Practice article

# Design of feedforward and feedback position control for passive bilateral teleoperation with delays 

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## H I G H L I G H T S

- Passive \& stable architecture even with any config in the middle \& varying time delay.
- Position and tracking errors are bounded.
- Adapt to varying time delay w/time varying gains using master ff \& fb position data.
- Passivity of proposed architecture proven through two Lyapunov like function theorems.
- Explicit position \& force tracking simulations under various control models \& gains.


## A R T I C L E I N F O

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#### Abstract

Bilateral teleoperation systems connected to computer networks such as the internet must be able to operate with varying time delays since such systems can easily become unstable. A passivity concept has been used as the framework to solve the stability problem in the bilateral control of teleoperation systems. Passivity and tracking performance are recovered using a control architecture that incorporates time varying gains into the transmission path, feedforward, and feedback position control. The proposed architecture has an inner component that can accommodate any configuration but still remain stable and passive even with varying time delay. The simulation results for a single degree of freedom master/slave system demonstrate the performance of the proposed control architecture.


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## 1. Introduction

Since the first master-slave teleoperator was built by Goertz and Thompson in 1954, a significant amount of research has been carried out in an attempt to understand and overcome specific problems in bilateral teleoperation [1]. The various potential applications of bilateral teleoperations, such as in mobile robots, telesurgery, space exploration, seabed operations, and virtual reality, continue to attract attention from researchers worldwide. However, a time delay is incurred in the transmission of data from one location to another whenever teleoperation is performed over a large distance. These time delays can easily destabilize a bilaterally-controlled teleoperation system if no extra control measures are employed.

Anderson and Spong [2] were the first to propose a solution to the time-delay problem in 1982 when they employed passivity [3] and scattering theory to overcome the instability caused by time delay. Niemeyer and Slotine [4] introduced the use of

[^0]wave variables in teleoperation, which extended the scattering theory proposed by Anderson and Spong. Lozano et al. [5] extended Anderson and Spong's previous approach for a varying time delay system by adding a small gain to compensate for the energy flow. Leeraphan et al. [6] proposed a method that could be used for a varying time delay by introducing a time-varying gain parameter, $b$, which varies with the power variables.

Several other methods have been proposed to address the effects of a constant time delay, including Forouzantabar's [7] extension of Lee and Spong's [8] work, and Hirche's [9] introduction of a distributed controller approach for networked control systems to address the problem of a constant time delay. Varying time delays have been addressed by Bouknifer [10,11], Fujita [12], Chopra [1315], Gu [16], Ryu [17-19], Ye [20], Chen [21], Farooq [22], and Wang [23]. Feedforward and feedback controllers have been employed by Hosseini-Suny [24], Hua [25], Lee [26], and Yang [27].

Later works considered nonlinear bilateral teleoperators with variable time delays that eliminated the need for velocity measurements in the position tracking problem. One such solution was proposed by Sarras [28], in which utilized the Immersion \& Invariance observer to derive a global exponentially convergent estimate of unmeasured velocities. Alternatively, Hua [29] addressed the lack
of velocity information by proposing an output-feedback adaptive SP+Sd-type controller where the unknown velocity was estimated by a new fast terminal sliding-mode velocity observer. The unknown gravity term was also approximated through an adaptive method. Sun [30] proposed a new wave-based time-domain passivity approach (TDPA) which achieved higher transparency while remaining stable even with random time delays. One of the most recent works on teleoperation was the development of KONTUR2by Artigas [31], where the Earth and International Space Station (ISS) were used to test and demonstrate a new technology for space-to-earth real-time manipulation by remotely operating a robot manipulator in Germany from a force feedback joystick in the ISS.

The standard approaches for the control of bilateral teleoperators with force feedback based either on Anderson and Spong's scattering approach or the equivalent wave variable formulation of Niemeyer and Slotine [32] generally preserve passivity only for a constant time delay. However, teleoperation over the Internet, which can have varying time delays due to several factors such as congestion, bandwidth, or distance, may severely degrade the performance or even result in an unstable system [2,4,32-34]. Lozano then carried out the passivation of force reflected bilateral teleoperations with time varying delays [5].

Since Anderson and Spong $[2,35]$ found that passivity does not guarantee good performance, in this paper we propose and analyze a modified control architecture similar to that proposed by Lawrence [36]. In the proposed architecture, time-varying gains are introduced to the transformation. These gains explicitly use the position data from the master to generate feedforward and feedback position control for the manipulator to recover passivity and achieve good tracking performance. Passivity and tracking performance can be recovered through such a configuration. Simulations using a 1-DOF teleoperator system demonstrated a tradeoff between position tracking and force tracking. Depending on the value combinations of the gain parameter $K$ and the impedance parameter $b$, satisfactory position tracking, force tracking, or both can be achieved.

The main contributions of this paper are the development of a control architecture for linear time-varying systems and demonstration that the proposed architecture possesses the following important properties:

- ability to remain passive and stable with any type of configuration in the middle even if there is variable time delay
- position and tracking errors are bounded
- adaptability to varying time delay with time varying control gains by using position data from the master feedforward and feedback.

This research work is presented as follows. Section 2 gives a brief background on the concept of passivity, while Section 3 presents the theoretical background on bilateral teleoperation. The proposed architecture for varying time delay is presented in Section 4 and the simulation results and corresponding discussion are given in Section 5 followed by the concluding remarks in Section 6.

## 2. Basic passivity concepts

Passivity formalism is motivated by the Lyapunov theory, and provides a powerful tool for system stability analysis and control law design. It represents a mathematical description of the intuitive physical concepts of power and energy. The "power" entering a system is defined as the scalar product between the input vector
and output vector $y$ of the system. A system is said to be passive if and only if it obeys (1) as follows [5,6]:
$P=x^{T} y=\frac{d E}{d t}+P_{\text {diss }}$
where $E$ is a lower-bounded "energy storage" function and $P_{\text {diss }}$ is a non-negative "power dissipation" function. Due to the lowerboundedness of $E$ without a loss of generality, $E$ is considered to be a non-negative function.
$\int_{0}^{t} P d \tau=\int_{0}^{t} x^{T} y d \tau=E(t)-E(0)+\int_{0}^{t} P_{\text {diss }} d \tau \geq-E(0)=$ constant
As can be seen in (1), the power is either stored or dissipated. This implies that the total energy supplied by the system up to time $t$ is limited to the initial stored energy, $E(0)$. If the power dissipation is zero for all time, the system is also termed lossless. In contrast, if the power dissipation is positive, provided the stored energy has not reached its lower bound, the system is strictly passive. Using the stored energy as a Lyapunov-like function, one can quickly analyze the stability and show that, without an external input, a passive system is stable.

## 3. Bilateral teleoperation

The passivity formalism is motivated by the Lyapunov theory, and provides a powerful tool for system stability analysis and control law design. It represents a mathematical description of the intuitive physical concepts of power and energy. The "power" entering a system is defined as the scalar product between the input vector and the output vector $y$ of the system.

A standard teleoperation system is presented in the block diagram shown in Fig. 1(a). The standard teleoperation system consists of five subsystems: the human operator, the master, the communication block, the slave, and the environment. In the block diagram, $\dot{x}$ represents the velocity and $F$ represents the force. The subscript $m$ represents the variables of the master at the local site, while subscript $s$ represents the variables of the slave at the remote location.

In general, the local site sends a position or velocity command to control the slave manipulator. Simultaneously, the remote force is transmitted back to the local site to provide the desired force command. The communication block connects the local and the remote systems, and also introduces time delays, which may be caused by physical transmission times or communication bandwidth limitations. Even small delays can cause a system to be unstable. Fig. 1(b) presents a model of communication with a constant time delay. Here, the power variables are given by
$\dot{x}_{s}(t)=\dot{x}_{m}(t-T), \quad F_{m}(t)=F_{s}(t-T)$
where $T$ is the time delay in the communication block, which is defined as a constant term. It can be easily proved that specific values for the input variables $\dot{\chi}_{m}$ and $F_{s}$ will cause negative power dissipation, which can make the overall system unstable. Niemeyeret al. [32] first described the stabilization of a time delay system by making the system sufficiently damped, whereby a damping element was inserted next to the communication out port to ensure the system could guarantee that positive power dissipation would result in a passive system. Fig. 1(c) shows a standard communication model with sufficient dissipation, in which power variables are transmitted with time delay $T$. For passive communication in Fig. 1(c), the implemented power variables are shown as follows:
$F_{m}^{*}=F_{m}+b \dot{x}_{m}, \quad \dot{x}_{s}^{*}=\dot{x}_{s}-\frac{1}{b} F_{s}$

a. Block diagram of a teleoperation system

b. Standard communication with constant time delay

c. Standard communication with sufficient dissipation

Fig. 1. Diagram and schematic of communication in a teleoperation system.

Thus, the modified power flowing into the system at any time $t$ is computed as: $P(t)=\dot{x}_{m}(t) F_{m}^{*}(t)-\dot{x}_{s}^{*}(t) F_{s}(t)$, that is

$$
\begin{align*}
& \dot{x}_{m}(t) F_{m}^{*}(t)-\dot{x}_{s}^{*}(t) F_{s}(t) \\
&= {\left[\frac{1}{2 b} F_{m}^{2}(t)+\dot{x}_{m}(t) F_{m}(t)+\frac{b}{2} \dot{x}_{m}^{2}(t)\right]+\left[\frac{1}{2 b} F_{s}^{2}(t)-\dot{x}_{s}(t) F_{s}(t)\right.} \\
&\left.+\frac{b}{2} \dot{x}_{s}^{2}(t)\right]+\left[\frac{b}{2} \dot{x}_{m}^{2}(t)-\frac{b}{2} \dot{x}_{s}^{2}(t)+\frac{1}{2 b} F_{s}^{2}(t)-\frac{1}{2 b} F_{m}^{2}(t)\right] \\
&= \frac{1}{2 b} F_{m}^{* 2}(t)+\frac{b}{2} \dot{x}_{s}^{* 2}(t)+\left[\frac{d}{d t} \int_{t-T}^{t} \frac{b}{2} \dot{x}_{m}^{2}(\tau) d \tau\right. \\
&\left.+\frac{d}{d t} \int_{t-T}^{t} \frac{1}{2 b} F_{s}^{2}(\tau) d \tau\right] \\
&= \frac{d E(t)}{d t}+P_{\text {disss }}(t) . \tag{5}
\end{align*}
$$

According to (1), the power dissipation $P_{\text {diss }}(t)$ and stored energy $E(t)$ are defined as
$P_{\text {diss }}(t)=\frac{1}{2 b} F_{m}^{* 2}(t)+\frac{b}{2} \dot{x}_{s}^{* 2}(t)$,
$\frac{d E(t)}{d t}=\frac{d}{d t} \int_{t-T}^{t} \frac{b}{2} \dot{x}_{m}^{2}(\tau) d \tau+\frac{d}{d t} \int_{t-T}^{t} \frac{1}{2 b} F_{s}^{2}(\tau) d \tau$
Therefore, the dissipated energy is non-negative and the system stays passive. However, the above result does not hold if $T=T(t)$, i.e. the delay is time-varying. The reason why stability is not guaranteed in the time-varying delay case is investigated as follows. The variable relationship between the master and slave at any time in a varying time delay case is given by:
$\dot{x}_{s}(t)=\dot{x}_{m}\left(t-T_{1}(t)\right), \quad F_{m}(t)=F_{s}\left(t-T_{2}(t)\right)$

The total energy expressed in (2) related to the communications during the signal transmission at any time can be written as follows:

$$
\begin{align*}
& \int_{0}^{t} P(\tau) d \tau=\int_{0}^{t}\left(\dot{x}_{m}(\tau) F_{m}^{*}(\tau)-\dot{x}_{s}^{*}(\tau) F_{s}(\tau)\right) d \tau \\
& =\int_{0}^{t}\left(\dot{x}_{m} F_{m}+b \dot{x}_{m}^{2}-\dot{x}_{s} F_{s}+\frac{1}{b} F_{s}^{2}\right) d \tau=\int_{0}^{t}\left(\frac{1}{2 b} F_{m}^{* 2}+\frac{b}{2} \dot{x}_{s}^{* 2}\right) d \tau \\
& \quad+\int_{0}^{t}\left(\frac{b}{2} \dot{x}_{m}^{2}+\frac{1}{2 b} F_{s}^{2}\right) d \tau \\
& \quad-\int_{0}^{t}\left(\frac{b}{2} \dot{x}_{m}^{2}\left(\tau-T_{1}(\tau)\right)+\frac{1}{2 b} F_{s}^{2}\left(\tau-T_{2}(\tau)\right)\right) d \tau . \tag{7}
\end{align*}
$$

From (7), it follows that it cannot be ensured that $\int_{0}^{t} P(\tau) d \tau$ is always lower-bounded, implying that a controller/manipulator setup always exists that can cause negative dissipation and can cause the system to lose passivity and become unstable.

## 4. Proposed new architecture for varying time delay and its properties

In order to overcome the potential destabilizing effects of the time-varying delay, we propose a new architecture shown in Fig. 2(a). The architecture can be divided into two components, as shown in Fig. 2(b) and 2(c).

The main difference of this architecture over that of Chopra's [37] is its ability to remain passive even when any type of configuration is utilized in the middle, represented by the box with broken line in Fig. 2(b). Derivations in the latter part of this section will show that, despite using proof from Chopra's [33] scheme, the results in the present study and those of Chopra's scheme

a. Modified architecture with modified gain, $g_{i}$, inserted in the communication channel

b. Modified architecture 1

c. Modified architecture 2

Fig. 2. Proposed new architecture for varying time delay.
do not coincide. This is because the variable time delay case was considered, while the delay in [33] is assumed to be constant and a different configuration of configuration is utilized in the middle, represented by the box with broken line in Fig. 2(b). Overlapping results also occurred with the time-varying delay version proposed by Nuno [38] since a similar approach was used even when derivations were made independently and the authors were not aware of existing results at the time.

The proposed system structure has the feedforward $F_{\text {feed }}=$ $K\left(x_{m}(t-T)-x_{s}\right)$ and does not utilize the scattering method to stabilize the system but relies on the variable gain similar to Lozano [5] and the impedance matching of Niemeyer [4]. Also, a stable passive system is achieved regardless of the type of configuration of the inner structure. Mathematical proof showing that the architecture will remain passive is given in succeeding sections of this paper.

We impose the following assumptions.
Assumption 1. Functions $T_{i}(t), i=1,2$ are continuous piecewisesmooth, and there exist numbers $T_{i \text { max }}, i=1,2$ such that $0 \leq$ $T_{i}(t)<T_{\text {max }}, t \geq 0, i=1,2$, and
$\dot{T}_{i}(t)<1, \quad i=1,2, t \geq 0$.

Assumption 2. Velocities $\dot{x}_{m}(t)$ and $\dot{x}_{s}^{*}(t)$ are 0 if $t<0$.

### 4.1. Passivity of the configuration represented in Fig. 2(c).

Theorem 1. Consider the configuration represented in Fig. 2(c). Suppose that Assumptions 1 and 2 hold true. If the gains $g_{i}(t), i=$ 1, 2 satisfy the inequalities
$g_{i}^{2}(t) \leq 1-\dot{T}_{i}(t), g_{i}(t) \geq 0, i=1,2$
then the configuration in Fig. 2(c) is passive, i.e. the following inequalities hold true
$\int_{0}^{t}\left(F_{m}^{*}(\tau) \dot{x}_{m}(\tau)-F_{s}(\tau) \dot{x}_{s}^{*}(\tau)\right) d \tau \geq 0 \quad$ for all $t \geq 0$.
Proof. According to the configuration,
$\begin{aligned} F_{m}^{*}(t) & =g_{2}(t) F_{s}\left(t-T_{2}(t)\right)+b \dot{x}_{m}(t)=F_{m}(t)+b \dot{x}_{m}(t), \\ F_{s}(t) & =b\left(g_{1}(t) \dot{x}_{m}\left(t-T_{1}(t)\right)-\dot{x}_{s}^{*}(t)\right)=b\left(\dot{x}_{s}(t)-\dot{x}_{s}^{*}(t)\right),\end{aligned}$
where $F_{m}(t)=g_{2}(t) F_{s}\left(t-T_{2}(t)\right)$ and $\dot{x}_{s}(t)=g_{1}(t)$ $\dot{x}_{m}\left(t-T_{1}(t)\right)$.

The total energy supplied by the system up to time $t$ can be calculated according to (5) as:

$$
\begin{align*}
& \int_{0}^{t}\left(\dot{x}_{m}(\tau) F_{m}^{*}(\tau)-\dot{x}_{s}^{*}(\tau) F_{s}(\tau)\right) d \tau \\
& =\int_{0}^{t}\left\{\begin{array}{l}
\frac{b}{2} \dot{x}_{m}^{2}(\tau)+\frac{b}{2} \dot{x}_{s}^{* 2}(\tau)+\frac{1}{2 b} F_{s}^{2}(\tau) \\
+\frac{1}{2 b} F_{m}^{* 2}(\tau)
\end{array}\right\} d \tau \\
& \quad-\int_{0}^{t}\left\{\frac{1}{2 b} g_{2}^{2}(\tau) F_{s}^{2}\left(\tau-T_{2}(\tau)\right)\right\} d \tau \\
& \quad-\int_{0}^{t}\left\{\frac{b}{2} g_{1}^{2}(\tau) \dot{x}_{m}^{2}\left(\tau-T_{1}(\tau)\right)\right\} d t . \tag{11}
\end{align*}
$$

Let us now transform the last two summands in (11) into another form. To this end, we introduce the functions $\sigma_{i}(\tau)=$ $\tau-T_{i}(\tau), \quad \tau \geq 0$.

By Assumption $1, \frac{d \sigma_{i}(\tau)}{d \tau}=1-\frac{d T_{i}(\tau)}{d \tau}>0, \tau \geq 0, i=1,2$.
Hence, according to the implicit function theorem, for the functions $\sigma_{i}(\tau), \tau \geq 0$, there exist the inverse functions $\sigma_{i}^{-1}(\alpha), \alpha \geq$ $0, i=1,2$. According to the definitions of these functions we have $\sigma_{i}\left(\sigma_{i}^{-1}(\alpha)\right) \equiv \alpha, \alpha \geq 0$, i.e. $\sigma_{i}^{-1}(\alpha)-T_{i}\left(\sigma_{i}^{-1}(\alpha)\right) \equiv \alpha$. Consequently, $\frac{d}{d \alpha}\left(\sigma_{i}^{-1}(\alpha)-T_{i}\left(\sigma_{i}^{-1}(\alpha)\right)\right) \equiv 1$, or equivalently, $\frac{d}{d \alpha}\left(\sigma_{i}^{-1}(\alpha)\right)-\bar{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right) \frac{d}{d \alpha}\left(\sigma_{i}^{-1}(\alpha)\right) \equiv 1$.
${ }^{\mathrm{d} \alpha}$ It follows from the previous identity that $\frac{d}{d \alpha}\left(\sigma_{i}^{-1}(\alpha)\right)=$ $\frac{1}{1-\dot{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right)}$. Hence,
$d\left(\sigma_{i}^{-1}(\alpha)\right)=\frac{1}{1-\dot{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right)} d \alpha$.
Let us consider the last two summands in (11) and show that

$$
\begin{align*}
\int_{0}^{t} g_{2}^{2}(\tau) F_{s}^{2}\left(\tau-T_{2}(\tau)\right) d \tau & =\int_{0}^{t-T_{2}(t)} \frac{g_{2}^{2}\left(\sigma_{2}^{-1}(\alpha)\right)}{1-\dot{T}_{2}\left(\sigma_{2}^{-1}(\alpha)\right)} F_{s}^{2}(\alpha) d \alpha \\
\int_{0}^{t} g_{1}^{2}(\tau) x_{m}^{2}\left(\tau-T_{1}(\tau)\right) d \tau & =\int_{0}^{t-T_{1}(t)} \frac{g_{1}^{2}\left(\sigma_{1}^{-1}(\alpha)\right)}{1-\dot{T}_{1}\left(\sigma_{1}^{-1}(\alpha)\right)} x_{m}^{2}(\alpha) d \alpha \tag{13}
\end{align*}
$$

First, consider the integral
$\int_{0}^{t}\left\{\frac{1}{2 b} g_{2}^{2}(\tau) F_{s}^{2}\left(\tau-T_{2}(\tau)\right)\right\} d \tau$.

In this integral, we make the variable substitution $\alpha \equiv \tau-T_{2}(\tau)$, where $\tau \equiv \sigma_{2}^{-1}(\alpha)$. After such a substitution, the integral (14) is calculated as follows:

$$
\begin{aligned}
& \int_{0}^{t}\left\{\frac{1}{2 b} g_{2}^{2}(\tau) F_{s}^{2}\left(\tau-T_{2}(\tau)\right)\right\} d \tau \\
& =\int_{-T_{2}(0)}^{t-T_{2}(t)}\left\{\frac{1}{2 b} g_{2}^{2}\left(\sigma_{2}^{-1}(\alpha)\right) F_{s}^{2}(\alpha)\right\} d\left(\sigma_{2}^{-1}(\alpha)\right) \\
& =\int_{-T_{2}(0)}^{t-T_{2}(t)}\left\{\frac{1}{2 b} g_{2}^{2}\left(\sigma_{2}^{-1}(\alpha)\right) F_{s}^{2}(\alpha)\right\} \frac{1}{1-\dot{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right)} d \alpha \\
& =\int_{0}^{t-T_{2}(t)} \frac{g_{2}^{2}\left(\sigma_{2}^{-1}(\alpha)\right)}{1-\dot{T}_{2}\left(\sigma_{2}^{-1}(\alpha)\right)} F_{s}^{2}(\alpha) d \alpha .
\end{aligned}
$$

Here, we took into account equality (12) and that $T_{2}(0)=0$. Therefore, we have proved the first equality in (13) while the second equality is derived analogously. Based on (13), the relationship (11) can be rewritten as:

$$
\begin{align*}
& \int_{0}^{t}\left(\dot{x}_{m}(\tau) F_{m}^{*}(\tau)-\dot{x}_{s}^{*}(\tau) F_{s}(\tau)\right) d \tau \\
& =\int_{0}^{t}\left\{\frac{b}{2} \dot{x}_{s}^{* 2}(\tau)+\frac{1}{2 b} F_{m}^{* 2}(\tau)\right\} d \tau+\int_{t-T_{1}(t)}^{t} \frac{b}{2} \dot{x}_{m}^{2}(\tau) d \tau \\
& \quad+\int_{t-T_{2}(t)}^{t} \frac{1}{2 b} F_{s}^{2}(\tau) d \tau+\frac{b}{2} \int_{0}^{t-T_{1}(t)} \xi_{1}(\alpha) \dot{x}_{m}^{2}(\alpha) d \alpha \\
& \quad+\frac{1}{2 b} \int_{0}^{t-T_{2}(t)} \xi_{2}(\alpha) F_{s}^{2}(\alpha) d \alpha, \tag{15}
\end{align*}
$$

where $\xi_{i}(\alpha)=\frac{1-\dot{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right)-g_{i}^{2}\left(\sigma_{i}^{-1}(\alpha)\right)}{1-\dot{T}_{i}\left(\sigma_{i}^{-1}(\alpha)\right)}, i=1,2$.
Hence, if the functions $g_{i}(t), t \geq 0, i=1,2$, satisfy the inequalities in (9), then $\xi_{i}(\alpha) \geq 0, \alpha \geq 0, i=1,2$, and it can be concluded from (15) that $\int_{0}^{t}\left(\dot{x}_{m}(\tau) F_{m}^{*}(\tau)-\dot{x}_{s}^{*}(\tau) F_{s}(\tau)\right) d \tau \geq 0, \forall t \geq$ 0 . The theorem is thus proven.

Remark. Eq. (7) defines the total energy for the configuration given by Fig. 1(c) while Eq. (11) defines the total energy for the configuration (Fig. 2(c)) proposed in this work. The difference between (7) and (11) is the presence of the variable gains $g_{i}(t), i=1,2$. In Theorem 1, we proved that Eq. (11) is equivalent to Eq. (15). Eqs. (11) and (15) are reduced to Eq. (7) when the gains $g_{i}(t)$ are equal to 1 .

Remark. The following rules can be used to satisfy the inequalities in (9):

$$
\begin{align*}
& g_{i}^{2}(t)=k_{i}\left(1-d T_{i}(t) / d t\right), \quad g_{i}(t) \geq 0 \forall t \geq 0, \quad i=1,2, \quad \text { or } \\
& g_{i}^{2}(t)=\min 1, k_{i}\left(1-d T_{i}(t) / d t\right), \quad g_{i}(t) \geq 0 \forall t \geq 0, \quad i=1,2, \tag{16}
\end{align*}
$$

with any fixed constants $0 \leq k_{i} \leq 1, i=1,2$.

### 4.2. Stability Property of the configuration in Fig. 2(b)

The theorem related to Fig. 2(b) will now be proved. Suppose that master and slave dynamics are described as:
$M_{m} \ddot{x}_{m}(t)+B_{m} \dot{x}_{m}(t)=F_{h}(t)+F_{\text {back }}(t)-F_{m}^{*}(t)$,
$M_{s} \ddot{x}_{s}^{*}(t)+B_{s} \dot{x}_{s}^{*}(t)=F_{s}(t)+F_{\text {feed }}(t)-F_{e}(t)$
where
$F_{\text {back }}(t)=K\left(x_{s}^{*}\left(t-T_{2}(t)\right)-x_{m}(t)\right)$,
$F_{\text {feed }}(t)=K\left(x_{m}\left(t-T_{1}(t)\right)-x_{s}^{*}(t)\right)$
and functions $F_{m}^{*}(t), F_{s}(t)$ are defined according to (10), with $\mathrm{g}_{i}(t)$, $t \geq 0, i=1,2$, satisfying (9).

Theorem 2. Suppose that Assumptions 1 and 2 hold true, and consider the system of (17) under the controls (10) and (18) satisfying condition (9), and
$K^{2}<\frac{\left(2 B_{m}+b \widetilde{\xi}_{1}\right)\left(2 B_{s}+b\right)}{\left(T_{1 \text { max }}+T_{2 \text { max }}\right)^{2}}$,
where $\tilde{\xi}_{1} \equiv \min 1, \xi_{1}(\tau), \tau \geq 0$. (Note that if the rules in (16) are used, then $\widetilde{\xi}_{1}=\left(1-k_{1}\right)$ ). Suppose a finite constant $d_{*}$ exists, such that
$\int_{0}^{t}\left(F_{e}(\tau) \dot{x}_{s}^{*}(\tau)-F_{h}(\tau) \dot{x}_{m}(\tau)\right) d \tau \geq d_{*}, \quad \forall t \geq 0$.
Then, the velocities $\dot{x}_{m}(t), \dot{x}_{s}^{*}(t)$, norms
$\left\|\dot{x}_{m}\right\|^{2}=\int_{0}^{t} \dot{x}_{m}^{2}(\tau) d \tau,\left\|\dot{x}_{s}^{*}\right\|^{2}=\int_{0}^{t} \dot{x}_{s}^{* 2}(\tau) d \tau$
and position tracking errors $e_{1}(t)=x_{m}\left(t-T_{1}(t)\right)-x_{s}^{*}(t), e_{2}(t)=$ $x_{s}^{*}\left(t-T_{2}(t)\right)-x_{m}(t), t \geq 0$, are bounded.

Proof. Assume $V: C \rightarrow R^{+}=[0, \infty)$ is a continuous semi-definite storage function for systems (17) (18) [39].
$V(t)=\frac{1}{2} M_{m} \dot{x}_{m}^{2}(t)+\frac{1}{2} M_{m} \dot{x}_{s}^{* 2}(t)+K\left(x_{m}(t)-x_{s}^{*}(t)\right)^{2}$.
Therefore,

$$
\begin{align*}
\dot{V}(t) & =\ddot{x}_{m}(t) M_{m} \dot{x}_{m}(t)+\ddot{x}_{s}^{*}(t) M_{s} \dot{x}_{s}^{*}(t) \\
& +K\left(\dot{x}_{m}(t)-\dot{x}_{s}^{*}(t)\right)\left(x_{m}(t)-x_{s}^{*}(t)\right) \\
& =\left(-B_{m} \dot{x}_{m}(t)+F_{h}(t)-F_{m}^{*}(t)+F_{\text {back }}(t)\right) \dot{x}_{m}(t) \\
& +\left(-B_{s} \dot{x}_{s}^{*}(t)+F_{s}(t)-F_{e}(t)+F_{\text {feed }}(t)\right) \dot{x}_{s}^{*} \\
& +\left(\dot{x}_{m}(t)-\dot{x}_{s}^{*}(t)\right) K\left(x_{m}(t)-x_{s}^{*}(t)\right) \\
& =-B_{m} \dot{x}_{m}^{2}(t)-B_{s} \dot{x}_{s}^{* 2}(t)+K\left(x_{s}^{*}\left(t-T_{2}(t)\right)-x_{s}^{*}(t)\right) \dot{x}_{m}(t) \\
& +K\left(x_{m}\left(t-T_{1}(t)\right)-x_{m}(t)\right) \dot{x}_{s}^{*}(t) \\
& +\left(F_{h}(t) \dot{x}_{m}(t)-F_{e}(t) \dot{x}_{s}(t)\right)+\left(-F_{m}^{*}(t) \dot{x}_{m}(t)+F_{s}(t) \dot{x}_{s}^{*}(t)\right) . \tag{23}
\end{align*}
$$

Integrating (21) over interval ( $0, \mathrm{t}$ ), we get

$$
\begin{align*}
& \int_{0}^{t} \dot{V}(\tau) d \tau=-B_{m}\left\|\dot{x}_{m}\right\|^{2}-B_{m}\left\|\dot{x}_{s}^{*}\right\|^{2} \\
& +K \int_{0}^{t}\left(x_{s}^{*}\left(\tau-T_{2}(\tau)\right)-x_{s}^{*}(\tau) \dot{x}_{m}(\tau) d \tau\right. \\
& +K \int_{0}^{t}\left(x_{m}\left(\tau-T_{1}(\tau)\right)-x_{m}(\tau)\right) \dot{x}_{s}^{*}(\tau) d \tau \\
& +\int_{0}^{t}\left(F_{h}(\tau) \dot{x}_{m}(\tau)-F_{e}(\tau) \dot{x}_{s}(\tau)\right) d \tau \\
& +\int_{0}^{t}\left(-F_{m}^{*}(\tau) \dot{x}_{m}(\tau)+F_{s}(\tau) \dot{x}_{s}^{*}(\tau)\right) d \tau . \tag{24}
\end{align*}
$$

where norms $\left\|\dot{x}_{m}\right\|^{2},\left\|\dot{x}_{s}^{*}\right\|^{2}$ are defined in (21).
Evaluate $\int_{0}^{t}\left(x_{s}^{*}\left(\tau-T_{2}(\tau)\right)-x_{s}^{*}(\tau)\right) \dot{x}_{m}(\tau) d \tau$, taking into account equality $\int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{*}(\tau-\mu) d \mu=x_{s}^{*}(\tau)-x_{s}^{*}\left(\tau-T_{2}(\tau)\right)$, to obtain $\int_{0}^{t}\left(x_{s}^{*}\left(\tau-T_{2}(\tau)\right)-x_{s}^{*}(\tau)\right) \dot{x}_{m}(\tau) d \tau=-\int_{0}^{t} \dot{x}_{m}(\tau) \int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{*}(\tau-$ $\mu) d \mu d \tau$.

Using the inequality $2 a^{T} b \leq \alpha_{2} a^{T} a+\frac{1}{\alpha_{2}} b^{T} b, \alpha_{2}>0$, and the above equation, we get the inequality

$$
\begin{align*}
& \int_{0}^{t}\left(x_{s}^{*}\left(\tau-T_{2}(\tau)\right)-x_{s}^{*}(\tau)\right) \dot{x}_{m}(\tau) d \tau \\
& \leq \frac{\alpha_{2}}{2} \int_{0}^{t} \dot{x}_{m}^{2}(\tau) d \tau+\frac{1}{2 \alpha_{2}} \int_{0}^{t}\left(\int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{*}(\tau-\mu) d \mu\right)^{2} d \tau \tag{25}
\end{align*}
$$

Using Holder's inequality, $\left(\int_{0}^{t} f(\tau) g(\tau) d \tau\right)^{2} \leq \int_{0}^{t} f^{2}(\tau) d \tau$ $\int_{0}^{t} g^{2}(\tau) d \tau$, we can evaluate the term $\left(\int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{*}(\tau-\mu) d \mu\right)^{2}$ as follows

$$
\begin{equation*}
\left(\int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{*}(\tau-\mu) d \mu\right)^{2} \leq T_{2}(\tau) \int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{* 2}(\tau-\mu) d \mu \tag{26}
\end{equation*}
$$

It follows from (25) and (26) that

$$
\begin{align*}
& \int_{0}^{t}\left(x_{s}^{*}\left(\tau-T_{2}(\tau)\right)-x_{s}^{*}(\tau)\right) \dot{x}_{m}(\tau) d \tau \\
& \leq \frac{\alpha_{2}}{2} \int_{0}^{t} \dot{x}_{m}^{2}(\tau)+\frac{1}{2 \alpha_{2}} \int_{0}^{t} T_{2}(\tau) \int_{0}^{T_{2}(\tau)} \dot{x}_{s}^{* 2}(\tau-\mu) d \mu d \tau \\
& \leq \frac{\alpha_{2}}{2}\left\|\dot{x}_{m}\right\|^{2}+\frac{1}{2 \alpha_{2}} \int_{0}^{T} T_{2 \max } \int_{0}^{T_{2} \max } \dot{x}_{s}^{* 2}(\tau-\mu) d \mu d \tau \\
& \leq \frac{\alpha_{2}}{2}\left\|\dot{x}_{m}\right\|^{2}+\frac{1}{2 \alpha_{2}} T_{2 \max } \int_{0}^{T_{2} \max } \int_{0}^{t} \dot{x}_{s}^{* 2}(\tau-\mu) d \tau d \mu \\
& \leq \frac{\alpha_{2}}{2}\left\|\dot{x}_{m}\right\|^{2}+\frac{1}{2 \alpha_{2}} T_{2 \max }^{2}\left\|\dot{x}_{s}^{*}\right\|^{2} \tag{27}
\end{align*}
$$

Similarly, it can be shown that
$\int_{0}^{t}\left(x_{m}\left(\tau-T_{1}(\tau)\right)-x_{m}(\tau)\right) \dot{x}_{s}^{*}(\tau) d \tau \leq \frac{\alpha_{1}}{2}\left\|\dot{x}_{s}^{*}\right\|^{2}+\frac{1}{2 \alpha_{1}} T_{1 \max }^{2}\left\|\dot{x}_{m}\right\|^{2}$.

It follows from (20), (27), (28), (24) and (15) that

$$
\begin{aligned}
& \int_{0}^{t} \dot{V}(\tau) d \tau \leq-B_{m}\left\|\dot{x}_{m}\right\|^{2}-B\left\|\dot{x}_{s}^{*}\right\|^{2}+K\left(\frac{\alpha_{1}}{2}\left\|\dot{x}_{s}^{*}\right\|^{2}\right. \\
& \left.+\frac{1}{2 \alpha_{1}} T_{1 \max }^{2}\left\|\dot{x}_{m}\right\|^{2}\right)+K\left(\frac{\alpha_{2}}{2}\left\|\dot{x}_{m}\right\|^{2}+\frac{1}{2 \alpha_{2}} T_{2 \max }^{2}\left\|\dot{x}_{s}^{*}\right\|^{2}\right) \\
& -\int_{0}^{t}\left\{\frac{b}{2} \dot{x}_{s}^{* 2}(\tau)+\frac{1}{2 b} F_{m}^{* 2}(\tau)\right\} d \tau-\int_{t-T_{1}(t)}^{t} \frac{b}{2} \dot{x}_{m}^{2}(\tau) d \tau \\
& -\int_{t-T_{2}(t)}^{t} \frac{1}{2 b} F_{s}^{2}(\tau) d \tau-\frac{b}{2} \int_{0}^{t-T_{1}(t)} \xi_{1}(\alpha) \dot{x}_{m}^{2}(\alpha) d \alpha \\
& =\int_{0}^{t}\left[-B_{m}+K T_{1 \max }^{2} \frac{1}{2 \alpha_{1}}+K \frac{\alpha_{2}}{2}-\frac{b}{2} \bar{\xi}_{1}(t, \tau)\right] \dot{x}_{m}^{2}(\tau) d \tau \\
& +\int_{0}^{t}\left[-B_{s}+K T_{2 \max }^{2} \frac{1}{2 \alpha_{2}}+K \frac{\alpha_{1}}{2}-\frac{b}{2}\right] \dot{x}_{s}^{* 2}(\tau) d \tau \\
& -\frac{1}{2 b} \int_{0}^{t} F_{m}^{* 2}(\tau) d \tau-\frac{1}{2 b} \int_{0}^{t} \bar{\xi}_{2}(t, \tau) F_{s}^{2}(\tau) d \tau-d_{*}
\end{aligned}
$$

where $\bar{\xi}_{i}(t, \tau)=\xi_{i}(\tau) \geq 0$ if $\tau \leq t-T_{i}(t), \bar{\xi}_{i}(t, \tau)=1 \mathrm{if} \tau>$ $t-T_{i}(t), t \geq 0, i=1,2$.

Suppose gain $K>0$ and satisfies the inequalities,
$\widetilde{B}_{m}:=-B_{m}+\frac{1}{2} K\left(\alpha_{2}+\frac{T_{1 \text { max }}^{2}}{\alpha_{1}}\right)-\frac{b}{2} \widetilde{\xi}_{1}<0$,
$\widetilde{B}_{s}:=-B_{s}+\frac{1}{2} K\left(\alpha_{1}+\frac{T_{2 \text { max }}^{2}}{\alpha_{2}}\right)-\frac{b}{2}<0$,
where $\tilde{\xi}_{1}=\min 1, \xi_{1}(\tau), \tau \geq 0$.

It follows from the latter inequalities that
$K<K^{*}\left(\alpha_{1}, \alpha_{2}\right):=\min \left\{\frac{\left(2 B_{m}+b \widetilde{\xi}_{1}\right) \alpha_{1}}{T_{1 \text { max }}^{2}+\alpha_{1} \alpha_{2}}, \frac{\left(2 B_{s}+b\right) \alpha_{2}}{T_{2 \text { max }}^{2}+\alpha_{1} \alpha_{2}}\right\}$.
It can then be shown that $\frac{\left(2 B_{m}+b \tilde{b}_{1}\right)\left(2 B_{s}+b\right)}{\left(T_{1 \text { max }}+T_{2} \max \right)^{2}}=$ $\max _{\alpha_{1}>0, \alpha_{2}>0} K^{*}\left(\alpha_{1}, \alpha_{2}\right)$.

Let ( $\alpha_{1}^{*}, \alpha_{2}^{*}$ ) be a solution of the latter optimization problem. Without losing generality, consider that the parameters $\alpha_{1}, \alpha_{2}$ take their best values, i.e. $\alpha_{1}=\alpha_{1}^{*}, \alpha_{2}=\alpha_{2}^{*}$. Inequality (29) is then equivalent to inequality (19). Hence,

$$
\begin{aligned}
\int_{0}^{t} \dot{V}(\tau) d \tau \leq & \widetilde{B}_{m}\left\|\dot{x}_{m}\right\|^{2}+\widetilde{B}_{s}\left\|\dot{x}_{s}^{*}\right\|^{2}-\frac{1}{2 b} \int_{0}^{t} F_{m}^{* 2}(\tau) d \tau \\
& -\frac{1}{2 b} \int_{0}^{t} \bar{\xi}_{2}(t, \tau) F_{s}^{2}(\tau) d \tau-d_{*}
\end{aligned}
$$

where $\widetilde{B}_{m}<0, \widetilde{B}_{s}<0, \bar{\xi}_{2}(t, \tau) \geq 0, \tau \geq 0$. Therefore,
$V(t) \leq V(0)+\widetilde{B}_{m}\left\|\dot{x}_{m}\right\|^{2}+\widetilde{B}_{s}\left\|\dot{x}_{s}^{*}\right\|^{2}-d_{*}$
where $V(t), V(0)$ are positive, $\widetilde{B}_{m}\left\|\dot{x}_{m}\right\|^{2}, \widetilde{B}_{s}\left\|\dot{x}_{s}^{*}\right\|^{2}$ are negative, and $d_{*}$ is a constant. It can be concluded from (22) and (30) that functions $\dot{x}_{m}(t), \dot{x}_{s}^{*}(t)$, and $\left(x_{m}(t)-x_{s}^{*}(t)\right), t \geq 0$, and norms $\left\|\dot{x}_{m}\right\|^{2},\left\|\dot{x}_{s}^{*}\right\|^{2}$ are bounded.

Note that, from (30), it follows that $\left(x_{m}(t)-x_{s}^{*}(t)\right)^{2} \leq\left(x_{m}(0)-\right.$ $\left.x_{s}^{*}(0)\right)^{2}+\frac{M_{m} \dot{x}_{m}^{2}(0)+M_{s} \dot{x}_{s}^{* 2}(0)-d_{*}}{2 K}$.

Then, to show that the tracking errors $e_{1}(t)=x_{m}\left(t-T_{1}(t)\right)-$ $x_{s}^{*}(t), e_{2}(t)=x_{s}^{*}\left(t-T_{2}(t)\right)-x_{m}(t), t \geq 0$, are bounded, the errors are presented in the following forms:

$$
\begin{align*}
e_{1}(t) & =x_{m}\left(t-T_{1}(t)\right)-x_{s}^{*}(t)=x_{m}(t)-x_{s}^{*}(t)-\int_{t-T_{1}(t)}^{t} \dot{x}_{m}(\tau) d \tau \\
e_{2}(t) & =x_{s}^{*}\left(t-T_{2}(t)\right)-x_{m}(t) \\
& =x_{s}^{*}(t)-x_{m}(t)-\int_{t-T_{2}(t)}^{t} \dot{x}_{s}^{*}(\tau) d \tau, \quad t \geq 0 \tag{31}
\end{align*}
$$

The functions $\dot{x}_{m}(t), \dot{x}_{s}^{*}(t), e(t)=\left(x_{m}(t)-x_{s}^{*}(t)\right), T_{i}(t), t \geq 0$, are bounded; consequently, from (31), it follows that $e_{i}(t), t \geq 0, i=$ 1,2 , are also bounded. The theorem is thus proven.

### 4.3. Some additional properties of the proposed configuration

Theorem 3. Let the assumptions of Theorem 2 be fulfilled.
(A) If the human operator and slave environment satisfy the inequalities
$\left|F_{h}(t)\right| \leq g_{h}\left(\dot{x}_{m}(t)\right),\left|F_{e}(t)\right| \leq g_{e}\left(\dot{x}_{s}^{*}(t)\right), \quad t \geq 0$,
where $g_{h}(s), g_{e}(s), s \in \mathbb{R}$, are continuous functions, then
$\dot{x}_{m}(t) \rightarrow 0, \quad \dot{x}_{s}^{*}(t) \rightarrow 0, \quad \dot{e}_{1}(t) \rightarrow 0, \quad \dot{e}_{2}(t) \rightarrow 0$ when $t \rightarrow \infty$.
(B) If $F_{h}(t)=0, F_{e}(t)=0 \forall t \geq 0$ then $e_{1}(t) \rightarrow 0, e_{2}(t) \rightarrow 0$ when $t \rightarrow \infty$.
(C) If $\dot{x}_{m}(t) \rightarrow 0, \ddot{\chi}_{m}(t) \rightarrow 0, \dot{\chi}_{s}^{*}(t) \rightarrow 0, \ddot{x}_{s}^{*}(t) \rightarrow 0$ when $t \rightarrow \infty$, then $F_{h}(t) \rightarrow F_{e}(t) \rightarrow-K\left(x_{m}(t)-x_{s}^{*}(t)\right)$ when $t \rightarrow \infty$.

Proof. (A). Show that functions $\frac{d}{d t} \dot{x}_{m}(t)=\ddot{x}_{m}(t), \frac{d}{d t} \dot{x}_{s}^{*}(t)=\ddot{x}_{s}^{*}(t)$, $t \geq 0$ are bounded. From (17) and (18), we have
$\ddot{x}_{m}(t)=\frac{1}{M_{m}}\left(-B_{m} \dot{x}_{m}(t)+F_{h}(t)+K e_{2}(t)-F_{m}^{*}(t)\right)$.

Relations (10), (32), and (34), and the boundedness of $\dot{x}_{m}(t)$, $\dot{x}_{s}^{*}(t), e_{2}(t)$ imply the boundedness of $\ddot{x}_{m}(t), t \geq 0$. Similarly, it can be shown that function $\ddot{x}_{s}^{*}(t), t \geq 0$, is also bounded. The boundedness of functions $\dot{x}_{m}(t), \dot{x}_{s}^{*}(t), \ddot{x}_{m}(t), \ddot{x}_{s}^{*}(t), \int_{0}^{t} \dot{x}_{m}^{2}(\tau) d \tau, \int_{0}^{t} \dot{x}_{s}^{* 2}(\tau) d \tau$ $\forall t \geq 0$, implies relation (33).

Proof. (B). From $F_{h}(t)=0, F_{e}(t)=0 \forall t \geq 0$, we have
$\ddot{\chi}_{m}(t)=\frac{1}{M_{m}}\left(-B_{m} \dot{x}_{m}(t)+K\left(x_{s}^{*}(t)-x_{m}(t)-\int_{t-T_{2}(t)}^{t} \dot{x}_{s}^{*}(\tau) d \tau\right)-F_{m}^{*}(t)\right)$,
$\dddot{x}_{s}^{*}(t)=\frac{1}{M_{s}}\left(-B_{s} \dot{x}_{s}^{*}(t)+K\left(x_{m}(t)-x_{s}^{*}(t)-\int_{t-T_{1}(t)}^{t} \dot{x}_{m}(\tau) d \tau\right)+F_{s}(t)\right)$.
Hence,
$\ddot{e}(t)=-S^{2} e(t)+q(t), \quad t \geq 0$,
where $e(t)=x_{m}(t)-x_{s}^{*}(t), S^{2}=K\left(\frac{1}{M_{m}}+\frac{1}{M_{s}}\right)$,

$$
\begin{aligned}
q(t)= & \frac{1}{M_{m}}\left(-B_{m} \dot{x}_{m}(t)-K \int_{t-T_{2}(t)}^{t} \dot{x}_{s}^{*}(\tau) d \tau-F_{m}^{*}(t)\right) \\
& -\frac{1}{M_{s}}\left(-B_{s} \dot{x}_{s}^{*}(t)-K \int_{t-T_{1}(t)}^{t} \dot{x}_{m}(\tau) d \tau+F_{s}(t)\right) .
\end{aligned}
$$

It follows from assertion (A) of this corollary that
$\dot{e}(t) \rightarrow 0, q(t) \rightarrow 0$ when $t \rightarrow \infty$.
It follows from (35) that for any $t \geq 0$ and $\Delta t \geq 0$,

$$
\begin{align*}
\dot{e}(t+\Delta t)= & -S \sin (S \Delta t) e(t)+\cos (S \Delta t) \dot{e}(t) \\
& +\int_{t}^{t+\Delta t} \cos (S(t+\Delta t-\tau)) q(\tau) d \tau \tag{37}
\end{align*}
$$

Suppose that $e(t), t \geq 0$, does not converge to 0 . (Note that the function $e(t), t \geq 0$, is bounded.) Then there exists a sequence $t_{i}, i=1,2, \ldots$, such that
$t_{i} \rightarrow \infty, \quad e\left(t_{i}\right) \rightarrow \beta \neq 0$ when $i \rightarrow \infty$.
Declaring that $\Delta t=\frac{\pi}{2 S}$, we get form (37)
$\dot{e}\left(t_{i}+\frac{\pi}{2 S}\right)=-S e\left(t_{i}\right)-\int_{t_{i}}^{t_{i}+\pi / 2 S} \sin \left(S\left(t_{i}-\tau\right)\right) q(\tau) d \tau, \quad i=1,2, \ldots$.

Passing to the limit in (39) and taking into account conditions (36) and (38), we obtain $0=-S \beta$. However, $S>0$, and by assumption $\beta \neq 0$. This contradiction proves that $e(t) \rightarrow 0$ when $t \rightarrow \infty$. Consequently $e_{1}(t) \rightarrow 0, e_{2}(t) \rightarrow 0$ if $t \rightarrow \infty$.

Proof. (C). Suppose that $\dot{x}_{m}(t) \rightarrow 0, \ddot{x}_{m}(t) \rightarrow 0, \dot{x}_{s}^{*}(t) \rightarrow$ $0, \ddot{x}_{s}^{*}(t) \rightarrow 0$ when $t \rightarrow \infty$. Then, $x_{m}\left(t-T_{1}(t)\right) \rightarrow x_{m}(t), x_{s}^{*}(t-$ $\left.T_{2}(t)\right) \rightarrow x_{s}^{*}(t)$, and from system dynamic (17), (18) and controls (11) and (19) we have $F_{h}(t) \rightarrow F_{e}(t) \rightarrow K\left(x_{m}(t)-x_{s}^{*}(t)\right)$ when $t \rightarrow \infty$. The corollary is thus proven.

## 5. Simulation

The proposed modified control architecture introduces timevarying gains into the transformation and explicitly uses the position data from the master to generate feedforward and feedback position control for the slave manipulator. In order to verify the efficacy of the proposed scheme, simulations to investigate the dependence of position tracking and force tracking on the parameter gain $K$ and impedance parameter $b$ were performed on a 1-DOF system (10), (17), (18).

Different values of $K$ and $b$ were used, where $b$ was chosen as arbitrary and non-negative, while $K$ is a positive number with an upper bound $\sqrt{K_{*}(b)}, K^{2}<K_{*}(b):=\frac{\left(2 B_{m}+b \tilde{\xi}_{1}\right)\left(2 B_{s}+b\right)}{\left(T_{1} \max +T_{2} \max \right)^{2}}$. If $b$ is large enough, then gain $K$ can be set to a large value, noting that $K_{*}(b) \rightarrow \infty$ when $b \rightarrow \infty$.

### 5.1. Position and force tracking error simulation

Three models (I-III) were set up for simulation with different combinations of human $\left(F_{h}(t)\right)$ and environmental $\left(F_{e}(t)\right)$ forces modeled as a spring and a damper, respectively. These models were designed to focus on the position and force tracking errors for the master and the slave, in addition to checking the compliance of the passivity condition (20).

The setup for Models I to III use $x_{m}(0)=-5, \dot{x}_{m}(0)=$ $10, x_{s}^{*}(0)=10, \dot{x}_{s}^{*}=0$ as the initial conditions, and in system (17) we set $M_{m}=M_{s}=B_{m}=B_{s}=1$. The human, $F_{h}(t), t \geq 0$, and environmental, $F_{e}(t), t \geq 0$, forces are modeled as follows:
for Model I: $F_{h}(t)=30 / x_{m}(t)-x_{m}(t)-2.5, \quad F_{e}(t)=x_{s}^{*}(t)$,
for Model II: $F_{h}(t)=x_{m}(t)-0.5 \dot{x}_{m}(t)-10-\sin t$,

$$
F_{e}(t)=5 x_{s}^{*}(t)+\dot{x}_{s}^{*}(t),
$$

for Model III: $F_{h}(t)=-x_{m}(t)-0.5 \dot{x}_{m}(t)-10$,

$$
F_{e}(t)=5 x_{s}^{*}(t)+\dot{x}_{s}^{*}(t) .
$$

The time-delay functions of Models I to III are the following
$T_{1}(t)=\frac{2 t}{5}+1, \quad t \in[0,10], \quad T_{1}(t)=\frac{-2 t}{5}+9, \quad t \in[10,20]$,
$T_{1}(t)=\frac{t}{5}-3, \quad t \in[20,30]$,
$T_{1}(t)=\frac{-t}{5}+9, \quad t \in[30,40]$,
$T_{1}(t)=\frac{2 t}{5}-15, \quad t \in[40,50]$,
$T_{1}(t)=\frac{-2 t}{5}+25, \quad t \in[50,60]$,
$T_{1}(t)=\frac{t}{5}-11, \quad t \in[60,70]$,
$T_{1}(t)=\frac{-3 t}{20}+\frac{27}{2}, \quad t \in[70,90]$,
$T_{1}(t)=\frac{t}{5}-18, \quad t \in[90,100] ;$
$T_{2}(t)=\frac{-t}{20}+1, \quad t \in[0,10]$,
$T_{2}(t)=\frac{t}{20}, \quad t \in[10,20]$,
$T_{2}(t)=\frac{3 t}{10}-5, \quad t \in[20,30]$,
$T_{2}(t)=\frac{-3 t}{20}+\frac{17}{2}, \quad t \in[30,50]$,
$T_{2}(t)=\frac{t}{10}-4, \quad t \in[50,100]$.
Different values for $K$ and $b$ were used to investigate the passivity and tracking performance using the proposed scheme, and the results for varying $K$ and $b$ values are shown in Fig. 3, 4, and 5. The position tracking is given in column (A) of Figs. 3-5 with the plots for $x_{m}\left(t-T_{1}(t)\right), x_{s}(t), t \in[0100]$, while plots in column (B) show the force tracking for the different $\mathrm{F}_{e}(t)$ and $\mathrm{F}_{h}(t)$ of each model, i.e. Fig. 3 for Model I, Fig. 4 for Model II, and Fig. 5 for Model III.

It can be analytically shown that for all values of $K$ and $b$, Models I and III satisfy the passivity condition (20). For $K$ and $b$ presented in Fig. 4, the simulation results showed that Model II also satisfies the


Fig. 3. Model I (A) Position \& (B) Force tracking.
passivity condition (20). However, there exists parameter values, e.g. $K=1, b=50$, such that the passivity condition (20) is not satisfied for Model II.

The simulation results for position tracking showed that there is a function $K^{*}(b), K_{*}(b)<K^{*}(b)<\infty$ such that for $K<K^{*}(b)$,
the limits $e_{i}^{*}(K), i=1,2$ : exists:
$\lim _{t \rightarrow \infty}\left|e_{i}(t)\right| \rightarrow e_{i}^{*}(K), \quad i=1,2$,
which depend on $K$. However, the speed of the convergence in (40) depends on $b$. For $K>K^{*}(b)$, no limit exists in (40). If $b$ is fixed,


Fig. 4. Model II (A) Position \& (B) Force tracking
a larger $K$ value corresponds to a smaller limit of $e_{i}^{*}(K), i=1,2$ \& larger vibration in
$x_{m}(t), x_{s}^{*}(t), e_{i}(t), \quad i=1,2$.
If $K$ is fixed, a larger $b$ value corresponds to a smaller vibration in (41) (i.e., the functions are more stable), and limits in (40) converge slowly. If $b$ is small, it is not possible to make the limits $e_{i}^{*}(K), i=$

1, 2 small, due to the restriction $K^{2}<K_{*}(b)$ on $K$. Therefore, large values for $K$ and $b$ are needed in order to achieve good position tracking.

Simulation results for force tracking showed that the limit
$\lim \left(F_{h}(t)-F_{e}(t)\right)$ when $t \rightarrow \infty$
depends on $K$ and $b$.


Fig. 5. Model III (A) Position \& (B) Force tracking.

If $b$ is fixed and $K$ decreases, then the force tracking error is improved and vibration in $F_{h}(t), F_{e}(t)$ is smaller. However, notice that the force tracking error can sometimes deteriorate if $K$ is too small. This can be explained by the following:

- for small values of $K$, position tracing error can be too large,
- forces $F_{h}(t), F_{s}(t)$ depend on positions $x_{m}(t), x_{s}(t)$.

For a fixed parameter $K$, the force tracking error is enhanced if $b$ is smaller (but the inequality $K<K^{*}(b)$ should be satisfied). If parameter value $b$ is large, it is not possible to achieve
$F_{h}(t)-F_{e}(t) \rightarrow 0$ when $t \rightarrow \infty$
Hence for (43) we need a small $b$.

## (A) Position Tracking Error <br> (B) Force Tracking Error








Fig. 6. Models IV-VI (A) Position \& (B) Force tracking errors.

The simulation results shows a trade-off between position and force tracking.
5.2. Investigation of position and force tracking dependence on gain coefficient values

To illustrate the advantage of using varying gain coefficients $g_{i}(t)<1, i=1,2$ over the traditionally used constant gain coefficients, let us present the results of the following experiment.

Using the same set of data, we simulate with the constant gain coefficients
$g_{i}(t)=1, \quad i=1,2$
which are used, e.g. in $[40,41]$, and with the varying gain coefficients proposed here:
$g_{i}(t)=\sqrt{1-\dot{T}_{i}(t)}$ if $\dot{T}_{i}(t)>0, \quad g_{i}(t)=1$ if $\dot{T}_{i}(t) \leq 0, i=1,2$.

In simulations, we consider different forces $F_{h}(t), F_{e}(t)$ and different values for $K, b$. The initial states $x_{m}(0)=-5, \dot{\chi}_{m}(0)=$ $-10, x_{s}(0)=10, \dot{x}_{s}(0)=50 \& M_{m}=M_{s}=1, B_{m}=B_{s}=0.1$ are used for all models.

Three Models (IV-VI) were considered with:
for Model IV: $F_{h}(t)=-3$,

$$
F_{e}(t)=-\left(5 x_{s}(t)+\dot{x}_{s}(t)\right), \quad K=4, b=10,
$$

for Mode lV: $F_{h}(t)=0.01\left(-\dot{x}_{m}(t)-x_{m}(t)-3\right)$,

$$
F_{e}(t)=0.01\left(5 x(t)-\dot{x}_{s}(t)\right), \quad K=0.2, b=2,
$$

for Model VI: $F_{h}(t)=-3.8\left(x_{m}(t)-\dot{x}_{m}(t)-3\right)$,

$$
F_{e}(t)=0.01\left(5 x(t)+\dot{x}_{s}(t)\right), \quad K=1, b=4
$$

The time-delay functions for Models IV-VI are:


Fig. 7. Schematic of Models I-III.
$T_{1}(t)=0.9 t-36 i, \quad t \in[40 i, 40 i+20], \quad i=0,1,2 ;$
$T_{1}(t)=-0.9 t+18 i+18, \quad t \in[20 i, 20 i+20], \quad i=1,2 ;$
$T_{2}(t)=0.95(t-30 i), \quad t \in[30 i, 30 i+20], \quad i=0,1,2 ;$
$T_{2}(t)=-1.9(t-30 i-20)+19, \quad t \in[30 i+20,30 i+00]$,

$$
i=0,1,2
$$

$T_{2}(t)=0.95(t-90), \quad t \in[90,100]$.
The results of the experiments are presented in Fig. 6, wherein the position tracking error is given in column (A) through plots of $\mathrm{x}_{m}\left(t-T_{1}(t)\right)-\mathrm{x}_{s}(t)$ with solid lines referring to the known Rule (44), while the proposed Rule (45) is represented by dashed lines. On the other hand, the force tracking error is shown in column (B) through the plots for $F_{h}(t)-F_{e}(t)$ with the Rule (44) and Rule (45) representation. It can be seen that the position and force tracking
performance of the proposed Rule (45) are superior to those of the known Rule (44). The schematics for Models I-III and Models IV-VI are given in Fig. 7 and Fig. 8 for reference, respectively.

## 6. Conclusion

A new modified control architecture composed of two major parts that can maintain passivity in a bilateral teleoperator was proposed with the following characteristics:

- a new architecture for varying time delay by using position data from the master feedforward and feedback;
- mathematical proof of the passivity for the proposed control architecture through Lyapunov-like function theorems; and
- explicit simulation experiments for position tracking control and force tracking control by using various models with various gains.

Passivity is preserved by dissipating energy via two time-varying gains inserted after the communication blocks on both the master


Fig. 8. Schematic of Models IV-VI.
and the slave sides. This architecture explicitly uses the position data from the master to generate feedforward and feedback position control to preserve passivity and for good tracking performance.

The simulation results for the proposed configuration, which modeled human and environmental forces as a spring and a damper, showed that passivity and good tracking performance can be obtained. The results verified the dependence of the position and force tracking errors on the parameter gain $K$ and the impedance parameter $b$. They also demonstrated the resulting trade-off between position tracking and force tracking, both of which depend on the value combinations of $K$ and $b$ to obtain satisfactory force tracking, position tracking or both. The feedback control scheme was developed to improve the position tracking performance of a system without a constant delay (without destabilization). A change in the value of the time-delay in a variable timedelay scenario will lead to changes in the velocity, force, and data which in turn will result in a lower tracking performance. Thus, extra effort should be used to improve the tracking performance.

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## Conflict of interest

None.

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