Dynamic model for the analysis of productive environments in a pseudo crisis phase: The case of demographic ageing

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Ponencia presentada en Impacts of Ageing on Public Finance and Labour Markets in Eu Regions realizado en 2013 por The Institute of Economic Research of the Slovak Academy of Sciences. Smolenice, Slovakia

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Abstract: In this paper, the hypothesis of an increasing elderly productive population is assumed in order to preserve the dynamical equilibrium between costs and production in the context of ageing on labour markets that takes place in demographic systems. The empirical Von Thünen equation based on the distance to the market, which had historically been useful in the past to evaluate the ground and to select the economical activity, is adopted here as a starting point for the transformation that allows obtaining a new dynamic point of view for the global estimation and preliminary analysis of demographic ageing impacts in productive environments.

Keywords: Ageing, Dynamical balance, stock, rent, mobility.

JEL classification: C00, I00, J00, R00.

1. Introduction

This work deals with the development of a dynamical model for the productivity analysis of a human system when the ageing and the population growth rate are increasing. This effort was thought profitable because the demographic ageing is of interest in the study of economical systems. Few months ago, the purpose of this work was to develop a theoretical connection with the historical Von Thünen (1826) model for the spatial distribution of economical activities (Beckmann, M. , 1972, Dickinson, H.O., 1969 and Peet, R. J., 1969) and the dynamic balance that takes place in every economical systems over time. Then, while developing the model a lot of interesting analogies with other fields of the knowledge were found. Now, the purpose of this paper is to provide a tool for the preliminary dynamical analysis of the ageing crisis. What is more, the underlined idea is to provide a developable tool that should quantitatively predict the evolution of the ageing systems from the real measurements of its intrinsic parameters. Although at the moment parameters are estimated, the idea of a balance, which is built from them, is clear and it should have persists. That is also to say that the Von Thünen balance should help in this dynamical development. The Von Thünen model was related to the empirical fact that higher transport costs of the products were balanced with the closer localization of the activities.

The balance equation has three terms: 1) the profit of an economical activity (rent), 2) the economical loss in the way to obtain the good (cost, transport, etc.), and 3) the gain (price minus costs multiplied by volume of production). The sum of the three is equal to zero. In this paper, life is considered as an economically productive balanced process developed over time. To this end, the spatial Von Thünen model is viewed as the first step in the way to define the dynamical equilibrium by using analogy as a strategy. The formalization is developed by focusing on the rent, the production cost and the ageing cost because of the demographical ageing of the active isolated system. Then, a new point of view for the analysis of the ageing is generated because another

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analogy is suspected: 1) the fruitful ground on the hearth related to the localization model and, 2) the productive human life associated with the dynamic model. Both items are interpreted as the essential terms to keep the production alive in their respective models. The city center is isolated in the localization model and the same occurs with the system (human, population,..) with no external interactions from the time-dependent model.

In the spatial model every economical activity is associated with a specific localization. Analogously, in the dynamical model every balance can be related to a period of time at a specific stage of people's life or at a level of cumulative production. Thus, to distribute space in the ground is similar to define periods in the life. Furthermore, in the dynamical model the life's cost (the cost that people need for grow old, or ageing cost) is associated with the gain in the agricultural activity. Once the analogy between models is obtained, it is useful to observe that as a person grow up the necessary family care (or state care) decreases. When people are young, it is very common to see that new activities are carried out in clubs, schools, teams and universities. Activities are developed just for educational purposes or for living into a community. But, at these stages, the cost is also supported for the individual. If the state or the neighborhood interacts, then the family partially, or totally, releases its initial charge. This charge may become reversible. In fact, at the midway of the life, the family's cost for supporting a healthy person is almost zero, and usually the healthy individual helps his family with goods.

How people obtain goods depend on the job. In analyzing works, there are a lot of cases. To give few typical examples: the person works in the state with dependent relationship, or the individual works for the world under humanitarian goals, or the employee who works to survive or people whose motivation is to work in order to save money or to live better. In any case, an old but economical healthy person gives to the world more movement or more cumulative energy than the necessary amount of it to survive. On the other hand, for elderly ill people that belong to an economical ill system, the costs or the ageing clearly makes difficult the production of goods and services. Accordingly, at the end of the life there are two possibilities: First, the representative individual has a positive rent. This person naturally decreases his costs as soon as he grows older. Second, the representative elderly man that has lost his or her work abilities or interests because of different reasons. This situation is associated with a collapse and with serious social consequences. This person increases his costs as it happens with the transport costs to land when it is placed far from the city. Sadly, the human case is totally different because of the irreversibility of time: "We cannot travel to the youth" while the opposite effect happens with agricultural activities that can travel near the city center when they have higher production costs far away. Hopefully, "After a life's decision we can belong to the group of elderly people with positive rent". The other case is the ill case which is as predictable as negative. So, the ill case will not be considered in this work. To deal with the ageing economical problem the underlying ideas are the energy balance and the evolution that takes place inside the dynamical system. The dynamical model is obtained throughout the semantic redefinition of the localization variables as if they were demographic variables. The new generated dynamical model is interpreted as a demographic system which can be useful for early and global predictions. The model is developed in five sections: 1) Introduction, 2) The Von Thünen model, 3) The analog dynamical system for productivity, 4) Discussion and 5) Conclusion. Finally, the presented model provides a local-global perspective and a dynamical tool, and can be applied to very different complex systems at different scales such as individuals, countries or the whole world.

2. The Von Thünen model

Von Thünen (1826) is considered the father of the economy of localization since his work broke with the contemporary English thought. In fact, his analysis was based on empirical and especial observations about cities, transports and farms localizations, and how they affect costs and salaries. The Von Thünen model was developed from an economical system in which there was no industry development. So, this fact explains why the historical model was based on the following assumptions:

1) The city was isolated and there is no external influence.
2) The state was isolated and all the ground around must be an isolated landscape
3) The landscape was plane without rivers and without mountains
4) The ground and the weather were the same in all the state
5) The farmers moved they products by wagons or carts and there were no roads
6) The farmers looked for maximizing their profits

Von Thünen presented in 1826 a theory of regional development based on four rings of activity around the isolated cities. In this theoretical context, some things appeared to be clear:

1) Milk derivatives and agriculture had been produced near the city because they are perishable. This is the first ring.
2) Wood products should also be produced near the city because of the weight. That is to say that the weight was directly associated with transport costs. Wood products are heavy, then, they were placed in the second ring.
3) Cereals and grains lasted more than milk. So, they can be placed farther in the third ring.
4) The last and fourth ring should contain animals due to the fact that they can move on they own at some point.
5) Far from these rings there was an uninhabited land. Also, it is assumed that the production inside the rings was consumed by the population of the city.

In this model both the cost of the ground and the use of the ground depend on the distance to the city. It can be interpreted as a balance between the transport cost and the cost of the ground. Farther from the city the cost of the land decreases its value. The model is also a balance between the cost of the transport and the localization of the activity due to the fact that the use of the ground depended on the distance. Activities with higher transport costs were located closer to the city and activities with lower transport cost are located far away. To formalize, the Von Thünen model can be expressed as

$$Q_i(p_i - c_i) = Q_i f_i k_j + LR_{IJ} \qquad (1)$$

where

$\mathrm{LR_{IJ}}$ = Localization rent of activity i in placement j

$\mathrm{Q_i}$ = Production volume

$\mathrm{p_i}$ = product price of activity i

$\mathrm{c_i}$ = product cost of activity i

$\mathrm{f_i}$ = transport cost of the product of activity i from the market (cost per km)

$\mathrm{k_j}$ = Distance to the market from location j

To summarize, the rent of activity i in placement j is equal to the gain in terms of physical quantity minus the cost of the transport of the good.

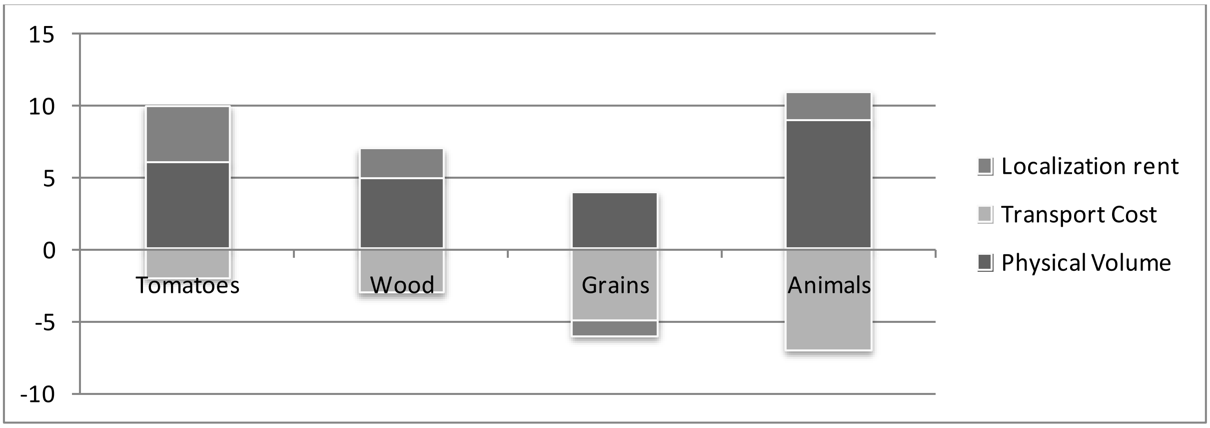
Table 1 shows a hypothetical case in which the rent of localization per unit of production can be viewed easily. The numerical values are adopted only for illustrative purposes, but there are neither real nor economical.

Table 1 – Example of balance from the product of activities: The flow of goods.

Product of Activity	$p_i - c_i$	$-f_i k_j$	$\frac{LR_{IJ}}{Q_i}$
Tomatoes	6	-2	4
Wood	5	-3	2
Grains	4	-5	-1
Animals	9	-7	2

Source: Author

Graph 1. Academic example of the activity distributions and their production, cost and localization rent



Source: Author

Graph 1 show that the location rent can be positive or negative. The transport cost is always negative and it is increasing in direct relationship with the distance to the city center. The difference between price and costs depend on the activity. It can be considered the case of no external customers, then $p_i = 0$. If $p_i = 0$, then $Q_i(0 - c_i) = Q_i f_i k_j + LR_{IJ}$. So, for internal transactions $Q_i(0 - c_i)$ is balanced with $Q_i f_i k_j + LR_{IJ}$. This observation is so essential in the following.

3. The dynamical system for a productivity model

The Von Thünen model is now adapted in order to obtain a dynamical model for the productivity analysis of complex systems. The system can be a person, a community, a country or the whole world. It is assumed that cumulative production of goods and experiences over time has the cost of system's ageing. To state a simple analogy, what is happen with a complex economical system over time is what occurs along the life of a person. All features included in this study are internal to the systems. So, the system has to be isolated. Isolation means that no external agent can give neither life nor grants or donations to the studied system. Furthermore, external agents are neither weak nor altruist, but indifferent. However, internal transactions are allowed because of economical, physical and biological reality. The model is now based on the following assumptions:

1) The dynamical system is isolated.
2) The state is isolated and all the world is isolated (ageing over all world populations)
3) The work for young and elderly people is the same type
4) The youth and elderly people have the same tools and information to work
5) The workers continue their labour along the time. There are neither privileged jobs nor healthier works.
6) The workers look for maximizing their profits and they are good people
7) The actual life expectance will not change significantly in the far future.

This is a theory of productivity development for people at different stages in their lives. In principle, it is based on four periods of time according to their ages. The four period of the life are:

1) The children are in the beginning of their life. They are near their parents, who take care of them. Children need their parents to survive. So, they have to live closer to their families. This is the first period.
2) The youth, in general, is near their family because they do not work. Young people need money for educational purposes but also need financial support to survive. Local and global laws protect the youth providing them the possibility to develop a career or profession. They need to stay close to their parents. They spent most of their time at clubs, schools and universities. The youth is placed in the second period.
3) Adults and senior people are independent, free and have legal responsibility. They are in the third period of the life. They have to work to support the youth and frequently to support more elderly people.
4) The last and fourth period contain the oldest people. Sometimes they can work with positive rent, but often an important amount of them are not in the labour market because they are ill or weak and receive social benefits. Frequently, policies "protect" elderly people against work. So, two kinds of elderly people are clearly distinguished.

   * Far from these periods there was unlimited time. However, it is assumed that people's life is consumed by producing goods or services for the population of the city, the community, the country, the world, etc.

As soon as healthy people are far from their original beginning, the cost of the necessary goods to support them decreases sharply. The model is also a balance between the cost of the support and the stage in people's life due to the fact that the use of the goods depended on the people's age. If everything is according to the model, periods of life with higher support costs (and lower productivity) are located closer to their social beginning and periods of life with lower cost are far away. In the past, activities with lower transport cost were located far away. Nowadays, this is the case of healthy elderly people.

Sequentially, the demographic system is formulated. The basal hypothesis is that the migration is banned (no external goods, no external youth, no external grants, etc.). This is the zero term "0" inside the parenthesis in the left member of Eq. (2). So, it is assumed a closed demographic system. The goods to be consumed for the system are only internal. That is to say that the system is going to be self-consumed.

The whole model is analog to a dissipative system. Also, it is analogous to the "dynamic" of a personal stock control problem where external system's donations are not allowed. However, internal transactions or interactions with the neighborhood are necessary to survive. This will be a kind of rent, but rent can be positive or negative with the hidden idea that it has to be as real as real life. To formalize an intermediate step, the preliminary stock model can be expressed as

$$M_i\left(0-\ddot{X}_\iota\right)=M_i\omega_i^2X_i+2\in\omega_iM_i\dot{X}_\iota \qquad (2)$$

Where, at stage i:

t_i $= time$
M_i $= Volume$
w_i^2 $= Price\ per\ unit\ of\ physical\ stock$
X_i $= Physical\ stock\ existence\ per\ unit\ of\ volume$
$M_iw_i^2X_i$ $= Physical\ stock\ price$
$\dot{X}_\iota$ $= \frac{\Delta X_i}{\Delta t}\ (Variation\ of\ X_i\ per\ unity\ of\ t).$
$\in$ $=$ Mobility of the system to improve its profit or its interactions
$2\in\omega M_i\dot{X}_i$ $=$ Stock price increment $(+$ or $-)$ because of transactions at period i.

$$\ddot{X}_i = \frac{\Delta \dot{X}_i}{\Delta t} \ (Variation\ of\ \dot{X}_i\ \ per\ unity\ of\ t).$$
$$M_i(0 - \ddot{X}_i) = -M_i \frac{\Delta \dot{X}}{\Delta t}\ \ Stock's\ \text{value}$$
$$i = The\ system\ is{:}\ 1\ (\text{too young}), 2\ (\text{young}), 3\ (\text{adult})\ \text{or}\ 4\ (\text{elderly}).$$

All parameters in Eq. (2) can be measured from a complex dynamical system. In particular, in this problem the measurements of complex systems parameters are possible. Population volume, costs per unit of production per individual are available in the economical literature at every moment. Mobility can be estimated. See Ferreyra, R.T., 2013, and Ferreyra, R.T., Ferreyra, M. A. 2013.

The dynamic balance is analog to a classical mass-spring damped system. In the discrete case, Eq. (2) is a balance for the stage "I" of the life. Thus, one can write Eq. (2) for the continuous case (An infinite number of stages). So, let us consider the continuous case, then $M_i \to M$, $\omega_i \to \omega$, and $X_i \to X$. Also, one can assume the initial conditions $X(0) = 0$ and $\dot{X}(0) = \dot{X}_0$. For simplicity, it is adopted the notation change

$$A\dot{X} - 2 \in \omega \dot{X} = 0 \tag{3}$$

Where A in Eq. (3) is associated with rent of the activity per unit of variation of the production. The term is also related to interactions, inversions, supplies, actions or movements that are implied by the activity. By simplifying M and by rearranging terms in Eq. (2) and by considering Eq. (3), it is finally obtained

$$(0 - \ddot{X}) = w^2 X + A\dot{X} \tag{4}$$

Where

$$\omega^2 = Production\ price\ per\ unit\ of\ production$$
$$X = Production$$
$$w^2 X = Production\ price$$
$$\dot{X} = \frac{dX}{dt}\ (Variation\ of\ X\ \ per\ unity\ of\ time).$$
$$A = Rent\ of\ the\ activity\ per\ unit\ of\ \dot{X}$$
$$A\dot{X} = Rent, (+)\ system's\ earnings\ \ and\ (-)\ system's\ neighborhood\ \ profits$$
$$\ddot{X} = \frac{d^2X}{dt^2}\ \ (Ageing\ cost)$$
$$t = t(q)\ time$$
$$q = cumulative\ production$$

Equation (4) is a dynamic balance. All terms have [rent or cost or price (in monetary units)] as the common unity. Since typical or representative economical parameters X^* , ω^{*2} and A^* can be taken or measured from the system, Eq. (4) is transformed to a dimensionless form. The balance in Eq. (4) ensures that the actual value of the production plus the rent is equal to the total cost which is read as the ageing cost because the independent variable is time and time has direct relationship with cumulative production. The solution of Eq. (4) is given by

$$X = e^{-\left(\frac{A}{2}\right)t}\left\{a\ cosh\left[\left(\sqrt{\left(\frac{A}{2}\right)^2 - \omega^2}\right)t\right] + b\ sinh\left[\left(\sqrt{\left(\frac{A}{2}\right)^2 - \omega^2}\right)t\right]\right\} \tag{5}$$

Where the parameters a and b are determined from initial conditions in the form

$$\binom{a}{b} = \left(\frac{\dot{X}_0}{\sqrt{\left(\frac{A}{2}\right)^2 - \omega^2}}\right) \tag{6}$$

By combining Eq. (5) and Eq. (6) it is obtained

$$X = e^{-\left(\frac{A}{2}\right)t}\left\{\left(\frac{\dot{X}_0}{\sqrt{\left(\frac{A}{2}\right)^2-\omega^2}}\right)sinh\left[\left(\sqrt{\left(\frac{A}{2}\right)^2-\omega^2}\right)t\right]\right\} \tag{7}$$

Equation (7) provides the economical production of an entity (preferable a complex system like a human being or like a community) along the cumulative production q which depends on time. According to Eq. (7), the extreme productivity value $X = X_m$ occurs whent $= t_m$

$$t_m = \left(\frac{1}{\sqrt{\left(\frac{A}{2}\right)^2-\omega^2}}\right)tanh^{-1}\left(\frac{\sqrt{\left(\frac{A}{2}\right)^2-\omega^2}}{\frac{A}{2}}\right) \tag{8}$$

By assuming $X = X_m$and by considering Eq. (7) and Eq. (8) it follows$\dot{X}_0$

$$\dot{X}_0\left(\frac{1}{|w|\sqrt{\epsilon^2-1}}\right) = sign(w)\frac{e^{\frac{\epsilon}{\sqrt{\epsilon^2-1}}.tanh^{-1}\left(\frac{\sqrt{\epsilon^2-1}}{\epsilon}\right)}}{sinh\left(tanh^{-1}\left(\frac{\sqrt{\epsilon^2-1}}{\epsilon}\right)\right)} \tag{9}$$

In this model, $w = |w|$ and $sign(w) = 1$. Since $w = sign(w)|w|$, then $\frac{\dot{X}_0}{w}$ depend only on ϵ which can be measured from the outside of the system. Furthermore, $\dot{X}_0$ only depends on the intrinsic parameters ϵ and w of the system. These parameters are alike in similar systems. So, systems or complex systems with similar intrinsic parameters can be viewed as a family. For instance, a representative complex system can be a person, (See Ferreyra, R.T., 2013, and Ferreyra, R.T., Ferreyra, M. A. 2013). More generally, it is also theoretically and practically possible the statistical measurements of the parameters of governments, societies and institutions.

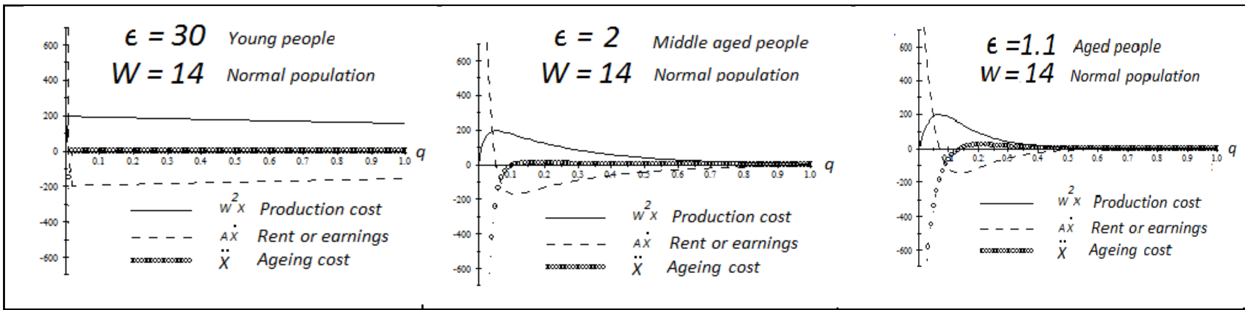
Equation (9) shows that $\dot{X}_0\left(\frac{1}{w\sqrt{\epsilon^2-1}}\right)$ depend only on ϵ. Therefore, by using the Eq. (3), Eq. (7) and Eq. (9), the production as a function of the two intrinsic parameters ϵ and ω is given by

$$X = \left(\frac{e^{\frac{\epsilon}{\sqrt{\epsilon^2-1}}.tanh^{-1}\left(\frac{\sqrt{\epsilon^2-1}}{\epsilon}\right)}}{sinh\left(tanh^{-1}\left(\frac{\sqrt{\epsilon^2-1}}{\epsilon}\right)\right)}\right)e^{-(\epsilon\omega)t}sinh\left[\left(w\sqrt{\epsilon^2-1}\right)t\right] \tag{10}$$

The production depends on time which is a variable. Since $t = t(q)$, then production also depends on cumulative production. In predictions, the estimated production in Eq. (10) does not take into account neither randomness nor catastrophic external events. But, frequently things go on their predictable way. This is mostly true after internal parameters have been measured. So, in this case, Eq. (10) can be a useful tool for the preliminary production estimation in the context of the global analysis. The basal model is an over damped system with inertia and energy storage.

In the casuistic, a typical population is considered first. The derived dynamical model is analog to the mass-spring over damped system. This observation helps for the reasoning when the size of the population will be taken into account. In fact, there is a non damped mass spring system (associated with the over-damped system) such that $W^2 = \frac{K}{M}$. In such systems, K is the rigidity and M is the mass or the inertia. The inertia has direct relationship with the population's size. For instance, large inertia implies that $M \rightarrow \infty$ and $W \rightarrow 0$. For the first example, the population is neither large nor small, so $W = 14$ is adopted just to see the curves. On the other hand, the ageing is controlled by the mobility parameter ϵ. In the bibliography, see Ferreyra, R.T., 2013, and Ferreyra, R.T., Ferreyra, M. A. 2013, it is showed that the mobility ϵ is proportional to the standard deviation of the normal distribution associated with the system. Also, in that work, it is showed that this standard deviation is like a measurement of the movement or vitality. In consequence, the ageing should imply positive lower ϵ values while positive large values are associated with the youth. It is important to note that negative ϵ values are not real in healthy systems, but they drive the ill systems to the economical death. Negative ϵ systems are not compatible with life. Also, it is assumed that the intrinsic parameters do not change over time. For a given population, the costs vary as a function of the mobility as it can be seen in Graph 2. The dynamic balance distributes the rent, the production costs and the ageing in the system.

Graph 2. Stages identification in the growth process for W=14 (normal system's inertia): a) $\in = 30$ there is no stages, b) $\in = 2$ there are at least four stages, c) $\in = 1.1$ earlier ageing effects in the growth stages. For illustrative purposes it is adopted t=q.



a) Young system | b) Middle aged system | c) Aged system

Source: Author

There are five representative points t_0, t_1, t_2, t_3, t_4 related to the cumulative production. These points define five different stages in the system's life:

0) t_0 such that $X(t_0) = 0$. The activities beginning,
1) t_1 such that $X(t_1) = A\dot{X}(t_1)$. Condition for the system's youth initiation,
2) t_2 such that $A\dot{X}(t_2 = t_m) = 0$. System's first adult period initiation,
3) t_3 such that $A\dot{X}(t_3) = \ddot{X}(t_3)$. System's second adult period initiation,
4) t_4 such that $\ddot{X}(t_4 = t_r) = 0$. End of the production, retirement or end of the activity.

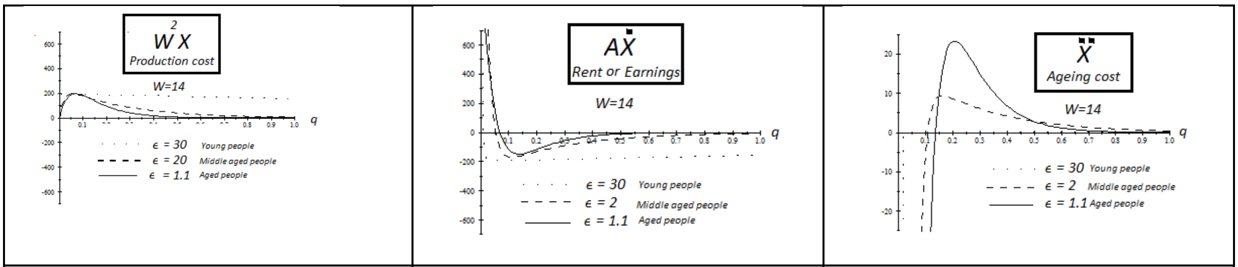
The t_r value is obtained as it is shown in Eq. (11)

$$t_r = \left(\frac{1}{w\sqrt{\epsilon^2 - 1}}\right) cotanh^{-1}\left[\frac{1 + \frac{\sqrt{\epsilon^2 - 1}}{\epsilon}}{\frac{\sqrt{\epsilon^2 - 1}}{\epsilon}}\right] \qquad (11)$$

If $\in \rightarrow 1^+$, then $t_r \rightarrow \infty$. This implies that the end of activity can be moved to the right or to the left by changing the intrinsic parameters. For example, in practical terms, the time for the end of the activity in a human is the retirement age. If the intrinsic parameters change over time, the retirement age will change.

The production capitalizes the system and its cost is a partial measurement of how much life the system has spent to obtain the product. The rent represents the rest of the life the system has invert in the production process. This rent improves the system's comfort during the ageing process. In other words, life can be thought as internal energy into a box. Afterwards, part of it had used to produce positive work while the rest had warmed the system. The system is isolated but life is not. So, life belongs to the system.

Graph 3. The effect of mobility $(\in = 30, 2, 1.1)$ in: a) production price, b) rent and c) ageing cost for a normal volume population $(W = 14)$. For illustrative purposes it is adopted t=q.



a) Production cost	b) Rent ((+) or (-))	c) Ageing cost

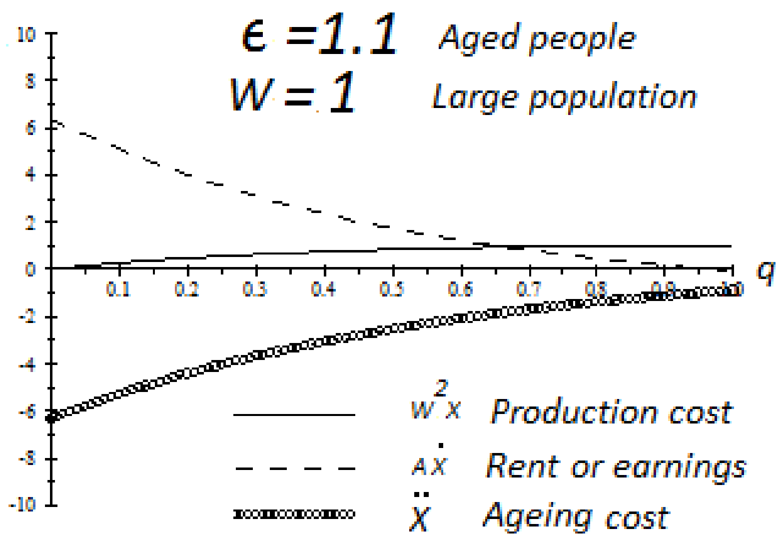
Source: Author

Graph 3 shows that as the mobility coefficient decreases, production cost decreases and the amount of rent also decreases. Positive rent is consumed for the system while negative rent is given to the internal neighborhood of the system. As the mobility is increased, the ageing cost decreases. This is reflexive because in the process of people's ageing their mobility decreases.

4. Discussion

The three terms considered (rent, production cost and ageing costs) interact by the control of the dynamical equilibrium. However, variations of each of the three can be compatible or not with a healthier economical system. If not, the system will collapse over time. Ageing cost can be positive or negative (this also can happen with people at a particular stage of the life and as a subset of every population). If the ageing costs for all cumulative productions are positive the process will be as follows: First, a weakness period. Second, the economical death is produced (costs tends to infinite). On the other hand, if ageing costs are negative for all cumulative production, then the aged population will take relevant risks (little by little to work more and more, and to receive less and less). If the retirement age occurs when the production price is comparable with the rent, or associated with null ageing costs, then the retirement age will be higher and higher over time. The retirement age will disappear under these circumstances. See Graph 4.

Graph 4. Production Cost, Rent and Ageing costs for $W = 1$ and $\in = 1.1$



Source: Authors

Thus, the old people in the population must play a more active role in the production system until near the end of their lives. In consequence, the retirement system will have less sense. Historically, the migration of the youth that belongs to young systems had been a kind of solution.

In fact, reciprocal help between ambitious (or poor) youth and elderly communities has always existed. However, the ageing process is a world process that means that, at the end of the day, this kind of temporal phenomena will disappear. In consequence, new policies of reciprocal collaboration should be developed or new policies to consume goods should be healthily standardized. The hypothesis of isolated system that only interacts with environment makes important the primary goods such as water, air, healthy food, warming, home, etc. So, in any way, primary goods should be quantified in isolated systems. But, to discuss into economical terms: How useful to the world will be the idea of increasing life expectance at the time that the work is increased at elderly stages of the life? Theoretically: Will retirement systems exist in the far future? How? What should change? What kind of goods do people should consume? What kind of things do they should dismiss? How do human biological limits can affect the predictions given by Graph 4? Let us assume an isolated system, ageing of the system, a growing population, growing production over time and no biological limits: Is there a maximum cumulative production? The answer is NO. Since, human life has an empirical limit: What message this model is offering to us? Work to death? [3]

5. Conclusion

A dynamical model for the ageing analysis is proposed. From the model it is possible to theoretically estimate how long the active period of a system will last once measurements of the intrinsic variables of the systems are assumed. The predictability will be much better if the intrinsic variables are measured from reality at the time that the set of original hypothesis remain valid over time. Practical issues are derived from the model. For example:

a) The ageing cost directly depend on the second variation of the production with respect to the cumulative production for a given system,
b) A mobility factor was identified for the rent cost which is related to the vitality of the population,
c) For a given cost per unit of production, the initial values of variation of the production with respect to cumulative production only depends on the intrinsic parameter of mobility of the system. For aged systems the absolute value of $\dot{X}_0$ decreases.
d) For large populations, the ageing cost clearly depends on the variation of the rent,
e) The calculated retirement age varies as a function of the ageing and the population growth rates for the given retirement criteria.
f) The obtained dynamical production model is analogous to a mass-spring over-damped system where the inertial term is related to the ageing process.

In addition, there were two possibilities in the analysis:

1) High positive ageing cost. In this case, for the hypothetical context it can be helpful to preliminary estimate how much time the system is going to have before this irreversible ageing crisis will start. This is due to the fact that simple and handy formulations are provided. But, probably, for this case the most important thing is related to this message: The ground is necessary to support agricultural activities in the same way that goods and

[3] The opinions in this work were developed from hypothetical and unrealistic situations and in the context of theoretical research. In consequence, opinions do not represent believes of neither academic institutions nor political organizations.

services are necessary to support the ageing. That is to say: "if there are no goods, then there is no ageing". This sounds drastic, unreal and ironic. However, the process of dismissing the no necessary goods for life probably will be one way to avoid or to put off or to postpone or to dismiss the ageing crisis associate with ill aged isolated economical systems.

2) Low positive or negative ageing costs. An increase number of elderly people that produce positive rent would attenuate the potential ageing crisis with a trivial economical consequence: retirement systems will have to change their policies due to the fact that they will have less sense. Mobility is very low and inertia is very high. The aged people will decrease their rent along the accumulative production o along the time. Securely, the concept of primary goods should be re-understood because of isolation. (For instance water, air, healthy food, warming, home,..). What is more, the actual life expectance will not change a lot because of actual biological facts at the time that the retirement age is higher and higher, the elderly population will have to be healthy for working more and more years. The previous reasoning is more or less the same as to say that "as it had happened in the past, people should work to death" whenever they belong to isolated economical systems, but ...they will be healthier and older than before. In practical terms, probably, "elderly people from this group will help and save their isolated systems".

By summarizing, the banned migration and ageing in the increasing population of inactive elderly people (negative mobility) implies the weakness and then the death of the economic system. Although this consequence had been historically predictable and also predictable for the formulation, the proposed model helps much more in considering active elderly people (positive mobility) or aged systems as well as their possibilities and vital quantities.

Practical results such us mobility, retirement age, ageing cost estimations as well as the relations and implies for a variety of intrinsic parameters were developed. The final result of this work was the given general dynamical balancing law. In addition, its analogies, its consequences and its potential applications were discussed. Thus, the proposed model can be applied to isolated systems as a tool for the economical ageing analysis.

Bibiliography:

BECKMANN, M. 1972, Von Thünen Revisited: A Neo-Classical Land Use Model, Swedish Journal of Economics, 74, 1972, pag 1-7.

DICKINSON, H.O., 1969, Von Thünen's Economics, Economic Journal,79, 1969,pag 894-902.

FERREYRA, R. T., 2013, Nonlinear Dynamics for the Prediction of Statistical Distributions: An Operational Research Perspective, Session to the Honour of Prof. Dr. A.N. Krylov and Prof. Dr. I.R. PRIGOGIN: Methodologies in Complex Multidisciplinary Systems Dynamics. Stream: Dynamical Systems and Mathematical Modeling in OR. XXVI EURO-INFORMS, 26th European Conference on Operational Research at "SAPIENZA" University of Rome, Rome 1-4 July, 2013.

FERREYRA, R. T., 2013, Nonlinear Dynamics for the Prediction of Statistical Distributions: An Operational Research Perspective. Journal of the International Federation of Nonlinear Analysis, Problems of Nonlinear Analysis in Engineering Systems, PNAES , 2013, Accepted for publication.

FERREYRA, R. T., FERREYRA, M. A. 2013, Dynamical Laws for Statistical Distributions: Application to Complex System Analysis. Journal of Mechanical Engineering and Automation, ISSN 2163-2405, e-ISSN: 2163-2413. Año 2013, vol 3 Nro 2, pag 46-53.

PEET, R. J., 1969, The Spatial Expansion of Commercial Agriculture in the 19th Century: A von Thünen Interpretation, Economic Geography 45, 1969, pag.283-301.