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Thermal conductivity coefficient via temperature-dependent elementary excitation spectra in thin liquid ^{4}He

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The coefficient of thermal conductivity (κ) in thin liquid ^{4}He is evaluated explicitly as a function of temperature via temperature-dependent two-dimensional elementary excitation spectra that are microscopic only in the long-wavelength limit. Below about 0.8 K, the coefficient increases exponentially with decreasing temperature. At temperatures below about 0.3 K, $\kappa(T)$ in the thin film case has an T^{-5} dependence with extra temperature-dependent terms which originate from three-phonon processes.

The study of thermal conductivity and first viscosity in superfluid liquid ^{4}He has been an important part of condensed matter physics, beginning with Landau and Khalatnikov's work on the kinetic phenomena in superfluid helium[1] and followed by others[2]. Recently Kirkpatrick and Dorfman[3] obtained the transport coefficients of bulk liquid ^{4}He for very low temperatures ($na\lambda^2 \gg 1$) and moderately low temperatures ($na\lambda^2 \ll 1$) on the basis of their kinetic equation for a dilute superfluid, where n, a and λ represent the number density, the s-wave scattering length and the thermal wavelength.

Concerning the elementary excitation spectrum, it is important to point out that an incorrect normal dispersion relation was used in the well-known Landau-Khalatnikov studies and other calculations mentioned above, whereas the correct dispersion is anomalous. We have derived these anomalous zero-temperature dispersion[4] in two and three dimensions, and have successfully explained the thermal conductivity and first viscosity in thin [5] and bulk liquid ^{4}He[6] and other transport coefficients[7] in both dimensions. In the present paper we give new results for the thermal conductivity of superfluid helium through the theory of kinetic phenomena developed by Landau and Khalatnikov[1],[8], using the temperature-dependent elementary excitation spectrum that we have obtained in thin helium films (this is Ref. 9 hereafter referred to as paper I). The spectrum is microscopic only in the long-wavelength limit. We will evaluate the thermal conductivity within the temperature ranges $T \leq 0.3K$ and $0.3\ K \leq T \leq 0.8\ K$, where the scattering depends on the nature of interactions between excitations, i.e., phonon-phonon and phonon-roton interactions. The scatterings which govern the transport processes and kinetic coefficients of thermal conductivity can be determined by the characteristic time τ of scattering.

In this paper we shall treat a thin helium film as two-dimensional (2D) – less than three atomic layers, namely one statistical layer of 3.6 Å – and neglect the substrate effects. In the calculations of the coefficients we will take the 2D temperature-dependent excitation spectrum [Eq. (4.9) in paper I], i.e., phonon and roton spectra,

$$\varepsilon_{ph}(p,T) = (A_0 + \frac{\alpha}{4A_0})p - \frac{\alpha}{A_0^2}p^2 + [B - \frac{\alpha}{8A_0^3}(A_0^2 + 1)]p^3 + \frac{\alpha A_1^2}{A_0^4}p^4 + \cdots, \tag{1}$$

$$\varepsilon_r(P) = \Delta + \frac{(P - P_0)^2}{2m^*}, \tag{2}$$

where Δ, m^* and P_0 are the roton parameters.

In considering the scattering of phonons by rotons, we may obtain the two-dimensional differential cross section[5] with scattering angle ψ as

$$d\sigma = \frac{P_0^2 p^2}{8\pi\rho_0^2 c^2}[(1 + \cos\psi)\cos^2\psi + \frac{1}{128}(\frac{P_0}{m^*c})^2(35\cos^4\psi + 3\sin^4\psi + 30\cos^2\psi)\sin^2\psi$$

$$+\frac{P_0 A}{4m^*c}(3\cos^2\psi + \sin^2\psi) + A^2]d\psi, \tag{3}$$

where A is a small constant which depends on the derivatives of the roton parameters[5].

The equilibrium distribution function n_0 of excitation satisfies the kinetic equation

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \vec{r}} \cdot \frac{\partial H}{\partial \vec{p}} - \frac{\partial n}{\partial \vec{p}} \cdot \frac{\partial H}{\partial \vec{r}} = J(n), \tag{4}$$

with vanishing collision integral. We assume that the nonequilibrium distribution function n deviates slightly from $n_0(n = n_0 + n_1, n_1 \ll n_0)$. This small deviation can be determined by the first derivatives of the velocities $\vec{v}_n$, $\vec{v}_s$ and the thermodynamic variables, since higher derivatives can be neglected. With the help of the continuity equation, the equation for conservation of entropy and the superfluid equation of motion, we can pick out the temperature gradient term in Eq. (4) as

$$\frac{n'}{k_B T^2}\frac{\partial T}{\partial x}\cos\theta[p\frac{ST}{\rho_n} - \varepsilon\frac{\partial\varepsilon}{\partial p}] = J(n_1), \tag{5}$$

where $n' = -n(n+1)$ and θ is the angle between $\vec{p}$ and ∇T. When there exists a temperature gradient in superfluid ^{4}He, there is not only the transport of heat but also an irreversible heat flow, which can be expressed by the coefficient of thermal conduction given as $\vec{q} = -\kappa\nabla T$. We may replace the collision integral by

$$J(n) \longrightarrow -\frac{n - n_0}{\tau}, \tag{6}$$

where τ is the characteristic time and depends on the collision integral. For example τ_{3PP} signifies the characteristic collision time for the three-phonon collision integral $J_{3PP}(n)$. Substitution of Eq. (6) into (5) yields

$$n - n_0 = -\frac{n'}{k_B T^2}\nabla T \cdot \{\vec{p}\frac{ST}{\rho_n} - \varepsilon\frac{\partial\varepsilon}{\partial\vec{p}}\}\tau, \tag{7}$$

and substituting Eq. (7) into the expression for the two-dimensional energy flow, we obtain

$$\vec{q} = \int \frac{\partial\varepsilon}{\partial\vec{p}}\varepsilon(p)(n - n_0)\frac{d\vec{p}}{(2\pi)^2}. \tag{8}$$

Considering the scattering processes, i.e., three-phonon processes, four-phonon processes, five-phonon processes and phonon-roton processes, the collision integral becomes

$$J(n) = J_{3PP}(n) + J_{4PP}(n) + J_{5PP}(n) + J_{ph-r}(n) \tag{9}$$

The $4PP$ do not change the total number of phonons, and the law of energy conservation holds but has the characteristic temperature T' in a given direction which is different from the temperature T in the equilibrium state.

The total number of phonons traveling in a given direction is changed by small-angle $3PP$ and $5PP$. The distribution function depends not only on T' but also on the chemical potential α', and can be written as

$$n = [\exp(\alpha' + \frac{pc}{k_B T'}) - 1]^{-1}. \tag{10}$$

Expanding the distribution function as a function of $(T' - T)$, we then obtain

$$\delta n = n - n_0 = -n_0(n_0 + 1)\{\alpha' - \frac{pc}{k_B T}\frac{T' - T}{T}\}. \tag{11}$$

Since θ is involved in the left-hand side of Eq. (5), α' and $T' - T$ depend on θ. To solve Eq. (5) we take the form

$$\alpha' = \alpha_1\cos\theta, \quad (T' - T)/T = \beta_1\cos\theta \tag{12}$$

where α_1 and β_1 are the constants to be determined from the kinetic equation. Considering the conservation of phonon numbers in a given direction and conservation of energy, we obtain the following integrals from Eq. (5) :

$$\int \frac{n'}{k_B T^2} \cos\theta \frac{\partial T}{\partial x} \{ p\frac{ST}{\rho_n} - \varepsilon\frac{\partial\varepsilon}{\partial p} \} p\, dp = \int [J_{3PP}(n) + J_{5PP}(n) + J_{ph-r}(n)] p\, dp, \qquad (13)$$

$$\int \frac{n'}{k_B T^2} \cos\theta \frac{\partial T}{\partial x} \{ p\frac{ST}{\rho_n} - \varepsilon\frac{\partial\varepsilon}{\partial p} \} \varepsilon p\, dp = \int J_{ph-r}(n)\varepsilon p\, dp. \qquad (14)$$

If we evaluate the collision integral for 3PP and 5PP[1], we get

$$J_{3PP}(n) = \frac{(u+1)^2}{8\pi\rho_0 c^3}(k_B T)^3 p[n_0(n_0+1)\alpha_1 \cos\theta]$$

$$\times [3!\zeta(3)(1 - \frac{\alpha}{4c^2}) + \frac{2\alpha}{c^3}(\frac{k_B T}{c})4!\zeta(4) - 3\gamma 5!\zeta(5)(\frac{k_B T}{c})^2]. \qquad (15)$$

$$\int J_{5PP}(n)p\, dp = 2\pi k_B \alpha' \Gamma_{ph}. \qquad (16)$$

Considering the probability per unit length that a particle undergoes collision is $N_r d\sigma$, where $d\sigma$ is given in Eq. (3), and making use of Eqs. (11)-(12), we obtain the collision integral

$$J_{ph-r}(n) = \cos\theta N_r c(\alpha_1 - \beta_1 \frac{pc}{k_B T})n_0(n_0+1) \times \frac{P_0^2}{8\rho_0^2 c^2}[\frac{1}{4} + \frac{9}{32}(\frac{P_0}{m^* c})^2 + A(\frac{P_0}{m^* c}) + 2A^2], \qquad (17)$$

where the roton distribution function N_r is number of rotons per unit area given by

$$N_r = (\frac{m^* k_B T}{2\pi})^{1/2} P_0 e^{\Delta/k_B T}. \qquad (18)$$

The characteristic times τ_{ph-r} and τ_{5PP} have the same values as those obtained for the zero-temperature excitation spectrum[5], but for τ_{3PP} :

$$\frac{1}{\tau_{3PP}} = \frac{(u+1)^2}{8\pi\rho_0 c^4}\frac{2!\zeta(2)6!\zeta(6)}{3!\zeta(3)4!\zeta(4)}(k_B T)^4[3!\zeta(3)(1 - \frac{\alpha}{4c^2}) + \frac{2\alpha}{c^3}4!\zeta(4)(\frac{k_B T}{c}) - 3\gamma 5!\zeta(5)]. \qquad (19)$$

Substituting Eqs. (15) and (16) into (13) and (14) and integrating over the momentum space, we obtain $\delta n = n - n_0$. Substitution of $\delta n = n - n_0$ in Eq. (11) and comparison of the ordinary thermal conduction equation gives the coefficient of thermal conductivity for the temperature variation of the elementary excitation:

$$\kappa_{ph}(T) = \frac{c^2}{4\pi T}\{[0.186 + \frac{4.792\alpha}{2c^3}(\frac{k_B T}{c})]\tau_{ph-r}^{-1} + \tau_{5PP}^{-1} + \tau_{3PP}^{-1}\}^{-1}\{[c(1 + \frac{\alpha}{2c^2}(2 - \frac{ST}{\rho_n c^2})) - \frac{ST}{\rho_n c}]$$

$$\times (16.659)(\frac{k_B T}{c})^3 - [(26.088)(\frac{ST}{\rho_n c}) + (141.667)c]\frac{\alpha}{c^3}(\frac{k_B T}{c})^4\} + \tau_{ph-r}\{[c(1 + \frac{\alpha}{2c^2}(2$$

$$- \frac{ST}{\rho_n c^2})) - \frac{ST}{\rho_n c}](7.212)(\frac{k_B T}{c})^3 + [(\frac{ST}{\rho_n c} - c)\frac{4\alpha}{c^3} - \frac{3\alpha}{c^3}](25.97)(\frac{k_B T}{c})^4\}. \qquad (20)$$

If we take temperature-dependent term α to be equal to zero, Eq. (20) reduces exactly to (3.31) in Ref. 5 as

$$\kappa_{ph}(T) = \frac{1.803 k_B^3 T^2}{\pi c^2}(c^2 - \frac{ST}{\rho_n})\{\tau_{ph-r} + 2.310/[\frac{0.186}{\tau_{ph-r}} + \frac{1}{\tau_{5PP}} + \frac{1}{\tau_{3PP}}]\}. \qquad (21)$$

We note that in a similar fashion to the above calculations, we can obtain the coefficient of the first viscosity as

$$\eta_{ph}(T) \sim 1.5 \times 10^4 \frac{1}{T^5}, \qquad T < 0.9K \qquad (22)$$
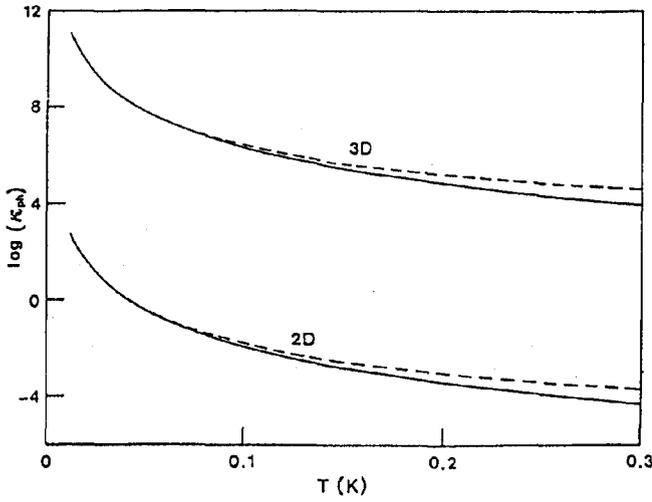
Fig. 1. Thermal conductivity coefficients due to the contribution from $3PP$. The solid and dotted lines are obtained from the zero and temperature-dependent excitation spectra in both dimensions.
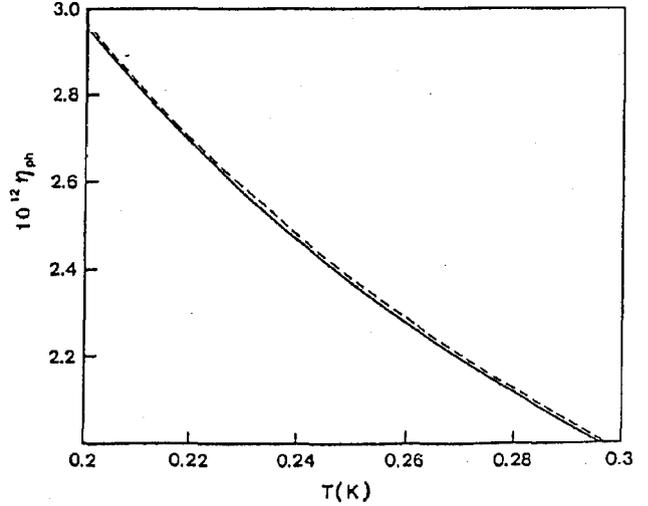
Fig. 2. First-viscosity coefficient due to the contribution from $3PP$ in thin helium films. The solid and dotted lines are obtained from the zero and temperature-dependent excitation spectra.

and

$$\eta_{ph}(T) \simeq 5.92 \times 10^{14} \frac{1}{T} \frac{1}{1 - 0.115\gamma(z)T + 0.128\gamma(z)T^2}, \quad T \leq 0.3K \tag{23}$$

To investigate the temperature variation of the coefficient $\kappa(T)$, the potential and roton parameters are chosen by analysis of the excitation spectrum[4),10)] for the bulk case and the specific heat data[11)] for the thin helium films and are listed in Table 1. The parameters u, a and A in two dimensions are taken to be 1.8[12)], 0.425[1)] and 1.0×10^{43} [1)], as used by previous workers for the bulk case, because these parameters are not known for thin helium films.

Near or just above $0.8\ K$, the $5PP$ and the scattering between phonon and roton will mainly contribute to the thermal conduction, and thus neglecting the $3PP$ contribution, Eq. (20) reduces exactly to Eq. (21). This is exactly the same form that we have obtained for the zero-temperature excitation spectrum[5)]. For temperature below about $0.8\ K$, only the interactions between phonons and rotons are important, and thus neglecting the $5PP$ and $3PP$, we can rewrite Eq. (20) as

$$\kappa_{ph}(T) = \frac{1.803(k_B T)^3}{\pi c^2}\left\{c^2 \left[1 + \frac{\alpha}{2c}\left(2 - \frac{ST}{\rho_n c^2}\right)\right] - \frac{ST}{\rho_n}\right\}\tau_{ph-r}, \quad T < 0.8\ K. \tag{24}$$

Comparing Eq. (24) with (5.3) in Ref. 5, we see that this equation includes a temperature-dependent α term. Therefore, Eq. (24) will vary with a slightly larger value than Eq. (5.3). Neglecting $\tau_{ph-r}^{-1}, \tau_{5PP}^{-1}$ and the temperature-dependent terms within the parantheses of Eq. (20), which are much smaller than unity for temperatures below about $0.3\ K$, this equation becomes

$$\kappa_{ph}(T) \simeq 1.093 \times 10^{-7}(1 + 0.461\gamma(z)T)T^{-5}, \quad T \leq 0.3\ K. \tag{25}$$

Note that the thermal conductivity for the zero-temperature spectrum below $0.3\ K$ [Eq. (5.4) in Ref. 5] is given by

$$\kappa_{ph}(T) \simeq 1.093 \times 10^{-7}T^{-5}, \quad T \leq 0.3\ K. \tag{26}$$

As temperature decreases ($\leq 0.8\ K$), Eq. (24) plays a dominant role with an exponential increase which is due to the characteristic time τ_{ph-r}.

At low temperatures below about $0.3\ K$, Eqs. (25) and (26) take part in the thermal conductivity. Figure 1 illustrates the thermal conductivities as a function of temperatures below about

Table 1. Parameters in thin and bulk liquid helium.

	ρ	$\Delta/k_BT(K)$	$P_0(\mathring{A}^{-1})$	m^*	$C(A_0)(m/s)$	$a(\mathring{A})$	$V_0(K)$	$E_0(K)$
2D	$2.79 \times 10^{-2}\mathring{A}^{-2}$	4.12	1.02	$0.75m_{He}$	164.4	3.581	8.369	7.140
3D	$2.18 \times 10^{-2}\mathring{A}^{-3}$	8.61	1.93	$0.153m_{He}$	238.2	2.76	30.58	19.03

0.3 K in both thin and bulk liquid helium in terms of both excitation spectra. At the moderately low temperatures below about 0.3 K, Eq. (25) varies with slightly larger values than Eq. (26). The coefficient $\kappa_{ph}(T)$ increases rapidly and diverges as temperature tends to zero.

At very low temperatures the two processes of a finite lifetime of an elementary excitation are (1) collisions between excitations and (2) spontaneous decay of a phonon into multiple phonons. As the temperature tends to zero, collisions are not important, but mainly spontaneous decay, i.e., three-phonon processes will contribute at very low temperatures and very small momenta. In previous works, Kirkpatrick and Dorfman introduced two isotropic linear collision operators: an excited particle scatters with a condensed particle to produce two excited particles or vice versa (L_{12}), and two excited particles scatter with each other and produce two new excited particles (L_{22}). These two processes are exactly three-phonon and four-phonon processes, respectively. The collision operator L_{12} holds only for small momenta. When the number of particles decreases with decreasing temperature, there are not enough collisions to keep a local equilibrium and thus this theory breaks down. This means that Kirkpatrick and Dorfman have not taken into account the three-phonon processes but only four-phonon processes.

Before Kirkpatrick and Dorfman, Ma and Popov obtained the same results, i.e., a damping factor on the order of q^2T^{-9}, where q is the wave number of the disturbance in the calculations of the hydrodynamic eigenvalue describing second-sound propagation. Comparing this result with that of Kirkpatrick and Dorfman, we find the relation $q^2\eta/\rho_n \sim q^2T^{-9}$ and thus have $\eta \sim T^{-5}$, where ρ_n is proportional to T^4. Popov has also obtained a T^{-5} dependence of η through fundamental integral analysis, which turns out to be mainly the same contribution from $4PP$ in Landau-Khalatnikov theory. All these previous works do not contain $3PP$, but only $4PP$. Therefore, our results for the coefficients of thermal conductivity and first viscosity involving the $3PP$ mechanism are new.

In conclusion, we confirm that the thermal conductivities obtained for the zero-temperature and temperature-dependent excitation spectra have very similar forms, and the behavior of these coefficients are very much like those of the bulk case. Below about 0.3 K, the coefficients $\kappa_{ph}(T)$ and $\eta_{ph}(T)$ have dependence of T^{-5} and T^{-1}, respectively, in the thin helium films. All these effects are due to the $3PP$ originating from the zero-temperature and temperature-dependent anomalous excitation spectra at low momenta.

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