,1,1,1,1)	diag.(0.01,0.01)	67.992
3	diag.(1,1,1,1,1,1,1)	diag.(0.001,0.001)	67.008
	without controller		252.328

The dynamic stability of the model system applied with these output feedback controllers has been checked by various methods described in section 5-3.

The eigenvalues of the closed-loop matrix ($\mathbb{A} - \mathbb{B}\cdot\mathbb{F}_w\cdot\mathbb{C}$) of the model system are shown in Table 5-5-8. In comparison with the eigenvalues of the original uncontrolled system shown in Table 5-5-1, all the eigenvalues are shifted to the left side apart from the imaginary axis, therefore the dynamic stability of the model system is improved by these controllers.

The values of Tr($\mathbb{L}$) of the model system applied with the above controllers are shown in Table 5-5-6. The value of Tr($\mathbb{L}$) of the model system is much decreased by the controllers, so it is evident that the system performance is much improved by the controllers.

Table 5-5-7 Feedback gain matrices $\mathbb{F}_s$ and $\mathbb{F}_w$

	Case 1 $\mathbb{F}_s^T$			Case 1 $\mathbb{F}_w^T$	
	u_1	u_2		u_1	u_2
Δz_1	-0.769	-1.203	$\Delta\delta$	-0.426	-3.826
Δz_2	2.528	-0.036	$\Delta\omega$	0.128	0.473
Δz_3	0.157	-0.031	ΔE_{fd}	0.985	-0.692
Δz_4	-0.062	0.922	V_s	0.358	-27.145
Δz_5	-0.726	-0.153	ΔP_v	-0.954	8.158
Δz_6	0.453	-0.030	Δv_t	1.892	-36.405
Δz_7	-0.461	-0.422	Δi_t	1.703	-7.938
	Case 2 $\mathbb{F}_s^T$			Case 2 $\mathbb{F}_w^T$	
	u_1	u_2		u_1	u_2
Δz_1	-3.321	-12.616	$\Delta\delta$	13.383	-34.355
Δz_2	25.807	4.780	$\Delta\omega$	-1.744	7.500
Δz_3	0.026	0.358	ΔE_{fd}	12.626	-3.322
Δz_4	-3.716	8.384	V_s	114.084	-231.093
Δz_5	-8.336	0.156	ΔP_v	-32.007	63.459
Δz_6	5.359	0.620	Δv_t	154.162	-300.943
Δz_7	-5.348	-6.024	Δi_t	35.667	-56.333
	Case 3 $\mathbb{F}_s^T$			Case 3 $\mathbb{F}_w^T$	
	u_1	u_2		u_1	u_2
Δz_1	-3.682	-43.928	$\Delta\delta$	43.960	-99.594
Δz_2	80.150	21.025	$\Delta\omega$	-8.657	24.652
Δz_3	-2.605	13.250	ΔE_{fd}	40.157	-6.983
Δz_4	-13.174	24.965	V_s	409.482	-689.713
Δz_5	-26.191	-0.229	ΔP_v	-109.414	185.873
Δz_6	17.650	1.382	Δv_t	546.970	-893.791
Δz_7	-16.321	-19.663	Δi_t	120.802	-161.433

Table 5-5-8 Eigenvalues of closed-loop system

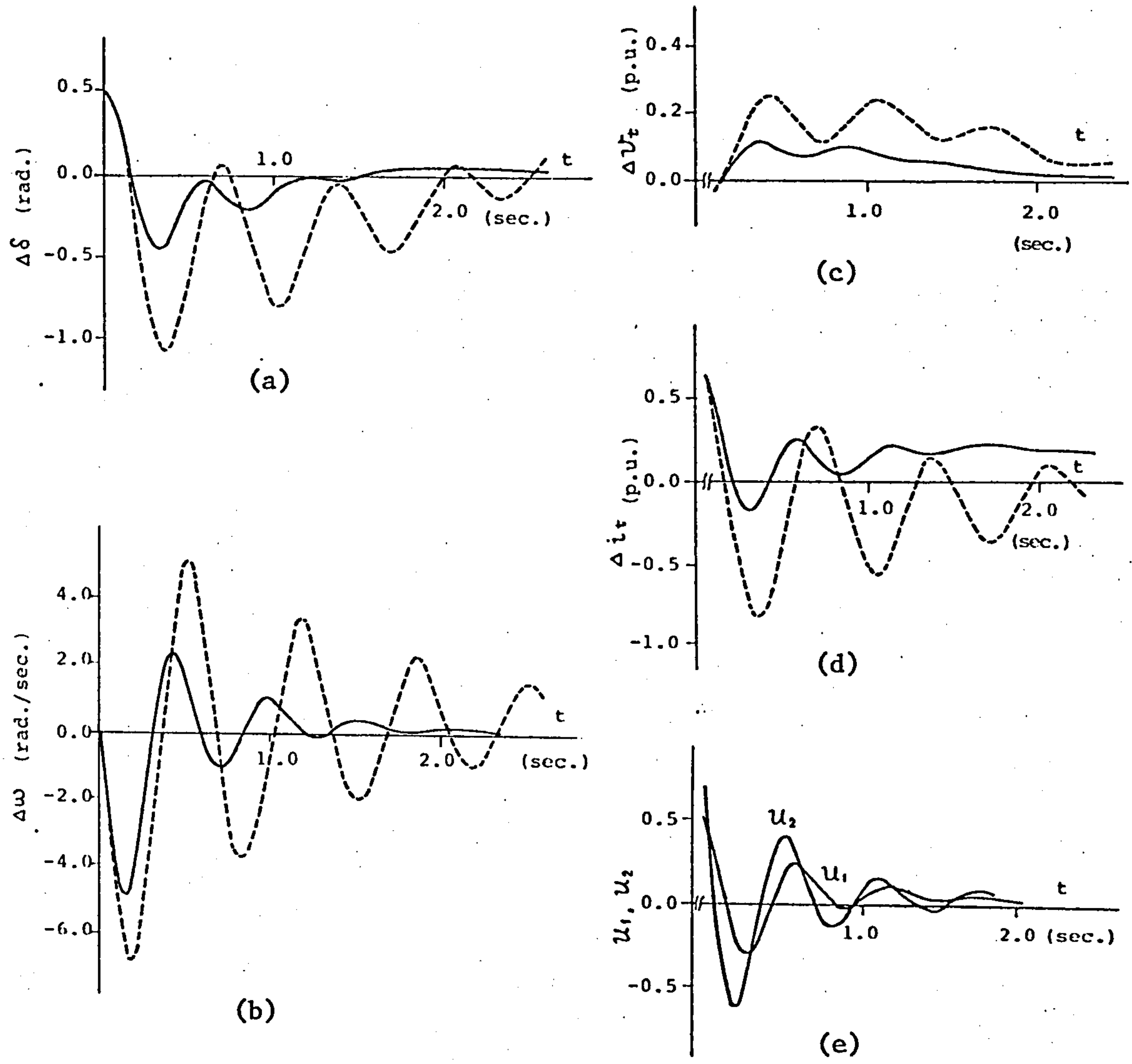
Case 1	Case 2	Case 3
-0.575	-0.587	-0.601
$-0.117\times10 \pm j0.195\times10$	$-0.113\times10 \pm j0.175\times10$	$-0.115\times10 \pm j0.177\times10$
$-0.157\times10 \pm j0.106\times10^2$	-0.201×10	-0.200×10
-0.201×10	$-0.214\times10 \pm j0.113\times10^2$	$-0.217\times10 \pm j0.113\times10^2$
-0.970×10	-0.989×10	-0.989×10
-0.155×10^2	-0.153×10^3	-0.491×10^3
-0.113×10^3	-0.113×10^4	-0.346×10^4
$-0.156\times10^3 \pm j0.202\times10^4$	$-0.155\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$

In the above calculations, the matrix $\mathbb{C}^T \cdot \mathbb{Q}_w \cdot \mathbb{C}$ has been selected as the matrix $\mathbb{N}$ in eqn.(5-29) instead of the (11×11) unit matrix. Consequently, the value of Tr($\mathbb{L}$) represents the expected value of the following quadratic performance index of the output variables Δw ; $Tr(\mathbb{L}) = n\hat{I}$, $I = \int_0^\infty (\Delta w^T \cdot \mathbb{Q}_w \cdot \Delta w) dt = \int_0^\infty (\Delta x^T \cdot \mathbb{C}^T \cdot \mathbb{Q}_w \cdot \mathbb{C} \cdot \Delta x) dt$.

The responses of the model system variables have been obtained by solving the closed-loop equation of the model system for the given initial deviations $\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$.

The typical responses are shown in Fig.5-5-3. As shown in this figure, the damping characteristic of the model system is much improved by the output feedback controller obtained here.

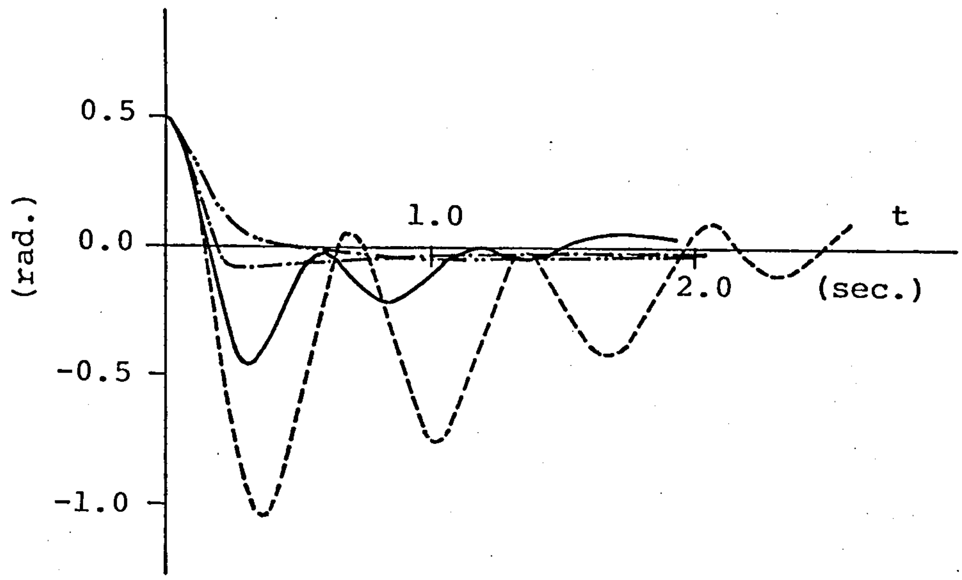
In Fig.5-5-4. the responses of $\Delta\delta$ of the model system applied with various controllers are shown. These controllers have been determined for the same weighting matrices $\mathbb{Q}_w$ and $\mathbb{R}$. As shown in this figure, the performance of the model system is much improved for the case $\Delta w = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v, \Delta v_t, \Delta i_t]^T$ than for the case $\Delta w = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_t, \Delta v_t, \Delta i_t]^T$. But, in comparison with the response of the model system applied with the state feedback optimal controller, the improvement is a little smaller.



——— : with output feedback controller (case 2)

----- : without controller

Fig.5-5-3 Typical responses of the model system for the initial deviations $\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$



$\mathbb{Q}_w$ = diag.(1,1,1,1,1,1,1)

$\mathbb{R}$ = diag.(0.01,0.01)

—— : with output feedback controller (case 2)
$\Delta \mathfrak{W} = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_v, \Delta v_t, \Delta i_t]^T$

—··— : with output feedback controller
$\Delta \mathfrak{W} = [\Delta\delta, \Delta\omega, \Delta E_{fd}, V_s, \Delta P_t, \Delta v_t, \Delta i_t]^T$

—·— : with state feedback optimal controller

----- : without controller

Fig.5-5-4 Responses of $\Delta\delta$ of the model system applied with various controllers

5-5-3. Numerical Results (III)

The output feedback controller for the model system has been determined by the methods described in section 5-2 and section 5-4-1.

In order to obtain the reduced system, the state variables Δx_1 and Δx_2 have been selected as shown in section 5-5-1. In this case, the reduced system is a 7-th order system, and the 7-th order measurable output variables Δw have also selected as shown in section 5-1-1.

The output feedback gains of the model system have been determined by the recursive formula shown in eqn.(5-20), eqn.(5-21), eqn.(5-22) and eqn.(5-24) for the given $\mathbb{Q}_S$ $(= \mathbb{C}_S^T \cdot \mathbb{Q}_w \cdot \mathbb{C}_S)$ matrix, where $\mathbb{Q}_w = \text{diag.}(1,1,1,1,1,1,1)$.

In Fig.5-5-5, the changes of the value of Tr($\mathbb{L}$) of the model system applied with the output controller obtained by the recursive formula above descrived are shown as the positive definite matrix $\mathbb{P}_j = \text{diag.}(K,K)$ varies. As shown in this figure, the optimal $\mathbb{P}_j$ matrix exists; $\mathbb{P}_1 = \text{diag.}(180,180)$, $\mathbb{P}_2 = \text{diag.}(2,2)$, $\mathbb{P}_3 = \text{diag.}(0.6,0.6)$. For these $\mathbb{P}_1$, $\mathbb{P}_2$, and $\mathbb{P}_3$ matrices the values of Tr($\mathbb{L}$) are shown in Table 5-5-9 and the feedback gain matrices $\mathbb{F}_w^{(1)}$, $\mathbb{F}_w^{(2)}$, and $\mathbb{F}_w^{(3)}$ are shown in Table 5-5-10. As shown in Table 5-5-9, the value of Tr($\mathbb{L}$) of the model system is much decreased by the output feedback controller obtained by the recursive formula above described, therefore it is evident that the system performance is improved by the controller. Furthermore the effectiveness of the recursive formula above described is assured.

In the above calculation, the matrix $\mathbb{C}^T \cdot \mathbb{Q}_w \cdot \mathbb{C}$ has been chosen as the matrix $\mathbb{N}$ in eqn.(5-29) insted of the (11×11) unit matrix as described in section 5-5-2. Consequently, the value of Tr($\mathbb{L}$) represents the expected value of the quadratic performance index of the output variables.

In Table 5-5-11, the eigenvalues of the closed-loop system are shown. The dynamic stability of the model system is much improved by the output feedback controller shown in Table 5-5-10, because all the eigenvalues are shifted to the left side apart from the imaginary axis.

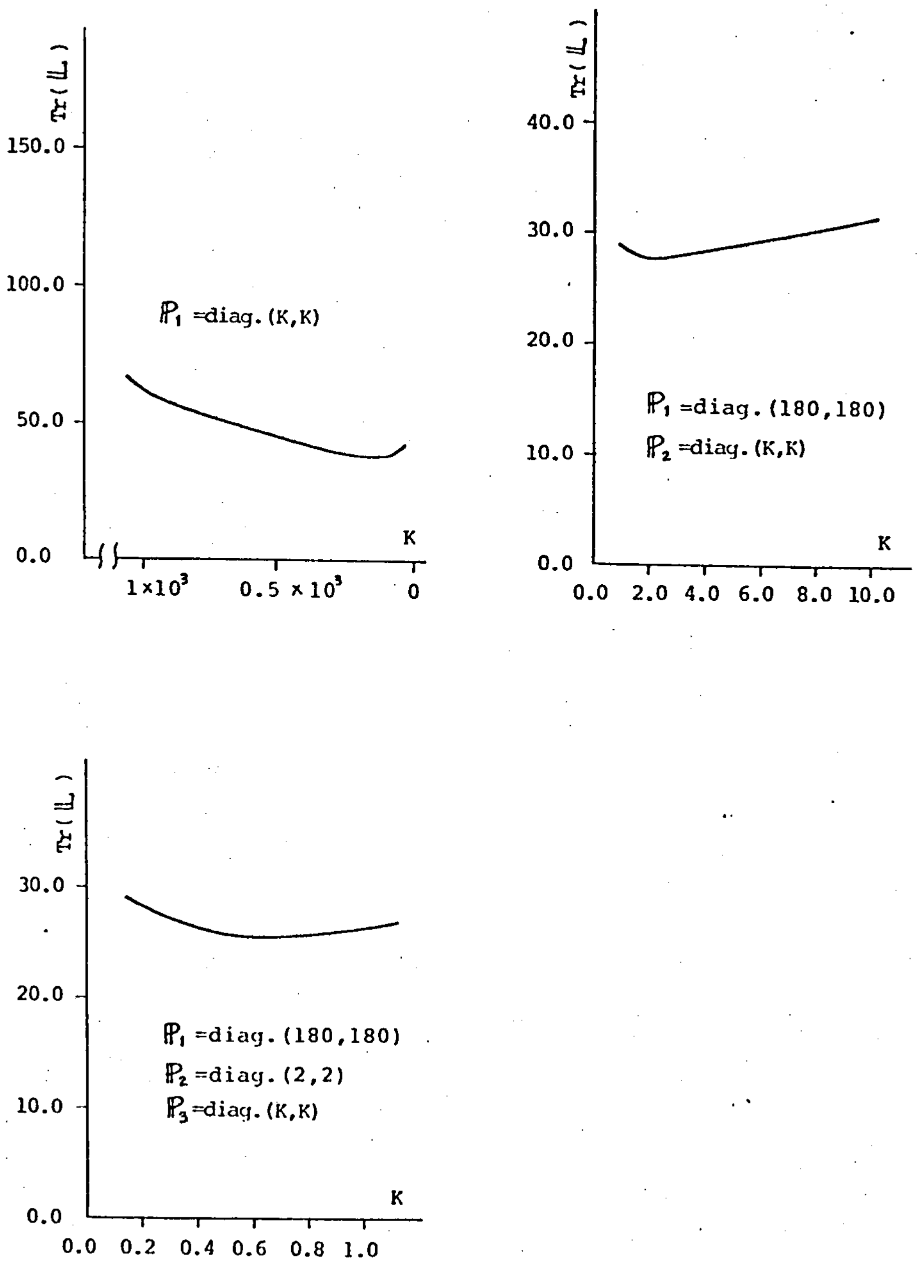


Fig.5-5-5 Changes of Tr($\mathbb{L}$) as $\mathbb{P}_j$ varies

Table 5-5-9 Value of $Tr(\mathbb{L})$

Case No.	matrix $\mathbb{P}_j$	$Tr(\mathbb{L})$
1	$\mathbb{P}_1$ =diag.(180,180)	36.292
2	$\mathbb{P}_1$ =diag.(180,180), $\mathbb{P}_2$ =diag.(2,2)	27.433
3	$\mathbb{P}_1$ =diag.(180,180), $\mathbb{P}_2$ =diag.(2,2), $\mathbb{P}_3$ =diag.(0.6,0.6)	25.580
	without controller	252.328

$\mathbb{Q}_w$=diag.(1,1,1,1,1,1,1) , $\mathbb{Q}_s = \mathbb{C}_s^T \cdot \mathbb{Q}_w \cdot \mathbb{C}_s$

Table 5-5-10 Feedback gain matrices $\mathbb{F}_w^{(1)}$, $\mathbb{F}_w^{(2)}$ and $\mathbb{F}_w^{(3)}$

$\mathbb{F}_w^{(j)T}$	$\mathbb{F}_w^{(1)T}$		$\mathbb{F}_w^{(2)T}$		$\mathbb{F}_w^{(3)T}$	
	u_1	u_2	u_1	u_2	u_1	u_2
$\Delta\delta$	-1.038	4.379	-4.244	-0.545	1.253	2.974
$\Delta\omega$	0.029	0.113	0.430	3.223	-0.123	0.597
ΔE_{fd}	0.301	-0.055	1.412	0.189	0.929	-0.000
V_s	-1.213	-0.968	2.552	1.923	-0.742	-0.719
ΔP_v	-0.243	5.097	0.610	3.236	-0.115	0.557
Δv_t	-0.459	-10.533	2.277	3.727	-0.400	-1.635
Δi_t	1.131	-9.808	1.298	2.826	0.003	-1.302
$\mathbb{P}_j$	$\mathbb{P}_1$ =diag.(180,180)		$\mathbb{P}_2$ =diag.(2,2)		$\mathbb{P}_3$ =diag.(0.6,0.6)	

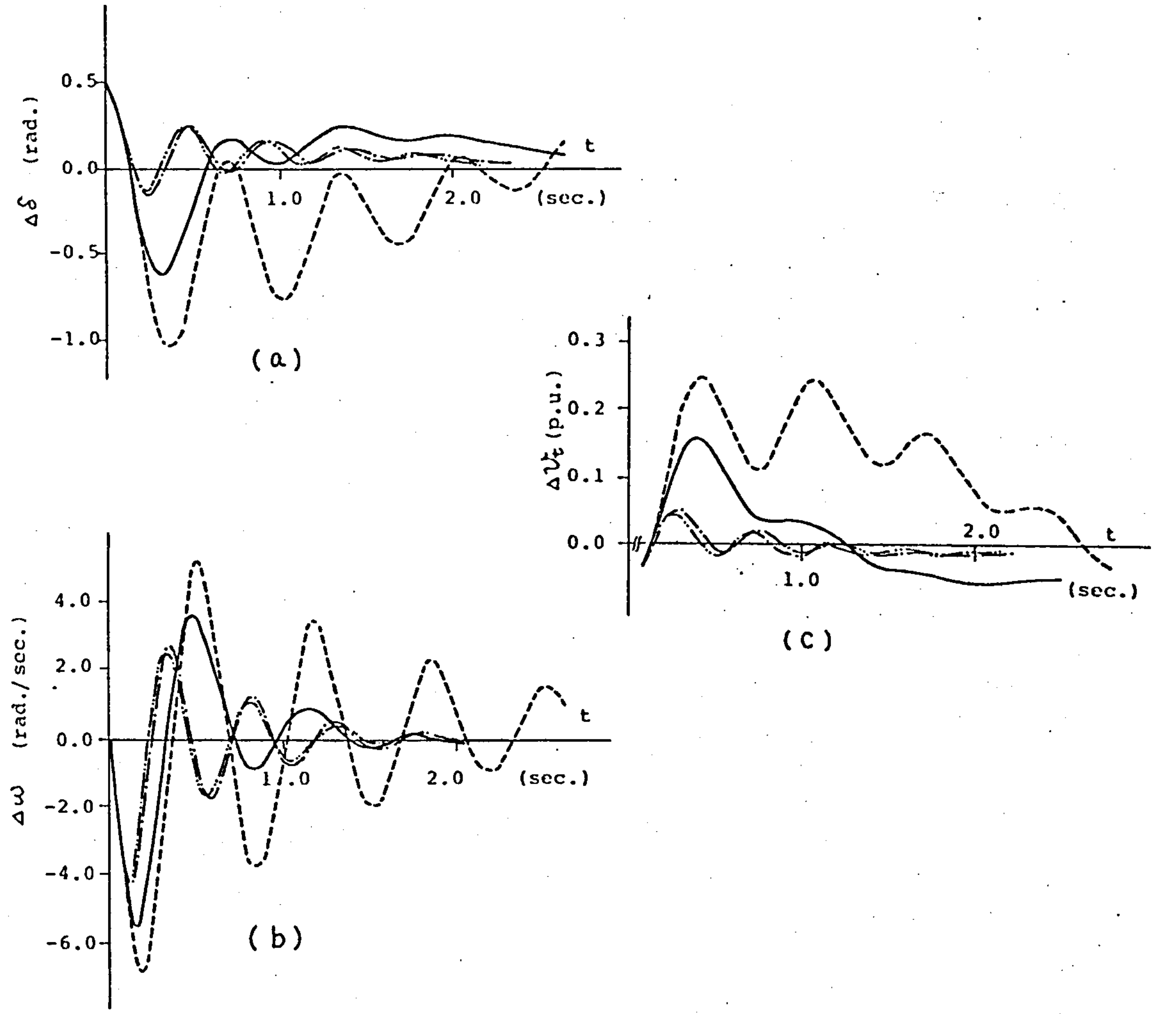
Table 5-5-11 Eigenvalues of closed-loop system

Case 1	Case 2	Case 3
$-0.936 \pm j0.987$	$-0.847 \pm j0.215\times10$	$-0.819 \pm j0.298\times10$
-0.175×10	-0.144×10	$-0.153\times10 \pm j0.139\times10^2$
$-0.213\times10 \pm j0.947\times10$	$-0.172\times10 \pm j0.135\times10^2$	-0.169×10
-0.234×10	-0.185×10	-0.190×10
-0.498×10	-0.962×10	-0.983×10
-0.103×10^2	-0.398×10^2	-0.569×10^2
-0.653×10^2	-0.107×10^3	-0.114×10^3
$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$	$-0.156\times10^3 \pm j0.202\times10^4$

The responses of the model system have been obtained by solving the closed-loop equation of the model system for the given initial deviations $\Delta\delta|_{t=0} = 0.5$ and $\Delta\Psi_{fd}|_{t=0} = 0.5$.

The responses are shown in Fig.5-5-6. As shown in this figure, the system performance is much improved by the output feedback controller obtained by the recursive calculation described above. And the improvement in case 3 is better than those in case 1 and case 2.

In Fig.5-5-7, the responses of the model system applied with various controllers are shown. It is evident that the system responses are much improved by the various output feedback controllers obtained in section 5-5-1, and in this section, but in comparison with that of the model system applied with the optimal state feedback controller obtained in section 4-3, the improvement of the system performance is a little smaller. Namely, these output feedback controllers act as suboptimal controllers for the model system.



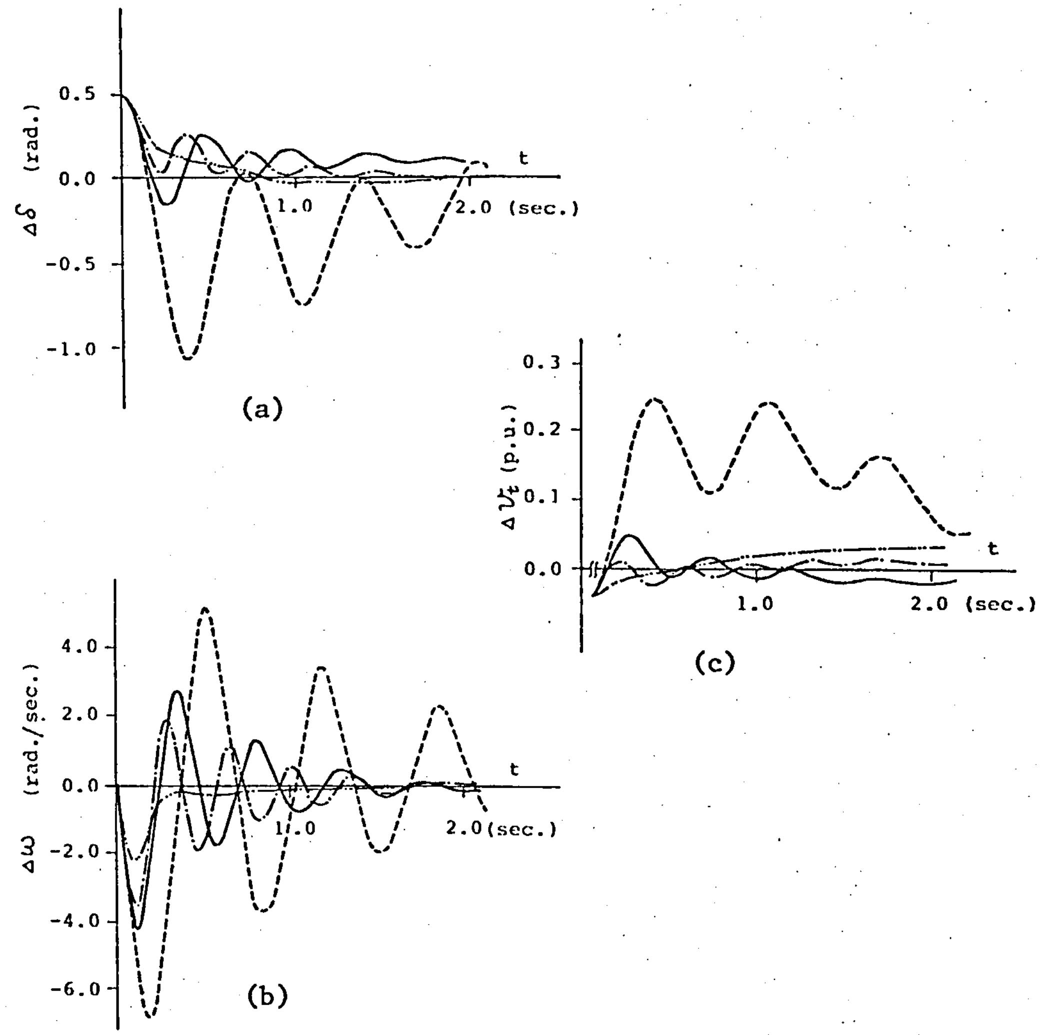
——— : with output feedback controller (case 1)

—·— : with output feedback controller (case 2)

—··— : with output feedback controller (case 3)

----- : without controller

Fig.5-5-6 Responses of the model system for the initial deviations $\Delta\delta|_{t=0}$= 0.5 and $\Delta\Psi_{fd}|_{t=0}$= 0.5



—— : with output feedback controller (case 2 in this section)

—·— : with output feedback controller (case 3 in section 5-5-1)

—··— : with state feedback optimal controller (case 2 in section 4-3)

----- : without controller

Fig.5-5-7 Responses of the model system applied with various controllers

Section 5-6. Summary

In this chapter, in order to construct a output feedback controller of the model system in terms of directly measurable output variables, the techniques of model reduction have been applied. The model reduction techniques used in this chapter are the state variable grouping technique and the eigenvalue grouping technique. In the numerical calculation, the 11-th order original system has been reduced to a 7-th order simplified system. By these model reduction, the dominant eigenvalues of the original system have been retained in the reduced system, so it has been assured that these techniques are useful to obtain a reduced system of a originally higher order system.

The procedures for utilizing the simplified model in deriving a output feedback controller have been also described in this chapter. The output feedback gains have been determined by the solution matrix of the matrix Riccati equation or Lyapunov's matrix equation of the reduced model system.

By these output feedback controllers the dynamic performance of the model system is much improved, but the improvement is a little smaller than that by the optimal state feedback controller. Furthermore, the originally unstable system has been stabilized by these output feedback controllers.

These output feedback controllers are constructed in terms of measurable output variables, so they are physically realizable and may be easily implemented.

CHAPTER 6 APPLICATION OF LYAPUNOV'S DIRECT METHOD TO CONTROL PROBLEM OF NON-LINEAR SYSTEM

The implementation of optimal controls, determined directly from a non-linear model through a standard optimization procedure, is extremely difficult[44] and thus few investigations regarding the practical realization of such control signals have been reported so far.

The possibility of obtaining a stabilizing controller for a power system using non-linear model will be demonstrated in this chapter. The direct method of Lyapunov is applied to determine the stabilizing controller for the non-linear system. Furthermore, the effectiveness of the proposed method is explained by the numerical analysis of the model multi-machine system.

Section 6-1. Introduction of Control Law through Lyapunov's Direct Method

Here, we consider the general case, where the system equation is represented by the following non-linear differential equation in vector form.

$$p\mathbf{x} = \mathbf{f}(\mathbf{x}) + \mathbf{B}\cdot\mathbf{u} \qquad (6\text{-}1)$$

where, $\mathbf{x}$: n-th order state variables vector

$\mathbf{u}$: r-th order control signals vector

$\mathbf{f}(\mathbf{x})$: n-th order non-linear functional vector

$\mathbf{B}$: $(n \times r)$ coefficient matrix

At the stable equilibrium point, it is assumed that the following relationships are satisfied.

$$\mathbf{f}(\mathbf{x}_0) = \mathbf{0} \quad , \quad \mathbf{u} = \mathbf{0} \qquad (6\text{-}2)$$

where, $\mathbf{x}_0$: stable equilibrium point

If we can obtain an appropriate Lyapunov function $V(x)$ for the given system described by eqn.(6-1), the control signals vector u can be determined as described below.

The time derivative of the Lyapunov function $V(x)$ along the trajectory expressed by eqn.(6-1) becomes:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x^T} \cdot f(x) + \frac{\partial V}{\partial x^T} \cdot B \cdot u \qquad (6\text{-}3)$$

where, $\frac{\partial V}{\partial x^T} = \left[\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \frac{\partial V}{\partial x_3}, \cdots\cdots, \frac{\partial V}{\partial x_n}\right]$

In order to improve the damping characteristic of the system, the control vector u is determined as follows:

$$u = - R^{-1} \cdot B^T \cdot \frac{\partial V}{\partial x^T} \qquad (6\text{-}4)$$

where, the matrix R is a $(r \times r)$ positive definite matrix.

Then, the time derivative of the Lyapunov function described by eqn.(6-3) becomes:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x^T} \cdot f(x) - u^T \cdot R \cdot u \qquad (6\text{-}5)$$

Now, let us suppose that the system starts from a point away from the equilibrium point and the system applied with no controller is asymptotically stable. Then, by the controller described by eqn.(6-4), the stable equilibrium point is approached faster since dV/dt is smaller due to the additional negative control term $-u^T \cdot R \cdot u$. Here, it is noted that the value of Lyapunov function is some measure of the distance from the stable equilibrium point.

Section 6-2. Determination of Control Law using Energy Function

In order to simplify the analysis following assumptions are made;

(1) Each machine in the system may be represented by a constant voltage behind a transient reactance.

(2) The mechanical angle of each machine rotor coincides with the electrical phase of the voltage behind the reactance.

Then, the dynamic performance of a n-machine power system can be described by the following electromechanical equations of motion.[45]

$$p\delta_j = \Delta\omega_j \qquad (j=1\sim n) \qquad (6\text{-}6)$$

$$p\Delta\omega_j = (P_{tj0} + \Delta P_{tj} - P_{ej} - P_{dj}\cdot\Delta\omega_j)/M_j \qquad (j=1\sim n) \qquad (6\text{-}7)$$

where, the subscript j denotes the j-th machine in the system, and ΔP_{tj} represents the deviation of mechanical input to the j-th machine rotor by the governor action of the j-th machine.

Here, the ideal governor, i.e. one which allows the prime-mover torque to be changed instantaneously, is considered and the deviation of the mechanical input to the j-th machine is considered as the control signal to the j-th machine.

$$u_j = \Delta P_{tj} \qquad (6\text{-}8)$$

From eqn.(6-6)-eqn.(6-8), the dynamic performance of the n-machine power system can be represented by the equation of the form described by eqn.(6-1).

One of the Lyapunov functions of this n-machine power system can be defined using the energy function of the system as follows:[46],[47]

$$V(\delta_1, \delta_2, \cdots\cdots, \delta_n, \Delta\omega_1, \Delta\omega_2, \cdots\cdots, \Delta\omega_n)$$
$$= \frac{1}{2M_T}\sum_{K=1}^{n-1}\sum_{j=K+1}^{n} M_j\cdot M_K\cdot(\Delta\omega_j - \Delta\omega_K)^2 + g(\delta_1, \delta_2, \cdots, \delta_n) \qquad (6\text{-}9)$$

where, $\quad M_T = \sum_{j=1}^{n} M_j$

The control signal u_j can be determined by the method described in section 6-1. The time derivative of the above Lyapunov function along the trajectory expressed by eqn.(6-6) and eqn.(6-7) becomes:

$$\frac{dV}{dt} = \sum_{j=1}^{n}\left(\frac{\partial V}{\partial\delta_j}\cdot\frac{d\delta_j}{dt} + \frac{\partial V}{\partial\Delta\omega_j}\cdot\frac{d\Delta\omega_j}{dt}\right) \qquad (6\text{-}10)$$

$$= \sum_{j=1}^{n} \left\{ \frac{\partial g}{\partial \delta_j} \cdot \Delta\omega_j + \frac{\partial V}{\partial \Delta\omega_j} \cdot (P_{tjo} - P_{ej} - P_{dj} \cdot \Delta\omega_j)/M_j \right\} + \sum_{j=1}^{n} \frac{\partial V}{\partial \Delta\omega_j} \cdot \frac{u_j}{M_j}$$

In order to improve the damping characteristic of the system, the control signal u_j is selected as:

$$u_j = -\frac{1}{R_j} \cdot \frac{\partial V}{\partial \Delta\omega_j} \cdot \frac{1}{M_j} \qquad (j=1\sim n) \qquad (6\text{-}11)$$

where, R_j : a positive constant

$$\frac{\partial V}{\partial \Delta\omega_j} = \frac{M_j}{M_T} \cdot \sum_{K=1}^{n} M_K \cdot (\Delta\omega_j - \Delta\omega_K)$$

Then, eqn.(6-10) becomes:

$$\frac{dV}{dt} = \sum_{j=1}^{n} \left\{ \frac{\partial g}{\partial \delta_j} \cdot \Delta\omega_j + \frac{\partial V}{\partial \Delta\omega_j} \cdot (P_{tjo} - P_{ej} - P_{dj} \cdot \Delta\omega_j)/M_j \right\} - \sum_{j=1}^{n} R_j \cdot u_j^2 \qquad (6\text{-}12)$$

As shown in the above equation, when the control signal u_j expressed by eqn.(6-11) is applied to the system, the equilibrium point is approached faster, since dV/dt is smaller due to the additional negative control term $-\sum_{j=1}^{n} R_j \cdot u_j^2$. Namely, the damping characteristic of the system can be improved by the control described by eqn.(6-11).

It is also possible to determine the control signal u_j as follows:

$$u_j = -u_{vj} \cdot sgn\left(\frac{\partial V}{\partial \Delta\omega_j}\right) \qquad (j=1\sim n) \qquad (6\text{-}13)$$

where, u_{vj} is a positive constant, and the control signal u_j is bounded by $|u_j| \leq u_{vj}$. By the control described above, the time derivative of the Lyapunov function becomes:

$$\frac{dV}{dt} = \sum_{j=1}^{n} \left\{ \frac{\partial g}{\partial \delta_j} \cdot \Delta\omega_j + \frac{\partial V}{\partial \Delta\omega_j} \cdot (P_{tjo} - P_{ej} - P_{dj} \cdot \Delta\omega_j)/M_j \right\} - \sum_{j=1}^{n} \left| \frac{\partial V}{\partial \Delta\omega_j} \right| \cdot \frac{u_{vj}}{M_j} \qquad (6\text{-}14)$$

As shown in eqn.(6-14), the damping characteristic of the system can also be improved by the controller described by eqn.(6-13), because dV/dt is smaller due to the additional negative control term $-\sum_{j=1}^{n} \left| \frac{\partial V}{\partial \Delta\omega_j} \right| \cdot \frac{u_{vj}}{M_j}$.

In order to improve the damping characteristic of the system, two types of controllers, i.e. eqn.(6-11) and eqn.(6-13), are proposed using the energy function as a Lyapunov function of the system. The control described by eqn.(6-11) is a proportional type control and the control described by eqn.(6-13) is a bang-bang type control.

Section 6-3. Application to a 3-machine Problem

The same 3-machine system shown in section 3-4 is used for the model multi-machine system. The system parameters and the initial conditions and the admittance matrices under several system conditions have already been shown in Table 3-4-1 ~ Table 3-4-3.

It is assumed that the internal induced voltage behind the transient reactance is constant for each machine. Namely, E'_{qj} has its steady state value during all the transient processes of the model system. Furthermore, each machine in the model system is the salient-pole one, so E'_{dj} becomes equal to zero during all the transient processes. Then the system equations are described by only the mechanical equations of motion of the form expressed by eqn.(6-6) and eqn.(6-7).

For the model 3-machine system, the control signal u_j ($= \Delta P_{tj}$) becomes:

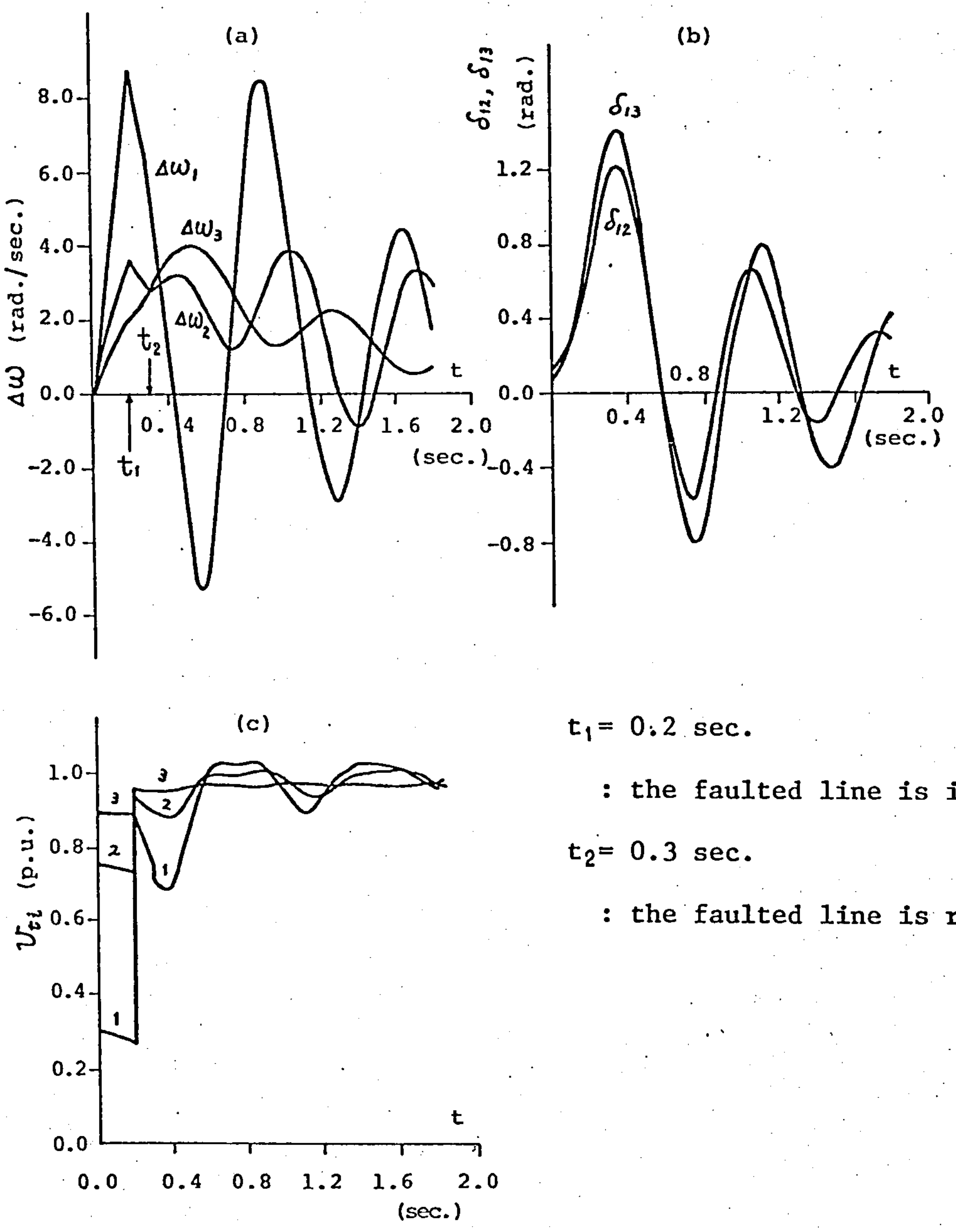
$$u_1 = -k_1 \cdot M_1 \cdot \{ M_2 \cdot (\Delta\omega_1 - \Delta\omega_2) + M_3 \cdot (\Delta\omega_1 - \Delta\omega_3) \}$$

$$u_2 = -k_2 \cdot M_2 \cdot \{ M_3 \cdot (\Delta\omega_2 - \Delta\omega_3) + M_1 \cdot (\Delta\omega_2 - \Delta\omega_1) \}$$

$$u_3 = -k_3 \cdot M_3 \cdot \{ M_1 \cdot (\Delta\omega_3 - \Delta\omega_1) + M_2 \cdot (\Delta\omega_3 - \Delta\omega_2) \}$$

where, $k_j = 1/R_j \cdot M_T$, $M_T = M_1 + M_2 + M_3$

The system responses have been obtained by solving the system equations using Runge-Kutta-Gill method under the following system conditions; (1) Three phase to ground fault occurrs at the point A in the model system at the time t=0.0 sec.. (2) The faulted line is isolated from the system by the circuit breakers at the time t=0.2 sec.. (3) The faulted line is reclosed at the time t=0.3 sec.. after clearing the fault.



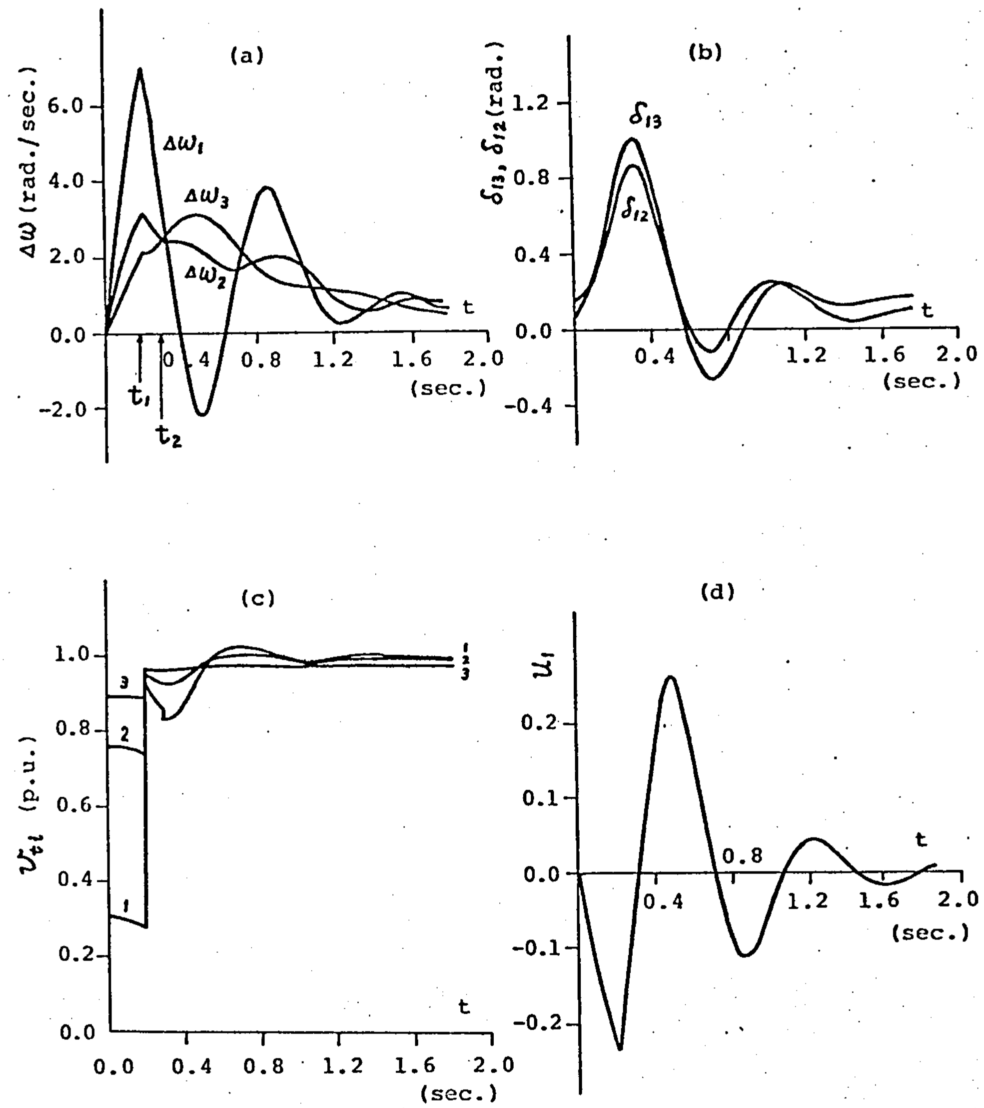
t_1 = 0.2 sec.

: the faulted line is isolated

t_2 = 0.3 sec.

: the faulted line is reclosed

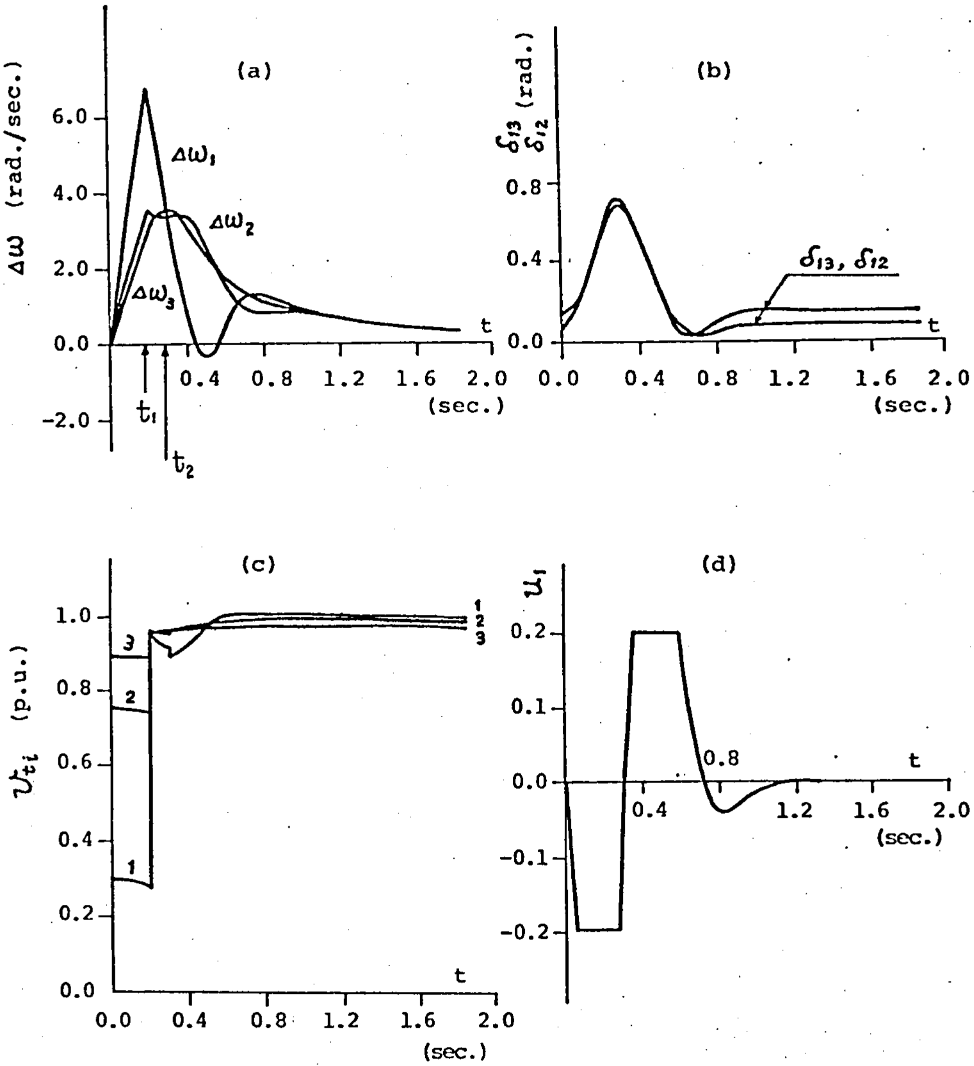
Fig.6-3-1 Responses of the model system applied with no controller



$t_1 = 0.2$ sec. : the faulted line is isolated

$t_2 = 0.3$ sec. : the faulted line is reclosed

Fig.6-3-2 Responses of the model system applied with the controller ($k_1 = 0.3$, $k_2 = 0.3$, $k_3 = 0.0$)



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

$|u_1| \leqq 0.2$, $|u_2| \leqq 0.2$, $|u_3| \leqslant 1.0$

Fig.6-3-3 Responses of the model system applied with the controller ($k_1 = 1.0$, $k_2 = 1.0$, $k_3 = 30.0$)

The responses of the model system applied with no controller are shown in Fig.6-3-1. The responses of the model system applied with the above controllers are shown in Fig.6-3-2 and Fig.6-3-3. In Fig.6-3-2, the positive constants k_1 , k_2 and k_3 have been selected as 0.3, 0.3 and 0.0, and in Fig.6-3-3 the positive constants k_1 , k_2 and k_3 have been selected as 1.0, 1.0 and 30.0, respectively. Furthermore, in Fig.6-3-3 the control signals u_1 , u_2 and u_3 have been bounded by $|u_1| \leq 0.2$, $|u_2| \leq 0.2$ and $|u_3| \leq 1.0$, so when the absolute values of the control signals u_1 , u_2 and u_3 are greater than 0.2, 0.2 and 1.0 respectively, the above control becomes a bang-bang type control and otherwise becomes a proportional type control.

As shown in these figures, the damping characteristic of the model system can be much improved by the above controller, and the improvement becomes greater for the greater values of the positive constants k_1 , k_2 and k_3 .

Section 6-4. Summary

In this chapter it has been shown that the Lyapunov's direct method is indeed applicable to the control problem of non-linear systems. A 3-machine power system has been used to test the effectiveness of the proposed method. But, it is noted that in order to simplify the analysis several assumptions have been made.

CHAPTER 7 IMPROVEMENT OF OVERALL STABILITY BY STABILIZING CONTROL

In the case of the linearized system, it is possible to obtain the controller as a linear function of the system variables as shown in the former chapters. Therefore, if the variables are measurable, the controller can be easily realized. However, it can not be guaranted that the controller obtained from the linearized system will always be applicable to the original non-linear system. In some cases such control, derived from the linearized system, when applied to the original non-linear system, results in undesirable system performance. Because, the controller is obtained from the small signal system equations, the improvement of the system performance is local in nature, and under large disturbance conditions, the system operation departs from the steady state operating point considerably and consequently the controller determined solely from the small signal consideration is inadequate in improving the system stability under these circumstances. Namely, in spite of its apparent advantages in implementation, the linearized analysis can not be considered fully satisfactory.

In this chapter, in order to improve the overall stability of the system the direct method of Lyapunov is applied as described in chapter 6, under the consideration of the results of the linearized analysis described in chapter 4 and chapter 5. The effectiveness of the proposed method is also shown by the numerical analysis of the model system.

Section 7-1. Introduction of Control Law using Krasovskii's Lyapunov Function

As shown in eqn.(6-1), the original non-linear equation becomes:

$$p\mathbf{x} = \mathbb{f}(\mathbf{x}) + \mathbb{B}\cdot\mathbf{u} \qquad (7\text{-}1)$$

where, $\mathbb{x}$ is the n-th order state variables vector, $\mathbb{u}$ is the r-th order control signals vector , $\mathbb{f}(\mathbb{x})$ is the n-th order non-linear functional vector and the matrix $\mathbb{B}$ is the $(n \times r)$ coefficient matrix.

The Lyapunov function of the above system is defined by the method of Krasovskii as follows:[48]

$$V(\mathbb{x}) = \frac{1}{2} \cdot \mathbb{f}(\mathbb{x})^T \cdot \mathbb{K} \cdot \mathbb{f}(\mathbb{x}) \tag{7-2}$$

where, $\mathbb{K}$: a $(n \times n)$ positive definite matrix

At the stable equilibrium point, i.e. at $\mathbb{x} = \mathbb{x}_o$, the value of the above Lyapunov function becomes equal to zero.

The time derivative of the above Lyapunov function along the trajectory expressed by eqn.(7-1) becomes:

$$\frac{dV}{dt} = \frac{1}{2} \cdot \mathbb{f}(\mathbb{x})^T \cdot (\mathbb{J}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{J}) \cdot \mathbb{f}(\mathbb{x}) + \frac{1}{2} \cdot \mathbb{u}^T \cdot \mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(\mathbb{x}) + \frac{1}{2} \cdot \mathbb{f}(\mathbb{x})^T \cdot \mathbb{K} \cdot \mathbb{J} \cdot \mathbb{B} \cdot \mathbb{u} \tag{7-3}$$

In order to improve the damping characteristic of the system, the control signal $\mathbb{u}$ is determined by the following equation.

$$\mathbb{u} = -\mathbb{F}(\mathbb{x}) \cdot \mathbb{f}(\mathbb{x}) , \quad \mathbb{F}(\mathbb{x}) = \mathbb{R}^{-1} \cdot \mathbb{B}^T \cdot \mathbb{J} \cdot \mathbb{K} \tag{7-4}$$

Then eqn.(7-3) becomes:

$$\frac{dV}{dt} = \frac{1}{2} \cdot \mathbb{f}(\mathbb{x})^T \cdot (\mathbb{J}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{J}) \cdot \mathbb{f}(\mathbb{x}) - \mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u} \tag{7-5}$$

If the matrix $-(\mathbb{J}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{J})$ becomes positive definite, the original uncontrolled system becomes asymptotically stable and the controller described by eqn. (7-4) improves the damping characteristic of the system because of the additional negative control term $-\mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u}$.

In the above equations, the matrix $\mathbb{J}$ is a. $(n \times n)$ functional matrix designated as Jacobian matrix and each term of this Jacobian matrix is a function of the state variables of the system. And the matrix $\mathbb{J}$ becomes:

$$\mathbb{J} = \frac{\partial \mathbb{f}}{\partial x^T} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, & \frac{\partial f_1}{\partial x_2}, & \frac{\partial f_1}{\partial x_3}, & \cdots\cdots & , \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1}, & \frac{\partial f_2}{\partial x_2}, & \frac{\partial f_2}{\partial x_3}, & \cdots\cdots & , \frac{\partial f_2}{\partial x_n} \\ & & & & \\ \frac{\partial f_n}{\partial x_1}, & \frac{\partial f_n}{\partial x_2}, & \frac{\partial f_n}{\partial x_3}, & \cdots\cdots & , \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad (7\text{-}6)$$

Therefore, the feedback gain matrix $\mathbb{F}(x)$ becomes a $(r \times n)$ functional matrix of the state variables of the system.

It is also possible to determine the control vector $\mathbb{U}$ as follows:

$$U_j = -U_{vj} \cdot sgn[\mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(x)]_j \qquad (j=1 \sim r) \qquad (7\text{-}7)$$

where, U_{vj} is a positive constant and the control signal U_j is bounded by $|U_j| \leq U_{vj}$, and the symbol $[\cdot]_j$ refers to the j-th component of the column matrix $[\cdot]$.

Introducing the vector function with vector argument $\mathbb{sgn}$, we obtain:

$$\mathbb{U}^T = -\mathbb{U}_v^T \cdot \mathbb{sgn}[\mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(x)] \qquad (7\text{-}8)$$

where, $\mathbb{U}_v = [U_{v1}, U_{v2}, \cdots\cdots, U_{vr}]^T$

$$\mathbb{sgn}[\mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(x)] = \text{diag.}[\cdots\cdots, sgn[\mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(x)]_j, \cdots\cdots]$$

By the controller described by eqn.(7-7) or eqn.(7-8), the time derivative of the Lyapunov function (7-2) becomes:

$$\frac{dV}{dt} = \frac{1}{2} \cdot \mathbb{f}(x)^T \cdot (\mathbb{J}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{J}) \cdot \mathbb{f}(x) - |\mathbb{f}(x)^T \cdot \mathbb{K} \cdot \mathbb{J} \cdot \mathbb{B}| \cdot \mathbb{U}_v \qquad (7\text{-}9)$$

As shown in the above equation, the controller described by eqn.(7-7) or eqn.(7-8) can improve the damping characteristic of the system because of the additional negative control term $-|\mathbb{f}(x)^T \cdot \mathbb{K} \cdot \mathbb{J} \cdot \mathbb{B}| \cdot \mathbb{U}_v$.

The effectiveness of the above two controllers expressed by eqn.(7-4) and eqn.(7-7) or eqn.(7-8) for the system can also explained as described below.

It is assumed that the performance index of the system will be defined as:

$$I(\mathbb{u}) = \frac{1}{2}\int_0^\infty \mathbb{f}(x)^T \cdot \mathbb{N} \cdot \mathbb{f}(x)\, dt \tag{7-10}$$

where, the matrix $\mathbb{N}$ satisfies the following relationship:

$$\mathbb{N} = -(\mathbb{J}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{J}) \tag{7-11}$$

Then the matrix $\mathbb{N}$ is a $(n \times n)$ functional matrix since the components of the Jacobian matrix $\mathbb{J}$ are the function of the state variables of the system.

If the matrix $\mathbb{N}$ becomes positive definite for a given initial system condition, the original uncontrolled system becomes asymptotically stable and eqn.(7-10) can be considered as the performance index of the system. Furthermore, we get the following relationship.

$$I(\mathbb{u}) = V(x)|_{t=0} + \int_0^\infty \mathbb{u}^T \cdot \mathbb{B}^T \cdot \mathbb{J}^T \cdot \mathbb{K} \cdot \mathbb{f}(x)\, dt \tag{7-12}$$

When the system is applied with the above two controllers expressed by eqn.(7-4) and eqn.(7-7) or eqn.(7-8), eqn.(7-12) becomes respectively as follows:

$$I(\mathbb{u}) = V(x)|_{t=0} - \int_0^\infty \mathbb{u}^T \cdot \mathbb{R} \cdot \mathbb{u}\, dt \tag{7-13}$$

$$I(\mathbb{u}) = V(x)|_{t=0} - \int_0^\infty |\mathbb{f}(x)^T \cdot \mathbb{K} \cdot \mathbb{J} \cdot \mathbb{B}| \cdot \mathbb{u}_v \tag{7-14}$$

As shown in the above two equations, the value of the performance index is smaller due to the additional negative control term $-\int_0^\infty \mathbb{u}^T \mathbb{R} \cdot \mathbb{u}\, dt$ or $-\int_0^\infty |\mathbb{f}(x)^T \cdot \mathbb{K} \cdot \mathbb{J} \cdot \mathbb{B}| \cdot \mathbb{u}_v$, conseguently the system performance can be improved by the above two controllers.

The controllers proposed, i.e. eqn.(7-4) and eqn.(7-7) or eqn.(7-8), are respectively a proportional type controller and a bang-bang type controller.

In this section, the controllers are determined by the Lyapunov function defined by Krasovskii, here it is noted that Krasovskii's theorem offers a sufficient condition on the asymptotic stability of the non-linear system, so the stable region obtained by this theorem is narrower than the practical stable region.

Section 7-2. Determination of Feedback Gain Matrix

The stabilizing controllers are determined by the method described in section 7-1 for the model systems shown later. In these model systems, the following relationship is satisfied.

$$\mathbb{B}^T \cdot \mathbb{J}^T = \mathbb{B}^T \cdot \mathbb{A}^T = \mathbb{B}_o^T \tag{7-15}$$

where, $\mathbb{A} = \mathbb{J}|_{x=x_o} = \partial \mathbb{f}/\partial x^T|_{x=x_o}$, x_o : stable equilibrium point

Consequently, eqn.(7-4) and eqn.(7-8) become:

$$\mathbb{u} = - \mathbb{F} \cdot \mathbb{f}(x) \quad , \quad \mathbb{F} = \mathbb{R}^{-1} \cdot \mathbb{B}_o^T \cdot \mathbb{K} \tag{7-16}$$

$$\mathbb{u}^T = - \mathbb{u}_v^T \cdot \mathrm{sgn}[\mathbb{R} \cdot \mathbb{F} \cdot \mathbb{f}(x)] \tag{7-17}$$

In the above two equations, the feedback gain matrix $\mathbb{F}$ becomes a constant matrix.

7-2-1. Complete Feedback Stabilizing Controller[49]

In order to determine the feedback gain matrix $\mathbb{F}$, it is necessary to choose an appropriate positive definite matrix $\mathbb{K}$. It should be noted that selecting the (n x n) unit matrix as the matrix $\mathbb{K}$ will often lead to success, but in this section we define the solution matrix of the following matrix Riccati equation as the matrix $\mathbb{K}$ under consideration of the results of the analysis of the linearized system shown in the former chapters.

$$\mathbb{A}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{A} - \mathbb{K} \cdot \mathbb{B}_o \cdot \mathbb{R}^{-1} \cdot \mathbb{B}_o^T \cdot \mathbb{K} + \mathbb{Q} = \mathbb{0} \tag{7-18}$$

where, the matrix $\mathbb{Q}$ is a positive definite or a positive semi-definite matrix.

When the disturbaces are sufficiently small, the original non-linear system described by eqn.(7-1) can be approximated by the following linearized system around the operating point $x = x_o$.

$$p\mathbb{X} = \mathbb{A}\cdot\mathbb{X} + \mathbb{B}_o\cdot\mathbb{u} \quad (7\text{-}19)$$

where, $\mathbb{X}$: n-th order state variables vector

$\mathbb{u}$: r-th order control signals vector

$\mathbb{A}, \mathbb{B}_o$: $(n \times n)$ and $(n \times r)$ coefficient matrices

$\mathbb{X} = \mathbb{A}\cdot\Delta\mathbb{x}$

$\Delta\mathbb{x} = \mathbb{x} - \mathbb{x}_o$

$\mathbb{B}_o = \mathbb{A}\cdot\mathbb{B}$

$\mathbb{A} = \mathbb{J}\,|_{\mathbb{x}=\mathbb{x}_o} = \partial\mathbb{f}/\partial\mathbb{x}^T\,|_{\mathbb{x}=\mathbb{x}_o}$

The Tailor expansion of $\mathbb{f}(\mathbb{x})$ around the operating point becomes:

$$\mathbb{f}(\mathbb{x}) = \mathbb{f}(\mathbb{x}_o) + \partial\mathbb{f}(\mathbb{x})/\partial\mathbb{x}^T\,|_{\mathbb{x}=\mathbb{x}_o}\cdot\Delta\mathbb{x} + \left(\begin{matrix}\text{higher terms}\\ \text{of } \Delta\mathbb{x}\end{matrix}\right) \quad (7\text{-}20)$$

So, the state variable $\mathbb{X}$ is the first approximation of $\mathbb{f}(\mathbb{x})$ around the operating point, and for sufficiently small $\Delta\mathbb{x}$, the following relationship is satisfied:

$$\mathbb{X} = \mathbb{f}(\mathbb{x}) \quad (7\text{-}21)$$

Then, eqn.(7-16) becomes:

$$\mathbb{u} = -\mathbb{F}\cdot\mathbb{X} \quad , \quad \mathbb{F} = \mathbb{R}^{-1}\cdot\mathbb{B}_o^T\cdot\mathbb{K} \quad (7\text{-}22)$$

The above controller minimizes the following quadratic performance index of the linearized system described by eqn.(7-19).

$$J = \frac{1}{2}\int_0^\infty (\mathbb{X}^T\cdot\mathbb{Q}\cdot\mathbb{X} + \mathbb{u}^T\cdot\mathbb{R}\cdot\mathbb{u})\,dt \quad (7\text{-}23)$$

Furthermore, the above controller also minimizes the expected value of J, i.e. the value of $\mathrm{Tr}(\mathbb{K})$.

As described above, the feedback gain matrix $\mathbb{F}$ of the controller expressed by eqn.(7-16) or eqn.(7-17) becomes the optimal state feedback gain matrix for the linearized system. Then, for sufficiently small disturbances the controller described by eqn.(7-16) and eqn.(7-18) becomes the optimal state feedback controller, and the system performance can be improved by the controller. Furthermore, with the reason described in section 7-1 for large disturbances the controller can also improve the system performance. Consequently, it may be possible to improve the overall stability of the original non-linear system by the controller described by eqn.(7-16), eqn.(7-17) and eqn.(7-18).

In this thesis, the controller is designated 'complete feedback stabilizing controller'. In order to construct the controller, it is necessary that all the system states f_j (j=1~n) are measurable. But, in practical power system it is almost impossible to measure all the system states, so it is necessary to construct the stabilizing controller using only the measurable system states, i.e. 'incomplete feedback stabilizing controller'.

7-2-2. Incomplete Feedback Stabilizing Controller[49]

As shown in the above section, the feedback gain matrix of the complete feedback stabilizing controller is equivalent to the optimal state feedback gain matrix for the linearized system.

In order to determine the feedback gain matrix of the incomplete feedback stabilizing controller of the form described by eqn.(7-16) or eqn.(7-17), the theoretical results which can be used to determine the optimal output feedback gain matrix for the linearized system are represented briefly. These results have already been represented by W.S.Levine and M.Athans,[50],[51] but here these results are extended as follows.

For the linearized system described by eqn.(7-19), it is assumed that each element of the control vector u_j (j=1~r) is represented by the linear combination of r_j outputs w_{jk} (k=1~r_j):

$$u_j = -\sum_{K=1}^{r_j} h_{jK} \cdot w_{jk} = -\mathbb{h}_j \cdot \mathbb{w}_j \qquad (j=1\sim r) \qquad (7\text{-}24)$$

$$\mathbb{w}_j = \mathbb{C}_j \cdot \mathbb{X} \qquad (j=1\sim r) \qquad (7\text{-}25)$$

where, the vector $\mathbb{h}_j$ is a r_j - dimensional row vector, and $\mathbb{C}_j$ is a (r_j x n) coefficient matrix. From the above two equations, the control vector $\mathbb{U}$

for the linearized system becomes:

$$\mathbb{U} = -\mathbb{F}\cdot\mathbb{X} \tag{7-26}$$

where, $\mathbb{F} = \begin{bmatrix}\mathbb{F}_1\\ \mathbb{F}_2\\ \vdots \\ \mathbb{F}_r\end{bmatrix}$, $\mathbb{F}_j = \mathbb{h}_j\cdot\mathbb{C}_j$ (j=1∼r)

The expected value of the quadratic performance index J described by eqn.(7-23) becomes:

$$\hat{J} = \frac{1}{2n}\mathrm{Tr}\left[\int_0^\infty exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T t\}\cdot(\mathbb{Q}+\mathbb{F}^T\cdot\mathbb{R}\cdot\mathbb{F})\cdot exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})t\}dt\right] \tag{7-27}$$

For simplicity, the constant $1/2n$ has been droped from eqn.(7-27), then the performance index, which has to be minimized, becomes:

$$\hat{J} = \mathrm{Tr}\left[\int_0^\infty exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T t\}\cdot(\mathbb{Q}+\mathbb{F}^T\cdot\mathbb{R}\cdot\mathbb{F})\cdot exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})t\}dt\right] \tag{7-28}$$

Here, we derivate eqn.(7-28) with respect to $\mathbb{h}_j$; the parameter sensitivity of $\hat{J}$ with respect to $\mathbb{h}_j$ becomes:

$$\frac{\partial\hat{J}}{\partial\mathbb{h}_j} = (\mathbb{r}_j\cdot\mathbb{F} - \mathbb{b}_j^T\cdot\mathbb{K})\cdot\mathbb{L}\cdot\mathbb{C}_j^T \qquad (j=1\sim r) \tag{7-29}$$

where, $\mathbb{r}_j$ is the j-th row of the matrix $\mathbb{R}$ and $\mathbb{b}_j$ is the j-th column of the matrix $\mathbb{B}_o$ and the matrices $\mathbb{K}$ and $\mathbb{L}$ become:

$$\mathbb{K} = \int_0^\infty exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T t\}\cdot(\mathbb{Q}+\mathbb{F}^T\cdot\mathbb{R}\cdot\mathbb{F})\cdot exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})t\}\,dt \tag{7-30}$$

$$\mathbb{L} = \int_0^\infty exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})t\}\cdot exp\{(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T t\}\,dt \tag{7-31}$$

Alternatively, the matrices $\mathbb{K}$ and $\mathbb{L}$ are the positive definite solutions of the following Lyapunov's matrix equations.

$$(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T\cdot\mathbb{K} + \mathbb{K}\cdot(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F}) + \mathbb{Q} + \mathbb{F}^T\cdot\mathbb{R}\cdot\mathbb{F} = \mathbb{0} \tag{7-32}$$

$$(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})\cdot\mathbb{L} + \mathbb{L}\cdot(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T + \mathbb{I} = \mathbb{0} \tag{7-33}$$

Here, it is assumed that the matrix $\mathbb{R}$ is a diagonal matrix shown below.

$$\mathbb{R} = \begin{bmatrix}\mathbb{r}_1\\ \mathbb{r}_2\\ \vdots\\ \mathbb{r}_r\end{bmatrix} = \mathrm{diag.}\,[r_1, r_2, \cdots\cdots, r_r] \tag{7-34}$$

Then the optimal parameter $\mathbb{H}_j$ is given by $\partial \hat{J}/\partial \mathbb{H}_j = \mathbb{0}$:

$$\mathbb{H}_j = \frac{1}{r_j} \cdot \mathbb{b}_j^T \cdot \mathbb{K} \cdot \mathbb{L} \cdot \mathbb{C}_j^T \cdot (\mathbb{C}_j \cdot \mathbb{L} \cdot \mathbb{C}_j^T)^{-1} \qquad (j=1\sim r) \qquad (7\text{-}35)$$

$$\mathbb{F}_j = \frac{1}{r_j} \cdot \mathbb{b}_j^T \cdot \mathbb{K} \cdot \mathbb{L} \cdot \mathbb{C}_j^T \cdot (\mathbb{C}_j \cdot \mathbb{L} \cdot \mathbb{C}_j^T)^{-1} \cdot \mathbb{C}_j \qquad (j=1\sim r) \qquad (7\text{-}36)$$

By the controller described above, the performance index $\hat{J}$ described by eqn.(7-28) becomes:

$$\hat{J} = Tr(\mathbb{K}) \qquad (7\text{-}37)$$

There are several remarks about the above theorm; First, it is noted that if we assume $\mathbb{C}_j^{-1}$ exists for $j=1\sim r$, eqn.(7-36) reduces to:

$$\mathbb{F}_j = \frac{1}{r_j} \cdot \mathbb{b}_j^T \cdot \mathbb{K} \qquad (j=1\sim r) \qquad (7\text{-}38)$$

i.e. $\quad \mathbb{F} = \mathbb{R}^{-1} \cdot \mathbb{B}_o^T \cdot \mathbb{K}$

Furthermore, eqn.(7-32) becomes the matrix Riccati equation described by eqn.(7-18), namely the above output feedback controller becomes the optimal state feedback controller for the linearized system. Second, eqn.(7-32), eqn.(7-33) and eqn.(7-36) show the necessary conditions for optimality.

The incomplete feedback stabilizing controller is constructed as follows using the optimal output feedback gain matrix for the linearized system obtained by eqn.(7-32), eqn.(7-33) and eqn.(7-36).

$$\mathcal{U} = -\mathbb{F} \cdot \mathbb{f}(x) \qquad (7\text{-}39)$$

7-2-3. Iterative Algorithm for Determination of Complete or Incomplete Feedback Gain Matrix

As shown in section 7-2-1 and section 7-2-2, the feedback gain matrix of the complete or incomplete feedback stabilizing controller is selected as the optimal state feedback gain matrix or the optimal output feedback gain matrix for the linearized system described by eqn.(7-19).

The feedback gain matrix $\mathbb{F}$,which minimizes the value of Tr($\mathbb{K}$), is researched,[52),(53)] thus the problem can be considered as:

$$\min_{\mathbb{F}} \hat{J} = \min_{\mathbb{F}} \mathrm{Tr}(\mathbb{K})$$

subject to

$$(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})^T\cdot\mathbb{K}+\mathbb{K}\cdot(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})+\mathbb{Q}+\mathbb{F}^T\cdot\mathbb{R}\cdot\mathbb{F}=\mathbb{0}$$

Then, the feedback gain matrix $\mathbb{F}$,which minimizes the value of Tr($\mathbb{K}$), is determined iteratively in the following steps.

Step 1: Assume an initial value of the feedback gain matrix $\mathbb{F}^{(o)}$ such that the linearized closed-loop system $p\mathbb{X}=(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F})\mathbb{X}$ becomes stable.

Step 2: In order to determine the matrices $\mathbb{K}^{(i)}$ and $\mathbb{L}^{(i)}$, solve eqn.(7-32) and eqn.(7-33), i.e.

$$(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F}^{(i)})^T\cdot\mathbb{K}^{(i)}+\mathbb{K}^{(i)}\cdot(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F}^{(i)})+\mathbb{Q}+\mathbb{F}^{(i)T}\cdot\mathbb{R}\cdot\mathbb{F}^{(i)}=\mathbb{0}$$

$$(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F}^{(i)})\cdot\mathbb{L}^{(i)}+\mathbb{L}^{(i)}\cdot(\mathbb{A}-\mathbb{B}_o\cdot\mathbb{F}^{(i)})^T+\mathbb{I}=\mathbb{0}$$

Step 3: Determine the direction of correction $\mathbb{D}^{(i)}$ for matrix $\mathbb{F}^{(i)}$ using the following equation.[54),(55]

$$\mathbb{D}^{(i)}=\overline{\mathbb{F}}^{(i)}-\mathbb{F}^{(i)}$$

where, $\overline{\mathbb{F}}^{(i)}$ is determined by eqn.(7-36), i.e.

$$\overline{\mathbb{F}}^{(i)}=\frac{1}{r_j}\cdot\mathbb{b}_j^T\cdot\mathbb{K}^{(i)}\cdot\mathbb{L}^{(i)}\cdot\mathbb{C}_j^T\cdot(\mathbb{C}_j\cdot\mathbb{L}^{(i)}\mathbb{C}_j^T)^{-1}\cdot\mathbb{C}_j$$

Step 4: Correct the feedback gain matrix $\mathbb{F}^{(i)}$ by the following equation.

$$\mathbb{F}^{(i+1)}=\mathbb{F}^{(i)}+\alpha^{(i)}\cdot\mathbb{D}^{(i)}$$

$$\alpha^{(i)}=\min_{\alpha}\mathrm{Tr}(\mathbb{K})$$

Step 5: Calculate $\|\nabla\hat{J}\|$ $(=\sum_{j=1}^{r}\|\partial\hat{J}/\partial\mathbb{h}_j^{(i)}\|)$ by eqn.(7-29).

Step 6: Check of convergence by $\|\nabla\hat{J}\|\leq\varepsilon$.

If the step 6 is not satisfied, return to the step 2.

Section 7-3. Application to a One-machine Problem

The power system under investigation consists of a synchronous machine unit connected to an infinite bus through a transmission line as shown in Fig.7-3-1.

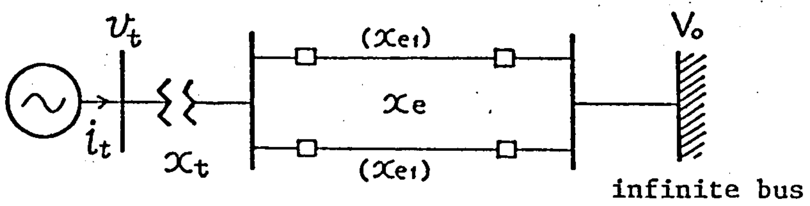


Fig.7-3-1 Model of one machine system

It has both voltage regulator and speed governor. The models shown in Fig. 2-3-2(b) and Fig.2-4-2(b) are used for these control systems, and the additional control signals u_1 and u_2 are determined by the method described in section 7-1 and section 7-2.

7-3-1. Non-linear First Order Differential Equations of Model System

The mathematical representation of synchronous machine described in section 2-2 is used in order to obtain the original non-linear equations of the model system.

The synchronous machine equations (2-26)-(2-29), (2-32) and (2-33) are rearranged as follows:

$$p\delta = \Delta\omega \qquad (7\text{-}40)$$

$$p\Delta\omega = (P_t - P_e - P_d \cdot \Delta\omega)/M \qquad (7\text{-}41)$$

$$pE_q' = \{E_{fd} - E_q' - (x_d - x_d') \cdot i_d\}/T_{do}' \qquad (7\text{-}42)$$

$$pE_d' = \{ \qquad - E_d' + (x_q - x_q') \cdot i_q\}/T_{qo}' \qquad (7\text{-}43)$$

where, $P_t = P_{to} + \Delta P_t$, $E_{fd} = E_{fdo} + \Delta E_{fd}$, $P_e = v_d \cdot i_d + v_q \cdot i_q$

From eqn.(2-35), the voltage regulator action becomes:

$$p\Delta E_{fd} = \{-K_f \cdot (\Delta v_t + V_s) - \Delta E_{fd}\}/T_f + K_f \cdot u_1/T_f \qquad (7\text{-}44)$$

$$pV_s = K_s\cdot\{-K_f\cdot(\Delta v_t + V_s) - \Delta E_{fd}\}/T_f\cdot T_s - V_s/T_s + K_f\cdot K_s\cdot u_1/T_f\cdot T_s \qquad (7\text{-}45)$$

where, $v_t = \sqrt{v_d^2 + v_q^2}$, $\Delta v_t = v_t - V_r$

$$\Delta E_{fd} = \begin{cases} \Delta E_{fdmax} & \text{for} \quad \Delta E_{fd} \geqslant \Delta E_{fdmax} \\ \Delta E_{fd} & \text{for} \quad \Delta E_{fdmin} < \Delta E_{fd} < \Delta E_{fdmax} \\ \Delta E_{fdmin} & \text{for} \quad \Delta E_{fd} \leqslant \Delta E_{fdmin} \end{cases}$$

From eqn.(2-38), the governor action becomes:

$$p\Delta P_v = (-K_g\cdot\Delta\omega/\omega_o - \Delta P_v)/T_g + K_g\cdot u_2/T_g \qquad (7\text{-}46)$$

$$p\Delta P_t = (\Delta P_v - \Delta P_t)/T_h \qquad (7\text{-}47)$$

where, $$\Delta P_v = \begin{cases} \Delta P_{vmax} & \text{for} \quad \Delta P_v \geqslant \Delta P_{vmax} \\ \Delta P_v & \text{for} \quad \Delta P_{vmin} < \Delta P_v < \Delta P_{vmax} \\ \Delta P_{vmin} & \text{for} \quad \Delta P_v \leqslant \Delta P_{vmin} \end{cases}$$

The terminal voltage and current become :

$$v_d = E_d' + x_q'\cdot i_q \qquad (7\text{-}48)$$

$$v_q = E_q' - x_d'\cdot i_d \qquad (7\text{-}49)$$

The equations of the transmission network become as follows in the steady state system configuration.

$$i_d = (E_q' - V_o\cdot\cos\delta)/(x_d' + x_t + x_e) \qquad (7\text{-}50)$$

$$i_q = (-E_d' + V_o\cdot\sin\delta)/(x_q' + x_t + x_e) \qquad (7\text{-}51)$$

For large disturbances in the model system various system conditions are considered as shown in Fig.7-3-2. For these conditions the equations of the transmission network become :

(a) $$i_d = E_q'/(x_d' + x_t) \qquad (7\text{-}52)$$

$$i_q = -E_d'/(x_q' + x_t) \qquad (7\text{-}53)$$

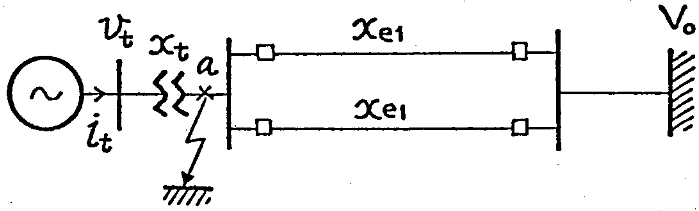
(b) $$i_d = \{E_q'\cdot(x_{e1}+x_{e2}) - x_{e2}\cdot V_o\cdot\cos\delta\}/\{(x_d'+x_t)\cdot(x_{e1}+x_{e2}) + x_{e1}\cdot x_{e2}\} \qquad (7\text{-}54)$$

$$i_q = \{-E_d' \cdot (x_{e1} + x_{e2}) + x_{e2} \cdot V_o \cdot \sin\delta\} / \{(x_q' + x_t) \cdot (x_{e1} + x_{e2}) + x_{e1} \cdot x_{e2}\} \qquad (7\text{-}55)$$

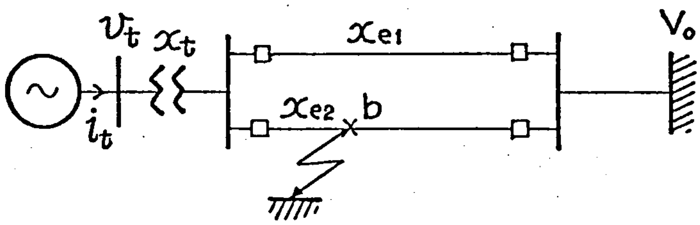
(c) $$i_d = (E_q' - V_o \cdot \cos\delta) / (x_d' + x_t + x_{e1}) \qquad (7\text{-}56)$$

$$i_q = (-E_d' + V_o \cdot \sin\delta) / (x_q' + x_t + x_{e1}) \qquad (7\text{-}57)$$

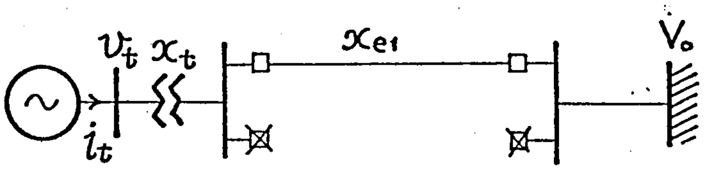
In the above equations, the armature and the transmission line resistances are neglected.



(a) Three phase to ground fault at the point a



(b) Three phase to ground fault at the point b



(c) One line operation

Fig.7-3-2 Various system conditions

From the above equations, the original non-linear equations of the model system can be described in vector form shown in eqn.(7-1), where the state variables vector $\mathbf{x}$, the control signals vector $\mathbf{u}$ and the non-linear functional vector $\mathbf{f}(\mathbf{x})$ become:

$$\mathbf{x} = [\delta, \Delta\omega, E_q', E_d', \Delta E_{fd}, V_s, \Delta P_v, \Delta P_t]^T \quad (7\text{-}58)$$

$$\mathbf{u} = [u_1, u_2]^T \quad (7\text{-}59)$$

$$\mathbf{f}(\mathbf{x}) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8]^T \quad (7\text{-}60)$$

where, $f_1 = p\delta$, $f_2 = p\Delta\omega$, $f_3 = pE_q'$, $f_4 = pE_d'$

$f_5 = \{-K_f\cdot(\Delta V_t + V_s) - \Delta E_{fd}\}/T_f$, $f_6 = K_s\{-K_f\cdot(\Delta V_t + V_s) - \Delta E_{fd}\}/T_f\cdot T_s - V_s/T_s$

$f_7 = (-K_g\cdot\Delta\omega/\omega_0 - \Delta P_v)/T_g$, $f_8 = p\Delta P_t$

7-3-2. Linearized Equations of Model System

In order to obtain the feedback gain matrix of the stabilizing controller, we need the following linearized equations around the operating point of the model system under the steady state system configuration.

From eqns.(7-40)-(7-51):

$$p\Delta\delta = \Delta\omega \quad (7\text{-}61)$$

$$p\Delta\omega = (\Delta P_t - \Delta P_e - P_d\cdot\Delta\omega)/M \quad (7\text{-}62)$$

$$p\Delta E_q' = \{\Delta E_{fd} - \Delta E_q' - (x_d - x_d')\cdot\Delta i_d\}/T_{do}' \quad (7\text{-}63)$$

$$p\Delta E_d' = \{-\Delta E_d' + (x_q - x_q')\cdot\Delta i_q\}/T_{qo}' \quad (7\text{-}64)$$

$$p\Delta E_{fd} = \{-K_f\cdot(\Delta V_t + V_s) - \Delta E_{fd}\}/T_f - K_f\cdot u_1/T_f \quad (7\text{-}65)$$

$$pV_s = K_s\cdot\{-K_f\cdot(\Delta V_t + V_s) - \Delta E_{fd}\}/T_f\cdot T_s - V_s/T_s + K_f\cdot K_s\cdot u_1/T_f\cdot T_s \quad (7\text{-}66)$$

$$p\Delta P_v = (-K_g\cdot\Delta\omega/\omega_0 - \Delta P_v)/T_g + K_g\cdot u_2/T_g \quad (7\text{-}67)$$

$$p\Delta P_t = (\Delta P_v - \Delta P_t)/T_h \quad (7\text{-}68)$$

where, $\Delta V_d = \Delta E_d' + x_q'\cdot\Delta i_q$, $\Delta V_q = \Delta E_q' - x_d'\cdot\Delta i_d$, $\Delta P_e = V_{do}\cdot\Delta i_d + V_{qo}\cdot\Delta i_q + i_{do}\cdot\Delta V_d + i_{qo}\cdot\Delta V_q$

$\Delta i_d = (\Delta E_q' + V_0\cdot\sin\delta_0\cdot\Delta\delta)/(x_d' + x_t + x_e)$

$\Delta i_q = (-\Delta E_d' + V_0\cdot\cos\delta_0\cdot\Delta\delta)/(x_q' + x_t + x_e)$

Let vector $\Delta\mathbf{y}$ be defined as follows:

$$\Delta\mathbf{y} = [\Delta V_d, \Delta V_q, \Delta i_d, \Delta i_q]^T \quad (7\text{-}69)$$

From the above equations, the linearized equations of the model system can be rewritten in vector form as:

$$p\Delta\mathbb{X} = \mathbb{A}_1 \cdot \Delta\mathbb{X} + \mathbb{A}_2 \cdot \Delta\mathbb{Y} + \mathbb{B} \cdot \mathbb{U} \quad (7\text{-}70)$$

$$\Delta\mathbb{Y} = \mathbb{A}_3 \cdot \Delta\mathbb{X} \quad (7\text{-}71)$$

From the above two equations, the linearized equation of the model system can be written in the form shown in eqn.(7-19), where the matrices $\mathbb{A}$ and $\mathbb{B}_0$ become as follows:

$$\mathbb{A} = \mathbb{A}_1 + \mathbb{A}_2 \cdot \mathbb{A}_3 \quad (7\text{-}72)$$

$$\mathbb{B}_0 = \mathbb{A} \cdot \mathbb{B} \quad (7\text{-}73)$$

In the above equations the subscript 0 denotes the steady state value, and the components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$, and $\mathbb{B}$ are shown in Table A-3 in appendix.

7-3-3. System parameters and Initial Conditions

The parameters of the model system are shown in Table 7-3-1. Before the construction of the matrix $\mathbb{A}$, it is necessary to find the steady state values of the system variables. After the load flow calculation, the steady state values of the system variables are determined using the phasor diagram shown in Fig.7-3-3. The initial conditions of the model system are shown in Table 7-3-2, where the operating point of the synchronous machine is selected as follows: P_0 (active power output or electrical output)=0.5 p.u., Q_0 (reactive power output)=0.1 p.u., and V_{t0} (terminal voltage)=1.0 p.u.

Table 7-3-1 System parameters

x_d =1.0,	x_d' =0.23,	x_q =1.08,	x_q' =0.23,	T_{d0}' =5.6 sec.,
T_{q0}' =4.5 sec.,	J (= $\omega_0 \cdot M$)=6.0 sec.,	P_d =0.01,	x_t =0.1,	
x_{e1} =0.6,	x_e =0.3,	K_g =10.0,	T_g =0.2 sec.,	T_h =0.2 sec.,
K_f =5.0,	K_s =0.01,	T_f =0.2 sec.,	T_s =0.1 sec.	

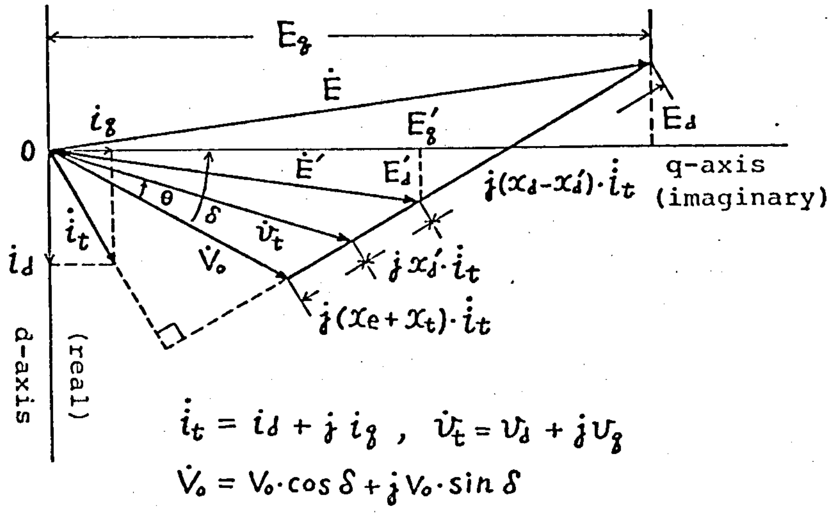


Fig.7-3-3 Phasor diagram for initial conditions

Table 7-3-2 Initial conditions

P_o (= P_{eo})=0.5,	Θ_o =0.1,	v_{to} =1.0
V_o =0.98062,	θ_o =0.20537 rad.,	δ_o =0.65881 rad.
v_{do} =0.43807,	v_{qo} =0.89894,	i_{do} =0.30887
i_{qo} =0.40562,	E'_{qo} =0.96998,	E'_{do} =0.34477
E_{fdo} =1.20781,	P_{to} =0.5,	

7-3-4. Numerical Results

The feedback gain matrix $\mathbb{F}$ of the model system, which minimizes the value of Tr($\mathbb{K}$), has been determined for each case shown in Table 7-3-3 using the iterative algorithm shown in section 7-2-3, and is shown in Table 7-3-4 for each case.

The convergence characteristic of the above iterative calculation for case 3 or case 5 is shown in Fig.7-3-4. The value of Tr($\mathbb{K}$) converges to its minimum value by about ten times iterations. The minimum value of Tr($\mathbb{K}$) for each case is shown in Table 7-3-3.

Table 7-3-3 Restriction of feedback places and minimum value of Tr($\mathbb{K}$)

Case	u_i \ f_j	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	min of Tr($\mathbb{K}$)
Case 1	u_1	*	*	*	*	*	*	*	*	13.144
	u_2	*	*	*	*	*	*	*	*	
Case 2	u_1	*	*	0	0	*	0	*	*	13.233
	u_2	*	*	0	0	*	0	*	*	
Case 3	u_1	0	*	0	0	*	0	0	0	13.438
	u_2	0	*	0	0	0	0	*	*	
Case 4	u_1	*	*	0	0	*	0	*	0	13.989
	u_2	*	*	0	0	*	0	*	0	
Case 5	u_1	*	*	0	0	*	0	0	0	14.014
	u_2	*	*	0	0	0	0	*	0	
Case 6	u_1	0	*	0	0	*	0	0	0	14.414
	u_2	0	*	0	0	0	0	0	*	
Case 7	u_1	0	0	0	0	0	0	0	0	177.934
	u_2	0	0	0	0	0	0	0	0	

$\mathbb{Q}$ = diag.(1 1 1 1 1 1 1 1)

$\mathbb{R}$ = diag.(1,1)

(* denotes the feedback place)

Table 7-3-4 Feedback gain matrix $\mathbb{F}$

	Case 1		Case 2		Case 3	
	u_1	u_2	u_1	u_2	u_1	u_2
f_1	0.4717×10^{-1}	0.1774×10	-0.2533×10^{-1}	0.4321×10^{-1}	0.0	0.0
f_2	-0.3163×10^{-1}	0.9593	-0.3322×10^{-1}	0.1321	-0.2763×10^{-1}	0.1084
f_3	0.1577	0.7338	0.0	0.0	0.0	0.0
f_4	-0.8551×10^{-2}	-0.1302×10	0.0	0.0	0.0	0.0
f_5	-0.9618	0.5515×10^{-1}	-0.3836	0.8501×10^{-1}	-0.4671	0.0
f_6	-0.3104×10^{-1}	-0.1337	0.0	0.0	0.0	0.0
f_7	-0.4154×10^{-1}	-0.9793	0.6874×10^{-2}	-0.1080	0.0	-0.8245×10^{-1}
f_8	-0.4639×10^{-1}	0.5690	-0.2960×10^{-1}	0.1935	0.0	0.1861
	Case 4		Case 5		Case 6	
	u_1	u_2	u_1	u_2	u_1	u_2
f_1	-0.2791×10^{-1}	0.5568×10^{-1}	-0.1632×10^{-1}	0.6746×10^{-1}	0.0	0.0
f_2	-0.4234×10^{-1}	0.6848×10^{-1}	-0.2427×10^{-1}	0.7504×10^{-1}	-0.1114×10^{-1}	0.2984×10^{-1}
f_3	0.0	0.0	0.0	0.0	0.0	0.0
f_4	0.0	0.0	0.0	0.0	0.0	0.0
f_5	-0.3582	0.5269×10^{-1}	-0.3665	0.0	-0.1955	0.0
f_6	0.0	0.0	0.0	0.0	0.0	0.0
f_7	0.3514×10^{-1}	-0.8702×10^{-1}	0.0	-0.1045	0.0	0.0
f_8	0.0	0.0	0.0	0.0	0.0	0.5520×10^{-1}

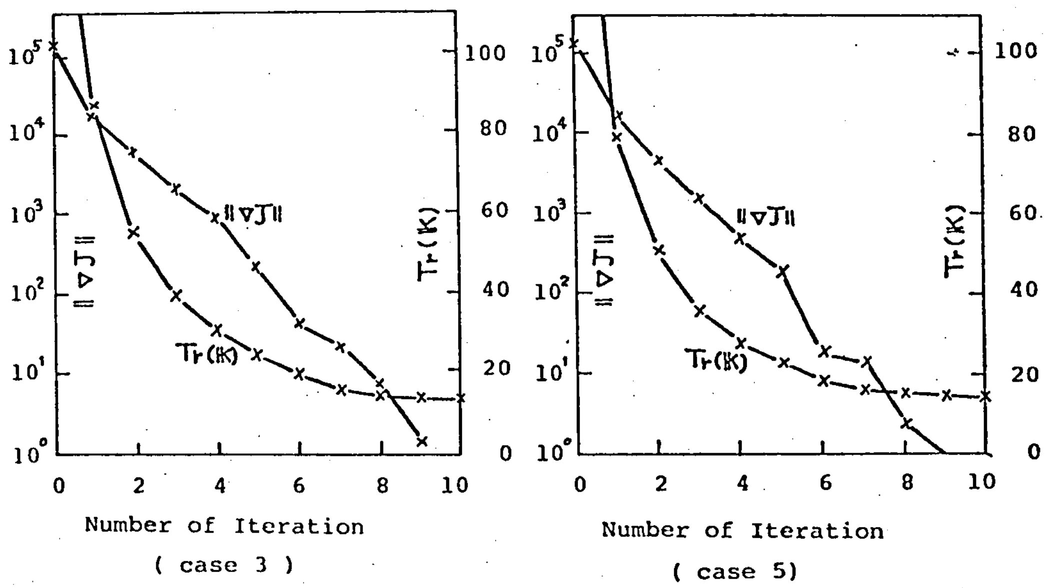


Fig.7-3-4 Convergence characteristic

The stabilizing controller of the model system is constructed using the feedback gain matrix $\mathbb{F}$ shown in Table 7-3-4 of the form $\mathbf{u}=-\mathbb{F}\cdot\mathbf{f}(\mathbf{x})$ for each case. When the system disturbances are sufficiently small, the stabilizing controller $\mathbf{u}=-\mathbb{F}\cdot\mathbf{f}(\mathbf{x})$ becomes equivalent to the optimal controller for the linearized system $\mathbf{u}=-\mathbb{F}\cdot\mathbb{X}$, which minimizes the expected value of the quadratic performance index, i.e. the value of Tr($\mathbb{K}$). As shown by the minimum value of Tr($\mathbb{K}$) in Table 7-3-3, the small signal performance of the model system is much improved by the stabilizing controller and the improvements are almost equal for both the complete feedback stabilizing controller (case 1) and the incomplete feedback stabilizing controllers (case 2$\sim$case 6).

In order to investigate the control effects by the above stabilizing controllers following three system disturbances have been considered.

(1) The three-phase to ground fault of 0.3 sec. duration occurrs at the point a in the model system.

(2) The three phase to ground fault occurrs at the point b in the model system at the time t=o.o sec., the faulted line is isolated at the

time t=0.2 sec. and the faulted line is reclosed at the time t=0.3 sec. after clearing the fault.

(3) One of the parallel transmission lines is isolated at the time t=0.0 sec..

The system responses following the disturbance (1) are shown in Fig. 7-3-5 ~ Fig.7-3-8. It is obvious that the stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ much improves the large signal performance of the model system, and the improvements by the incomplete feedback stabilizing controllers are almost equal to those by the complete feedback stabilizing controller.

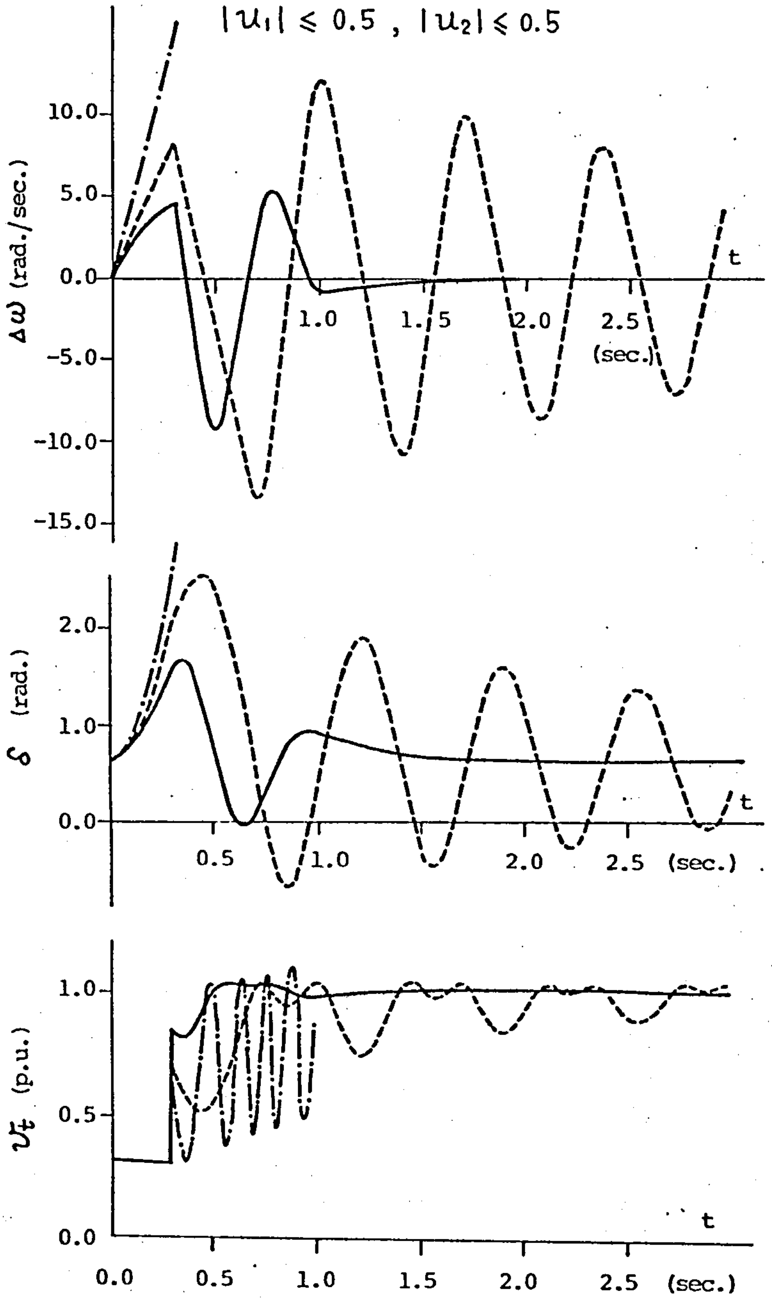
As shown in Fig.7-3-5, the optimal controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system, when applied to the original non-linear system, results in undesirable system performance, i.e. it makes the original non-linear system unstable under the large disturbance condition.

In Fig.7-3-5 and Fig.7-3-6 the control signals u_1 and u_2 are bounded by $|u_1| \leq 0.5$ and by $|u_2| \leq 0.5$, so when the absolute values of the control signals u_1 and u_2 are greater than 0.5, the stabilizing controllers become the bang-bang type controller and otherwise become proportional type controller.

In Fig.7-3-7, the control signals u_1 and u_2 are bounded by $|u_1| \leq 0.5$ and $|u_2| \leq 0.5$ or by $|u_1| \leq 2.0$ and $|u_2| \leq 2.0$, and for the larger values of the bounded values of the control signals, the improvements of the system performance become greater.

In Fig.7-3-8 the phase plane trajectory of the model system applied with the bang-bang type stabilizing controller is shown. The control effect by the bang-bang type controller is almost equal to that by the proportional type stabilizing controller.

The system responses following the disturbance (2) or (3) are shown in Fig.7-3-9 or Fig.7-3-10 respectively.

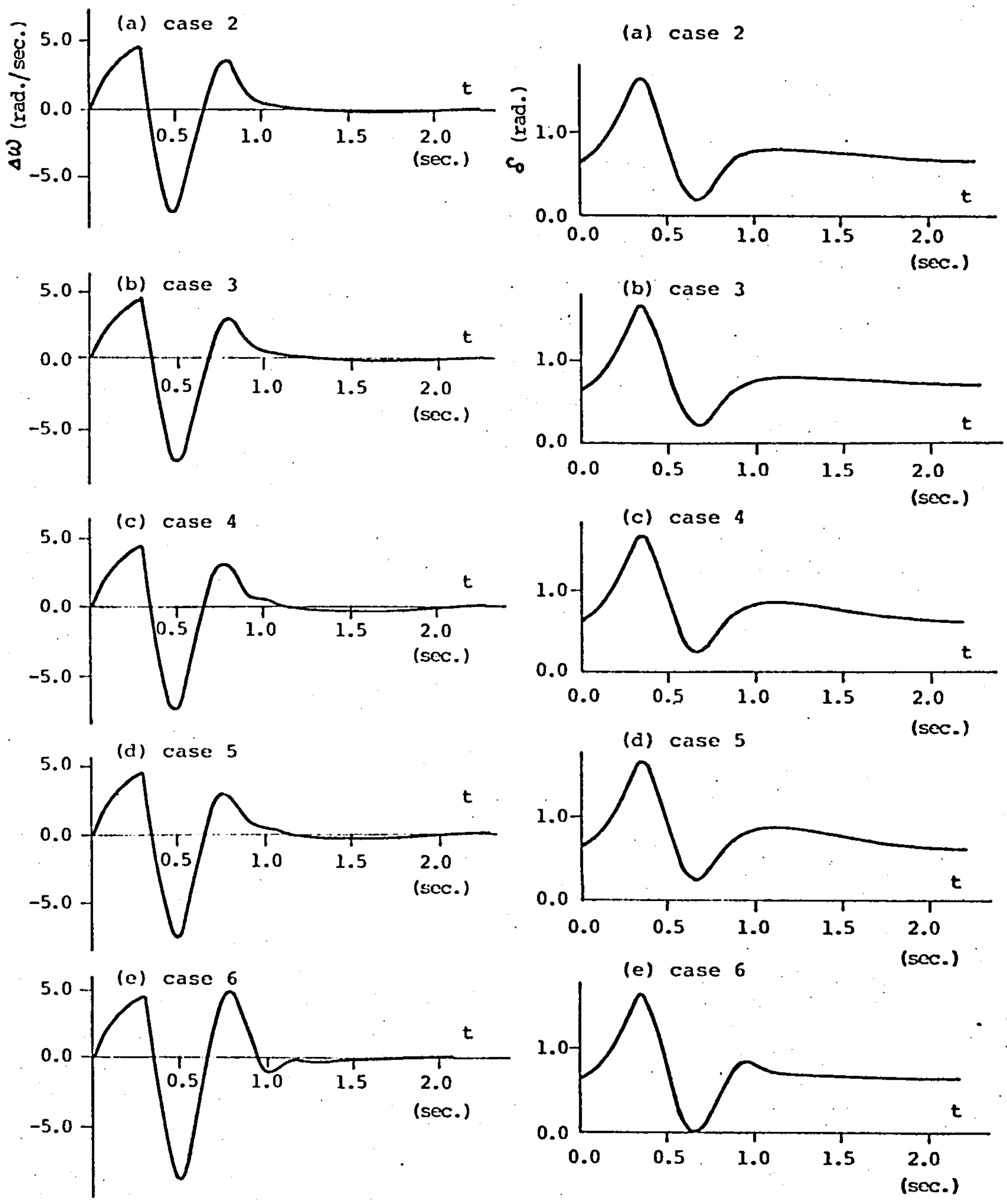


——— : with the stabilizing controller $\mathbb{U} = -\mathbb{F}\cdot\mathbb{f}(x)$ (case 1)

–·–·– : with the optimal controller $\mathbb{U} = -\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

- - - - - : without controller

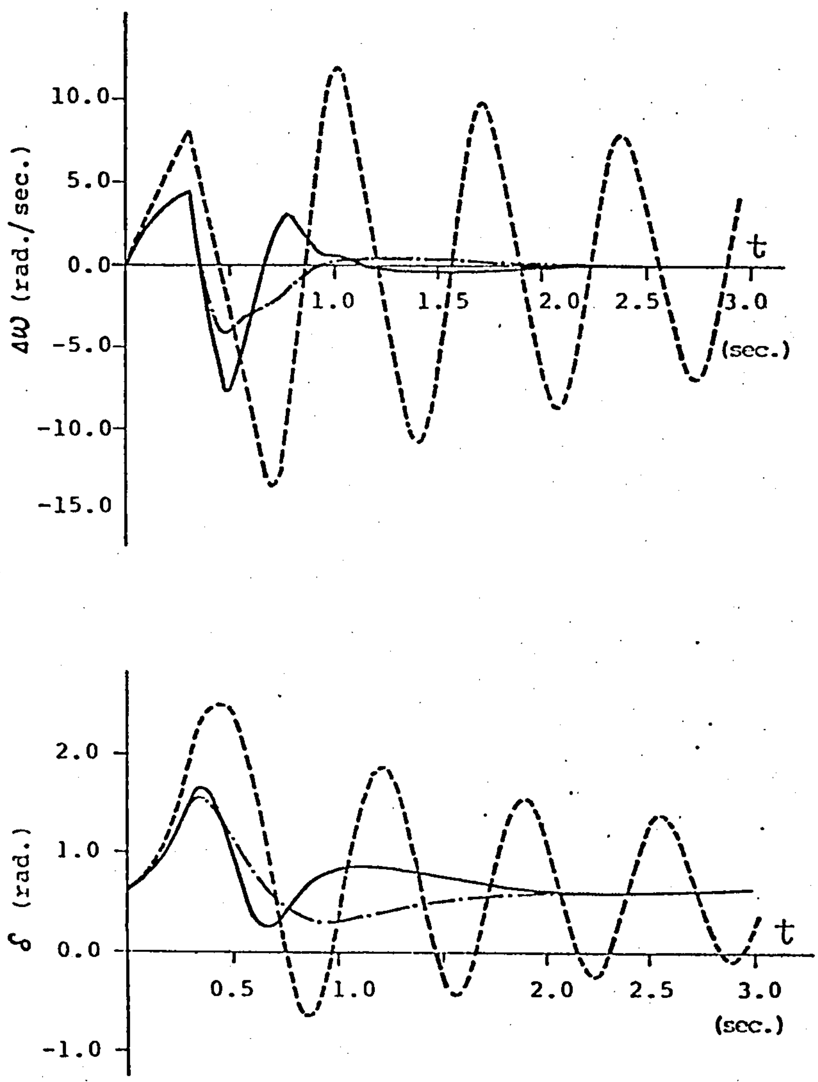
Fig.7-3-5 Control effect by the complete feedback stabilizing controller



$|u_1| \leqslant 0.5, \ |u_2| \leqslant 0.5$

$u = -F \cdot f(x)$ (case 2 ~ case 6)

Fig.7-3-6 Control effects by the incomplete feedback stabilizing controllers



——— : with the stabilizing controller (case 5) $|u_1| \leqslant 0.5$, $|u_2| \leqslant 0.5$

—·—· : with the stabilizing controller (case 5) $|u_1| \leqslant 2.0$, $|u_2| \leqslant 2.0$

----- : without controller

Fig.7-3-7 Effect of the bounded values of the control signals

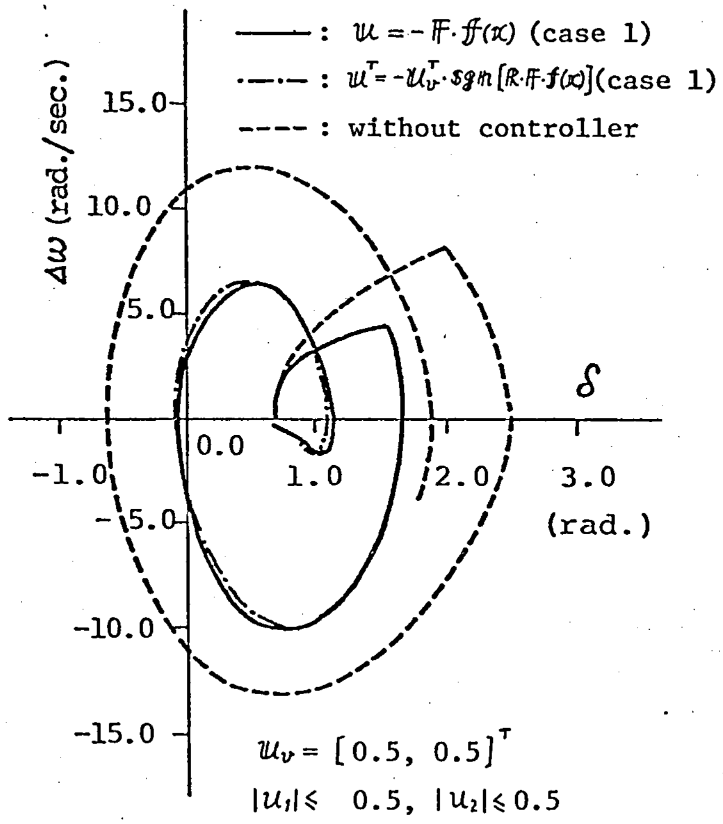


Fig.7-3-8 Control effect by the bang-bang type controller

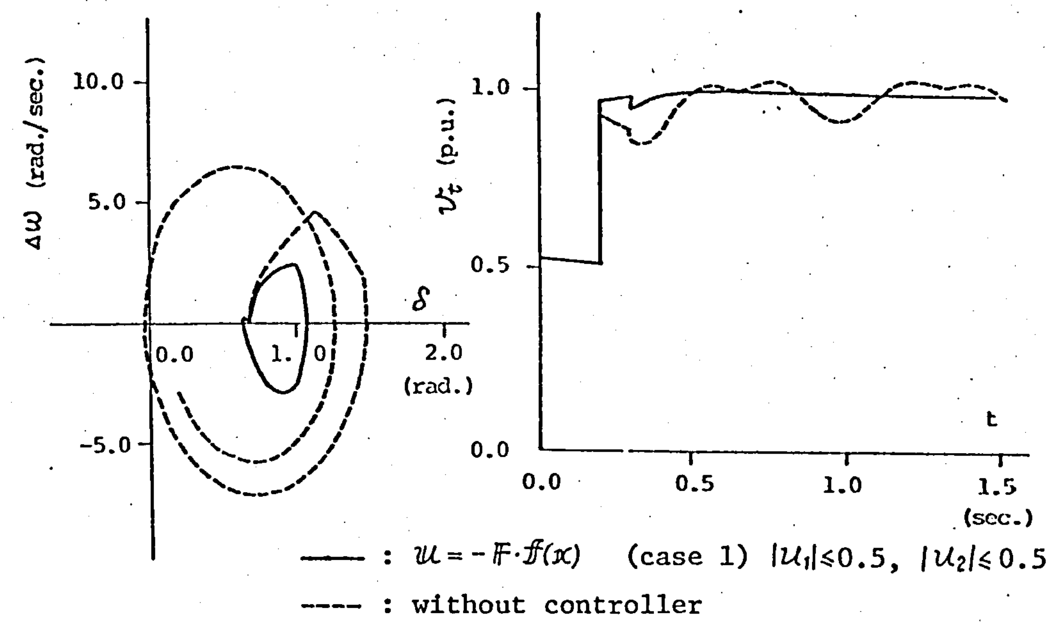


Fig 7-3-9 Control effect by the complete feedback stabilizing controller

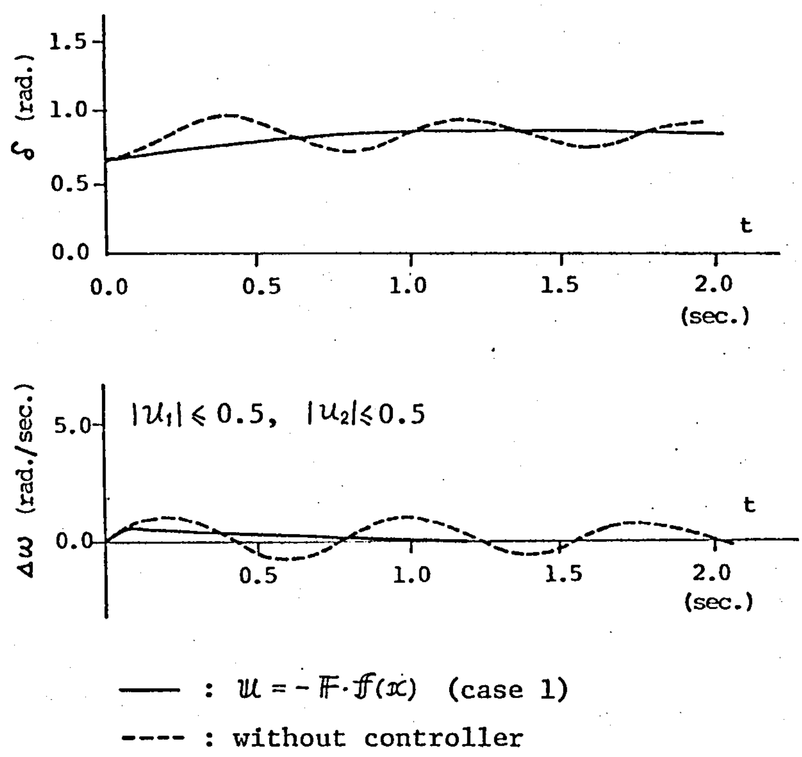


Fig.7-3-10 Control effect by the complete feedback stabilizing controller

The system performance is also improved by the stabilizing controller.

In the above calculations, the same feedback gain matrix $\mathbb{F}$, which is determined in the steady state system condition, has been used during the all the processes of the system transients.

As described above the stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(x)$ can improve the system performance following both large and small disturbances. The improvement by the well selecting incomplete feedback stabilizing controller is almost equivalent to that by the complete feedback stabilizing controller, so it is possible to construct the stabilizing controller using only the measurable states of the system.

Section 7-4. Application to a 3-machine Problem

The same 3-machine system shown in section 3-4 is used for the model multi-machine power system. The system parameters, the initial conditions and the admittance matrices under several system conditions have been shown in Table 3-4-1 $\sim$ Table 3-4-3.

The control systems for each machine are shown in Fig.2-3-2(a) and Fig. 2-4-2(a). In the model system, No.3 machine is conventionally used to represent a large scale power system, and the control systems for No.3 machine are not considered here.

7-4-1. Non-linear First Order Differential Equations of Model System

In section 6-3, the stabilizing controller of the model system has been introduced using the simplified model, namely only the mechanical equations of motion have been considered and the deviation of mechanical input has been considered as a control signal under the assumption that the governor action is ideal.

In this section, the stabilizing controller of the model system is determined using the detailed equations of the model system. These equations have already been represented in section 3-4, and a little modified forms of these equations are used.

From eqn.(2-34) and eqn.(2-37), the voltage regulator and the governor actions of the j-th machine become :

$$p\Delta E_{fdj} = (-K_{fj}\cdot\Delta V_{tj} - \Delta E_{fdj})/T_{fj} + K_{fj}\cdot u_{1j}/T_{fj} \qquad (7\text{-}74)$$

$$p\Delta P_{tj} = (-K_{gj}\cdot\Delta\omega_j/\omega_0 - \Delta P_{tj})/T_{gj} + K_{gj}\cdot u_{2j}/T_{gj} \qquad (7\text{-}75)$$

where, j=1,2 and

$$\Delta E_{fdj} = \begin{cases} \Delta E_{fdj\,max} & \text{for} \quad \Delta E_{fdj} \geq \Delta E_{fdj\,max} \\ \Delta E_{fdj} & \text{for} \quad \Delta E_{fdj\,min} < \Delta E_{fdj} < \Delta E_{fdj\,max} \\ \Delta E_{fdj\,min} & \text{for} \quad \Delta E_{fdj} \leq \Delta E_{fdj\,min} \end{cases}$$

$$\Delta P_t = \begin{cases} \Delta P_{tmax} & \text{for } \Delta P_t \geq \Delta P_{tmax} \\ \Delta P_t & \text{for } \Delta P_{tmin} < \Delta P_t < \Delta P_{tmax} \\ \Delta P_{tmin} & \text{for } \Delta P_t \leq \Delta P_{tmin} \end{cases}$$

The original non-linear equations of the model system can be written in vector form shown in eqn.(7-1), where the state variables vector $\mathbf{x}$, the control signals vector and the non-linear functional vector $\mathbf{f}(\mathbf{x})$ become:

$$\mathbf{x} = [\delta_{13}, \delta_{23}, \Delta\omega_1, \Delta\omega_2, \Delta\omega_3, E'_{q1}, E'_{q2}, E'_{q3}, \Delta E_{fd1}, \Delta E_{fd2}, \Delta P_{t1}, \Delta P_{t2}]^T \quad (7\text{-}76)$$

$$\mathbf{u} = [u_{11}, u_{12}, u_{21}, u_{22}]^T \quad (7\text{-}77)$$

$$\mathbf{f}(\mathbf{x}) = [f_1, f_2, f_3, \cdots\cdots, f_{12}]^T \quad (7\text{-}78)$$

where, $f_1 = p\delta_{13} = \Delta\omega_1 - \Delta\omega_3$, $f_2 = p\delta_{23} = \Delta\omega_2 - \Delta\omega_3$

$f_3 = p\Delta\omega_1$, $f_4 = p\Delta\omega_2$, $f_5 = p\Delta\omega_3$, $f_6 = pE'_{q1}$

$f_7 = pE'_{q2}$, $f_8 = pE'_{q3}$

$f_9 = (-K_{f1}\cdot\Delta V_{t1} - \Delta E_{fd1})/T_{f1}$, $f_{10} = (-K_{f2}\cdot\Delta V_{t2} - \Delta E_{fd2})/T_{f2}$

$f_{11} = (-K_{g1}\cdot\Delta\omega_1/\omega_0 - \Delta P_{t1})/T_{g1}$, $f_{12} = (-K_{g2}\cdot\Delta\omega_2/\omega_0 - \Delta P_{t2})/T_{g2}$

The state variable $\mathbf{x}$ of the model system used here is a little modified in comparison with that used in section 3-4; the internal induced voltage E'_{dj} in direct axis is neglected here since all the synchronous machines in the model system are salient-type ones, and the difference angles δ_{13} ($= \delta_1 - \delta_3$) and δ_{23} ($= \delta_2 - \delta_3$) are considered instead of the rotor angles δ_1 , δ_2 and δ_3 .

7-4-2. Linearized Equations of Model System

In order to obtain the feedback gain matrix of the stabilizing controller of the model system, we need following linearized equations of the model system.

$$p\Delta\delta_{13} = \Delta\omega_1 - \Delta\omega_3 \quad (7\text{-}79)$$

$$p\Delta\delta_{23} = \Delta\omega_2 - \Delta\omega_3 \quad (7\text{-}80)$$

$$p\Delta E'_{gj} = \{ \Delta E_{fdj} - \Delta E'_{gj} - (x_{dj} - x'_{dj}) \cdot i_{dj} \} / T'_{doj} \quad (j=1,2) \tag{7-81}$$

$$p\Delta E'_{g3} = \{ - \Delta E'_{g3} - (x_{d3} - x'_{d3}) \cdot i_{d3} \} / T'_{do3} \tag{7-82}$$

$$p\Delta \omega_j = (\Delta P_{tj} - \Delta P_{ej} - P_{dj} \cdot \Delta \omega_j) / M_j \quad (j=1,2) \tag{7-83}$$

$$p\Delta \omega_3 = (- \Delta P_{e3} - P_{d3} \cdot \Delta \omega_3) / M_3 \tag{7-84}$$

$$p\Delta E_{fdj} = (-K_{fj} \cdot \Delta V_{tj} - \Delta E_{fdj}) / T_{fj} + K_{fj} \cdot U_{1j} / T_{fj} \quad (j=1,2) \tag{7-85}$$

$$p\Delta P_{tj} = (-K_{gj} \cdot \Delta \omega_j / \omega_o - \Delta P_{tj}) / T_{gj} + K_{gj} \cdot U_{2j} / T_{gj} \quad (j=1,2) \tag{7-86}$$

where,
$$\Delta V_{tj} = \frac{V_{djo}}{V_{tjo}} \cdot \Delta V_{dj} + \frac{V_{gjo}}{V_{tjo}} \cdot \Delta V_{gj}$$

$$\Delta P_{ej} = V_{djo} \cdot \Delta i_{dj} + V_{gjo} \cdot \Delta i_{gj} + i_{djo} \cdot \Delta V_{dj} + i_{gjo} \cdot \Delta V_{gj}$$

$$\Delta E'_{gj} = \Delta V_{gj} + x'_{dj} \cdot \Delta i_{dj} + r_j \cdot \Delta i_{gj}$$

The transmission network equation becomes:

$$\Delta \mathring{i}_{dj} = \sum_{K=1}^{3} \{ Y'_{jK} \Big|_{\substack{\delta_j = \delta_{jo} \\ \delta_K = \delta_{Ko}}} \cdot \Delta V_{dK} + \frac{\partial Y'_{jK}}{\partial \delta_j} \Big|_{\substack{\delta_j = \delta_{jo} \\ \delta_K = \delta_{Ko}}} V_{dKo} \cdot \Delta \delta_j + \frac{\partial \dot{Y}'_{jK}}{\partial \delta_K} \Big|_{\substack{\delta_j = \delta_{jo} \\ \delta_K = \delta_{Ko}}} \times V_{dKo} \cdot \Delta \delta_K \} \tag{7-87}$$

Let vector $\Delta \mathbb{X}$, $\Delta \mathbb{Y}$ and $\Delta \mathbb{Z}$ be defined as:

$$\Delta \mathbb{X} = [\Delta \delta_{13}, \Delta \delta_{23}, \Delta \omega_1, \Delta \omega_2, \Delta \omega_3, \Delta E'_{g1}, \Delta E'_{g2}, \Delta E'_{g3}, \Delta E_{fd1}, \Delta E_{fd2}, \Delta P_{t1}, \Delta P_{t2}]^T \tag{7-88}$$

$$\Delta \mathbb{Y} = [\Delta V_{d1}, \Delta V_{g1}, \Delta V_{d2}, \Delta V_{g2}, \Delta V_{d3}, \Delta V_{g3}, \Delta i_{d1}, \Delta i_{g1}, \Delta i_{d2}, \Delta i_{g2}, \Delta i_{d3}, \Delta i_{g3}]^T \tag{7-89}$$

$$\Delta \mathbb{Z} = [\Delta \delta_{13}, \Delta \delta_{23}, \Delta E'_{g1}, \Delta E'_{g2}, \Delta E'_{g3}]^T \tag{7-90}$$

Then, from eqn.(7-79)-eqn.(7-87):

$$p\Delta \mathbb{X} = \mathbb{A}_1 \cdot \Delta \mathbb{X} + \mathbb{A}_2 \cdot \Delta \mathbb{Y} + \mathbb{B} \cdot \mathbb{U} \tag{7-91}$$

$$\mathbb{A}_3 \cdot \Delta \mathbb{Y} = \mathbb{A}_4 \cdot \Delta \mathbb{Z} \tag{7-92}$$

$$\Delta \mathbb{Z} = \mathbb{A}_5 \cdot \Delta \mathbb{X} \tag{7-93}$$

From the above three equations, the linearized equations of the model system can be written in the form shown in eqn.(7-19), where the matrices $\mathbb{A}$ and $\mathbb{B}_o$ become:

$$\mathbb{A} = \mathbb{A}_1 + \mathbb{A}_2 \cdot \mathbb{A}_3^{-1} \cdot \mathbb{A}_4 \cdot \mathbb{A}_5 \qquad (7\text{-}94)$$

$$\mathbb{B}_0 = \mathbb{A} \cdot \mathbb{B} \qquad (7\text{-}95)$$

The components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$, $\mathbb{A}_4$, $\mathbb{A}_5$ and $\mathbb{B}$ are shown in Table A-4 in appendix.

7-4-3. Numerical Results

The parameters of the control systems have been selected as shown in Table 7-4-1.

The feedback gain matrix $\mathbb{F}$ of the model system, which minimizes the value of Tr($\mathbb{K}$), has been determined using the iterative algorithm shown in section 7-2-3. The complete feedback gain matrix and an incomplete feedback gain matrix are shown in Table 7-4-2.

The stabilizing controller of the model system is constructed using the feedback gain matrix $\mathbb{F}$ shown in Table 7-4-2 of the form $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(x)$. When the disturbances in the system are sufficiently small, the stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(x)$ becomes equivalent to the optimal controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system, so the small signal performance of the model system can be improved by the stabilizing controller as described before.

In order to investigate the control effects by the stabilizing controller for large disturbances, the following system conditions have been considered; (1) Three-phase to ground fault occurrs at the point A in the model system at the time t=0.0 sec., (2) The faulted line is isolated at the time t=0.2 sec., (3) The faulted line is reclosed at the time t=0.3 sec., after clearing the fault.

The responses of the model system applied with various controllers, i.e. no controller, optimal controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system, complete or incomplete feedback stabilizing controll $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(x)$, are shown in Fig. 7-4-1 ∼ Fig.7-4-5.

In this case, the model system is stabilized by the optimal controller

Table 7-4-1 Parameters of control systems

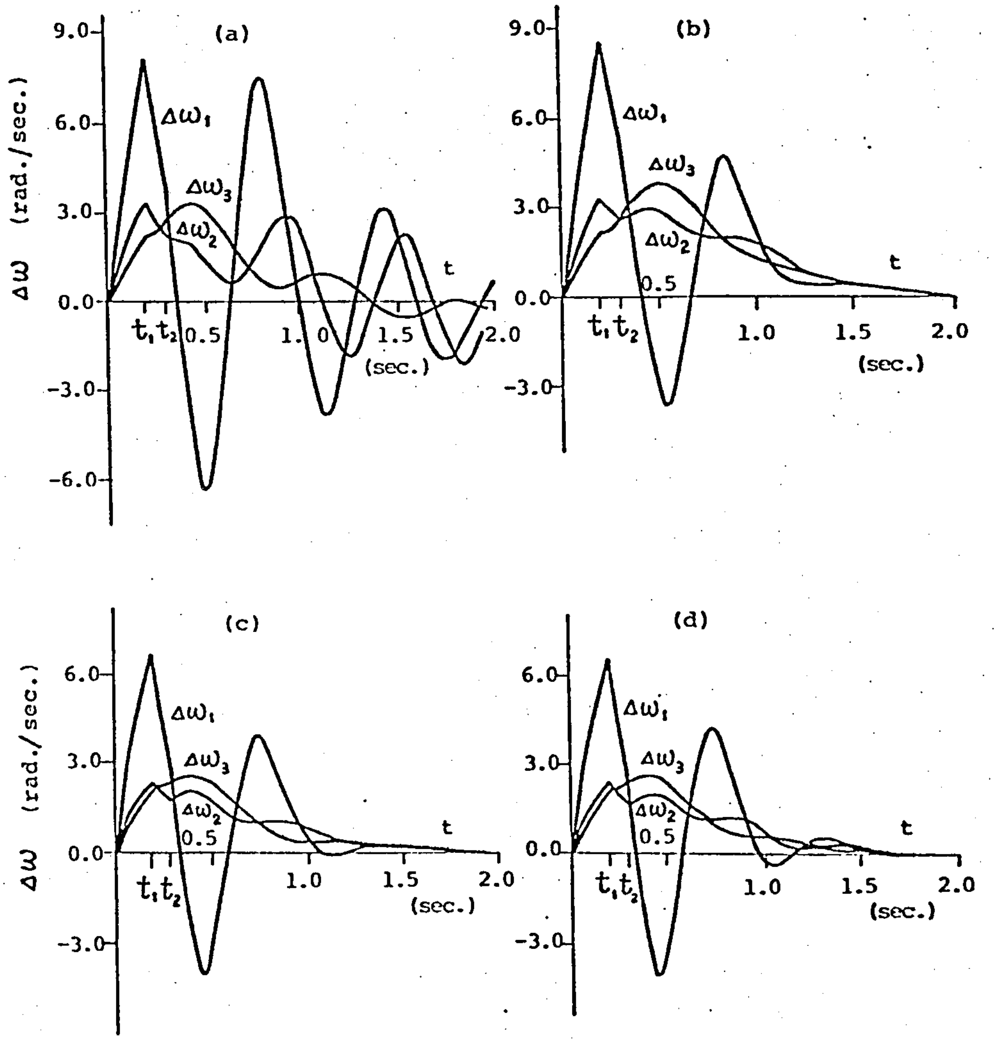
$K_{fj} = 5.0$,	$T_{fj} = 0.2$ sec.
$K_{gj} = 25.0$,	$T_{gj} = 0.3$ sec.
$j=1,2$	

Table 7-4-2 Feedback gain matrix $\mathbb{F}$

	Case 1 (complete feedback)			
	u_{11}	u_{12}	u_{21}	u_{22}
f_1	-0.1433×10^{-1}	-0.3712×10^{-2}	0.7584×10^{-2}	-0.3486×10^{-3}
f_2	0.4427×10^{-4}	-0.6468×10^{-2}	-0.4896×10^{-3}	0.7380×10^{-2}
f_3	-0.1938×10^{-2}	-0.1649×10^{-3}	0.1795×10^{-2}	-0.3163×10^{-4}
f_4	0.9411×10^{-5}	-0.1754×10^{-2}	-0.2057×10^{-4}	0.1783×10^{-2}
f_5	0.8086×10^{-3}	0.4847×10^{-3}	-0.8504×10^{-3}	-0.8975×10^{-3}
f_6	0.7897×10^{-1}	0.1797×10^{-1}	0.5424×10^{-2}	0.6384×10^{-2}
f_7	0.2025×10^{-1}	0.6569×10^{-1}	0.6668×10^{-3}	0.4720×10^{-2}
f_8	0.5795×10^{-1}	0.6473×10^{-1}	0.3235×10^{-3}	0.2082×10^{-2}
f_9	-0.2788	-0.2629×10^{-4}	0.2968×10^{-3}	0.1994×10^{-4}
f_{10}	0.5510×10^{-4}	-0.2788	0.3160×10^{-4}	0.2626×10^{-3}
f_{11}	-0.1016×10^{-1}	-0.1141×10^{-2}	-0.5436×10^{-2}	-0.1108×10^{-3}
f_{12}	-0.6160×10^{-3}	-0.8976×10^{-2}	0.1125×10^{-3}	-0.5440×10^{-2}
	Case 2 (incomplete feedback)			
	u_{11}	u_{12}	u_{21}	u_{22}
f_1	-0.5545×10^{-1}	0.0	0.7472×10^{-2}	0.0
f_2	0.0	-0.4315×10^{-1}	0.0	0.7940×10^{-2}
f_3	-0.5543×10^{-2}	0.0	0.8518×10^{-3}	0.0
f_4	0.0	-0.4857×10^{-2}	0.0	0.3973×10^{-3}
f_5	0.0	0.0	0.0	0.0
f_6	0.0	0.0	0.0	0.0
f_7	0.0	0.0	0.0	0.0
f_8	0.0	0.0	0.0	0.0
f_9	-0.4968	0.0	0.0	0.0
f_{10}	0.0	-0.4914	0.0	0.0
f_{11}	0.0	0.0	-0.1010×10^{-1}	0.0
f_{12}	0.0	0.0	0.0	-0.1030×10^{-1}

$\mathbb{Q}$ = diag.(0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)

$\mathbb{R}$ = diag.(1.0,1.0,1.0,1.0)



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

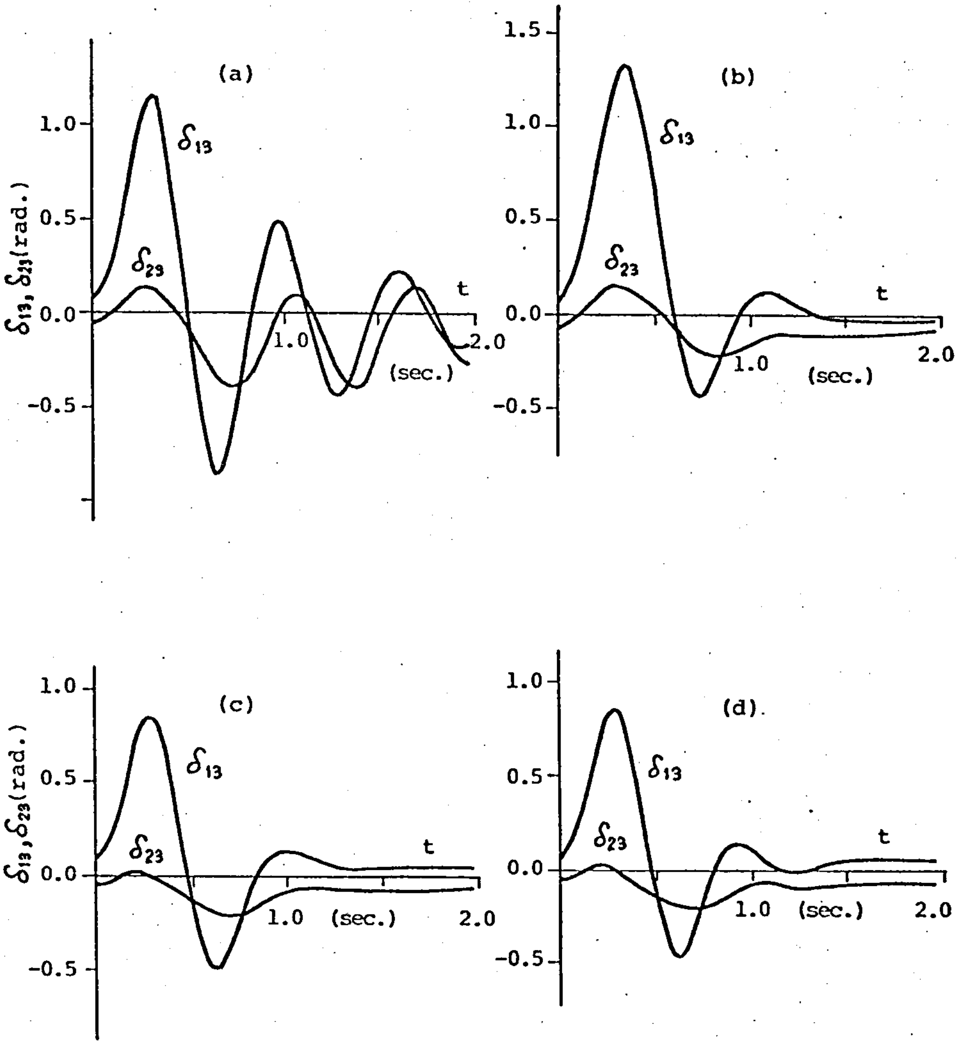
(a) : without controller

(b) : with the optimal controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

(c) : with the complete feedback stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 1)

(d) : with the incomplete feedback stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 2)

Fig.7-4-1 Responses of the angular velocities $\Delta\omega_1$, $\Delta\omega_2$ and $\Delta\omega_3$



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

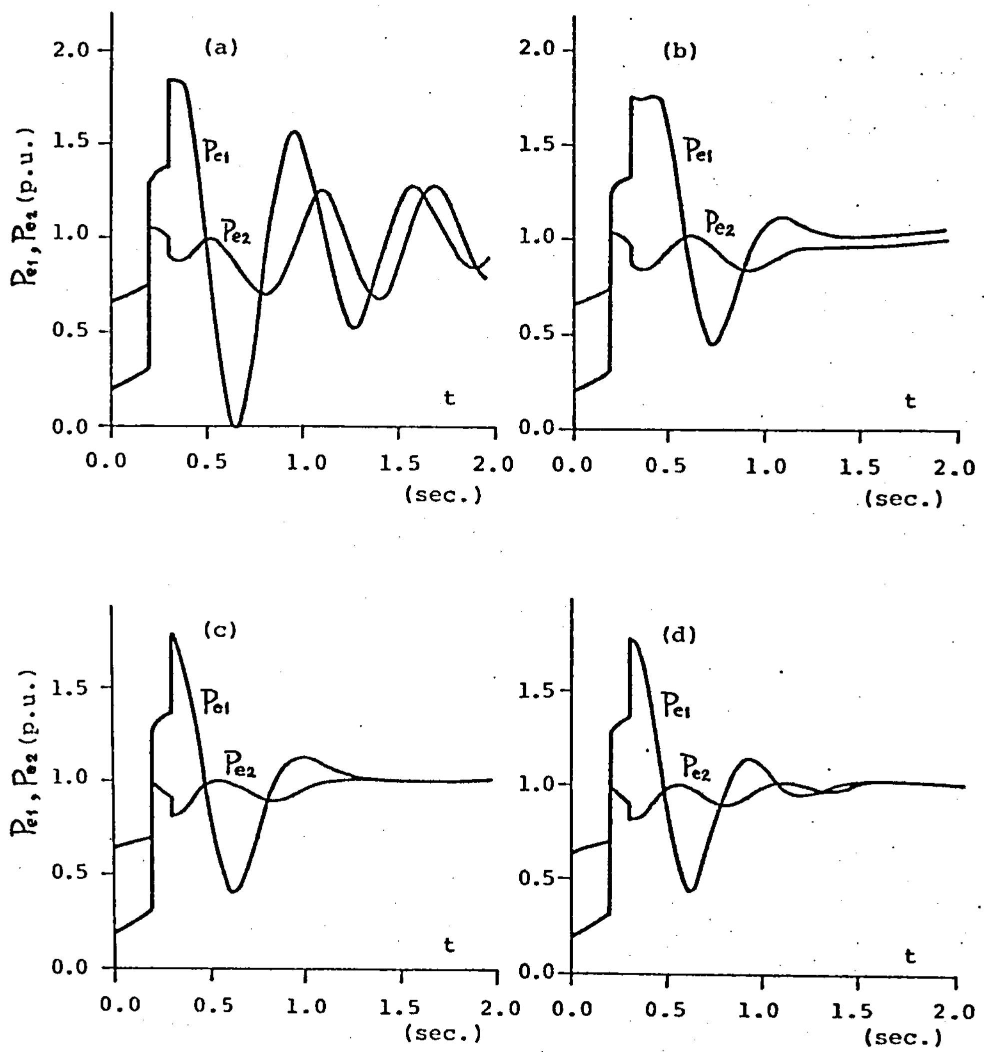
(a) : without controller

(b) : with the optimal controller $\mathcal{U}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

(c) : with the complete feedback stabilizing controller $\mathcal{U}=-\mathbb{F}\cdot\mathbb{f}(x)$ (case 1)

(d) : with the incomplete feedback stabilizing controller $\mathcal{U}=-\mathbb{F}\cdot\mathbb{f}(x)$ (case 2)

Fig.7-4-2 Responses of the difference angles δ_{13} and δ_{23}



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

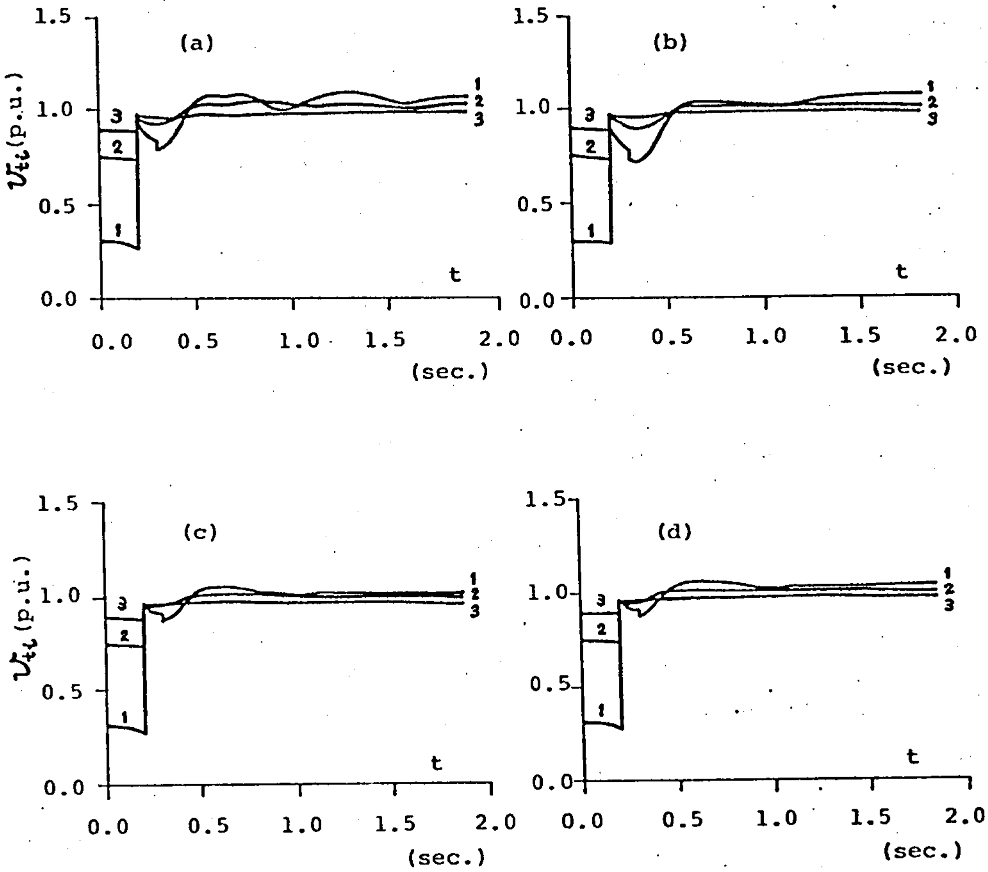
(a) : without controller

(b) : with the optimal controller $\mathbb{u} = -\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

(c) : with the complete feedback stabilizing controller $\mathbb{u} = -\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 1)

(d) : with the incomplete feedback stabilizing controller $\mathbb{u} = -\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 2)

Fig.7-4-3 Responses of the electrical outputs P_{e1} and P_{e2}



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

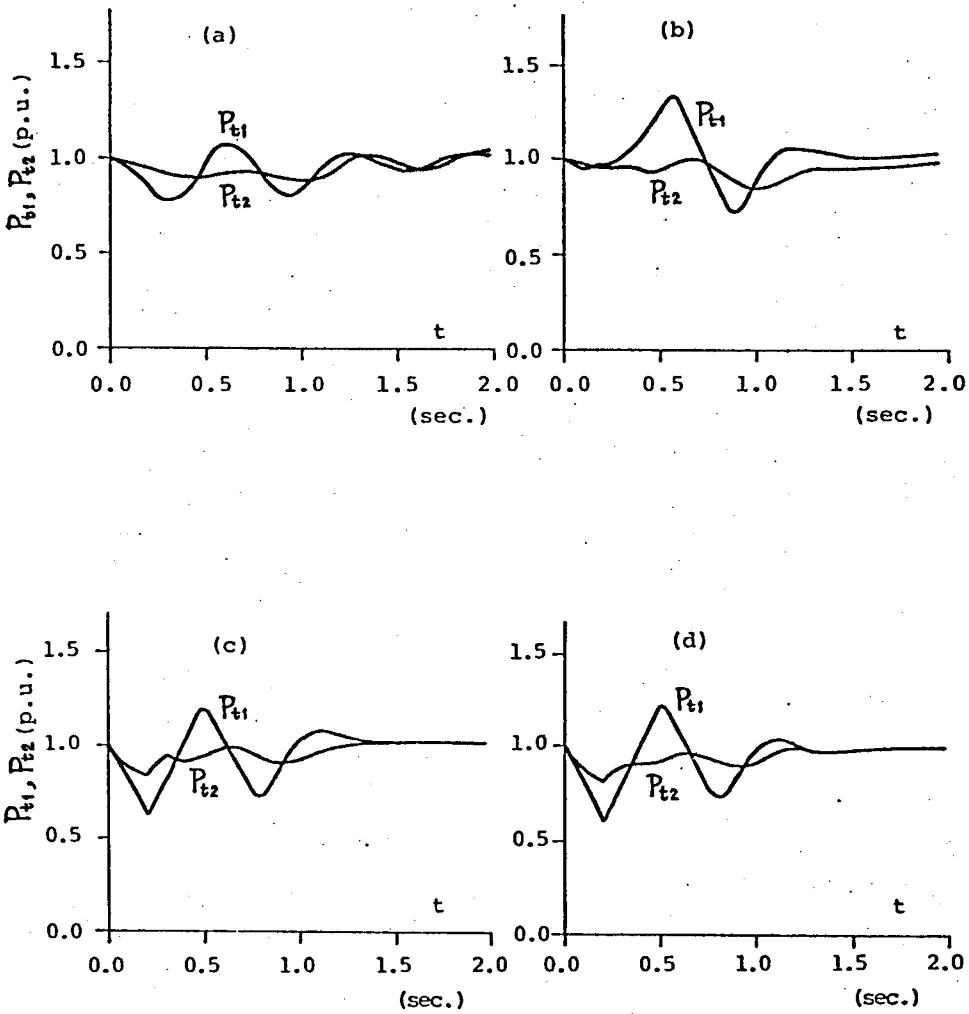
(a) : without controller

(b) : with the optimal controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

(c) : with the complete feedback stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 1)

(d) : with the incomplete feedback stabilizing controller $\mathbb{u}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 2)

Fig.7-4-4 Responses of the terminal voltages v_{t1} , v_{t2} and v_{t3}



t_1 = 0.2 sec. : the faulted line is isolated

t_2 = 0.3 sec. : the faulted line is reclosed

(a) : without controller

(b) : with the optimal controller $\mathbb{U}=-\mathbb{F}\cdot\mathbb{X}$ for the linearized system (case 1)

(c) : with the complete feedback stabilizing controller $\mathbb{U}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 1)

(d) : with the incomplete feedback stabilizing controller $\mathbb{U}=-\mathbb{F}\cdot\mathbb{f}(\mathbb{x})$ (case 2)

Fig.7-4-5 Responses of the mechanical inputs to rotors P_{t1} and P_{t2}

$\mathbb{U} = -\mathbb{F} \cdot \mathbb{X}$ for the linearized system, but the disturbances during the fault are increased by the controller, namely the improvement is local in nature, and is smaller than that by the complete or incomplete feedback stabilizing controller $\mathbb{U} = -\mathbb{F} \cdot \mathbb{f}(\mathbb{X})$.

The large signal performance of the model system is much improved by the stabilizing controller, and the improvement by the incomplete feedback stabilizing controller is almost equivalent to that by the complete feedback stabilizing controller, so it is possible to construct the stabilizing controller using only measurable states of the model system.

In the above incomplete feedback stabilizing controller, each machine is almost controlled using the informations of each machine respectively. Consequently, the possibility of the decentralized control of the large scale power system can be explained by the above numerical results.

In the above calculations, it is noted that the same feedback gain matrix has been used throughout all the transient processes of the model system.

Section 7-5. Summary

In this chapter, the stabilizing controller of the form $\mathbb{U} = -\mathbb{F} \cdot \mathbb{f}(\mathbb{X})$ has been proposed for the non-linear system $p\mathbb{X} = \mathbb{f}(\mathbb{X}) + \mathbb{B} \cdot \mathbb{U}$.

By the numerical results shown in this chapter it is recognized that; The optimal controller $\mathbb{U} = -\mathbb{F} \cdot \mathbb{X}$ for the linearized system is not always applicable to the original non-linear system. In some case, such control, when applied to the original non-linear system, results in undesirable system condition, i.e. such control makes the system unstable.

But, the stabilizing controller proposed in this chapter, can much improve the system performance following both large and small disturbances of the system, i.e. the overall stability of the system can be improved by the stabilizing controller, Furthermore, the improvement by the well-selecting incomplete feedback stabilizing controller is almost equivalent to that by the complete feedback stabilizing controller, so it is possible to construct the stabilizing controller using only the measurable state of the system, and such control can be easily realized.

The possibility of the decentralized control of the large scale power system can also be explained here.

CHAPTER 8 CONCLUSION

In this thesis, the studies about the stability analysis and the developement of compensating controllers for the required system stabilization have been considered.

The mathematical representations of the electrical power systems have been described in a form of a mathematical model on a general-purpose digital computer in chapter 2. By these methematical representations, we can handle a number of machines connected to a transmission network of any topological form, and these representations include the model of a round-rotor machine or the model of a salient-pole one, and the automatic voltage regulators and the speed governors. Furthermore, these representations allow the inclusion of any alternative governors or voltage regulators that act continuously.

Throughout this thesis, the small signal performance, the dynamic stability, of the system has been analyzed using the eigenvalues, the system responses and the direct method of Lyapunov, and the large signal performance, the transient stability, of the system has been investigated using the system responses as shown in chapter 3. Furthermore, in this chapter a new stability measure has been proposed using the expected value of the quadratic performance index, and the stability margin has also proposed by the restriction of the real parts of the eigenvalues of the system.

In chapter 4, the optimal state feedback controller has been derived in order to improve the dynamic stability of the system. The dynamics of the system can be much stabilized by the controller, but the controller is represented by the linear function of all the state variables, namely the controller usually requires the complete measurements of the system states, so especially large-scale power system it is almost impossible to have all the informations about the system states. With the reason above described the implementation of the controller to the practical power system may be difficult. Furthermore, the selection of the weighted matrices in the quadratic performance index is very important in order to improve the system performance as shown in this chapter.

In chapter 5, the procedures for utilizing the reduced model in deriving the output feedback controller of the system have been proposed, and the possibility of the application of Lyapunov's direct method to the control problem of the linear system has also been explained. The dynamic stability of the model system can be much improved by the obtained controller, but the improvement is a little smaller than that by the optimal state feedback controller. The output feedback controller is constructed in terms of directly measurable output variables, so the controller obtained may be easily implemented to the practical power system.

In chapter 6, the possibility of the application of Lyapunov's direct method has been explained, and the controller has been determined using the well known Lyapunov function, the energy function, of the system under several assumptions. The transient stability of the model system can be much improved by the controller obtained.

By the numerical results shown in chapter 7, it is recognized that the optimal state feedback controller and the output feedback controller obtained from the linearized model is not always applicable to large disturbances conditions. In some cases, such controllers, when applied to the original non-linear model, result in undesirable system performance, namely such controllers make the original non-linear system unstable under large disturbances conditions. Because, such controllers are obtained from the linearized equations, the improvement is local in nature, and under large disturbances conditions, the system operation departs considerably from the steady state operating point, and consequently such controllers are inadequate in improving the system stability under these circumstances.

In order to improve the overall stability, the stabilizing controller has been proposed in chapter 7. The stabilizing controller is determined using the Lyapunov function proposed by Krasovskii under consideration of the theoretical results of the control problem of linearized system described in former chapters. The stabilizing controller can much improve the system stability under large and small disturbances conditions, namely the overall stability of the system can be much improved by the controller.

Furthermore, it is possible to construct the stabilizing controller using the only measurable states of the system, consequently such stabilizing controller can be easily implemented to the practical power system.

The possibility of the decentralized control of the large-scale power system has also shown in this chapter by the numerical results of the model multi-machine system.

APPENDIX A COMPONENTS OF COEFFICIENT MATRICES

Table A-1 Components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$ and $\mathbb{A}_4$

(a) Matrix $\mathbb{A}_1$

	$\Delta\delta_{13}$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	ΔE_{fd1}	$\Delta\omega_1$	ΔP_{t1}	$\Delta\delta_{23}$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	ΔE_{fd2}	$\Delta\omega_2$	ΔP_{t2}	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta\omega_3$
$p\Delta\delta_{13}$					1.0										-1.0
$p\Delta E'_{q1}$		$\frac{-1}{T'_{d01}}$		$\frac{1}{T'_{d01}}$											
$p\Delta E'_{d1}$			$\frac{-1}{T'_{q01}}$												
$p\Delta E_{fd1}$				$\frac{-1}{T_{f1}}$											
$p\Delta\omega_1$					$\frac{-P_{d1}}{M_1}$	$\frac{1}{M_1}$									
$p\Delta P_{t1}$					$\frac{-K_{g1}}{\omega_0 \cdot T_{g1}}$	$\frac{-1}{T_{g1}}$									
$p\Delta\delta_{23}$											1.0				-1.0
$p\Delta E'_{q2}$								$\frac{-1}{T'_{d02}}$		$\frac{1}{T'_{d02}}$					
$p\Delta E'_{d2}$									$\frac{-1}{T'_{q02}}$						
$p\Delta E_{fd2}$										$\frac{-1}{T_{f2}}$					
$p\Delta\omega_2$											$\frac{-P_{d2}}{M_2}$	$\frac{1}{M_2}$			
$p\Delta P_{t2}$											$\frac{-K_{g2}}{\omega_0 \cdot T_{g2}}$	$\frac{-1}{T_{g2}}$			
$p\Delta E'_{q3}$													$\frac{-1}{T'_{d03}}$		
$p\Delta E'_{d3}$														$\frac{-1}{T'_{q03}}$	
$p\Delta\omega_3$															$\frac{-P_{d3}}{M_3}$

(b) Matrix $\mathbb{A}_2$

	Δv_{d1}	Δv_{q1}	Δi_{d1}	Δi_{q1}	Δv_{d2}	Δv_{q2}	Δi_{d2}	Δi_{q2}	Δv_{d3}	Δv_{q3}	Δi_{d3}	Δi_{q3}
$p\Delta\delta_{13}$												
$p\Delta E'_{q1}$			$\frac{x'_{d1}-x_{d1}}{T'_{do1}}$									
$p\Delta E'_{d1}$				$\frac{x_{q1}-x'_{q1}}{T'_{qo1}}$								
$p\Delta E_{fd1}$	$\frac{-K_{f1}\cdot v_{d10}}{T_{f1}\cdot v_{t10}}$	$\frac{-K_{f1}\cdot v_{q10}}{T_{f1}\cdot v_{t10}}$										
$p\Delta\omega_1$	$\frac{-i_{d10}}{M_1}$	$\frac{-i_{q10}}{M_1}$	$\frac{-v_{d10}}{M_1}$	$\frac{-v_{q10}}{M_1}$								
$p\Delta P_{t1}$												
$p\Delta\delta_{23}$												
$p\Delta E'_{q2}$							$\frac{x'_{d2}-x_{d2}}{T'_{do2}}$					
$p\Delta E'_{d2}$								$\frac{x_{q2}-x'_{q2}}{T'_{qo2}}$				
$p\Delta E_{fd2}$					$\frac{-K_{f2}\cdot v_{d20}}{T_{f1}\cdot v_{t10}}$	$\frac{-K_{f2}\cdot v_{q20}}{T_{f2}\cdot v_{t20}}$						
$p\Delta\omega_2$					$\frac{-i_{d20}}{M_2}$	$\frac{-i_{q20}}{M_2}$	$\frac{-v_{d20}}{M_2}$	$\frac{-v_{q20}}{M_2}$				
$p\Delta P_{t2}$												
$p\Delta E'_{q3}$										$\frac{x'_{d3}-x_{d3}}{T'_{do3}}$		
$p\Delta E'_{d3}$											$\frac{x_{q3}-x'_{q3}}{T'_{qo3}}$	
$p\Delta\omega_3$									$\frac{-i_{d30}}{M_3}$	$\frac{-i_{q30}}{M_3}$	$\frac{-v_{d30}}{M_3}$	$\frac{-v_{q30}}{M_3}$

(c) Matrix $\mathbb{A}_3$

$\Delta\delta_{13}$	$\Delta E'_{q1}$	$\Delta E'_{d1}$	ΔE_{fd1}	$\Delta\omega_1$	ΔP_{t1}	$\Delta\delta_{23}$	$\Delta E'_{q2}$	$\Delta E'_{d2}$	ΔE_{fd2}	$\Delta\omega_2$	ΔP_{t2}	$\Delta E'_{q3}$	$\Delta E'_{d3}$	$\Delta\omega_3$
$[-jT_{13}v_{d30} - j\bar{Z}_{11}T_{12}i'_{d10}]^*$				$[j\omega_0 L_{11} i'_{d10} - \bar{Z}_{11}Y'_1 v_{d10}]^*$		$[j\bar{Z}_{12}T_{12}i'_{d20}]^*$				$[(j\omega_0 L_{12} i'_{d20} - \bar{Z}_{12}Y'_2 v_{d20})T_{12}]^*$				
	0 1	1 0												
$[j\bar{Z}_{21}T_{21}i_{d10}]^*$				$[(j\omega_0 L_{21} i'_{d10} - \bar{Z}_{21}Y'_1 v_{d10})T_{21}]^*$		$[-jT_{23}v_{d30} - j\bar{Z}_{21}T_{21}i_{d10}]^*$				$[j\omega_0 L_{22} i'_{d20} - \bar{Z}_{22}Y'_2 v_{d20}]^*$				
							0 1	1 0						$[Y'_3 \cdot v_{d30}]^*$
$[-jT_{31}i_{d10}]^*$				$[T_{31}Y'_1 v_{d10}]^*$		$[-jT_{32}i_{d20}]^*$				$[T_{32}Y'_2 v_{d20}]^*$				
												0 1	1 0	

(d) Matrix $\mathbb{A}_4$

Δv_{d1}	Δi_{d1}	Δv_{d2}	Δi_{d2}	Δv_{d3}	Δi_{d3}
$[1+Z_{11}Y_1]^*$	$-Z_{11}^*$	$[Z_{12}T_{12}Y_2]^*$	$[-Z_{12}T_{12}]^*$	$-T_{13}^*$	
1 1	$r_1 \; -x'_{q1}$ $x'_{d1} \; r_1$				
$[Z_{21}T_{21}Y_1]^*$	$[-Z_{21}T_{21}]^*$	$[1+Z_{22}Y_2]^*$	$-Z_{22}^*$	$-T_{23}^*$	
		1 1	$r_2 \; -x'_{q2}$ $x'_{d2} \; r_2$		
$[-T_{31}Y_1]^*$	T_{31}^*	$[-T_{32}Y_2]^*$	T_{32}^*	Y_3^*	1 1
				1 1	$r_3 \; -x'_{q3}$ $x'_{d3} \; r_3$

The absence of the zero-sequence equations allows the voltage vectors and the current vectors to be denoted by the complex variables. Choosing the rotor-pole axis as the real axis, the transformation matrices $\mathbb{T}(\delta_{ij})$, $\mathbb{T}'(\delta_{ij})$ and the admittance matrix $\mathbb{Y}_j(\omega_j)$ together with its derivative $\mathbb{Y}_j'(\omega_j)$ are denoted with complex values T_{ij} , T_{ij}' , Y_j and Y_j' ,respectively , for their representation in the steady state.

In Table A-1 (c) and (d),

$$T_{ij} = exp(-j\,\delta_{ijo}) \quad , \quad T_{ij}' = exp(-j(\delta_{ijo} - \tfrac{\pi}{2})) = j\,T_{ij}$$

$$Y_j = G_j - j(\omega_o C_j - 1/\omega_o L_j), \quad Y_j' = j\,(1/\omega_o^2 \cdot L_j + C_j)$$

$$Z_{ij} = R_{ij} + j\,\omega_o \cdot L_{ij} \quad , \quad \bar{Z}_{ij} = R_{ij} - j\,\omega_o \cdot L_{ij}$$

and in Table A-1 (c), $[\,\cdot\,]^*$ denotes $[\mathrm{Re}.[\cdot], \mathrm{Im}.[\cdot]]$,

and in Table A-1 (d), $[\,\cdot\,]^*$ denotes $\begin{bmatrix} \mathrm{Re}.[\cdot], & -\mathrm{Im}.[\cdot] \\ \mathrm{Im}.[\cdot], & \mathrm{Re}.[\cdot] \end{bmatrix}$.

Table A-2 Components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$, $\mathbb{B}$ and $\mathbb{C}$

(a) Matrix $\mathbb{A}_1$

	$\Delta\Psi_{fd}$	$\Delta\Psi_d$	$\Delta\Psi_{kd}$	$\Delta\Psi_q$	$\Delta\Psi_{kq}$	$\Delta\delta$	$\Delta\omega$	ΔE_{fd}	V_s	ΔP_v	ΔP_t
$p\Delta\Psi_{fd}$	$\frac{-\omega_o \cdot r_{fd}}{x_{fl}}$							$\frac{\omega_o \cdot r_{fd}}{x_{ad}}$			
$p\Delta\Psi_d$		$\frac{-\omega_o \cdot r}{x_{al}}$		ω_o			Ψ_{qo}				
$p\Delta\Psi_{kd}$			$\frac{-\omega_o \cdot r_{kd}}{x_{kdl}}$								
$p\Delta\Psi_q$		$-\omega_o$		$\frac{-\omega_o \cdot r}{x_{al}}$			$-\Psi_{do}$				
$p\Delta\Psi_{kq}$					$\frac{-\omega_o \cdot r_{kq}}{x_{kql}}$						
$p\Delta\delta$							1.0				
$p\Delta\omega$		$\frac{-i_{qo}}{M}$		$\frac{i_{do}}{M}$			$\frac{-P_d}{M}$				$\frac{1}{M}$
$p\Delta E_{fd}$								$\frac{-1}{T_f}$	$\frac{-K_f}{T_f}$		
pV_s								$\frac{-K_s}{T_f \cdot T_s}$	$\frac{-1}{T_s}\left(1+\frac{K_s \cdot K_f}{T_f}\right)$		
$p\Delta P_v$							$\frac{-K_g}{\omega_o \cdot T_g}$			$\frac{-1}{T_g}$	
$p\Delta P_t$										$\frac{1}{T_h}$	$\frac{-1}{T_h}$

(b) Matrix $\mathbb{A}_2$

	$\Delta\Psi_{ad}$	$\Delta\Psi_{aq}$	Δi_d	Δi_q	Δv_d	Δv_q
$p\Delta\Psi_{fd}$	$\frac{\omega_o \cdot r_{fd}}{x_{fl}}$					
$p\Delta\Psi_d$	$\frac{\omega_o \cdot r}{x_{al}}$				ω_o	
$p\Delta\Psi_{kd}$	$\frac{\omega_o \cdot r_{kd}}{x_{kdl}}$					
$p\Delta\Psi_q$		$\frac{\omega_o \cdot r}{x_{al}}$				ω_o
$p\Delta\Psi_{kq}$		$\frac{\omega_o \cdot r_{kq}}{x_{kql}}$				
$p\Delta\delta$						
$p\Delta\omega$			$\frac{\Psi_{qo}}{M}$	$-\frac{\Psi_{do}}{M}$		
$p\Delta E_{fd}$					$\frac{-K_f \cdot v_{do}}{T_f \cdot v_{to}}$	$\frac{-K_f \cdot v_{qo}}{T_f \cdot v_{to}}$
pV_s					$\frac{-K_s \cdot K_f \cdot v_{do}}{T_s \cdot T_f \cdot v_{to}}$	$\frac{-K_s \cdot K_f \cdot v_{qo}}{T_s \cdot T_f \cdot v_{to}}$
$p\Delta P_v$						
$p\Delta P_t$						

(c) Matrix $\mathbb{B}$

	u_1	u_2
$p\Delta\Psi_{fd}$		
$p\Delta\Psi_d$		
$p\Delta\Psi_{kd}$		
$p\Delta\Psi_q$		
$p\Delta\Psi_{kq}$		
$p\Delta\delta$		
$p\Delta\omega$		
$p\Delta E_{fd}$	$\frac{K_f}{T_f}$	
pV_s	$\frac{K_s \cdot K_f}{T_s \cdot T_f}$	
$p\Delta P_v$		$\frac{K_g}{T_g}$
$p\Delta P_t$		

(d) Matrix $\mathbb{A}_3$

	$\Delta\Psi_{fd}$	$\Delta\Psi_d$	$\Delta\Psi_{kd}$	$\Delta\Psi_q$	$\Delta\Psi_{kq}$	$\Delta\delta$	$\Delta\omega$	ΔE_{fd}	V_s	ΔP_v	ΔP_t
$\Delta\Psi_{ad}$	$\frac{1}{K_1\cdot X_{fl}}$	$\frac{1}{K_1\cdot X_{al}}$	$\frac{1}{K_1\cdot X_{kdl}}$								
$\Delta\Psi_{aq}$				$\frac{1}{K_2\cdot X_{al}}$	$\frac{1}{K_2\cdot X_{kql}}$						
Δi_d	$\frac{1}{K_1\cdot X_{fl}\cdot X_{al}}$	$\frac{1}{K_1\cdot X_{al}^2}-\frac{1}{X_{al}}$	$\frac{1}{K_1\cdot X_{kdl}\cdot X_{al}}$								
Δi_q				$\frac{1}{K_2\cdot X_{al}^2}-\frac{1}{X_{al}}$	$\frac{1}{K_2\cdot X_{kql}\cdot X_{al}}$						
Δv_d	$(\mathbb{I}+\mathbb{Z}_e\cdot\mathbb{Y})^{-1}\cdot\mathbb{Z}_e\cdot\begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix}$					$(\mathbb{I}+\mathbb{Z}_e\cdot\mathbb{Y})^{-1}\times\mathbb{T}'(\delta_0)\cdot\mathbb{V}_0$					
Δv_q											

(e) Matrix $\mathbb{C}$

	$\Delta\Psi_{fd}$	$\Delta\Psi_d$	$\Delta\Psi_{kd}$	$\Delta\Psi_q$	$\Delta\Psi_{kq}$	$\Delta\delta$	$\Delta\omega$	ΔE_{fd}	V_s	ΔP_v	ΔP_t
$\Delta\delta$						1.0					
$\Delta\omega$							1.0				
ΔE_{fd}								1.0			
V_s									1.0		
ΔP_v										1.0	
Δv_t	$\left[\frac{v_{d0}}{v_{t0}},\frac{v_{q0}}{v_{t0}}\right]\cdot(\mathbb{I}+\mathbb{Z}_e\cdot\mathbb{Y})^{-1}\cdot\{\mathbb{Z}_e\cdot\Delta\mathbf{i}_d+\mathbb{T}'(\delta_0)\cdot\mathbb{V}_0\cdot\Delta\delta\}$										
Δi_t	$\left[\frac{i_{d0}}{i_{t0}},\frac{i_{q0}}{i_{t0}}\right]\cdot\Delta\mathbf{i}_d$										

Table A-3 Components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$ and $\mathbb{B}$

(a) Matrix $\mathbb{A}_1$

	$\Delta\delta$	$\Delta\omega$	$\Delta E_q'$	$\Delta E_d'$	ΔE_{fd}	V_s	ΔP_v	ΔP_t
$p\Delta\delta$		1.0						
$p\Delta\omega$		$-P_d/M$						$1/M$
$p\Delta E_q'$			$-1/T_{do}'$		$1/T_{do}'$			
$p\Delta E_d'$				$-1/T_{qo}'$				
$p\Delta E_{fd}$					$-1/T_f$	$-K_f/T_f$		
pV_s					$-K_s/T_f\cdot T_s$	$-K_f\cdot K_s/T_f\cdot T_s$ $-1/T_s$		
$p\Delta P_v$		$-K_g/\omega_0\cdot T_g$					$-1/T_g$	
$p\Delta P_t$							$1/T_h$	$-1/T_h$

(b) Matrix $\mathbb{A}_2$

	Δv_d	Δv_q	Δi_d	Δi_q
$p\Delta\delta$				
$p\Delta\omega$	$-i_{do}/M$	$-i_{qo}/M$	$-v_{do}/M$	$-v_{qo}/M$
$p\Delta E_q'$			$\frac{x_d'-x_d}{T_{do}'}$	
$p\Delta E_d'$				$\frac{x_q-x_q'}{T_{qo}'}$
$p\Delta E_{fd}$	$\frac{-K_f\cdot v_{do}}{T_f\cdot v_{to}}$	$\frac{-K_f\cdot v_{qo}}{T_f\cdot v_{to}}$		
pV_s	$\frac{-K_f\cdot K_s\cdot v_{do}}{T_f\cdot T_s\cdot v_{to}}$	$\frac{-K_f\cdot K_s\cdot v_{qo}}{T_f\cdot T_s\cdot v_{to}}$		
$p\Delta P_v$				
$p\Delta P_t$				

(c) Matrix $\mathbb{B}$

	u_1	u_2
$p\Delta\delta$		
$p\Delta\omega$		
$p\Delta E_q'$		
$p\Delta E_d'$		
$p\Delta E_{fd}$	K_f/T_f	
pV_s	$K_f\cdot K_s/T_f\cdot T_s$	
$p\Delta P_v$		K_s/T_g
$p\Delta P_t$		

(d) Matrix $\mathbb{A}_3$

	$\Delta\delta$	$\Delta\omega$	$\Delta E_q'$	$\Delta E_d'$	ΔE_{fd}	V_s	ΔP_v	ΔP_t
Δv_d	$\frac{x_q'\cdot V_o\cdot\cos\delta_o}{x_q'+x_e+x_t}$			$\frac{x_e+x_t}{x_q'+x_e+x_t}$				
Δv_q	$\frac{x_d'\cdot V_o\cdot\sin\delta_o}{x_d'+x_e+x_t}$		$\frac{x_e+x_t}{x_d'+x_e+x_t}$					
Δi_d	$\frac{V_o\cdot\sin\delta_o}{x_d'+x_e+x_t}$		$\frac{1}{x_d'+x_e+x_t}$					
Δi_q	$\frac{V_o\cdot\cos\delta_o}{x_q'+x_e+x_t}$			$\frac{-1}{x_q'+x_e+x_t}$				

Table A-4 Components of the matrices $\mathbb{A}_1$, $\mathbb{A}_2$, $\mathbb{A}_3$, $\mathbb{A}_4$, $\mathbb{A}_5$ and $\mathbb{B}$

(a) Matrix $\mathbb{A}_1$

	$\Delta\delta_{13}$	$\Delta\delta_{23}$	$\Delta\omega_1$	$\Delta\omega_2$	$\Delta\omega_3$	$\Delta E_{q1}'$	$\Delta E_{q2}'$	$\Delta E_{q3}'$	ΔE_{fd1}	ΔE_{fd2}	ΔP_{t1}	ΔP_{t2}
$p\Delta\delta_{13}$			1.0		-1.0							
$p\Delta\delta_{23}$				1.0	-1.0							
$p\Delta\omega_1$			$-P_{d1}/M_1$								$1/M_1$	
$p\Delta\omega_2$				$-P_{d2}/M_2$								$1/M_2$
$p\Delta\omega_3$					$-P_{d3}/M_3$							
$p\Delta E_{q1}'$						$-1/T_{do1}'$			$1/T_{do1}'$			
$p\Delta E_{q2}'$							$-1/T_{do2}'$			$1/T_{do2}'$		
$p\Delta E_{q3}'$								$-1/T_{do3}'$				
$p\Delta E_{fd1}$									$-1/T_{f1}$			
$p\Delta E_{fd2}$										$-1/T_{f2}$		
$p\Delta P_{t1}$											$-K_{g1}/\omega_o\cdot T_{g1}$	
$p\Delta P_{t2}$												$-K_{g2}/\omega_o\cdot T_{g2}$

(b) Matrix $\mathbb{A}_2$

	ΔV_{d1}	ΔV_{q1}	ΔV_{d2}	ΔV_{q2}	ΔV_{d3}	ΔV_{q3}	Δi_{d1}	Δi_{q1}	Δi_{d2}	Δi_{q2}	Δi_{d3}	Δi_{q3}
$p\Delta\delta_{13}$												
$p\Delta\delta_{23}$												
$p\Delta\omega_1$	$\frac{-i_{d10}}{M_1}$	$\frac{-i_{q10}}{M_1}$					$\frac{-V_{d10}}{M_1}$	$\frac{-V_{q10}}{M_1}$				
$p\Delta\omega_2$			$\frac{-i_{d20}}{M_2}$	$\frac{-i_{q20}}{M_2}$					$\frac{-V_{d20}}{M_2}$	$\frac{-V_{q20}}{M_2}$		
$p\Delta\omega_3$					$\frac{-i_{d30}}{M_3}$	$\frac{-i_{q30}}{M_3}$					$\frac{-V_{d30}}{M_3}$	$\frac{-V_{q30}}{M_3}$
$p\Delta E'_{q1}$							$\frac{x'_{d1}-x_{d1}}{T'_{d01}}$					
$p\Delta E'_{q2}$									$\frac{x'_{d2}-x_{d2}}{T'_{d02}}$			
$p\Delta E'_{q3}$											$\frac{x'_{d3}-x_{d3}}{T'_{d03}}$	
$p\Delta E_{fd1}$	$\frac{-K_{f1}\cdot V_{d10}}{T_{f1}\cdot V_{t10}}$	$\frac{-K_{f1}\cdot V_{q10}}{T_{f1}\cdot V_{t10}}$										
$p\Delta E_{fd2}$			$\frac{-K_{f2}\cdot V_{d20}}{T_{f2}\cdot V_{t20}}$	$\frac{-K_{f2}\cdot V_{q20}}{T_{f2}\cdot V_{t20}}$								
$p\Delta P_{t1}$												
$p\Delta P_{t2}$												

(c) Matrix $\mathbb{A}_3$

ΔV_{d1}	ΔV_{q1}	ΔV_{d2}	ΔV_{q2}	ΔV_{d3}	ΔV_{q3}	Δi_{d1}	Δi_{q1}	Δi_{d2}	Δi_{q2}	Δi_{d3}	Δi_{q3}
1.0						r_1	$-x'_{q1}$				
	1.0					x'_{d1}	r_1				
		1.0						r_2	$-x'_{q2}$		
			1.0					x'_{d2}	r_2		
				1.0						r_3	$-x'_{q3}$
					1.0					x'_{d3}	r_3
$\mathbb{Y}'_{11}$		$\mathbb{Y}'_{12}$		$\mathbb{Y}'_{13}$							
$\mathbb{Y}'_{21}$		$\mathbb{Y}'_{22}$		$\mathbb{Y}'_{23}$							
$\mathbb{Y}'_{31}$		$\mathbb{Y}'_{32}$		$\mathbb{Y}'_{33}$							

(d) Matrix $\mathbb{A}_4$

$\Delta\delta_{13}$	$\Delta\delta_{23}$	$\Delta E'_{g1}$	$\Delta E'_{g2}$	$\Delta E'_{g3}$
		1.0		
			1.0	
				1.0
α_1	α_2			
α_3	α_4			
α_5	α_6			

$$\alpha_1 = \left.\frac{\partial \mathbb{Y}'_{12}}{\partial \delta_{13}}\right|_0 \cdot \upsilon_{d20} + \left.\frac{\partial \mathbb{Y}'_{13}}{\partial \delta_{13}}\right|_0 \cdot \upsilon_{d30}$$

$$\alpha_2 = \left.\frac{\partial \mathbb{Y}'_{12}}{\partial \delta_{23}}\right|_0 \cdot \upsilon_{d20}$$

$$\alpha_3 = \left.\frac{\partial \mathbb{Y}'_{21}}{\partial \delta_{13}}\right|_0 \cdot \upsilon_{d10}$$

$$\alpha_4 = \left.\frac{\partial \mathbb{Y}'_{21}}{\partial \delta_{23}}\right|_0 \cdot \upsilon_{d10} + \left.\frac{\partial \mathbb{Y}'_{23}}{\partial \delta_{23}}\right|_0 \cdot \upsilon_{d30}$$

$$\alpha_5 = \left.\frac{\partial \mathbb{Y}'_{21}}{\partial \delta_{13}}\right|_0 \cdot \upsilon_{d10}$$

$$\alpha_6 = \left.\frac{\partial \mathbb{Y}'_{32}}{\partial \delta_{23}}\right|_0 \cdot \upsilon_{d20}$$

(f) Matrix $\mathbb{B}$

	u_{11}	u_{12}	u_{21}	u_{22}
$p\Delta\delta_{13}$				
$p\Delta\delta_{23}$				
$p\Delta\omega_1$				
$p\Delta\omega_2$				
$p\Delta\omega_3$				
$p\Delta E'_{g1}$				
$p\Delta E'_{g2}$				
$p\Delta E'_{g3}$				
$p\Delta E_{fd1}$	K_{f1}/T_{f1}			
$p\Delta E_{fd2}$		K_{f2}/T_{f2}		
$p\Delta P_{t1}$			K_{g1}/T_{g1}	
$p\Delta P_{t2}$				K_{g2}/T_{g2}

(d) Matrix $\mathbb{A}_5$

	$\Delta\delta_{13}$	$\Delta\delta_{23}$	$\Delta\omega_1$	$\Delta\omega_2$	$\Delta\omega_3$	$\Delta E'_{g1}$	$\Delta E'_{g2}$	$\Delta E'_{g3}$	ΔE_{fd1}	ΔE_{fd2}	ΔP_{t1}	ΔP_{t2}
$\Delta\delta_{13}$	1.0											
$\Delta\delta_{23}$		1.0										
$\Delta E'_{g1}$						1.0						
$\Delta E'_{g2}$							1.0					
$\Delta E'_{g3}$								1.0				

APPENDIX B Eigenvector Solution of Matrix Riccati Equation[56],[57],[58]

Consider the following linear system along with its cost function given by the following equation.

$$p\Delta x = A \cdot \Delta x + B \cdot u \tag{B-1}$$

$$J = \frac{1}{2}\int_0^\infty (\Delta x^T \cdot Q \cdot \Delta x + u^T \cdot R \cdot u)\, dt \tag{B-2}$$

Let λ be the costate vector of Δx. Then λ obeys the differential equation:

$$p\lambda = -Q \cdot \Delta x - A^T \cdot \lambda \tag{B-3}$$

Then, it is well known that the optimal control, which minimizes the cost function, is given by:

$$u = -R^{-1} \cdot B^T \cdot \lambda \tag{B-4}$$

From eqn.(B-1), eqn.(B-3) and eqn.(B-4), it is obtained:

$$p\begin{bmatrix} \Delta x \\ \lambda \end{bmatrix} = \begin{bmatrix} A, & -B \cdot R^{-1} \cdot B^T \\ -Q, & -A^T \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \lambda \end{bmatrix} \tag{B-5}$$

Let the system matrix of eqn.(B-5) be denoted by M.

$$M = \begin{bmatrix} A, & -B \cdot R^{-1} \cdot B^T \\ -Q, & -A^T \end{bmatrix} \tag{B-6}$$

It has been shown that the eigenvalues of M must be symmetric with respect to the imaginary axis of the complex plane and there are no pure imaginary eigenvalues. It will be further assumed that the eigenvalues of M are distinct.

Let D be $(2n \times 2n)$ diagonal matrix of the eigenvalues of M arranged so that $-\Lambda$ is the $(n \times n)$ diagonal matrix of left half plane eigenvalues, which are the eigenvalues of the optimally controlled system, where n is the order of the state variables vector Δx :

$$\mathbb{D} = \begin{bmatrix} -\Lambda , & 0 \\ 0 , & \Lambda \end{bmatrix} = \mathbb{W}^{-1} \cdot \mathbb{M} \cdot \mathbb{W} \tag{B-7}$$

Hence here we can make use of the eigenvalue grouping technique to arrive at a model

$$p \Delta x = \mathbb{A}^{*} \cdot \Delta x \tag{B-8}$$

by eliminating the eigenvalues Λ in eqn.(B-6). This is justified because $\varepsilon^{\Lambda t}$ tends to zero as the Riccati equation is to be solved backwards in time. If the matrix $\mathbb{W}$ is partitioned properly as follows:

$$\mathbb{W} = \begin{bmatrix} \mathbb{W}_{11} , & \mathbb{W}_{12} \\ \mathbb{W}_{21} , & \mathbb{W}_{22} \end{bmatrix} \tag{B-9}$$

Then we get:

$$\mathbb{A}^{*} = \mathbb{A} - \mathbb{B} \cdot \mathbb{R}^{-1} \cdot \mathbb{B}^{T} \cdot \mathbb{W}_{21} \cdot \mathbb{W}_{11}^{-1} \tag{B-10}$$

Defining

$$\mathbb{K} = \mathbb{W}_{21} \cdot \mathbb{W}_{11}^{-1} \tag{B-11}$$

it can easily seen that the matrix $\mathbb{K}$ is the solution of the following matrix Riccati equation:

$$\mathbb{A}^{T} \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{A} - \mathbb{K} \cdot \mathbb{B} \cdot \mathbb{R}^{-1} \cdot \mathbb{B}^{T} \cdot \mathbb{K} + \mathbb{Q} = 0 \tag{B-12}$$

From eqn.(B-11) it follows that the eigenvectors of the matrix $\mathbb{M}$ corresponding to those eigenvalues with negative real part only are needed to get the required solution of the matrix Riccati equation (B-12).

APPENDIX C Eigenvector Solution of Lyapunov's Matrix Equation

By the assumption that $\mathbb{R}^{-1} = \mathbb{0}$, the matrix Riccati equation (B-12) becomes following Lyapunov's matrix equation.

$$\mathbb{A}^T \cdot \mathbb{K} + \mathbb{K} \cdot \mathbb{A} + \mathbb{Q} = \mathbb{0} \qquad \text{(C-1)}$$

In this case, eqn.(B-6) becomes as follows:

$$\mathbb{M} = \begin{bmatrix} \mathbb{A} , & \mathbb{0} \\ -\mathbb{Q} , & -\mathbb{A}^T \end{bmatrix} \qquad \text{(C-2)}$$

It has been shown that the eigenvalues of the matrix $\mathbb{M}$ must be symmetric with respect to the imaginary axis of the complex plane. It will be further assumed that the eigenvalues of the matrix $\mathbb{M}$ are distinct and there are no pure imaginary eigenvalues.

Let $\mathbb{D}$ be $(2n \times 2n)$ diagonal matrix of the eigenvalues of $\mathbb{M}$ arranged so that $-\Lambda$ is the $(n \times n)$ diagonal matrix of the left half plane eigenvalues of the matrix $\mathbb{M}$:

$$\mathbb{D} = \begin{bmatrix} -\Lambda , & \mathbb{0} \\ \mathbb{0} , & \Lambda \end{bmatrix} = \mathbb{W}^{-1} \cdot \mathbb{M} \cdot \mathbb{W} \qquad \text{(C-3)}$$

where the matrix $\mathbb{W}$ is constructed using the eigenvectors of the matrix $\mathbb{M}$ and is properly partitioned as:

$$\mathbb{W} = \begin{bmatrix} \mathbb{W}_{11} , & \mathbb{W}_{12} \\ \mathbb{W}_{21} , & \mathbb{W}_{22} \end{bmatrix} \qquad \text{(C-4)}$$

Then, the solution matrix $\mathbb{K}$ of the above Lyapunov's matrix equation becomes:

$$\mathbb{K} = \mathbb{W}_{21} \cdot \mathbb{W}_{11}^{-1} \qquad \text{(C-5)}$$

APPENDIX D Solution of Lyapunov's Matrix Equation using Companion Matrix

Consider the Lyapunov's matrix equation (C-1) shown in Appendix C. Let $\mathbb{C}_a$ be the (n x n) companion matrix[43] of the matrix $\mathbb{A}$ and Let $\mathbb{T}_a$ be the (n x n) transformation matrix, then the following relationship is satisfied:

$$\mathbb{C}_a^T = (\mathbb{T}_a^{-1})^T \cdot \mathbb{A}^T \cdot \mathbb{T}_a = \begin{bmatrix} 0 & \cdots & \cdots & -a_1 \\ 1 & \ddots & & \vdots \\ & \ddots & 0 & -a_{n-1} \\ \mathbb{0} & & 1 & -a_n \end{bmatrix} \qquad \text{(D-1)}$$

where, a_1 , a_2 , ... , and a_n are the coefficients of the following characteristic polynomial:

$$det|\mathbb{A}-\lambda\mathbb{I}| = \lambda^n + a_n \cdot \lambda^{n-1} + \cdots\cdots + a_2 \cdot \lambda + a_1 \qquad \text{(D-2)}$$

Eqn.(C-1) can be written using the matrices $\mathbb{C}_a$ and $\mathbb{T}_a$ as follows:

$$\mathbb{C}_a^T \cdot \mathbb{Y} + \mathbb{Y} \cdot \mathbb{C}_a + \mathbb{R} = 0 \qquad \text{(D-3)}$$

where,

$$\mathbb{R} = (\mathbb{T}_a^{-1})^T \cdot \mathbb{Q} \cdot \mathbb{T}_a^{-1} \qquad \text{(D-4)}$$

$$\mathbb{Y} = (\mathbb{T}_a^{-1})^T \cdot \mathbb{K} \cdot \mathbb{T}_a^{-1} \qquad \text{(D-5)}$$

Then the solution $\mathbb{K}$ of the Lyapunov's matrix equation (B-1) can be given by the solution $\mathbb{Y}$ of eqn.(D-3) as follows:

$$\mathbb{K} = \mathbb{T}_a^T \cdot \mathbb{Y} \cdot \mathbb{T}_a \qquad \text{(D-6)}$$

Here, it is noted that eqn.(D-3) can be easily solved.

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