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Option Package Bundling[*]

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Abstract

This paper analyzes the optimality of package bundling by focusing on the "main and accessory" relationship between two goods. In particular, we consider option package bundling in which an *optional* good is valuable *only if* it is consumed together with a certain (nonoptional) base good. We develop a model of option package bundling for a monopolist in which buyers' valuations are independently and uniformly distributed. We also allow *inter-relationship* between valuations by assuming that the reservation value of the bundle can be greater or less than the sum of the innate value of both goods. Our analysis observes that mixed bundling, in which the base good is sold with or without the optional good, yields a higher profit than pure bundling *if and only if* the range of the optional good valuation exceeds a threshold value. We then conduct a welfare analysis of the bundling choice. The result is surprising: pure bundling is *always* desirable from the social welfare viewpoint when a monopolist chooses mixed bundling.

Keywords: Multiproduct monopoly; Bundling; Optional goods; Interdependent valuations.

JEL classification: D42, L11.

1 Introduction

Bundled sales are not as unusual as one may think. While "explicit" bundling is sometimes denounced as being *per se* illegal in the U.S.,[1] many bundling strategies are often "implicit" and abound in daily life. Examples include cable TV subscription packages, text editors or web browsers in a computer operating system, and basic and optional plans for cell phone services. Outside the sales of traditional goods and services, basic and improvement patents are often sold as a bundle.

The characterizing feature of these implicit bundled sales is that the relationship between the bundled goods is *not symmetric*: casual observations suggest that many bundled goods offered in the real world display "main and accessory" relationship. More specifically, in these cases, an *optional* good is bundled, which is valuable *only if* it is consumed together with a certain (nonoptional) base good. Surprisingly, however, this characteristic of bundling has been overlooked both by economists and antitrust authorities. While the asymmetric relationship plays a significant role in the sale of bundled goods, we know relatively little about this type of *option package bundling*.

Option package bundling involves complex decision-making for the profit maximizing firm: sellers must first decide whether to sell the two goods only in a bundle (*pure* bundling) or separately (*mixed* bundling) and then decide the pricing scheme for the chosen method of selling. While a typical example of pure bundling is that of "free" software provided with a computer operating system, mixed bundling is prevalent in many other cases, such as the cable TV subscription. In the case of pure bundling not a few consumers may have thought, "Many electronic gadgets have optional features and software that are of little use. Why don't sellers offer the main good without optional features at a lower price?"

In this paper, we focus on the role of option package bundling as a method of sorting consumers with different willingness to pay, to address the following two related questions: (1) Why do we observe two different patterns (pure and mixed) in option package bundling? (What makes pure or mixed bundling an optimal strategy

for a firm?), and (2) Are the above complaints justifiable in light of improving consumer welfare? (On average, does the availability of the purchase of the base good alone (mixed bundling) benefit the consumers?)

One distinct feature of this study is that we also obtained analytical results for the monopolist's problem. Our main result is the derivation of the optimal bundling prices for the basic model and to verify the following claim: *mixed bundling gives the monopolist higher profit than pure bundling does if and only if the range of optional good valuation exceeds a threshold value.* This answers Question (1) mentioned above. Our result suggests an interesting testable implication: the *smaller* the diversity of valuation of an optional good, the *more* likely that the monopolist adopts pure bundling.

We then conduct a welfare analysis of the bundling choice, and the result is surprising: pure bundling is *always* desirable from the social welfare viewpoint when a monopolist chooses mixed bundling. Thus, our answer to Question (2), mentioned above, is *No*. The reason for this result is that under mixed bundling the monopolist can exercise its monopoly in a less restrictive way, resulting in inefficient, discriminatory pricing.

In this paper, we consider a model beyond a standard setting in which the reservation value of a bundle is equal to the sum of the innate value of each good (i.e., *independent* valuations). A consumer may value the optional good more as he or she values the base good more. This applies to the case of basic and improvement patents. In other cases, the optional good may have a diminishing value as a consumer's reservation value of the main good becomes higher. Note here that while our model entails "structural complementarity," in the sense that an optional good is never consumed solely, the reservation value of the bundle can be greater ("*complementary*") or less ("*substitute*") than the sum of the innate value of both goods. It can be predicted that the *higher* the degree of complementarity, the *more* likely that mixed bundling is as an optimal strategy. Intuitively, a consumer is, *ceteris paribus*, more willing to purchase both goods (precisely because they are complements) even with pure bundling. We verify this claim analytically as well as

numerically in a tractable yet formal model.

The situation we consider is described as follows. A monopolist produces base and optional goods, with constant marginal costs (zero for simplicity; see Section 6). Each consumer demands up to one unit of each good. In this two-goods setup, the sum of the two prices can be interpreted as the bundling price: generally, pure bundling corresponds to a *zero* optional good price, while mixed bundling corresponds to a *positive* optional good price. In general, we can view our model as a variant of vertical product differentiation because all consumers share the same ranking with regard to consumption patterns (no good, base good alone, and bundle, evaluated from the lowest to the highest). However, owing to the multidimensionality of the valuations, which considerably affects consumer screening, the demand for each good depends on *both* prices of base and optional goods. For example, an increase in the optional good price not only reduces its own demand but also that of the base good. This is because consumers who only slightly prefer a bundle to no good will stop buying any good following the price change.

To discuss the mechanism underlying option package bundling, we provide several graphical arguments and insights, following the custom in the literature since Adams and Yellen (1976). Our analysis observes that monopolistic screening is important to change the prices of the base and optional goods by keeping the sum (i.e., the bundle price) constant. For a marginal increase in the optional good price, this form of screening has the following three effects: (a) *newcomer gain* from consumers who start buying the base good alone because of the reduced price of the base good, (b) *reduced price loss* from consumers who continue to buy the base good alone, and (c) *switching consumers loss* from switching from buying a bundle to the base good alone. Note that those consumers who continue to buy the bundle pay the same bundle price and therefore have no effect on profit. Importantly, these effects are endogenous with both prices: we verify that while the newcomer gain increases in both prices, the reduced price loss and the switching loss decrease in the base good price but increase in the optional good price.

To understand the result that mixed bundling outperforms pure bundling if

and only if the range of optional good valuation exceeds a certain threshold, it is instructive to observe how strongly the monopolist is motivated to screen consumers from pure bundling. Recall that the larger the range of optional good valuation, the higher the average willingness to pay for a bundle; this necessitates a higher bundle price for profit maximization. In pure bundling, the bundle price equals the price of the base good. Therefore, as the pure bundle price increases, the newcomer gains from screening *increases* while the switching loss *decreases* (the reduced price loss is zero with pure bundling). Thus, the monopolist's motivation to screen consumers becomes stronger, and pure bundling eventually becomes suboptimal.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model with interdependent valuations. We then derive the optimal bundle prices for the basic model in Section 4. In Section 5, we provide a welfare analysis. Section 6 discusses our assumption that the (constant) marginal costs are zero for both goods. Lastly, Section 7 concludes the paper.

2 Related Literature

This paper is related to the literature on bundling under a monopoly, initiated by Stigler (1963) and Adams and Yellen (1976), with two-goods, discrete-type consumer cases.[2,3] Our basic model differs from these previous bundling models in that we focus on the *asymmetric* nature of the goods by considering one of the goods as an *optional* good.[4]

McAfee, McMillan and Whinston (1989) provided the primary results that if consumers' valuations are independently distributed, then mixed bundling always dominates component sales. McAfee, McMillan and Winston (1987) do not compare mixed bundling with pure bundling, because they rule out pure bundling, stating that it is dominated by mixed bundling. Their argument presumes that the (constant) marginal costs of two goods are positive (more precisely, higher than the lower bounds of valuation support).[5]

In contrast, by noting that pure bundling is often observed in reality, Pierce and

Winter (1996) construct a simple model where pure bundling is chosen as an optimal strategy. Although they do not explicitly mention it, Pierce and Winter (1996) assume what our paper calls base and optional goods.[6] This feature enables Pierce and Winter (1996) to directly compare mixed bundling with pure bundling. They focus on the role of consumer heterogeneity for determining the relative profitability of pure bundling. In particular, they illustrate that pure bundling is chosen *if and only if* consumer heterogeneity in the valuation of an optional good is lower than some threshold value and vice versa in case of mixed bundling.

To illustrate this result, Pierce and Winter (1996) rely on the specific example of a setup with two goods and two types of consumers. Indeed, they adopt the assumption that one good is optional, as in our analysis, primarily because assuming that both goods are regular is not necessary to screen consumers in mixed bundling, and also because the exposition becomes simpler. They conclude that pure bundling is optimal when one type of consumer's valuation for the optional good is similar to the other's, as long as a monopolist serves both types (no exclusion).

The present paper generalizes Pierce and Winter's (1996) results by analyzing a model with (a uniformly distributed) continuous valuation, as in McAfee, McMillan and Whinston (1989).[7] Although Pierce and Winter's (1996) model is similar to our approach in many respects, there are several important differences: first, our result holds without the qualification of exclusion; second, their discrete-type setup typically involves the correlation of valuations between types, whereas our continuous-type model adopts the independence of valuations; and finally, their analysis comprises a one-dimensional screening between two types, whereas our analysis comprises a multidimensional screening with deterministic mechanisms.

The present paper's model also considers *interdependent valuations*. Venkatesh and Kamakura (2003) set up a monopolistic model where a consumer's reservation value is not merely the sum of the component prices when considering complements (superadditive) or substitutes (subadditive). Venkatesh and Kamakura (2003) compare pure bundling with mixed bundling by considering complementarity and substitutability, an uncovered topic in papers such as McAfee, McMillan and Whinston

(1989). Venkatesh and Kamakura (2003) focus on two parameters: one is the measure of interdependency, and the other is the ratio of the constant marginal cost to the common upper bound of the valuation for each good. They argue that pure bundling should be employed if two goods are strong complements. An important difference between this work and our model is that the two goods are asymmetric in our model. Moreover, while Venkatesh and Kamakura (2003) rely on a numerical analysis of the relative profitability of mixed bundling compared to pure bundling, we provide distinct results for the optimal bundling strategy of a monopolistic seller.[8]

3 Model

3.1 Setup

We consider a multi-product monopolist's profit maximization problem. Following convention in the literature, we use the same setup as in McAfee, McMillan and Whinston (1989) to model the problem, with one important departure: there is a *base* good and an *optional* good in the following sense, while a base good has its own value irrespective of the consumption of other goods, an associated optional good can only have value if it is consumed together with the base good.

More specifically, suppose that a monopolist produces two types of goods, good 1 (base) and good 2 (optional). Each consumer consumes up to one unit of each good. Let (v_1, v_2) denote a vector of valuations of goods 1 and 2 for a consumer. Note that v_2 should be interpreted as the *conditional* utility that is realized *only if* good 1 is consumed together with good 2.

Consumers are heterogenous in the sense that (v_1, v_2) is distributed uniformly on $[0, 1] \times [0, \tau]$, where $0 < \tau \leq 1$, with independence between v_1 and v_2 (the density is thus $1/\tau$). Two remarks are in order. First, the reason for employing a uniform distribution is that it enables us to derive an explicit solution and to provide an intuitive explanation. Second, it appears natural that the highest valuation of the optional good is no greater than that of the base good.[9] The parameter τ is thus a measure of the *diversity* of consumers' valuations of the optional good.

A consumer with (v_1, v_2) then obtains utility of v_1 if good 1 alone is consumed, and $v_B \equiv v_1 + v_2 + \alpha v_1 v_2$ if both goods are consumed together,[10] where we call α the *degree of contingency* (following Venkatesh and Kamakura (2003)). We assume that α is common for all consumers. If $\alpha > 0$, then the higher v_1, the higher an incremental value of v_2 for v_B. In this case, the relationship between a base good and an optional is *complementary*. If $\alpha < 0$, an optional good has a negative impact on the value of the base good. This case applies, for example, to subscription of the print edition of a magazine and its online version. In the case, an optional good is an (imperfect) *substitute* to a base good. Lastly, if $\alpha = 0$, the base and the optional goods are *independent*.[11] To ensure that $1 + \alpha v_1 > 0$ and $1 + \alpha v_2 > 0$ for all $(v_1, v_2) \in [0, 1] \times [0, \tau]$, we assume that $\alpha > -1$ and $1 + \alpha \tau > 0$.

Now, let p_B be the price that the monopolist charges for a unit of the "composite good" and p_1 be the price of good 1. For expositional convenience, we define "price" for good 2 as $p_2 \equiv p_B - p_1$. The costs of producing each good are constant and normalized to zero. It is thus clear that in this setting, welfare maximization requires everyone in $[0, 1] \times [0, \tau]$ consumes both goods 1 and 2. These assumptions concerning distribution support and the zero marginal costs are relaxed in Section 5.

A consumer prefers a bundle to the base good only, if and only if

$$
\begin{aligned}
v_B - p_B &\geq v_1 - p_1 \\
&\Leftrightarrow v_2 \geq \frac{p_B - p_1}{1 + \alpha v_1} \\
&\Leftrightarrow v_2 \geq \frac{p_2}{1 + \alpha v_1} (\equiv v_2^{B1}(v_1; \alpha)).
\end{aligned}
$$

Similarly, a consumer prefers a bundle to nothing if and only if

$$
\begin{aligned}
v_B - p_B &\geq 0 \\
&\Leftrightarrow v_2 \geq \frac{-v_1 + p_1 + p_2}{1 + \alpha v_1} (\equiv v_2^{B0}(v_1; \alpha)).
\end{aligned}
$$

Given good 2 has a non-negative value for any consumer, $p_2 = 0$ implies that a consumer buys both goods or nothing. Also, if $p_1 = 1$ and $p_2 > 0$, a generic consumer buys either both or nothing (because $\alpha > -1$ and $v_1 \leq 1$). We identify these pricing strategies as *pure bundling* (see Figure 1). On the other hand, if $p_2 > 0$

and $p_1 < 1$, some consumers buy only good 1 and others bundle goods 1 and 2. We refer to this pricing strategy as *mixed bundling*. In this sense, *pure bundling is nested in mixed bundling*. A consumer whose (v_1, v_2) does not satisfy either of the inequalities buys nothing.

[Insert Figures 1, 2 and 3.]

Given the prices and nature of the goods, the demand for the goods is depicted in Figures 2 (complementarity: $\alpha > 0$) and 3 (substitutability: $\alpha < 0$).[12] The shapes of the two curves are determined by the following derivatives:

$$
\begin{cases}
\dfrac{dv_2^{B1}}{dv_1} = -\dfrac{\alpha p_2}{(1 + \alpha v_1)^2} \lesseqgtr 0 \text{ if and only if } \alpha \gtreqless 0 \\[4mm]
\dfrac{d^2 v_2^{B1}}{dv_1^2} = \dfrac{2\alpha^2 p_2}{(1 + \alpha v_1)^3} > 0
\end{cases}
$$

and

$$
\begin{cases}
\dfrac{dv_2^{B0}}{dv_1} = -\dfrac{1 + \alpha(p_1 + p_2)}{(1 + \alpha v_1)^2} < 0 \\[4mm]
\dfrac{d^2 v_2^{B0}}{dv_1^2} = \dfrac{2\alpha[1 + \alpha(p_1 + p_2)]}{(1 + \alpha v_1)^3} \gtreqless 0 \text{ if and only if } \alpha \gtreqless 0.
\end{cases}
$$

In particular, the curve of $v_2^{B1}(v_1)$ is convex, with the slope being negative in the case of $\alpha > 0$, and positive in the case of $\alpha < 0$. For the shape of v_2^{B0}, note that $1 + \alpha(p_1 + p_2) > 0$ for relevant (p_1, p_2).[13] Hence, we have:

$$
\begin{cases}
-(1 + \alpha(p_1 + p_2)) < -1 \text{ if } \alpha > 0 \\[2mm]
0 > -(1 + \alpha(p_1 + p_2)) > -1 \text{ if } \alpha < 0.
\end{cases}
$$

The curve of $v_2^{B0}(v_1)$ is convex if $\alpha > 0$, and concave if $\alpha < 0$. Finally, it is verified that the two curves intersect at $(p_1, p_2/(1 + \alpha p_1))$.

Intuitively, the shape for $v_1 < p_1$ is derived from the fact that as v_1 and v_2 move closer, a consumer obtains more utility from a bundle in the case of $\alpha > 0$, and vice versa in the case of $\alpha < 0$. The shape for $v_1 \geq p_1$ is explained as follows: for a fixed value of v_2, as v_1 increases, a bundle becomes more (resp. less) attractive if $\alpha > 0$ (resp. $\alpha < 0$).

3.2 Modeling Interdependent Valuations

Notice the difference between Venkatesh and Kamakura (2003) and our formulation, apart from their setting of two normal goods. As Eckalbar (2010) suggests, in general, utility with non-independence is expressed by

$$v_B \equiv v_1 + v_2 + H$$

where $H > 0$ indicates complementarity, and $H < 0$ indicates substitutability. Venkatesh and Kamakura (2003) set $H = \theta(v_1 + v_2)$, so that the consumption value of consuming both goods is modeled as

$$v_B \equiv (1 + \theta)(v_1 + v_2),$$

where θ is a parameter of the degree of contingency/ As per Venkatesh and Kamakura (2003), the goods are complements if $\theta > 0$ and substitutes if $\theta < 0$.

In their formulation, complementarity/substitutability is incorporated *additively*, which provides linear boundaries between consumers with different purchase patterns. They primarily rely on numerical analyses and presume a formulation that significantly eases the computational burden. In contrast, our formulation allows *multiplicability*: $H = \alpha v_1 v_2$. The two approaches differ in the manner that they treat complementarity/substitutability. In particular, the cross-partial derivative of the utility of one good is not independent of the consumption of the other good, given as

$$\frac{\partial v_B}{\partial v_i v_j} = \alpha \ (i, j = 1, 2, \ i \neq j),$$

while it is zero in Venkatesh and Kamakura's (2003) formulation. Note that our model encompasses super- and sub-modularity in valuation, while Venkatesh and Kamakura's (2003) does not.

4 Pure vs. Mixed Bundling

We now analyze an optimal pricing strategy for the monopolist. Let $p_1 \in [0, 1]$ and $p_2 \in [0, \tau]$ be the prices of goods 1 and 2 in the regime of mixed bundling,

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respectively. As discussed, $p_1 = 1$ or $p_2 = 0$ is interpreted as pure bundling. Let b denote the bundling price in the regime of pure bundling (that is, b is defined as p_B with $p_2 = 0$). And, let b^*, p_1^* and p_2^* denote the optimal prices.

To obtain the heuristics for an optimal solution with complementarity and substitutability between valuations for the two goods, consider the following screening in the case of independence ($\alpha = 0$). For a given price (p_1, p_2), let the optional good price increase by a small amount, $\varepsilon > 0$, and the base good price decrease by the same amount ε, keeping the price of the composite good constant (see Figure 4 for the case of $p_2 > 0$). Notice that the inframarginal consumers purchasing both goods have no effects on the profits.

[Insert Figure 4.]

Three first-order effects are then identified. First, there are new consumers who start buying the base good alone ("*Newcomer Gain*"). This effect is evaluated by $\varepsilon p_1 p_2/\tau$ (the selling price, p_1, multiplied by the density, $\varepsilon p_2/\tau$, in the first-order change). Second, those who continue to buy a base good alone pay less ("*Reduced Price Loss*"). This is evaluated by $-\varepsilon(1-p_1)p_2/\tau$ (the loss in the price per customer, $-\varepsilon$, multiplied by the density, $(1 - p_1)p_2/\tau$). Lastly, there are consumers who switch from buying a bundle to a base good alone ("*Switching Loss*"). This is also evaluated by $-\varepsilon p_2(1-p_1)/\tau$ (the loss in the revenue per customer, $-(p_B - p_1) = -p_2$, multiplied by the density, $(1 - p_1)\varepsilon/\tau$, ignoring the second-order change). Hence, the gain exceeds the losses if and only if:

$$p_1 p_2 \geq 2(1 - p_1)p_2 \iff p_1 \geq 2/3 \text{ and } p_2 > 0, \text{ or } p_2 = 0.$$

This shows that as long as $p_2 > 0$, this screening (starting from $p_1 = 1$) should be continued until $p_1 = 2/3$ in order to increase the profits for the given bundle price. We then need to check what to do for $p_2 = 0$. In this case, the effects include only the newcomer gain and the switching loss, and these are merely the second-order change (see Figure 5).

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[Insert Figure 5.]

The newcomer gain from this change is evaluated by $\varepsilon^2 p_1 / (2\tau)$ (ignoring any higher order changes), whereas the switching loss is by $(1 - p_1)\varepsilon^2/\tau$. Hence the gain is greater if and only if $p_1/2 \geq 1 - p_1 \Leftrightarrow p_1 \geq 2/3$. Therefore, the optional good price $p_2 = 0$ should be increased if the base good price is greater than $2/3$.

Combining the previous arguments, we can find the optimal prices by (i) solving the optimal pure bundling price, and if it is less than $2/3$, which defines the optimal prices, otherwise (ii) setting $p_1 = 2/3$ and finding $p_2 > 0$ that maximizes profit.[14]

Adachi, Ebina and Hanazono (2010) show that pure bundling is suboptimal for $\tau > 2/3$ because the optimal pure bundling price is greater than $2/3$. An intuitive explanation is as follows. It is straightforward to discern that the newcomer gain is increasing in both prices, and the reduced rate and switching losses are decreasing in p_1 but increasing in p_2. For a small p_2, the gain from screening is thus more likely to exceed the losses if p_1 is higher. When τ is large, the bundle price $b = p_1 + p_2$ is likely to be high because the average willingness to pay for the bundle is large. In this case, the monopolist finds it profitable to invite consumers who are willing to buy a base good at a high price for the cost of the switching consumers and the reduced rate, both of which only slightly affect profits for a small p_2.[15]

4.1   Analytical Results

We now characterize the optimal bundling strategy of the monopolist in the presence of complementarity or substitutability between the two goods ($\alpha \neq 0$). In this subsection, we observe that even in the presence of interdependent valuations, the above argument of screening applies. In addition, the optimal scheme is such that the monopolist adopts mixed bundling if and only if τ exceeds a threshold value. We first obtain the following proposition about the optimal mixed bundle prices.

Proposition 1. *If mixed bundling is optimal, then the optimal price of the base good p_1^* is independent of $\tau \in (0, 1]$ and the optimal price of the optional good p_2^*.*

Proof. See Appendix A1. □

From Proposition 1, we obtain $p_1^* = p_1^*(\alpha)$. Now, let

$$F(p_1, \alpha) \equiv \frac{p_1}{1 + \alpha p_1} - 2\frac{(\ln(1 + \alpha) - \ln(1 + \alpha p_1))}{\alpha}.$$

As Appendix A1 shows, p_1^* is defined by $F(p_1^*, \alpha) = 0$. The next proposition provides another characterization of p_1^* with respect to the degree of contingency, α. Namely, complementarity makes the optimal price of the base good *lower* under mixed bundling.

Proposition 2. *The optimal price of the base good under mixed bundling, $p_1^*(\alpha)$, is decreasing in α.*

Proof. See Appendix A2. □

It is also shown that $p_1^*(\alpha) > 1/2$ because

$$
\begin{aligned}
F(p_1^*(\alpha), \alpha) &= 0 \\
&\Leftrightarrow \frac{p_1^*(\alpha)}{1 + \alpha p_1^*(\alpha)} = \frac{2}{\alpha}\left(\ln\left(\frac{1 + \alpha}{1 + \alpha p_1^*(\alpha)}\right)\right) \\
&\Leftrightarrow \frac{p_1^*(\alpha)}{2} = \left(\frac{1}{\alpha} + p_1^*(\alpha)\right)\left(\ln\left(\frac{1/\alpha + 1}{1/\alpha + p_1^*(\alpha)}\right)\right),
\end{aligned}
$$

and by taking $\alpha \to \infty$, we have

$$\frac{p_1^*(\infty)}{2} = p_1^*(\infty)\ln\left(\frac{1}{p_1^*(\infty)}\right),$$

such that $p_1^*(\infty) = \exp(-0.5) \cong 0.607 > 1/2$. We now characterize the optimal bundling strategy. Let b^* denote the optimal price under pure bundling (it is not necessarily optimal if the scheme of mixed bundling is optimal).

Proposition 3. *If the optimal pure bundle price b^* is greater than $p_1^*(\alpha)$, then mixed bundling is optimal.*

Proof. See Appendix A3. □

Propositions 1 and 3 imply that the optimal price pair for mixed bundling and pure bundling (i.e., bundling with $p_2 = 0$) lies on the bold lines in Figure 6. To better understand this, first note that $b^* > p_1^*$ cannot constitute an optimal strategy (Proposition 3). Second, the price pair with $p_2^* = \tau$ cannot be optimal because the monopolist only sells the base good. The optimal strategy, in this case, is to set the price of the base good to one half (the profit is $\tau/4$). Last, the price pair (p_1, p_2) with $p_1 \neq p_1^*(\alpha)$ and $p_2 \in (0, \tau)$ cannot be an optimal strategy under mixed bundling (Proposition 1).

[Insert Figure 6.]

Thus, we can avoid considering the optima globally, because we know that the optimal pricing strategy is either "$p_2 = 0$ and $p_1 < p_1^*(\alpha)$" (in this case b is used for p_1), or "$p_2 > 0$ and $p_1 = p_1^*(\alpha)$." Hence, we thus focus on the prices on the bold lines in Figure 6: we treat the optimization problem as a one dimensional problem for each region to check the global second-order condition, along with the first-order condition.

Proposition 4. *If the optimal pure bundle price b^* is no greater than $p_1^*(\alpha)$, then pure bundling is optimal.*

Proof. See Appendix A4. □

Now, let $b^* = b^*(\alpha, \tau)$. We also illustrate the following proposition.

Proposition 5. *The optimal pure bundle price $b^*(\tau, \alpha)$ is increasing in τ.*

Proof. See Appendix A5. □

To complete our argument, we need to show that $b^*(\tau, \alpha)$ exceeds $p_1^*(\alpha)$ for $\tau > \widetilde{\tau} \in (0, 1)$. It is clear that for any $\alpha > -1$, $b^* \downarrow 1/2 < p_1^*(\alpha)$ as $\tau \downarrow 0$. Figure 7 shows that as τ increases, the marginal gain from increase in the pure bundle price $b = p_1^*(\alpha)$ (area B) increases, while the marginal cost (area C multiplied by C)

remains the same. Thus, for a large τ, the marginal gain exceeds the marginal cost, and hence, b^* is larger than $p_1^*(\alpha)$. Note here that b^* may not be able to exceed $p_1^*(\alpha)$ if $\tau \leq 1$. Although we have assumed $\tau \leq 1$, our argument does not crucially depend on this assumption.[16] If we do not assume $\tau \leq 1$, we expect the threshold to exist for any $\alpha > -1$. Summarizing the argument so far, we establish the following claim.

[Insert Figure 7.]

Claim 1. *From the seller's viewpoint, mixed bundling outperforms pure bundling if and only if the range of optional good valuation exceeds a certain threshold. If the degree of contingency α is sufficiently negative, then pure bundling may always be optimal, depending on the range of other parameters.*

4.2   Numerical Examples

We now provide some numerical examples. Figures 8 and 9 demonstrate how the optimal prices change as τ increases (from 0.3 to 1, with increments of 0.05) depending on the nature of both goods. For this purpose, we set $\alpha = -0.1$ (Figure 8) or -0.3 (Figure 9) for substitutes, $\alpha = 0$ for independency, and $\alpha = 0.1$ (Figure 8) or 0.3 (Figure 9) for complements). In all cases, *mixed bundling is more desirable for the monopolist as τ increases.* It is also observed that the greater the value of α, the *lower* the threshold of τ.[17] That is, if the goods are complements, then the seller is willing to provide mixed bundling for a *smaller* value of τ and vice versa (as seen in Proposition 2). Intuitively, when the goods are complements, the monopolist loses less from mixed bundling (some consumers purchase the base good only) because many consumers are willing to purchase both goods, precisely because of complementarity.

[Insert Figures 8 and 9.]

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Considering real world examples, it can be believed that Claim 1 captures the causes of option package bundling well. "Free" software is considered an example, where τ would be presumably low (at least relative to the valuation from that of the software itself). On the other hand, special cable TV channels would have a high value for interested viewers. Claim 1 suggests that this explains why cable TV subscription packages are often offered as options.

5   Welfare Analysis

In this section, we investigate whether allowing mixed bundling enhances social welfare, maintaining the assumption that the monopolist charges profit-maximizing prices. From the analysis in the previous section, we know when the monopolist chooses mixed bundling and pure bundling. Remember that pure bundling is a special case of mixed bundling. Thus, when the monopolist chooses pure bundling, prohibiting mixed bundling by antitrust authorities has no effect on the monopolist's decision-making process.

As mixed bundling is a means of price discrimination, it should primarily cause efficiency distortions, as in standard monopoly pricing. Thus, when the monopolist chooses mixed bundling, does the prohibition of mixed bundling improve welfare? For $\alpha = 0$, we can derive an analytical result, as will be subsequently illustrated. However, it is not possible to use the same argument for $\alpha \neq 0$, and thus we conduct a numerical analysis.

5.1   The Case of Independency ($\alpha = 0$)

By prohibiting mixed bundling, there are welfare gains and losses (see Figure 10). The gains come from (i) consumers who switch from buying no good to a bundle, as the bundle price is reduced, and (ii) those who switch from good 1 alone to a bundle. The losses are from the consumers who switch from buying good 1 alone to no good. The gains from (ii) are closely related to the efficiency distortion in the standard monopoly (or vertical differentiation) model, as these consumers are

screened through the price of an optional good p_2. However, there are some changes that affect welfare that are absent in the standard monopoly model, namely the changes in p_1 (reduced) and in b (increased). As pure bundling is optimal for $\tau \leq 2/3$, this restriction on mixed bundling has bite only when $\tau > 2/3$. We now obtain the following result.

[Insert Figure 10.]

Proposition 6. *When the monopolist chooses mixed bundling, pure bundling is desirable from the social welfare viewpoint.*

Proof. Using the optimal prices derived in Propositions 1 and 2 in Adachi, Ebina and Hanazono (2010), we know that for $\tau > 2/3$,

$$2/3 = p_1^*(\tau) < b^*(\tau) = \sqrt{2\tau/3} < p_1^*(\tau) + p_2^*(\tau) = \tau/2 + 1/3,$$

i.e., the price for good 1 in mixed bundling, $p_1^*(\tau)$, is smaller than the optimal pure bundle price $b^*(\tau)$, but the bundle price in mixed bundling, $p_1^*(\tau) + p_2^*(\tau)$, is higher than $b^*(\tau)$.

Now consider the welfare gains from the consumers who switch from buying no good to a bundle (part (i) above), whose level is denoted by A, and the welfare losses, denoted by B.

We maintain that A is greater than B for any τ, so that the total welfare gains (A plus the standard gains) must be larger through the prohibition of mixed bundling. Although calculating the exact welfare gains for A involves the integration of the consumers' willingness to pay, we only require a lower bound to support the claim.

The consumers associated with this gain are in the parallelogram, surrounded by the lines drawn by $v_1 = 0$, $v_1 = 2/3$, $v_1 + v_2 = b^*(\tau)$ and $v_1 + v_2 = p_1^*(\tau) + p_2^*(\tau)$. The area of this parallelogram is

$$\frac{2}{3}\left(\left\{\frac{2}{3} + \frac{\tau}{2} - \frac{1}{3}\right\} - \sqrt{\frac{2\tau}{3}}\right) = \frac{2}{3}\left(\frac{1}{3} + \frac{\tau}{2} - \sqrt{\frac{2\tau}{3}}\right)$$

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$$= \frac{2}{9} + \frac{\tau}{3} - \frac{2}{3}\sqrt{\frac{2\tau}{3}}.$$

Recall that each consumer in this parallelogram lies in the northeast of $b^*(\tau)$ line and thus its willingness to pay for a bundle is at least as much as $b^*(\tau)$. The welfare gains A are therefore greater than $b^*(\tau)$ times the area. Now we compute an upper bound of B, the welfare losses. The consumers associated with this loss are in the triangle surrounded by the horizontal axis, the lines drawn by $v_1 = 2/3$, and $v_1 + v_2 = b^*(\tau)$. The area of this triangle is

$$\frac{1}{2}\left(\sqrt{\frac{2\tau}{3}} - \frac{2}{3}\right)^2 = \frac{1}{2}\left(\frac{2\tau}{3} + \frac{4}{9} - \frac{4}{3}\sqrt{\frac{2\tau}{3}}\right)$$

$$= \frac{2}{9} + \frac{\tau}{3} - \frac{2}{3}\sqrt{\frac{2\tau}{3}},$$

which is exactly the *same* as the parallelogram above. Each consumer in this region will switch to buying nothing when mixed bundling is prohibited. Here, each consumer's willingness to pay for good 1 alone is at most $b^*(\tau)$ such that the welfare losses B are smaller than $b^*(\tau)$ times this area. We now compute the upper bound of the losses and the lower bound of the gains,

$$A \geq \left(\frac{2}{9} + \frac{\tau}{3} - \frac{2}{3}\sqrt{\frac{2\tau}{3}}\right)\sqrt{\frac{2\tau}{3}} \geq B.$$

Hence, the welfare gains from prohibiting mixed bundling, which must be strictly larger than A, are greater than the total losses B. $\square$

As τ increases, does the welfare loss become larger or smaller? The above proposition does not answer this question. Based on the explicit forms for social welfare (available upon request), we verify that the welfare loss becomes larger as τ increases, as depicted in Figure 11.

[Insert Figure 11.]

The result that pure bundling is socially desirable for $\tau > 2/3$ is worth mentioning because it would at first appear counter-intuitive. To understand this, remember

that our model is a monopolistic model, and hence mixed bundling is used only for inefficient, discriminatory pricing. Consider instead the case where firms compete in the market for the optional good, while a base good is monopolistically provided. If the monopolistic base good producer is forced to adopt mixed bundling, then some consumers are able to choose from which firm they buy an optional good. On the contrary, if the monopolistic base good producer commits to pure bundling, then the firm can exclude other optional good producers from the market, and hence acquire stronger market power. Overall, competition affects the consequences of pure bundling, and thus the above result on social welfare should be taken with caution.

5.2 Numerical Analysis when $\alpha \neq 0$

In the case of $\alpha \neq 0$, we cannot apply the previous argument. Thus, we conduct a numerical analysis. In Appendix A6, four tables are shown in the region of τ, where the monopolist optimally chooses mixed bundling. Figure 12 is a representative case ($\alpha = 0.3$). It is observed that social welfare under pure bundling is higher than that under mixed bundling in these four cases. We conjecture that Proposition 6 holds in the presence of contingent valuations.

[Insert Figure 12.]

6 Discussion

In this paper, we assume zero marginal costs for both goods. In the case of independent valuations ($\alpha = 0$), Adachi, Ebina and Hanazono (2010) examine the robustness of the results to parametric changes in marginal costs and distribution supports. In particular, they consider the cases with positive marginal costs or with the lowest valuation of a good that exceeds the marginal cost (an upward shift in the valuation support). They then illustrate that the main result (that mixed bundling outperforms pure bundling if and only if the range of optional good valuation exceeds a certain threshold) holds, *except when the (constant) marginal cost of an*

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optional good is higher than its lowest valuation. In this case, mixed bundling is *always* optimal if the optional goods are priced at least as high as the marginal cost. That is, it is always beneficial to avoid serving inefficient consumers (i.e., those who have less valuation than the marginal cost).

To better understand this, consider a pure bundling with $p_1 > 0$ and $p_2 = 0$. Let $c_2 > 0$ be the (constant) marginal cost of an optional good. Then, it becomes clear that this price is dominated by $p'_1 = p_1 - c_2$, $p'_2 = c_2$: with this change, the monopolist avoids serving inefficient consumers with optional goods without incurring any loss, and attracts new consumers of the base good with $v_1 \in [p'_1, p_1]$, thereby earning positive profits. Note that this argument does not depend on the assumption of uniform distribution; therefore, the conclusion is robust for any continuous distribution. This case may be applied to a real-world example of luxurious leather seats for cars, which are costly to produce and which some consumers may not appreciate enough to justify their cost. In fact, we would rarely observe pure bundles in the case of such luxurious optional goods. We conjecture that this argument holds in the presence of complementarity or substitutability.

7   Concluding Remarks

This paper develops a model of a monopolist's two-goods option package bundling problem with the assumption of a uniform distribution of consumers' valuations. We then study the monopolist's problem of choosing pure or mixed bundling in the context of base and optional goods by considering complementarity and substitutability. This paper is a generalization of Pierce and Winter (1996) in the sense that we consider continuous valuation. We find that pure bundling can generate more profits than mixed bundling *if and only if* the diversity of a consumer's valuation for the optional good is small. As for welfare implications, our analysis shows that pure bundling is *always* desirable from the social welfare viewpoint, with a caution that competition in the market of optional goods may alter the result.

Because of its specific assumptions, our model is certainly special. In particular,

it raises the question that if there are many optional goods, as in reality, can we distinguish between bundled features and options for buying by merely looking at the main goods and each feature individually? Although we may be able to generate interesting testable implications by considering such a many-optional-goods case, it would certainly involve difficult problems because of the presence of more than two goods in bundling problems. A more tractable way to ask the question is to determine, first, whether it is more profitable to bundle all optional goods as a package or to sell each of them separately, and second, whether options bundled with a base good are more profitable. For this direction of research, the framework of Armstrong (1999), Bakos and Brynjolfsson (1999), Fang and Norman (2006), Chu, Leslie and Sorensen (2011), and Crawford and Yurukoglu (2011), who analyze specific multi-good cases, would be helpful.

Appendices

A1. Proof of Proposition 1

For any optimal prices (p_1^*, p_2^*) under mixed bundling, consider a small reduction in p_1 by ε, keeping p_B^* held constant. As in Figure 4, the newcomer gain is given by

$$(p_1^* \times \varepsilon \frac{p_2^*}{1 + \alpha p_1^*})/\tau,$$

while the reduced price loss and the switching loss are given by

$$\left(\varepsilon \times \int_{p_1^*}^1 \frac{p_2^*}{1 + \alpha v_1} dv_1 \right) / \tau$$

and

$$\left(p_2^* \times \int_{p_1^*}^1 \varepsilon \frac{1}{1 + \alpha v_1} dv_1 \right) / \tau$$

respectively. Because of the optimality, the marginal gain must coincide with the marginal loss:

$$\frac{p_1^* p_2^*}{1 + \alpha p_1^*} = 2 \frac{p_2^*(\ln(1 + \alpha) - \ln(1 + \alpha p_1^*))}{\alpha}.$$

It is clear that the above equality is *independent* of τ and p_2 as in the case of $\alpha = 0$.[18]

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A2. Proof of Proposition 2

Because

$$\frac{\partial F(p_1, \alpha)}{\partial p_1} = \frac{1}{(1 + \alpha p_1)^2} + \frac{2}{1 + \alpha p_1} > 0,$$

and $F(0, \alpha) < 0 < F(1, \alpha)$, $p_1^*(\alpha)$ is uniquely determined as a function that satisfies $F(p_1^*(\alpha), \alpha)$. We use the implicit function theorem to verify

$$p_1'(\alpha) = -\frac{F_\alpha}{F_{p_1}} < 0$$

where $F_{p_1} > 0$ is positive.

We can proceed:

$$
\begin{aligned}
F_\alpha &= \frac{-p_1^2}{(1 + \alpha p_1)^2} + \frac{2}{\alpha^2}\left[\ln(1 + \alpha) - \ln(1 + \alpha p_1)\right] - \frac{2}{\alpha}\left(\frac{1}{1 + \alpha} - \frac{p_1}{1 + \alpha p_1}\right) \\
&= \frac{-p_1^2}{(1 + \alpha p_1)^2} + \frac{1}{\alpha}\frac{p_1}{1 + \alpha p_1} - \frac{2}{\alpha}\left(\frac{1}{1 + \alpha} - \frac{p_1}{1 + \alpha p_1}\right) \\
&= \frac{-p_1^2}{(1 + \alpha p_1)^2} + \frac{3}{\alpha}\frac{p_1}{1 + \alpha p_1} - \frac{2}{\alpha(1 + \alpha)} \\
&= \frac{1}{(1 + \alpha p_1)^2 \alpha(1 + \alpha)}\left[-p_1^2\alpha(1 + \alpha) + 3p_1(1 + \alpha p_1)(1 + \alpha) - 2(1 + \alpha p_1)^2\right] \\
&= \frac{1}{(1 + \alpha p_1)^2 \alpha(1 + \alpha)}\left[p_1^2(-\alpha(1 + \alpha) + 3\alpha(1 + \alpha) - 2\alpha^2) + p_1(3(1 + \alpha) - 4\alpha) - 2\right] \\
&= \frac{1}{(1 + \alpha p_1)^2 \alpha(1 + \alpha)}\left[2\alpha p_1^2 + (3 - \alpha)p_1 - 2\right].
\end{aligned}
$$

By the mean-value theorem, it is verified that

$$\ln(1 + \alpha) - \ln(1 + \alpha p_1) = \frac{1}{1 + \alpha p_1 + \varepsilon_\alpha}(\alpha(1 - p_1))$$

for some $\varepsilon_\alpha \in (\min\{0, \alpha(1 - p_1)\}, \max\{0, \alpha(1 - p_1)\})$. Thus, we have

$$
\begin{aligned}
F(p_1(\alpha), \alpha) &= \frac{p_1(\alpha)}{1 + \alpha p_1(\alpha)} - 2\frac{(\ln(1 + \alpha) - \ln(1 + \alpha p_1(\alpha)))}{\alpha} \\
&= \frac{p_1(\alpha)}{1 + \alpha p_1(\alpha)} - \frac{2}{\alpha}\frac{\alpha(1 - p_1(\alpha))}{1 + \alpha p_1(\alpha) + \varepsilon_\alpha} = 0,
\end{aligned}
$$

which is equivalent to

$$(2\alpha p_1^2 + (3 - \alpha)p_1 - 2) + (\varepsilon_\alpha - \alpha(1 - p_1))p_1 = 0.$$

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Because $(\varepsilon_\alpha - \alpha(1 - p_1))p_1 < 0$ (resp. > 0) if $\alpha > 0$ (resp. < 0), we must have

$$\frac{2\alpha p_1^2 + (3 - \alpha)p_1 - 2}{\alpha} > 0,$$

and thus $F_\alpha > 0$, which leads to $p_1'(\alpha) < 0$.

A3. Proof of Proposition 3

Consider how screening works for $p_2 = 0$. Note that in this case there are no first-order changes and thus we need to focus on second-order changes.

Next, consider a small reduction in p_1 by ε (with the associated change in p_2 from zero to $\varepsilon/(1 + \alpha p_1)$). Then, the newcomer gain is given by:

$$
\begin{aligned}
p_1 \int_{p_1-\varepsilon}^{p_1} \frac{-v_1 + p_1}{1 + \alpha v_1} dv_1 &= p_1 \left[\frac{1}{\alpha^2} \left\{ (1 + p_1\alpha)\ln(1 + \alpha v_1) - \alpha v_1 \right\} \right]_{p_1-\varepsilon}^{p_1} \\
&= \frac{p_1}{\alpha^2} \left[(1 + p_1\alpha)\{\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))\} - \alpha\varepsilon \right] \\
&= \frac{p_1}{\alpha^2} \left[(1 + p_1\alpha)\frac{1}{1 + \alpha p_1 - \delta_\varepsilon}\alpha\varepsilon - \alpha\varepsilon \right] \\
&= \frac{p_1}{\alpha^2} \left[(1 + p_1\alpha)\frac{1}{1 + \alpha p_1 - \delta_\varepsilon}\alpha\varepsilon - \frac{1 + \alpha p_1 - \delta_\varepsilon}{1 + \alpha p_1 - \delta_\varepsilon}\alpha\varepsilon \right] \\
&= \frac{p_1}{\alpha^2} \frac{\delta_\varepsilon}{1 + \alpha p_1 - \delta_\varepsilon}\alpha\varepsilon \\
&= \frac{p_1}{\alpha} \frac{\delta_\varepsilon \varepsilon}{1 + \alpha p_1 - \delta_\varepsilon},
\end{aligned}
$$

for some $\delta_\varepsilon \in (\min\{0, \alpha\varepsilon\}, \max\{0, \alpha\varepsilon\})$, while the switching loss is:

$$
\begin{aligned}
&\varepsilon \left(\int_{p_1-\varepsilon}^{p_1} \left\{ \frac{\varepsilon}{1 + \alpha(p_1 - \varepsilon)} - \frac{-v_1 + p_1}{1 + \alpha v_1} \right\} dv_1 + \int_{p_1}^{1} \frac{\varepsilon}{1 + \alpha(p_1 - \varepsilon)} dv_1 \right) \\
&\approx \varepsilon \int_{p_1}^{1} \frac{\varepsilon}{1 + \alpha v_1} dv_1 = \frac{\varepsilon^2}{\alpha} \left\{ \ln(1 + \alpha) - \ln(1 + \alpha p_1) \right\}.
\end{aligned}
$$

Note that the first term has third-order changes of ε and is thus ignored. The gain is weakly greater than the loss if and only if[19]

$$
\begin{aligned}
\frac{p_1}{\alpha} \frac{\delta_\varepsilon \varepsilon}{1 + \alpha p_1 - \delta_\varepsilon} &\geq \frac{\varepsilon^2}{\alpha} \left\{ \ln(1 + \alpha) - \ln(1 + \alpha p_1) \right\} \\
\Leftrightarrow \frac{p_1}{1 + \alpha p_1 - \delta_\varepsilon} &\geq \frac{\varepsilon}{\delta_\varepsilon} \left\{ \ln(1 + \alpha) - \ln(1 + \alpha p_1) \right\}.
\end{aligned}
$$

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As $\varepsilon \to 0$, the above condition coincides with the first-order argument, i.e.,

$$\frac{p_1}{1 + \alpha p_1} \geq \frac{2}{\alpha} \left\{ \ln(1 + \alpha) - \ln(1 + \alpha p_1) \right\}.$$

To see this, we need to verify that

$$\lim_{\varepsilon \to 0} \frac{\delta_\varepsilon}{\varepsilon} = \frac{\alpha}{2}.$$

Recall that

$$\frac{1}{1 + \alpha p_1 - \delta_\varepsilon} \alpha \varepsilon = \ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))$$

$$\Leftrightarrow \delta_\varepsilon = \frac{-\alpha \varepsilon}{\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))} + (1 + \alpha p_1).$$

Thus, we obtain

$$
\begin{aligned}
\lim_{\varepsilon \to 0} \frac{\delta_\varepsilon}{\varepsilon} &= \lim_{\varepsilon \to 0} \left\{ \frac{-\alpha}{\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))} + \frac{(1 + \alpha p_1)}{\varepsilon} \right\} \\
&= \lim_{\varepsilon \to 0} \frac{-\alpha \varepsilon + (1 + \alpha p_1)\left(\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))\right)}{\left(\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))\right)\varepsilon} \\
&= \lim_{\varepsilon \to 0} \frac{-\alpha + (1 + \alpha p_1)\frac{\alpha}{(1 + \alpha(p_1 - \varepsilon))}}{\left(\ln(1 + \alpha p_1) - \ln(1 + \alpha(p_1 - \varepsilon))\right) + \frac{\alpha}{(1 + \alpha(p_1 - \varepsilon))}\varepsilon} \quad \text{(by l'Hospital's rule)} \\
&= \lim_{\varepsilon \to 0} \frac{(1 + \alpha p_1)\frac{\alpha^2}{(1 + \alpha(p_1 - \varepsilon))^2}}{\frac{\alpha}{(1 + \alpha(p_1 - \varepsilon))} + \frac{\alpha}{(1 + \alpha(p_1 - \varepsilon))} + \frac{\alpha^2 \varepsilon}{(1 + \alpha(p_1 - \varepsilon))^2}} \quad \text{(by l'Hospital's rule, again)} \\
&= \frac{(1 + \alpha p_1)\frac{\alpha^2}{(1 + \alpha p_1)^2}}{\frac{2\alpha}{(1 + \alpha p_1)}} \\
&= \frac{\alpha}{2},
\end{aligned}
$$

which is the desired condition. This implies that for any $b^* > p_1^*(\alpha)$, the marginal gain from mixed bundling exceeds the marginal cost.

A4. Proof of Proposition 4

It suffices to show that $b^* < p_1^*(\alpha)$ implies that no mixed bundling price $(p_1^*(\alpha), p_2)$ is more profitable than $b^*(\tau, \alpha)$.

Step 1: Characterization of the optimal pure bundle price[20]

Suppose $b < p_1^*(\alpha)$. The profit under pure bundling is written as

$$\Pi^{PB}(b) = \begin{cases} b \cdot \left(\int_0^b \left[\tau - \dfrac{-v_1 + b}{1 + \alpha v_1} \right] dv_1 + \tau(1 - b) \right) / \tau & \text{for } b \leq \tau \\[3mm] b \cdot \left(\int_{\frac{b-\tau}{1+\alpha\tau}}^b \left[\tau - \dfrac{-v_1 + b}{1 + \alpha v_1} \right] dv_1 + \tau(1 - b) \right) / \tau & \text{for } b \geq \tau. \end{cases}$$

For $b \leq \tau$, it is equal to

$$\begin{aligned} \Pi_{b \leq \tau}^{PB}(b) &\equiv b \cdot \left(\int_0^b \left[\tau - \frac{-v_1 + b}{1 + \alpha v_1} \right] dv_1 + \tau(1 - b) \right) / \tau \\ &= b \cdot \left(\tau - \int_0^b \frac{-v_1 + b}{1 + \alpha v_1} dv_1 \right) / \tau, \end{aligned}$$

where

$$\begin{aligned} \int_0^b \frac{-v_1 + b}{1 + \alpha v_1} dv_1 &= \left[\frac{\ln(1 + \alpha v_1)}{\alpha}(b - v_1) \right]\Big|_0^b + \int_0^b \frac{\ln(1 + \alpha v_1)}{\alpha} dv_1 \\ &= \frac{\ln(1 + \alpha b)}{\alpha}(b - b) - \frac{\ln(1)}{\alpha} b + \frac{1}{\alpha} \int_0^b \ln(1 + \alpha v_1) dv_1 \\ &= \frac{1}{\alpha} \int_0^b \ln(1 + \alpha v_1) dv_1 \\ &= \frac{1}{\alpha^2} \int_1^{1+\alpha b} \ln(x) dx \\ &= \frac{1}{\alpha^2} \left[x(\ln x - 1) \right]\Big|_1^{1+\alpha b} \\ &= \frac{1}{\alpha^2} \left\{ (1 + \alpha b) \ln(1 + \alpha b) - \alpha b \right\}. \end{aligned}$$

Thus, we have

$$\Pi_{b \leq \tau}^{PB}(b) = b \cdot \left\{ 1 + \frac{b}{\alpha\tau} - \frac{1 + \alpha b}{\alpha^2 \tau} \ln(1 + \alpha b) \right\}$$

for $b \leq \tau$.

Next, for $b \geq \tau$, the profit under pure bundling is

$$\Pi_{b \geq \tau}^{PB}(b) \equiv b \cdot \left(\int_{\frac{b-\tau}{1+\alpha\tau}}^b \left[\tau - \frac{-v_1 + b}{1 + \alpha v_1} \right] dv_1 + \tau(1 - b) \right) / \tau$$

$$= b \cdot \left(\tau \left[1 - \frac{b-\tau}{1+\alpha\tau} \right] - \int_{\frac{b-\tau}{1+\alpha\tau}}^{b} \frac{-v_1+b}{1+\alpha v_1} dv_1 \right) / \tau$$

where

$$\int_{\frac{b-\tau}{1+\alpha\tau}}^{b} \frac{-v_1+b}{1+\alpha v_1} dv_1 = \left[\frac{\ln(1+\alpha v_1)}{\alpha}(b-v_1) \right] \Bigg|_{\frac{b-\tau}{1+\alpha\tau}}^{b} + \int_{\frac{b-\tau}{1+\alpha\tau}}^{b} \frac{\ln(1+\alpha v_1)}{\alpha} dv_1$$

$$= \frac{1+\alpha b}{\alpha^2} \left[\ln(1+\alpha\tau) - \frac{\alpha\tau}{1+\alpha\tau} \right].$$

Thus, we have

$$\Pi_{b \geq \tau}^{PB}(b) = b \cdot \left\{ 1 + \frac{1}{\alpha} - \frac{1+\alpha b}{\alpha^2 \tau} \ln(1+\alpha\tau) \right\}$$

for $b \geq \tau$. Notice that $\Pi^{PB}(b)$ is continuous at $b = \tau$ because

$$\Pi_{b \leq \tau}^{PB}(\tau) = \tau \cdot \left\{ 1 + \frac{1}{\alpha} - \frac{1+\alpha\tau}{\alpha^2 \tau} \ln(1+\alpha\tau) \right\} = \Pi_{b \geq \tau}^{PB}(\tau).$$

Now, we have

$$(\Pi^{PB})'(b) = \begin{cases} 1 + \dfrac{b}{\alpha\tau} - \dfrac{1+2\alpha b}{\alpha^2 \tau} \ln(1+\alpha b) & \text{for } b \leq \tau \\[3mm] 1 + \dfrac{1}{\alpha} - \dfrac{1+2\alpha b}{\alpha^2 \tau} \ln(1+\alpha\tau) & \text{for } b \geq \tau \end{cases}$$

and

$$(\Pi^{PB})''(b) = \begin{cases} -\dfrac{1}{\alpha\tau} \left[\dfrac{\alpha b}{1+\alpha b} + 2\ln(1+\alpha b) \right] < 0 & \text{for } b \leq \tau \\[3mm] -\dfrac{2\ln(1+\alpha\tau)}{\alpha\tau} < 0 & \text{for } b \geq \tau, \end{cases}$$

where the sign of $(\Pi^{PB})''$ is determined because $\ln(1+\alpha\tau)$ is positive (resp. negative) if and only if $\alpha > 0$ (resp. $\alpha < 0$). Note also that $(\Pi^{PB})'(b)$ is continuous at $b = \tau$. Thus, as long as $(\Pi^{PB})'(0) > 0$ and $(\Pi^{PB})'(p_1^*(\alpha)) < 0$ (remember that we assume that $b^* < p_1^*(\alpha)$), the optimal pure bundle price $b^*(\tau, \alpha)$ is uniquely determined by $(\Pi^{PB})'(b^*) = 0$. It is observed that $b^*(\tau, \alpha) \gtreqless \tau$ if and only if

$$1 + \frac{1}{\alpha} \gtreqless \frac{1+2\alpha\tau}{\alpha^2 \tau} \ln(1+\alpha\tau).$$

Thus, if $1 + 1/\alpha > (1+2\alpha\tau)\ln(1+\alpha\tau)/\alpha^2\tau$, then

$$b^*(\tau, \alpha) = \frac{\dfrac{\alpha\tau(1+\alpha)}{\ln(1+\alpha\tau)} - 1}{2\alpha},$$

while if $1 + 1/\alpha < (1 + 2\alpha\tau)\ln(1 + \alpha\tau)/\alpha^2\tau$, then $b^*(\tau, \alpha)$ is determined by

$$1 + \frac{b^*}{\alpha\tau} = \frac{1 + 2\alpha b^*}{\alpha^2\tau}\ln(1 + \alpha b^*).$$

Step 2: Profit changes by an increase in p_2 at $\left(p_1^*(\alpha), p_2\right)$

Now, we consider the marginal gain and loss from a small increase in p_2 at a point $\left(p_1^*(\alpha), p_2\right)$.

[Insert Figure 13.]

Case A: $p_1^*(\alpha) \geq \tau$ (see Figure 13). Area A is given by

$$
\begin{aligned}
A &= \int_{\frac{p_1^* + p_2 - \tau}{1 + \alpha\tau}}^{p_1^*} \left[\tau - \frac{-v_1 + p_1^* + p_2}{1 + \alpha v_1}\right] dv_1 \\
&= \tau \cdot \frac{\alpha\tau p_1^* - p_2 + \tau}{1 + \alpha\tau} - \frac{p_2}{\alpha}\ln(1 + \alpha p_1^*) + \frac{\tau(1 + \alpha(p_1^* + p_2))}{\alpha(1 + \alpha\tau)}\ln\left(\frac{1 + \alpha(p_1^* + p_2)}{1 + \alpha\tau}\right) \\
&\quad - \frac{1 + \alpha p_1^*}{\alpha^2}\ln(1 + \alpha p_1^*) + \frac{1 + \alpha(p_1^* + p_2)}{\alpha^2(1 + \alpha\tau)}\ln\left(\frac{1 + \alpha(p_1^* + p_2)}{1 + \alpha\tau}\right) + \frac{\alpha\tau p_1^* + \tau - p_2}{\alpha(1 + \alpha\tau)} \\
&= \frac{\alpha\tau p_1^* + \tau - p_2}{\alpha} - \frac{1 + \alpha(p_1^* + p_2)}{\alpha^2}\ln\left(\frac{(1 + \alpha p_1^*)(1 + \alpha\tau)}{1 + \alpha(p_1^* + p_2)}\right),
\end{aligned}
$$

multiplied by $1/\tau$, while Area B is given by

$$
\begin{aligned}
B &= \int_{p_1^*}^{1} \left[\tau - \frac{p_2}{1 + \alpha v_1}\right] dv_1 \\
&= \tau \cdot (1 - p_1^*) - \frac{p_2}{\alpha}\ln\left(\frac{1 + \alpha}{1 + \alpha p_1^*}\right),
\end{aligned}
$$

multiplied by $1/\tau$. In the following, we ignore the term $1/\tau$ for notational simplicity (because it does not affect the result).

The marginal gain is given by

$$A + B = \frac{(1 + \alpha)\tau - p_2}{\alpha} - \frac{1 + \alpha(p_1^* + p_2)}{\alpha^2}\ln\left(\frac{(1 + \alpha p_1^*)(1 + \alpha\tau)}{1 + \alpha(p_1^* + p_2)}\right) - \frac{p_2}{\alpha}\ln\left(\frac{1 + \alpha}{1 + \alpha p_1^*}\right)$$

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while the marginal loss is given by

$$
\left[\frac{\partial A}{\partial p_2} \cdot (p_1^* + p_2) + \frac{\partial B}{\partial p_2} \cdot p_2 \right] \times (-1)
$$

$$
= \frac{p_1^* + p_2}{\alpha} \ln \left(\frac{(1 + \alpha p_1^*)(1 + \alpha \tau)}{1 + \alpha(p_1^* + p_2)} \right) + \frac{p_2}{\alpha} \ln \left(\frac{1 + \alpha}{1 + \alpha p_1^*} \right).
$$

so that the marginal profit is

$$
\frac{\alpha \tau p_1^* - p_2 + \tau}{\alpha} - \frac{1 + 2\alpha(p_1^* + p_2)}{\alpha^2} \ln \left(\frac{(1 + \alpha p_1^*)(1 + \alpha \tau)}{1 + \alpha(p_1^* + p_2)} \right)
$$

$$
+ \tau \cdot (1 - p_1^*) - \frac{2 p_2}{\alpha} \ln \left(\frac{1 + \alpha}{1 + \alpha p_1^*} \right) \quad (\quad \equiv \quad MP).
$$

Then,

$$
MP|_{p_2=0} = \tau \left(1 + \frac{1}{\alpha} - \frac{1 + 2\alpha p_1^*}{\alpha^2 \tau} \ln \left(1 + \alpha \tau \right) \right) < 0
$$

because $p_1^* > b$. Now, we have

$$
\frac{\partial MP}{\partial p_2} = \frac{p_1^* + p_2}{1 + \alpha(p_1^* + p_2)} - \frac{2}{\alpha} \ln \frac{1 + \alpha}{1 + \alpha(p_1^* + p_2)} - \frac{2}{\alpha} \ln(1 + \alpha \tau).
$$

Recall that p_1^* is determined by

$$
\frac{p_1^*}{1 + \alpha p_1^*} - \frac{2}{\alpha} \ln \frac{1 + \alpha}{1 + \alpha p_1^*} = 0,
$$

which leads to

$$
\left. \frac{\partial MP}{\partial p_2} \right|_{p_2=0} = -\frac{2}{\alpha} \ln(1 + \alpha \tau) < 0.
$$

We also have

$$
\frac{\partial^2 MP}{\partial p_2^2} = \frac{1}{(1 + \alpha(p_1^* + p_2))^2} + \frac{2}{1 + \alpha(p_1^* + p_2)} > 0 \text{ for } p_2 > 0,
$$

which implies that $\partial MP / \partial p_2$ is monotonically increasing. Because $MP|_{p_2=0} < 0$, the graph of MP is drawn as in Figure 14.

[Insert Figure 14.]

27

Thus, there are no points that locally maximize the profit for $p_2 \in (0, \tau + p_1^*(1 + \alpha\tau))$, and therefore, the profit is highest at either of the endpoints $(p_1^*, 0)$ or $(p_1^*, \tau + p_1^*(1 + \alpha\tau))$. However, the profit at either endpoint is lower than with the optimal pure bundle price b^* because (i) it is already seen that $p_2 = 0$ is suboptimal, and (ii) the profit at $(p_1, p_2) = (p_1^*, \tau + p_1^*(1 + \alpha\tau))$ is lower than with $(p_1, p_2) = (1/2, \tau + (1/2)(1 + \alpha\tau))$. Notice that, by charging $(1/2, \tau + (1/2)(1 + \alpha\tau))$, the monopolist sells the base good only, at the component-wise monopoly price. However, by Proposition 3, this price is dominated by a mixed bundle price. Thus, the optimality of pure bundling is established.

Case B: $p_1^*(\alpha) < \tau$. As long as $p_2 \leq \tau - p_1^*(\alpha)$ (now, $(0, v_2^{B0}(0; \alpha))$ is below $(0, \tau)$), Area A is given by

$$
\begin{aligned}
A &= \int_0^{p_1^*} \left[\tau - \frac{-v_1 + p_1^* + p_2}{1 + \alpha v_1} \right] dv_1 \\
&= \tau p_1^* - \frac{1 + \alpha(p_1^* + p_2)}{\alpha^2} \ln(1 + \alpha p_1^*) + \frac{p_1^*}{\alpha}.
\end{aligned}
$$

Thus, the marginal gain is given by

$$
A + B = \tau + \frac{p_1^*}{\alpha} - \frac{1 + \alpha(p_1^* + p_2)}{\alpha^2} \ln(1 + \alpha p_1^*) - \frac{p_2}{\alpha} \ln\left(\frac{1 + \alpha}{1 + \alpha p_1^*}\right),
$$

while the marginal loss is given by

$$
\begin{aligned}
&\left[\frac{\partial A}{\partial p_2} \cdot (p_1^* + p_2) + \frac{\partial B}{\partial p_2} \cdot p_2 \right] \times (-1) \\
&= \frac{p_1^* + p_2}{\alpha} \ln(1 + \alpha p_1^*) + \frac{p_2}{\alpha} \ln\left(\frac{1 + \alpha}{1 + \alpha p_1^*}\right).
\end{aligned}
$$

Clearly, the marginal gain is decreasing in p_2 while the marginal loss is increasing in p_2, meaning that the marginal profit falls as p_2 increases (with $p_1 = p_1^*$ held fixed: the argument holds either for $\alpha > 0$ or for $\alpha < 0$). When p_2 is sufficiently large that $p_1^*(\alpha) + p_2 > \tau$, it is verified there are no local interior maxima on $(p_1^*(\alpha), p_2)$ as in Case A, and that the boundary points are not optimal. Thus, the optimality of pure bundling is also established in this case because, as in Case A, the net marginal gain at $(p_1^*, 0)$ coincides with $(\Pi^{PB})'(b^*)$ in the case of $b^* < \tau$ (i.e., $1 + b^*/\alpha\tau - (1 + 2\alpha b^*)\ln(1 + \alpha b^*)/\alpha^2\tau)$.

A5. Proof of Proposition 5

If $1 + 1/\alpha \geq (1 + 2\alpha\tau)\ln(1 + \alpha\tau)/\alpha^2\tau$, then

$$\frac{\partial b^*(\tau, \alpha)}{\partial \tau} = \frac{1 + \alpha}{2(1 + \alpha\tau)}\frac{(1 + \alpha\tau)\ln(1 + \alpha\tau) - \alpha\tau}{(\ln(1 + \alpha\tau))^2}.$$

By the mean-value theorem, we have

$$
\begin{aligned}
&(1 + x)\ln(1 + x) - x \\
=\ &(1 + x)\ln(1 + x) - (1 + 0)\ln(1 + 0) - x \\
=\ &x \cdot \{\ln(1 + \widetilde{x}) + 1\} - x \\
=\ &x\ln(1 + \widetilde{x}) > 0
\end{aligned}
$$

for $1 + x > 0$ and $\widetilde{x}$ is some value between 0 and x (which is either positive or negative). This shows that $\partial b^*(\tau, \alpha)/\partial \tau > 0$.

Next, if $1 + 1/\alpha < (1 + 2\alpha\tau)\ln(1 + \alpha\tau)/\alpha^2\tau$, let

$$G(b, \tau) = \frac{1 + 2\alpha b}{\alpha}\ln(1 + \alpha b) - (b + \alpha\tau).$$

Then, we have

$$
\begin{aligned}
\frac{\partial b^*(\tau, \alpha)}{\partial \tau} &= -\frac{\partial G/\partial \tau}{\partial G/\partial b} \\
&= \frac{\alpha}{\partial G/\partial b},
\end{aligned}
$$

where

$$\frac{\partial G}{\partial b} = 2\ln(1 + \alpha b^*) + \frac{\alpha b^*}{1 + \alpha b^*}.$$

Either for $\alpha > 0$ or for $\alpha < 0$, $\alpha/(\partial G/\partial b)$ is positive.

A6. Numerical Values in Welfare Comparison $(\alpha \neq 0)$

[Insert Tables 1, 2, 3 and 4.]

Notes

[1] A well-known example is the legal case of Microsoft's attempt to bundle its Windows operating system and its Internet browser (Internet Explorer) (see e.g., Evans, Nichols, and Schmalensee (2005) and Rubinfeld (2009)). In the EU countries, bundling can be illegal once it is judged to be a predatory strategy under Article 82 of the Treaty Establishing the European Community.

[2] Schmalensee (1984), McAfee, McMillan, and Whinston (1989), and Salinger (1995) then study two-goods, continuum-type cases (Salinger's (1995) work also permits discrete types). Many-goods, continuum-type cases are analyzed by, e.g., Armstrong (1999), Bakos and Brynjolfsson (1999), Fang and Norman (2006), Crawford and Cullen (2007), and Crawford (2008). An important common idea in this body of work is that bundling reduces the dispersion of the buyers' average willingness to pay for the goods, unless the buyer's valuations for bundled goods are perfectly positively correlated. In this situation, a monopolist can extract more profits through bundling. Excellent surveys on bundling include Varian (1989, Subsection 2.6), Shy (1996, Chapter 14), Motta (2004, Chapter 7), Shy (2008, Chapter 4), and Belleflamme and Peitz (2010, Chapter 11).

[3] We confine our attention to the case of a monopoly. See, e.g., Nalebuff (2004), Thanassoulis (2007), Armstrong and Vickers (2010), and Jeon and Menicucci (2011) for analyses of bundling and strategic interaction. Cheng and Nahm (2007) consider a similar problem where there is a main good and an optional good, and each good is produced by a different producer. In a different vein, Ellison (2005) considers price competition in a Hotelling model with base and optional goods, and studies the consequence of "add-on pricing," in which the price of the optional good is hidden upon the purchase of the base good. He observes that, under certain conditions, add-on pricing reduces firms' competition.

[4] Similar to this paper, Adachi and Ebina (2011) study a model of package discount for add-ons, by adopting the main-accessory relationship between two goods as in our model. They find that discounting for a package relative to the (exogenous) market price is profitable if the relative number of price-sensitive consumers is large. Our analysis is different from theirs in several ways. First, we assume no outside suppliers that determine the market price for a package, and thus we have no implication for discounts. Second, they assume only two types of consumers heterogeneity (high- and low-valued consumers). More importantly, Adachi and Ebina (2011) assume a specific demand structure that abstracts the tradeoff between a bundle and no purchase. Specifically, they assume myopic consumers who decide whether to buy a base good at the base good price, without considering potential purchase of an option. This feature misses the relationship between the optional good price and the demand for base goods, which is key in our multidimensional screening analysis.

[5] Eckalbar (2010) obtains closed-form solutions for mixed bundling with independent valuations. In an older version of this paper (Adachi, Ebina and Hanazono (2010)), independent of Eckalbar (2010), we analyze the problem of bundling with the independency of base and optional goods. Both Eckalbar (2010) and Adachi, Ebina and Hanazono (2010) allow for the support of valuation of one good to be different from the other (which Eckalbar (2010) calls "asymmetric cases").

[6] This assumption fits the data used by Pierce and Winter (1996) (advertisement on newspapers with large circulation as a base good, and those with small circulation as an optional good).

[7] In the context of option package bundling, pure bundling is a special case of mixed bundling, and thus, the two regimes can be considered as one "connected" regime. This enables us to consider

mixed bundling or pure bundling only, without worrying about separate sales. In addition, the nature of the option significantly reduces complexity associated with the analysis of mixed bundling.

[8]Most studies explore a demand-based explanation on bundling decisions. Salinger (1995) and Evans and Salinger (2008) analyze cost savings from bundling. In particular, Evans and Salinger (2008) assume cost reduction by joint production: the constant marginal cost of joint production is no greater than the sum of the constant marginal cost of separate production ($c_B \leq c_1 + c_2$). As in Adachi and Ebina (2011), Evans and Salinger (2008) do not start with utility. Evans and Salinger (2008) study welfare analysis in terms of cost minimization.

[9]The restriction on $\tau \leq 1$ should not be important: in the case of $\alpha = 0$, the main results in Adachi, Ebina and Hanazono (2010) hold as long as $\tau \leq 3/2$ (because the optimal pure bundling price does not exceed the upper bound for v_1).

[10]An alternative setting would be to assume a one-dimensional index support $[0, 1]$ for the mass of consumers, and valuation mappings $v_1 : [0, 1] \rightarrow R_+$ and $v_B : [0, 1] \rightarrow R_+$. However, this setting would be better suited to analyzing issues surrounding quality upgrading. Although they share similar features, our focus is on discriminatory pricing behavior rather than on quality upgrading.

[11]In the present paper, we do not focus on the case of $\alpha = 0$ because we have extensively analyzed this situation in an older version (Adachi, Ebina and Hanazono (2010)).

[12]Both figures are depicted for the case of $p_1 + p_2 < \tau$. If $p_1 + p_2 \geq \tau$, the curve $v_2 = v_2^{B0}(v_1; \alpha)$ intersects the boundary $v_2 = \tau$ at $v_1 = (p_1 + p_2 - \tau)/(1 + \alpha\tau)$.

[13]Note that if $\alpha < 0$ and $p_1 + p_2 = -1/\alpha$, even a consumer with $(v_1, v_2) = (1, \tau)$ has negative utility by purchasing a bundle, given as

$$1 + \tau + \alpha\tau - (p_1 + p_2) = 1 + \tau + \alpha\tau + \frac{1}{\alpha} = \frac{1}{\alpha}(1 + \alpha)(1 + \alpha\tau) < 0 \ (\because \alpha < 0).$$

[14]For an analytical verification of this claim, see Adachi, Ebina and Hanazono (2010).

[15]Note that τ increases the variance of the willingness to pay for a bundle along with its mean. We later consider variance-preserving upward shifts of the support for the valuation of good 2, and verify that the main thrust of this argument holds.

[16]The only concern is that the term $\ln(1 + \alpha\tau)$, which appears several times in Appendices, might not be well-defined because $1 + \alpha\tau$ can be negative if $\tau > 1$. However, the term $\ln(1 + \alpha\tau)$ appears from the restriction $\tau \leq 1$, and thus the results hold even if we allow $\tau > 1$.

[17]In Figure 8, the thresholds for τ are $0.55 - 0.60$ for $\alpha = 0.1$ and $0.67 - 0.70$ for $\alpha = -0.1$. In Figure 9, the thresholds are $0.45 - 0.50$ for $\alpha = 0.3$ and $0.75 - 0.80$ for $\alpha = -0.3$ (in both cases, the threshold with $\alpha = 0$ is 0.67).

[18]Note that if $\alpha \rightarrow 0$, then we have the same equality in the case of $\alpha = 0$ ($p_1 p_2 = 2(1 - p_1)p_2$) because

$$\lim_{\alpha \to 0} \frac{\ln(1 + \alpha) - \ln(1 + \alpha p_1^*)}{\alpha} = \lim_{\alpha \to 0} \frac{\dfrac{1}{1 + \alpha} - \dfrac{p_1^*}{1 + \alpha p_1^*}}{1} = 1 - p_1^*$$

by l'Hôpital's rule.

[19]Note that $\delta_\varepsilon > 0$ (resp. < 0)for $\alpha > 0$ (resp. > 0).

[20]Note that the case with $b \geq 1$ is excluded because by Proposition 3, mixed bundling is optimal in such a case.

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τ \ Welfare	Pure Bundling	Mixed Bundling
0.6	0.6587239	0.6587238
0.65	0.6864614	0.6863702
0.7	0.7126905	0.7124764
0.75	0.7333987	0.7326998
0.8	0.7542818	0.7528568
0.85	0.7753236	0.7729592
0.9	0.7965104	0.7930161
0.95	0.8178302	0.8130346
1	0.8392726	0.8330204

Table 1: Complementarity ($\alpha = 0.1$)

τ \ Welfare	Pure Bundling	Mixed Bundling
0.7	0.68374068	0.68373951
0.75	0.70238105	0.70221359
0.8	0.72119633	0.72060564
0.85	0.74017041	0.73893015
0.9	0.75928942	0.75719838
0.95	0.77854144	0.77541923
1	0.79791612	0.79359976

Table 2: Substitutability ($\alpha = -0.1$)

$\tau \setminus$ Welfare	Pure Bundling	Mixed Bundling
0.5	0.590960336	0.590959389
0.55	0.615930270	0.615789169
0.6	0.641493938	0.640991632
0.65	0.667646519	0.666582887
0.7	0.694383140	0.692574555
0.75	0.718483483	0.717029168
0.8	0.738369256	0.735913390
0.85	0.758413503	0.754747837
0.9	0.778602489	0.773540881
0.95	0.798924264	0.792299031
1	0.819368465	0.811027511

Table 3: Complementarity ($\alpha = 0.3$)

$\tau \setminus$ Welfare	Pure Bundling	Mixed Bundling
0.8	0.68834084	0.68828550
0.85	0.70526943	0.70492892
0.9	0.72234370	0.72150010
0.95	0.73955170	0.73801042
1	0.75688305	0.75446902

Table 4: Substitutability ($\alpha = -0.3$)
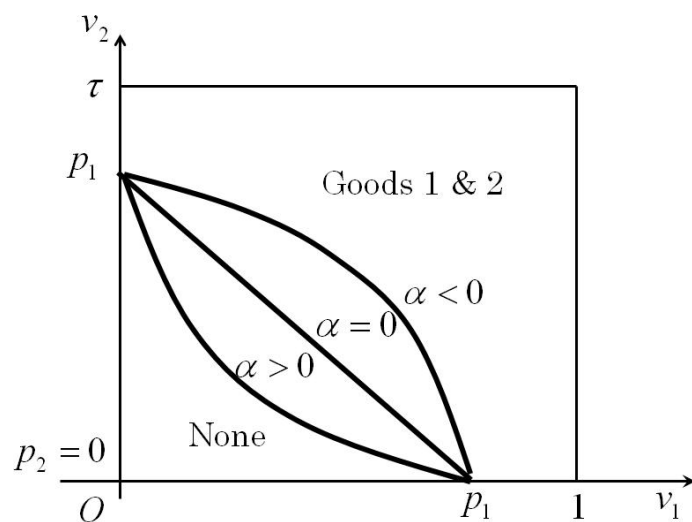
Figure 1: Demand Structure with Pure Bundling
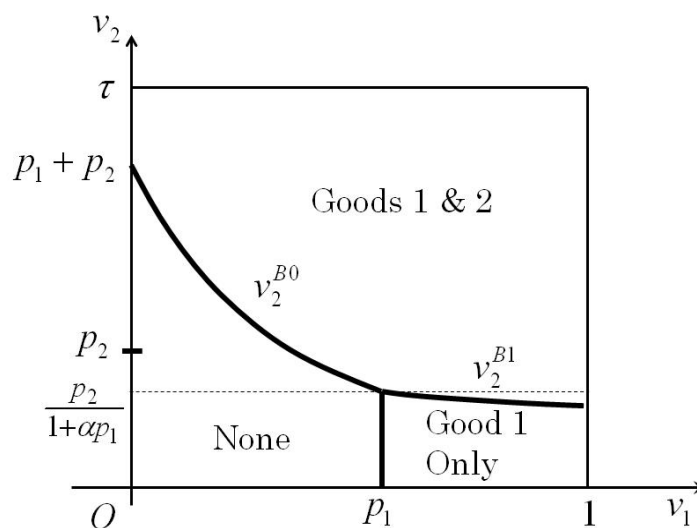
Figure 2: Demand Structure with Mixed Bundling: The Case of Complements $(\alpha > 0)$
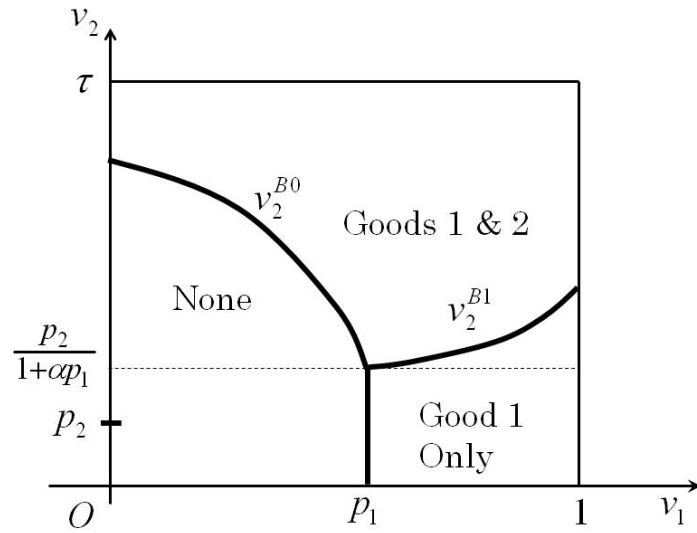
Figure 3: Demand Structure with Mixed Bundling: The Case of Substitutes ($\alpha < 0$)
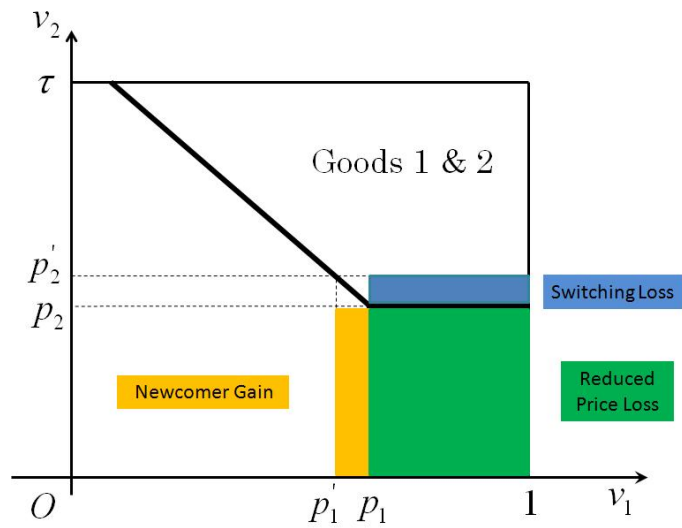
Figure 4: The Effects of Screening ($\alpha = 0$)
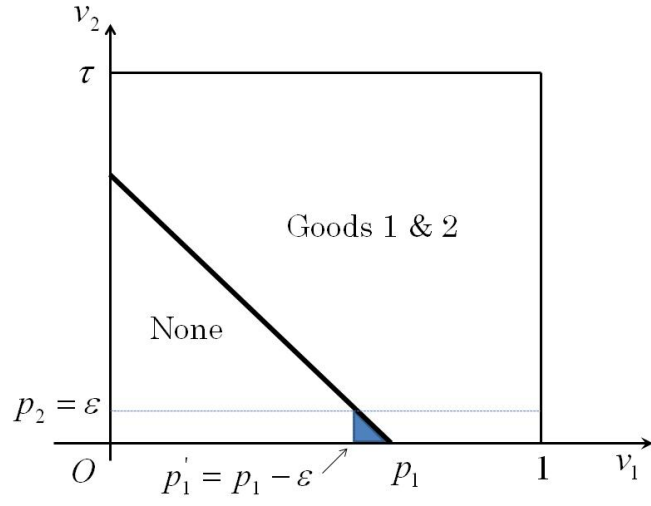
Figure 5: Suboptimality of Pure Bundling ($\alpha = 0$)
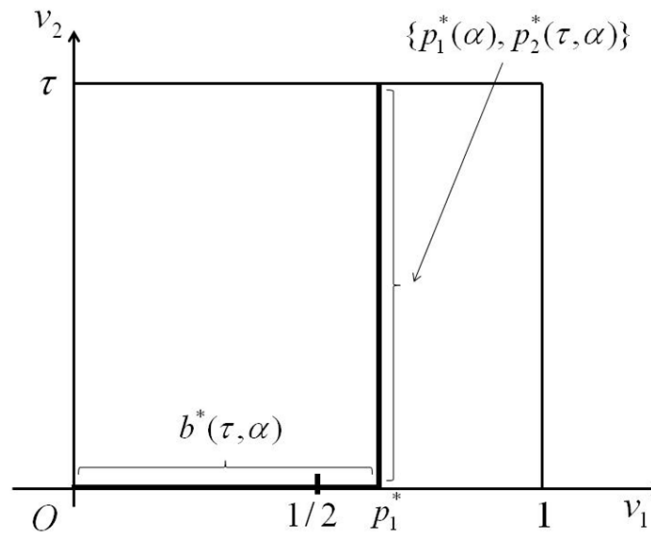
Figure 6: Possible Pairs for Optimal Mixed Bundle Prices
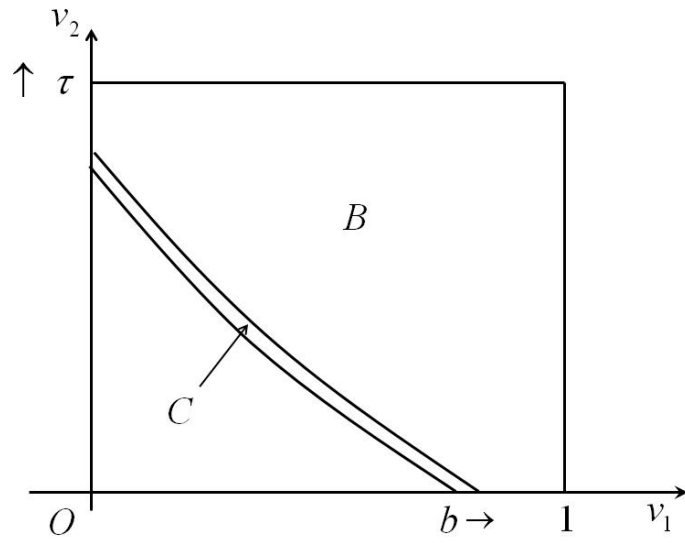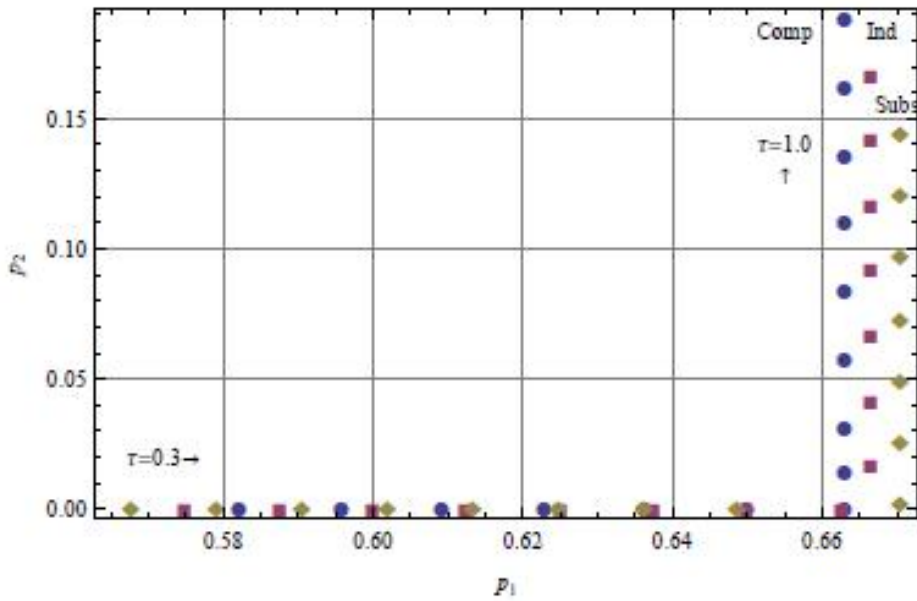
Figure 7: Marginal Gain and Loss in Pure Bundling

Figure 8: Changes in the Optimal Prices as τ Increases under Independency ($\alpha = 0$; squares), Complementarity ($\alpha = 0.1$; circles), and Substitutability ($\alpha = -0.1$; diamonds)
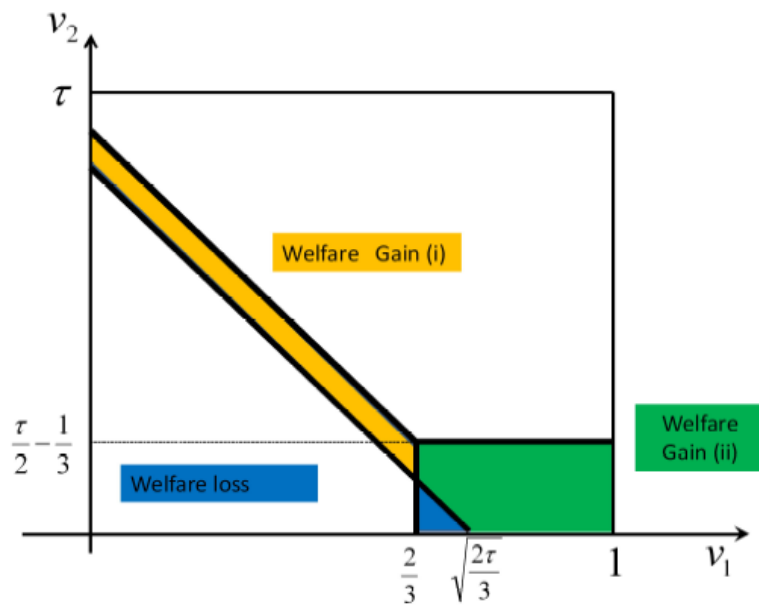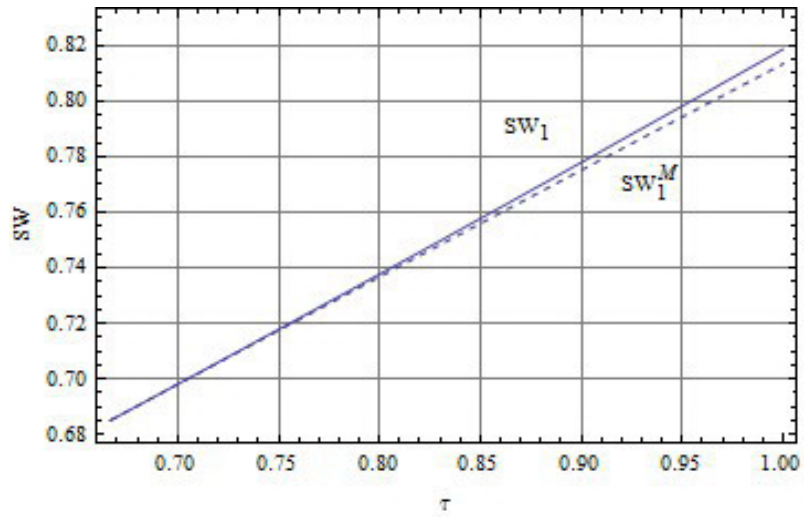
40
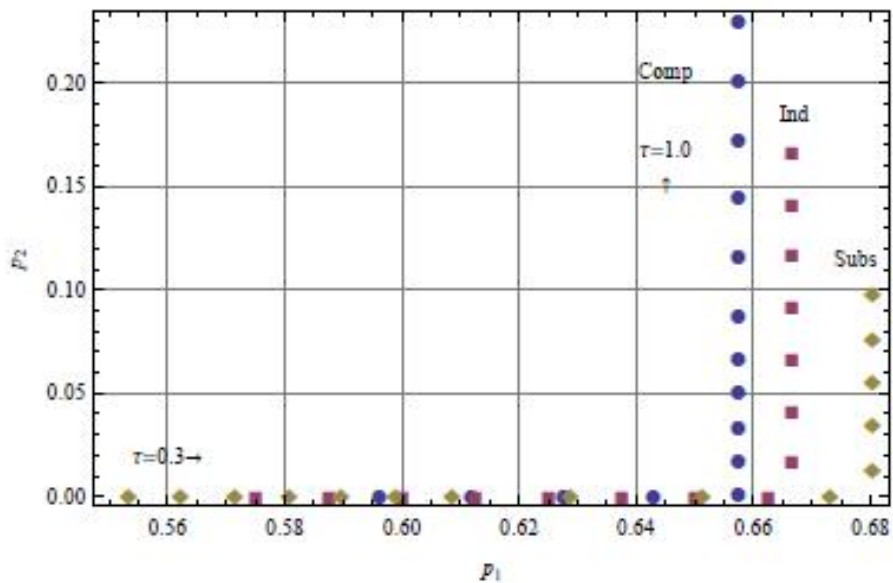
Figure 9: Changes in the Optimal Prices as τ Increases under Independency ($\alpha = 0$; squares), Complementarity ($\alpha = 0.3$; circles), and Substitutability ($\alpha = -0.3$; diamonds)

Figure 10: Effects of Prohibiting Mixed Bundling

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Figure 11: Social Welfare Loss by Mixed Bundling (SW_1: Pure Bundling; SW_1^M: Mixed Bundling)
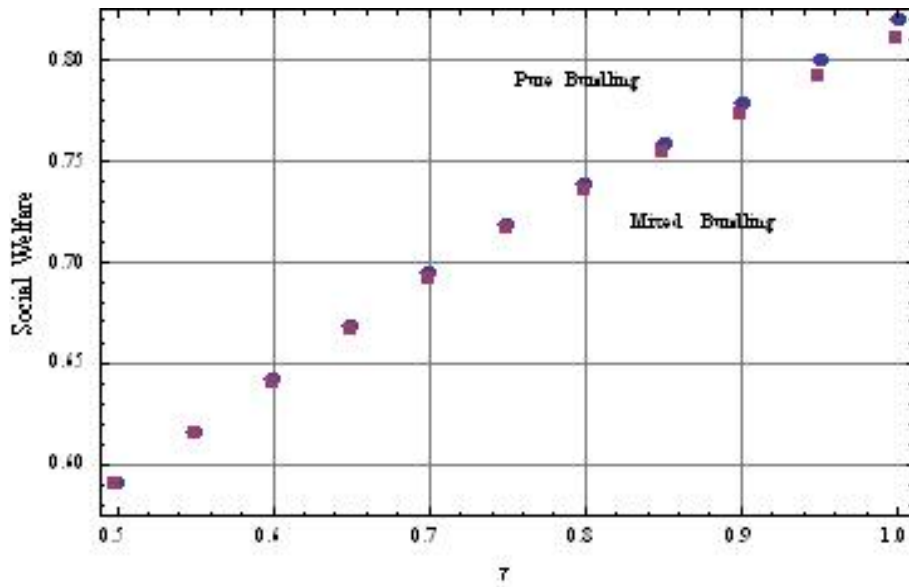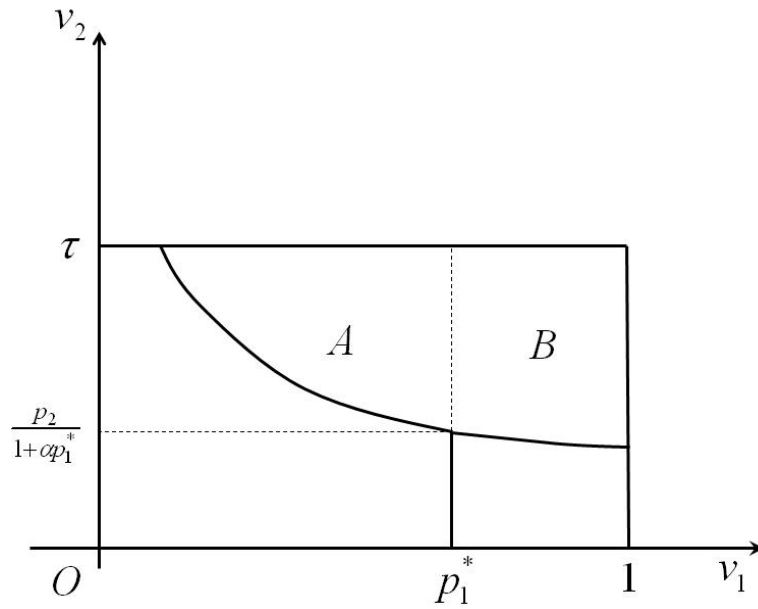

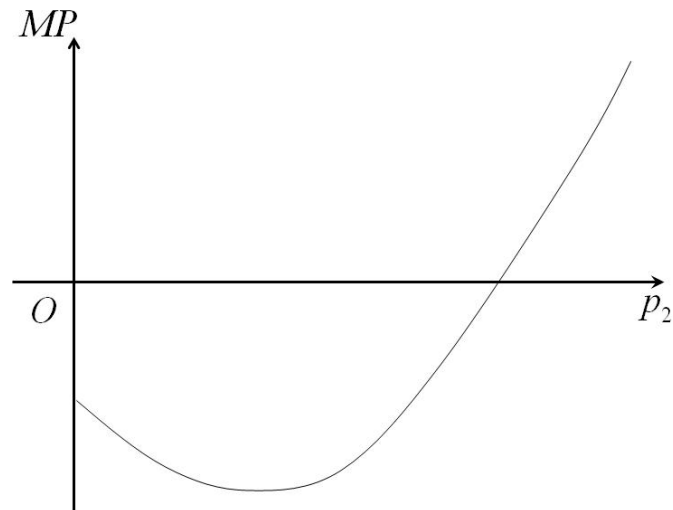Figure 12: Welfare Comparison ($\alpha = 0.3$)

Figure 13: $p_1^*(\alpha) \geq \tau$ (in the case of $\alpha > 0$)

Figure 14: Marginal Profits

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