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On the Vortex Excited Oscillation of a Square Cylinder in Smooth Flow

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Abstract

The vortex excited oscillation of a canti-levered square cylinder in smooth flow was studied experimentally. The flow velocity-oscillation amplitude relations were obtained for cylinders with 3 combinations of mass and natural frequency and with 3 kinds of damping. From these results, the critical flow velocity and the lift force-oscillation amplitude relation at the critical flow velocity were obtained. The critical flow velocity was higher than that of the two dimensional cylinder and the lift force was much reduced compared with that of the two dimensional cylinder.

1. Introduction

A cylinder placed in a smooth flow oscillates in the lateral direction to the flow at and around the critical flow velocity due to the dynamic lift force associated with the vortex shedding from the cylinder. Thus, to design cylinder-like structures such as stacks and slender tall buildings, knowledge of the critical flow velocity and the characteristics of the dynamic lift force are needed. The critical flow velocity for a circular cylinder was studied by Scruton (1965)[1] and those for rectangular cylinders were studied by Parkinson (1971).[2] However, the characteristics of the dynamic lift force, which depends on the flow velocity and oscillational behavior due to the interaction between the flow and the oscillating cylinder, is not known well.

For a lightly damped cylinder, the oscillation is nearly simple harmonic with the natural frequency of the cylinder at and around the critical flow velocity.

$$x = X_o \sin 2\pi f_o t \tag{1}$$

where x is the lateral displacement and X_o is the oscillation amplitude. Scruton assumed the dynamic lift force $F(t)$ as the combination of in-phase and out-of-phase component of the displacement.

$$F(t) = h_a(X_o, V)\rho D^2 f_o x + k_a(X_o, V)\rho D^2 f_o \dot{x} \tag{2}$$

where V, ρ and D are the flow velocity, fluid density and the cylinder width, respectively. Scruton found $k_a(X_o, V)$ to be negligible and studied $h_a(X_o, V)$ for a circular cylinder. Bishop & Hassan (1964)[3] assumed F(t) as follows

$$F(t) = C_L(X_o, V)\frac{1}{2}\rho V^2 D \sin(2\pi f_o t + \phi) \tag{3}$$

where ϕ is the phase lag between x and $F(t)$. They studied $C_L(X_o, V)$ and ϕ for a circular cylinder by forced oscillation technique. In the present work, we assumed $F(t)$ in general as is given in eq (3). However, as we confined our study only with the resonant oscillation of a cylinder at and around the critical flow velocity, ϕ in eq. (3) turns out to be $\pi/2$.

$$F(t)=C_L(X_o, V)\cdot\frac{1}{2}\cdot\rho V^2 D\sin(2\pi f_o t+\pi/2) \tag{4}$$

Slender tall buildings commonly have a square cross section, but there is little work on the dynamic lift force on the square cylinder. Therefore, we studied $C_L(X_o, V)$ for a square cylinder by free oscillation technique.

2. Experimental Equipment and Procedure

Fig. 1 shows the model on a dynamic balance. The vertical rod which supports

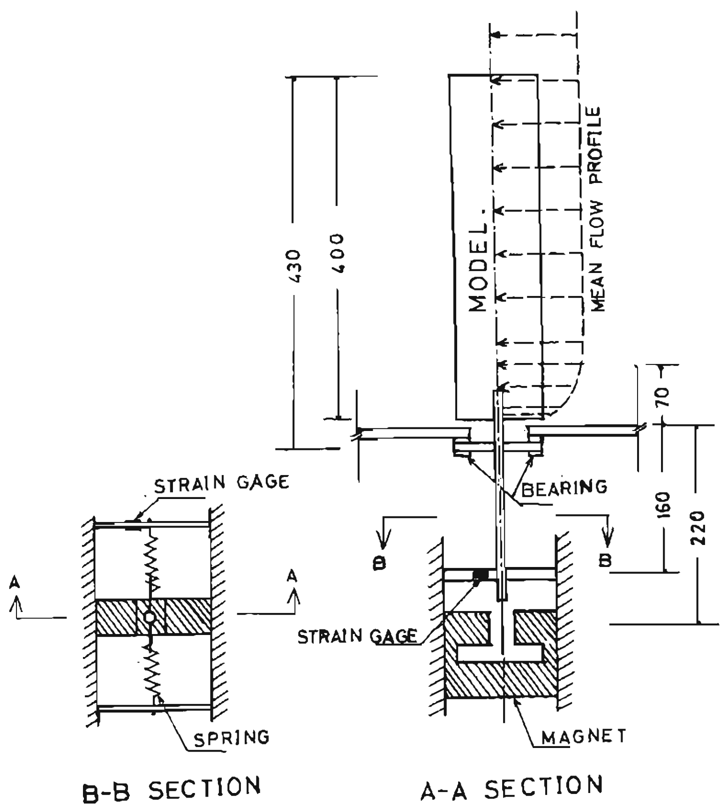

Fig. 1 Dynamic Balance and Model

the model is connected near the top to a horizontal rod and this horizontal rod is held by two bearings in such a way that the model can oscillate just in the lateral direction to the flow without any friction. The vertical rod at the bottom is fixed to coil springs and these to beams. Strain gages are attached on one of these beams for displacement measurement. The beams were so rigid as not to cause any higher mode oscillation. An electric magnet below the vertical rod provided necessary damping for the system. The model was a square cylinder of 10 cm width and 40 cm height and was made of plexiglass.

Taking the rotation of the model at the bearings θ as the variable, the equilibrium of moment is

$$M\frac{d^2\theta}{dt^2}+C\frac{d\theta}{dt}+K\theta=\int_0^H F(z,t)z\cdot dz \tag{5}$$

where $M=\int m(z)\cdot z^2\cdot dz$ in which $m(z)$ is the mass distribution from the rod bottom to the model top; C, K and H are damping coefficient, spring constant and the model height, respectively. Choosing the model top displacement as the variable, the equilibrium equation becomes

$$M_o\left(\frac{d^2x}{dt^2}+2\zeta\cdot2\pi f_o\cdot x+4\pi^2 f_o^2\cdot x\right)=\frac{1}{H}\int_0^H F(z,t)z\cdot dz \tag{6}$$

where $2\pi f_o=\sqrt{K_o/M_o}$ and $\zeta=C_o/(2\sqrt{K_oM_o})$, in which $M_o=M/H^2$, $C_o=C/H^2$ and $K_o=K/H^2$.

The mean velocity profile of the flow is also shown in fig. 1. The boundary layer thickness of the flow was about 7cm and the turbulence in the flow above the boundary layer was negligible.

Parkinson reports that a circular cylinder oscillation can have two different values of the oscillation amplitude at and around the critical velocity. To check this aspect, for a cylinder at a flow velocity, two initial displacement of $X_o/D=0.0$ and 0.2, which was the maximum displacement measurable, were given and the resulting stationary amplitude of simple harmonic oscillation was obtained.

3. Experimental Results and Analysis

3-1 Flow velocity-response amplitude relation

For square cylinders with 3 kinds of combinations of M_0 and f_0 and with various critical damping ratio ζ, the flow velocity-response amplitude relation were obtained and are shown in Fig. 2. As is seen from the figure, at a flow velocity a little lower than the critical one, the oscillation amplitude can take either one of the two values depending upon the initial displacement. In a velocity range higher than the critical one, the same phenomenon occurs though the two amplitudes are less easy to distinguish.

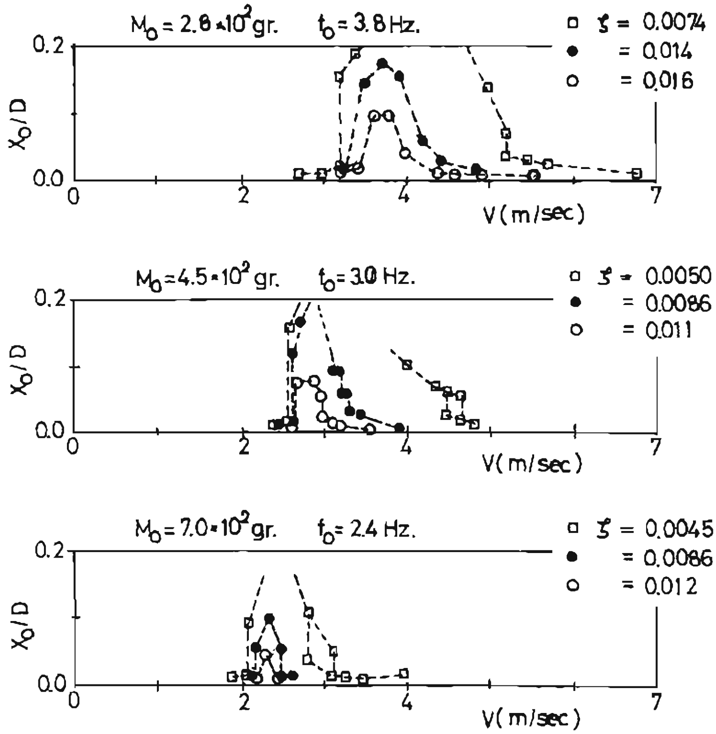

Fig. 2 Flow Velocity-Response Amplitude Relation

3-2 Critical flow velocity

The reduced critical flow velocity V_c/f_oD was 9.8 for all the cylinders tested. Scruton says V_c/f_oD to be 6.6 for two dimensional square cylinder. The higher value of V_c/f_oD for the present cylinder is due to the three dimensional condition as was pointed out by Vickery (1968).[4] Bishop & Hassan found that the critical flow velocity depends on the oscillation amplitude for a circular cylinder. However, for the present square cylinder, the critical flow velocity does not show any marked dependency on the oscillation amplitude.

3-3 Analysis of the oscillation

The equation of the motion for a canti-levered cylinder is given in eq. (6) and is rewritten here.

$$M_o\left(\frac{d^2x}{dt^2}+2\zeta\cdot 2\pi f_o x+4\pi^2 f_o^2 x\right)=\frac{1}{H}\int_0^H F(z,t)z\cdot dz \tag{7}$$

For lightly damped cylinders tested, the oscillations were nearly simple harmonic motion with the natural frequency of the cylinder at and around the critical flow velocity

$$x = X_o \sin 2\pi f_o t \tag{8}$$

The oscillation of eq. (8) means that the lift force $F(t)$ is also simple harmonic and that the phase between the displacement and force is $\pi/2$. Thus, $F(t)$ was expressed here as follows

$$F(z,\ t)=C_L\cdot\frac{1}{2}\rho V^2 D\sin\left(2\pi f_o t+\pi/2\right) \tag{9}$$

The coefficient C_L depends upon the flow characteristics, cylinder shape and cylinder oscillation.

$$C_L=C_L(V,\ \rho,\ \mu,\ D,\ H,\ \psi,\ X_o,\ f_o) \tag{10}$$

where μ is the flow viscosity and ψ is the cross sectional shape of the cylinder ; the other symbols are defined elsewhere. Arranging the parameters in dimensionless form

$$C_L=C_L\left(\frac{\rho VD}{\mu},\ \frac{V}{f_oD},\ \frac{H}{D},\ \frac{X_o}{D},\ \psi\right) \tag{11}$$

In the present study, the sectional shape is square and the Reynolds number influence will be negligible. H/D was 4. Thus, for this particular cylinder

$$F(z,\ t)=C_L\left(\frac{X_o}{D},\ \frac{V}{f_oD}\right)\frac{1}{2}\rho V^2 D\sin\left(2\pi f_o t+\pi/2\right) \tag{12}$$

Putting eqs. (12) and (8) into eq. (7), we obtain

$$\frac{X_o}{D}=\frac{1}{16\pi^2}C_L\left(\frac{X_o}{D},\ \frac{V}{f_oD}\right)\frac{\rho D^2H}{2\zeta M_o}\left(\frac{V}{f_oD}\right)^2 \tag{13}$$

The equation of motion for two dimensional cylinder is

$$M_o\left(\frac{d^2x}{dt^2}+2\zeta\cdot 2\pi f_o\frac{dx}{dt}+4\pi^2 f_o^2 x\right)=\int_0^H F(z,\ t)\cdot dz \tag{14}$$

where $M_o=\int m(z)dz\cdot$ The relation between the flow velocity, mechanical property of the cylinder and the oscillation amplitude is

$$\frac{X_o}{D}=\frac{1}{16\pi^2}C_L\left(\frac{X_o}{D},\ \frac{V}{f_oD}\right)\frac{\rho D^2H}{\zeta M_o}\left(\frac{V}{f_oD}\right)^2 \tag{15}$$

The difference in the mechanical condition between the cantilevered cylinder and two dimensional cylinder appears in 2 in front of ζM_o in eq. (13) instead of 1 in eq. (15).

From eq. (13), we can see that $2\zeta M_o/\rho D^2H$ value which gives a certain flow amplitude at a flow velocity is uniquely determined. Fig. 3 shows $2\zeta M_o/\rho D^2H$ values which give $X_o/D=0.05$ at various flow velocities for all the models tested. The data fall onto a curve confirming the above discussions. Taking $X_o/D=0.05$ as the permissible maximum response, we can refer to the figure as a stability diagram. The

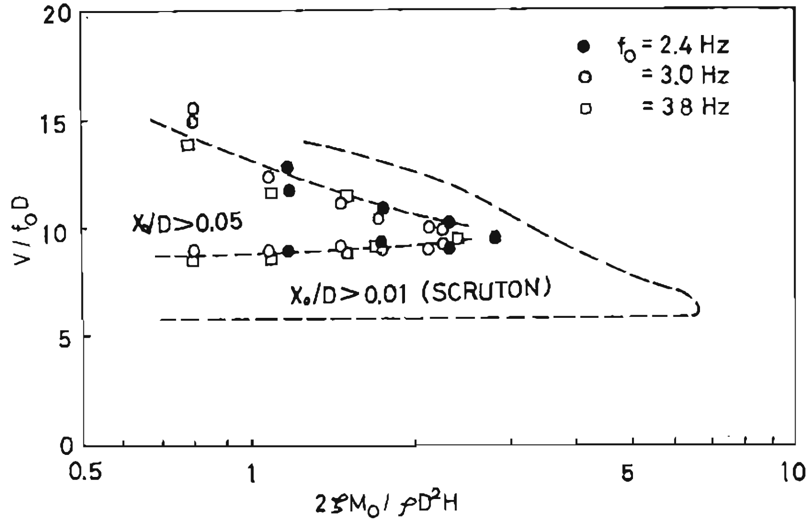

Fig. 3 Stability Diagram. The abscissa is $\zeta M/\rho D^2H$ for Scruton's data.

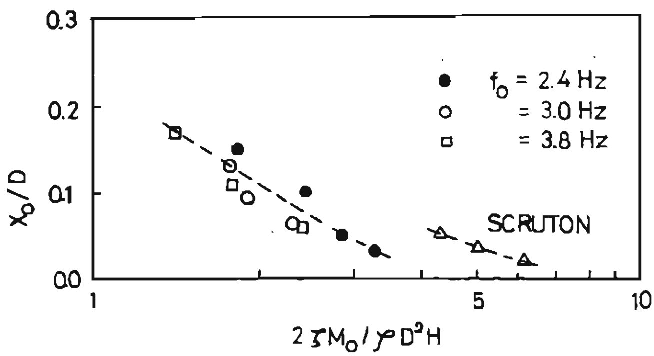

Fig. 4 Damping-Maximum Amplitude Relation. The abscissa is $\zeta M/\rho D^2H$ for Scruton's data.

stability diagram for a two dimensional square cylinder obtained by Scruton is also shown in the figure, in which $X_0/D=0.01$ is the permissible maximum response and the abscissa is $\zeta M_0/\rho D^2H$. The figure shows that the critical flow velocity for two dimensional cylinder is lower than that of the canti-levered cylinder. Apart from that, the difference of the oscillation between the two conditions is not clear.

Eq. (13) also shows that, with V/f_0D fixed, there is a relation between X_0/D and $2\zeta M_0/\rho D^2H$, though the relation may not be one-to-one. Fig. 4 shows this relation at $V/f_0D=9.8$. As there is only one oscillation amplitude at the critical velocity, the date falls into one curve. As the critical flow velocity was independent of the oscillation amlitude and was alway $V_c/f_0D=9.8$, the figure presents $2\zeta M_0/\rho D^2H$-maximum amplitude relation. Scruton's data at $V_c/f_0D=6.6$, which is the critical flow velocity for two dimensional cylinder, is also shown in the figure in which the absissa is

$\zeta M_o/\rho D^2H$. The figure shows that the maximum oscillation for a cantilevered cylinder is much reduced compared with that of the two dimensional cylinder.

From eq. (13), the lift force coefficient can be obtained from the oscillation amplitude as follows

$$C_L\left(\frac{X_o}{D}, \frac{V}{f_oD}\right)=16\pi^2\frac{X_o}{D}\frac{2\zeta M_o}{\rho D^2H}\left(\frac{f_oD}{V}\right)^2 \tag{16}$$

For two dimensional cylinder. the relation becomes

$$C_L\left(\frac{X_o}{D}, \frac{V}{f_oD}\right)=16\pi^2\frac{X_o}{D}\frac{\zeta M_o}{\rho D^2H}\left(\frac{f_oD}{V}\right)^2 \tag{17}$$

At $V_c/f_oD=9.8$, C_L was obtained from eq. (16) for the present cylinder and is shown in fig. 5. The data falls onto a straight line and C_L can be expressed as follows

$$C_L(X_o/D)=0.1+2.0\cdot X_o/D \tag{18}$$

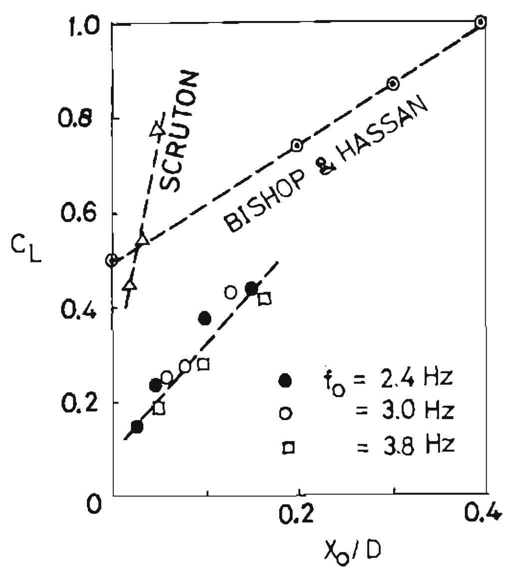

Fig. 5 Amplitude-Lateral Force Relation at the Critical Flow Velocity.

Also shown in the figure is C_L for a two dimensional square cylinder obtained from Scruton's data using eq. (17), and C_L for a two dimensional circular cylinder at the critical velocity taken from Bishop & Hassan. The figure shows that the lift force for a canti-levered cylinder is much reduced compared with that of a two dimensional cylinder.

The results suggests that C_L at the critical flow velocity is a linear function of X_o/D.

$$C_L(X_o/D)=\alpha+\beta\cdot X_o/D \tag{19}$$

Putting eq. (19) into eq. (12) and making use of eq. (8)

$$F(t)=\alpha\cdot\frac{1}{2}\rho V^2D\sin(2\pi f_0t+\pi/2)+\frac{\beta}{2\pi S^2}\cdot\frac{1}{2}\rho f_0D^2\dot{x} \tag{20}$$

where $S=f_0D/V$. As is seen from eq. (3–14), the linear dependency of C_L on X_0/D can be considered to be the source of aerodynamic negative damping.

4. Application

For the prediction of the lateral oscillation of buildings and stacks in the wind, the effect of turbulence present in the wind should be taken into account. However, the effect of the turbulence on the lateral oscillation of cylinder has not been studied much so far and is not known well. Surry (1971)[5] says that the presence of turbulence does not much reduce the intensity of the dynamic lift force due to the vortex shedding and Scruton & Rogers (1971)[6] say that the response oscillation in a turbulent flow is far from the simple harmonic oscillation associated with vortex shedding in smooth flow. Here we will just show the way the lateral response of a tall building in wind can be predicted from the exprimental results above, if the turbulence present in wind does not change the dynamic lift force characteristics found in the smooth flow.

We consider a building response $y(z, t)$ in the first mode $\mu(z)$ which is approximately a linear function of z,

$$y(z,\ t)=x(t)\mu(z)=X(t)\cdot z/H \tag{21}$$

For this generalized coordinate $x(t)$, the equation of motion takes the following form

$$M_0(\ddot{x}+2\zeta\cdot 2\pi f_0\dot{x}+4\pi^2f_0^2x)=\frac{1}{H}\int_0^H F(z,\ t)zdz \tag{22}$$

where $M_0=\int_0^H m(z)\mu^2(z)dz$. Comparing eq. (22) with eq. (7), it is seen that the rigid canti-levered cylinder oscillation represents the oscillation of a building in the first mode. Thus the results obtained for the response of canti-levered square cylinder in a wind tunnel can be applied to the first mode response of a tall building of a similar shape.

We take a building with following characteristics; $D=20$m, $H=40$m, $f_0=0.4\ Hz$, $\zeta=0.02$ and average mass=20kg·sec^2/m^4. From the experimental results we can see that $V_c=80$m/sec for this building, and, as $M_0/\rho D^2H=50$, we can see that the oscillation amplitude at the building top is 200cm if the wind blows at this speed.

4. Conclusion

The vortex excited oscillation of a canti-levered square cylinder of $H/D=4$ in a smooth flow was studied experimentally. The main findings are summarized as follows;

(1) The lift force at and around the critical flow velocity of a lightly damped cylinder will be expressed as follows:

$$F(t) = C_L\left(\frac{X_o}{D},\ \frac{V}{f_oD}\right)\frac{1}{2}\rho V^2 D \sin\left(2\pi f_o t + \pi/2\right)$$

(2) The critical flow velocity of this square cylinder is higher than that of the two dimensional one and is given below:

$$V_c/f_oD = 9.8$$

(3) The lift force coefficient at the critical flow velocity is much reduced than that of the two dimensional one and is given as follows in the oscillation amplitude range of $X_o/D < 0.2$.

$$C_L(X_o/D) = 0.1 + 2.0 \cdot X_o/D$$

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