

Title	A Note on the Dugundji Extension Property (可算乗法的空間族)
Author(s)	OKUYAMA, AKIHIRO
Citation	数理解析研究所講究録 (1978), 330: 9-10
Issue Date	1978-08
URL	http://hdl.handle.net/2433/104142
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

A note on the Dugundji extension property

A. Okuyama

For any topological space X , let $C^*(X)$ be the vector space of continuous, bounded, real-valued functions on X , equipped with the sup-norm. Let us say that a space X has the Dugundji extension property, if for any closed subset A of X there is a linear transformation $\eta : C^*(A) \rightarrow C^*(X)$ such that for each $f \in C^*(A)$, $\eta(f)$ extends f and $\sup \{ |\eta(f)(x)| : x \in X \} \leq \sup \{ |f(a)| : a \in A \}$.

It is known that every metric space has the Dugundji extension property (cf. [3], [4]) and every stratifiable space also has such property (cf. [1]). On the other hand, E. van Douwen [2] constructed a first countable cosmic space H_1 which did not have that property.

For any space Y , let τY be the topology of Y . A space X is said to be a K_1 -space, if for every subspace F of X there exists a function $\kappa : \tau F \rightarrow \tau X$ such that

- (1) $F \cap \kappa(U) = U$ for every $U \in \tau F$,
- (2) if $U \cap V = \phi$ in τF , then $\kappa(U) \cap \kappa(V) = \phi$.

It has shown that every space with the Dugundji extension property is always a K_1 -space (cf. [2]), and van Douwen [2] also proved that the above space H_1 was not a K_1 -space.

The space H_1 is expressed as the union of two subspaces F and C of X such that F is stratifiable as a subspace of X and that each point of C is isolated in X . Modeling the space H_1 , the following result is obtained :

Theorem 1. Let X be a topological space and A a closed, G_δ -subset of X such that A is stratifiable as the subspace of X and $X \setminus A$ is a discrete subspace of X . Then the following conditions are equivalent :

- (1) X is stratifiable.
- (2) X has the Dugundji extension property.
- (3) X is a K_1 -space.

For a space X and a closed subset A of X , let us say that (X, A) is a semi-canonical pair, if there is an open cover \mathcal{V} of $X \setminus A$ such that, for each point $a \in A$ and any neighborhood U of a in X , there exists a neighborhood W of a in X provided with $\cup\{V \in \mathcal{V} : W \cap V \neq \emptyset\} \subset U$.

Relating with the semi-canonical pair, the following result is obtained and it seems to suggest that the condition being semi-canonical pair works on a half part in the extension theorem :

Theorem 2. Let X be a paracompact σ - and K_1 -space, and let A be a closed subset of X such that A is stratifiable as the subspace of X . Then there exists a linear transformation $\eta : C^*(A) \rightarrow C^*(X)$ which satisfies all conditions required in the Dugundji extension property.

References

1. C. R. Borges , On stratifiable space, Pacific J. Math. 17(1966), 1 - 16.
2. E. K. van Douwen, Simultaneous extension of continuous functions, Ph. D. Thesis, Free University of Amsterdam, 1975.
3. J. Dugundji, An extension of Tietze's theorem, Pacific J. Math. 1 (1951), 353 - 367.
4. E. Michael, Some extension theorems for continuous functions, Pacific J. Math., 3 (1953), 789 - 806.

Department of Mathematics

Osaka Kyoiku University