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CMB Fluctuations and String Compactification Scales

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Abstract

We propose a mechanism for the generation of temperature fluctuations of cosmic microwave background. We consider a large number of fields, such as Kaluza-Klein modes and string excitations. Each field contributes to the gravitational potential by a small amount, but an observable level of temperature fluctuations is achieved by summing up the contribution of typically of order 10^{14} fields. Tensor fluctuations are hardly affected by these fields. Our mechanism is based on purely quantum effects of the fields which are classically at rest, and is different from the one in slow-roll inflation. Using the observed data, we find constraints on the parameters of this model, such as the size of the extra dimensions and the string scale. Our model predicts a particular pattern of non-gaussianity with a small magnitude.

Keywords: string theory, compactification scale, cosmic microwave background

Introduction.—. Observation of cosmic microwave background (CMB) [1] provides a great opportunity for testing fundamental theories. It is generally believed that the fluctuations of CMB originate from quantum fluctuations generated at energy much higher than present accelerators can achieve.

It is hoped that string theory gives predictions for observational cosmology. But at the current stage of its development, string theory is defined only in limited classes of background spacetimes. The analysis of cosmology has been limited to those based on low energy effective field theory, and it has been difficult to make concrete predictions. (For attempts to formulate cosmology beyond the level of effective field theory, see e.g. [2] and [3].)

In this Letter we will focus on an aspect of fundamental theories, namely the presence of a large number of fields. These could be Kaluza-Klein (KK) modes from the compactification of extra dimensions or excited states of strings. Our analysis is mostly based on effective field theory, but we will also use a general property of perturbative string theory as an input.

It would be possible that the size of extra dimensions L is large in unit of Hubble scale of inflation H^{-1}. We already know a hierarchy of scales between the Planck scale m_{pl} and H: The fact that tensor perturbation (B-mode polarization of CMB) has not been observed implies H is at least 4 orders of magnitude smaller than m_{pl} [1]. The question is whether string scale m_s is close to m_{pl} or H. We will see below that with m_s slightly smaller than

$H \sim 10^{-4} m_{pl}$, L can be as large as $L \sim 10^8 H^{-1}$. In such a case, there will be many KK modes with $m \lesssim H$. We will show that the quantum effects of these fields lead to interesting observable effects.

For concreteness, we shall consider a collection of N free fields ϕ_A ($A = 1, \ldots, N$) with mass m_A, which are classically at the bottom of the potential $\phi_A = 0$. We will not consider inflaton field, and take the background to be pure de Sitter, and find the temperature fluctuations generated during inflation. At the end of this Letter, we make comments on the effect of time dependence of Hubble, e.g. near the end of inflation.

In this Letter we point out that temperature fluctuations $\delta T/T$ can be generated by a mechanism different from the standard one based on slow-roll inflation. In the latter case, the classical value of a scalar field (inflaton) provides a preferred time slicing, and its fluctuation is interpreted as the difference of duration of inflation at different points in space, which results in the density (or temperature) anisotropy. In our case, the fields ϕ_A are classically at rest, and the above argument does not apply. It has been assumed that these fields do not contribute to $\delta T/T$. We will show that by keeping quadratic terms in ϕ_A, the gravitational potential Φ is generated through quantum effects of these fields. Each field gives small contribution, but when $N \gg 1$, this effect becomes important. The large ratio between scalar and tensor contributions to $\delta T/T$ is usually assumed to be the consequence of the smallness of the slope of the inflaton potential. In our approach, it is the consequence of a large number of fields.

In the following, we first review quantization in de Sitter space, and recall that fields with $m < \frac{3}{2}H$ do not oscillate at super-horizon scales, because the "friction" due to the

cosmic expansion overdumps the oscillation. These are the fields that we will focus on. We then study Einstein equations and solve for the gravitational potential Φ in terms of the matter fields. Using this, we find the contribution to CMB fluctuations from KK modes. We will make an argument based on string perturbation theory that if string scale m_s is lower than H, the summation over the mass is effectively cut off at m_s. We then use the observed data on the amplitude of $\delta T/T$ and the tensor-to-scalar ratio $r_{t/s}$ to constrain the parameters, such as m_s and L.

There have been inflationary models involving a large number of fields. In "N-flation" [4] (or "assisted inflation" [5]), many fields (such as axions) classically roll down the potential, collectively producing an effect similar to chaotic inflation. Our mechanism is different from this, since we are not assuming classical motion. Also note that our mechanism is different from the curvatons scenario [6]. We demonstrate that curvature fluctuations are generated during inflations without such a late time mechanism.

Quantization in de Sitter background.—. We consider background de Sitter space

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2, \quad a(t) = H^{-1}e^{Ht}, \quad (1)$$

and a collection of N free massive scalar fields,

$$S = \sum_{A=1}^{N} \int d^4x \sqrt{-g} \left\{ \partial_\mu \phi_A \partial^\mu \phi_A - m_A^2 \phi_A^2 \right\}. \quad (2)$$

In the following we will often use the conformal time, $\tau = \int dt/a(t) = -e^{-Ht}$ ($-\infty \le \tau \le 0$), and the rescaled field $\chi_A = a\phi_A$ which has the standard kinetic term. We will suppress the label A hereafter for brevity.

Fourier component of χ satisfies the equation of motion (prime denotes ∂_τ),

$$\chi_{\vec{k}}''(\tau) + \left\{ |\vec{k}|^2 + \left(H^{-2}m^2 - 2\right)\frac{1}{\tau^2} \right\} \chi_{\vec{k}}(\tau) = 0. \quad (3)$$

Quantization is done by setting

$$\chi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2|\vec{k}|}} \left[u_{\vec{k}}(\tau) a_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} + u_{\vec{k}}^*(\tau) a_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}} \right] \quad (4)$$

where $u_{\vec{k}}(\tau)$ is the solution of (3) which approaches $u_{\vec{k}}(\tau) \to e^{-i|\vec{k}|\tau}$ at early time $\tau \to -\infty$. This condition ensures that the flat space result is recovered in the short distance limit. The solution is given by $u_{\vec{k}}(\tau) = \sqrt{\frac{\pi}{2}} e^{i\frac{\pi}{2}(\nu+\frac{1}{2})} \sqrt{-|\vec{k}|\tau} H_\nu^{(1)}(-|\vec{k}|\tau)$ with $\nu = \sqrt{\frac{9}{4} - m^2 H^{-2}}$. Asymptotic behavior at the late times (in the super-horizon $|\vec{k}|/a \ll H$ limit) is $u_{\vec{k}} \sim (-|\vec{k}|\tau)^{-\nu+\frac{1}{2}}$. Time dependence factorizes from space dependence, as is clear from the fact that $|\vec{k}|$ dependence drops out from (3) in this limit. Fields with small mass, $mH^{-1} < \frac{3}{2}$, do not oscillate in time. We will take the $mH^{-1} \ll 1$ limit in the

following formulas, since this is the case of importance for our applications, as explained below.

The equal time two-point function of ϕ at late times is

$$\langle \phi(\tau, \vec{x})\phi(\tau, \vec{x}') \rangle = \frac{1}{a^2(\tau)} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2|k|} |u_k(\tau)|^2 e^{i\vec{k}(\vec{x}-\vec{x}')}$$
$$\sim \frac{H^2}{4\pi^2\beta} (Ha|\vec{x}-\vec{x}'|)^{-\beta}, \quad (5)$$

where we have taken the $m^2 H^{-2} \ll 1$ limit, and defined

$$\beta = \frac{2}{3} m^2 H^{-2}. \quad (6)$$

Einstein equations.—. Quantum fluctuations of the fields ϕ induce gravitational potential. Let us look at the $(0, i)$ and (i, j) components of the Einstein equation,

$$(\Psi' + \mathcal{H}\Phi)_{,i} = 4\pi G \delta T_{0i}^{(S)}, \quad (7)$$

$$\left[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{\triangle}{2}(\Phi - \Psi) \right] \delta_{ij}$$
$$-\frac{1}{2}(\Phi - \Psi)_{,ij} = 4\pi G \delta T_{ij}^{(S)} \quad (8)$$

where $\mathcal{H} = \frac{a'}{a} = -\frac{1}{\tau}$. The l.h.s is the Einstein tensor expanded to the first order in metric fluctuations; Φ and Ψ are the two gauge invariant combinations constructed from the scalar modes (See e.g. [8]). On the r.h.s. we have energy momentum tensor which is quadratic in ϕ,

$$\delta T_{\mu\nu} = \sum \left\{ \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial^\rho \phi \partial_\rho \phi - m^2 \phi^2) \right\}. \quad (9)$$

The sum is taken over the species of the fields. The superscript (S) denotes the scalar part (the part which can be written as derivatives of a scalar), e.g.,

$$\delta T_{0i}^{(S)} = \partial_i \left(\frac{1}{\triangle} \partial_k \delta T_{0k} \right) = \frac{1}{\triangle} \partial_i \partial_k (\phi' \partial_k \phi) \quad (10)$$

We shall use the Einstein equation to express Φ and Ψ in terms of the matter fields ϕ. We first find the difference $\Phi - \Psi$ from the $(i \neq j)$ component of (8),

$$\Phi - \Psi = -8\pi G \sum s, \quad s \equiv \frac{3}{2\triangle^2} \partial_i \partial_j (\partial_i \phi \partial_j \phi - \frac{\delta_{ij}}{3} \partial_k \phi \partial_k \phi)$$

and substitute it into (7) to get

$$\Phi' + \mathcal{H}\Phi = 4\pi G \sum \left\{ -2s' + \frac{1}{\triangle} \partial_i (\phi' \partial_i \phi) \right\}. \quad (11)$$

At late times, the r.h.s. goes like $(-\tau)^{\beta-1}$, which implies $\Phi \sim (-\tau)^\beta$. The part containing s can be dropped when $\beta \ll 1$, since this gives smaller contribution in correlation functions than the second term[1]. Using the fact that

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[1] In correlation functions, $\frac{\partial}{\partial x^i} \langle \phi(x)\phi(x') \rangle$ gives a factor of order β (and further differentiation only gives order 1 factors like $-(\beta+1)$). Correlators involving the first term of (11) necessarily contain more of these factors than the ones involving only the second term of (11).

time dependence ($\phi \sim (-\tau)^{\beta/2}$) enters as a multiplicative factor, and $\partial_i(\phi\partial_i\phi) = \frac{1}{2}\triangle\phi^2$, we find

$$\Phi = -\sum \pi G\beta\phi^2. \qquad (12)$$

at late times and $\beta \ll 1$. This satisfies all the components of Einstein equations. The connected part of two-point function becomes[2]

$$
\begin{aligned}
\langle\Phi(\tau,x)\Phi(\tau,x')\rangle &= \sum 2(\pi G\beta)^2\langle\phi(\tau,x)\phi(\tau,x')\rangle^2 \\
&= \sum \frac{H^4 G^2}{8\pi^2}(Ha|\vec{x}-\vec{x'}|)^{-2\beta}.(13)
\end{aligned}
$$

Next we consider tensor fluctuations. The transverse-traceless (TT) mode of gravitons ($\nabla^i h_{ij}=h_i^i=0$) is sourced by the TT part of energy-momentum tensor,

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \triangle h_{ij} = 8\pi G\delta T^{(T)}_{ij}. \qquad (14)$$

The general solution to this equation is the sum of the solution $h^{(0)}_{ij}$ for the homogeneous equation and a particular solution $h^{(1)}_{ij}$ which depends on $\delta T^{(T)}_{ij}$. This means that the tensor fluctuation consists of the usual gravitational wave $h^{(0)}_{ij}$ on top of the piece $h^{(1)}_{ij}$ which is determined by $\delta T^{(T)}_{ij}$. Since the homogeneous equation is equivalent to massless scalar equation, $h^{(0)}_{ij}$ scales logarithmically (in space and time), giving rise to the scale invariant spectrum with the amplitude H/m_{pl}. On the other hand, $h^{(1)}_{ij}$ decays in time ($h^{(1)}_{ij} \sim (-\tau)^{\beta+2}$, since $\delta T^{(T)}_{ij} \sim (-\tau)^\beta$), so the effect of $\delta T^{(T)}_{ij}$ is unimportant at late times.

KK modes and string states.— Let us now assume that a part of the extra dimensions is compactified on a manifold with the size $L \gg H^{-1}$, and find temperature fluctuations produced by the KK modes. For simplicity, we assume D dimensions are compactified on T^D with the same periodicity L in all directions, and the other dimensions are compactified on a space with the string scale size. KK modes from T^D have mass $m^2 = \sum_{a=1}^D (2\pi n_a)^2/L^2$. Assuming the level is sufficiently dense, the number of states is given by $S_{D-1}|n|^{D-1}d|n| = S_{D-1}(L/2\pi)^D m^{D-1}dm$, where $S_{D-1} = 2\pi^{D/2}/\Gamma(D/2)$ is the volume of the $D-1$ dimensional unit sphere.

In this Letter, we only consider scalar fields. There are also vector and tensor KK modes, but as long as we are considering quantum fluctuations around vanishing expectation values, they behave similarly to massive scalars, and just contribute extra multiplicities[3].

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[2] The expectation value (one-point function) of Φ is UV divergent, but the two-point function is not. As long as we are studying the connected part of two-point functions, we do not need renormalization.

[3] We do not have problems of anisotropy, since vectors and tensors have zero expectation values. Even if they aquire non-zero expectation values after inflation, the anisotropy will be suppressed as $1/\sqrt{N}$ when there are N independent fields. See [7] for discussion in a closely related context.

The two point function of Φ (13) becomes

$$\langle\Phi\Phi\rangle = c_D L^D \left(\frac{H}{m_p}\right)^4 \int_0^{m_c} dm\, m^{D-1}(Ha|\vec{x}-\vec{x'}|)^{-2\beta}, \qquad (15)$$

where $c_D = S_{D-1}/(4(2\pi)^{D+2})$. The upper limit m_c of the integration should be $m_c \sim \frac{3}{2}H$ as long as we are working in Einstein gravity. However, if string scale is less than Hubble scale, $m_s < H$, string states should be taken into account. In this case, we expect that the sum over the mass is effectively cut off at $m_c \sim m_s$ for the following reason. Let us assume the two-point function of Φ comes from the one-loop diagram in string theory. String theory can be regarded as a field theory with infinitely many fields, except that one-loop amplitude effectively has UV cutoff due to modular invariance; Schwinger proper time is cut off at string scale. There is no physical meaning to time interval shorter than string scale, or oscillations much higher than string scale. This means the internal states in the loop which has mass much larger than string scale do not have physical effect[4].

CMB fluctuations.— Temperature fluctuation of CMB is related to Φ at recombination (at redshift $z \sim 1100$) by $\delta T/T = -\Phi/3$. The angle θ on the sky corresponds to the distance $d_r = 2R_r\sin(\theta/2)$ where $R_r \sim H_0^{-1}$ is the radius of the surface of last scattering. We are interested in the modes outside the horizon at recombination, which correspond to $3^o \leq \theta$ ($l \leq 60$). For these modes, Φ will be frozen (remain constant at fixed comoving distance) after inflation. Thus we will look at the correlator at distance $(a_e/a_r)d_r = 2R\sin(\theta/2) = a_e|\vec{x}-\vec{x'}|$ at the end of inflation, where a_e and a_r are the scale factors at the end of inflation and at recombination. The radius $R = (a_e/a_r)R_r$ will depend on the scale of inflation. We will assume $RH \sim 10^{29} \sim e^{67}$ in the following.

The power spectrum C_l is defined by $\langle\frac{\delta T}{T}(\theta)\frac{\delta T}{T}(0)\rangle = \sum_{l=1}^\infty (2l+1)C_l P_l(\cos\theta)$. To find it, let us expand

$$(Ha|\vec{x}-\vec{x'}|)^{-2\beta} \sim (2RH)^{-2\beta}(1 - 2\beta\log(\sin(\theta/2)),$$

and recall that $-2\log(\sin(\theta/2))$ is $1/(l(l+1))$ in harmonic space. The square amplitude $\delta_T^2 \equiv l(l+1)C_l$ becomes

$$
\begin{aligned}
\delta_T^2 &= \frac{2}{3}c_D\frac{L^D}{H^2}\left(\frac{H}{m_{pl}}\right)^4 \int_0^{m_s} dm\, m^{D+1}(2RH)^{-\frac{4}{3}m^2 H^{-2}} \\
&= \frac{2}{3}c_D\left(\frac{m_s}{H}\right)^2 (Lm_s)^D \left(\frac{H}{m_{pl}}\right)^4 \mathcal{M}_D(\zeta_0), \qquad (16)
\end{aligned}
$$

where we have taken the upper limit to be $m_c = m_s$. This is a good approximation even when $m_s > H$, since the

---

[4] This argument is based on string theory in flat background, and it is not clear whether this can be applied to an arbitrarily curved background, but we believe this is a reasonable estimate. In fact, the precise value of this cutoff does not affect our final conclusion: The preferred value of the string scale, being about one order lower than Hubble scale, is not modified even if we take the upper limit to be $(3/2)H$ instead of m_s.

contribution from the region $m \gtrsim H/10$ will be strongly suppressed due to the factor $(2RH)^{-\frac{4}{3}m^2 H^{-2}}$ in any case. We have defined

$$\mathcal{M}_D(\zeta) = \int_0^1 dt\, e^{-\zeta t^2} t^{D+1}, \quad \zeta_0 = \frac{4}{3}\frac{m_s^2}{H^2}\log(2RH). \quad (17)$$

To see the qualitative behavior of δ_T^2, it would be helpful to note $\mathcal{M}_D(\zeta_0) \sim \zeta_0^{-\frac{D+2}{2}}$ when $\zeta_0 \gg 1$. In this limit, we have $\delta_T^2 \sim (\frac{H}{m_{pl}})^4 (LH)^D (\log(2RH))^{-\frac{D+2}{2}}$ up to constant factors. δ_T^2 is enhanced when extra dimensions are large, $(LH)^D \gg 1$, since many fields contribute to it. δ_T^2 becomes small if $\log(2RH)$ were larger, since massive fields weakly decay in time during inflation.

Fixing parameters from the data.—. We will now use observational data [1],

$$\delta_{\mathrm{T}} \sim 2.6 \times 10^{-5}, \quad r_{t/s} \lesssim 0.22, \quad (18)$$

to constrain the parameters in our model. This implies $\frac{H}{m_{pl}} = \sqrt{\frac{9\pi}{2}\delta_{\mathrm{T}}^2 r_{t/s}} \lesssim 0.81 \times 10^{-4}$. Let us first assume this inequality is saturated. Then the amplitude (16) provides the relation between the two parameters m_s and L, or equivalently, between m_s and the string coupling g_s, since L is written as $(Lm_s)^D = 8\pi^6 g_s^2 (m_{pl}^2/m_s^2)$.

Figs. 1 and 2 show (Lm_{pl}) and g_s as functions of m_s/H, respectively. It is easier to have weak coupling with small D, while it is easier to keep L not too large with large D. Typical values that are consistent with (18) would be: $\{D = 2, m_s/H = 0.2, Lm_{pl} = 10^{12}, g_s = 3\}$, $\{D = 3, m_s/H = 0.2, Lm_{pl} = 10^{10}, g_s = 5\}$, $\{D = 4, m_s/H = 0.1, Lm_{pl} = 10^9, g_s = 7\}$. The number of the fields that participate in $\delta T/T$ is roughly $N \sim (Lm_s)^D$. For the above choice of parameters, $10^{14} \lesssim N \lesssim 10^{16}$. The fact that m_s is close to H (or slightly smaller) in our model can be understood as follows: If we include quantum expectation value of the scalar fields on the r.h.s. of Friedmann equation during inflation, we should have $H^2 \gtrsim NH^4/m_{pl}^2$. Our values of H and N do not satisfy this. This implies corrections (characterized by H/m_s) are so large that Friedmann equation is not applicable.

Since $\delta T/T$ originates from the fluctuations of massive fields, the spectrum is slightly stronger in the UV, thus the spectral index n_s is larger than 1 in our model,

$$
\begin{aligned}
n_s &= 1 - \frac{d}{d\log(Ha|\vec{x}-\vec{x'}|)}\log\left\langle\frac{\delta T}{T}(\tau,\vec{x})\frac{\delta T}{T}(\tau,\vec{x'})\right\rangle \\
&= 1 + \frac{4m_s^2}{3H^2}\frac{\mathcal{M}_D(\zeta_0)}{\mathcal{M}_{D-2}(\zeta_0)},
\end{aligned}
\quad (19)
$$

This is in the range $1 \lesssim n_s \lesssim 1.02$ when $D = 2$, and $1 \lesssim n_s \lesssim 1.05$ for $D \leq 6$.

Non-gaussianities.—. The three-point function of Φ can readily be calculated. It is given by the triangle diagram
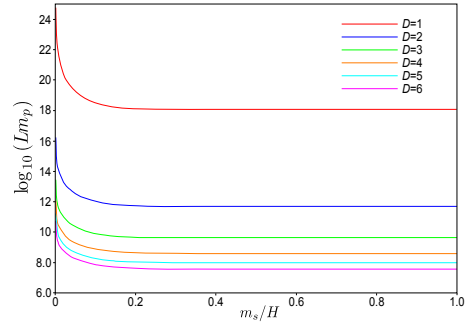

Figure 1: $\log_{10}(Lm_{pl})$ as a function of m_s/H, with $\delta_T = 2.6 \times 10^{-5}$, $r_{t/s} = 0.22$, $RH \sim 10^{67}$. If $r_{t/s}$ is smaller, (Lm_{pl}) moves up; the $r_{t/s}$ dependence is roughly $(Lm_{pl}) \sim r_{t/s}^{-(1/2+2/D)}$.
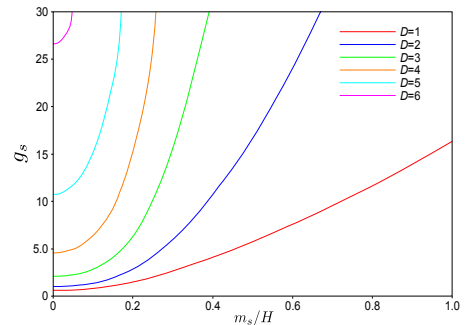

Figure 2: g_s as a function of m_s/H, with $\delta_T = 2.6 \times 10^{-5}$, $r_{t/s} = 0.22$, $RH \sim 10^{67}$. The dependence on $r_{t/s}$ and m_s/H at large m_s/H is $g_s^2 \sim (m_s/H)^{D+2}/r_{t/s}$.

with each pair of points connected by $\langle\phi\phi\rangle$,

$$
\begin{aligned}
&\langle\Phi(\tau,\vec{x})\Phi(\tau,\vec{y})\Phi(\tau,\vec{z})\rangle \quad (20)\\
&= \frac{1}{8\pi^3}\left(\frac{H}{m_{pl}}\right)^6 \sum \left(H^3 a^3 |\vec{x}-\vec{y}||\vec{y}-\vec{z}||\vec{x}-\vec{z}|\right)^{-\beta}.
\end{aligned}
$$

The non-linearity parameter f_{NL} is defined by $\Phi \to \Phi_g + f_{\mathrm{NL}}\Phi_g^2$ [9] with a gaussian field Φ_g. Our three-point function cannot be written this way with a local function f_{NL}, but we can estimate f_{NL} by considering $\langle\Phi(x)\Phi(y)\Phi(z)\rangle$ at generic angular separation,

$$f_{\mathrm{NL}} \sim \frac{1}{32}r_{t/s}\left(\frac{m_s}{H}\right)^2 \frac{\mathcal{M}_D(\frac{3}{2}\zeta_0)}{\mathcal{M}_{D-2}(\zeta_0)}. \quad (21)$$

This is proportional to $r_{t/s}$, and further suppressed. (Note $\left(\frac{m_s}{H}\right)^2 \mathcal{M}_D(\frac{3}{2}\zeta_0)/\mathcal{M}_{D-2}(\zeta_0) \sim (\log(2RH))^{-1} < 1$ when $\frac{m_s}{H} \gg 1$.) For $r_{t/s} = 0.22$, we have $f_{\mathrm{NL}} < 10^{-4}$.

Conclusions.—. We have studied the contribution to the CMB fluctuations from quantum effects of many fields with $m < H$. If Hubble is large, there could be many such fields. The effect studied in this Letter is important when there are large extra dimensions.

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The simple model studied here gives $n_s > 1$, but n_s can be made smaller if H decreases with time. Time dependence of Hubble is also needed at the end of inflation. We can study the dynamics of time dependent Hubble effectively by considering an inflaton field φ. By treating the inflaton fluctuation $\delta\varphi$ to be of the same order as Φ, Ψ, we obtain Φ in the super-horizon limit and in the slow-roll approximation,

$$
\begin{aligned}
\Phi \quad \sim \quad & -\left(\frac{V_{,\varphi}}{V}\right)^2 \left\{ H_* \left(\frac{V}{V_{,\varphi}}\right)_* + \frac{1}{4}\sum(\phi_*^2 - \phi^2) \right\} \\
& - \sum \pi G \beta \phi^2
\end{aligned}
\tag{22}
$$

where the star denotes the quantities evaluated at the horizon crossing, and V is the inflaton potential. The first term is the part induced by the inflaton fluctuation $\delta\varphi$, and the second term is the effect of the matter ϕ_A that we have been studying. The relative importance of the two terms depends on the details of the model, such as the slope of the inflaton potential and the time between horizon crossing and the end of inflation. The effect of the large number of fields ϕ_A will be important unless the slope is fine tuned to a small value.

More detailed analysis will be presented in a forthcoming paper [10].

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