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On Canonical Partition of Edge Set

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(Abstract)

Let E be the edge set of a nonseparable graph G. For each edge e , let t(e) be the number of trees which contain e , and $\overline{t}(e)$ be the number of trees which does not contain e . Here, let E_+ , E_o , E_- be edge sets which satisfy $t(e) < \overline{t}(e)$, $t(e) = \overline{t}(e)$, $t(e) > \overline{t}(e)$, respectively, then we have the tripartition (E_+ , E_o , E_-) of E. We call this tripartition the <u>canonical partition</u> of edge set E.

This paper presents properties in connection with the structure of graphs which satisfy $E = E_o$, $E_+ = E_- = \emptyset$. G is called to have the complementary tree structure when E is partitioned into E_1 , E_2 and each E_i is the edge set of a tree of G.

[Theorem 1] Let ($\emptyset$, E , $\emptyset$) be the canonical partition of E, then G have the complementary tree structure.

[Example 1] The canonical partitions of E of Graphs L_2, K_4 and $K_{3,4}$ shown in Fig. 1 are all (ϕ , E , ϕ) , and they also have the complementary tree structures.

[Theorem 2] Let G_1, G_2 be graphs whose canonical partitions are (ϕ , E_1 , ϕ) and (ϕ , E_2 , ϕ), respectively. If we delete any edges $e_i = (u_i, v_i) \in E_i$ (i=1,2) and connect u_1 and u_2, v_1 and v_2, respectively, then the canonical partition of the obtained graph G is (ϕ , E , ϕ), where $E = E_1 \cup E_2 - \{e_1, e_2\}$. (END)

Fig. 2 explains the manner obtaining G from G_1 and G_2 by the procedure stated in Theorem 2.

The converse of this theorem is stated as follows.

[Theorem 3] Let G be the graph shown in Fig. 2(b) which is connected by two vertices u and v, and its canonical partition is (ϕ , E , ϕ). If we delete $E-E_i'$ and add an edge e_i between u and v, then the canonical partition of the obtained graph G_i is (ϕ , E_i, ϕ), respectively, where $E_i' = E_i - \{e_i\}$ (i=1,2).

[Theorem 4] Let G be the graph whose canonical partition is (ϕ , E , ϕ). The canonical partition of any graph G' which contains G as a partial subgraph is not (ϕ , E' , ϕ). Also, the canonical partition of G'' which is contained in G as a partial subgraph is not (ϕ , E'' , ϕ). (END)

The following conjecture is of interest.

[Conjecture] The canonical partition of edge set E of three connected graph G is (ϕ , E , ϕ) if and only if G is K_4 or $K_{3,4}$.

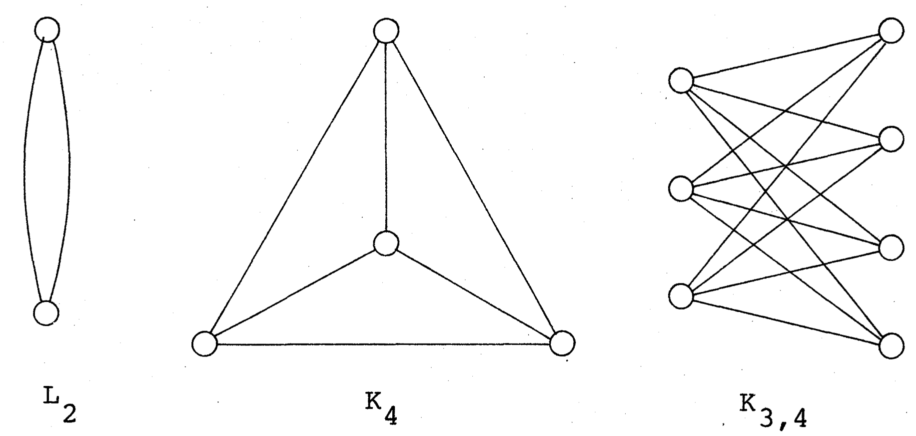

Fig. 1

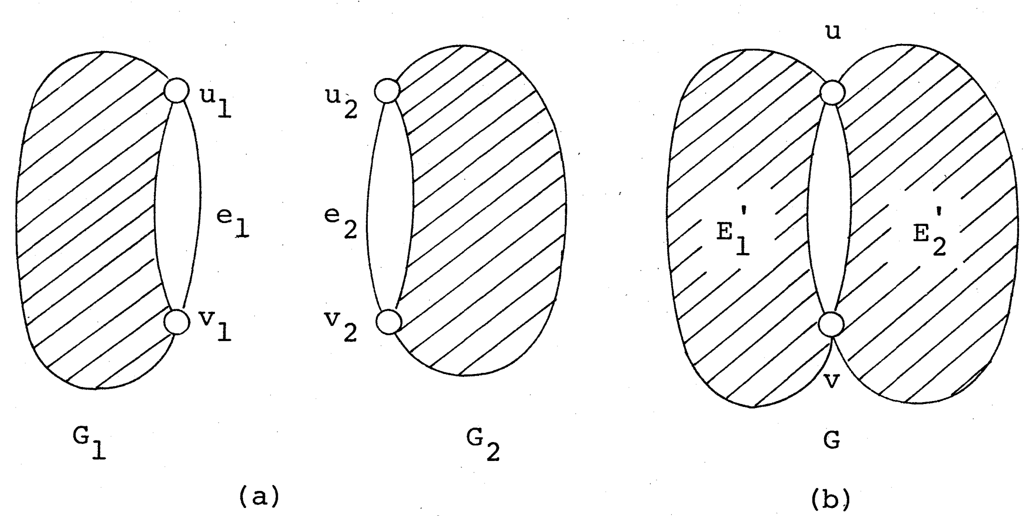

Fig. 2