МАТЕМАТИКА, АЛГОРИТМЫ

ON THE ALGORITHM OF EXTENSION OF A COORDINATE NEIGHBORHOOD ON A COMPACT MANIFOLD

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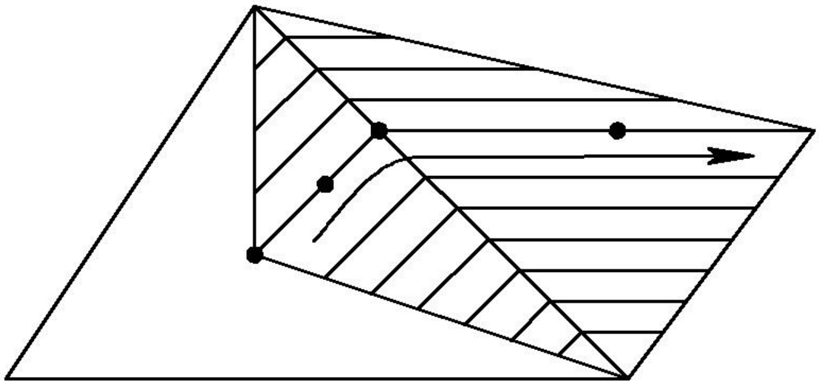
Let M^n be a connected, compact, closed and smooth manifold of dimension *n.* It is well known that there exists a smooth triangulation of the manifold M^n. Let δ_0^n be some simplex of the triangulation. We paint the inner part $Int\,\delta_0^n$ of the simplex δ_0^n white and the boundary $\partial\delta_0^n$ of δ_0^n black. There exist coordinates on $Int\,\delta_0^n$ given by some diffeomorphism φ_0. A subsimplex $\delta_{01}^n \subset \delta_0^n$ is defined by a black face $\delta_{01}^{n-1} \subset \delta_0^n$ and the center c_0 of δ_0^n. We connect c_0 with the center d_0 of the face δ_{01}^{n-1} and decompose the subsimplex δ_{01}^n as a set of intervals which are parallel to the interval c_0d_0. The face δ_{01}^{n-1} is a face of some simplex δ_1^n that has not been painted. We draw an interval between d_0 and the vertex v_1 of the subsimplex δ_1^n which is opposite to the face δ_{01}^{n-1} then we decompose δ_1^n as a set of intervals which are parallel to the interval $d_0 v_1$. The set $\delta_{01}^n \cup \delta_1^n$ is a union of such broken lines every one from which consists of two intervals where the endpoint of the first interval coincides with the beginning of the second interval (in the face δ_{01}^{n-1}) the first interval belongs to δ_{01}^n and the second interval belongs to δ_1^n. We construct a homeomorphism (extension) $\varphi_{01}^1 : Int\,\delta_{01}^n \to Int(\delta_{01}^n \cup \delta_1^n)$. Let us consider a point $x \in Int\,\delta_{01}^n$ and let *x* belong to a broken line consisting of two intervals the first interval is of a length of s_1 and the second interval is of a length of s_2 and let *x* be at a distance of *s* from the beginning of the first interval. Then we suppose that $\varphi_{01}^1(x)$ belongs to the same broken line at a distance of $\dfrac{s_1 + s_2}{s_1} \cdot s$ from the beginning of the first interval. It is clear that φ_{01}^1 is a homeomorphism giving coordinates on $Int(\delta_{01}^n \cup \delta_1^n)$. We paint points of $Int(\delta_{01}^n \cup \delta_1^n)$ white. Assuming the coordinates of points of white initial faces of subsimplex δ_{01}^n to be fixed we obtain correctly introduced coordinates on $Int(\delta_0^n \cup \delta_1^n)$. The set $\sigma_1 = \delta_0^n \cup \delta_1^n$ is called a *canonical polyhedron*. We paint faces of the boundary $\partial\sigma_1$ black. The situation in dimension two can be illustrated below.



We describe the contents of the successive step of the algorithm of extension of coordinate neighborhood. Let us have a canonical polyhedron σ_{k-1} with white inner points (they have introduced *white coordinates*) and the black boundary $\partial\sigma_{k-1}$. We look for such an *n*–simplex in σ_{k-1}, let it be δ_0^n that has such a black face, let it be δ_{01}^{n-1} that is simultaneously a face of some *n*–simplex, let it be δ_1^n, inner points of which are not painted. Then we apply the procedure described above to the pair δ_0^n, δ_1^n. As a result we have a polyhedron σ_k with one simplex more than σ_{k-1} has. Points of $Int\ \sigma_k$ are painted white and the boundary $\partial\sigma_k$ is painted black. The process is finished in the case when all the black faces of the last polyhedron border on the set of white points (the cell) from two sides.

After that all the points of the manifold M^n are painted in black or white, otherwise we would have that $M^n = M_0^n \cup M_1^n$ (the points of M_0^n would be painted and those of M_1^n would be not) with M_0^n and M_1^n being unconnected, which would contradict of connectivity of M^n.

Thus, we have proved the following

Theorem. *Let M^n be a connected, compact, closed, smooth manifold of dimension n. Then $M^n = C^n \cup K^{n-1}$, $C^n \cap K^{n-1} = \varnothing$, where C^n is an n–dimensional cell and K^{n-1} is a union of some finite number of (n–1)–simplexes of the triangulation.*

Список использованных источников:

1. Ermolitski, A.A. On a geometric black hole of a compact manifold, arXiv : 0901.0528 / A. A. Ermolitski. – 2009.