LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

MASTER
MATHEMATICAL FINANCE

MASTER´S FINAL WORK
DISSERTATION

ASSET-LIABILITY MANAGEMENT IN PENSION FUNDS

TIAGO LIMA DE CARVALHO

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SUPERVISION
ONOFRE ALVES SIMÕES

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GLOSSARY

ALM – Asset Liability Management.
DB – Defined Benefit
DC – Defined Contribution
FR – Funding Ratio
GDP – Gross Domestic Product
LBP – Liability Benchmark Portfolio
LDI – Liability Driven Investment.

ABSTRACT

Pension funds have a very representative role in the financial markets, considering investments made and the asset allocations profile. In defined benefit pension schemes, the major focus is to secure the participants future payments with the accumulated contributions. Or, in other words, to cover the liabilities with the assets.

Asset Liability Management (ALM) is a collection of methodologies and tools structured to guide the assets investments in order to protect the liabilities. This concept has been used largely in insurance companies and pension funds. It analyzes market expectations, scheme risks and objectives, in order to create the best asset investment option.

The purpose of this project is, using a Liability Driven Investment (LDI) technique, recover the Funding Ratio of a pension fund, achieve the scheme goals and minimize the risk. Project liabilities, interest rate and inflation are the bases of this work. Build the asset portfolio, project the fund cashflow and track the risk are the principal steps to achieve the goal. To check the results adherence, the output will be compared with a bold recovery strategy.

To conclude, after setting the context (theoretical and practical perspectives), the work will show how to recover a Funding Ratio using a developed model and keeping the risk inside pension plan limits. However, comparing with a bolder strategy, the recovery from a bigger Funding Ratio gap will call for more risk and the ALM model chosen maybe not so adequate.

KEYWORDS: Defined Benefit Schemes; Asset Liability Management; Funding Ratio; Liability Driven Investment; Portfolio Selection

RESUMO

Os fundos de pensão têm uma participação representativa nos mercados financeiros, seja considerando o capital investido ou o perfil de escolha de ativos. Nos planos de pensão de benefício definido, o foco é assegurar os pagamentos futuros, definidos previamente, dos participantes de acordo com o capital acumulado no tempo, em outras palavras, cobrir os passivos com os ativos existentes.

A gestão de ativos e passivos (em inglês ALM) é o conjunto de métodos e ferramentas projetadas com a finalidade de orientar como os fundos devem investir seus ativos a fim de que, em determinada data, seja possível pagar seus passivos. Este conceito é amplamente utilizado em empresas seguradoras e fundos de pensão. O portfolio de investimentos é construído de acordo com análises de mercado, definição dos riscos em que o fundo deseja se expor e os objetivos de retorno.

O propósito deste projeto é, aplicando a teoria de investimentos orientados a passivos, recuperar o nível de financiamento de um fundo de pensões, a fim de cumprir com as metas do esquema e se expondo ao menor risco possível. Este projeto terá como informação base a estimativa dos passivos, da taxa de juros e da inflação. A partir deles, contruiremos o portfolio de investimentos, projetaremos o fluxo de caixa e monitoraremos o risco de não cumprimento dos objetivos. Para validar a consistência do modelo, iremos comparar contra uma estratégia mais arriscada.

As conclusões, após contextualização (prática e teórica), demonstram que é possível recuperar o nível de financiamento, de acordo com prazos estabelecidos e com um nível moderado de risco. Entretanto, quando comparamos com a estratégia arriscada, pode-se entender que para recuperar grandes diferenças no financiamento será preciso assumir maiores riscos em investimentos, portanto o modelo de ALM escolhido pode não ser o ideal.

PALAVRAS-CHAVE: Fundos de Pensão de Benefício Definido; Gestão de Ativos e Passivos; Nível de financiamento; Investimentos Orientados por Passivos; Seleção de Portfólio

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1. INTRODUCTION

1.1. Motivation

During my master's in Mathematical Finance, topics involving optimal portfolio theory, stochastic processes and assets pricing were the most interesting, in my point of view. After the first year, I started to work with pension plans and pension funds. This woke up the issue about the importance and complexity of connecting assets and liabilities in the framework of a pension scheme. So, the challenge here is not only how to obtain the highest return with a portfolio of assets, but also how to build the portfolio that best covers the liabilities when they enter in payment. Liabilities that depend on a large number of variables, in particular, on the longevity of pensioners, which is now a sensitive concern.

At present, longevity increases and reduced rates of return on investments are the origin of more and more discussions about the capability of assets to cover liabilities. Covering liabilities is an essential goal from investments perspective and a challenge. Today, on average, OECD members have above 100% of funding in pension schemes (OECD, 2019), but there are cases below this level. One of them is the UK, where total assets in defined benefit pension funds do not cover total liabilities, with around 13 billion pounds of gap (Pension Protection Fund, 2019).

In the United Nations website (United Nations, 2020) it is possible to see a clear decline in birth rates per women, coming from almost five children in 1970 to 2.5 in 2020, a decrease of 50%. This scenario becomes worse as forecasts points to less than two children in 2090. In the same period, the death rate also decreased considerably, going from 13 deaths per 1,000 people in 1970 to around 7.5 in 2020. Because of this, even with the birth's slowdown, the world population is estimated to grow above 1% in 2020 (and expected to grow 0% in 2100).

According to the same source, life expectation at birth increased from 60 years in 1980 to above 70 in 2020, and in 2060 it is expected that 18% of world population will be above 65 years and the population between 20 and 65 will be around 56%, compared to the actual 9% and 58%, respectively. In the "Pensions at Glance" report (OECD, 2019, p.18) is estimated that "the old-age to working-age ratio – the number of people older than 65 years per 100 people of working age (20 to 64 years) – has increased by a little more than 50% in the OECD, on average, going from 20 in 1980 to 31 in 2020. Over the next 40 years, it will almost double to a projected 58 in 2060."

In the design of Pension Schemes the population trend has to be considered, due to its important effects over the liabilities. Improvements in mortality, particularly after the age of retirement, are one of the reasons for the value of the assets in the funds to increase continuously and so their relevance in the international market. OCDE numbers in the report "Pension Markets in Focus" (OECD, 2019a) state that the total amount of pension assets has grown from USD 23.6 trillion in 2008 to USD 44.1 trillion in 2018, increasing more than 86% in ten years. The United States represent 64.8% of the world pension funds' assets, followed by the United Kingdom with 6.6% and Canada with 5.9%. Other countries, like Iceland, the Netherlands and Denmark have more than 100% of their GDP in pension assets, showing that pension funds are relevant to their economies. Beyond the size of pension funds, another important indicator is that private schemes are paying around 6% of the world GDP in retirees' pensions (OECD, 2019).

With more than 400% of growth on S&P index since last crisis in 2008, it is possible to assume a "bull" market in equities (yahoo finance, 2020). However, the yields of US 30-year Treasury bonds have reached the lowest values in 20 years and Germany 30-year bonds have a yield of 0.25% (Bloomberg, 2020) and negative yields for all other maturities. Nevertheless, following (OECD, 2019a), on average, pension plans hold 44.9% of their assets in bonds and bills and 24.4% in equities. In this regard, (OECD 2019a, p.32) holds a clear point about pension funds migrating investments from the traditional applications in bonds, to other assets: "adjustments of the portfolio of pension providers, potentially as a search for yield to meet the pension promise, is not intrinsically bad as long as it does not imply an excessive increase in the risk profile of the portfolio."

Furthermore, the 2020 scenario adds a world pandemic to the problem, reducing consumption and economic performance, a flat world GDP growth being expected (The World Bank, 2020).

After the 2008 financial crisis, the pension funds were involved in an uncertainty scenario, where their investments in equities suffered a considerable decrease. This motivated a movement towards the bond market and, as an example, United Kingdom pension funds had 61% of their investment in equities at that time, however in 2019 they hold only 24% (Pension Protection Fund, 2019). This time, in 2020, lower yields will not be sufficient to maintain funds liabilities obligations. That is why OECD defends "adjustments of the portfolio".

This work has the purpose to, using an Asset-Liability model, study the real impact of a world economic crisis in pension funds and simulate an investment portfolio.

In the end of 2019, the market was debating about crisis in pension funds schemes, because of lower interest rates. The bond yields fall is a specific problematic for Defined Benefit pension plans, because they use the yields of high-grade bonds to calculate the present value of their liabilities, so a small downgrade can increase their funding challenges (Cumbo and Wigglesworth, 2019). However, UK pension funds have increased their hedge against interest rates downwards shifts, and it is possible to assume their funding position will not suffer with interest rates volatility (Flood, 2019).

Based on the firsts publications in 1950's decade, we can assume that the Asset Liability Management is a methodology that has been developed along the past 50 years at least. With models adapting to countries legislation and economy movements. Therefore, in this work we will study classical approaches and apply a recent popular one, in order to identify possible issues and developments.

1.2. Thesis structure

To achieve the goal, this thesis is structured in two main parts, a theoretical one (Chapters 2, 3 and 4) and an applied study, in the framework of the Dutch Pension Funds (Chapter 5).

In Chapter 2, we do a literature review and introduce the main concepts on the subject of Pension Schemes, giving particular detail to the risks involved. In Chapter 3, we review Assets and Liabilities Models, the classical approach about how pension funds manage their assets in purpose to match the liabilities resulting from the pension schemes. As a continuation of Chapter 3, in the Chapter 4 we will go further in the Liability Driven Investment approach. The case study is presented in Chapter 5, involving a specific pension fund environment. Chapter 6, the final chapter, contains the conclusions and other thoughts, namely a few open questions requiring future investigation works.

2. PENSION FUNDS

Impavido (2013) defines pension funds as a basket of money savings accumulated during an individual work life. Complementing this definition Dickson, Hardy and Waters (2020) state that in a pension plan the employee pays a fixed contribution, which added with the employer contribution, is obtained a future retirement benefit.

To make clear the difference between a pension fund and a pension plan, Garcia and Simões (2010) define a pension plan as a set of rules and conditions which allow a person to be eligible to receive benefits, generally in pension format. The authors also define different types of pensions: early, normal or late retirement pension, survivors' pension, disability pension or deferred pension. Dickson, Hardy and Waters (2009) also said that pension plans offer their members retirement payments as lump sum or/and annuity benefits.

In Willmore (1998), the discussion about the pillars of pension systems points to three different aims: redistribution, forced savings and voluntary savings. The first one guarantees a minimum income and should be taken care by State. The third one is personal, so voluntary and can be related to a private market. Pilar 2 is controversial with regard to the funding responsibility, depending on Government vision about economy. In liberal economies both the employer and the employee (not always) hold contributions, not the State.

This discussion is important to understand that historically State pensions are unfunded and called "pay as you go", which means, keeping Willmore (1998, page 2), words, "*contributions from today's workers finance the pensions of today's retirees*". It is a risk, because there is a danger that active members' contributions are not enough to match pensions in payment. On private pension funds, more possibilities exist and Garcia and Simões (2010) list three types of pension plans:

I. **Defined Benefit (DB)**: in this type of pension the benefit is clearly defined. For example, we have a defined pension to be paid if the member follows all scheme rules.
II. **Defined Contribution (DC)**: in this type of pension only the contributions are defined and not the final pension, which is dependent from the accumulated contributions.

III. **Mixed or Hybrid plans**: in this type of plan is possible to identify two parts, one following the defined benefit and other following the defined contribution approaches. A modern approach is the called Collective Defined Contribution plan. In this approach, all the members and employers share the risk making their contributions to one unique money pot. This pot is invested, and the gains and losses are shared by the participants (Sagay-Yusuf, 2019).

The two "pure" kinds of plans (I and II) have also as a major difference resulting from who takes risk associated to the ultimate payments of pensions. For DB schemes, the sponsor (Dickson et. al., 2009) is the legal responsible to guarantee future payments. This responsibility is an obvious source of risk, because the funds must have enough assets to fulfill their promises when retirement age comes. Contrasting, in DC schemes the risk is integrally with the employee. The retirement income will depend exclusively on the performance of the fund investments.

In UK, Defined Benefit schemes follow a specific structure, where they are separated from the sponsoring company. To run all schemes decisions and take care of pension fund assets a group called Trustees is defined and made responsible. This Trustee figure can be a person, or a company and the existence is required by UK law. The main goal is to ensure that the pension scheme assets are completely separated from the company assets and the members benefits are secure (https://www.thepensionsregulator.gov.uk/). To guide and support the Trustees work, an actuary or a third part actuarial company must be chosen to, at least once in each three years (Check Pensions Act 2004), analyze the scheme liabilities calculations and the funding level (Ndayong, 2016).

As the focus of this work is on Defined Benefit schemes, it is relevant to understand that the funding level is the connection between assets and liabilities and that all DB schemes should be funded. Funding Level is the ratio of the current value of the scheme's assets and the present value of its liabilities, usually expressed as a percentage (The Pensions Regulator, 2018). When this ratio is under 100% the scheme is underfunded (Pension Protection Fund, 2019). In UK scenario, when a scheme does not meet their statutory funding objective (SFO), which means that the scheme does not have assets to cover all liabilities, a recovery plan must be elaborated with the sponsor and

submitted to the regulatory body. In exceptional cases when the scheme is considered insolvent, the Pension Protection Fund (PPF) takes responsibilities for the payments, however following specific rules and stablishing new pension amounts for future retirees.[1]

2.1. Risks in DB schemes

Risk management in DB schemes is always connected with a funding purpose and the solvency aspect. As in DB schemes the sponsor holds the risk and the level of benefit is defined, the sponsor contributions, and in some schemes contributions from the active employees too, are accumulated to match the future benefits. If an adverse economic scenario drops down the assets market value, the company must make additional contributions to cover the loss. On the other hand, if assets rise, the company can have their contributions revised. So, this "off balance sheets", funding level, is the thermometer which indicates that changes must be done in contributions amounts (Dickson et. al., 2009).

Actuarial valuation of liabilities is made using the pension plan specifications and actuarial premises, but the assets are priced following market values. It is important for the funding level if the actuarial calculations pursue conditions consistent with the market, to reach a result close to reality. Exley (1997) suggested that if actuaries uses as discount rate the forward interest rates, the precision would be better. However, this idea is not heavily used because of the asset volatility and the necessity to work in a continuous time.

In Ai, Brockett and Jacobson (2015), AON survey results were analyzed. Conclusions show a list of most common perceived risks by pension sponsors and insurers. More than 50% of respondents indicate Interest Rate Risk as the most feared, followed by Longevity Risk, Inflation Risk and Investment Risk. (Gallo 2009) lists six major risks that are faced by pension schemes and besides the ones listed above there are risks with respect to future contributions and reinsurance (crisis risk) and the model (assumptions) risk. In her point of view the Interest Rate Risk and Longevity Risk can be joined in a Liability Risk. However, in this work we will split them and keep six issues to be dealt with, as described next.

[1] To know more, access: https://www.thepensionsregulator.gov.uk/

I. ***Interest Rate Risk***: As in liabilities calculations the analyst must bring all future payments to the present, changes in long term yields will impact this final result. For a group annuity, the key risk is the risk of interest rate falling below the assumed reinvestment rate for long enough to turn possible a balance change between assets and liabilities (Ai et al., 2015). Reinvestment rate is related to the cases when the fund must choose other investment. For instance, when interest rate fall, bond prices increase and their yield fall. Then, the reinvestment rate will be lower than anticipated.

II. ***Longevity Risk***: This risk is, likewise the previous one, directly connected with the liabilities. It measures the mortality improvements at old ages and if there are very significant differences in the survival of pensioners when compared with the forecasted, the fund will have funding problems. Living more indicates more time in payment after retirement than the scheme calculated.

III. ***Inflation Risk***: Analysing pension increases, it is common to face different indexations as Limited Price Index (LPI), Retail Price Index (RPI) or Consumer Price Index (CPI) . This means that the retired or deferred members have their pension upgrades following those rates, however this gives a complexity to match market asset allocations. It is common to find schemes with some limitations as RPI limited to 2.5%, 3% or 5%. For instance, it will depend from the pension plan specifications.

IV. ***Investment risk***: Every kind of investment holds a risk and naturally the returns on investments are correlated to the inherent risks. Even bonds, which are the most used to match liabilities, have issuer default risk. Equity shares, foreign currency and investment funds have the market volatility. So, all assets returns are hard to predict, which raises complex questions in an Assets Liabilities management framework.

V. ***Crisis risk*:** A pension fund is composed of contributions, assets movements and sometimes of an insurance for errors and unexpected problems. In a world crisis the default risk increases for everyone. Meanwhile the stocks volatility also increases, impacting assets invested. On the other hand, some companies must hold a currency risk because of the global market and employees in different countries, so local crisis can impact investments.

VI. ***Assumptions risk*:** This is related with Asset Liability Modelling and the assumptions established, any significant mistake can seriously impact final results.

Also, foreign-exchange rate can be a concern about liabilities valuation. In case of multinational companies, pension benefits can be denominated in foreign currencies for foreign employees (Shang and Hossen, 2019).

Dutch government, in the Financial Assessment Framework (FTK), rises the so called Concentration Risk, where it is specified the risk of disproportionally invest the assets in few asset classes (stocks, countries or companies). In this analysis, bonds are out of evaluation, assuming they do not represent risk. This is a mistake because in countries with high credited default risk, can be an issue (Spaan, 2012).

Following Exley (1997), to obtain the best results for pension funds and reach the funding level as expected, and with the lowest risk possible, assets and liabilities must be looked together. Using this stretch, it is possible to identify that pension funds are not just interested in obtaining the best portfolio return for a given risk. They must understand the liabilities behavior, like payments dates, to plan portfolio structure. Then, the liabilities can be treated as constraints' in portfolio selection.

As form to understand and mitigate risks related to liability management rises the concept of Asset Liability Management, or ALM. The International Association of Insurance Supervisors (IAIS) defines ALM as the management business practice to coordinate assets and liabilities to turn possible to take decisions.[2] The ALM policy has the goal to protect the pension fund acting in the underfund probability, which needs to be kept between a defined level range (Dert, 1995).

[2] Access for more information: https://www.iaisweb.org/home

3. ASSET LIABILITY MANAGEMENT IN PENSION FUNDS

3.1. Overview

As already mentioned, see Ryan (2013), although insurance companies have used ALM for a long time, in part because of the regulations they have to observe, it was in the late 1970s and early 1980s that ALM practices gained increased popularity, due to interest rates becoming more and more volatile. The kick off was connected with a period of high interest rates, with impact on the matching of cashflows from assets and liabilities - "*the dedicated bond portfolio, as it is frequently called, is a strategy that matches monthly cash flows from a portfolio of bonds to a pre specified set of monthly cash requirements of liabilities.*" (Fabozzi and Mann, 2012, p. 1103). This model holds some important limitations: it conveys a static approach and 'ignores' the complexity to forecast liabilities and the fact that it may not be possible to invest/reinvest in bonds for all required maturities. Finding bonds that cover all different liability maturities and reinvesting coupons are difficulties faced by the model which were not considered (Martellini 2006).

As a method to follow an interest rate trend and reduce volatility impacts, the multi-period immunization concept gained increasing popularity since de 1980s, making use of the duration concept to match assets and liabilities (Macaulay (1938), Fabozzi (2012), Ryan (2013)). A celebrated definition of immunization was given by Reddington (1952), according to which one must invest in assets in a way that the current existing business will be immune to changes in interest rates. According to the definition, Redington derived three conditions for the immunization to take place, see Reddington (1952).

One shortcoming of the model is that there is no difference between short and long-term interest rates, making way for arbitrage opportunities (Shiu 1990). Setting the appropriate discount rate to obtain the present value of future amounts is a key issue (Ryan, 2013). Another problem is that establishing what the long term is, and matching duration for that term, will not work because (again) it could not be possible to purchase bonds to match all existing liabilities. To rebalance the portfolio almost on a yearly basis

is very often a necessity. Furthermore, as pension schemes can be very complex, the (average) duration may be quite insufficient to guide the investment process (Gallo, 2009).

History shows that the interest rates fall in 1982 increased the matching difficulties using only the plane Reddington model, which gave the final push to the rise of "contingent immunization". This technique allows the portfolio manager to set the minimum return needed and operates freely to obtain higher returns (Martellini, 2006).

The model consists of building a bond portfolio with duration based on the investors plan and interest rate expectations. This portfolio duration can be under or over liabilities duration, being impacted by the expected interest rate. If the investor expects an interest rate moving higher than the market expectation, then the bond portfolio should have a duration smaller than the one of liabilities. The opposite movement is also true. However, if the real interest rate moves in a different direction than the previewed by the investor, and the portfolio value falls below a target minimum level, also called stop loss, it starts an immunization trigger and with that the portfolio will be rebalanced to an immunization profile (Diaz et al., 2009).

Even though, uncertainty continues to be an issue, so to increase the assets returns may be necessary to include new asset classes in the pension fund's portfolio. However, the new portfolio of assets may not behave in a way that will replicate perfectly the liabilities, rising the flag about how to find methods to optimize investments (Martellini, 2006). Naturally it requires the use of stochastic models to improve how existing risks can be treated.

ALM models can be divided in two different groups (Dert 1995): the dynamic models and the static models. The dynamic models very often make use of simulation to forecast what will happen after a possible current decision is made. On the other hand, the static models deal equally with long and short-term, lacking sometimes sensibility to short run effects of the decisions. Then, is possible to assume that static models are limited versions of the dynamic models.

Gallo (2009) holds a different perspective when comparing the existing approaches to ALM. Again, this author splits the contributions in the academic literature in two sets: the one that deals with ALM problem in a continuous-time framework (Merton (1969, 1971), Boulier et al. (1995)); another that is focused on developing more

comprehensive models of uncertainty in an ALM context, leading to the development of a stochastic programming approach (Kallberg *et al*. (1982), Kusy and Ziemba (1986), Mulvey and Vladimirou (1992)).

According to Platanakis and Sutcliffe (2015) the different forms to apply ALM models can be joined in four groups: stochastic programming, dynamic programming, portfolio theory and stochastic simulation.

In the early 2000's a "new" ALM approach called Liability Driven Investment rises and Ehrentreich (2009) explains the main causes are: the pension regulation, which makes harder to invest in equities; the absence of incentive to an overfund the pension fund; and possible equities shortfalls. In 2005, a growing crisis in pension funds motivates alternative approaches. This year, Ryan and Fabozzi propose a solution where the liabilities are priced at the market value, different from the previous actuarial discount rate approach, and then an index is created as a benchmark portfolio (Ryan and Fabozzi, 2005).

The major difference between classic ALM and the LDI concept is about discount rates. LDI uses the rates of long-term bonds and is focused on the volatility risk of interest in a short-term perspective. On the other hand, classic ALM models use the interest rates of from a pre-defined plan yield, and, in a risk perspective, it is focused to reach returns in a long-term period. The fixed plan yield turns the traditional ALM models relatively less flexible. Also, LDI is known for allowing the use of derivatives, like swaps and options, and the use the idea of risk budgeting (Watanabe, 2009).

3.2. Modern approaches to ALM

In the last chapter we prepared an overview of ALM models. This classic discrete time models use bonds to cover liabilities, however, as liabilities suffers other risks than inflation and interest rate, these models have faced mistrust.

To address "new" risks and uncertainty, the use of stochastic programming can be a practical solution. These techniques use the same discretized time to build decision points and create scenarios with a tree format. In these scenarios it will be possible to consider different factors and probabilities of occurrence. This requires time series for each one of the variables, which usually have heavy tails and stochastic volatility. Also,

these characteristics will be capturated by time series methods and incorporating stable distributions (Rachev and Grebeck, 2005).

Multistage Stochastic Programming (MSP), which is common in Netherlands, was described by Dart (1995) and makes use of account information and some policy constraints to forecast the probability to have an underfunded issue. It uses also Boender (1994) stochastic programming application. MSP originally had the mission to keep the Funding Ratio at 100% using a dynamic approach to anticipate future decisions. It creates key points in time *t*, and inputs estimated liabilities, contributions and assets information to estimate ratio. Gondzio and Kowenberg (2001) explain that one of the biggest challenges is the number of portfolio rebalancing dates, which makes the analysis explode. This can be minimized if we break this decision points and eliminate unrealistic scenarios. The model, aims punishing deficit scenarios. Stochastic dynamic models in principle can be adapted to different risk measures and different returns distributions (Grothey, 2011).

Continuous time analysis is usually related to Markovitz Mean-Variance theory. This theory bases itself in expectation of returns and standard deviation as a risk measure. Applying it in the ALM scenario the assets part follows the same rules as any portfolio, because the goal is to build a portfolio to maximize the expected return and minimize the volatility. However now we have the expectation of liability grow, so the entire portfolio return will be dependent on assets and liabilities. The difference in investing decisions using ALM lies in the variance of the entire portfolio, which will depend on the covariance between returns of assets and liabilities growth (Dart 1995). The use of the standard deviation as risk measure is not the best way, in the sense that the Trustees are interested in the risk of underfund and not the risk of surplus.

Following these steps and optimal portfolio selection theory (see Merton 1971), Merton (1990) applies the original concept in a pension fund environment. At that time, he used the pension liabilities as a constraint in the formula of asset allocation. The major problem with portfolio theory was related with errors estimation and consequently risk understanding. The author lists three ways to reduce this error impact: First option is changing the estimation of the mean vector and the covariance matrix, using Bayes estimation or James-Stein shrinkage estimation. The second is to constrain the assets proportions so we can keep the extremes out and control the total error. The third is using

the average of simulated scenarios, see Michaud (1999) (Platanakis and Sutcliffe, 2015). Nevertheless, these are not good solutions, the first two approaches do not guarantee a good risk-return performance of the generated portfolio and the last one becomes too complex when the number of assets grows (Goldfarb and Iyengar 2003).

Another contribution using Portfolio Theory was given by Ben-Tal and Nemirovsky (1999), called robust optimization. The main idea is applying linear programming to solve problems under an uncertain environment. An interesting approach in this method is the use of upper and lower bounds as constraints to keep the market stochastic movements "under control".

Gulpinar and Pachamanova (2013) use robust optimization in pension funds, and this method is easier to apply than stochastic programming because the simulations size grows linearly and not exponentially, but the authors agree that in more complex problems it would be better to use stochastic programming.

In a bit different approach, Platanakis and Stucliffe (2015) worked using the same technique but now taking Sharpe ratio maximization as objective, while Gulpinar and Pachamanova the expected difference between the assets and contributions. This Sharpe ratio was calculated dividing the assets expected return per liabilities standard deviation (Sharpe, 1994).

The European movement to close DB schemes for new entrances, a necessity to have a simpler methodology and also the market approach to evaluate liabilities turns the Liability Driven Investment model interesting. It takes the main ALM idea to use assets as a liabilities match, but it also uses assets to obtain leverage and recover the Funding Ratio. The LDI percentage will depend on the current fund level, how mature is the scheme and if it is closed for new members. In extreme scenarios, where the scheme has low funding level it will be necessary to invest in riskier assets. This model has been questioned because it is difficult to deal with underfunded pension funds, or liabilities with high risk and volatility.

4. Liability Driven Investment (LDI)

4.1. Fundamentals

DB pension funds are based on contributions and investment returns to keep their funding position as expected. However, this type of management does not always provide safety by keeping a low risk. This means that funding level volatility may be high, even though the asset market keeps increasing in general value. Therefore, the LDI strategy rises with the purpose of reducing the volatility of the funding level (Insight Investments, 2019).

LDI is structured to monitor the performance of the asset portfolio compared with the liability portfolio, focusing on two main objectives: to minimize the probability of assets value dropping below liabilities and to minimize the probability of insufficiency liquidity for payments (Shang and Hossen, 2019). The LDI approach is popular as it seeks to use the liabilities behavior to create a portfolio that replicates their movement. This need resulted of an equity bear market which has forced the migration from other assets to bonds. On the other hand, bonds were not necessarily a good match for deferred and pensioner liabilities (Chambers, 2005).

As Shang and Hossain (2019) say, to achieve the scheme objectives it is important to analyze six principal factors:

1) **Market Condition:** It will rule the cost of hedging the liabilities risk. A low interest rate environment, for example, can reduce the fear about the future.
2) **Regulation:** Will impact mostly in how heavy can be the effect on the sponsors' contributions, in case of a bad scenario.
3) **Current funding level:** This can allow the sponsor to take more risk, in case of a high funding level, or a lower appetite in the opposite case.
4) **Target funding level:** This target will guide about which return will be needed, and about the hedge level and contributions level.
5) **Plan sponsor's financial strength:** The possibility of an extra contribution from the sponsor increases the risk appetite, because the pension fund has high possibility to be saved from a difficult situation.

6) **Plan sponsors' risk appetite:** This gives the necessary impulse, or not, for hedge level and risky investments.

To evaluate the LDI strategies and the LDI performance we have the Liability Benchmark model. This model starts from a given asset allocation and economic scenario to forecast the Funding Ratio. To develop a benchmark model, we have three phases (Shang and Hossain, 2019):

1) **An Economic Scenario Generation (ESG)** – In this step a future macroeconomic scenario is simulated with the goal of having future assets returns estimated. Other factors are also simulated, specifically wage inflation and the sponsor equity return. So, by the end, the relationship between assets and liabilities is determined.
2) **Plan Projection** – From the ESG results, assets plan, future benefits, future contributions and pension liabilities discount rates are defined.
3) **Investment Strategy Optimization** – The LDI strategy is generated based on the scheme rules, the sponsors risk appetite and the plan projection results. The model chosen is based on risk and return tradeoff.

The use of a benchmark portfolio turns it possible to provide a cash-flow and estimated return to be pursued by the investment manager. The benchmark portfolio, in case of a 100% Funding Ratio, is typically composed of bonds, with guaranteed fixed and/or inflation linked income (Chambers, 2005).

To build the benchmark portfolio we first must create a custom liability index with the liability movement, and accounting for the scheme risk appetite. After that, we will choose the assets group with closer correlation to the risk/reward behavior of this index. The portfolio created to match the index behavior is called Liability Beta Portfolio (LBP). Differently from the immunization approach, which is focused on matching the average duration, the Liability Beta Portfolio must match the benefit payments for a given time horizon, usually set ten years ahead. To make this match possible the Beta Portfolio must have the same yield curve shape (or term structure), as the custom liabilities (Ryan and Fabozzi, 2011).

As the Beta Portfolio purpose is to match and fund the payments for a set time horizon, we have the figure of the Alfa Portfolio as a way to solve funding issues. This portfolio is responsible to, outside the liability average duration, reduce any economic

deficit from the scheme. Therefore, its return must overperform the liabilities increases; by transferring the extra earn to the Beta Portfolio, it can raise the Funding Ratio or reduce the future contributions. This transference will be automatically responsible to match more future benefit payments, beyond the time set previously decided, in the Beta Portfolio. Then, is possible to conclude that the Alfa Portfolio holds the objective to erase the fund deficit (Ryan and Fabozzi, 2011).

For a fully funded scheme, the Liability Benchmark Portfolio (LBP) was defined as the portfolio responsible for, independently of any economic changes, keeping the funding level. This must happen even in the absence of future contributions, benefit accrual or changes in demographic assumptions (Chambers et al, 2005). For schemes with funding below 100%, the LBP should be defined as that part of the liability that is covered through the priorities between beneficiaries of the scheme on winding up (Speed et al, 2003).

In general, the key decision for investments is the split between bonds and equities and it comes from "risk-seeking" and "hedging level" positions. Scheme maturity, funding level, sponsor wishes, and Trustees' risk appetite are, among others, key factors to decide this split. Depending on scheme sizes, budget and capabilities, more asset classes and more statistical tools can be applied (Chambers et al, 2005). Martellini (2006) indicated that the LDI asset portfolio can be resumed as three different portfolios: the standard optimal growth portfolio, the liability hedging portfolio and the risk-free asset. The difference here is that the author decided to have a risk-free portfolio which is not inside the hedging portfolio.

4.3. Liability Beta Portfolio (Liability Hedging portfolio)

Owens and Wilson (2020) discuss an important indicator called Hedge Ratio, responsible to monitor the interest rate exposure. It measures the sensitivity of the assets to an interest rate change and compares it with the liabilities sensitivity. It is build based on the product of three factors: funding status, hedging portfolio allocation and duration ratio. The first factor is based on the contributions, so it is the quotient between assets and liabilities. The second term is the percentage of the total assets which are in the hedging portfolio (or matching portfolio), usually an investment in long duration bonds. The last one is the ratio between the LDI portfolio duration and the liabilities duration.

Shang and Hossain (2019) have described that an LDI hedging strategy can be applied following five different models and rules. The most common and direct approach is related with duration and convexity (see Fabozzi (2012)). For an ALM purpose we have the goal to match the duration impact in money (surplus or deficit). However, for schemes where the liabilities won't be fully protected, we have the Hedging Ratio which is set as percentage by the Trustees' board. Therefore, we have the equality:

$$PV_A \times D_A = PV_L \times D_L \times H \qquad (1)$$

where

$$PV_A = Assets\ present\ value$$
$$D_A = Assets\ duration$$
$$PV_L = Liabilities\ present\ value$$
$$D_L = Liabilities\ duration$$
$$H = Hedging\ factor$$

As the duration can be described as the first derivative of the price/yield formula, the convexity is the second - and can be considered the next step from the duration. In our work, the Asset convexity will be equaling the Liability convexity times the hedging rate, following Shang and Hossain (2019).

The convexity is a better indicator to measure the impacts on the portfolio when the interest rate suffers large fluctuations. One hypothesis is to use duration analysis (method one) and convexity analysis (method two) simultaneously. Other LDI approach cited by Shang and Hossain (2019) is to use the key-rate duration matching, where the duration is analyzed for an interest rate at specific maturity. This can catch different changes in assets and liabilities from a non-parallel yield curve.

The fourth approach is the well-known cash flow matching, which was explained before. Further, Chambers (2005) provides an easier approach where the liability will be divided in time buckets and the matching portfolio will be composed in order to reach the duration for each one of these buckets. Applying this model, the scheme owner can decide to not match cashflows completely, only during the first 20 years, for example. Deciding to not use the complete asset value to match the cashflow, the portfolio manager will have margin to invest in risky assets.

As fifth and last, we have the Liability replicating portfolio. This portfolio is created with the purpose to mimic the value and sensitivity from the interest rate curve. Shang and Hossain (2019) indicate that in this approach we have to simulate scenarios changing the interest rate expectations. These changes will provide new value for the asset portfolio and a new present value for the liabilities. They also indicate that is possible to weight the scenarios, based on the probability of occurring. Therefore, the replication portfolio will catch small and big changes in interest rate levels and the yield curve shape with the following minimization formula:

$$min_{RP} \sum_{i,t} w_i \left(V_{RP}^i(t) - V_L^i(t) \right)^2 \qquad (2)$$

Where

$$w_i = weight\ assigned\ to\ scenario\ i$$
$$V_{RP}^i(t) = value\ of\ replicating\ portfolio\ at\ time\ t\ and\ scenario\ i$$
$$V_L^i(t) = value\ of\ liability\ at\ time\ t\ and\ scenario\ i$$

With the purpose of matching liabilities with lowest risk possible, fixed interest rate securities are the most appropriated for cash-flow matching or immunization. For instance, government bonds index-linked and non-index-linked, mortgage loans, credit and short-term receivable.

Bonds are agreements between lender and borrower where a fixed income will be provided as interest. Fabozzi (2012) describes three major bond issuers: domestic corporations, municipal governments and federal governments. The key factor about bonds is the term-to-maturity, which represents the number of years where the "agreement" will be valid and when it will be closed. The bond return to the holder is given by the redemption amount and the coupons. The connection between maturity and return can vary, but in general a long-term bond normally offers a greater yield.

Hull (2018) defines a bond duration as a measure related to how long the investor has to wait before receiving the price paid for it. The author also explains that in bond portfolios, the final duration can be calculated doing the weighted average of the individual durations, but each weight must be proportional to the bond prices.

According to Fabozzi (2012), duration is a good measure for small shifts in the yield curve, however for large changes this is not true. In these cases, the convexity is a best measure because accounts for the shape of the curve.

In cases of corporate bonds, the credit risk spread, which changes based on the company rating, has importance. This classification indicates the default risk, therefore, in case of downgrade, the bond price will also fall (Fabozzi, 2012).

The use of derivatives can improve the portfolio protection against interest rate shifts. We have two main instruments used in pension funds: interest rate swaps and repurchase agreements (repos) (Shang and Hossain, 2019).

Interest rate swaps are products where two parties agree to exchange interest payments based on some predeterminate amount (called notional amount). In this relationship the notional does not change hands, it works as baseline for the payments. The most common contract is one party receive a fixed interest rate return and the other party receive a floating interest rate return. (Fabozzi, 2012). The use of this product by pension funds, in order to have better liabilities protection, demands to find swap curves similar to the scheme liabilities discount rate curve.

In the repurchase agreements, one party (seller) sells a product to the other party with the commitment to buy it back by the set price and date (Fabozzi, 2012). Pension funds use this product to sell long government bonds and buy them back before maturity. Therefore, this instrument allows the fund to keep his hedge with the bond, but also gives cash at the present. This cash will be free to be reinvested in more bonds or even in riskier assets. One possible complication is related to the repurchase agreement term, usually on short periods and mostly cases in months, that will need to be rolled over after expiration (Shang and Hossain, 2019).

Other (less common) derivatives, explained by Shang and Hossain (2019), are the credit spread options or forex (FX) futures and forwards. These kinds of derivatives can protect the scheme from inflation, credit spread risk or exchange rate, but opting for them will depend on the Trustees' risk appetite. To protect from the longevity risk, the author suggests the use of longevity swaps, longevity bonds emitted by the plan sponsor, or the so-called buy out, where a third party (usually an insurance company) becomes responsible for some liabilities for a defined price.

4.4. Alfa portfolio (Risk seeking portfolio)

To structure the Alfa portfolio, we have some models largely applied in the market. One of them is the Modern Portfolio Theory (MPT) (see Markovitz 1952), using the concept

of variance as risk measure and the portfolio selection using Efficient Frontier. To structure the benchmark alfa portfolio is commonly used index funds from stocks, debts, real state and commodities.

Real estate are investments connected with physical assets, as commercial or residential buildings for example. The investors can buy directly the asset or can invest in a fund composed of a pool of assets. These pool of assets in many cases involves periodical payments, as they usually are connected with long term returns. Because of this characteristic, we can expect low volatility and moderate returns. In some countries, investors connect real estate development to the GDP and to inflation. However, Shang and Hossen (2019) state that for pension funds this hedge strategy may not be valid, because the liabilities from wage growth are partially connected with the whole economy growth, and are also correlated with interest rate growth, which can offset the inflation impact.

Similar to real estate, investments on commodities are also connected with physical assets as crude oil, gold and silver, among others. Transactions on these assets indicate historically higher volatility than company's equity and this can be an issue for pension funds. On the other hand, these assets hold positive correlation with inflation and negative correlation with equities and bonds (Shang and Hossen, 2019); thus, they can provide a good diversification for a pension fund portfolio. One issue is the direct connection between supply and demand and government involvement.

As a shareholder is an owner of the company, the price of stocks oscillates following the firm results, market expectations about the future and strategic plans. The Standard and Poor's 500 (S&P 500) is the index composed of the top 500 companies traded in the Nasdaq and NYSE stock markets. This index is a good guide about the equity market because it is composed of top American companies, divided by sectors and weighted by their total equity. In 2020, especially after Covid-19 crisis has erupted, we saw an increase of value of tech companies and a decrease in other sectors, as retail for example.

In line with S&P, other well know index provider is the MSCI. The company provides index for different pools of countries, as emerging markets, and grouped by companies, as the large caps. The difference between small and large caps is subjective and can vary among the investment advisor. However, we can assume that small caps are

firms with market value below 2 billion dollars. Invest in small caps can, in theory, represent major volatility, but higher expected returns. In the case of crisis scenario, these companies tend to more affected, as most of the time they have a high leverage ratio and bold growth plan. In case of pension funds, where portfolio rebalances are less frequent, we can assume large caps as the best investment to do.

Considering a Europe based company, the use of emerging countries market index focused on European companies tends to be a common choice for investors. Investing in South America and Asia sometimes carries an extra risk.

Debt indexes are sometimes considered fixed income investments and for others they are a risky asset. In this work we will assume the index related to emerging markets, where their debt is part of funds offered by investment banks. These assets are related to the countries credit default risk and usually hold low volatility, as changes in the risk rating tend to take time.

In this work we will not go deeper in alternative investments as private equities or hedge funds. The first one is related with OTC investments, and can be more attractive to investment funds, to beat the market index. These types of investments hold higher risk and low liquidity, because the investor works directly with the company. Hedge funds are also complex options, as they operate freely using leverage positions, many derivatives and OTC investments. They can give higher returns, but for a higher "cost" in terms of volatility.

4.5. Portfolio optimization

To obtain the final asset allocation, we have to think the Alfa and Beta portfolios return as one and compare it with the liabilities return. The best asset allocation will depend on different Trustees' positions. They can be focused on minimizing the portfolio volatility, minimizing the liability shortfall probability or even maximizing the Funding Ratio for given risk (Shang and Hossain, 2019).

Sharpe and Tint (1990) provide an optimization approach based on the maximization of the surplus (difference between assets and liabilities). This approach can be considered an extension of Markovitz' MPT (1952) replacing the classical wealth utility function with the surplus. The authors use the portfolio return variance in order to discount the maximum expected return, demonstrating that a higher return is important,

but with the minimum risk possible, as the original Mean-Variance theory. However, they include a final term considering the covariance between assets and liabilities return, highlighting the importance of their relationship.

Martellini (2006) used the Expected Utility Theory as a way to set the fund investment goal. As pension funds' assets are meant to, at least, match liabilities, the author adjusts the theory by replacing the wealth with the Funding Ratio. The investment policy should maximize the expectation of the utility function related to funding purposes. The utility function is connected to the investor preferences about investment, then in this case the author concludes that the Trustees preferences will depend on how the funding status is. Therefore, to adapt the model, the author uses a power utility function, with the degree of risk aversion as a discount factor.

Ang et al. (2013) show that in Sharpe and Tint (1990) solution the use of covariance can be not enough for ALM cases, because it is assuming in the same way a downside or upside related to the covariance term. Additionally, in cases with assets and liabilities uncorrelated, it turns in a simple portfolio selection. To mitigate these issues, the authors replace the covariance related term by the called downsize risk. This new approach considers only penalization for losses probability and this term is connected with the Liability Hedge Portfolio. An important factor about downsize risk is when it is zero, then it corresponds to a regular Mean-Variance case. However, if the downside risk is infinity, it means the objective is to hedge the liabilities regardless the Mean-Variance performance.

Jiang, Smith and Malloy (2020) suggest building the optimal portfolio based on an adapted Sharpe ratio. The original ratio is changed to have the Funding Ratio in the denominator and not the portfolio standard deviation. Additionally, they apply a multiplicative factor based on the shortfall probability. In their words, it selects a more ambitions portfolio because the focus is not on the minimum risk portfolio, but in the better Funding Ratio. Additionally, they explain that it gives a longer term thinking and a better diversification purpose. This can be a solution to improve the LDI use in cases with low Funding Ratio.

5. CASE STUDY – A DUTCH SCENARIO

In this chapter, we will apply a LDI model to a Dutch pension fund with the purpose of illustrating how to perform the recovery of a pension fund with minimum risk. This case study is not a real case although based on real data. Because of the necessary data transformation due to confidentiality constraints, we can face challenges in consistency, and, besides that, the information provided in this work may be insufficient for a complete understanding of the process. Furthermore, the graphs will not have any information on their axes. Before entering the case, we will have a brief portrayal of Dutch pension funds structure and regulation rules.

5.1. Overview

Following Bovenberg and Nijman (2009), the pension fund system in Netherlands is structured in three pillars. The first one is the called "pays as you go", denominated "Algemene Ouderdomswet" (AOW), and it is provided by the State in their public pension scheme. It is based on the minimum wage and if compared with European Union countries, can be considerate low for those who receive medium to high income during their work life. This opens a necessity for the second pillar, made of private companies' scheme benefits for their employers. In 90% of the cases, these are the regular DB schemes (Goudswaard, 2013). By last, the third pillar is for personal savings, where people are saving their own future pension in banks or insurance companies from their own choice. These investment returns will also depend on their choice and it is a completely "independent" pillar.

On the AOW, the State pension age is now 66 years and 4 months with the goal to reach 67 in 2025 for those with date of birth later than 31/12/1958. The pension full amount is now fixed, in the best scenario with tax credit, in 1,201.42 euros/month (73% of current minimum wage). Also, this amount can be reduced depending on work years in Netherlands. (Sociale Verzekeringsbank, 2020)

The assets hold by the second pillar, occupational pension schemes, represent 125% of the country GDP and their liabilities involve 90% of the country labor force. With the new Financial Assessment Framework[3], the pension funds must match their cashflows with 97.5% of certainty. After the recent crisis, some pension funds start to change how they define the pensionable salary. Before that, was common to see the final salary as reference wage, but nowadays the career average salary becomes more frequently used.

Other important decision was about the indexation factors, which are, in most cases, connected with interest rates or inflation. The indexation factors are not guaranteed because they will depend on the fund performance (Bovenberg and Nijman, 2009). Therefore, Dutch pension funds must follow the solvency rules and Telkamp (2019) describes that if the Funding Ratio average for the previous five years is below 104% some measures must be taken; most of the cases the liabilities must be reduced in six months. Also, in this case, the indexation is not provided. If the one-year Funding Ratio is between the inferior limit and 110%, a 12-year recovery plan should be presented and implemented. In these cases, the indexation and other scheme premiums are adjusted to make the recovery possible. On the cases where the one-year Funding Ratio is above 110% the indexation can be full covered, and pensions can be repaired. This repair means to adjust pensions that suffered indexation penalties in the past. If no repairs are needed, the scheme can fix ten years of fully indexation and also premium reduction.

Concerning risk, the Financial Assessment Framework indicates ten major classes: Interest rate, equity and real state, currency, commodity, credit, technical insurance, liquidity, concentration, operational and active management. The Required Own Fund (ROF) is based on the square root of the sum of the square of the risk percentages, which are calculated with 97.5% certainty level for the next year. To this scenario be true must be assumed that diversification brings correlation 1 between them (Telkamp, 2019).

5.2. Liability Driven Investment model

In order to define a time cutoff, all liability and assets information are based on the second quarter of 2020 (30/06/2020). As premises we assume that this scheme is closed for new

[3] Access: https://www.dnb.nl/

entries and will not receive any further contributions. Other assumption is that any pension will be repaired in case the Funding Ratio increases and we will not have a partial indexation - just the full, when the Funding Ratio becomes higher than 120%. It is important to address that this work will not analyze the risk shocks for minimum Funding Ratio because it is out of my scope.

The application of the LDI model will follow the continuous time approach, using the Mean Variance theory to select the optimal risky portfolio. As a form to minimize the error and keep the portfolio risk under control we will use a limit to bound the size of equities portfolio.

We will start from current year with assets allocation done and the rebalance analysis will be made mostly at each three years. The goal of this work is to recover this pension fund from its 97% Funding Ratio and reach 120% in 12 years, as it is the Dutch recovery time limit. After stabilizing, the purpose will be reaching the fully indexation and cover the liabilities increased by inflation. Also, we will assume a greater risk until achieving the first goal, after that we will reduce the risky portfolio and pursue a full hedging portfolio. Even with this initial risk appetite we will adopt equity participation oscillating between 20% and 30%, already counting with others as private equity, commodities or real state. One important constraint for the whole period is to keep the hedging ratio above 50%, which is the minimum accepted by the Trustees.

As explained in Chapter 4, LDI first step is creating the Economic Scenario, where we have to define the equity assets estimated mean returns and standard deviations. To have indicators which can be replicated for long time periods, the 2020 returns are excluded from the exercise. The result can be seen in Table 1.

	MSCI world	MSCI EMEA	MSCI small caps	MSCI emerging markets	S&P 500	S&P comodities	MSCI USA	JP Morgan Real State	Morgan Stanley EM Debt
Annual return	8,19%	3,95%	8,05%	7,19%	11,01%	0,79%	10,97%	5,46%	6,22%
Standard Deviation	9,67%	12,64%	10,12%	15,29%	5,11%	3,72%	6,85%	6,47%	8,93%

Table 1 - Equity assets mean and variance
Source: MSCI, JP Morgan and Bloomberg (2015 – 2019)

Additionally, in table 2 we have the correlation matrix indicating the relationship between the assets.

Correlation	MSCI world	MSCI EMEA	MSCI small caps	MSCI emerging markets	S&P 500	S&P comodities	MSCI USA	JP Morgan Real State	Morgan Stanley EM Debt
MSCI world	1								
MSCI EMEA	0,62	1							
MSCI small caps	0,95	0,53	1						
MSCI emerging markets	0,77	0,84	0,68	1					
S&P 500	0,96	0,52	0,92	0,69	1				
S&P comodities	0,53	0,28	0,55	0,34	0,51	1			
MSCI USA	0,97	0,51	0,94	0,68	0,99	0,51	1		
JP Morgan Real State	0,56	0,37	0,59	0,47	0,54	0,21	0,57	1	
Morgan Stanley EM Debt	0,46	0,65	0,39	0,65	0,37	0,24	0,37	0,51	1

Table 2 - Equity assets correlation
Source: MSCI, JP Morgan and Bloomberg (2015 – 2019)

To project 100 years inflation, the Nelson and Siegel model was used, as in Gallo (2009); however, in this work we allow negative inflation, as the current scenario indicates. Additionally, the interest rate forecast was made using the Ultimate Forward Rate (UFR) model, as recommended by the European Insurance and Occupational Pensions Authority (EIOPA) in the Solvency II document. This methodology to calculate interest rates for longer terms provides a higher rate than the market, because the impact of a cumulative inflation rate. To comparison effects, the graph below was made with both projections, inflation and interest rates.

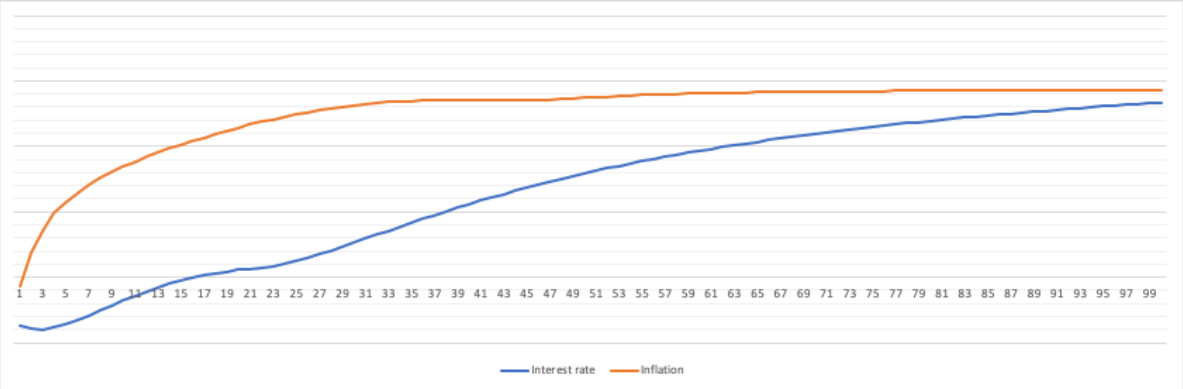

Figure 1 - Inflation and Interest Rate curves
Source: Author's calculations

These curves result from the country's and world's economic expectations. The estimation of negative interest rate for more than ten years ca be considered a consequence of the actual crisis. Additionally, the expectation is that the inflation and interest rate become closer to each other in the long run.

In the second step, called Plan Projection, we first built the liabilities curve. We will work with two types of curves, one is the nominal and the other is the inflation indexed. In Figure 2, we have both curves fitted together and divided per year.

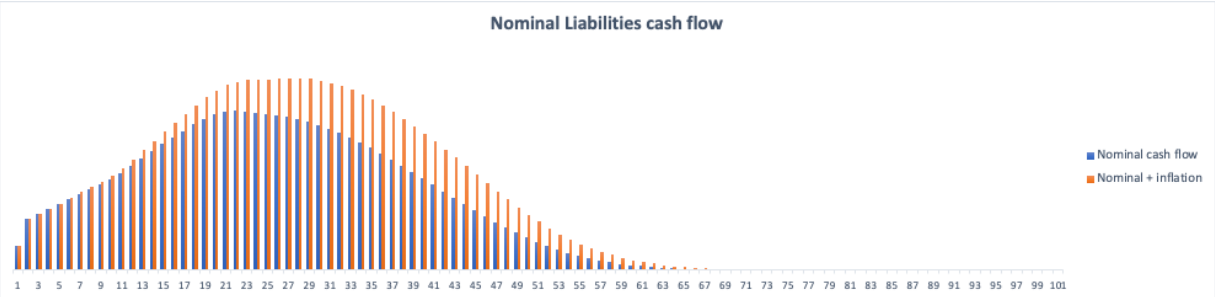

Figure 2 - Liabilities curves
Source: Author's calculations

As we can see, the nominal cashflow will be almost equal than the one with indexation over the first 10 years. When the scheme achieves a Funding Ratio above 120%, we can expect a greater cashflow. The following table (Table 3) shows the payment split and when we have more than 95% of the liabilities paid.

Payment buckets	Nominal amount	Amount with inflation
1-5 years	5,4%	3,9%
6-10 years	7,7%	5,9%
11-15 years	10,7%	8,6%
16-20 years	13,7%	11,9%
21-25 years	14,6%	13,8%
26-30 years	13,8%	14,1%
31-35 years	12,0%	13,3%
36-40 years	9,3%	11,1%
41-45 years	6,4%	8,2%
46-50 years	3,7%	5,1%

Table 3 - Liabilities per time bucket
Source: Author's calculations

We can see in the previous table that most of the payments will be made in more than ten years, giving time to increase the current Funding Ratio. In Table 4 we have all the scheme information at time $t = 0$ and the pension plan relevant assumptions.

Type	Value
Funding ratio (Nominal liabilities)	94%
Funding ratio (liabilities with inflation)	82%
Duration nominal liabilities	23,61
Duration liabilities w/ inflation	25,86
Modified duration for 0.01% changes in interest rate	20,31
Minimum hedging portfolio	50%
Risky assets lower bound (recovery period)	20%
Risky assets upper bound	30%
Maximum portfolio standard deviation	1.5%

Table 4 - Pension plan premises
Source: Author's calculations

Based on this information, the constraints were set, and the assets will be split between the two groups: the hedging portfolio (Beta portfolio) and the risk-seek portfolio (Alfa Portfolio). In the third step we will build these two portfolios.

The approach used to build the Hedging portfolio is the fourth described in Section 4.2, but following Chambers (2005) time bucket idea. Although we will focus in matching the cash-flow for first six years the duration will also be controlled. Then, we will choose high coupons and high yields bonds, as they usually have low degree of convexity and greater duration. Consolidating the strategy, we will mix Dutch bonds with high coupon rates and long maturity with a swap portfolio. The use of swaps will be made to protect the investments from interest rate shifts in the short term and cover the long time cashflows (above 30 years duration). In this strategy, we will sell swaps with short time to maturity and buy swaps with long time to maturity. Table 5 represents the initial hedging portfolio structure.

Type	Present Value
Bonds	88%
Swaps sold	-16%
Swaps bought	28%

Table 5 - Liability Hedging Portfolio
Source: Author's calculations

To forecast the future market value of the matching portfolio, we apply the interest rate curve to discount the cash flows of each one of bonds and swaps. It means that if an interest rate decreases it will impact directly on a bond price, increasing it. On the other hand, the presence of coupons reduces volatility and keeps the prices more stable and predictable.

The transference of funds from the Alfa portfolio will be invested in a bond portfolio, and not new swaps instruments. The Alfa portfolio will have a direct impact in the Funding Ratio recovery. Because of this, we will need to use an optimization model. The chosen model was the Funding Ratio maximization, described in Section 4.3. Therefore, we reach a surplus growth target and it reflects on the equity expected return. With this expected return we used the Markovitz (1952) Mean-Variance theorem to have the best risk budgeting. In the first ten years we choose the target portfolio return, however after reaching 110% of Funding Ratio we opt for the best return per risk.

Due to the big downfalls in early 2020, we will assume a possible risk increase in emerging countries. Based on that, we have as constraints the upper limit of 40% in emerging markets in total, in debts index or companies index. Besides that, real state and commodities will not be used at the beginning, because their 2020 instability and the lack of need to protect the portfolio against inflation movements. Also, we will assume an upper limit of 33% for each asset and short selling will not be allowed.

For the asset portfolio rebalance in year 10, we have changed the constraint for emerging markets, allowing until 10% invested in Emerging Markets Debt.

Table 6 shows the changes in the portfolio structure, at the end of the ten-year period.

Assets	Wheight year 0	Wheight year 10
MSCI world	32%	33%
MSCI EMEA	0%	0%
MSCI small caps	0%	0%
MSCI emerging markets	0%	2%
S&P 500	33%	33%
S&P comodities	0%	5%
MSCI USA	33%	0%
JP Morgan Real State	0%	17%
Morgan Stanley EM Debt	2%	10%
Expected return	10,0%	8,1%
Expected Standard deviation	4,1%	3,8%

Table 6 - Equity asset portfolio
Source: Author's calculations

As an equity premise we just have access to open index funds, then we will use them as the benchmark to be followed by the portfolio manager. It is further assumed that 2020 second semester will have a really low growth in equity, and for the next three years will be 5%. This scenario is based on the hypothesis that the Covid-19 vaccine will be

available in two years, with the impacts this might have on the economic crisis. Although we know a bond market fall may shift money to equities, we are afraid of a possible bubble making the two to fall together.

We will keep 2% of the total assets reserved to cash and insurance investments, which won't be detailed in this work. After using the surplus to increase payment cash and reinvest on the hedging portfolio, we will have the following results. Table 7 displays the changes of the scheme investments, following the results obtained.

Assets	Year 0	Year 11	Year 15	Year 30
Matching Portifolio	70,2%	74,2%	91,2%	91,8%
Equity	27,7%	18,6%	4,6%	4,2%
Real State	0,0%	4,1%	1,0%	0,9%
Commodities	0,0%	1,2%	0,3%	0,3%
Others	2,0%	2,0%	2,9%	2,8%

Table 7 - Alfa and beta portfolio
Source: Author's calculations

The portfolio changes have direct impact on the average surplus and on hedging. The first ten years we have the lower return because of the chosen lower equity growth after covid-19. In Table 8, we can see the fund results with the LDI model application.

Indicator	Year 10	Year 15	Year 21	Year 30
Average surplus return	1,6%	1,6%	1,5%	1,4%
Hedging nominal liabilities	60,4%	72,2%	105,1%	148,2%
FR nominal liabilities	110,0%	119,4%	129,4%	144,2%
Hedging liabilities w/ inflation	46,1%	53,5%	74,6%	97,7%
FR liabilities w/ inflation	92,9%	97,8%	100,9%	102,9%

Table 8 - Simulation results
Source: Author's calculations

In the first ten years, there was a clear focus on accumulation, however keeping the portfolio standard deviation below 4%. After reaching the 120% Funding Ratio, we can see the change in the portfolio diversification where we include other assets to reduce the risk. On 130% of Funding Ratio, the portfolio becomes 100% hedged to interest rate shifts. The final change was when we reach above 103% of Funding Ratio for indexed liabilities, therefore we move to 100% of hedging these liabilities.

After reaching 100% of Funding Ratio for the liabilities with inflation, we assumed that that the pension fund could afford for a buyout or a fully protection for the liabilities.

As the portfolio was adjusted to have a better fit with the liabilities, for the first 30 years we will have the tracking error of 2.38%, which corresponds to what Fabozzi (2006, p.18) classifies as a "moderated risk strategy". As expected from a moderated risk strategy, the duration will take time to be fully hedged and in the case of indexed liability this will not happen in the 30-year simulation.

Applying the Solvency Sharpe Ratio, we have the result of 0.84, which comparing with Jiang et al. (2020) would be an intermediated position, not the optimal portfolio which in their exercise had a ratio of 0.95, but in the second place.

To conclude the risk analysis, the next table holds the average standard deviation for the surplus and the Value at Risk.

Indicator	Value
Surplus SD	0,45%
VaR 95% surplus return	-0,74%
VaR 99% surplus return	-1,05%

Table 9 - Risk analysis
Source: Author's calculations

The table 10 above shows how adherent the model is, because shows a standard deviation lower than a Dutch bond for example. Additionally, in the 1% tail we have the 1% of downfall. This shows that even in extreme scenarios, the liabilities payment won't be compromised.

6. CONCLUSION

In this work, besides setting the theoretical framework of the research topic, we essentially deal with a pensions scheme affected by a crisis scenario with Funding Ratio below 105%. According to the Dutch rules, the fund should be in a recovery plan. The Government gives 12 years (DnB, 2020) for the plan to achieve the adequate Funding Ratio, which is related to the maturity position and the risk appetite. On the other hand, the fund in this example has the goal to reach 120% of Funding Ratio in six years. With this target we did simulations to understand if it is possible to accomplish this objective with the initial premises.

The possibility is based on keeping the hedging level above 50% and an average 25% of assets invested in equity. The required return to achieve the goal is on average 13.92% for the first six years. This result means that the investor has to beat in almost 3% the best asset return in the chosen list. Increase the percentage invested in risky assets will impact the payments risk, which has an established limit. Therefore, as a last choice, the investor will have to choose a riskier asset mix and increase the portfolio deviation.

In any solution we can see a direct reflection in the tracking error, which will increase to 11% and can be assumed as a very aggressive strategy, according to Fabozzi (2006, p.18).

In 2011, Cambridge associates did a simulation using four different portfolios and following their track for 10 years. The less risky of them reached 130% in four years and 100% hedging in the same year. Also, it was the best performer in a crisis environment. However, this test assumes a tracking error of 11.1% for a 100% funded scheme, which means a high investment in risky assets (40% of total assets), and estimated surplus of 3% per year. Although these results show that is possible to reach the expected Funding Ration in short time, it may not be applicable under Dutch pensions rules.

We can conclude in this simulation that the defined LDI model, using mean variance to optimize the risky portfolio, will require a number of rebalances higher than the initially expected. In addition, we can improve the model to consider costs of transactions and have a more real maintenance cost.

This example was constructed on an underfunded scheme, but an underfunded close to 100% of Funding Ratio. As the result indicates, we can assume low to medium risk in order to recover. In cases where the start Funding Ratio is lower, we estimate close to 80%, we will have two options: to increase the stocks return with more volatile assets, or to reduce the hedging and the matching portfolio. In both scenarios, high tracking error will be faced and the risk taken will be higher. The LDI model can be not the ideal in this scenario because it will not be possible to match the duration, and in some cases even the long run cashflows. Additionally, to cover the cashflows and reinvest the risk portfolio return, more rebalances will be required and more equity transactions will be made. Therefore, the use of a unique portfolio with more risky assets and derivatives can be the choice.

For future analysis, we would like to see the result of an approach using the concept of solvency Sharpe ratio, described by Jiang et al. (2020). As this theory claims to have a better fit to ALM purposes, it would be interesting to compare the portfolio using such approach with the portfolio structured in this work.

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