LISBON SCHOOL OF ECONOMICS & MANAGEMENT
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MASTER

MATHEMATICAL FINANCE

MASTER'S FINAL WORK

DISSERTATION

PREDICTIVE PERFORMANCE OF VALUE-AT-RISK MODELS

COVID-19 "Pandemonium"

DIOGO RICARDO VIEIRA RAMALHO

OCTOBER 2020

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ONOFRE ALVES SIMÕES

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Glossary

ARMA — AutoRegressive-Moving Average.

BMM — Block Maxima (Minima).

CAViaR — Conditional Autoregressive Value-at-Risk.

EGARCH — Exponential GARCH.

ES — Expected Shortfall.

EVT — Extreme Value Theory.

EWMA — Exponentially Weighted Moving Average.

FHS — Filtered HS.

G7 — Group of the 7 most industrialized countries in the world.

GARCH — Generalized Autoregressive Conditional Heteroskedasticity.

GEV — Generalized Extreme Value Distribution.

GFC — Global Financial Crisis.

GPD — Generalized Pareto Distribution.

HS — Historical Simulation.

HHS — Hybrid HS.

LB — Ljung-Box test.

MA — Moving Average.

MC — Monte Carlo.

MTM — Market value.

NVaR — Normal Distributed Value-at-Risk.

P&L — Profit and Loss.

POT — Peak Over Threshold.

QML — Quasi-Maximum Likelihood.

RM — RiskMetrics.

tVaR — Student-t Distributed Value-at-Risk.

VaR — Value-at-Risk.

VC — Variance-Covariance.

WHO — World Health Organisation.

Abstract

Nowadays, Value-at-Risk (VaR) models play a crucial role in Financial Markets, being one of the most widely risk management tools used by financial analysts, to estimate market risk. Informally, VaR is a statistic that measures the maximum possible change in value (loss) of a portfolio of financial instruments, with a given probability over a certain time horizon.

In this thesis, three widely used approaches to estimate VaR, namely Historical Simulation, $GARCH(1,1)$ and Dynamic EVT-POT, were applied. The purpose is to estimate VaR models for six of the G7 countries, as well as Chinese, Spanish and Portuguese stock market indices, more specifically the S&P500 (United States of America), CAC40 (France), DAX (Germany), FTSE MIB (Italy), NKY225 (Japan), FTSE100 (United Kingdom), SHSZ300 (China), IBEX35 (Spain) and PSI20 (Portugal), with a time horizon from 1st of January of 2007 to 31st of August 2020. It was chosen a confidence level of 99%. These estimations will then be backtested in order to see if the estimated models are accurate enough to represent the real movement of the stock indices. With this backtest it is possible to highlight when most of the exceedances occurred, enabling a conclusion of when those exceedances happen (if in a "normal" period or in a crisis period, e.g. COVID-19 Pandemic). Further, it is studied if there is any relation between the mortality number in each country and the movement in returns or volatility of stock indices.

The model that showed to be the most accurate when estimating crisis periods is Dynamic EVT-POT model. This conclusion was already expected, since this is a model known to estimate well the tails of the distribution. The model that showed less accuracy is the HS, even though being a good estimator for tranquil periods. It is possible to see that the majority of the exceedances, caused by outlier observations, occur during years 2008, 2011, 2013, 2018 and 2020 which are years known to be crisis periods. It was also possible to conclude that the movement in the stock indices is influenced with the increase of deaths related with contagious infectious diseases (COVID-19, in this case), showing therefore that there is some sort of relation between the two phenomena (when the number of deaths increase, the markets are more volatile).

Keywords: VaR, HS, GARCH, EVT-POT, Bivariate Regression, COVID-19.

RESUMO

Atualmente, os modelos Value-at-Risk (VaR), têm um papel muito importante a nível dos Mercados Financeiros, sendo uma das ferramentas mais utilizadas, por analistas financeiros, para gestão estimação de risco de mercado. Informalmente, VaR é uma estatística que mede a mudança máxima em valor (neste caso perda) de um portfolio de instrumentos financeiros, com uma determinada probabilidade e sob um determinado período de tempo.

Nesta tese, três métodos de estimação de VaR, nomeadamente o método de Simulação Histórica, $GARCH(1,1)$ e o método EVT-POT Dinâmico, foram aplicados. O propósito deste trabalho é estimar modelos VaR para seis países do Grupo dos 7 (G7), assim como para os índices de stocks da China, Espanha e Portugal, especificamente os índices S&P500 (EUA), CAC40 (França), DAX (Alemanha), FTSE MIB (Itália), NKY225 (Japão), FTSE100 (Reino Unido), SHSZ300 (China), IBEX35 (Espanha) e PSI20 (Portugal), com um intervalo de tempo desde 1 de Janeiro de 2007 até 31 de Agosto de 2020. Para esta análise foi tido em conta um nível de confiança de 99%. Estas estimações serão então testadas de modo a ver se os modelos estimados são precisos o suficiente para representar o movimento real dos índices de stocks. Este teste permitirá identificar quando ocorreram a maioria das falhas, e se estas ocorrências se deram mais em períodos normais ou de crise (por exemplo, a Pandemia COVID-19). Adicionalmente, é estudado se existe alguma relação entre o número de mortos por país e o movimento dos retornos ou da volatilidade dos índices de stocks.

O modelo que mostrou ter maior precisão aquando da estimação de períodos de crise foi o EVT-POT dinâmico. Esta conclusão era expectável, visto que este tipo de modelos é conhecido por estimar bem as caudas das distribuições. O modelo que mostrou menos precisão aquando da estimação foi a Simulação Histórica, apesar de ser um bom estimador para períodos normais/não crise. É possível observar que a maioria das falhas, causadas por observações incomuns, ocorreram durante os anos 2008, 2011, 2013, 2018 e 2020, que são considerados períodos de crise. Foi também possível concluir que o movimento dos índices de stocks é influenciado pelo aumento do número de mortes por infeções contagiosas (neste caso, COVID-19), mostrando assim que existe uma relação entre ambos (quando o número de mortes aumenta, os mercados tornam-se mais voláteis).

Palavras-Chave: VaR, HS, GARCH, EVT-POT, Regressão Bivariada, COVID-19.

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"To give away money is an easy matter and in any man's power. But to decide to whom to give it, and how large and when, and for what purpose and how, is neither in every man's power nor an easy matter."

Aristotle (384 BC - 322 BC)

1. INTRODUCTION

1.1. Motivation

Facing an increase in the unpredictability of financial stock markets movement, due to the new pandemic COVID-19, it was of utmost interest to try to explore further the thematic that estimates the risk of losses. VaR models play a crucial role in nowadays prediction of risk. The main purpose of this work is therefore to deepen the knowledge about how to predict the risk of a stock index and additionally to test if there is any correlation between stock indices and real-life crises (this case, COVID-19 pandemic).

1.2. COVID-19

COVID-19, also known as coronavirus disease, is an infectious disease caused by a newly discovered coronavirus. Supposedly to have started in China, 31 *December* 2019, later alleged to have emerged almost a year before in Europe based on the study of the wastewater of European (Italian and Spanish) sewage, (Miró, Estrada & Guix 2020; Naujokaitytė 2020), SARS-CoV-2 is a virus with a rapid spread, having dramatic impacts not only on common health but on financial markets as well, all over the world.

Being such an easily spread virus, it was during the New Year's Season, a period with a mass migration to other countries by tourists, that the disease ended up being too difficult to contain, spreading silently to the whole globe. This "theory" can be sustained by just analysing the data and information presented by the World Health Organization (WHO) and in the works of (He et al., 2020). On 11 *March* 2020, the rapid increase in the number of cases outside China led WHO to announce that the outbreak could be now characterized as a pandemic, World Health Organization (WHO) (2020).

Several articles and studies point out that this pandemic is exhibiting tremendous impact on the economies of the affected countries, and because being such a trigger to markets' volatility and even to countries' economies, it was necessary to take some precautions, (Ali *et al.,* 2020; Schell *et al.,* 2020). Regarding monetary and fiscal policies, each country took their own safeguard measures and protocols, in order to sustain their economic development, preventing collapse. Since in this study only the six of the G7 countries and three more countries were taken under analysis (USA, China, United Kingdom, France, Italy, Germany, Japan, Spain and Portugal), it is reasonable to present only about their major measures to detain the spread of the virus.

In response to this outbreak, some of the key policy responses of these countries were the implementation of travelling restrictions, border closures, social distancing practice, closure of schools and non-essential businesses premises and increased testing. Reflecting the impact of all these containment measures, the economies of these countries contracted an annualized rate varying from 2.2% in the UK to 5.8% in France, in the first quarter of 2020, leading to an increase of the unemployment rate. Even though the change was not that noticeable for the European countries and Japan, the USA rate increased 10.4% since the start of this year. China was not represented in the data set, therefore no conclusions could be taken. Furthermore, there was also the need to allocate extra funds, enhance liquidity and debt relief to the healthcare systems and to support households and small businesses, creating a stability mechanism, (Eurostat n.d.; International Monetary Fund (IMF), 2020).

1.3. Risk and Value-at-Risk

Experience has shown that certain events (known as Black Swans, which can be pandemics, terrorist attacks or wars) cause an increase in risk exposure for certain organizations. This is certainly true considering the present pandemic. According to Garcia-Arenas (2020), the pandemic we are currently passing is considered a Black Swan, being consistent with the idea of increase in risk exposure, (He *et al.,* 2020).

Concerning financial context, risk is typically associated with the volatility of unexpected outcomes and it can broadly be defined as the biggest possible loss of capital on an investment or business venture, (Jorion & Garp, 2010; Joshi, 2008). Since in Finance, the focus is set on the case of negative outcomes (how much *the loss* will be), it is usually thought only as the possibility of loss, even though risk can as well be seen as the possibility of gains. Volatility is potentially dangerous, because when presented in high range and with unpredictable movement, the risk of that security increases substantially. Financial risk can be classified in different categories: (1) *market risk/price risk*, associated to the asset price uncertainty, when assets are traded on competitive markets or due to changes in market conditions; (2) *credit risk/default risk*, when counterparties may be unable or unwilling to fulfil their contractual obligations, causing payment defaults; (3) *operational risk*, takes into account the risks arising from human and technical errors or accidents, as well as fraud and regulatory risks; (4) *liquidity* *risk/risk of counterparty*, arises if it becomes difficult to trade, buy or sell quickly the necessary amount of assets at fair prices, due to lack of buyers or sellers; and finally (5) *model risk/estimation risk*, which is a problem that appears when risks are measured and priced using flawed mathematical models (theoretical models always contain some kind of misspecification, or parameter estimation errors), (Katajisto Rami, 2008; Manganelli & Engle, 2001).

For example, because of the COVID-19 pandemic default risk has been increasing, exceeding even the 2008 financial crisis, (Choi *et al.,* 2020; Welburn & Strong, 2020).

As aforementioned, volatility has increased steeply in the last months, showing a larger impact on stock markets than any other similar disease, which affects negatively almost all financial markets (Onali, 2020).

Risk management can be defined as "*the process by which financial risks are identified, assessed, measured, and managed in order to create economic value*", (Jorion & Garp, 2010). According to the BIS Basel Committee on Banking Supervision (2013), the two most used metrics to measure and manage financial risks for internal bank models are Value-at-Risk (VaR) and Expected Shortfall (ES). In Manganelli and Engle (2001), VaR is defined as "a measure that gives the maximum amount an investor or financial institution can lose over a given time horizon, with a specified probability" defining as well ES as "the expected loss, given that the return exceeded the VaR". In this study, because of time and size constraints, only VaR measures will be used, since it is more applied in practice.

1.4. Research Questions

The main goal of this thesis is to find the most accurate models to estimate the market risk exposure through the comparison among various VaR methodologies. Through the analysis of different works (Allen et al. 2011; Andersen and Frederiksen 2010; Baur and Schulze 2005; Katajisto Rami 2008; Manganelli and Engle 2001; Singh, Allen, and Powell 2011), it was possible to conclude that analyses follow different approaches and use different models, depending on the period under consideration (before, during or after market crisis). For this work, makes sense to examine market settings under "normal" conditions and extreme/"non-normal" conditions, checking in which conditions the model predicts more accurately, giving better results. With this in view, some stock markets indices were chosen, as this type of indices show a more pronounced impact every time a strong change occurs (Katajisto Rami, 2008). The particular emphasis was on the impacts from the COVID-19 pandemic.

Hence, following (Ali, Alam, and Rizvi 2020; Andersen and Frederiksen 2010; Duda and Schmidt 2009), the ultimate objective of this study is to answer the following questions:

1. Which are the most accurate VaR models under normal and extreme market conditions and when the majority of the failures of prediction occur?
2. Is there any type of impact on index stock markets facing the number of COVID-19 related deaths?

1.5. Data Set

Since the main concern is the current pandemic, the crisis/extreme period to be analysed is the pandemic period. It is already evident that index stock markets show a huge fluctuation in volatility of their return sample series graphs, Appendix 2 - Section 1 - Returns Graphs and Histogram.

The choice for the data was based not only on the countries being more industrialized and a reference to other countries, but also because some of them are among the ones that showed a bigger range in mortality due to COVID-19, (Elflein, 2020). As of, the stock market indices used represent 6 of the G7 countries, namely USA, France, Germany, Italy, Japan and United Kingdom and China, Spain and Portugal, specifically the index stock markets, S&P500, CAC40, DAX, FTSE MIB, NKY225, FTSE100, SHSZ300, IBEX35 and PSI20.

The data set will comprise business daily closing prices with start at 01/01/2007 up until 31/08/2020, offering 13 years and 8 months of working data. The greater quantity of data is available to analyse, more robust the estimation will be.

1.6. Structure of the text

The structure of the study is as follows. Chapter 2 contains the literature review concerning the topic, including a survey of existing methodologies and their strengths and weaknesses. In Chapter 3, precise definitions, applied equations and the data to be used are briefly detailed. Chapter 4 is the core part, with the practical application, giving the answers to the first question above. In Chapter 5 the last question is answered, searching for a correlation between the number of COVID-19 deceases and the change in financial stock markets indices. Lastly, in Chapter 6, conclusions, final considerations and suggestions for further studies are presented.

“True wisdom comes to each of us when we realize how little we understand about life, ourselves, and the world around us.”

Socrates

2. Literature Review

Regarding the variety of existing studies, the literature review is presented in two sections. Thus, Section 2.1 includes works on VaR models, to provide some details about the diversity of the existing models, highlighting which ones provide better results in different market settings, while Section 2.2 contains works that focus on the COVID-19 pandemic, enhancing knowledge about it and about the risks it causes to financial markets.

2.1. VaR models

VaR measures have many applications, (Allen *et al.,* 2011). Manganelli & Engle (2001) is one of the first comprehensive studies about this subject and follows the impositions denoted by the Basel Committee on Banking Supervision. Financial institutions, such as banks and investment firms, must meet capital requirements based on VaR estimates, (Basle Committee on Banking Supervision, 1996). The performance of the most popular univariate VaR methodologies, namely the RiskMetrics (RM) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) parametric models, the nonparametric Historical Simulation (HS), the Hybrid HS (HHS), that combines RM with HS, and the semiparametric Extreme Value Theory (EVT), employed with a Quasi-Maximum Likelihood-GARCH (QML-GARCH), and (Conditional Autoregressive Value-at-Risk (CAViaR), were analysed in Manganelli & Engle (2001). Moreover, two methodological contributions were made, one being the introduction of EVT into the CAViaR model and the other the estimation of ES. The performance of the models was evaluated by means of a MC approach, through the generation of 1000 samples of 2000 observations each, using three different distributions (the Normal and the Student’s t with 3 and 4 degrees of freedom) for seven processes: five different GARCH processes, a process with a GARCH variance and a CAViaR process. Results showed that CAViaR models performed better. Furthermore, a regression-based method was introduced to estimate the ES. Conclusions underline that the regression method tends to outperform the EVT approach at very common confidence levels (99% and 95%).

Systemic risk in different market conditions was examined in Baur and Schulze (2005), suggesting that this risk can contribute to the intensification of financial crises, being also contagious. The stock indices analysed are from the Emerging Markets Free Asia and Latin America, and the Europe and North America Developed Markets, by means of quantile regression and Exponential GARCH (EGARCH) modelling. Findings acknowledged constant (or decreasing) impact of systemic risk in extreme market circumstances as a fundamental condition for financial market stability. Results also display the fact that systemic shocks become more important and predictive for emerging markets in times of stress (high volatility regimes), contrasting with developed markets that exhibit a constant dependence to it, being an essential condition for financial market stability. It was also pointed out that many emerging markets do not exhibit financial stability.

The predictiveness of VaR models in periods before, during and after market crisis were compared in Katajisto Rami (2008). The effects of market turbulence on the precision of VaR models to forecast risk were studied, aiming to help financial institutions to select the most appropriate ones to use as their in-house models. The data comprises daily closing prices of S&P 500 Composite (United States), CDAX General 'Kurs' (Germany), HEX General (Finland), and India BSE National (India) indices, from 01/01/1988 to 31/12/2004. Even though with different intensity, all these indices were affected by the 2000 bubble crisis. The VaR approaches used were RM, Variance-Covariance (VC) and HS, each with two extensions, and Monte Carlo (MC) and EVT, followed with one extension, each. When using a VaR significance level of 5%, no model outperformed the benchmark model (RM). However, as the significance was lowered to 1%, extensions of HS models, like volatility updating or filtered return series, and an EVT model using the Generalized Pareto Distribution (GPD), outpaced RM. The author suggested in this way that the normality assumption in the RM model loses progressively its precision, when the confidence level changes, thus making the extensions of HS or EVT more interesting.

The predictive performance of market losses using one day out-of-sample traditional models [HS, Normal distributed VaR (NVaR) and Student t distributed VaR (tVaR)] was compared with the predictive performance of market losses using a CAViaR model in Duda & Schmidt (2009). The data consisted in ten years of daily returns from US (NYSE Composite), as a representation of a mature market, whilst Hong Kong (FTSE W Hong Kong) and Russian (Russia RTS) indices represent emerging markets, comprising both normal and extreme periods. Main conclusions point to the fact that the traditional methods behave well during the tranquil period, even though when unfiltered, these models fail to produce reliable results. For the crisis period, symmetric and asymmetric specifications of CAViaR showed generally better and more stable results, than traditional approaches. Overall, CAViaR was found to work better on 95% than on 99% confidence level. However, this model class was in most cases outperformed by conventional filtered models in tranquil periods. There was a scarce evidence that different markets have impact when choosing the best VaR model.

In the same year, a VaR approach was made in Gustafsson & Lundberg (2009), by examining the HS, GARCH and the Moving Average (MA) methods. Their purpose was not only to test and compare the accuracy of the approaches, but also to analyse the results and see if it is possible to relate the characteristic of the underlying assets with the accuracy of the models. The indices used were the Brent Oil (Daily Oil Prices), OMXs30 (Stocholm index) and Swedish Treasury Bills. Results showed that, among the three, there was no superior approach (even though HS was more appropriate when using higher levels of confidence), that more complex approaches do not mean better results and that the choice of VaR to be used must be evaluated depending on the assets.

A set of models, which effectively estimate the future VaR in normal and extreme conditions were developed in Andersen & Frederiksen (2010). The work focuses on the overall volatility of returns and the impact that adverse market movements have on a portfolio value, studying the ability of the models to estimate the risk exposure, given the empirical distribution of returns. The methods implemented were the Basel II model, taken as benchmark, the $GARCH(1,1)$ model and the EVT approach, where a conditional Peaks over threshold (POT) model (combination of standard POT and GARCH) was introduced. The usage of both normal model (GARCH) and EVT (POT) in normal markets and the employment of only the EVT (POT) model in extreme markets was recommended.

According to Allen et al. (2011), EVT was assumed to be an effective estimator when applied to the computation of extreme risk measures as return level, Value-at-Risk and Expected Shortfall. The Univariate EVT was employed to model the extreme market risk for the Australian (ASX-All Ordinaries) and USA (S&P-500) indices (data includes the crash of 1987 and the GFC of 2007/2008). Hence, the authors implement a Block Maxima Method (BMM), POT and a two-step dynamic POT method. To test the accuracy of the models, a backtesting methodology was applied. The results showed that EVT can be effectively useful regarding the Australian stock market return series for predicting next day VaR, by the usage of a $GARCH(1,1)$ based dynamic EVT approach. It was once again pointed out that, EVT is better when assessing extreme tail events, presenting a better dynamic, when compared with other methods like Normal $GARCH(1,1)$ and RM, not only in normal but also in extreme market conditions.

In Avdulaj (2011), a multivariate market risk estimating method that employs MC to estimate VaR models for a portfolio of four stock exchange indices from Central Europe (Austria (ATX), Germany (DAX), the Czech Republic (PX50) and Switzerland (SSMI)), was proposed. The estimation procedure consists of two steps: univariate modelling (step 1) and multivariate modelling (step 2). The univariate modelling involves AR-GARCH models (standard conditional constant mean-GACRH and another model that allows asymmetry) and EVT, while the multivariate involves t-copulas and their capability to conduct multivariate MC simulations. Non-parametric distributions were selected, with the goal of capturing small risks, while EVT would be the one to capture large and rare risks. The method estimations were then compared with HS and VC approaches, under low and high volatility samples of data. The results obtained allow to conclude that while the HS method overestimates the VaR for extreme events, VC underestimates it. It was also possible to conclude that the method proposed in the paper gave a result in between, because not only it considers the historical performance of the stocks, but also corrects the heavy tails of the distribution, highlighting that both EVT and his estimate method show to be beneficial as a satisfactory risk measurement tool for extreme events, especially for high volatility times.

A study with three goals was conducted in López Martín (2015). The first purpose was to provide a comprehensive theoretical review of some VaR methodologies (HS and Non-parametric Density Estimation methods, GARCH, Stochastic Volatility and Realized Volatility, Volatility-Weighted HS, Filtered HS (FHS), CAViaR and EVT (BMM, POT and MC)), presenting the pros and cons of each one. Secondly, the accuracy of the distributions was evaluated, conducting in this way a comparison between two symmetric distributions with several skewed and fat tailed (asymmetric) distributions. The last goal was to examine whether the comparison of VaR models depends on the loss functions specified and used in the work. The dataset used was the closing daily returns of the indices Nikkei (Japan), Hang Seng (Hong Kong), Tel Aviv (100) (Israel), Merval (Argentina), S&P 500 and Dow Jones (US), FTSE100 (UK), CAC40 (France), IBEX-35 (Spain), the closing daily data of a spot crude oil price (Brent) and the Dow Jones Industrials stock index. Through the analysis of the state of art, it was concluded that the best methods to be used are EVT and FHS. For the comparison between the asymmetric and the symmetric distributions, the accuracy test indicates that between the asymmetric and the normal one, the former ones outperform the Normal one in fitting financial returns and forecasting VaR, while when compared with the Student-t ones, it is possible to infer that the majority of the skewed and fat-tailed distributions fit the data better, outperforming also the symmetric ones in terms of VaR accuracy. For the last task, concerning the loss functions, two possible points of view were mentioned. From the regulator's perspective, Student-t distribution is the best one in forecasting VaR. However, from the firm's point of view, the skewed distributions outperform the Student-t distribution. It was, concerning the latter analysis, concluded that the best VaR model always depends on the family functions used: regulator´s and/or firm´s loss functions.

More recently, in Jobayed (2017), Normal, HS and Exponentially Weighted Moving Average (EWMA) methods to estimate VaR models for three Nordic indices, specifically OMXH25 (Finland), OMXS30 (Sweden) and OMXC20 (Denmark) were applied. To better know the accuracy of the selected models, eight backtesting tests were applied. These can be categorized as frequency, independence and joint tests. Empirical results showed that the Nordic markets behave somewhat similarly when exposed to global market conditions. Moreover, the models were ranked by level of performance, as follows: EWMA, HS and Normal. In general, while comparing the predictive performance of VaR Models between 2008 (crisis year) and 2010 (tranquil year), most models perform poorly when exposed to extreme events such as the GFC, while being relatively accurate during normal market conditions. This raises an important uncertainty about the suitability of VaR as a risk management tool, though, despite its limitations, still continues to be a well-accepted measure of market risk.

2.2. Pandemic times

In this subsection, several works that consider the context of the COVID-19 pandemic are presented. Although being a very recent phenomenon, there is already a significant number of researches, making it possible to have a basis for the study of COVID-19 social, economic and financial impacts in each country.

In Zhang, Hu & Ji (2020), the overall patterns of country specific and systemic risks in the global financial markets were mapped and as well the analysis of possible consequences and uncertainties, financial and economic policies might have into the financial markets. This analysis focus on the impact of the COVID-19 pandemic on the stock markets of the ten countries with the most confirmed cases of COVID-19 (according to the date $27\ March\ 2020$), along with Japan, Korea and Singapore. Since markets are presenting higher levels of volatility and unpredictability, the results allowed to conclude that the global financial market risk has increased considerably concerning this pandemic. Even though there was a need for the world to take measures to contain the virus and level the stock markets, non-conventional financial policies interventions as the US' Unlimited Quantitative Easing, and their decision to implement a zero-percent interest rate, creates even more uncertainty and is expected to cause long-term problems. Additionally, because the countries group studied responds differently to national-level policies and the development of the pandemic, it was possible to infer that countries are working individually.

The repercussions of global financial markets, in terms of their decay and volatility, as Coronavirus epicentre moved from China to Europe and then to the US were examined in Ali *et al.* (2020). To better understand volatility, an EGARCH model was applied, and a bivariate regression model between the returns and volatility of the different financial securities and COVID-19 deaths was employed. Results allowed to determine that, when analysing China, USA, UK, Italy, Spain, France, Germany, Switzerland and South Korea, along with World (WRLD), Europe (EU) and Asia indices, as well as corporate bonds index (S&P 500), US treasury bonds core index (ICE core), Bitcoin, Oil (WTI spot) and Gold, the global markets have gone into a freefall, while Chinese markets stabilized and recovered. Also noted that safe commodities, like gold, have been affected as this pandemic crossed continental boundaries, although being found to be the least volatile. As far as equity markets are concerned, the European stock markets showed the highest sensitivity towards the pandemic. The authors also found that most of financial securities returns can be negatively and significantly related to the number of COVID-19 deceases. On the other hand, the volatility of most of the securities is found to be positively related to the deaths, which means that securities become more volatile as the number of deaths due to COVID-19 pandemic increase.

Bhutada and Mrinal (2020) addressed some of the challenges on modelling market risk factors and compared market situations between the COVID-19 and Spanish flu pandemics. This study was done under the analysis of the DJIA index and some extracted data from the Spanish flu and COVID-19 deaths. The results show that the two pandemics have a high degree of similarity and impact on banks from a market-risk perspective, but while the Spanish flu did not result in widespread pandemic modelling at banks, the situation regarding the COVID-19 pandemic shows the opposite behaviour.

In Gunay (2020a), the impact of the COVID-19 pandemic on six different stock markets (US, Italy, Spain, China, UK and Turkey), more specifically, the indices DJI, FMIB, IBEX, SHC, UKX, and XU100, was explored. A unit root test, a ICSS test, a M- ICSS test, a DCC-MVGARCH and DCC-MVFIGARCH models (the latter models illustrated the effect of the COVID-19 pandemic on dynamic conditional correlations), were performed. According to the results obtained from the M-ICSS test, the pandemic has led to structural breaks, mostly in February, in the stock indices volatility, except for the Turkish and Chinese indices. The former showed no breaks, while the latter presented earlier breaks compared to the other countries. For the results of the DCC-MVGARCH and DCC-MVFIGARCH, although the weak relationship of the Chinese and Turkish stock markets during past years, it was shown an increase in co-movements following the beginning of the pandemic. One of the conclusions of the study is that other markets also exhibit rising correlation, although lower increases, possibly related with the recent deterioration of the Turkish economy.

In another study, Gunay (2020b), the relation between the current pandemic status in foreign exchange market rates (USD/EUR, USD/GBP, USD/JPY, USD/NCY, USD/BRL, and USD/TRY) with the turmoil lived in the GFC of 2008/2009 was studied. The tests used were the Kapetanios m-break unit root test, downside variance, upside risk, volatility skewness, NVaR, HS, and the modified (Cornish-Fisher) VaR. According to most of the results, the turmoil in exchange markets is not yet as bad as in the GFC. Nonetheless, the pessimism employed by the media, regarding the future of financial markets, is analogous to the study of the volatility skewness, showing a different picture about the severity of the COVID-19 pandemic. Furthermore, it is observed that the Japanese yen, out the of the six currencies, presents a higher risk through the COVID-19 pandemic than the one observed during the GFC.

Furthermore, conventional t-tests and non-parametric Mann–Whitney tests were used to examine daily return data from the stock markets of the People's Republic of China, Italy, South Korea, France, Spain, Germany, Japan and USA, in He *et al.* (2020). The goal was to explore the direct impacts of the current pandemic on stock markets. Firstly, it was checked if the COVID-19 pandemic stirred the stock markets, finding a negative but short-term impact on them. Secondly, it was investigated the spill-over effects of China's stock market on the other stock markets and the reverse interaction by defining domestic and non-domestic COVID-19 timelines. It was proved that although there is no evidence that COVID-19 negatively affects these countries' stock markets more than it does the global average, it was still shown that it has bidirectional spill-over effects between Asian countries and European and American countries.

In Schell, Wang & Huynh (2020), an evaluation of the different reactions in global stock markets to the same kind of disease-related news, during Public Health Risk Emergency of

International Concern (PHEIC) announcements, by analysing abnormal stock markets returns, was conducted. For this work, 26 stock indices were studied, each displaying reactions when passing through any of the six PHEIC pointed out. This article followed the methodology implemented in MacKinlay, 1997. Although PHEIC announcements can be classified as the same type of events, there were no reliable patterns found in the reactions of financial markets to infer that these are treated similarly (investors tend to distinguish the different PHEIC). Besides for the COVID-19 pandemic, that showed a significative negative effect on stock markets, all other diseases did not show presence of significant impact on the markets, suggesting relatively low economic impact of the diseases on a global scale during that period.

The impact of COVID-19 cases and related deaths on the US stock market (S&P500 and Dow Jones indices) was checked, allowing in this way changes in trading volume and volatility expectations, as well as day-of-the-week effects, (Onali, 2020). For the number of reported cases and deaths, the countries used for the dataset were the most affected during the first three months of 2020 (Italy, Spain, China, US, France, Iran and UK). For this study, it was employed a $GARCH$ (1,1) model, with robust standard errors, being the impact of COVID-19 cases and deaths estimated by extending the common GARCH(1,1) model with a multiplicative heteroscedasticity component and Markov-Switching models. It was suggested that the number of cases and deaths from the pandemic do not present impact on the US stock market returns, showing evidence of shocks only on the conditional heteroscedasticity of the Dow Jones and S&P500 returns. By the analysis of the VaR study results allow to infer that the number of reported deaths in Italy and France have a positive impact on the VIX benchmark index, and a negative impact on stock market returns. To conclude, Markov-Switching models propose that at the end of February 2020, the magnitude of the negative impact on the VIX index increased threefold.

Summarizing the presented literature, it is settled that no ideal VaR model exists, concluding that the accuracy of those models depends accordingly to the conditions of the markets, chosen confidence level and time horizon. This research applies an HS, $GARCH(1,1)$ and Dynamic EVT-POT methodology to predict the P&L movements of index stock markets. These models were chosen because, the HS is simple and easy to implement, the $GARCH(1,1)$ completely characterizes the distribution of returns, improving as well the accuracy of other models, and the EVT-POT model is the method that shows more accurate results when dealing with extreme observations (crisis periods). Also, it is possible to conclude that the COVID-19 pandemic showed a negative effect on stock and index stock markets.

“The man who has anticipated the coming of troubles takes away their power when they arrive”
Seneca, Consolation to Marcia 9.2

3. Methodology

The current chapter presents and explains the chosen methodologies from the last chapter, having as purpose to provide an idea of how the processes are implemented and how VaR models can be applied by financial institutions. We follow, as main references, (Alexander, 2008; Andersen & Frederiksen, 2010; Christoffersen, 2003; Danielsson, 2011; Goorbergh & Vlaar, 1999; Hull, 2013; Jorion & Garp, 2010; Mcneil, 1999).

3.1. VaR backgroud

Before Value-at-Risk models, risks were measured by employing a variety of ad hoc tools (market value amounts, sensitivity measures and scenario analysis) that always showed to be unsatisfactory. Their deficient results arise by the non-measuring of the downside risk for the total portfolio, failing to take into account differences in volatilities across markets, correlations across risk factors, as well as the probability of adverse movements in the risk factors, (Jorion & Garp, 2010). To defy these shortcomings, VaR models were introduced.

J.P. Morgan, in the late eighties, launched a methodology known as RiskMetrics, that turned to be one of the most successful risk management approaches at that time. In this way, VaR models were, assumingly, pioneered by this company. These models started to be taken more into account after its endorsement by the Group of Thirty (https://group30.org/).

Later on, mostly due to the early 1990's recession, the Basel Committee created an amendment in 1996 that imposes some requirements to financial institutions, in order to provide a more secure financial system, (Manganelli and Engle 2001). These guidelines allow them to use internal models for VaR estimation. In the Capital Requirements Regulation (CRR), is stated that all banks should keep enough cash to be able to cover potential losses in their trading portfolios over a ten-day horizon, 99% of the time, given an observation period based on at least a year of historical data updated quarterly, (Basel Committee on Banking Supervision, 2013).

VaR is a statistical method that measures the total portfolio risk, considering for example diversification, being the most commonly accepted and used measure of market risk, (Duda & Schmidt, 2009; Manganelli & Engle, 2001). Roughly, it converts the (market) risk associated with a portfolio into just one number, either a money value or a percent value that represents the loss, with a given probability.

VaR is broadly defined as the "the loss over a target horizon such that there is a low, pre-specified, probability that the actual loss will be larger", (Jorion and Garp 2010), than that amount. Mathematically, this definition corresponds to the p-quantile of the portfolio's profit and loss (P&L) for a given portfolio.

$$VaR_t(p) = -F^{-1}(p|\Omega_t) \tag{1}$$

In (1), $VaR_t(p)$ represents the VaR of a portfolio at probability p and time $t, F^{-1}(p|\Omega_t)$ is the quantile function of P&L distribution (continuous distribution). It is time varying regarding the change of the portfolio's composition, Ω_t. The negative sign will ensure that VaR will be a positive number even though it represents a loss, (Campbell, 2005).

One of the shortcomings of VaR is that it does not satisfy the properties required to be a coherent risk measure. In Artzner et al. (1999), four desirable properties of a coherent risk measure $\rho(X)$ were set, for capital adequacy purposes:

1. *Monotonicity*: if a portfolio X_1 has systematically lower values than a portfolio X_2 , it must show a greater risk, i.e., $X_1 \leq X_2 \Longrightarrow \rho(X_1) \geq \rho(X_2)$;
2. *Translation Invariance*: adding a certain amount of cash λ to a portfolio X, should reduce its risk by λ, i.e., $\rho(X + \lambda) = \rho(X) - \lambda$;
3. *Homogeneity:* increasing the size of a portfolio X by a factor β, should scale its risk measure by the same factor β, i.e., $\rho(\beta X) = \beta\rho(X)$;
4. *Subadditivity*: the risk of a sum of two portfolios X_1 and X_2 must not exceed the sum of the separate risks, i.e., $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$.

VaR is not a coherent measure because it does not fulfil the fourth property (this property states that diversification helps reducing risk). Because of this, it is implied that VaR is a somewhat more conservative measure, overestimating the risk of the total portfolio. In addition, they argue that VaR gives only an upper bound of losses that occur with a given frequency. According to Danielsson (2011), more downsides exist since sometimes the homogeneity property is also not satisfied. Further VaR represents a quantile that measures a potential loss when facing an unfavourable situation, but it does not take into account the significance of other potential losses. This author acknowledges that VaR can be easily manipulated. However, when assuming normal distributions, the volatility–based VAR satisfies the four properties.

3.2. Statistical approaches to VaR and Backtesting

As seen in Chapter 2, VaR models can be categorized into: parametric, non-parametric and semi-parametric. The parametric models are based on estimating the underlying distribution of returns and then obtaining risk forecasts from it. In the non-parametric methods, no model is specified, and no parameters are estimated, the empirical distribution being traditionally used to estimate VaR, (Huang et al. 2020). In some special cases it is possible to see a combination of the two. Those special cases correspond to semi-parametric methods, (Manganelli and Engle, 2001). The three approaches used in this chapter will be summarized in the next three subsections, and all followed the same ideology: Compute Market Value (MTM) of the index, estimate the distribution of the returns and finally, compute VaR.

3.2.1. Historical Simulation approach

Historical Simulation (HS) is a method that relies on the empirical distribution and the assumption that history repeats itself. This means that, by using a large amount of data, it is possible to assume that the historical distribution of the returns is adequate to represent the distribution of future returns. Although controversial, it is popular among banks, (Pérignon and Smith 2010). In our application, we follow Goorbergh and Vlaar (1999), slightly adapted.

The first step is to calculate the daily returns of the assets from the historical prices, obtaining a sample of $T - 1$ observations, assuming T as the total number of prices. The second step is to calculate a daily VaR, by taking a 99% percentile of the returns with a rolling yearly window, leaving us now with a VaR sample of $T - n + 1$ observations, where $n = 252$. Each VaR estimation must finally be multiplied by the Market value (MTM).

The main advantage of HS is the direct usage of the observed data, not requiring any explicit distributional assumptions. Moreover, the method is simple, easy to implement and bases risk factor dependencies on experienced risk factor returns. When comparing with parametric models, HS does not rely on variance estimations to generate returns, which can be seen not only as an advantage (lack in estimation risk), but as well as a drawback, since it does not take into account the changes in volatility. The main limitation of HS is the constraint in the sample size (the amount of data must be as large as possible). Because returns have fixed weights, the procedure responds slowly to structural changes of risk (when occurs a structural break in volatility. Summarizing, this method implies that the predicted losses cannot exceed the historical losses, (Alexander, 2008; Christoffersen, 2003; Danielsson, 2011; Jorion & Garp, 2010).

3.2.2. GARCH(1,1) approach

The family of GARCH models was firstly introduced by Engle (1982), in the eighties of the 20th century, with the Autoregressive Conditional Heteroscedasticity (ARCH) model, later extended in Bollerslev (1986), to a Generalized ARCH (GARCH) model. The latter belongs to the category of conditional volatility models (models that relate their movement in volatility, with random shocks occurring at a certain day). This model intends to keep track of the variations in the volatility through daily lags. For this implementation, Hull (2013) showed to be convenient.

$GARCH(p,q)$ model, stands for Generalised Autoregressive Conditional Heteroskedasticity model, of order (p,q). This model computes σ_t^2 from the p lagged terms on historical returns and q terms of previous variance estimates, (Jorion & Garp, 2010).

Broadly, $GARCH(1{,}1)$ is believed to be a good forecasting model when accounting for volatility estimation, being considered "by far as the most popular of the GARCH models", (Hull, 2013). It is by many considered to be "unnecessary to include more than one lag in the conditional variance and in the squared innovations", (Goorbergh & Vlaar, 1999). Hence, $GARCH(1{,}1)$ model is employed, to estimate volatility of stock indices through time. Must be noted that $GARCH(1{,}1)$ is not considered to be the best VaR method, but rather to be taken as a way to forecast the volatility in returns, improving other VaR models, (Gustafsson & Lundberg, 2009).

Defining σ_t^2 as the conditional variance rate at day t and r_t as the continuously compounded return at day t, the $GARCH(1{,}1)$ can be described as:

$$r_t = \mu_t + \varepsilon_t \tag{2}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

where: $\varepsilon_t = \sigma_t \cdot Z_t$ and $Z_t \sim N(0,1)\ i.i.d. random\ walk$. The residual Z_t is assumed to be a white noise variable, that follows a standard normal distribution $\Phi(z)$; $\omega = \gamma V_L$, γ being the weight that measures the reaction of the conditional variance to the long-run average variance rate V_L (also known as the average unconditional variance); α is the weight that measures the reaction of the conditional variance to market shocks; β is the lag parameter that measures the persistence of the conditional volatility.

For the returns, r_t, we assume they are stationary in mean (check Appendix 1 - Section 1 - Mean Stationarity to see closely the repercussions of this statement).

To have an accurate estimation, four constraints are needed:

$$\begin{cases} \gamma + \alpha + \beta = 1 \\ \alpha + \beta < 1 \\ \alpha, \beta > 0 \end{cases} \tag{4}$$

The first one states that the weights must sum 1, as usual, the second will ensure that the process is stable, not allowing $\gamma < 0$, and all together guarantee that the weights are positive. This model assigns weights that decline exponentially at rate β, not only for the past r^2 but also for the unconditional volatility, V_L, (Appendix 1 - Section 2 - $GARCH(1,1)$ decay rate).

The GARCH parameters above are estimated using a Quasi-Maximum log-likelihood (QML) Method. QML is denoted as being a pseudo Maximum Likelihood method, since for this work the data to fit the model does not strictly follow a Normal distribution. For financial return data this assumption of Normality is not always true, (Allen *et al.,* 2011).

According to the assumptions of the model, for each observation $r_t, t = 1, \dots, m,$ of a sample of m observations, and assuming normality, the probability density function is

$$f(r_t|\omega, \alpha, \beta) = \frac{e^{-\frac{r_t{}^2}{2v_t}}}{\sqrt{2\pi v_t}} \tag{5}$$

taking v_t as the variance at day t.

Ignoring the additive constants, the log-likelihood function is

$$ln\mathcal{L}(\omega, \alpha, \beta|r_t) = \sum_{\mathrm{t}=1}^{m} \left[-ln(v_t) - \frac{r_t{}^2}{v_t} \right] \tag{6}$$

Once ω, α and β have been estimated, we can compute $\gamma = 1 - \alpha - \beta$, and then we estimate the long-term variance $V_L = \frac{\omega}{\gamma}$.

Being the final purpose of this study to estimate VaR, when using the $GARCH(1,1)$ approach, this estimate will take the form, (Angelidis *et al.,* 2004).

$$\mathrm{VaR}^p_{t+1|\mathrm{t}} = -\mathrm{MTM} \cdot \hat{\sigma}_{t+1|\mathrm{t}} \cdot \Phi^{-1}(p) \tag{7}$$

Where MTM denotes the Market Value, $\hat{\sigma}_{t+1|t} = \sqrt{\sigma^2_{t+1|t}}$ is the forecasted one-step-ahead conditional volatility at time $t+1$, given the information at time t, and $\Phi^{-1}(p)$ is the p-th quantile of the standard normal probability distribution.

The main advantage of this method is that it allows a complete characterisation of the distribution of returns, as it assumes a pre-existed distribution. Unfortunately, this can also be taken as a drawback, since it assumes that the used sample follows certain distributions, which sometimes is not true. Moreover, this model will effectively model the volatility cluster of stock returns, (Gustafsson & Lundberg, 2009). These models may lead to other disadvantages:

1. The fact that it considers negative and positive movements in returns, to have the same effect in volatility (negative and positive shocks increase volatility in the same way), (Duda & Schmidt, 2009);
2. The specification of the variance equation and assumption of the distribution chosen to build the log-likelihood may not be the best (the assumption of conditional normality does not seem to always hold for real data). In this way, difficulties may also appear when a skewed distribution is assumed to be Normal;
3. The independent identically distributed ($i.i.d$) assumption may not be verified, meaning that the returns would be correlated, (Hull, 2013).

Even though all these drawbacks, $GARCH(1,1)$ is considered to be a good volatility estimator and plays a huge role when estimating parametric and semi-parametric VaR, (Gustafsson & Lundberg, 2009).

3.2.3. Extreme Value Theory (Dynamic POT)

Even though the aforementioned models show to be accurate while estimating stock index movements in the majority of times, whenever there is a volatility increase resulted by sudden shocks, the models tend to not be able to predict those shocks effectively. These extreme returns tend to occur rarely, therefore they appear represented in the tails of the distribution. For the need to model these rare events, Extreme Value Theory (EVT) was created. EVT focuses explicitly in those extreme events, particularly in the case of this study, only the extreme negative returns are considered (left tails). The two main methods of this way of estimating risk are the Block Maxima (BM) model, based on the Generalized Extreme Value distribution (GEV), and the Peaks-Over-Threshold (POT) model, based on the Generalized Pareto Distribution (GPD). Both methods are used to model high quantiles of the underlying data's distribution.

According to the literature review (Chapter 2), the second model shows to be more efficient in levels of accuracy, when using both extrapolation and interpolation. Therefore, this thesis will only serve itself of the POT model. The POT approach is based on the idea that EVT holds sufficiently far out in the tails, enabling to model all the data that exceeds some predetermined threshold (u). Since it is based on a limit theorem, the EVT distribution is only asymptotically valid (as u grows larger). This research takes into account, Mcneil (1999) methodology.

For this implementation, it was considered periodic returns $R_1, R_2, \dots$ as random variables, identically distributed to a random variable R with unknown underlying distribution $F(R)$. The distribution of the excess loss over a threshold u, $Y_t = R_t - u, t = 1,2, \dots$, is

$$F_u(y) = P(R - u \leq y | R > u) = \frac{F(y+u) - F(u)}{1 - F(u)} \tag{8}$$

for $y \in [0, R_0 - u]$, where R_0 is the right endpoint of F. The excess distribution represents the probability that a loss exceeds the threshold u by at most an amount y, given the information that it exceeds the threshold.

In Balkema & de Haan (1974), it is verified that for a large class of underlying distribution functions F (Normal, Lognormal, $\mathcal{X}^2$, t-Student, F, Gamma, Exponential,...) the excess of loss distribution F_u can be well approximated by a Generalized Pareto Distribution (GPD) $G_{\xi,\beta}(y)$, for an increasing threshold u:

$$F_u(y) \approx G_{\xi,\beta}(y) \quad \text{as} \quad u \to \infty \tag{9}$$

In this way, our model for a risk R_t having distribution F assumes that, for a given u, the excess distribution above this threshold follows a GPD with parameters ξ and β. Hence, for a large class of underlying distributions F, as the threshold u is progressively raised, the excess distribution F_u converges to a Generalized Pareto Distribution.

The general form of the GPD distribution is

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}} & \text{, if } \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\beta}\right) & \text{, if } \xi = 0 \end{cases} \tag{10}$$

where $\beta > 0$ and $y > 0$, for $\xi \geq 0$, and $0 \leq y \leq -\frac{\beta}{\xi}$, for $\xi < 0$.

The two parameters of this distribution are β, the scale parameter, and ξ, the shape parameter. According to the value of the shape parameter, three special cases might rise. When $\xi > 0$, the $G_{\xi,\beta}(y)$ resembles heavy-tailed distributions, therefore, is a model used for large losses, whose tails decay like power functions (the upper bound will be $-\frac{\sigma}{\xi}$). The case when $\xi = 0$ corresponds to distributions whose tails decay exponentially. For $\xi < 0$, the GPD is similar to a group of distributions that present short tails, known as a Pareto type II distribution, with finite endpoint (no upper bound).

To determine the threshold u several approaches can be used. The choice of this threshold is one of the disadvantages of this implementation, since many possible errors might rise from it, (Rydman, 2018). One should always be careful between choosing a sufficiently high value, so that the asymptotic theorem can be applied, and choosing a sufficiently low one, so that the available information is enough to estimate the parameters. The most common approach, and the one used for this study, is the eyeball method. To choose the threshold u, the analysis of the Daily Backtesting graphs, the number of exceptions and the QMLE showed to be useful. This method can be seen as a trial error method.

Supposing that N_u is the total data points that exceed the threshold, and once the threshold is chosen, the GPD is fitted to the N_u excess points by some statistical fitting method, to obtain estimates for ξ and β, say $\hat{\xi}$ and $\hat{\beta}$. In Rydman (2018) the QML is estimated by the following forms (depending on the value of ξ):

$$\begin{aligned} ln\mathcal{L}(\xi,\beta;y_1,\dots,y_{N_u}) &= \sum_{t=1}^{N_u} lnG_{\xi,\beta}(y_t) \\ &= -N_u\ln(\beta)-\left(1+\frac{1}{\xi}\right)\sum_{t=1}^{N_u} ln\left(1+\frac{\xi y_t}{\beta}\right), \quad for\ \xi \neq 0 \end{aligned} \tag{11}$$

$$ln\mathcal{L}(\beta;y_1,\dots,y_{N_u}) = -N_u\ln(\beta)-\left(\frac{1}{\beta}\right)\sum_{t=1}^{N_u} ln(y_t), \quad for\ \xi = 0 \tag{12}$$

subject to the constraints $\beta > 0$ and $1+\frac{\xi y_t}{\beta} > 0, \forall t$.

Combining (8) and (9), and rearranging the equation, the model can be written as

$$F(r) = \big(1-F(u)\big)G_{\xi,\beta}(r-u)+F(u), \quad \mathrm{r} > u \tag{13}$$

To obtain an estimate of $F(u)$, it was used the following method of Historical Simulation

$$\widehat{F(u)} = \frac{n-N_u}{n} \tag{14}$$

The reason not to estimate the whole tail by the HS method is because it would be unreliable since HS predicts poorly the tails distribution. In fact, the purpose is to estimate the tail of the distribution where the number of observations is small and is often impossible to have enough data, (Mcneil, 1999).

Including (11) or (12) and (14) in (13), then the next formula is obtained

$$\widehat{F(r)} = 1-\frac{N_u}{n}\left(1+\hat{\xi}\frac{r-u}{\hat{\beta}}\right)^{-1/\hat{\xi}} \quad , \quad r > u \tag{15}$$

With a given probability $p > F(u)$, the VaR estimate is given by the inversion of (15)

$$VaR_p^{EVT} = u + \frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right] \tag{16}$$

When applying (16) directly into raw data, one would get a static model, (Allen et al. 2011). In Mcneil (1999), a dynamic method was proposed, that serves itself from the model GARCH above implemented. With this extension to the model, the dynamic EVT provides accurate estimates under the tail distributions for extreme samples with numerous returns, that are disperse and unevenly distributed, (Andersen & Frederiksen, 2010). The VaR estimate will then take the form:

$$\mathrm{VaR}_{t+1|t}^{p} = -\mathrm{MTM} \cdot \hat{\sigma}_{t+1|t} \cdot VaR_p^{EVT} \tag{17}$$

were MTM is the Market value, $\hat{\sigma}_{t+1}$ is the volatility estimation for day $t+1$ (estimated by mean of GARCH forecasting method) and $\cdot VaR_p^{EVT}$ is the p-quantile of the noise variable obtained from the $GARCH(1,1)$ estimation, since to attain a dynamic procedure, the GPD estimation procedure is applied to the random variables Z_t rather for the returns. Once again, the stationarity in the conditional mean and the $i.i.d$ assumption are assumed to hold.

This study favours the POT approach as the EVT tail estimation because it uses data more efficiently (better adapted to the risk measurement of tail losses), (Jorion & Garp, 2010). As drawbacks, the worst one is the difficulty when choosing the threshold value, since this method focuses on the distribution of exceedances over a threshold (only works for low probability levels), (Danielsson, 2011). Moreover, this method only makes sense if the returns over the given threshold are $i.i.d.$. Note that this approach is meant for the tails, not being able to conclude anything about the rest of the distribution, (Jorion & Garp, 2010).

3.3. Backtesting

To assess the accuracy of a given model, a backtesting analysis must be performed, (Campbell, 2005; Christoffersen, 2003). Backtesting is a procedure used to compare the various risk models and it aims to take ex-ante VaR forecasts from a particular model and compare them with ex-post realized return (historical observations). Broadly speaking, these tests do a comparison between the VaR estimation and the realized profit and loss (P&L) distribution. Whenever actual losses are greater than the projected VaR value, we say that a violation has occurred. These violations can also be referred as failures and exceptions. One should always note that the time horizon selection is negatively correlated with the power of the backtesting done, (Jorion & Garp, 2010). To count the exceptions indicator functions are defined as follows,

$$I_{t+1|t}^{p} = \begin{cases} 1, & \text{if } r_{t+1|\text{t}} \leq -VaR_{t+1|t}^{p} \\ 0, & \text{if } r_{t+1|\text{t}} > -VaR_{t+1|t}^{p} \end{cases} \quad (18)$$

where $r_{t+1|t}$ denotes the P&L on the portfolio over a fixed time interval, in this case, daily.

In Christoffersen (2003), the problem of determining the accuracy of a VaR model can be reduced to the problem of studying whether the hit sequence fulfils the next two properties:

1. *Unconditional Coverage property*, that states that the probability of a loss exceeding $VaR_{t+1|t}^{p}$ to occur must be p;
2. *Independence property*, that states that the indicator sequence should be unpredictable, therefore distributed independently over time.

As a consequence, the sequence of the indicator functions is associated to a sequence of $i.i.d.$ Bernoulli random variables, with parameter p (H_0: $I_{t+1|t}^{p} \sim iid\ Bernoully(p)$).

Summarizing, these two properties ensure that the theoretical confidence level p matches the empirical probability of violation and that there are no clusters in the data, making an outcome in $t+1$ to be independent from the outcome at time t.

In this work we will use the Unconditional coverage test, introduced by Kupiec, according to which the model should not present a number of exceptions bigger than $p \cdot 100\%$ of the time, taking p as the probability level, and an Independence test for the cluster issue, (Campbell 2005; Christoffersen 2003). The $number\ of\ observations \cdot p$ is defined as being the target number of VaR breaks. If the number of exceptions exceeds the target, then the model is not well specified and requires improvement.

Following Campbell (2005) and Christoffersen (2003), and using a sample of T observations, Kupiec's test statistic follows the next form:

$$H_0: p = \hat{p}\ vs\ H_1: p \neq \hat{p}$$

$$LR_{uc} = 2ln\left(\left(\frac{1-\hat{p}}{1-p}\right)^{T-\sum_{t=1}^{T} I(p)} \cdot \left(\frac{\hat{p}}{p}\right)^{\sum_{t=1}^{T} I(p)}\right) \sim \mathcal{X}^2(1) \quad (19)$$

$$\hat{p} = \frac{\sum_{t=1}^{T} I_t(p)}{T}$$

As the proportion of VaR violations differs from the target number, the Likelihood Ratio LR_{uc} test statistic grows, indicating that the proposed VaR measure either overestimates or underestimates the portfolio's underlying level of risk.

To test the independence of the VaR estimate, the next definitions hold:

- n_{01} as the number of observations where a non-failure is followed by a failure;

- n_{11} as the number of observations where a failure is followed by a failure;
- n_{00} as the number of observations where a non-failure is followed by a non-failure;
- n_{10} as the number of observations where a failure is followed by a non-failure.

Remind that $n_{01} = n_{10}$, since the order is not taken into account. It is possible to estimate the probability of tomorrow being a violation knowing that today was also a violation, and the probability of tomorrow being a violation knowing that today violation has not occurred by:

$$\hat{p}_{11} = \frac{n_{11}}{n_{10} + n_{11}} \qquad and \qquad \hat{p}_{01} = \frac{n_{01}}{n_{00} + n_{01}} \tag{20}$$

Since the probabilities must sum to the unity, we take:

$$\hat{p}_{00} = 1 - \hat{p}_{01} \qquad and \qquad \hat{p}_{10} = 1 - \hat{p}_{11} \tag{21}$$

The null hypothesis is that

$$H_0: \hat{p}_{00} = \hat{p}_{10} = p_2$$
$$\text{where } p_2 \approx \hat{p}_2 = \frac{n_{01} + n_{11}}{n} \tag{22}$$
$$\text{having } n = n_{10} + n_{11} + n_{00} + n_{01}$$

With the results above, we get the next two Markov chains:

$$\hat{P}_1 = \begin{bmatrix} \hat{p}_{00} & \hat{p}_{01} \\ \hat{p}_{10} & \hat{p}_{11} \end{bmatrix} \quad and \quad P_2 = \begin{bmatrix} 1 - \hat{p}_2 & \hat{p}_2 \\ 1 - \hat{p}_2 & \hat{p}_2 \end{bmatrix} \tag{23}$$

It is now possible to compute the independency test, using the next likelihood ratio test

$$LR_{ind} = 2ln\left(\frac{\hat{p}_{00}{}^{n_{00}} \cdot \hat{p}_{01}{}^{n_{01}} \cdot \hat{p}_{10}{}^{n_{10}} \cdot \hat{p}_{11}{}^{n_{11}}}{(1 - \hat{p}_2)^{(n_{00}+n_{10})} \cdot \hat{p}_2{}^{(n_{01}+n_{11})}}\right) \sim \mathcal{X}^2(1) \tag{24}$$

Since, according to (Christoffersen 2003), to know the accuracy of a model, it is needed to consider the results from both tests, it was employed in this way, the Conditional Coverage test which jointly tests if the VaR violations are independent and if the percentage of failures is statistically equal to the expected one. The ratio of this test will be the sum of the Likelihood ratio of the above presented tests,

$$LR_{cc} = (LR_{uc} + LR_{in}) \sim \mathcal{X}^2(2) \tag{25}$$

Even though this process helps identifying some drawbacks of the risk forecasting models, it does not identify the causes of the weaknesses, being only possible to conclude that the models should be reassessed and revaluated in terms of faulty assumptions, wrong parameters or inaccurate modelling, (Danielsson, 2011). Another problem with these tests is the clustering in time of the VaR exceptions, (Christoffersen, 2003). Besides those drawbacks, backtesting helps reduce the likelihood of overestimating VaR estimations, that can lead to excessive conservatism, (Danielsson, 2011).

3.4. Bivariate Linear Regression

Lastly, inspired by Ali, Alam & Rizvi (2020), we will employ, a simple linear regression analysis between the volatility (or the returns) of the different financial securities and COVID-19 deaths, with the goal of evaluating if there is any relationship between them. For this model we will take COVID-19 deaths as our independent variable and the returns and the volatility as the dependent variables. The models will take the form:

$$Y_{DCt} = \beta_0 + \beta_1 X_{Ct} + \varepsilon_t \quad (26)$$

where $D = \{r, v\}$ that represent the returns or the volatility of the stock index from the country C at time t and X_{Ct} represents the number of COVID-19 related deaths in the country C at time t. β_0, β_1 and ε_t represent, respectively, the intercept, the slope and the error term/residuals of our equation.

With this study one can evaluate the returns and the volatility changes of financial markets at the expenses of the pandemic related deaths.

"The most reliable way to forecast the future is to try understand the present"

John Naisbitt

4. Data Analysis

In this chapter, a brief description of the used data is given, displaying the results obtained according to each VaR approach used. From these empirical results, one can deduce which is the model that shows to be more accurate when predicting risk, according to different scenarios (normal and crisis periods), allowing to check also when most exceptions occurred. It is also concluded which are the stock indices that present bigger risk. Due to the length constraints, most of the tables and figures displaying the results are in the Appendix 2.

4.1. Data Specification

This section starts by pointing out that the dataset for the first study includes the business daily closing prices of the stock market indices from the G7 group, more specifically the S&P500 (USA), CAC40 (France), FTSE MIB (Italy), FTSE100 (United Kingdom), DAX30 (Germany) and NKY (Japan), adding further the PSI20 (Portugal), IBEX 35 (Spain) and the SHSZ300 (China). Henceforth these indices will be referred by their respective country names. All the data was collected from Bloomberg database. The testing window (sample period) chosen was from 1st of January of 2007 up until 31st of August 2020. The total number of observations vary per country stock index, since each country presents different business days. For the stock indices that had currencies different than the Euro, namely China, Japan, UK and USA, the currencies were exchanged based on the daily fixing rates extracted from the European Central Bank database (https://sdw.ecb.europa.eu/browse.do?node=9691296).

Logarithmic returns are used and calculated as the continuously compounded returns using the adjusted closing prices in the following way,

$$R_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{27}$$

where R_t is considered to be the daily returns at the day t and P_t and P_{t-1} are the closing prices of the stock market index at days t and $t-1$, respectively.

For this analysis was assumed that an investor would buy 1000 shares per stock index (the Market Value is taken as the number of shares bought, times the price of the stock per share), and for the computation of all VaR measures a confidence level of 99% was chosen, giving, in this way, more importance to the most extreme events.

4.2. Descriptive Statistics

In this research, the above-mentioned country stock indices were chosen majorly because those countries presented the highest impact concerning COVID-19 deaths, which will enable a better analysis to be done in the last subsection. It makes sense of using these indices not only for the above mentioned need, but also because the majority of these countries are considered to be the world's leading industrial nations corresponding as well to have the best well developed economies in the world, having in this way a strong worldwide political influence, (European Comission n.d.). The chosen indices are known as being reference indices of the capital markets of each country.

To analyse the datasets it was used, Microsoft Excel, that took advantage of the Solver add-in program, the Eviews and RStudio software. The following table contains the descriptive statistics of the business daily returns of the stock indices used for this first analysis.

	Descriptive Statistics of the Returns in Euros								
	CHI	***FR***	***GER***	***IT***	***JPN***	***PT***	***SP***	***UK***	***USA***
Mean	3,1E-04	-3,3E-05	2,0E-04	-2,2E-04	1,4E-04	-2,7E-04	-2,0E-04	-9,4E-05	2,8E-04
Medn	0,0007	0,0004	0,0008	0,0004	0,0005	0,0001	0,0004	0,0003	0,0006
Max	0,1077	0,1059	0,1080	0,1087	0,0991	0,1020	0,1348	0,0961	0,1052
Min	-0,1131	-0,1310	-0,1305	-0,1854	-0,1045	-0,1038	-0,1515	-0,1257	-0,1324
Stdv.	0,0178	0,0145	0,0142	0,0167	0,0134	0,0131	0,0155	0,0134	0,0136
Skew	-0,5379	-0,2895	-0,2169	-0,6534	-0,4333	-0,3938	-0,3926	-0,4647	-0,4054
Kurt	7,3337	10,9308	11,1222	12,1850	9,0888	10,0856	12,2484	12,3668	13,6250
J-B test	2941,7	9210,9	9546,0	12430,1	5596,4	7403,6	12513,2	12932,0	16691,5
Prob	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
N. Obs	3541	3496	3463	3466	3551	3496	3486	3503	3528

Table 1: Descriptive Statistics of the compounded business daily log-returns from 01/01/2007 to 31/08/2020.

Analysing the above presented table, all the unconditional means of the daily log-returns are close to zero. The maximum and minimum returns are in between 13,48% and −18,54%, respectively. The standard deviations tend towards values in between 13,10% and 17,80%. The skewness statistics are negatively close to zero for all the returns, implying that they're not far from being symmetrical, even though indicating that the returns are skewed to the left. The kurtosis will measure the thickness of the tails of each distribution. Since all the series show a Kurtosis bigger than three, it is possible to infer that the distributions show a leptokurtic behaviour (fat tails). This can also be checked by the analysis of the histograms in 40Section 1 - Returns Graphs and Histogram. A normality Jarque-Bera test was also employed. It is possible to affirm, based on the p-value, that the null hypothesis is rejected, having enough statistical evidence of defending the non-normality of the series.

In Section 1 - Returns Graphs and Histogram, the graphs from the returns of the stock indices are also presented. Analysing the graphs, it is visible the presence of volatility clusters, meaning that the volatility shocks are always followed by other volatility shocks (high return movements are followed by high return movements, happening the exact opposite relation for small return movements).

The following table shows the estimates of the weights for the $GARCH(1,1)$ model.

	Weights for GARCH(1,1) estimate								
	CHI	**FR**	**GER**	**IT**	**JPN**	**SP**	**PT**	**UK**	**USA**
ω	1,7E-06	2,9E-06	3,1E-06	4,5E-06	4,4E-06	3,8E-06	3,3E-06	2,4E-06	3,7E-06
α	0,061	0,100	0,095	0,100	0,096	0,100	0,100	0,100	0,100
β	0,936	0,885	0,890	0,885	0,879	0,884	0,879	0,883	0,872
γ	0,0029	0,0147	0,0153	0,0149	0,0249	0,0155	0,0215	0,0170	0,0284
VL	0,0006	0,0002	0,0002	0,0003	0,0002	0,0002	0,0002	0,0001	0,0001

Table 2: GARCH(1,1) weights.

For the $GARCH(1,1)$ model to be accurate, the autocorrelation of the data must be removed. Following Hull (2013), to test the autocorrelation of the data, a Ljung-Box statistic test was employed. One must assume that r_t^2 exhibits autocorrelation and check if Z_t^2 are uncorrelated. We obtain the random variables, Z_t^2, by:

$$r_t^2 = \varepsilon_t^2 = \sigma_t^2 \cdot Z_t^2 \Leftrightarrow Z_t^2 = \frac{r_t^2}{\sigma_t^2} \tag{28}$$

Note that Z_t is assumed to be a random walk process.

For this test, we considered 15 lags (degrees of freedom) and a confidence level of 99%. There is enough statistical evidence to reject zero autocorrelation for a Ljung-Box (LB) statistic greater than 30,5. From the analysis of the Table 7: Ljung-Box test, it is possible to conclude that the LB statistic for the r^2 shows strong evidence of correlation for all the financial stock indices. On the other hand, after the $GARCH(1,1)$ model is used, the LB statistic for the Z^2 suggests that the autocorrelation has been largely removed for all the stock indices (except for the Italian index), enhancing the idea of the Z_t to follow a random walk process.

The next table shows the estimates of the weights for the EVT-POT model

	Threshold and estimated weights through QMLE for EVT-POT								
	CHI	**FR**	**GER**	**IT**	**JPN**	**SP**	**PT**	**UK**	**USA**
u	1,59	1,89	1,90	1,73	1,31	1,70	1,81	1,79	1,90
ξ	0,049	0,063	0,102	0,055	0,084	0,097	0,006	0,016	0,120
β	0,716	0,635	0,528	0,606	0,641	0,586	0,615	0,684	0,600

Table 3: EVT-POT threshold and weights.

4.3. VaR Analysis

Through the implementation of the models presented in the third chapter (Statistical approaches to VaR and Backtesting), into the pre-worked data as aforementioned (log-returns), many conclusions can be taken.

First, it makes sense to state that the number of expected failures is obtained by multiplying the number of observations by the chosen probability level, $p = 0{,}01$.

$$VaR\ expected\ Number\ of\ failures = number\ obs \cdot 0{,}01$$

Check Table 8: VaR expected failures., for the expected failures per country index.

Defining the number of expected failures, it is possible to do a comparison between that number and the real number of fails. The next table presents some outputs of the analysis:

	VaR			Number of fails			$\hat{p}$		
Countries	**HS**	**GARCH**	**EVT**	**HS**	**GARCH**	**EVT**	**HS**	**GARCH**	**EVT**
CHI	47 784	59 780	72 978	44	20	11	1,24%	0,56%	0,31%
USA	217 369	420 238	495 229	49	24	10	1,39%	0,68%	0,28%
PT	466 064	901 393	1 075 677	60	16	12	1,72%	0,46%	0,34%
FR	312 897	588 342	697 933	56	16	10	1,60%	0,46%	0,29%
GER	736 750	1 440 971	1 654 690	60	15	10	1,73%	0,43%	0,29%
JPN	6 644	15 305	18 309	51	17	12	1,44%	0,48%	0,34%
SP	756 587	1 376 256	1 608 532	52	15	8	1,49%	0,43%	0,23%
IT	1 629 090	3 175 153	3 759 037	56	15	7	1,62%	0,43%	0,20%
UK	439 782	819 855	975 617	55	21	11	1,57%	0,60%	0,31%

Table 4: VaR, Number of fails and probability of happening a violation.

Table 4 presents the VaR analysis per implementation used, the number of fails per each model, regarding P&L movements, and the estimated probability of a failure to occur.

According to Jobayed (2017), VaR estimates by themselves are insufficient to take any type of conclusions. Therefore, these estimates will be compared with the actual P&L returns following the backtesting method presented at the end of Chapter 3 - Backtesting. For this analysis makes sense to conclude separately for each model and then take a general conclusion of all the estimations.

Table 9: Expected loss rate has the purpose of enabling the comparison of the worst expected loss rate. To do this we divide the VaR estimate with the MTM price observed on 31/08/2020. In this way, it is possible to conclude which countries show the worst expected possible return. For this analysis the next structure will be followed:

1. The most important question, when analysing the estimated models, is if they show to be accurate enough, while describing stock index movements. In this way, and basing this analysis on Table 4 and Table 8: VaR expected failures., a comparison between the number of VaR expected fails and the observed number of fails is done;

2. Based on the analysis of Table 9, and since the currencies of all the countries are the same, a comparison of the expected (loss) returns can be done, allowing to decide which stock indices are riskier to invest;
3. Concerning Table 10: Backtesting LRuc, LRind and LRcc, it is possible to conclude for the Unconditional Coverage, Independence and Conditional Coverage tests, if the models are accurate.
4. Through the analysis of the graphs in Section 3 - Backtesting Graphs, it can be seen which are the models that predict better the falls in P&L defined as outliers, helping as well to check which was the period where the models showed more difficulty when estimating the P&L movements, with specific focus on the period starting from 01/01/2020 until 31/08/2020.

4.3.1. Historical Simulation

From the analysis of the HS models, it is possible to conclude that the number of observed VaR failures ranges between 44 and 60, from China and Germany, respectively (China was the best model to be predicted and Germany the worst). When comparing these results with the expected amount of failures, it is observable that there are at least more 25 VaR fails than the ones expected. From this, one can conclude that HS is not the best when modelling the data because it is a model that underestimates risk.

When checking which are the indices that give the worst expected rate of loss, the results from Table 9 should be considered. From this table, the stock index that presents the biggest risk of investment loss is the Spanish, closely followed by Portugal, and the index that presents the smallest risk is the Japanese one. Therefore, if an investor wants to invest in any of these countries, Japan may be the safest choice, since it is the country that shows a smaller risk of investment loss.

From the analysis of Table 10: Backtesting LRuc, LRind and LRcc, in terms of the Unconditional Coverage test, for the Chinese, American and Japanese stock indices the model is accepted, being the rest of the indices rejected (the empirical probability of a violation to occur diverges too much from the theoretical probability level). For the Independence test, the Chinese, French, Spanish and Italian stock indices are unpredictable and independently correlated through time. The other indices do not present independency between the number of failures, rejecting H_0. The Conditional Coverage test, test that considers both the UC and Ind tests, only accepts the accuracy of the model for the Chinese stock index.

4.3.2. GARCH(1,1) model

$GARCH(1,1)$ modelling was introduced to improve the VaR estimates under the assumption of the Normality of the data. When comparing the GARCH model with HS, it can be easily seen that the number of failures has decreased substantially, being even lower than the number of expected failures. For this model, the number of VaR failures ranges between 15 and 24, from the German, Spanish and Italian and American indices, respectively. Because the number obtained failures is smaller than the expected amount of violations, it is possible to conclude that the $GARCH(1,1)$ instead of underestimating risk, overestimates it.

For this approach, when checking Table 9 it can be seen that the riskiest index to invest is the Portuguese, followed by the Spanish, while the safest being still the Japanese index.

Analysing Table 10: Backtesting LRuc, LRind and LRcc, the Unconditional test accepts the Unconditional property for the UK and USA, rejecting all the others. In terms of the independence property, only Japan, USA and UK have been rejected. Once again, the Conditional Coverage test accepts the accuracy only for the Chinese model.

4.3.3. EVT-POT (Dynamic) model

Since the above presented model still failed to predict the most extreme movements in P&L, the Dynamic EVT-POT model was implemented. As aforementioned this model is known to be a good estimator of the tails distribution, therefore it is expected a decrease in the number of failures. Analysing Table 4, even less exceedances were observed, ranging from 7 to 12 failures, being seven failures from the Italian stock index, and 12 from both Portuguese and Japanese indices. As the $GARCH(1,1)$ model, the EVT model overestimates risk.

The stock indices that show to be less risky are the Japanese, Chinese and German, while the riskiest continue to be the Portuguese and Spanish.

From the analysis of Table 10: Backtesting LRuc, LRind and LRcc, it can be concluded that the Unconditional test rejects the Unconditional property for all the indices. This deduction makes sense since the probability of a failure to occur, for these kind of Extreme Value Theory models, is substantially low and far from the expected one. In terms of the independence property, all the countries presented signs of independency when the outcome of a failure occurred, being in accordance with the low number in exceedances. Because of the results from the UC test, the Conditional Coverage test rejects all the accuracy of these models.

4.4. Backtetsing analysis

Checking the graphs in Section 3 - Backtesting Graphs, it can be seen a representation of the discrete P&L movements of each country's stock index (represented as a blue cloud of dots) and the daily VaR estimates of the HS, $GARCH(1,1)$ and EVT-POT models (represented as the orange, grey and yellow lines, respectively). It can be easily concluded that the model that shows to be the most accurate, regarding extreme movements in P&L is, as expected, the EVT-POT model, even though failing to predict a residual number of movements.

The HS model shows promising accuracy when predicting the P&L movements for certain time periods (normal periods). Every time an extreme movement in P&L occurs, the HS model fails to predict it because it responds slowly to abrupt changes (this happens because, as seen in 3.2.1. Historical Simulation approach, the predicted loss can never exceed historical losses). The $GARCH(1,1)$ model improved the HS VaR estimates, since it captures the Heteroskedasticity of the data. The number of exceedances decreased, but the model still failed to predict many movements in P&L. For the Dynamic EVT-POT model, the estimations improved substantially, not only because of taking into account the Heteroskedasticity of the data, but also because it relied only on the information of the left side of the distribution of returns.

By the analysis of these graphs it is possible to infer that the P&L movements, from the different countries, seem to be correlated with each other in many time periods (whenever a shock is observed in a stock index, that same shock can generally be observed in the other countries).

Based on the tables in Section 7 - Number of exceptions per year, it can be checked the number of exceedances that the three models presented per year. For this analysis it was only considered the prediction made by the least accurate model (HS), the one that underestimates risk, since the others just simply show a better risk estimation for the most unpredictable periods. In 2008, USA, Portugal, France, Spain, Italy and the UK presented a big amount of exceptions, making sense because this is a period that is known for having huge volatility movements. This period corresponds to the Subprime and the Lehman crises, that were felt all around the world. In 2011, the biggest amount of exceptions was observed in France, Germany and Italy. This period is known as being a crisis period, more specifically, the European Sovereign Crisis, majorly felt by the European countries. In 2013, only Japan presented a big number of exceptions, probably due to the Emerging Markets Crisis, that occurred in

2013/2014. In 2018, the Chinese, Portuguese, Japanese and Italian stock indices showed an increase in unpredictability thanks to the announcement of Brexit.

Turning the attention to the results occurring in the time period from 01/01/2020 until 31/08/2020, the crisis period that this study focuses the most (COVID-19 Pandemic), it is possible to conclude that many countries experience absurd movements in their stock index returns, being Portugal, France, Japan and Spain the countries that show a bigger number of exceptions. From the graphs in Section 3 - Backtesting Graphs, can be as well checked that even though this period is not as big as the one represented in the other years, it still shows a big number of P&L movements failed to be predicted.

Since this crisis is still far from being over, possibly with the passing of time, the stock indices will continue to show signs of more unpredictability. This conclusion is only based trough the analysis of the number of exceptions, that enables to speculate the movements in volatility (unpredictability) of the financial stock index markets.

Generally, all the indices suffered repercussions from those events, enabling a bigger risk in those periods. The years that showed the least exceedances, considered to be the tranquil/normal ones, were years 2009, 2012 and 2017.

Lastly, it is possible to conclude that the HS and $GARCH(1,1)$ models predict well the movements of the stock indices for the tranquil periods, while the EVT-POT model should be the one to be used when estimating the most abrupt movements of these indices. Therefore, a combination of both models should give the most accurate results. Not to forget that the principal objective of these models is to predict the movements in P&L, therefore it is always advisable to use the model that shows better accuracy when predicting extreme observations (bigger risk).

"There is as yet insufficient data for a meaningful answer."
— Isaac Asimov (The Last Question)

5. Bivariate Regression Analysis

Following (Ali, Alam, and Rizvi 2020), since it was considered to be an interesting and curious study, a Bivariate Regression Analysis was also employed in order to find if there is a detectible relationship between the deaths related to COVID-19 and the returns (or the volatility) of the different financial stock index markets. By doing so, the purpose is to further deepen knowledge on the topic and somehow to complete and give a different perspective to the analysis previously done.

5.1. Data

For this study it was needed to collect additional data about the number of COVID-19 deaths. The data used consists of the number of deaths occurred in the nine countries analysed before, specifically China, United Kingdom, United States of America, Italy, France, Spain, Portugal, Germany and Japan, and it comprises the daily deaths since 01/01/2020 until 30/01/2020. The complementary data to be used in this analysis was extracted from the WHO database (https://covid19.who.int/table).

The methodology employed follows the one from Ali, Alam, and Rizvi (2020), with some differences:

1. It was performed two linear regression analysis: one looks for a relationship between the number of deaths caused by Covid-19 (independent variable) and the returns of the index stock market (dependent variable) and the other looks for the relationship between the first and the index stock market's volatility (dependent variable).
2. Since it was used business daily prices, all the deaths occurring during weekends and holidays will be cumulative to the next working day, because the cumulative impact would be seen on "Monday's" price.
3. To compute the volatility, the authors employed an Exponential GARCH (EGARCH) variance model. However, in (Hull 2013) is stated that "$GARCH(1,1)$ is by far the most popular of the GARCH models", and since this research already employed the $GARCH(1,1)$ to estimate the volatilities of the stock market (in order to compute the VaR), for the sake of consistency, the same model is used.

4. For the COVID-19 deaths dataset, the data was extracted from the WHO database. A comparison was performed between this dataset and some specific data from each country's official health department to confirm consistency. Apart from some daily differences (total numbers are equal), the two sources provide comparable observations.

Remark that some of the countries in the original study were not included, and the cases of Portugal and Japan were added (we have chosen only countries included in the previous chapters).

In Appendix 3 we can see, the graphs from the returns of the securities and their histograms, the indices Returns and Volatility, as functions of the COVID-19 related deaths, and tables with the weights for the $GARCH(1,1)$ model, as well as the Ljung-Box test. Below are presented the descriptive statistics of the data and the results from the estimation.

	Descriptive Statistics of the Returns in Euros (Regression study)								
	CHI	*FR*	*GER*	*IT*	*JPN*	*PT*	*SP*	*UK*	*USA*
Mean	0,00069	-1,11E-03	-0,00014	-0,0011	-3,39E-04	-0,0011	-0,0019	-1,68E-03	0,00011
Medn	0,00114	-0,00004	-0,00007	0,0013	0,00035	0,000	-0,0008	-0,00009	0,00157
Max	0,05276	0,08056	0,10414	0,0855	0,07166	0,075	0,0753	0,09607	0,10098
Min	-0,09527	-0,13098	-0,13055	-0,1854	-0,08198	-0,103	-0,1515	-0,12571	-0,13241
Stdv.	0,01704	0,02324	0,02363	0,0259	0,01746	0,019	0,0240	0,02298	0,02569
Skew	-1,46140	-1,23583	-0,85432	-2,5484	0,07390	-1,314	-1,5588	-1,00188	-0,68702
Kurt	9,39589	9,6607	10,33336	19,7890	7,38057	11,731	12,3244	9,56813	9,71537
J-B test	354,3933	357,5268	399,2456	2180,591	138,4807	588,8569	684,7094	335,9824	336,7197
Prob	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Obs	172	170	169	170	173	170	170	171	172

Table 5: Descriptive statistics of the continuously compounded business daily log-returns from 01/01/2020 to 31/08/2020.

Bivariate Regression Analysis (Independent variable: COVID-19 deaths)						
Dependent Variable	**Returns**			**Volatility**		
Countries	**Coefficient**	**t-statistic**	**P-value**	**Coefficient**	**t-statistic**	**P-value**
CHI	4,23E-06	0,361	0,719	3,08E-06	1,352	1,78E-01
FR	5,97E-06	1,637	0,104	8,26E-06	5,303	3,55E-07
GER	2,72E-05	1,688	0,093	3,58E-05	5,857	2,44E-08
IT	1,85E-06	0,385	0,700	1,48E-05	8,448	1,35E-14
JPN	5,90E-06	0,056	0,955	1,67E-05	0,602	5,48E-01
PT	1,19E-04	1,641	0,103	1,01E-04	3,115	2,17E-03
SP	2,15E-06	0,478	0,634	1,41E-05	8,505	9,57E-15
UK	4,61E-06	1,368	0,173	6,70E-06	4,742	4,48E-06
USA	1,97E-06	1,478	0,141	1,53E-07	0,229	8,19E-01

Table 6: Bivariate regressions.

The main comment is that results are quite aligned with those in Ali, Alam, and Rizvi (2020), even if this research has more observations than those in the original paper. In fact, from the analysis of the coefficients and p-values, we can conclude that most of the remarks made by the authors are still valid. In Ali, Alam, and Rizvi (2020), the authors concluded that there was statistical evidence, with a confidence level of 99%, to infer that the movement in the volatility in most of the stock market indices showed a positive dependent relation with the number of COVID-19 related deaths, excepting for the Italian, Spanish and Chinese indices. For the returns model, it was found that Germany, France and the UK were negatively related with the number of deaths.

Taking only into account the analogous countries used in both studies, therefore Italy, Spain, China, Germany, France, USA and UK, it was possible to conclude that, for the Returns Bivariate model, there were some inconclusive cases at a level of 99% confidence, more explicitly for Germany, UK and France, where there were only statistical evidences (for confidence levels of 95% for the two first and 90% for the latter) that the returns of those indices were negatively related with the deaths. For the other countries there was no statistical evidence of a being related. Concerning the Volatility Bivariate model, most of the indices present enough evidence of a relation occurring (this with a confidence level of 99%), except for Germany, France and the UK.

Finally, there are only a few cases where conclusions are different. In the original paper, the authors observed that returns were not significantly dependent on the number of deaths, the only exceptions being France, UK and Germany. In our model, this is no longer completely true: there is no statistical evidence that returns vary according with the level of deaths, for France and UK. On the other hand, Germany still shows statistical evidence, even though only for a confidence level of 90%, of a positive relation between both deaths and returns movements. In our Volatility model, Italy and Spain started showing significant evidence of a relation, meaning that the index stock markets become more volatile as the number of COVID-19 deaths increase. Portugal could be pointed out as showing a relation between both variables. The opposite happened with the index from USA, which lost its statistical evidence. For Japan there were also, any signs of a relation to exist.

In a way, this also confirms that our results are aligned with theirs. The results obtained do not contradict the works from Onali (2020), where it is stated that the COVID-19 deaths that occurred in Italy and France presented a positive impact on the benchmark volatility index, VIX.

Checking Table 15: Ljung-Box test, it must be taken into account that the $GARCH(1,1)$ estimates for the Portuguese, Italian and UK indices show evidence of autocorrelation.

To summarize, it is reassuring to conclude that our results do not contradict the results in Ali, Alam, and Rizvi (2020). The obtained results go in line with the ones obtained from the VaR analysis, where more exceptions were pointed out.

"In literature and in life we ultimately pursue, not conclusions, but beginnings."

Sam Tanenhaus, Literature Unbound

6. Summary and Conclusions

This chapter summarizes all the results and conclusions taken out during the research, giving, as well, suggestions of topics and improvements for further research.

6.1. Main findings

Through the analysis of the literature review, it is possible to conclude that, even though there exists a huge number of ways to estimate VaR, some of them with a high degree of complexity, there is not an approach that can be considered «the best». In the majority of the analyzed researches, it is possible to conclude that the Historical Simulation, Generalized Autoregressive Conditional Heteroskedasticity and Dynamic EVT-Peaks Over Threshold methods were the ones that presented the most accurate results, when predicting the movement of the stock index returns. Hence, the same methods were implemented for this study.

From the analysis of the VaR estimates, it was found, as expected, that the least accurate method is the HS, which is the simplest of all the implemented models, and the Dynamic EVT-POT model was the most accurate, predicting well the tails of the distribution of returns. The majority of the exceptions occurred in periods of more volatility of the financial stock indices, namely the Subprime crisis (2007), Lehman crisis (2008), European Sovereign crisis (2011), Emerging Markets crisis (2013/2014), Brexit (2018) and COVID-19 Pandemic (2020). It was also pointed out that the stock indices that seem to be the riskiest to invest in are the Spanish and the Portuguese, while the least risky is the Japanese.

For the regression study it is possible to conclude that the volatilities of the stock indices tend to increase, making the management of the risk an even more difficult task, as the number of COVID-19 related deaths increases. On the other hand, there is in general no statistical evidence of the same happening when considering stock indices returns.

6.2. Further research

One possible idea for further research is testing the parametric $GARCH(1,1)$ with a t-student distribution, as seen in other works. This distribution is similar to the Gaussian, with the difference that the tails are thicker. As referred before, the stock index returns present fatter tails than the Normal distribution, thus, this modification may increase the accuracy of the estimation. Still in the parametric estimation, instead of assuming the stationarity of the conditional mean, it could be interesting to check the difference in results one might have when estimating the mean and the residuals. For this estimation, an EWMA or an $ARMA(p, q)$ models are the most common and appropriate.

Another suggestion is to estimate VaR models with different confidence levels (90% and 95%), allowing a comparison between the changes the models might have, when facing different probability levels.

Most importantly, the analysis of the pandemic impact on the financial stock indices should continue to be stressed, since with the upcoming of the second wave the indices may to show more signs of unpredictability.

Finally, since VaR is known to have certain limitations, Expected Shortfall (ES) can be employed simultaneously, to complement the analysis.

Appendices

1. Appendix 1

1.1. Section 1 - Mean Stationarity

Assuming that the returns r_t are assumed to be stationary in mean during time, implies that the mean will be taken as a constant value ($\mu_t = \mu$). This constant value is assumed to be $\mu = 0$, since in all stock indices, the average rounded to three decimal cases is null.

Mathematically, this assumption will imply:

$$r_t = \mu_t + \varepsilon_t = \mu + \varepsilon_t = 0 + \varepsilon_t = \varepsilon_t$$

And since we know that $\varepsilon_t = \sigma_t \cdot Z_t$, taking $Z_t \sim N(0,1)\ i.i.d.\ random\ walk$, we can define

$$r_t = \sigma_t \cdot Z_t$$

being σ_t the estimated volatility at day t.

1.2. Section 2 - $GARCH(1,1)$ decay rate

Exponential decline explained

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Substituting σ_{t-1}^2 in the equation, we get

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta(\omega + \alpha r_{t-2}^2 + \beta \sigma_{t-2}^2) \Leftrightarrow$$

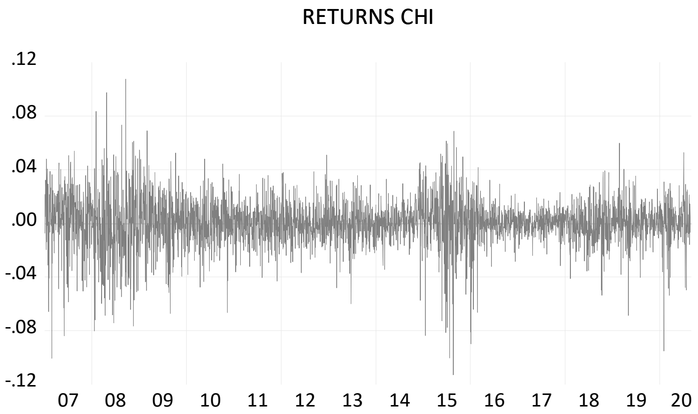
$$\Leftrightarrow \sigma_t^2 = \omega + \beta\omega + \alpha r_{t-1}^2 + \alpha\beta r_{t-2}^2 + \beta^2 \sigma_{t-2}^2 \Leftrightarrow$$

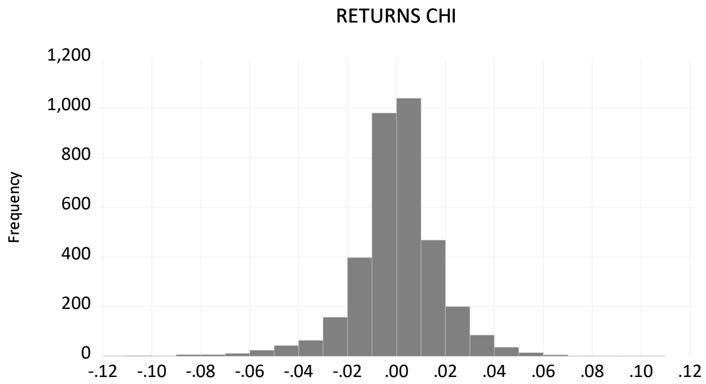
$$\Leftrightarrow \sigma_t^2 = \omega + \beta\omega + \beta^2\omega + \alpha r_{t-1}^2 + \alpha\beta r_{t-2}^2 + \alpha\beta^2 r_{t-3}^2 + \beta^3 \sigma_{t-3}^2 \Leftrightarrow \cdots$$

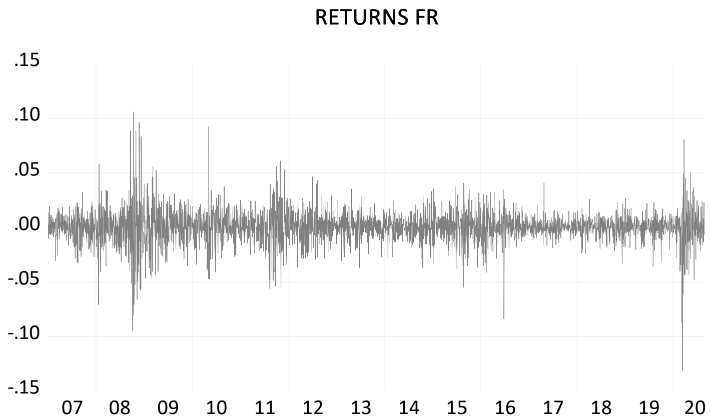
And so forth. That is why we say that β can be interpreted as a “decay rate”, because it assigns weights that decline exponentially to the past squared returns and to the long-run average volatility. For the long-run average volatility, we will get a geometric distribution.

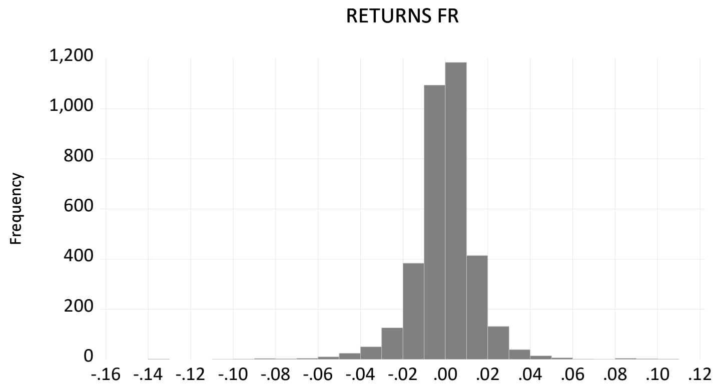
2. Appendix 2

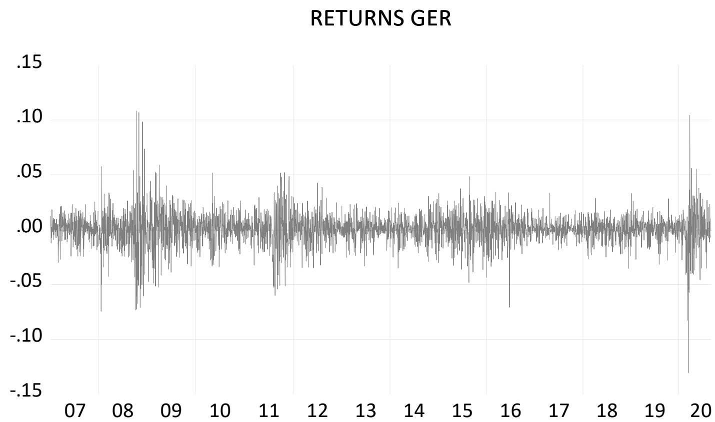
2.1. Section 1 - Returns Graphs and Histograms

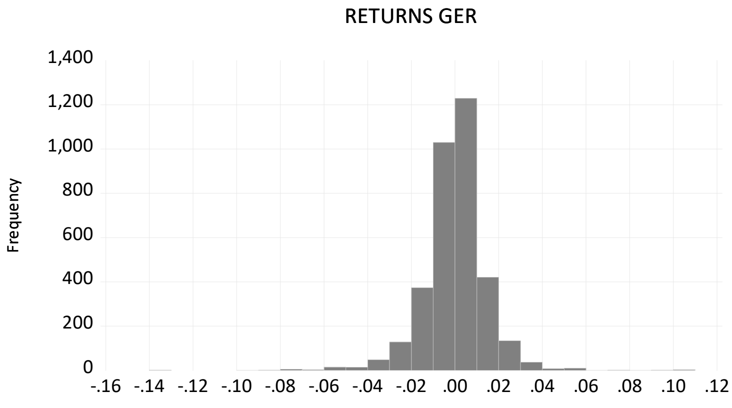


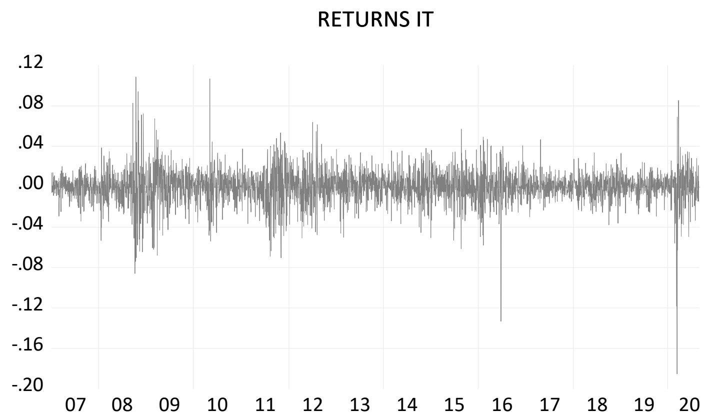


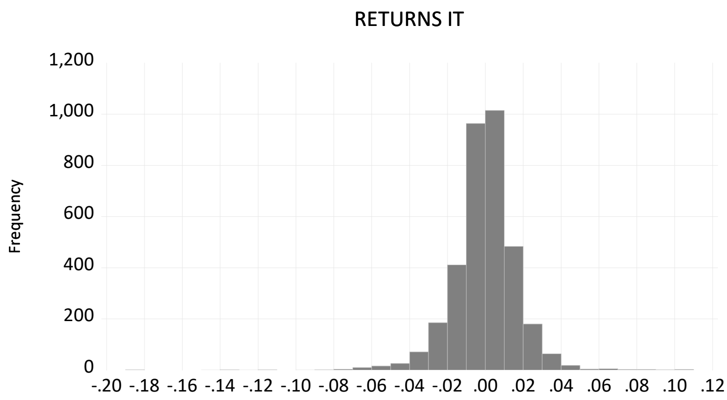


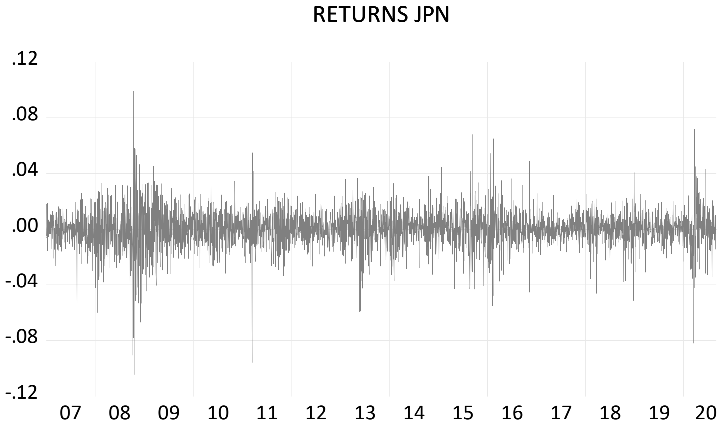








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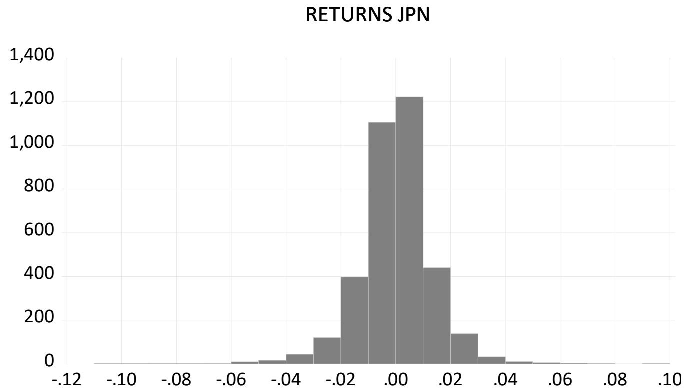
.08
.04
.00
-.04
-.08
-.12
07 08 09 10 11 12 13 14 15 16 17 18 19 20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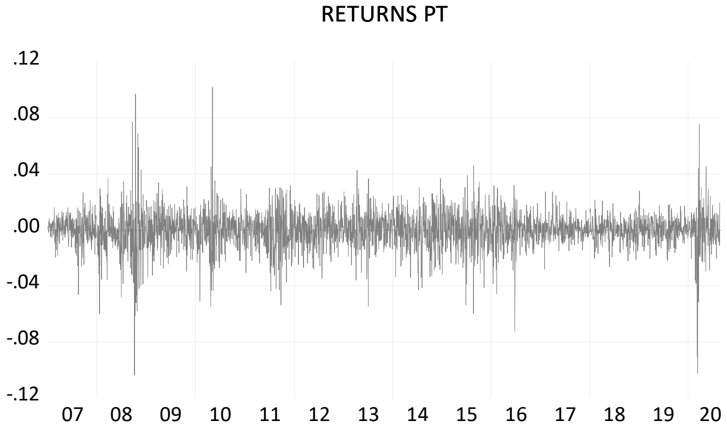
1,000
800
600
400
200
0
-.12 -.10 -.08 -.06 -.04 -.02 .00 .02 .04 .06 .08 .10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.00
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-.08
-.12
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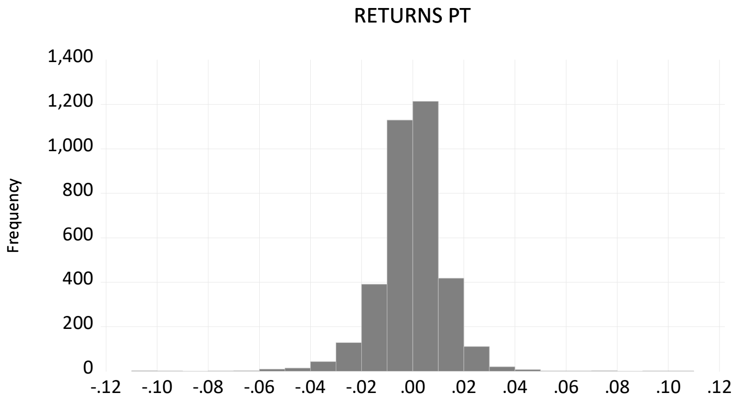
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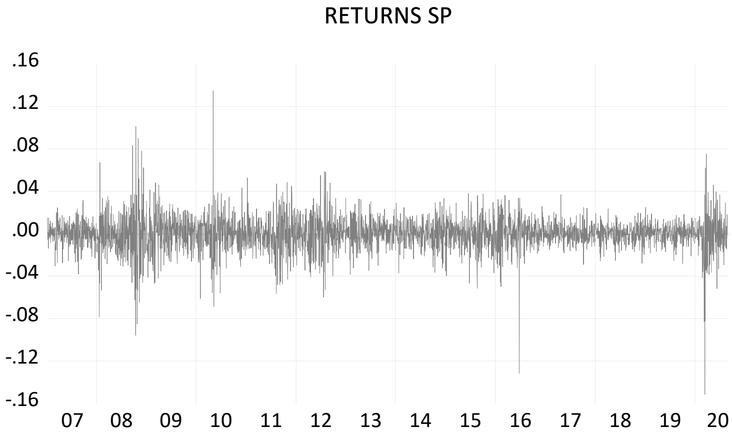
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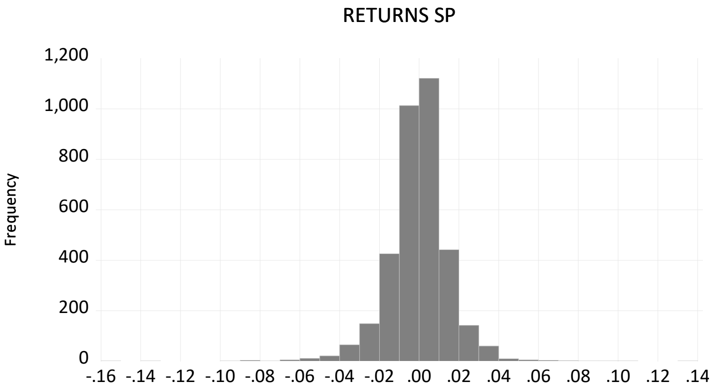
.08
.04
.00
-.04
-.08
-.12
-.16
07 08 09 10 11 12 13 14 15 16 17 18 19 20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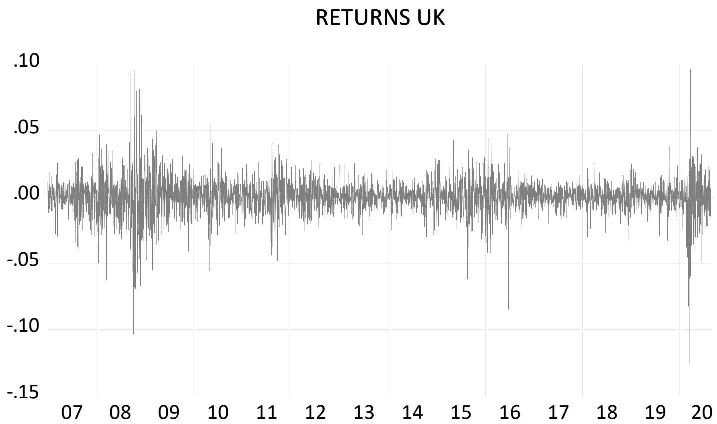
800
600
400
200
0
-.16 -.14 -.12 -.10 -.08 -.06 -.04 -.02 .00 .02 .04 .06 .08 .10 .12 .14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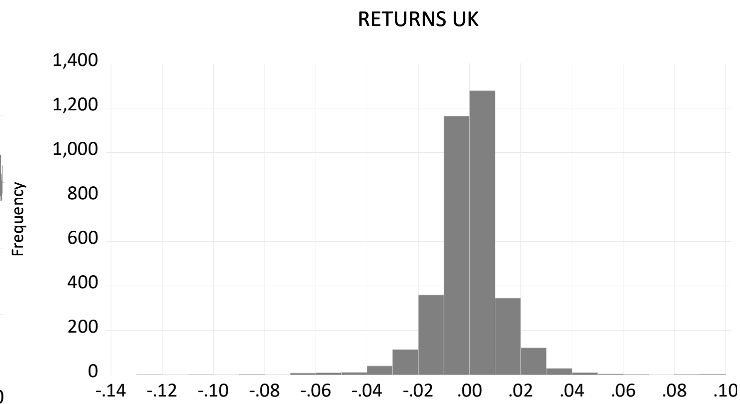
.00
-.05
-.10
-.15
07 08 09 10 11 12 13 14 15 16 17 18 19 20


RETURNS UK
Frequency
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1,000
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600
400
200
0
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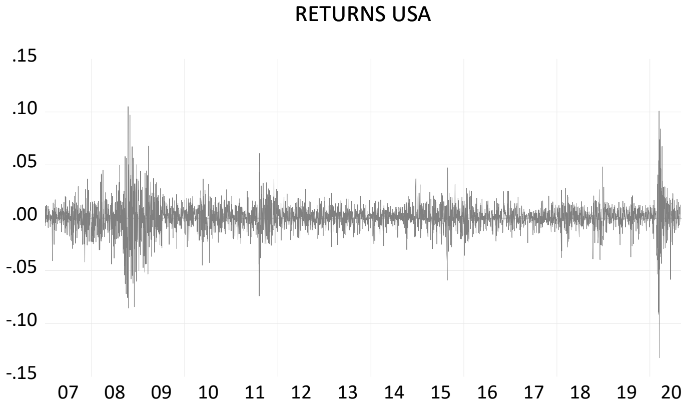
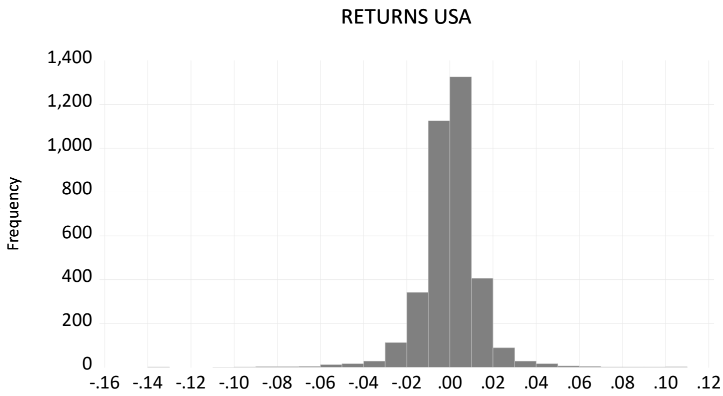
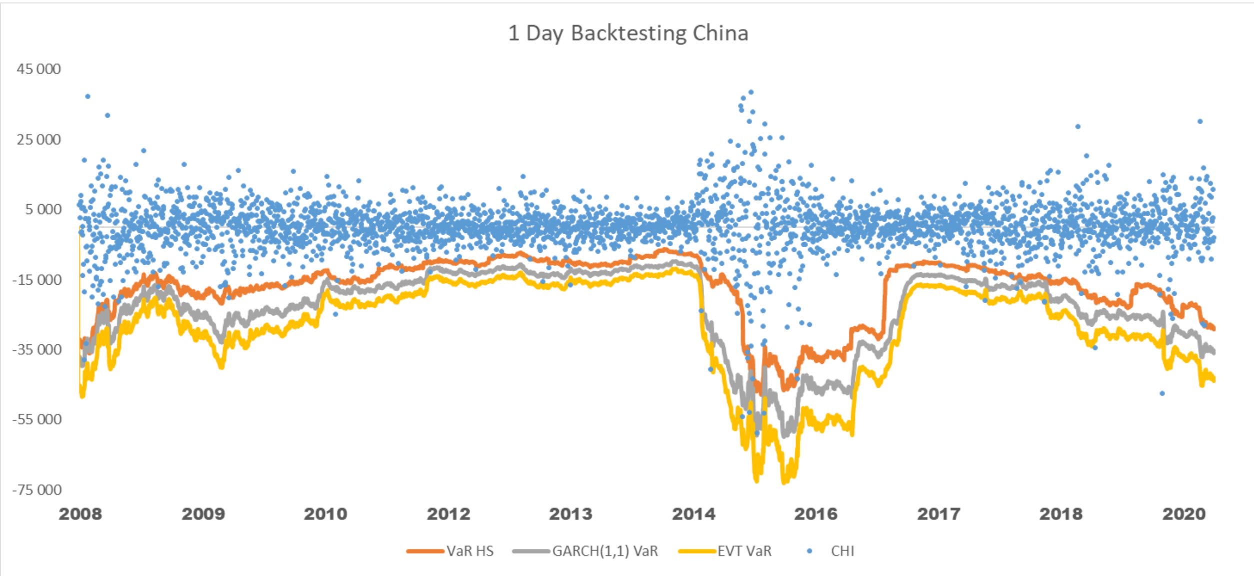
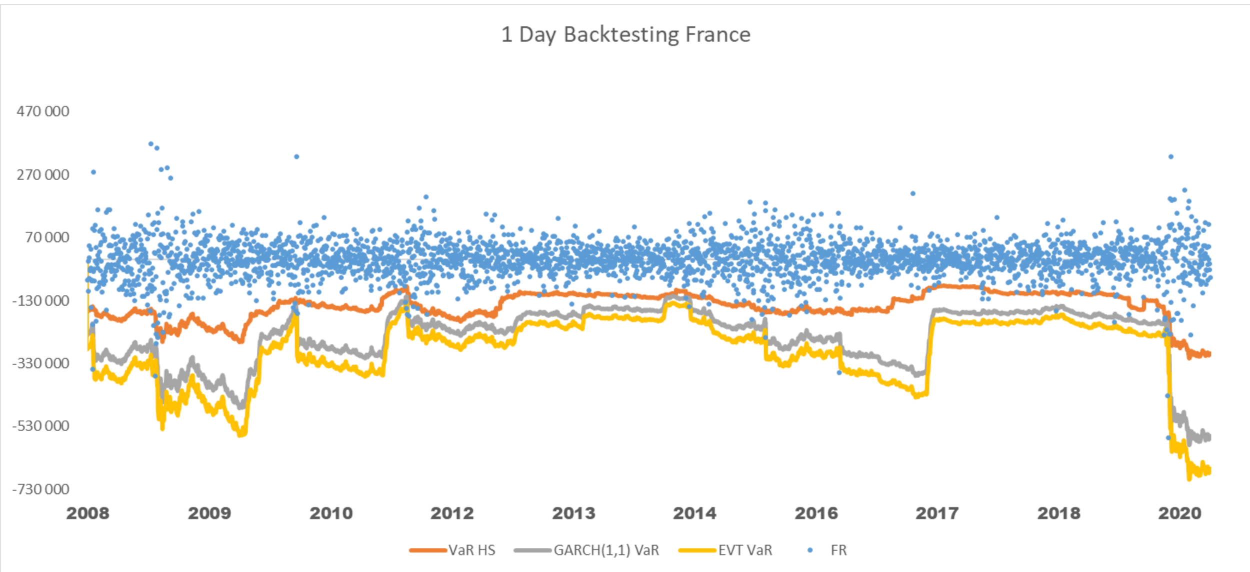
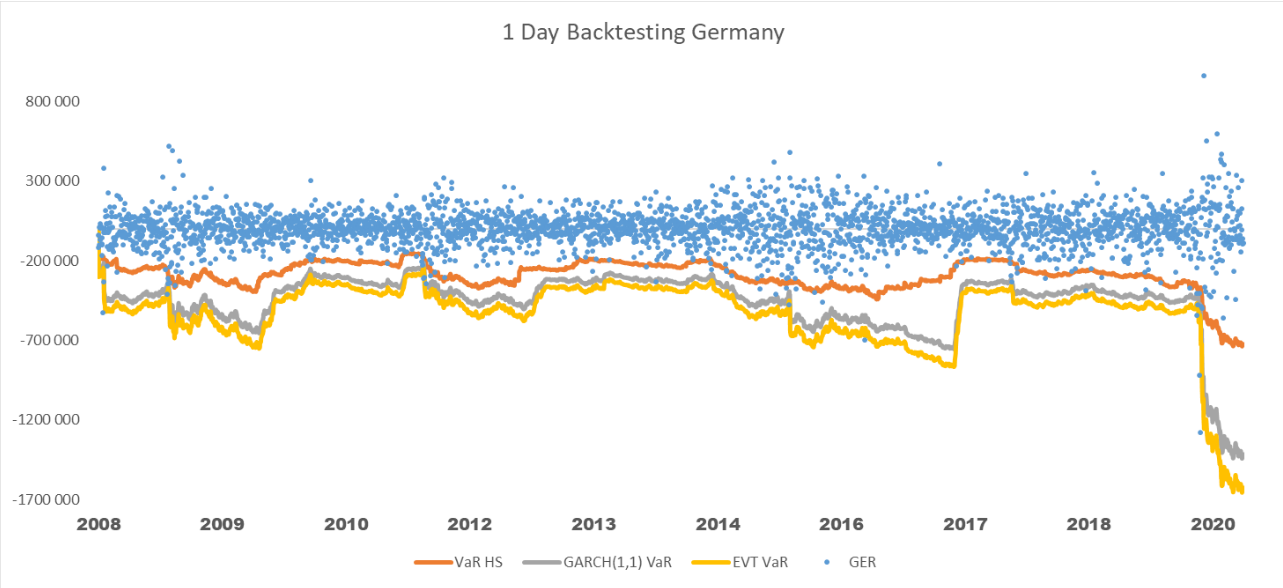




2.2. Section 3 - Backtesting Graphs



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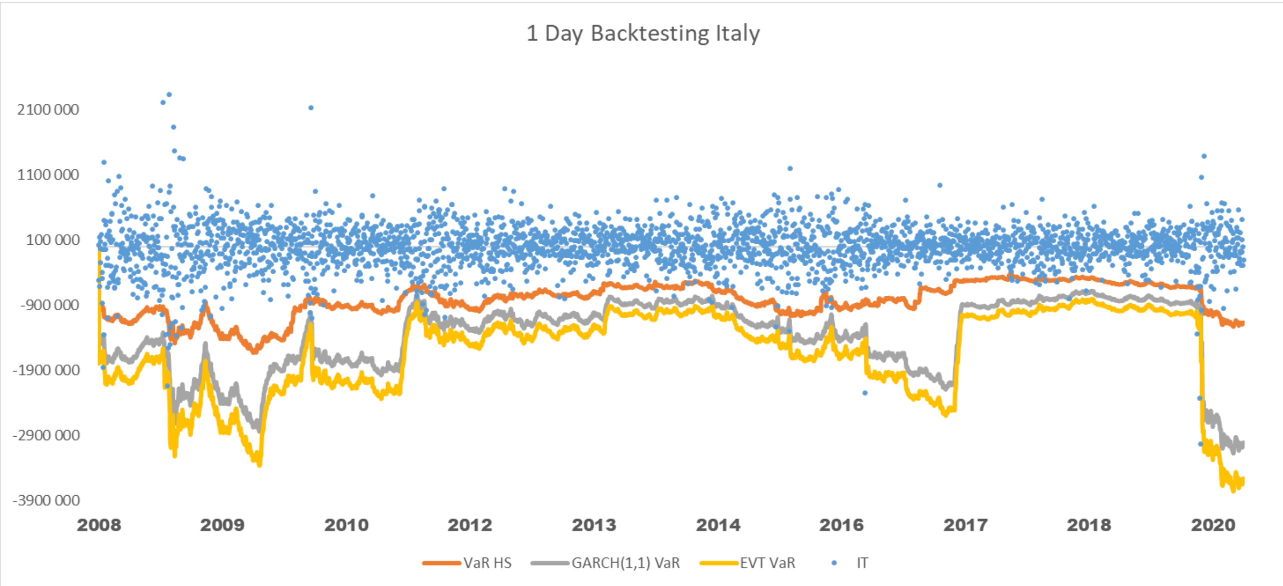
800 000
300 000
-200 000
-700 000
-1200 000
-1700 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
GER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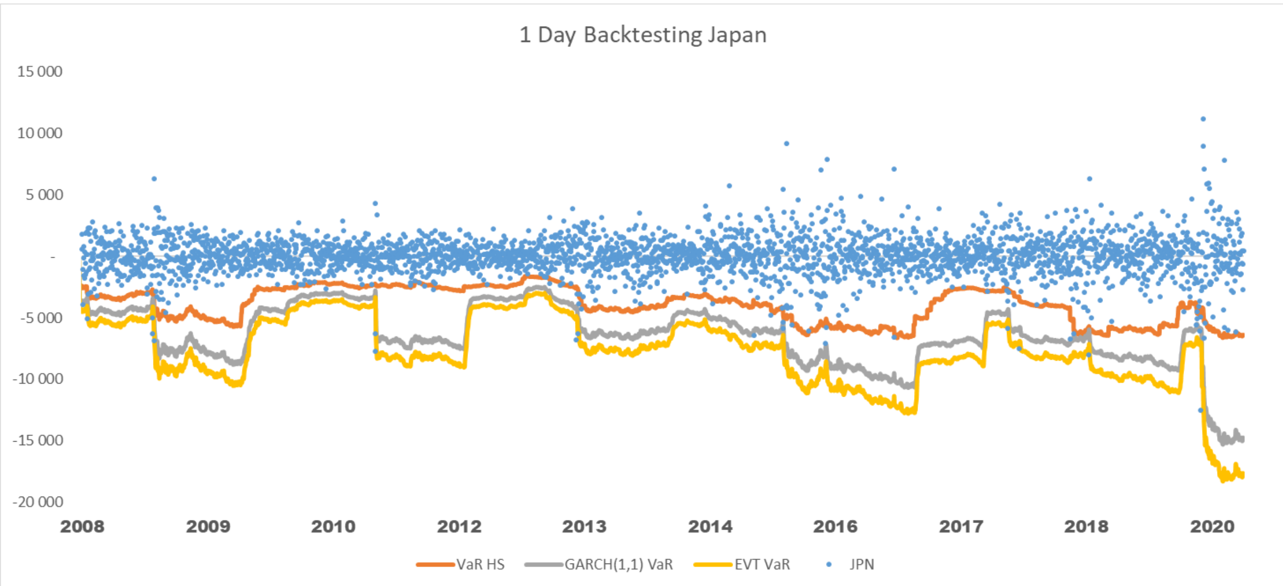
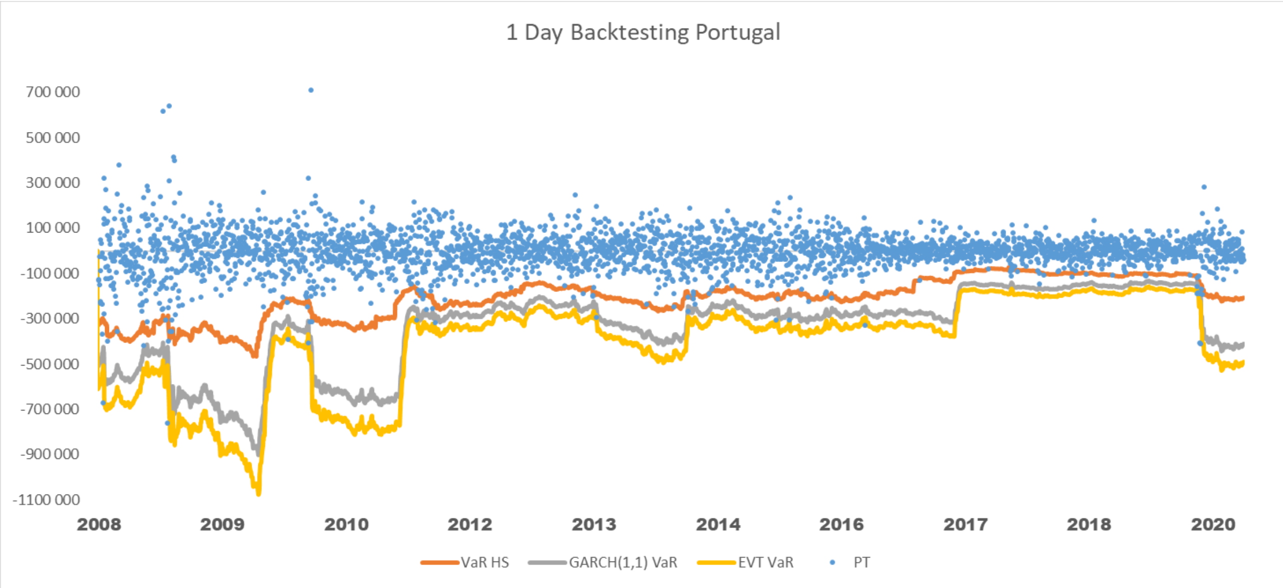
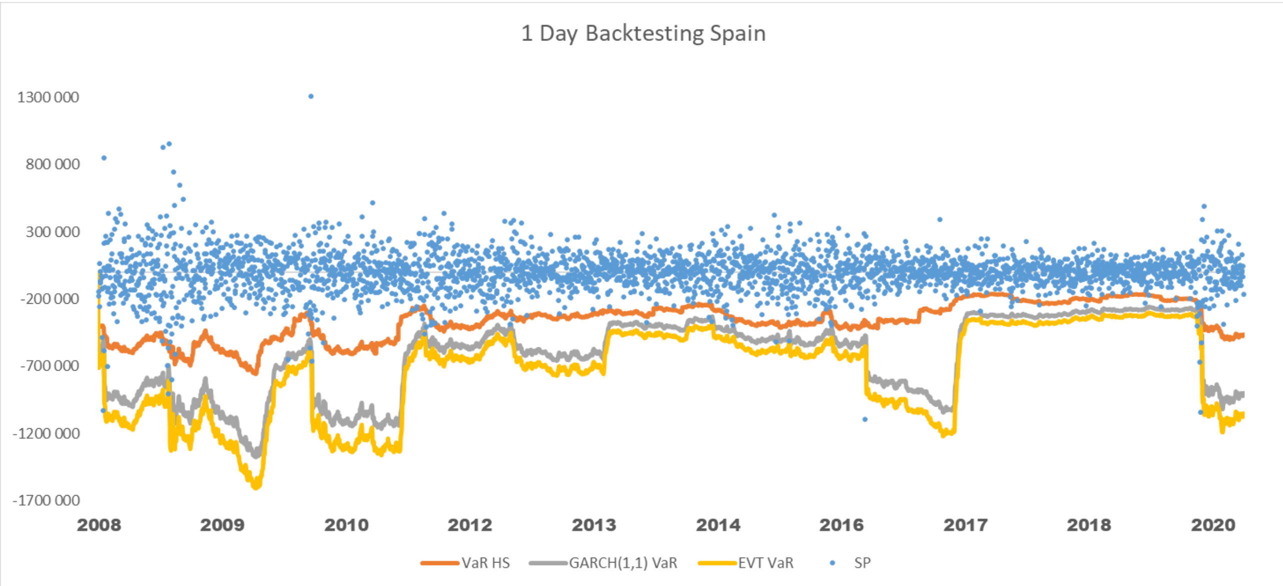
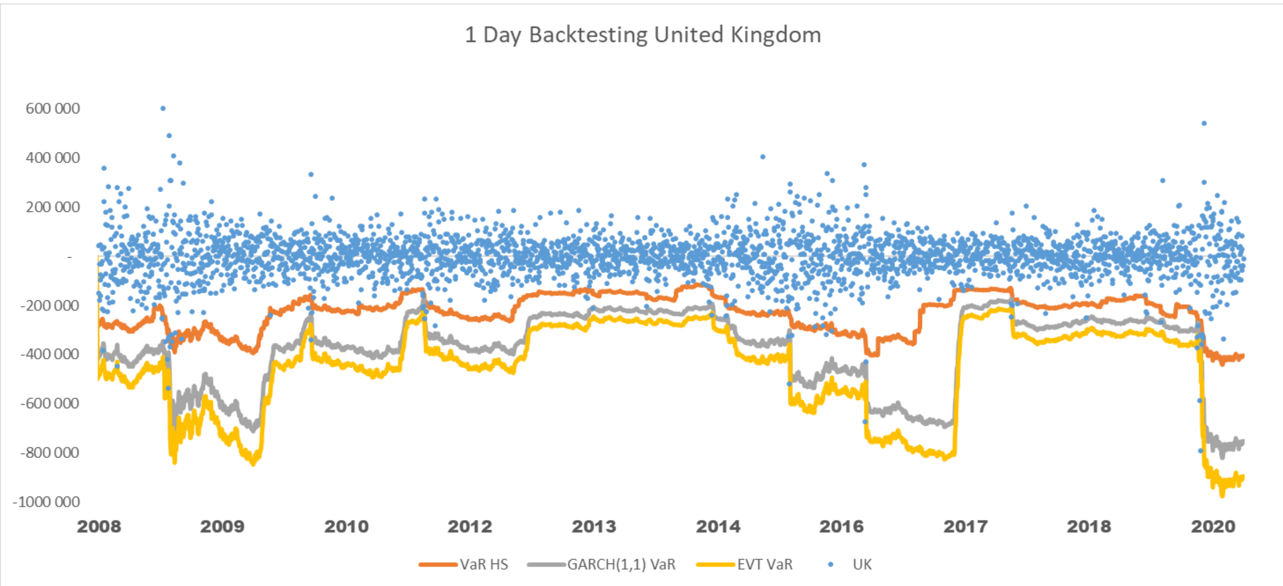
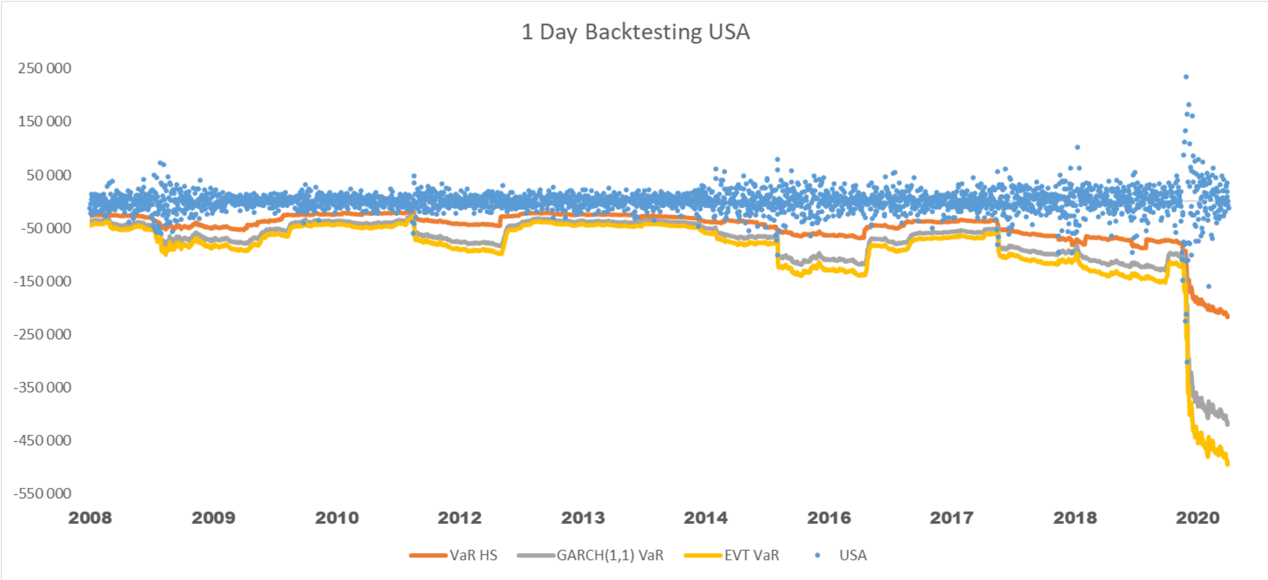
2100 000
1100 000
100 000
-900 000
-1900 000
-2900 000
-3900 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
IT


1 Day Backtesting Japan
15 000
10 000
5 000
-
-5 000
-10 000
-15 000
-20 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
JPN


1 Day Backtesting Portugal
700 000
500 000
300 000
100 000
-100 000
-300 000
-500 000
-700 000
-900 000
-1100 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
PT


1 Day Backtesting Spain
1300 000
800 000
300 000
-200 000
-700 000
-1200 000
-1700 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
SP


1 Day Backtesting United Kingdom
600 000
400 000
200 000
-
-200 000
-400 000
-600 000
-800 000
-1000 000
2008
2009
2010
2012
2013
2014
2016
2017
2018
2020
VaR HS
GARCH(1,1) VaR
EVT VaR
UK



2.3. Section 3 - Ljung-Box test

	Ljung-Box test	
Countries	r^2	r^2/var
CHI	803,50	18,30
FR	1914,81	22,87
GER	1611,68	26,90
IT	814,89	39,19
JPN	1801,66	20,66
PT	1297,08	30,50
SP	993,12	28,67
UK	2842,26	23,87
USA	4315,26	12,35

Chi(15) 99%	30,578

Table 7: Ljung-Box test

2.4. Section 4 - VaR expected failures

	CHI	USA	PT	FR	GER	JPN	SP	IT	UK
p	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
N	3541	3528	3496	3496	3463	3551	3486	3466	3503
Expected Failures	35	35	34	34	34	35	34	34	35

Table 8: VaR expected failures.

2.5. Section 5 - Expected loss rate

	VaR/Price at day 31/08/2020		
Countries	**HS**	**GARCH**	**EVT-POT**
CHI	8,11%	10,14%	12,38%
USA	7,41%	14,33%	16,89%
PT	10,84%	20,96%	25,01%
FR	6,32%	11,89%	14,11%
GER	5,69%	11,13%	12,78%
JPN	3,63%	8,37%	10,01%
SP	10,86%	19,75%	23,08%
IT	8,30%	16,17%	19,15%
UK	6,61%	12,32%	14,66%

Table 9: Expected loss rate

2.6. Section 6 - Backtesting

	Backtesting								
	LRuc			LRind			LRcc		
Countries	**HS**	**GARCH**	**EVT-POT**	**HS**	**GARCH**	**EVT-POT**	**HS**	**GARCH**	**EVT-POT**
CHI	1,95	8,04	23,27	1,19	0,24	0,07	3,15	8,28	23,34
USA	4,81	4,10	25,53	11,45	12,07	5,23	16,26	16,17	30,76
PT	14,92	13,01	20,41	19,52	3,34	0,09	34,44	16,36	20,50
FR	10,82	13,01	25,07	2,88	3,34	0,06	13,70	16,36	25,13
GER	15,40	14,27	24,59	15,11	0,14	0,06	30,51	14,41	24,66
JPN	6,01	12,07	21,14	10,78	9,14	4,51	16,79	21,21	25,65
SP	7,40	14,54	30,38	6,60	0,14	0,04	13,99	14,68	30,42
IT	11,19	14,31	33,15	2,84	0,14	0,03	14,03	14,45	33,18
UK	9,80	6,63	22,74	32,61	20,87	0,07	42,41	27,50	22,82

Table 10: Backtesting LRuc, LRind and LRcc

2.7. Section 7 - Number of exceptions per year

	HS								
	CHI	**USA**	**PT**	**FR**	**GER**	**JPN**	**SP**	**IT**	**UK**
2020	5	8	10	9	9	10	8	7	8
2019	2	1	2	4	3	0	3	3	3
2018	8	6	9	6	7	9	5	9	6
2017	2	2	1	1	2	1	1	0	1
2016	2	1	2	2	2	3	3	4	4
2015	7	7	3	5	7	5	4	3	3
2014	1	3	6	5	6	2	4	6	5
2013	5	2	5	3	1	8	0	2	3
2012	1	0	0	0	0	1	2	0	0
2011	2	5	7	8	12	5	7	8	7
2010	3	2	6	5	2	1	6	4	3
2009	1	0	0	0	0	0	0	1	1
2008	5	12	9	8	9	6	9	9	11
TOTAL	44	49	60	56	60	51	52	56	55

Table 11: Number of exceptions per year for the Historical Simulation Model.

	GARCH(1,1)								
	CHI	**USA**	**PT**	**FR**	**GER**	**JPN**	**SP**	**IT**	**UK**
2020	1	6	5	4	4	2	5	3	5
2019	1	0	0	1	0	0	0	0	1
2018	5	5	0	1	1	3	0	1	2
2017	1	0	0	0	0	0	0	0	0
2016	0	0	1	1	1	0	1	1	2
2015	5	2	2	1	1	3	2	3	2
2014	1	2	1	2	1	0	1	2	1
2013	2	0	1	0	0	0	0	0	1
2012	0	0	0	0	0	2	0	0	0
2011	1	3	2	3	3	0	1	3	3
2010	1	2	2	1	0	2	2	0	1
2009	0	0	0	0	0	0	0	0	0
2008	2	4	2	2	4	5	3	2	3
TOTAL	20	24	16	16	15	17	15	15	21

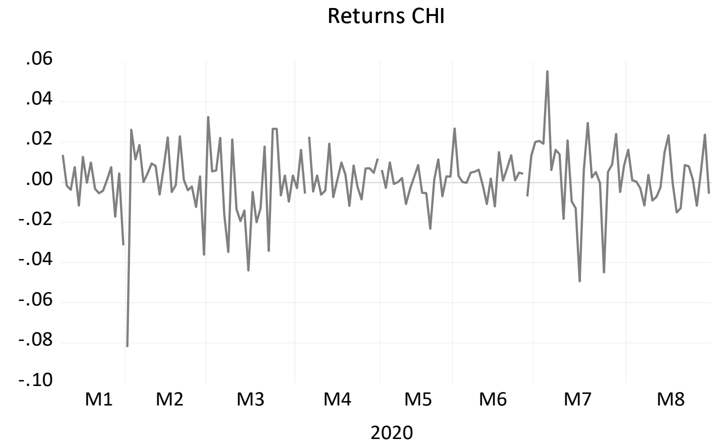
Table 12: Number of exceptions per year for the GARCH(1,1) Model.

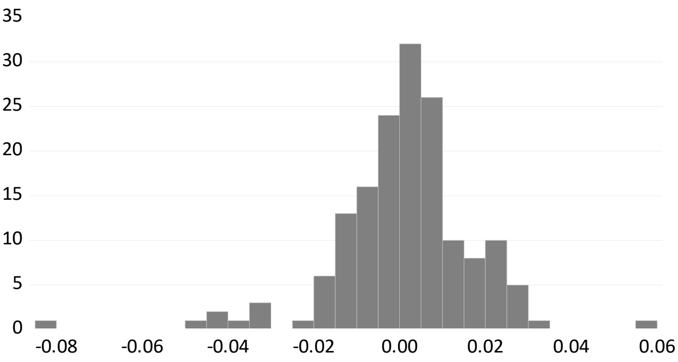
	EVT								
	CHI	USA	PT	FR	GER	JPN	SP	IT	UK
2020	1	4	4	3	3	1	3	3	3
2019	1	0	0	0	0	0	0	0	0
2018	3	1	0	0	0	3	0	0	1
2017	0	0	0	0	0	0	0	0	0
2016	0	0	1	1	1	0	1	1	1
2015	3	2	1	1	1	0	1	0	1
2014	1	0	0	1	0	0	0	0	1
2013	1	0	1	0	0	2	0	0	0
2012	0	0	0	0	0	0	0	0	0
2011	0	3	1	2	2	2	1	1	1
2010	1	0	2	0	0	0	0	0	1
2009	0	0	0	0	0	0	0	0	0
2008	0	0	2	2	3	4	2	2	2
TOTAL	11	10	12	10	10	12	8	7	11

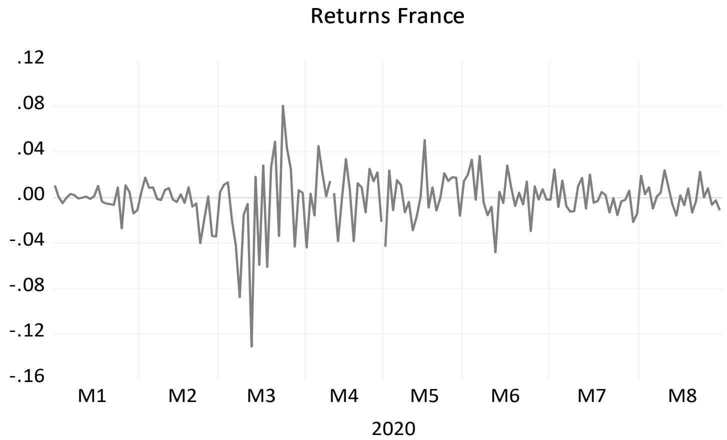
Table 13: Number of exceptions per year for the EVT-POT Model.

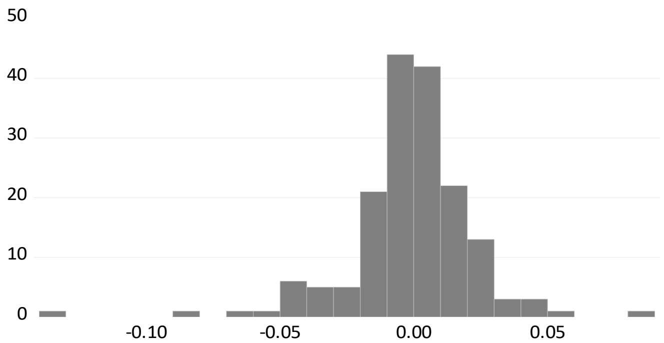
3. Appendix 3

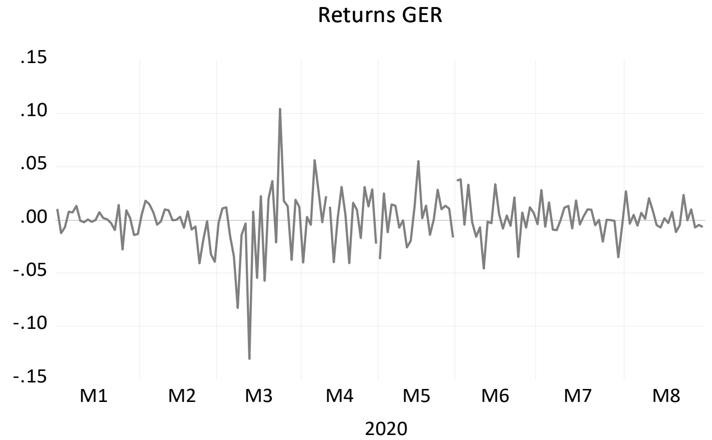
3.1. Section 1 - Returns Graphs and Histograms

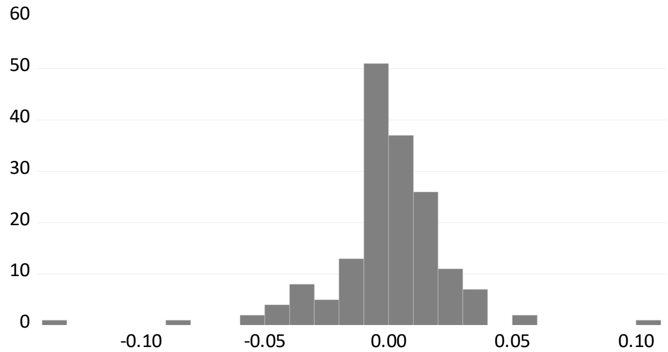


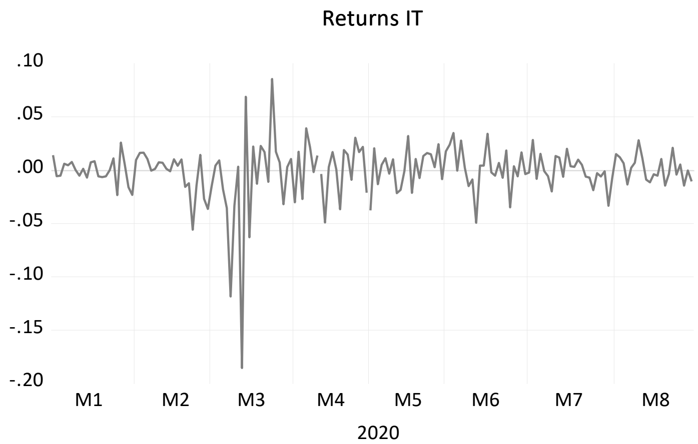


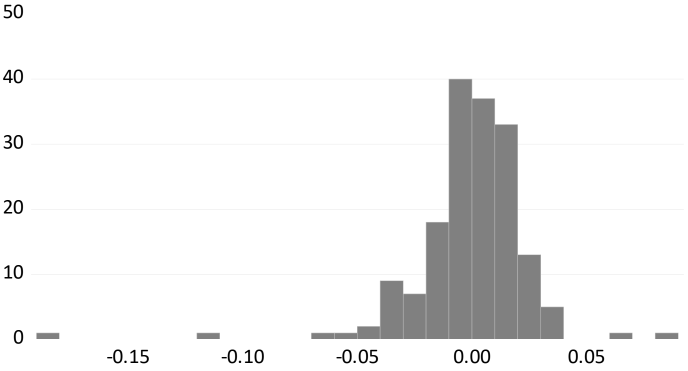


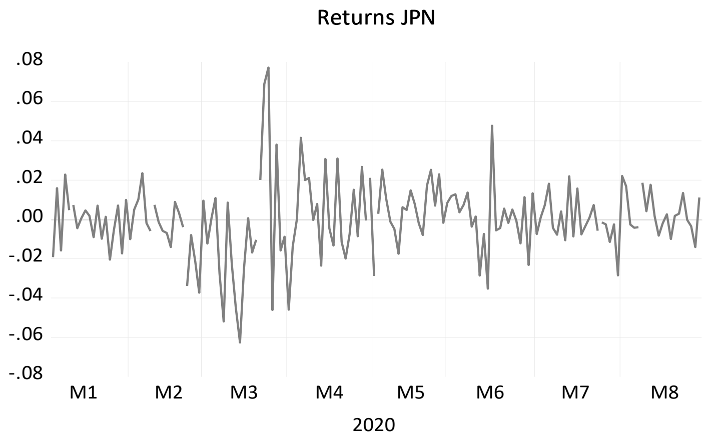


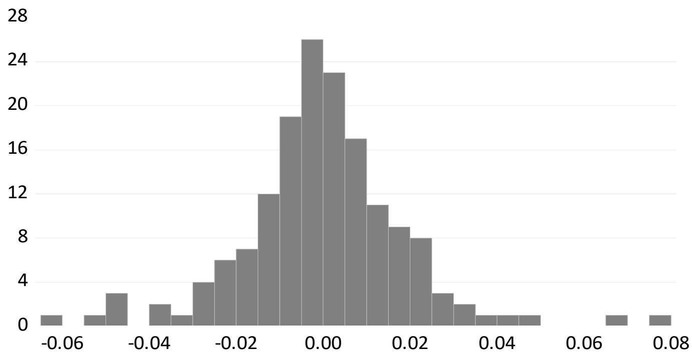


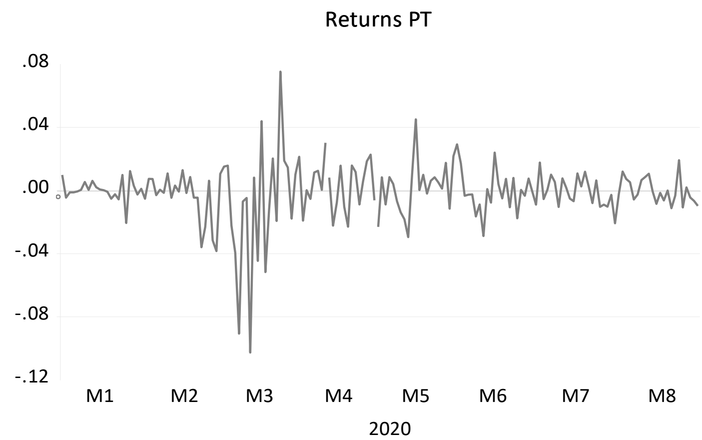


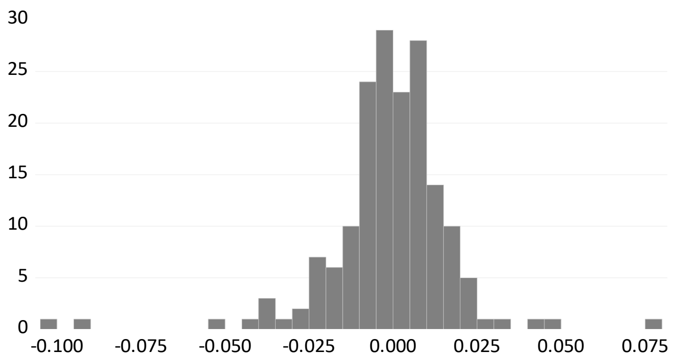

Returns IT
.10
.05
.00
-.05
-.10
-.15
-.20
M1 M2 M3 M4 M5 M6 M7 M8
2020

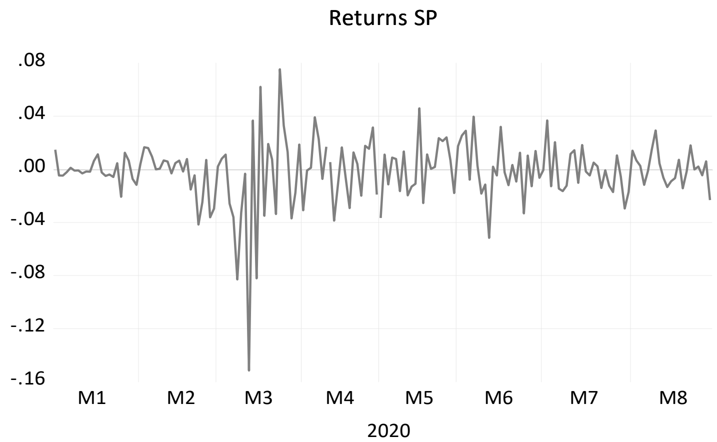

50
40
30
20
10
0
-0.15 -0.10 -0.05 0.00 0.05

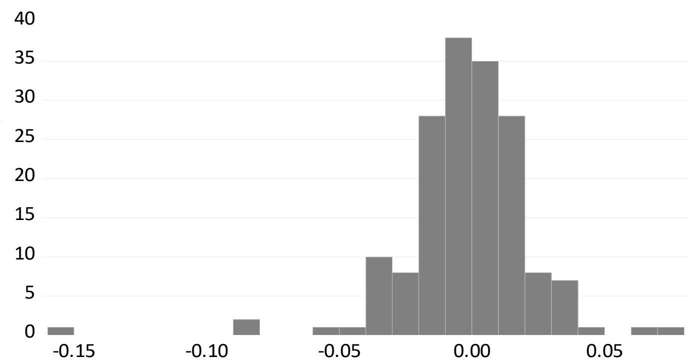

Returns JPN
.08
.06
.04
.02
.00
-.02
-.04
-.06
-.08
M1 M2 M3 M4 M5 M6 M7 M8
2020

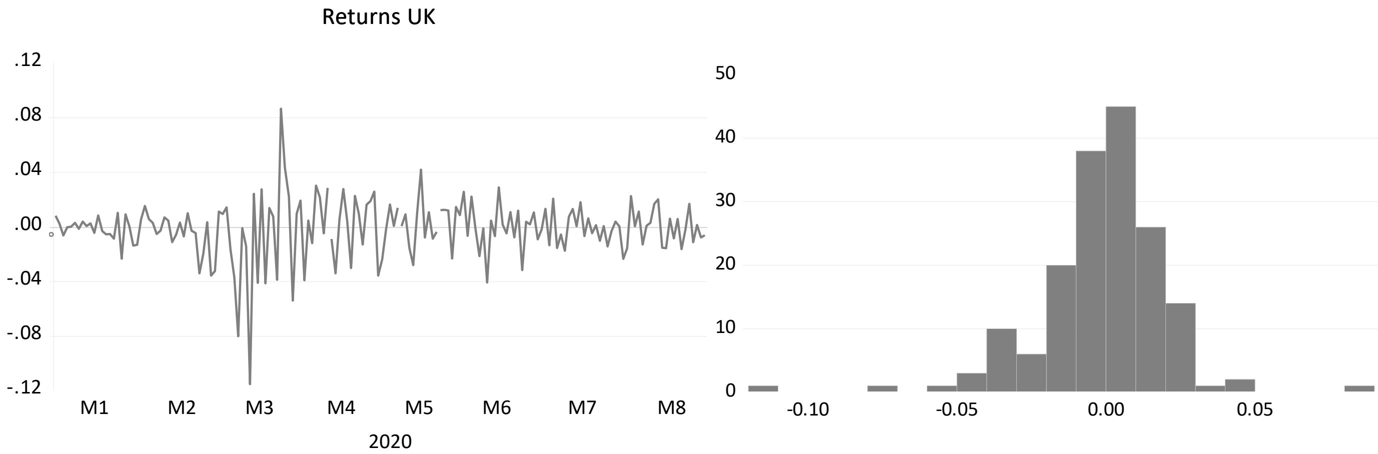

28
24
20
16
12
8
4
0
-0.06 -0.04 -0.02 0.00 0.02 0.04 0.06 0.08

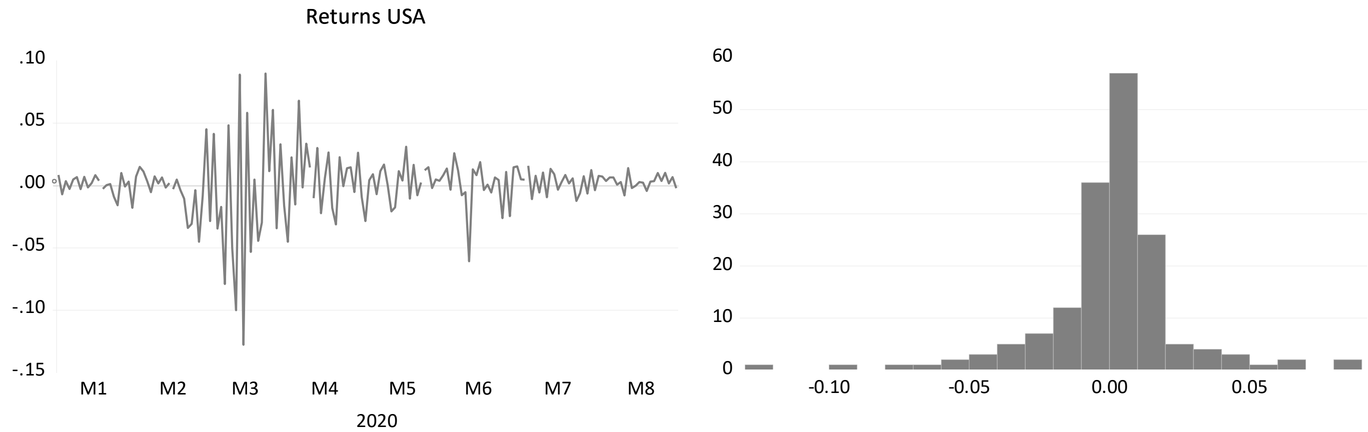

Returns PT
.08
.04
.00
-.04
-.08
-.12
M1 M2 M3 M4 M5 M6 M7 M8
2020


30
25
20
15
10
5
0
-0.100 -0.075 -0.050 -0.025 0.000 0.025 0.050 0.075

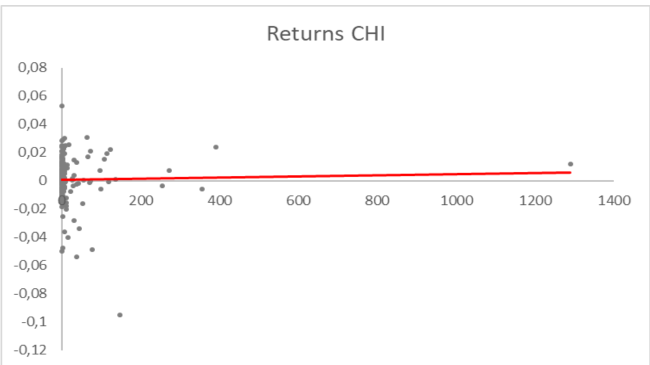

Returns SP
.08
.04
.00
-.04
-.08
-.12
-.16
M1 M2 M3 M4 M5 M6 M7 M8
2020

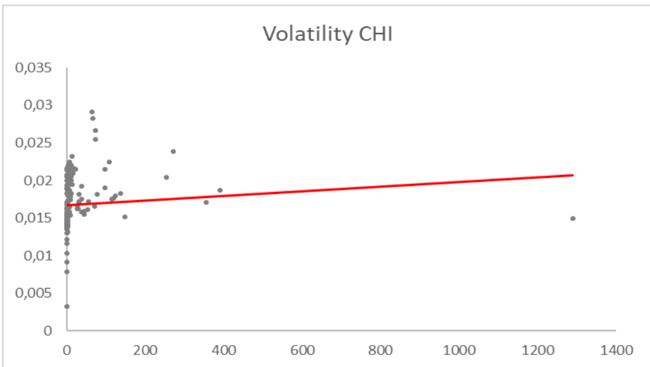

40
35
30
25
20
15
10
5
0
-0.15 -0.10 -0.05 0.00 0.05

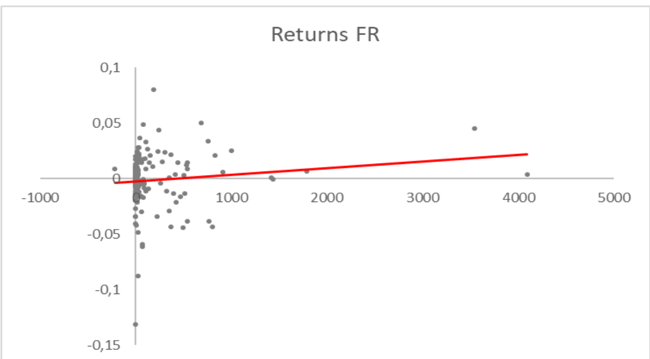


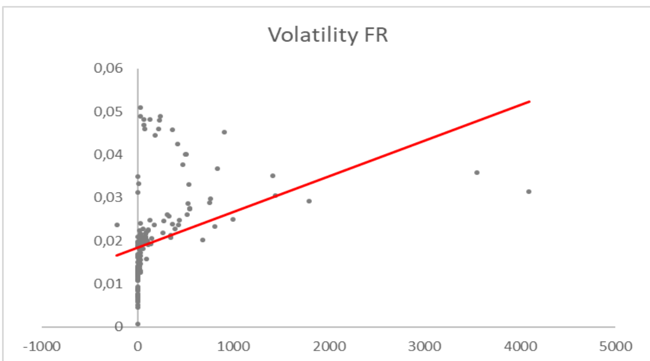


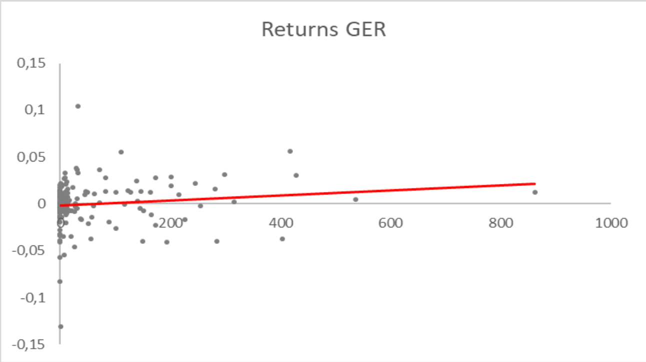
3.2. Section 2 - Bivariate Regression Graphs

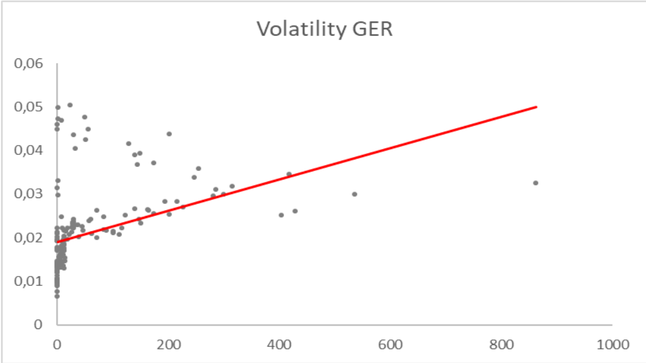


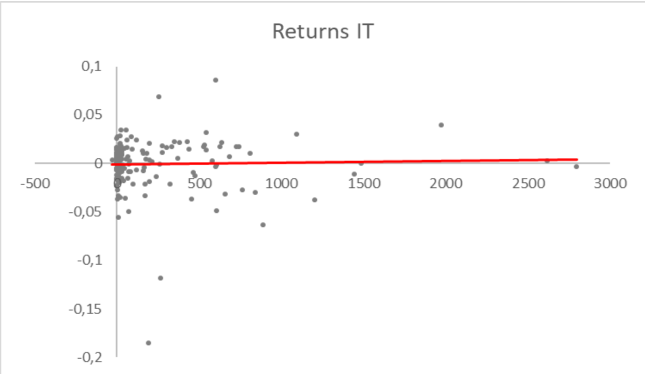


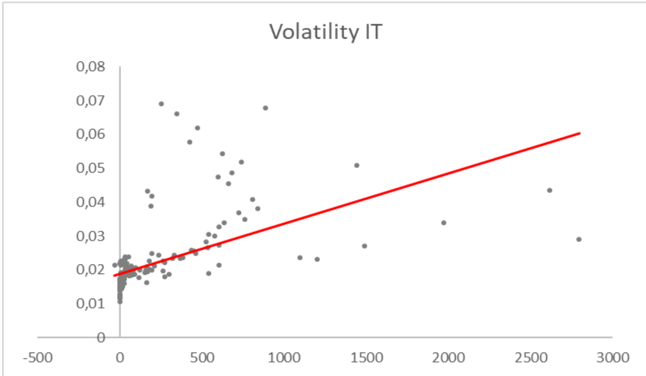


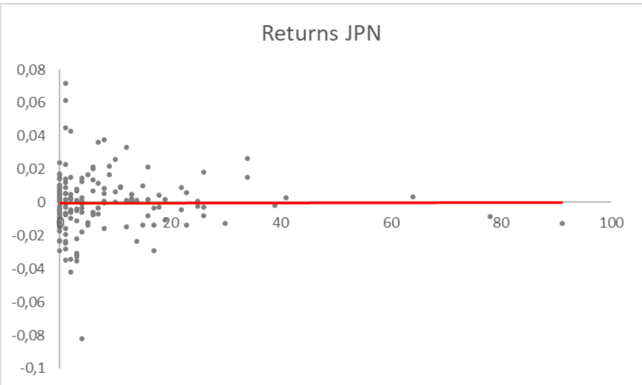


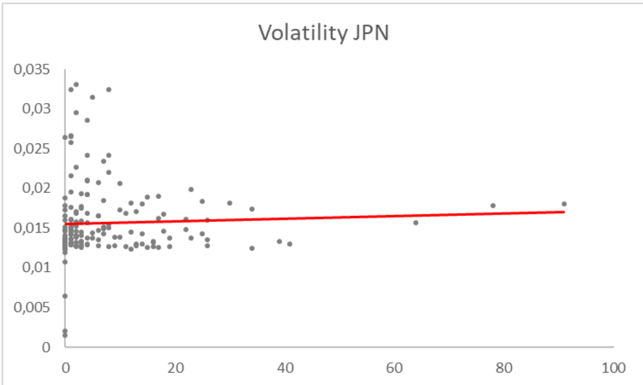

Returns GER
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
200
400
600
800
1000

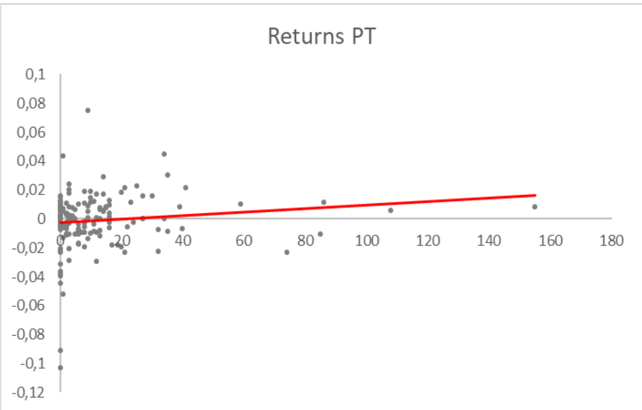

Volatility GER
0,06
0,05
0,04
0,03
0,02
0,01
0
0
200
400
600
800
1000

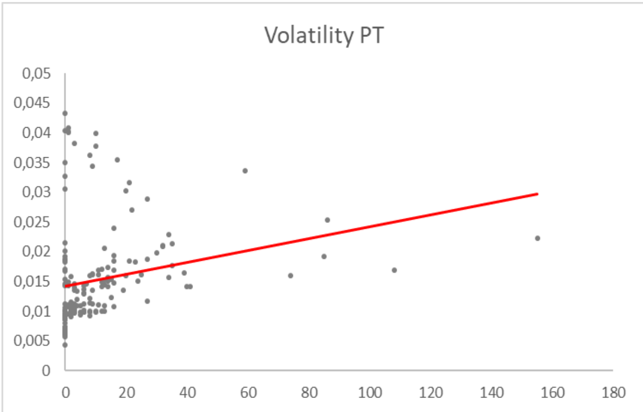

Returns IT
0,1
0,05
0
-0,05
-0,1
-0,15
-0,2
-500
500
1000
1500
2000
2500
3000

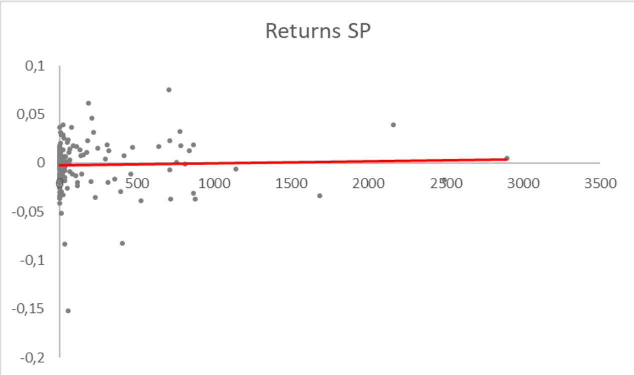

Volatility IT
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
-500
0
500
1000
1500
2000
2500
3000

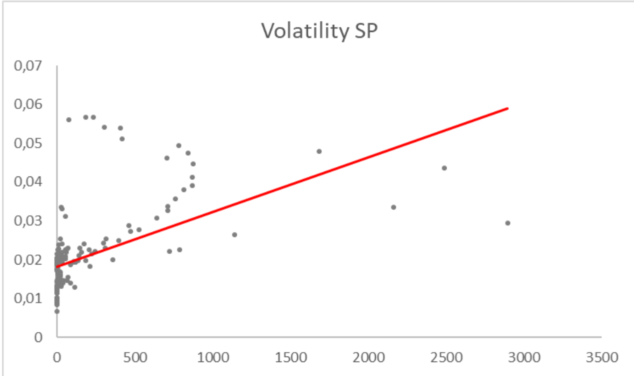

Returns JPN
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
-0,1
20
40
60
80
100

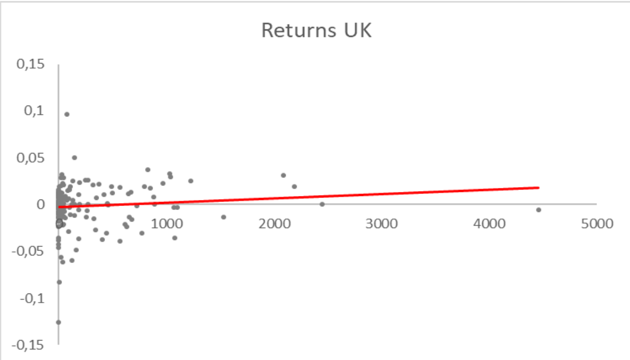

Volatility JPN
0,035
0,03
0,025
0,02
0,015
0,01
0,005
0
0
20
40
60
80
100

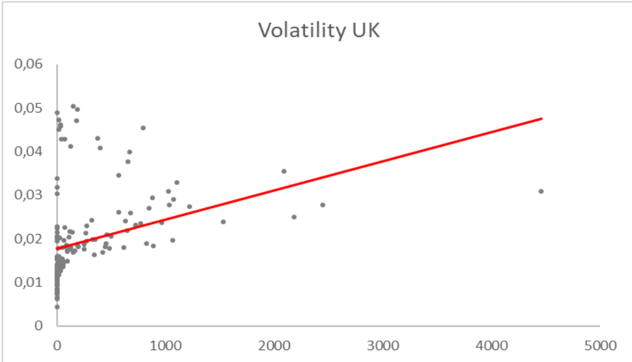

Returns PT
0,1
0,08
0,06
0,04
0,02
0
-0,02
-0,04
-0,06
-0,08
-0,1
-0,12
20
40
60
80
100
120
140
160
180

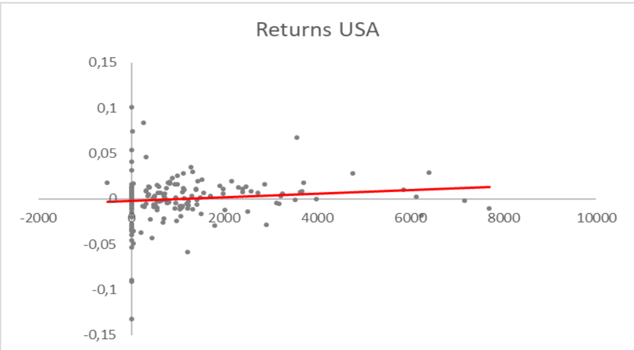

Volatility PT
0,05
0,045
0,04
0,035
0,03
0,025
0,02
0,015
0,01
0,005
0
0
20
40
60
80
100
120
140
160
180

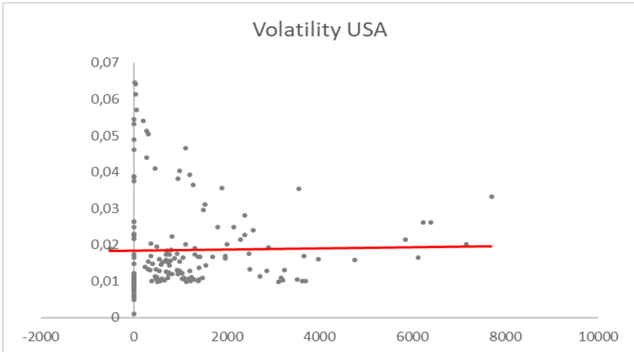












3.3. Section 3 - *GARCH* (1,1) weights for the Regression model

	Weights for GARCH(1,1) estimate Regression Analysis								
	CHI	FR	GER	IT	JPN	SP	PT	UK	USA
ω	3,3E-05	8,6E-06	1,2E-05	1,8E-05	3,0E-05	1,1E-05	6,5E-06	9,4E-06	9,3E-06
α	0,069	0,100	0,100	0,100	0,100	0,100	0,100	0,100	0,100
β	0,823	0,880	0,877	0,862	0,777	0,874	0,866	0,870	0,853
γ	0,1078	0,0204	0,0233	0,0385	0,1235	0,0261	0,0338	0,0296	0,0467
VL	0,0003	0,0004	0,0005	0,0005	0,0002	0,0004	0,0002	0,0003	0,0002

Table 14: GARCH(1,1) Weights

3.4. Section 4 - Ljung-Box test

	Ljung-Box test			
Countries	r^2	r^2/var		
CHI	3,81	1,81		
FR	65,81	1,78		
GER	48,84	26,87		
IT	26,93	34,44		
JPN	76,20	13,55		
PT	75,04	34,00		
SP	42,52	29,13		
UK	75,01	40,53		
USA	211,74	2,35	***Chi(15) 99%***	30,578

Table 15: Ljung-Box test

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