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Primordial non-Gaussianity from G inflation

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We present a comprehensive study of primordial fluctuations generated from G inflation, in which the inflaton Lagrangian is of the form $K(\phi, X) - G(\phi, X)\Box\phi$ with $X = -(\partial\phi)^2/2$. The Lagrangian still gives rise to second-order gravitational and scalar field equations, and thus offers a more generic class of single-field inflation than ever studied, with a richer phenomenology. We compute the power spectrum and the bispectrum, and clarify how the non-Gaussian amplitude depends upon parameters such as the sound speed. In so doing we try to keep as great generality as possible, allowing for non slow-roll and deviation from the exact scale invariance.

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I. INTRODUCTION

Cosmological inflation [1] is now a widely accepted paradigm explaining the flatness, homogeneity, and isotropy of the observed Universe. In the most common scenario, inflation occurs when the inflaton, a scalar field driving the accelerated expansion, rolls down a nearly flat potential slowly. During this slow-roll stage fluctuations in the inflaton field are generated quantum-mechanically and stretched outside the Hubble horizon, which eventually reenter the Hubble radius in a later epoch to be a seed for the large-scale structure of the Universe. The detailed shape of the potential can be probed by observing the power spectrum of fluctuations in terms of the cosmic microwave background anisotropies [2]. As to theoretical approaches, much effort has been made to determine the inflaton potential in the particle physics context. However, single-field inflation with a canonical kinetic term and a nearly flat potential is not the only option to induce the accelerated expansion and to produce almost scale-invariant perturbations with an appropriate amplitude. Liberating inflation models from the standard assumption, one may consider a variety of interesting scenarios: multiple scalar fields might participate the inflationary dynamics, the kinetic term of the inflaton(s) might be noncanonical [3], and a scalar field other than the inflaton might be responsible for the density perturbation [4]. From a high-energy physics point of view, supersymmetric theories naturally provide many scalar fields with flat potentials [5], and the Dirac–Born–Infeld (DBI)-type noncanonical kinetic term naturally arises from D3-brane motion in a warped compactification [6].

Different inflationary scenarios can be distinguished by future and ongoing experiments such as Planck [7], aiming to obtain better constraints on the amount of non-Gaussianities in the primordial curvature perturbations as well as on the spectral index n_s, its running, and the tensor-to-scalar ratio r. The standard canonical slow-roll inflation models produce negligible non-Gaussianity [8], while exotic inflationary scenarios are expected to predict measurable non-Gaussian signals. In the context of single-field inflation, non-Gaussian perturbations have been computed for the Lagrangian of the form [9,10]

$$\mathcal{L}_\phi = K(\phi, X), \tag{1}$$

where ϕ is the inflaton and $X := -\partial_\mu\phi\partial^\mu\phi/2$. This class of models yields a sound speed c_s different from the speed of light in general, and large non-Gaussianity is generated for $c_s \ll 1$. A significant non-Gaussian signal together with the confirmation of the consistency relation $r = -8c_s n_T$, where n_T is the spectral index of primordial tensor perturbations, is a smoking gun of the inflaton Lagrangian (1).

In this paper, we consider a more general Lagrangian [11,12]

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \tag{2}$$

where K and G are some generic functions of the inflaton ϕ and X. The new term $G(\phi, X)\Box\phi$ in the Lagrangian (2) is inspired by the Galileon interaction [13,14] and reduces to the one having the Galilean shift symmetry, $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$, in the Minkowski background in the case $G \propto X$. One of the most important properties of the Galileon Lagrangian is that the field equations do not contain derivatives higher than 2. The interaction $G(\phi, X)\Box\phi$ is a generalization of the Galileon term

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$X\Box\phi$ while maintaining the second-order property. In this sense, the Lagrangian (2) defines a more generic class of single-field inflation than ever studied. Here, the Galilean shift symmetry is abandoned in exchange for generality, but one should note that the symmetry does not make sense already upon covariantization for any interaction that is Galilean invariant in the flat background.[1] (The name "Galileon" is therefore no longer appropriate when covariantized.) Cosmological applications of the Galileon interaction can be found in [15] with emphasis on dark energy and modified gravity. Primordial inflation based on the generic Lagrangian (2) was first proposed very recently by [12,16], and is dubbed G inflation. Almost simultaneously the same Lagrangian was used to explain the late-time cosmic acceleration rather than the primordial one [11,17]. In [18,19] the effective-field-theory approach [20] was employed to see the consequences of imposing the approximate Galilean shift symmetry on the Lagrangian of primordial perturbations. Interestingly, the scalar field theory with the $G\Box\phi$ term can violate the null energy condition stably. This fact motivates the authors of Refs. [21,22] to propose a radical scenario of the earliest Universe alternative to inflation. Some specific form of the above type of interaction arises from a probe brane action in higher dimensions [23] and from the Kaluza-Klein reduction of Lovelock gravity [24,25]. A supersymmetric completion of Galileons is explored in [26].

The purpose of the present paper is to understand the nature of cosmological perturbations generated from G inflation. We rederive the power spectrum and the tilt of the spectrum without assuming slow roll, clarifying how the (approximate) scale invariance is achieved in G inflation. We then calculate the cubic action for the curvature perturbation and evaluate the full non-Gaussian amplitude, again without assuming slow roll and the exact scale invariance. Throughout the paper we try to make our formulas as general as possible, which we hope maximizes the usefulness of the results. Recently, non-Gaussianity from G inflation was calculated neglecting a number of terms working in the de Sitter limit [27] and in the slow-roll limit [28]. See also a recent work by Naruko and Sasaki, in which the superhorizon evolution of the nonlinear curvature perturbation from G inflation is addressed [29].

This paper is organized as follows. In the next section we review the basic properties of G inflation and derive the power spectrum of the curvature perturbation. In Sec. III we compute the cubic action for the curvature perturbation to evaluate the three-point function in G inflation.

[1]Concerning this point, one may worry about the naturalness of G-inflation models discussed in the present paper because there is no symmetry to protect the Lagrangian. However, it should be noted that symmetry, if present, must be broken at least to end inflation. We therefore will not provide a symmetry-based argument but rather take a phenomenological approach, assuming that some UV complete theory would give the (in some sense fine-tuned) Lagrangian that leads to second-order field equations.

II. G INFLATION

We start with a brief review on the basics of G inflation [12,16]. The scalar field Lagrangian for G inflation is given by Eq. (2). Assuming that ϕ is minimally coupled to gravity, the total action we are going to study is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_\phi\right]. \tag{3}$$

In the following we will set $M_{\rm Pl} = 1$. The energy-momentum tensor $T_{\mu\nu}$ of the scalar field is given by

$$T_{\mu\nu} = K_X\nabla_\mu\phi\nabla_\nu\phi + Kg_{\mu\nu} - 2\nabla_{(\mu}G\nabla_{\nu)}\phi + g_{\mu\nu}\nabla_\lambda G\nabla^\lambda\phi - G_X\Box\phi\nabla_\mu\phi\nabla_\nu\phi. \tag{4}$$

Here and hereafter we use the notation K_X for $\partial K/\partial X$ etc. Varying the action with respect to ϕ, we obtain the scalar field equation of motion,

$$\begin{aligned} &K_X\Box\phi - K_{XX}(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\phi\nabla^\nu\phi) \\ &\quad - 2K_{\phi X}X + K_\phi - 2(G_\phi - G_{\phi X}X)\Box\phi \\ &\quad + G_X[(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) - (\Box\phi)^2 + R_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi] \\ &\quad + 2G_{\phi X}(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\phi\nabla^\nu\phi) + 2G_{\phi\phi}X \\ &\quad - G_{XX}(\nabla^\mu\nabla^\lambda\phi - g^{\mu\lambda}\Box\phi)(\nabla_\mu\nabla^\nu\phi)\nabla_\nu\phi\nabla_\lambda\phi = 0, \end{aligned} \tag{5}$$

which is of course equivalent to the conservation equation $\nabla_\nu T_\mu{}^\nu = 0$. One verifies from Eqs. (4) and (5) that the gravitational and scalar field equations are indeed of second order.

Higher order Galileon terms (with a ϕ-dependent coefficient) such as $f(\phi)X[2\Box(\phi)^2 - 2\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi + RX]$ can be added to the scalar field Lagrangian while keeping the field equations of second order. Although the effect of such higher order Galileons might be interesting in the context of primordial inflation, we leave the issue for future study and concentrate on the Lagrangian of the form (2) in the present paper.

A. The background equations

Let us consider homogeneous and isotropic background:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \qquad \phi = \phi(t). \tag{6}$$

Although the energy-momentum tensor (4) cannot be recast in a perfect-fluid form in general [11], for the above cosmological ansatz it takes the desirable form $T_\mu{}^\nu = \mathrm{diag}(-\rho, p, p, p)$ with

$$\rho = 2K_XX - K + 3HG_X\dot\phi^3 - 2G_\phi X, \tag{7}$$

$$p = K - 2(G_\phi + G_X\ddot{\phi})X. \tag{8}$$

The gravitational field equations are thus

$$3H^2 = \rho, \tag{9}$$

$$-3H^2 - 2\dot{H} = p, \tag{10}$$

and the scalar field equation of motion is given by

$$K_X(\ddot{\phi} + 3H\dot{\phi}) + 2K_{XX}X\ddot{\phi} + 2K_{X\phi}X - K_\phi - 2(G_\phi - G_{X\phi}X)(\ddot{\phi} + 3H\dot{\phi}) + 6G_X[(HX)^{\cdot} + 3H^2X] - 4G_{X\phi}X\ddot{\phi} - 2G_{\phi\phi}X + 6HG_{XX}X\dot{X} = 0. \tag{11}$$

If, for example, K is given by the standard, canonical kinetic term with a potential, $K = X - V(\phi)$, one can consider an inflationary scenario in which the energy density is dominated by the potential as in the standard case, while the dynamics of the scalar field is modified by the $G\Box\phi$ term, changing the potential that ϕ effectively feels. This is the scenario proposed in [16] and called potential driven G inflation. Another possible scenario is that inflation is driven by ϕ's kinetic energy which is kept almost constant with a nontrivial functional form of K and G. In models with the exact shift symmetry, $\phi \to \phi + c$, i.e., $K = K(X)$ and $G = G(X)$, it is easy to obtain an exactly de Sitter background satisfying $H = \text{const}$ and $\dot{\phi} = \text{const}$. This may be regarded as a generalization of k inflation [3], and we call the class of models kinematically driven G inflation [12]. Deferring the summary of these two specific classes of G inflation to Sec. II C, we now move on to describe the general properties of the power spectrum of primordial perturbations from G inflation.

B. Power spectrum

In this section we derive a series of general formulas for linear cosmological perturbations without assuming any specific form of K and G. We work in the unitary gauge, $\phi(t, \mathbf{x}) = \phi(t)$.[2] Using the remaining gauge degree of freedom the linearly perturbed metric is taken to be

$$ds^2 = -(1 + 2\alpha_1)dt^2 + 2a^2\partial_i\beta_1 dt dx^i + a^2(1 + 2\mathcal{R})d\mathbf{x}^2. \tag{12}$$

Expanding the action to second order in perturbations and then varying with respect to α_1 and β_1, we obtain the following constraint equations:

$$\dot{\mathcal{R}} = \Theta\alpha_1, \tag{13}$$

$$\frac{\partial^2}{a^2}(\mathcal{R} + a^2\Theta\beta_1) = X\mathcal{G}\alpha_1, \tag{14}$$

where $\partial^2 := \delta^{ij}\partial_i\partial_j$,

$$\Theta := H - \dot{\phi}XG_X, \tag{15}$$

$$\mathcal{G} := K_X + 2XK_{XX} + 6G_XH\dot{\phi} + 6G_X^2X^2 - 2(G_\phi + XG_{\phi X}) + 6G_{XX}HX\dot{\phi}. \tag{16}$$

Substituting the constraints (13) and (14) to the action, we arrive at the quadratic action for $\mathcal{R}$ [11,12]:

$$S_2 = \int dt d^3x a^3\sigma\left[\frac{1}{c_s^2}\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\partial\mathcal{R})^2\right], \tag{17}$$

where

$$c_s^2 := \frac{\mathcal{F}}{\mathcal{G}}, \tag{18}$$

$$\sigma := \frac{X\mathcal{F}}{\Theta^2}, \tag{19}$$

and

$$\mathcal{F} := K_X + 2G_X(\ddot{\phi} + 2H\dot{\phi}) - 2G_X^2X^2 + 2G_{XX}X\ddot{\phi} - 2(G_\phi - XG_{\phi X}). \tag{20}$$

One can verify that setting $G(\phi, X) = 0$ the quadratic action (17) reproduces the expression obtained for k inflation [30]. It is useful to notice that σ can also be expressed as

$$\sigma = -\frac{\dot{\Theta}}{\Theta^2} + \frac{\dot{\phi}XG_X}{\Theta}. \tag{21}$$

Let us define three parameters that characterize the rate of change of three background quantities:

$$\epsilon := -\frac{\dot{H}}{H^2}, \qquad s := \frac{\dot{c}_s}{Hc_s}, \qquad \delta := \frac{\dot{\sigma}}{H\sigma}. \tag{22}$$

In this paper we assume that

$$\frac{\dot{\epsilon}}{H\epsilon} \simeq 0, \qquad \frac{\dot{s}}{Hs} \simeq 0, \qquad \frac{\dot{\delta}}{H\delta} \simeq 0, \tag{23}$$

but we do not neglect ϵ, s, and δ. (In the next section, however, we will assume some stronger conditions to evaluate the bispectrum.) It should be noted, in particular, that σ is not necessarily small, in contrast to the usual (k) inflation models in which σ is degenerate, i.e., $\sigma = \epsilon < 1$ [30]. Even in the slow-roll limit we may have $\sigma \gtrsim 1$ in G inflation.

Under the assumption that the parameters defined in (22) are constant (but not necessarily very small), it is straightforward to solve the equation of motion derived from the action (17) and compute the power spectrum of $\mathcal{R}$ [12]. For this purpose it is convenient to define a new time

[2] The unitary gauge does not coincide with the comoving gauge, $\delta T_i^{\ 0} = 0$, in the case of G inflation [12]. This fact stems from the imperfect-fluid nature of the energy-momentum tensor (4).

coordinate y by $dy = c_s dt/a$ [31]. In terms of y, the scale factor, the sound speed, and σ are written as

$$a = \frac{c_{s*}(y/y_*)^{-1/(1-\epsilon-s)}}{(-y_*)H_*(1-\epsilon-s)},$$
$$c_s = c_{s*}(y/y_*)^{-s/(1-\epsilon-s)}, \tag{24}$$
$$\sigma = \sigma_*(y/y_*)^{-\delta/(1-\epsilon-s)},$$

where the quantities with $*$ are those evaluated at some reference time $y = y_*$. Using a new variable $u := \tilde{z}\mathcal{R}$ with $\tilde{z} := a\sqrt{2\sigma/c_s}$, the equation of motion can be written in the Fourier space as

$$u_k'' + \left(k^2 - \frac{\tilde{z}''}{\tilde{z}}\right)u_k = 0, \tag{25}$$

where the prime denotes differentiation with respect to y, and we find

$$\tilde{z} \propto (-y)^{1/2-q},$$
$$\frac{\tilde{z}''}{\tilde{z}} = \frac{q^2 - 1/4}{y^2}, \quad \text{with} \quad q := \frac{3-\epsilon-2s+\delta}{2(1-\epsilon-s)}. \tag{26}$$

The normalized mode solution to Eq. (25) corresponding to the Minkowski vacuum in the high frequency limit is then given in terms of the Hankel function by

$$u_k = \frac{\sqrt{\pi}}{2}\sqrt{-y}H_q^{(1)}(-ky). \tag{27}$$

We thus write the operator $\mathcal{R}$ using the creation and annihilation modes as

$$\mathcal{R}(\mathbf{k}, y) = \psi(k, y)\hat{a}_{\mathbf{k}} + \psi^*(-k, y)\hat{a}^\dagger_{-\mathbf{k}}, \tag{28}$$

$$\psi(k, y) = \frac{u_k(y)}{\tilde{z}}, \tag{29}$$

with the commutation relation $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3 \times \delta^{(3)}(\mathbf{k} - \mathbf{k}')$. This immediately leads to the power spectrum [12],

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\left|\frac{u_k}{\tilde{z}}\right|^2 = 2^{2q-3}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2 \frac{(1-\epsilon-s)^2}{4\pi^2}\frac{H^2}{2\sigma c_s}\bigg|_{ky=-1}. \tag{30}$$

The scalar spectral index is found to be

$$n_s - 1 = 3 - 2q = -\frac{2\epsilon + s + \delta}{1-\epsilon-s}. \tag{31}$$

The above formula has been derived without assuming the smallness of ϵ, s, and δ, though we have assumed that they are constant. In this sense, the above expression is more general than that given in [12,16,27,28]. To ensure the scale invariance we require $2\epsilon + s + \delta \simeq 0$. However, this does not force *each* parameter to be as small as $\mathcal{O}(n_s - 1)$; each can be large, ϵ, s, $\delta \gg \mathcal{O}(n_s - 1)$, but the three may cancel each other out to produce an almost scale-invariant spectrum. This possibility was first pointed out by [31] in the less generic context of DBI inflation, for which $\sigma = \epsilon$ and consequently $\delta = 0$. We leave this interesting possibility open, and will complete the following calculation without taking the slow-roll limit. We would stress again that even if we consider the slow-roll limit, σ is not necessarily slow-roll suppressed.

Since the inflaton field is minimally coupled to gravity, the nature of tensor perturbations is the same as the standard one and is dependent only on the geometrical quantity $H = H(t)$. In the slow-roll limit, $\epsilon = 0$, the tensor power spectrum is given by $\mathcal{P}_h = 8(H/2\pi)^2$. The tensor-to-scalar ratio r is thus given by

$$r = 16\sigma c_s, \tag{32}$$

where just for simplicity the scalar power spectrum is evaluated also in the slow-roll limit, $\epsilon = s = \delta = 0$.

For later convenience we introduce the following quantity:

$$\nu := \frac{\dot{\phi}XG_X}{H}, \tag{33}$$

or, equivalently, $\Theta = H(1-\nu)$. From Eq. (21) we obtain

$$\sigma = \frac{\dot{\nu}}{H(1-\nu)^2} + \frac{\nu}{1-\nu} + \frac{\epsilon}{1-\nu}. \tag{34}$$

For $\epsilon =$ const, $s =$ const, and $\delta =$ const, the above equation can be integrated to yield

$$\frac{H}{\Theta} = \frac{1}{1-\nu(y)} = \frac{1}{1+\epsilon} + \frac{\sigma(y)}{1+\epsilon+\delta} + \left(\frac{1}{1-\nu_*} - \frac{1}{1+\epsilon} - \frac{\sigma_*}{1+\epsilon+\delta}\right)(y/y_*)^{(1+\epsilon)/(1-\epsilon-s)}. \tag{35}$$

If we assume $\nu =$ const then we have $\sigma =$ const. In this case the two quantities are related as

$$\nu = \frac{\sigma - \epsilon}{1+\sigma}. \tag{36}$$

Note in passing that the opposite is not in general true: for $\sigma =$ const Eq. (34) still admits time-dependent ν.

C. *G* inflation examples

1. Kinematically driven G inflation

Inflation can be driven by kinetic energy of ϕ. This possibility was explored in [12]. Let us consider for simplicity the Lagrangian with exact shift symmetry $\phi \to \phi + c$, i.e.,

$$K = K(X), \qquad G = G(X), \tag{37}$$

and look for an exact de Sitter background satisfying $H = \text{const}$ and $\dot{\phi} = \text{const}$. It follows from the field equations that

$$3H^2 = -K, \tag{38}$$

$$K_X + 3G_X H\dot{\phi} = 0. \tag{39}$$

For this background we have

$$\mathcal{F} = -\frac{K}{3X}\nu(1-\nu), \tag{40}$$

$$\mathcal{G} = -\frac{K}{X}\nu\left(1+\nu-2\frac{XK_{XX}}{K_X}+2\frac{XG_{XX}}{G_X}\right), \tag{41}$$

$$\sigma = \frac{\nu}{1-\nu}, \tag{42}$$

where $\nu = \dot{\phi}XG_X/H = XK_X/K = \text{const}$. In evaluating the above equations we used the background Eqs. (38) and (39).

The concrete toy model presented in [12] is given by

$$K = -X + \frac{X^2}{2M^3\mu}, \qquad G = \frac{X}{M^3}, \tag{43}$$

where M and μ are parameters. In this case, c_s and σ can be expressed in terms of μ. It turns out that the tensor-to-scalar ratio $r = 16\sigma c_s = 16\sigma c_s(\sigma)$ is an increasing function of σ, and $\sigma \simeq \nu \ll 1$ is required in order for r not to exceed the observationally allowed value. Explicitly, one finds $r \simeq (8/\sqrt{3})\sigma^{3/2} \simeq (16\sqrt{6}/3)(\sqrt{3}\mu)^{3/2}$ [12].

Note, however, that $\nu \ll 1$ is not necessary to get a stable, prolonged de Sitter phase. As already emphasized above, $\sigma \gtrsim 1$ is made possible by a suitable choice of $K(X)$ and $G(X)$, provided that $r = 16\sigma c_s$ remains not too large. In [27] Mizuno and Koyama have studied the case with $\sigma \simeq \nu \ll 1$ focusing their attention on the model (43). In contrast, the analysis in the present paper can apply to more general cases with $\sigma \gtrsim 1$.

In the presence of exact shift symmetry, the exact de Sitter solution is an attractor. Along this attractor the scalar fluctuations acquire an exactly scale-invariant spectrum. Making K and/or G weakly dependent on ϕ, one obtains a quasi-de Sitter attractor and thereby the spectrum can be tilted. Though we do not provide corresponding concrete examples here, more generic, possibly complicated, choices of $K(\phi, X)$ and $G(\phi, X)$ would lead to the interesting situation mentioned above: $n_s - 1 \ll 1$ with ϵ, s, $\delta \gg \mathcal{O}(n_s - 1)$.

2. Potential driven G inflation

In [16] a novel class of inflation models was proposed in which the energy density is dominated by ϕ's potential but its dynamics is nontrivial due to the $G\Box\phi$ term. In particular, it was shown that slow-roll inflation can proceed even if the potential is too steep to support standard slow-roll inflation. The model examined in [16] is described by

$$K = X - V(\phi), \qquad G = -g(\phi)X. \tag{44}$$

For $gV_\phi \gg 1$, the effect of the $G\Box\phi$ term dominates in the slow-roll equation of motion for ϕ, and the potential is effectively flattened, leading to slow-roll G inflation. In this regime one finds

$$\sigma \simeq \frac{4}{3}\epsilon \quad \text{and} \quad c_s^2 \simeq \frac{2}{3}. \tag{45}$$

Though σ could be free from the slow-roll constraint in principle, in the present case it is actually related to ϵ in a way different from standard slow-roll inflation. Since $c_s^2 \simeq \text{const}$, the scale-invariant spectrum requires that $\epsilon \ll 1$, and hence $\sigma \ll 1$.

III. BISPECTRUM

In order to evaluate the bispectrum, we compute the cubic action for $\mathcal{R}$ working in the Arnowitt-Deser-Misner (ADM) formalism [8–10],

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{46}$$

where

$$\begin{aligned} h_{ij} &= a^2(t)e^{2\mathcal{R}}\delta_{ij}, \\ N &= 1 + \alpha_1 + \alpha_2 + \cdots, \\ N_i &= a^2\partial_i(\beta_1 + \beta_2 + \cdots) + \tilde{N}_{1i} + \cdots, \end{aligned} \tag{47}$$

with $\partial^i\tilde{N}_{ni} = 0$. Here, α_n and β_n are $\mathcal{O}(\mathcal{R}^n)$. The fluctuation of the scalar field vanishes in this gauge. At linear order the above metric reduces to Eq. (12).

As pointed out in [8], we only need to consider first-order perturbations in N and N^i to get the cubic action. (This holds true even in the presence of the $G\Box\phi$ term.) Therefore, it suffices to use the first-order solution of the constraint equations, Eqs. (13) and (14), supplemented with a vanishing first-order vector perturbation, $\tilde{N}_{1i} = 0$.

We plug the solution for α_1 and β_1 into the action and expand it to third order in $\mathcal{R}$. After cumbersome multiple integrations by parts, one ends up with

$$\begin{aligned} S_3 = \int dt^3 x a^3 \bigg[& \frac{\mathcal{C}_1}{H}\dot{\mathcal{R}}^3 + \mathcal{C}_2\mathcal{R}\dot{\mathcal{R}}^2 + \frac{\mathcal{C}_3}{a^4H^2}\partial^2\mathcal{R}(\partial\mathcal{R})^2 \\ & + \frac{\mathcal{C}_4}{a^2H^2}\dot{\mathcal{R}}^2\partial^2\mathcal{R} + \mathcal{C}_5 H\mathcal{R}^2\dot{\mathcal{R}} + \frac{\mathcal{C}_6}{a^4H}\partial^2\mathcal{R}(\partial\mathcal{R}\cdot\partial\chi) \\ & + \frac{\mathcal{C}_7}{a^4}\partial^2\mathcal{R}(\partial\chi)^2 + \frac{\mathcal{C}_8}{a^2}\mathcal{R}(\partial\mathcal{R})^2 \\ & + \frac{\mathcal{C}_9}{a^2}\dot{\mathcal{R}}(\partial\mathcal{R}\cdot\partial\chi) + \frac{2}{a^3}f(\mathcal{R})\frac{\delta L}{\delta\mathcal{R}}\bigg|_1 \bigg], \end{aligned} \tag{48}$$

where $\chi := \partial^{-2}\Lambda$ with

$$\Lambda := \frac{a^2}{\Theta^2}X\mathcal{G}\dot{\mathcal{R}} = \frac{a^2\sigma}{c_s^2}\dot{\mathcal{R}}. \tag{49}$$

The dimensionless coefficients are given by

$$\mathcal{C}_1 = -\frac{H}{\Theta}\frac{\sigma}{c_s^2}\left(1+2\frac{\mathcal{I}}{\mathcal{G}}\right) - 2\dot{\phi}X(G_X + XG_{XX})\frac{H\sigma}{c_s^2\Theta^2} + \frac{H^2\sigma}{c_s^4\Theta^2}, \quad (50)$$

$$\mathcal{C}_2 = \frac{\sigma}{c_s^2}\left[3 - \frac{H^2}{c_s^2\Theta^2}\left(3+\epsilon+\frac{2\dot{\Theta}}{H\Theta}\right)\right], \quad (51)$$

$$\mathcal{C}_3 = -\frac{H^2\dot{\phi}XG_X}{\Theta^3}, \quad (52)$$

$$\mathcal{C}_4 = \frac{2H^2\dot{\phi}X(G_X + XG_{XX})}{\Theta^3}, \quad (53)$$

$$\mathcal{C}_5 = \frac{\sigma}{2c_s^2 H}\frac{d}{dt}\left(\frac{H^2\delta}{c_s^2\Theta^2}\right), \quad (54)$$

$$\mathcal{C}_6 = \frac{2H\dot{\phi}XG_X}{\Theta^2}, \quad (55)$$

$$\mathcal{C}_7 = \frac{\sigma}{4} - \frac{\dot{\phi}XG_X}{\Theta}, \quad (56)$$

$$\mathcal{C}_8 = -\sigma + \frac{H^2}{\Theta^2}\frac{\sigma}{c_s^2}\left(1-\epsilon-2s-\frac{2\dot{\Theta}}{H\Theta}\right), \quad (57)$$

$$\mathcal{C}_9 = \frac{\sigma}{c_s^2}\left(-\frac{2H}{\Theta}+\frac{\sigma}{2}\right), \quad (58)$$

where

$$\mathcal{I} := XK_{XX} + \frac{2X^2}{3}K_{XXX} + H\dot{\phi}G_X + 6X^2G_X^2 + 5H\dot{\phi}XG_{XX} + 6X^3G_XG_{XX} + 2H\dot{\phi}X^2G_{XXX} - \frac{2X}{3}(2G_{\phi X} + XG_{\phi XX}). \quad (59)$$

The last term is the field equation which follows from the quadratic action,

$$\frac{\delta L}{\delta\mathcal{R}}\bigg|_1 = a\left[\frac{d\Lambda}{dt} + H\Lambda - \sigma\partial^2\mathcal{R}\right], \quad (60)$$

multiplied by

$$f(\mathcal{R}) = \frac{H\dot{\sigma}}{4c_s^2\Theta^2\sigma}\mathcal{R}^2 + \frac{H}{c_s^2\Theta^2}\mathcal{R}\dot{\mathcal{R}} + \frac{1}{4a^2\Theta^2}[-(\partial\mathcal{R})^2 + \partial^{-2}\partial^i\partial^j(\partial_i\mathcal{R}\partial_j\mathcal{R})] + \frac{1}{2a^2\Theta}[\partial\chi\cdot\partial\mathcal{R} - \partial^{-2}\partial^i\partial^j(\partial_i\mathcal{R}\partial_j\chi)]. \quad (61)$$

In deriving the above cubic action we have not performed any slow-roll expansion, so that we have kept full generality up to here. Taking the limit $G\to 0$, $\Theta\to H$, and $\sigma\to\epsilon$, we can verify that the above equations reproduce the previous result derived for generic k-inflation models, $\mathcal{L}_\phi = K(\phi, X)$ [9,10]. In particular, the $\mathcal{C}_3$, $\mathcal{C}_4$, and $\mathcal{C}_6$ terms are absent in that case. The $\mathcal{C}_5$ term is clearly a higher order term so that we will neglect it in the following.

Employing the in-in formalism, the 3-point function can be computed from the following formula:

$$\langle\mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = -i\int_{t_0}^{t}dt'\langle[\mathcal{R}(\mathbf{k}_1,t)\mathcal{R}(\mathbf{k}_2,t)\mathcal{R}(\mathbf{k}_3,t), H_{\rm int}(t')]\rangle, \quad (62)$$

where t_0 is some early time when the fluctuation is well inside the horizon, t is a time several e-foldings after the horizon exit, and the interaction Hamiltonian is given by

$$H_{\rm int}(t) = -\int d^3x a^3\left[\frac{\mathcal{C}_1}{H}\dot{\mathcal{R}}^3 + \mathcal{C}_2\mathcal{R}\dot{\mathcal{R}}^2 + \cdots\right]. \quad (63)$$

We use Eqs. (27) and (28) to evaluate each contribution, which can be conventionally expressed as

$$\langle\mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^7\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}_\mathcal{R}^2\frac{\mathcal{A}}{k_1^3k_2^3k_3^3}, \quad (64)$$

$$\mathcal{A} = \sum_M \mathcal{A}_M. \quad (65)$$

The power spectrum $\mathcal{P}_\mathcal{R}$ here is to be calculated for the mode with $k_t = k_1 + k_2 + k_3$.

To proceed, we assume that $\nu = \text{const}$, which holds in a wide class of G-inflation models as described in Sec. II C. We then immediately see that $\sigma = \text{const}$, and $\mathcal{C}_3$, $\mathcal{C}_6$, and $\mathcal{C}_7$ are all constant in time as well. The coefficients are explicitly given by

$$\mathcal{C}_3 = -\frac{(1+\sigma)^2(\sigma-\epsilon)}{(1+\epsilon)^3}, \quad \mathcal{C}_6 = \frac{2(1+\sigma)(\sigma-\epsilon)}{(1+\epsilon)^2}, \quad \mathcal{C}_7 = \frac{4\epsilon-\sigma(3-\epsilon)}{4(1+\epsilon)}. \quad (66)$$

In order to evaluate the contributions from the $\dot{\mathcal{R}}^3$ ($\mathcal{C}_1$) and $\dot{\mathcal{R}}^2\partial^2\mathcal{R}$ ($\mathcal{C}_4$) terms, we further assume that $\mathcal{I}/\mathcal{G}$ and $\dot{\phi}X^2G_{XX}/H$ are of the form

$$\frac{\mathcal{I}}{\mathcal{G}} = \mathcal{J}_1 + \frac{\mathcal{J}_2}{c_s^2}, \quad (67)$$

$$\frac{\dot{\phi}X^2G_{XX}}{H} = \varrho_1 + \frac{\varrho_2}{c_s^2}, \quad (68)$$

where $\mathcal{J}_1$, $\mathcal{J}_2$, ϱ_1, and ϱ_2 are constants. In kinematically driven G inflation [12], we indeed have $\mathcal{I}/\mathcal{G} = \text{const}$ and $\dot{\phi}X^2G_{XX}/H = \text{const}$ in the de Sitter limit. In potential driven G inflation [16] $\mathcal{I}/\mathcal{G} \simeq \mathrm{const}$ and $G_{XX} = 0$. Therefore, the assumptions made here are sufficiently general and reasonable. It then follows that $\mathcal{C}_1$ and $\mathcal{C}_4$ take the form

$$\mathcal{C}_1 = \frac{\mathcal{D}_1}{c_s^2} + \frac{\mathcal{E}_1}{c_s^4}, \tag{69}$$

$$\mathcal{C}_4 = \mathcal{D}_4 + \frac{\mathcal{E}_4}{c_s^2}, \tag{70}$$

where $\mathcal{D}_1$, $\mathcal{E}_1$, $\mathcal{D}_4$, and $\mathcal{E}_4$ are constant and are given by

$$\mathcal{D}_1 = -\frac{\sigma(1+\sigma)}{1+\epsilon}\left[1 + 2\mathcal{J}_1 + 2\frac{\sigma - \epsilon + (1+\sigma)\varrho_1}{1+\epsilon}\right], \tag{71}$$

$$\mathcal{E}_1 = -\frac{\sigma(1+\sigma)}{1+\epsilon}\left[2\mathcal{J}_2 - \frac{1+\sigma}{1+\epsilon}(1-2\varrho_2)\right], \tag{72}$$

$$\mathcal{D}_4 = 2\frac{(1+\sigma)^3}{(1+\epsilon)^3}\left[\frac{\sigma-\epsilon}{1+\sigma} + \varrho_1\right], \tag{73}$$

$$\mathcal{E}_4 = 2\frac{(1+\sigma)^3}{(1+\epsilon)^3}\varrho_2. \tag{74}$$

Each contribution can now be evaluated as

$$\mathcal{A}_1 = \frac{3}{2\sigma}(1-\epsilon-s)\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} \times\left[\mathcal{D}_1 I_1(n_s-1) + \frac{\mathcal{E}_1}{c_{s*}^2}I_1(q')\right], \tag{75}$$

$$\mathcal{A}_2 = \frac{1}{4}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} \times\left[3I_2(n_s-1) - \frac{3-\epsilon}{c_{s*}^2}\left(\frac{1+\sigma}{1+\epsilon}\right)^2 I_2(q')\right], \tag{76}$$

$$\mathcal{A}_3 = \frac{1}{2}\frac{\mathcal{C}_3}{\sigma c_{s*}^2}(1-\epsilon-s)^2\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_3(q'), \tag{77}$$

$$\mathcal{A}_4 = \frac{3}{\sigma}(1-\epsilon-s)^2\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} \times\left[\mathcal{D}_4 I_4(n_s-1) + \frac{\mathcal{E}_4}{c_{s*}^2}I_4(q')\right], \tag{78}$$

$$\mathcal{A}_6 = \frac{\mathcal{C}_6}{8c_{s*}^2}(1-\epsilon-s)\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_6(q'), \tag{79}$$

$$\mathcal{A}_7 = \frac{\mathcal{C}_7}{4}\frac{\sigma}{c_{s*}^2}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_7(q'), \tag{80}$$

$$\mathcal{A}_8 = \frac{1}{8}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} \times\left[-I_8(n_s-1) + \frac{1+\epsilon-2s}{c_{s*}^2}\left(\frac{1+\sigma}{1+\epsilon}\right)^2 I_8(q')\right], \tag{81}$$

$$\mathcal{A}_9 = \frac{\mathcal{C}_{9*}}{8}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_9(q'), \tag{82}$$

where c_{s*} and $\mathcal{C}_{9*}$ are evaluated at sound horizon crossing, $k_t y = -1$, and

$$q' := \frac{s-2\epsilon}{1-\epsilon-s}. \tag{83}$$

The k-dependent functions I_M are given by

$$I_1(z) := \frac{k_1^2k_2^2k_3^2}{k_t^3}\cos\left(\frac{\pi z}{2}\right)\frac{\Gamma(3+z)}{2}, \tag{84}$$

$$I_2(z) := \cos\left(\frac{\pi z}{2}\right)\left[\frac{2+z}{k_t}\sum_{i>j}k_i^2k_j^2 - \frac{1+z}{k_t^2}\sum_{i\neq j}k_i^2k_j^3\right]\Gamma(1+z), \tag{85}$$

$$I_3(z) := \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\frac{2+z}{2}\left\{\Gamma(1+z) + \Gamma(2+z) \times\left[\frac{k_1k_2+k_2k_3+k_3k_1}{k_t^2} + (3+z)\frac{k_1k_2k_3}{k_t^3}\right]\right\} + \mathrm{sym.}, \tag{86}$$

$$I_4(z) := \frac{k_1^2k_2^2k_3^2}{k_t^3}\cos\left(\frac{\pi z}{2}\right)\frac{(6+z)\Gamma(3+z)}{12}, \tag{87}$$

$$I_6(z) := \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[(3+z)\Gamma(1+z) + (3+z)\Gamma(2+z)\frac{k_3}{k_t} - \Gamma(3+z)\frac{k_3^2}{k_t^2}\right] + \mathrm{sym.}, \tag{88}$$

$$I_7(z) := \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[\Gamma(1+z) + \Gamma(2+z)\frac{k_3}{k_t}\right] + \mathrm{sym.}, \tag{89}$$

$$I_8(z) := \cos\left(\frac{\pi z}{2}\right)\left(\sum_i k_i^2\right)\left[\frac{k_t}{1-z} - \frac{1}{k_t}\sum_{i>j}k_ik_j - \frac{1+z}{k_t^2}k_1k_2k_3\right]\Gamma(1+z), \tag{90}$$

$$I_9(z) := \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[(3+z)\Gamma(1+z) - \Gamma(2+z)\frac{k_3}{k_t}\right] + \mathrm{sym.} \tag{91}$$

In computing the above we have used the approximation

$$\psi(k,y) \simeq \sqrt{2}\pi \mathcal{P}_{\mathcal{R}}^{1/2}(k_t) k_t^{q-3/2} k^{-q}(1+iky)e^{-iky}, \quad |ky| \ll 1. \tag{92}$$

One can check that by setting $\sigma \to \epsilon$ the above equations reproduce the result of [32]. The field redefinition $\mathcal{R} \to \mathcal{R}_n + f(\mathcal{R}_n)$ gives rise to the non-Gaussian amplitude proportional to δ, which can be ignored in the present approximation.

The shapes of $\mathcal{A}_3(1,k_2,k_3)/k_2k_3$ and $\mathcal{A}_6(1,k_2,k_3)/k_2k_3$ are very similar to the equilateral one, as was already pointed out by [27]. The contributions from $\dot{\mathcal{R}}^3$ ($\mathcal{C}_1$) and $\dot{\mathcal{R}}^2\partial^2\mathcal{R}$ ($\mathcal{C}_4$) give the same momentum dependence because the two terms are essentially equivalent after using the first-order equation of motion.

The size of the three-point correlation function is conventionally parameterized by $f_{\rm NL}$ defined as

$$f_{\rm NL} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{k_t^3}, \tag{93}$$

which can be computed straightforwardly by evaluating the amplitude $\mathcal{A}$ at $k_1 = k_2 = k_3 = k_t/3$. The exact expression for $f_{\rm NL}$ is given by

$$f_{\rm NL} = 30\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{1}{54}\right)^{n_s-1}\left\{\frac{3(1-\epsilon-s)}{2\sigma}\left[\mathcal{D}_1 I_1^{\rm equi}(n_s-1) + \frac{\mathcal{E}_1}{c_{s*}^2}I_1^{\rm equi}(q')\right] + \frac{3(1-\epsilon-s)^2}{\sigma}\left[\mathcal{D}_4\frac{n_s+5}{6}I_1^{\rm equi}(n_s-1)\right.\right.$$
$$\left. + \frac{\mathcal{E}_4}{c_{s*}^2}\frac{6+q'}{6}I_1^{\rm equi}(q')\right] + \frac{3}{4}I_2^{\rm equi}(n_s-1) + \frac{1}{4c_{s*}^2}\left[\frac{3\sigma^2}{8} - \frac{\sigma-\epsilon}{2(1+\epsilon)} - \frac{1+\sigma}{1+\epsilon}\left(\sigma + (3-\epsilon)\frac{1+\sigma}{1+\epsilon}\right)\right]I_2^{\rm equi}(q')$$
$$\left. - \frac{(1-\epsilon-s)^2}{2\sigma c_{s*}^2}\frac{(1+\sigma)^2(\sigma-\epsilon)}{(1+\epsilon)^3}I_3^{\rm equi}(q') + \frac{3}{8(n_s-2)}I_6^{\rm equi}(n_s-1) + \frac{1-\epsilon-s}{8c_{s*}^2}\frac{1+\sigma}{1+\epsilon}\left(3\frac{1+\sigma}{1+\epsilon} + 2\frac{\sigma-\epsilon}{1+\epsilon}\right)I_6^{\rm equi}(q')\right\}, \tag{94}$$

where we have defined $I_M^{\rm equi}(z) := k_t^{-3}I_M(z)|_{k_1=k_2=k_3}$, i.e.,

$$I_1^{\rm equi}(z) := \cos\left(\frac{\pi z}{2}\right)\frac{\Gamma(3+z)}{1458}, \tag{95}$$

$$I_2^{\rm equi}(z) := \cos\left(\frac{\pi z}{2}\right)\frac{(4+z)\Gamma(1+z)}{81}, \tag{96}$$

$$I_3^{\rm equi}(z) := \cos\left(\frac{\pi z}{2}\right)\frac{(2+z)(39+13z+z^2)\Gamma(1+z)}{2916}, \tag{97}$$

$$I_6^{\rm equi}(z) := \cos\left(\frac{\pi z}{2}\right)\frac{(17+9z+z^2)\Gamma(1+z)}{243}, \tag{98}$$

and used the fact that $I_4^{\rm equi}(z) = (1+z/6)I_1^{\rm equi}(z)$, $I_7^{\rm equi}(z) = I_2^{\rm equi}(z)/2$, $I_8^{\rm equi}(z) = 3I_6^{\rm equi}(z)/(1-z)$, and $I_9^{\rm equi}(z) = I_2^{\rm equi}(z)$ to shorten the expression. The above generic formula is involved, but an order of estimate of $f_{\rm NL}$ is found to be

$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_s^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2\frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right),$$
$$\tilde{\sigma} := \max\{1, \sigma\}. \tag{99}$$

This is one of the main results of the present paper.

The sound speed at horizon crossing can be written in terms of k_t as $c_{s*}^2 \propto k_t^{s/(1-\epsilon-s)}$. Under our assumptions we see that $f_{\rm NL}$ can be expressed as $f_{\rm NL} = f_1 + f_2/c_{s*}^2$, where f_1 and f_2 depend on ϵ, s, σ, $\mathcal{D}_{1,4}$, and $\mathcal{E}_{1,4}$, but are independent of k_t. Therefore, the wave number dependence of $f_{\rm NL}$ appears only through c_{s*}^2, so that the tilt $n_{\rm NG}$ is given by

$$n_{\rm NG} - 1 = -\frac{f_2 c_{s*}^{-2}}{f_1 + f_2 c_{s*}^{-2}}\frac{2s}{1-\epsilon-s}. \tag{100}$$

If ϵ, $s \ll 1$ and the main contribution to $f_{\rm NL}$ is due to a small sound speed, then we recover the result of [10], $n_{\rm NG} - 1 \simeq -2s$, even in the presence of the $G\Box\phi$ term.

We close this section by illustrating several examples of non-Gaussian shapes $\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1} \times (k_3/k_1)^{-1}$ in G inflation. First, let us consider the de Sitter limit of kinematically driven G inflation. The functions K and G may be written as

$$K = -X + c_1X^2 + c_2X^3 + \cdots,$$
$$G = \frac{X}{M^3} + d_2X^2 + d_3X^3 + \cdots, \tag{101}$$

where c_i, d_i, and M are arbitrary in principle. Given $\nu = \dot{\phi}XG_X/H$ and $\varrho = \dot{\phi}X^2G_{XX}/H$, the former is related to XK_X/K through the background equations, which in turn fixes the value of σ. The latter is related to c_s^2, but since the expression for c_s^2 contains both of the second derivatives K_{XX} and G_{XX}, c_s^2 can be chosen independently of ϱ. Third derivatives K_{XXX} and G_{XXX} appear only in the function $\mathcal{I}$. In summary, in the case of kinematically driven G inflation, the non-Gaussian amplitude in the de Sitter limit is completely determined by the four parameters

$$\sigma, \quad c_s, \quad \varrho, \quad \frac{\mathcal{I}}{\mathcal{G}}. \tag{102}$$

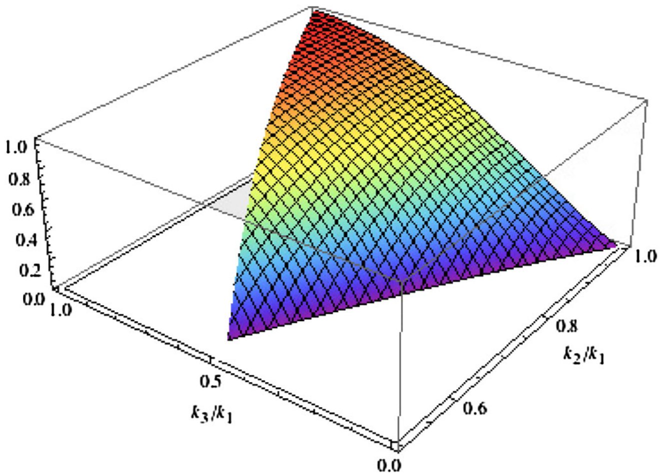

FIG. 1 (color online). The non-Gaussian amplitude $\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$ as a function of k_2/k_1 and k_3/k_1 for kinematicallly driven *G* inflation. The amplitude is normalized to unity at an equilateral configuration, $k_2/k_1 = k_3/k_1 = 1$. The parameters are given by $\sigma = 0.36$, $c_s = 0.03$, $\varrho = 1$, and $\mathcal{I}/\mathcal{G} = 1$, so that $r \simeq 0.17$. The size of non-Gaussianity is $f_{\rm NL} \simeq 210$.

The four parameters can be written in terms of c_i, d_i, and M, but in practice the expressions are quite involved. We plot in Figs. 1–3 the shapes of non-Gaussianity for different parameters.

Another example of the non-Gaussian shapes we explicitly compute is given by potential driven *G* inflation [16] in the slow-roll approximation. In this case we have $\sigma \simeq 4\epsilon/3$, $c_s^2 \simeq 2/3$, $\Theta \simeq H(1 - \epsilon/3)$, and

$$\mathcal{I} = -gH\dot{\phi} + 6g^2X^2 + \frac{4}{3}Xg_\phi \simeq \frac{1}{6}\mathcal{G}. \qquad (103)$$

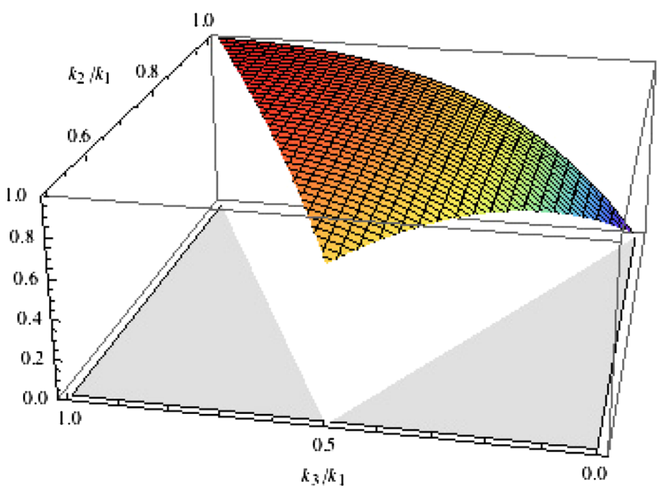

FIG. 2 (color online). The non-Gaussian amplitude $\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$ as a function of k_2/k_1 and k_3/k_1 for kinematicallly driven *G* inflation. The amplitude is normalized to unity at an equilateral configuration, $k_2/k_1 = k_3/k_1 = 1$. The parameters are given by $\sigma = 0.1$, $c_s = 0.1$, $\varrho = 60$, and $\mathcal{I}/\mathcal{G} = 1$. The size of non-Gaussianity is $f_{\rm NL} \simeq 204$.

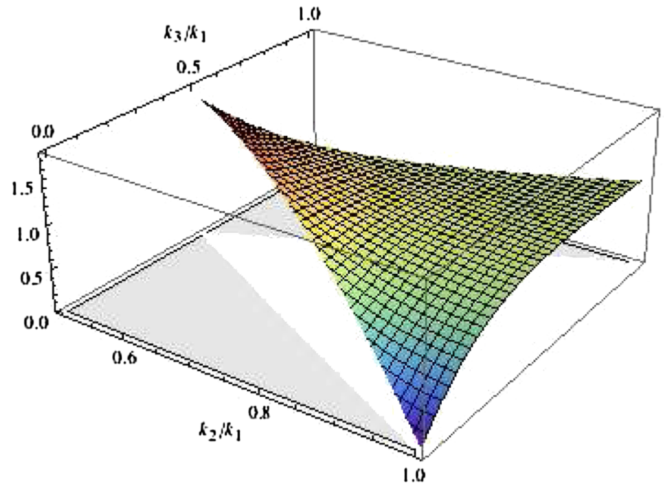

FIG. 3 (color online). The non-Gaussian amplitude $\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$ as a function of k_2/k_1 and k_3/k_1 for kinematicallly driven *G* inflation. The amplitude is normalized to unity at an equilateral configuration, $k_2/k_1 = k_3/k_1 = 1$. The parameters are given by $\sigma = 0.1$, $c_s = 0.1$, $\varrho = 1$, and $\mathcal{I}/\mathcal{G} = 300$. In this case the shape peaks in the folded configuration $k_1 = 2k_2 = 2k_3$.

The contributions relevant at leading order in slow roll are

$$\mathcal{A}_1 \simeq \frac{1}{4}I_1(0), \qquad \mathcal{A}_2 \simeq -\frac{3}{8}I_2(0), \qquad \mathcal{A}_3 \simeq -\frac{3}{16}I_3(0),$$
$$\mathcal{A}_4 \simeq \frac{3}{2}I_4(0), \qquad \mathcal{A}_8 \simeq \frac{1}{16}I_8(0). \qquad (104)$$

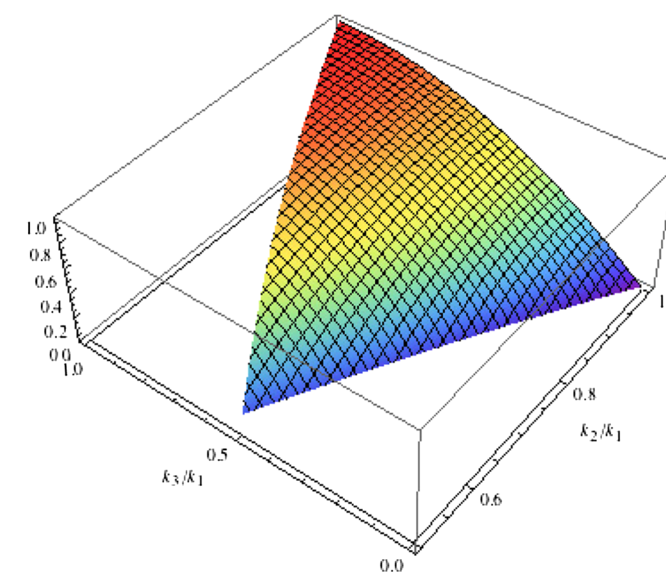

FIG. 4 (color online). The non-Gaussian amplitude $\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$ as a function of k_2/k_1 and k_3/k_1 for potential driven *G* inflation. The amplitude is normalized to unity at an equilateral configuration, $k_2/k_1 = k_3/k_1 = 1$. The size of non-Gaussianity is $f_{\rm NL} = 235/3888 \simeq 0.06$.

More explicitly, the non-Gaussian amplitude is given by

$$\mathcal{A} = \frac{7}{4}\frac{k_1^2k_2^2k_3^2}{k_t^3} - \frac{3}{8}\left(\frac{2}{k_t}\sum_{i>j}k_i^2k_j^2 - \frac{1}{k_t^2}\sum_{i\neq j}k_i^2k_j^3\right) - \frac{3}{16}\left(\frac{1}{2k_t}\sum_i k_i^4 - \frac{1}{k_t}\sum_{i>j}k_i^2k_j^2\right) \times\left(1 + \frac{1}{k_t^2}\sum_{i>j}k_ik_j + 3\frac{k_1k_2k_3}{k_t^3}\right) + \frac{1}{16}k_t\left(\sum_i k_i^2\right)\left(1 - \frac{1}{k_t^2}\sum_{i>j}k_ik_j - \frac{k_1k_2k_3}{k_t^3}\right), \tag{105}$$

which is plotted in Fig. 4. Taking the equilateral limit, the size of non-Gaussianity is found to be $f_{\rm NL} = 235/3888 \simeq 0.06$. The above result is insensitive to the inflaton potential.

IV. CONCLUSION

In this paper, we have studied G inflation, i.e., generic single-field inflation obtained from the Lagrangian (2). We have revisited the power spectrum and the spectral index to clarify how the (approximate) scale invariance can be achieved in this class of inflation models and determined the possible non-Gaussian amplitude without assuming slow roll and the exact scale invariance. The nonlinearity parameter $f_{\rm NL}$ in G inflation can be summarized schematically as

$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{2}\right) + \mathcal{O}\left(\tilde{\sigma}^2\frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right), \qquad \tilde{\sigma} := \max\{1, \sigma\}. \tag{106}$$

It should be emphasized that we have in principle no dynamical constraints that require σ to be very small. If the first term dominates and $\sigma \gtrsim 1$, then we have $f_{\rm NL} \sim \sigma^2/c_s^2$. Therefore, large $f_{\rm NL}$ and large $r = 16\sigma c_s$ are compatible. The situation should be contrasted with the models without the $G\Box\phi$ term, for which $\sigma = \epsilon$ follows, and hence large $f_{\rm NL}(\sim 1/c_s^2)$ implies small r.

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