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Neutrino oscillations in intensive magnetic fields

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The evolution of the neutrino flux traveling through condensed matter and intensive magnetic field is considered. As the examples of the intensive magnetic field the magnetic fields of the coupled sunspots and the collapsar jets are considered. It is assumed that the neutrinos possess both the dipole magnetic moment and the anapole moment while the magnetic field may takes the values $\geq 10^5$ Gs and has the twisting nature. The problem is investigated within three neutrino generations. The possible resonance conversions of the neutrino flux are examined.

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1. Introduction

Neutrinos are neutral particles and their total Lagrangian does not contain any multipole moments (MM's). These moments are caused by the radiative corrections. Interaction of neutrinos with external electromagnetic fields is defined by the MM's. Due to smallness of the neutrino MM's this interaction becomes essential in the case of intensive fields only. The examples of such fields are the Sun's magnetic fields. In that case of special interest are the magnetic fields of the solar sunspots in the period preceding the solar flare (SF). It is believed that the magnetic field is the main energy source of the SF's [1, 2]. During the years of the active Sun, the magnetic flux $\sim 10^{24}$ Gauss $\cdot$ cm^2 [3] erupts from the solar interior and accumulates within the sunspots giving rise to the stored magnetic field. The SF formation starts from pairing big sunspots of opposite polarity (coupled sunspots). Then the process of magnetic energy storage of the CS's begins. The duration of this period (the SF initial phase) varies from several to dozens of hours. At this phase the magnetic field value for coupled sunspots could be increased from $\sim 10^4$ Gs up to $\sim 10^5$ Gs and upwards. It should be noted that flare events also occur in other first-generation stars (solar-like stars). For example, the Kepler mission [4], which surveyed $\sim 10^5$ stars of M-, K-, and G-types and produced detailed statistics about large flares with energies of order 10^{33} erg, has yielded a great deal of data concerning the flare mechanism. So, the study of the SF's is sheding light on the structure and evolution of the Universe.

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Neutrino interaction with the magnetic fields also allows to explain the deficit of high-energetic muon neutrinos arising at Gamma-ray bursts (GRBs) [5]. Short GRBs seem to be the result of the final merger of two compact objects, whereas long GRBs are probably associated with the gravitational collapse of very massive stars. These imploding stars are called collapsars. The collapse of the stellar core produces a black hole, which accretes material from the inner layers of the star. An ultradense magnetized accretion disk is formed during the accretion process. Part of the plasma that surrounds the black hole is ejected producing two relativistic jets. The magnetic fields within the jets of collapsars could be as high as $10^7 - 10^8$ G. Each jet pushes the stellar material outwards. The energy radiated during the GRBs could be very large $10^{51} - 10^{54}$ erg. Besides producing electromagnetic emission the GRBs could also be sources of three important non-electromagnetic signals: cosmic rays, neutrinos, and gravitational waves. The neutrino energies are monstrous large. They lie in the range PeV to EeV ($10^{15} - 10^{18}$ eV). There are a lot of works devoted to studying the neutrino production in different scenarios of GRBs, however, the upper limit on the muon neutrino flux obtained with the data collected with the 59-string configuration of IceCube is 3.7 times below existing theoretical predictions. Recall that the IceCube is sensitive to muon neutrinos.

Intensive magnetic field ($B = 10^{11-13}$ Gs) also exist at the surface of neutron stars's that are extremely compact remnants of high-mass stars. Besides, there is evidence of neutron stars's with ultra strong magnetic field's ($B \geq 10^{14-15}$ G) called magnetars.

The purpose of the present work is to consider the evolution of the neutrino flux in the intensive magnetic field and the condensed matter. Within three neutrino generations in the next section we deduce the corresponding evolution equation and identify all the possible resonance conversions. Our treatment of the problem carries rather general character, namely, it holds for any standard model (SM) extensions in which neutrinos have masses and possess both the magnetic dipole and anapole moments [6]. Finally, in section 3, some conclusions are drawn.

2. Evolution equation

To deduce the evolution equation we shall use the standard technique (see, for example, the book [7]). The basic idea of this approach consists in the reduction of the totality of the neutrino interactions in matter and magnetic field to the motion in a field with a potential energy. For this purpose the Lagrangian describing the interaction of the gauge bosons with the neutrinos is required

$$\mathcal{L}_g = \frac{g}{2\sqrt{2}}\left[\overline{\nu}_l(x)\gamma^\mu(1-\gamma_5)l(x)W^*_\mu(x) + \bar{l}(x)\gamma^\mu(1-\gamma_5)\nu_l(x)W_\mu(x)\right] +$$

$$+\frac{g}{4\cos\theta_W}\left\{\overline{\nu}_l(x)\gamma^\mu[1-\gamma_5]\nu_l(x) + \bar{l}(x)\gamma^\mu[4\sin^2\theta_W - 1 + \gamma_5]l(x)\right\}Z_\mu(x). \qquad (1)$$

In its turn the existence of electromagnetic multipole moments caused by the radiative corrections (RC's) could be taken into account in terms of the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{i}{2}\overline{\nu}_l(x)\left[\mu_{ll'}\sigma^{\mu\lambda} + a_{ll'}(\partial^\mu\gamma^\lambda - \partial^\lambda\gamma^\mu)\right](1-\gamma_5)\nu_{l'}(x)F_{\lambda\mu}(x) + \text{conj.} =$$

$$= \frac{i}{2}\overline{\nu}_a(x)\left[\mu_{ab}\sigma^{\mu\lambda} + a_{ab}(\partial^\mu\gamma^\lambda - \partial^\lambda\gamma^\mu)\right](1-\gamma_5)\nu_b(x)F_{\lambda\mu}(x) + \text{conj.}, \qquad (2)$$

where the indexes l, l' refer to the flavor basis ($l, l' = e, \mu, \tau$) while the indexes a and b refer to the mass eigenstate basis ($a, b = 1, 2, 3$), μ_{ab} (a_{ab}) are the dipole magnetic (anapole) moments of the mass eigenstates, and $F_{\lambda\mu} = \partial_\lambda A_\mu - \partial_\mu A_\lambda$.

The system under consideration must include both the left-handed and right-handed neutrinos, that is, it must consist of $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ and their anti-particles $(\nu_{eL})^c, (\nu_{\mu L})^c, (\nu_{\tau L})^c$. Then, by virtue of the fact that $(\nu_{eL})^c, (\nu_{\mu L})^c$ and $(\nu_{\tau L})^c$ are right-handed neutrinos, we shall use for them the following designations $\overline{\nu}_{eL}, \overline{\mu}_{eL}$ and $\overline{\nu}_{\tau L}$.

We shall consider that magnetic field has the nonpotential character

$$(\text{rot } \mathbf{B})_z = 4\pi j_z, \tag{3}$$

and exhibits the geometrical phase $\Phi(z)$

$$B_x \pm iB_y = B_\perp e^{\pm i\Phi(z)}, \tag{4}$$

which is defined by the simple expression

$$\Phi(z) = \frac{\alpha\pi}{L_{mf}} z.$$

So, we assume that the magnetic field is in existence over a distance L_{mf} and twists by an angle $\alpha\pi$.

In order to make the results physically more transparent, we transform to the following basis

$$\begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \\ \overline{\nu}'_1 \\ \overline{\nu}'_2 \\ \overline{\nu}'_3 \end{pmatrix} = \mathcal{U}' \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \overline{\nu}_{eL} \\ \overline{\nu}_{\mu L} \\ \overline{\nu}_{\tau L} \end{pmatrix},$$

where

$$\mathcal{U}' = \begin{pmatrix} \mathcal{D}' & 0 \\ 0 & \mathcal{D}' \end{pmatrix},$$

$$\mathcal{D}' = \exp\left(-i\lambda_5\phi\right)\exp(-i\lambda_7\psi) = \begin{pmatrix} c_\phi & 0 & s_\phi \\ -s_\phi s_\psi & c_\psi & c_\phi s_\psi \\ -s_\phi c_\psi & -s_\phi & c_\phi c_\psi \end{pmatrix}.$$

In the new basis the evolution equation will look like

$$i\frac{d}{dz}\begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \\ \overline{\nu}'_1 \\ \overline{\nu}'_2 \\ \overline{\nu}'_3 \end{pmatrix} = \mathcal{H}' \begin{pmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \\ \overline{\nu}'_1 \\ \overline{\nu}'_2 \\ \overline{\nu}'_3 \end{pmatrix} \tag{5}$$

where the Hamiltonian $\mathcal{H}'$ has the form

$$\mathcal{H}' = \begin{pmatrix} \mathcal{B}_v + \Lambda & \mathcal{M} \\ \mathcal{M} & \mathcal{B}_v + \tilde{\Lambda} \end{pmatrix} \tag{6}$$

$$\mathcal{B}_v = \begin{pmatrix} -\delta^{12}c_{2\omega} & \delta^{12}s_{2\omega} & 0 \\ \delta^{12}s_{2\omega} & \delta^{12}c_{2\omega} & 0 \\ 0 & 0 & \delta^{31}+\delta^{32} \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} V^{eff}_{eL}c^2_\phi & 0 & V^{eff}_{eL}s_{2\phi}/2 \\ 0 & 0 & 0 \\ V^{eff}_{eL}s_{2\phi}/2 & 0 & V^{eff}_{eL}s^2_\phi \end{pmatrix},$$

$$\tilde{\Lambda} = \begin{pmatrix} \mathcal{A}_{\overline{\nu}\overline{\nu}} - \mathcal{A}_{\nu\nu} - V_{\mu L} & 0 & 0 \\ 0 & \mathcal{A}_{\overline{\nu}\overline{\nu}} - \mathcal{A}_{\nu\nu} - V_{\mu L} & 0 \\ 0 & 0 & \mathcal{A}_{\overline{\nu}\overline{\nu}} - \mathcal{A}_{\nu\nu} - V_{\mu L} \end{pmatrix},$$

$$\mathcal{M} = \begin{pmatrix} (\mu_0 + \mu_{12}s_{2\omega})B_\perp & \mu_{12}c_{2\omega}B_\perp & (\mu_{13}c_\omega + \mu_{23}s_\omega)B_\perp \\ \mu_{12}c_{2\omega}B_\perp & \mu_{12}c_{2\omega}B_\perp & (\mu_{13}c_\omega + \mu_{23}s_\omega)B_\perp \\ 0 & 0 & \mu_{33}B_\perp \end{pmatrix},$$

$$\delta^{ik} = \frac{m_i^2 - m_k^2}{4E}, \qquad \mu_{11} = \mu_{22} = \mu_0/2, \qquad V_{eL}^{eff} = \sqrt{2}G_F n_e.$$

$$\mathcal{A}_{\nu_l\nu_l} = 4\pi a_{\nu_l\nu_l} j_z - \dot{\Phi}/2, \qquad \mathcal{A}_{\overline{\nu}_l\overline{\nu}_l} = -4\pi a_{\overline{\nu}_l\overline{\nu}_l} j_z + \dot{\Phi}/2,$$

$\psi = \theta_{23}$, $\phi = \theta_{13}$, $\omega = \theta_{12}$, $c_\phi = \cos\phi$, $s_\phi = \sin\phi$, and so on, the λ's are Gell-Mann matrices corresponding to the spin-one matrices of the $SO(3)$ group, V_{eL} ($V_{\mu L}$) is a matter potential describing interaction of the ν_{eL} ($\nu_{\mu L}, \nu_{\tau L}$) neutrinos with a solar matter,

$$V_{eL} = \sqrt{2}G_F(n_e - n_n/2), \qquad V_{\mu L} = V_{\tau L} = -\sqrt{2}G_F n_n/2,$$

n_e and n_n are electron and neutron densities, respectively, $a_{\nu_l\nu_{l'}}$ ($\mu_{\nu_l\nu_{l'}}$) is an anapole (dipole magnetic) moment between ν_l and $\nu_{l'}$ states, and, for the sake of simplicity, we have assumed that the nondiagonal neutrino AM's are equal to zero, while $a_{\nu_e\nu_e} = a_{\nu_\mu\nu_\mu} = a_{\nu_\tau\nu_\tau} = a_{\nu\nu}$.

Now, using the expression for $\mathcal{H}'$, we can establish all possible resonance conversions. We shall assume that the resonance localization places are situated rather far from one another. That allows us to consider them as independent ones. In what follows we shall be limited by consideration of the resonance transition with participation of the ν_1' neutrinos only. Further, making numerical estimates, we shall take the following values for the DMM [8]

$$\mu_{\nu_l\nu_{l'}} = 10^{-10}\ \mu_B,$$

and for the AM [9]

$$|a_{\nu_l\nu_{l'}}| = 3 \times 10^{-40}\ \text{esu} \cdot \text{cm}^2.$$

Let us start with the $\nu_1' \leftrightarrow \nu_2'$ transitions. The resonance condition is given by the expression

$$-2\delta^{12}c_{2\omega} + V_{eL}^{eff}c_\phi^2 = 0, \tag{7}$$

while the transition width is as follows

$$\Gamma(\nu_{eL} \leftrightarrow \nu_{\mu L}) \simeq \frac{\sqrt{2}\delta^{12}s_{2\omega}}{G_F}. \tag{8}$$

For the $\nu_1' \leftrightarrow \nu_2'$ oscillations to be appeared the neutrino beam must pass a distance comparable with oscillation length which is of the form

$$L_{\nu_1'\nu_2'} = \frac{2\pi}{\sqrt{(2\delta^{12}c_{2\omega} - V_{eL}^{eff}c_\phi^2)^2 + (\delta^{12}s_{2\omega})^2}}. \tag{9}$$

From Eq.(9) it follows that the oscillation length reaches its maximal value at the resonance. It is clear that this resonance belongs to the kind of the matter-induced resonances. Note, when $\phi = 0$ then the expressions (7)- (9) convert to the corresponding ones for $\nu_{eL} \leftrightarrow \nu_{\mu L}$ resonance transition found in two flavor approximation (FA).

Now we discuss this resonance in two interesting cases. First we consider the motion of the solar neutrino flux. Using

$$E_\nu = 10 \text{ MeV}, \qquad \Delta m^2_{12} = 7.37 \times 10^{-5} \text{ eV}^2, \qquad \sin^2\theta_{12} = 0.297, \tag{10}$$

we get the realization of the resonance condition at $V^{eff}_{eL} \simeq 10^{-12}$ eV and the value of the maximum oscillation length equal to $\simeq 3.5 \times 10^7$ cm. Therefore, this resonance transition will be fulfilled before the convective zone. For the collapsar jet the situation is as follows. The matter potential has the order 10^{-21} eV while the neutrinos energy could be as high as 10^{18} eV. Then, since the minimal value of $\delta^{12}c_{2\omega}$ is 10^{-23} eV the $\nu'_1 \leftrightarrow \nu'_2$ resonance transition seemingly may occur for the neutrinos with the energies of the order of 10^{15} eV. However, the oscillation length proves to be equal to $\sim 10^{15}$ cm while the size of the collapsar jet is as short as $\sim 10^8$ cm. So under these conditions this resonance is not observed.

Further we shall consider the $\nu'_1 \to \overline{\nu}'_2$ resonance. It may be realized at the condition

$$-2\delta^{12}c_{2\omega} + V^{eff}_{eL}c^2_\phi + V_{\mu L} + 4\pi(a_{\nu\nu} + a_{\overline{\nu}\overline{\nu}})j_z - \dot{\Phi} = 0. \tag{11}$$

The corresponding expressions for the transition width and the maximum oscillation length are as follows

$$\Gamma(\nu'_1 \to \overline{\nu}'_2) \simeq \frac{\sqrt{2}\mu_{12}c_{2\omega}B_\perp}{G_F}, \tag{12}$$

$$(L_{\nu'_1\to\overline{\nu}'_2})_{max} \simeq \frac{2\pi}{\mu_{12}c_{2\omega}B_\perp}. \tag{13}$$

Comparing the expressions (11) - (13) with the corresponding ones describing the $\nu_{eL} \to \overline{\nu}_{\mu L}$ resonance in two flavor approximation (FA) [10] one could be convinced that the formulas for three neutrino generations are evident from those of two FA under substitutions

$$n_e \to n_e c^2_\Phi, \tag{14}$$

$$\mu_{\nu_e\overline{\nu}_\mu} \to \mu_{12}c_{2\omega}. \tag{15}$$

For the solar neutrinos $(\delta^{12})_{min} \simeq 10^{-12}$ eV which is much more bigger than the matter potential even in photosphere ($V_{ph} \simeq 10^{-20}$ eV). Therefore, in the Sun's conditions the resonance $\nu'_1 \to \overline{\nu}'_2$ may occur only at the cost of magnetic field, that is, when the sum

$$2\delta^{12}c_{2\omega} + \dot{\Phi} - 4\pi(a_{\nu\nu} + a_{\overline{\nu}\overline{\nu}})j_z \tag{16}$$

has the same order as $V^{eff}_{eL}c^2_\phi + V_{\mu L}$. Therefore, it falls to the kind of the magnetic-induced resonances. We emphasize that for this resonance to exist the magnetic field must be twisted and/or has a nonpotential character. Using $B = 10^5$ Gs we get $(L_{\nu'_1\to\overline{\nu}'_2})_{max} \simeq 1.8 \times 10^9$ cm. Then the resonance condition and the equality $L_{mf} = (L_{\nu_{eL}\to\overline{\nu}_{\mu L}})_{max}$ will be fulfilled provided the twist frequency is equal to $\simeq -50\pi/L_{mf}$, where we have assumed for simplicity

$$\dot{\Phi} \gg 4\pi(a_{\nu\nu} + a_{\overline{\nu}\overline{\nu}})j_z. \tag{17}$$

On the other hand when the magnetic field reaches the value of 10^6 Gs what will be possible for the super-SF's, the fulfillment above mentioned requirements will be effected at the twist frequency being equal to $\simeq -5\pi/L_{mf}$ ($L_{mf} \simeq 1.8 \times 10^8$). So, we see that under the specific conditions the $\nu'_1 \to \overline{\nu}'_2$ resonance may be in existence in the Sun conditions.

For the case of the collapsar jets the $\nu'_1 \to \overline{\nu}'_2$ resonance could occur even in the case

$$\dot{\Phi} = j_z = 0, \qquad \text{and} \qquad -2\delta^{12}c_{2\omega} + V^{eff}_{eL}c^2_\phi + V_{\mu L} = 0. \tag{18}$$

Really, for example, when one assumes that $\dot{\Phi}$ or $4\pi(a_{\nu\nu}+a_{\overline{\nu}\overline{\nu}})j_z$ have the same order as $V_{eL}^{eff}\simeq 10^{-21}$ eV, then the jet magnetic field must be extended over the distance $\simeq 10^{15}$ cm while the current density values must reach 10^7 A/cm^2. So, for the collapsar jets the conditions (18) are more realistic and the $\nu'_1\to\overline{\nu}'_2$ resonance will be referred to the matter induced resonance.

The next resonance conversion is $\nu'_1\to\overline{\nu}'_1$. The corresponding formulas will look like

$$V_{eL}^{eff}c_\phi^2+V_{\mu L}+4\pi(a_{\nu\nu}+a_{\overline{\nu}\overline{\nu}})j_z-\dot{\Phi}=0 \tag{19}$$

$$\Gamma(\nu'_1\to\overline{\nu}'_1)\simeq\frac{\sqrt{2}(\mu_{11}+\mu_{12}s_{2\omega})B_\perp}{G_F}, \tag{20}$$

$$(L_{\nu'_1\to\overline{\nu}'_1})_{max}\simeq\frac{2\pi}{(\mu_{11}+\mu_{12}s_{2\omega})B_\perp}. \tag{21}$$

We see that all the expressions which govern the $\nu'_1\to\overline{\nu}'_1$ resonance conversion may be deduced from the expressions for $\nu_{eL}\to\overline{\nu}_{eL}$ resonance obtained in two FA [10] provided the replacement

$$n_e\to n_e c_\Phi^2, \tag{22}$$

$$\mu_{\nu_e\overline{\nu}_e}\to\mu_{11}+\mu_{12}s_{2\omega}. \tag{23}$$

That admits us to consider the $\nu_{eL}\to\overline{\nu}_{eL}$ resonance as the analog of the $\nu'_1\to\overline{\nu}'_1$ resonance in two FA.

For the solar neutrinos the situation with

$$j_z=0,\qquad \text{but}\qquad V_{eL}^{eff}c_\phi^2+V_{\mu L}\simeq\dot{\Phi}, \tag{24}$$

is excluded, since in this case the twisting magnetic field must exist over the distance which is much more even the solar radius. So, only when the requirements

$$\dot{\Phi}=0,\qquad \text{but}\qquad V_{eL}^{eff}c_\phi^2+V_{\mu L}\simeq-4\pi(a_{\nu\nu}+a_{\overline{\nu}\overline{\nu}})j_z \tag{25}$$

will be obeyed this resonance is observed. Let us estimate the value of j_z which is needed to realize the $\nu'_1\to\overline{\nu}'_1$ resonance in the chromosphere (corona). Taking into account $n_n\simeq n_e/6$ we obtain the following value for the matter potential $\sim 10^{-27}$ eV ($\sim 10^{-30}$ eV). Then the resonance condition (19) will be fulfilled provided

$$j_z\simeq 6\ \mathrm{A/cm^2}\qquad (j_z\simeq 0.06\ \mathrm{A/cm^2}). \tag{26}$$

We see that, in point of fact, the resonance condition (19) is the distance function. On the other hand, since both the resonance condition and the transition width do not display the dependence on the neutrino energy, then all the electron neutrinos produced in the center of the Sun (*pp*-, ^{13}N-,...and *hep*-neutrinos) may undergo $\nu'_1\to\overline{\nu}'_1$ resonance transition.

As far as the collapsar jets are concerned, here the resonance condition (19) cannot be fulfilled and, as a result, the $\nu'_1\to\overline{\nu}'_1$ resonance is forbidden.

Now we proceed to consideration of the $\nu'_1\to\nu'_3$ and $\nu'_1\to\overline{\nu}'_3$ transitions. The quantity proportional to $\Sigma=\delta^{31}+\delta^{32}$ is the dominant term in the Hamiltonian (6) and this leads to the decoupling both of ν'_3 and of $\overline{\nu}'_3$ states from the remaining ones. This means that the oscillations $\nu'_1\to\nu'_3$ and $\nu'_1\to\overline{\nu}'_3$ which are driven by the Σ term can be simply averaged out in the final survival probability for neutrinos of any flavor at the Earth. This is valid both for the Sun and for the collapsar jets as well.

3. Conclusions

The goal of this work was to investigate behavior of the neutrino flux in the condensed matter and intensive magnetic field in three flavor approximation. One was assumed that the neutrinos possess both the DMM and the AM while the magnetic field has a twisting nature and the strength of this field may be $\geq 10^5$ Gs. For the description of the magnetic field twisting has been used the simple model $\Phi = \exp \alpha\pi/L_{mf}$. As the examples of the intensive magnetic field we have considered the magnetic fields of the coupled sunspots and the collapsar jets. In order to make the results physically more transparent we have passed to new basis in which one of the states ν_1' was predominantly the ν_{eL} state. In this basis the possible resonance conversions of the neutrinos have been examined. In so doing we have restricted be the investigation of the resonance transitions of ν_1' neutrinos. It was shown that for the Sun conditions three resonance transitions with the ν_1' participation may be possible while for the collapsar jet conditions only one resonance transition may be observed.

In summary, we emphasize that the conditions for emergence of the investigated resonances contains two uncertainties, namely, the value of the magnetic field and the values of the neutrino multipole moments. Therefore, investigation of the neutrino fluxes coming from the stellar objects will allow us to obtain information not only about neutrino properties but about stelar object structure too.

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