AN IMPROVED FORMING ALGORITHM OF ERROR-PATTERNS FOR TWO-DIMENSIONAL CODES

Ren Xunhuan, Ma Jun, Konopelko V. K.
Department of Infocommunication Technologies ,BSUIR
Minsk, Belarus
E-mail: rxh1549417024@gmail.com

Dvelopments of techniques for processing two-dimensional information have aroused interest in two-dimensional codes. [1] *have investigated the two-dimensional codes called - $\beta\gamma$ array codes. Nonzero codewords of these codes are two-dimensional versions of M-sequences and have interesting properties that are essentially two-dimensional.*[2] *has made studies on product codes from a two-dimensional viewpoint and has shown that the product of an (n_1, k_1) cyclic code and an (n_2, k_2) cyclic code permits the correction of every two dimensional burst (or spot) of area $(n_1 - k_1)$ or (n_2, k_2) or less. In this paper, we analyze the methods for forming error vectors and the properties of error vectors, which are allowed to determining the type of error vector in two-dimensional coding of information.*

Introduction

Error-correcting codes have their origin in Shannon's seminal publication from 1948, where he proved that nearly error-free discrete data transmission is possible over any noisy channel when the code rate is less than the channel capacity. This statement is nowadays called the (noisy) channel coding theorem. The channel capacity depends on the physical properties of the channel and it is an active research area to determine the capacity of non-trivial communication channels. However, the proof of the channel coding theorem is non - constructive and therefore it is not clear how to construct error - correcting codes which actually achieve the Shannon limit. Noise - resistant codes are designed for synchronous noise and noise control in real channels of digital information. Nowadays, they have become an inevitable attribute of almost all mobile telecommunication systems (TCS). The history of the rapid development of the theory and practice of error -correcting coding has a little more than 70 years and Hamming R. laid the foundations of noise-resistant coding, proposed the first real noise - resistant codes [3].

In this paper, we analyze the methods for classifying error vectors and the properties of error vectors, which are allowed to determining the type of error vector in two-dimensional coding of information. On the basis of [4], a new method and algorithm for classifying error vectors are proposed.

The rest of the paper is organized as follows. In Section 1, a brief description of product codes. Section 2, different algorithms for generating the library of t = 2:6 error patterns. The conclusion is given in Section 3.

I. Principle of product code

Product code is a technique to form a long length code with higher ECC capabilities using small length constituent codes. Compared to plain long length codes, it has high performance from cross parity check and low circuitry overhead since the constituent codewords are of low error correction capability. Let C_1 be a (n_1, k_1) linear code, and let C_2 be a linear code. Then, a $(n_1 n_2, k_1 k_2)$ linear code can be formed such that each codeword is a rectangular array of n_1 columns and n_2 rows in which every row is a code word of , and every column is a code word in , as shown in Figure 1. This two-dimensional code is called the direct product (or simply the product) of C_1 and C_2 . The $k_1 k_2$ digits in the upper right corner of the array are information symbols. The digits in the upper left corner of this array are computed from the parity-check rules for C_1 on rows, and the digits in the lower right corner are computed from the parity-check rules for C_2 of columns. Now should we compute the check digits in the lower-left corner by using the parity-check rules for C_2 on columns or the parity-check rules for C_1 on rows.It turns out that either way yields the same $(n_1 - k_1) \times (n_2 - k_2)$ check digits, and it is possible to have row code words in C_1 and all column code words in C_2 simultaneously [3].

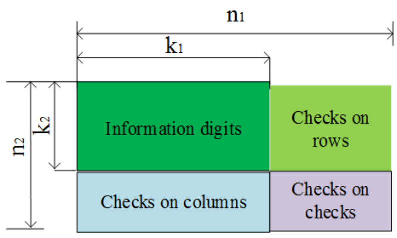

Рис. 1 – A typical two-dimensional product code

Here we take two Hamming codes as the code and illustrate the encoding process of the product code, for example, the $(7 \times 4) \times (7 \times 4)$ coding matrix is shown in the figure 2, where m represents the information bit, p indicates the corresponding parity bit, and the subscript i and j represent the number of rows respectively.

For example, firstly encoding the row (column) and then encoding the column (row), the row encoder encodes the first row of information bits

$(m_{11}, m_{21}, m_{31}, m_{41})$ according to the encoding rule of the $(7, 4)$ Hamming code, calculates the corresponding check bit (p_{51}, p_{61}, p_{71}) and adds it to the right of $(m_{11}, m_{21}, m_{31}, m_{41})$, and then the encoder turns to the next row. Every line performs the same encoding process until the fourth line of the information bit is completed, that's $(m_{14}, m_{24}, m_{34}, m_{44})$ encoding which can form a (4×7) matrix; then the column encoder also encodes the first line of information bits $(m_{11}, m_{21}, m_{31}, m_{41})$ according to the coding rules of the $(7, 4)$ Hamming code, calculate the corresponding check bits (p_{15}, p_{16}, p_{17}) and added under the information bits $(m_{14}, m_{24}, m_{34}, m_{44})$. The column encoder then moves to the next column to perform the same encoding process. After completion the information bit encoding in the fourth column, the column encoder continues to encode the third column of the check bits in the right to obtain the checks on checks bits, and finally completes the product code. When the code is transmitted in the channel, it will be transmitted serially according to the sequence $(m_{11}, m_{21}, m_{31}, m_{41}, p_{51}, p_{61}, p_{71}, m_{12}, m_{22}, ..., p_{71})$. At the receiving end, the decoder will still arrange the received serial sequence according to the above way, convert it into a two-dimensional matrix, and then decode it according to the matrix structure. Product codes are usually encoded by simple system codes, and decoding is usually done by first column decoding. Therefore, the complexity of decoding increases linearly with the decoding complexity of its subcodes.

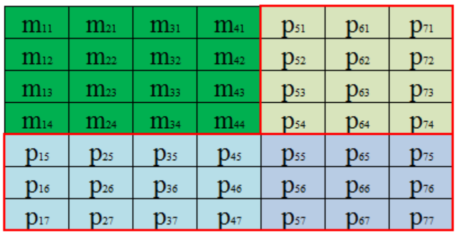

Рис. 2 – A schematic view of the (7×7) product code encoding

II. Algorithm for generating error pattern

The known classification algorithm of generation error vector [4] The error pattern can be equivalent to a matrix of $(t \times t)$. For each square matrix containing t error modes, according to the definition, we consider that the two matrices are equivalent if they can be transformed into each other through the elementary transformation. The rules which can generate the library error patterns [4].

1. Generating all of the original vectors V_o on length $n_1 = (t \times t)$;
2. From all of the vector V_o calculate the vector V_f;
3. If there is a class whose vectors formed the same error pattern, the vector is excluded from consideration;
4. If there is no such class, the vector becomes the pattern of the new class ;
5. Generating all of the original vectors V_o on the length of n_1 ;
6. Compare the patterns of classes in order to generate typical patterns classes .

According to the algorithm of [4], we proposed an improved algorithm which uses the structure of trees. Firstly, based on the error pattern of t, we adding one random error and generating the error patterns of $t + 1$, finally, we get an error pattern library (Fig.3).

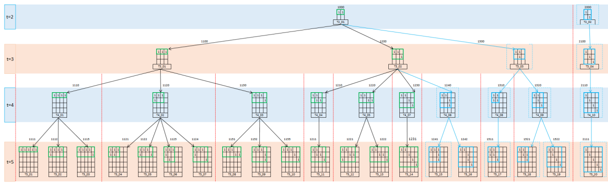

Рис. 3 – The Structure generating error patterns for $t = 2 : 5$

Таблица 1 – Performance comparison of the methods of forming the error vector

t	V_f	Fast method V_f	Acceleration factor
2	6	3	2
3	84	14	6
4	1820	52	35
5	53130	210	253
6	1947792	1054	3141

III. Conclusion

In this paper, we analyze different methods for generating the library of error patterns on the basis of two-dimension coding. The improved method has reduced the time of generating the error library. The experiment results indicate that the new method has a good performance.

1. Nomura, T. H. A theory of two-dimensional linear recurring arrays / T. H. Nomura,I. Miyakawa, A. Fukuda //IEEE Trans. Inform. –Nov.1972. – № IT-18. – С. 775–785.
2. Elspas, B. Notes on multidimensional burst-error correction / B. Elspas //IEEE Int. Svmn. Information Theorv. San Remo.Italy. – 1967.
3. Daniel, J, C. Error Control Coding, Second Edition / J. C. Danie,l, SHU //(ISBN 0-13-042672-5). – 2004. – . С. 44–89.
4. Smolyakov, O. G.Classification of error vectors in two-dimensional information coding / O. G.O. GSmolyakova ,V. K. Konopelko // – 2008. – . С. 19–28.
5. Smolyakova, .O. G. Correction of errors and erasures in two-dimensional coding of information / O. G.O. GSmolyakova ,V. K. Konopelko // – 2008. – . С. 142–153.
6. Sung, W. Y.Two-Dimensional Error-Pattern-Correcting Codes / W. Y. Sung,M. Jaekyun //IEEE Trans. Inf.Theory, vol. 63 . – Aug.2015. – . С. 2725–2740.